# Abstracts for the MAA Undergraduate Student Paper Session

## MAA MathFest 2021
## August 4–7, 2021


MAA
MATHEMATICAL ASSOCIATION OF AMERICA

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# Wednesday, August 4 – Session 1 – Computational Mathematics

**Wednesday, 11:00 am**

## Filling in Missing Entries in a Matrix

**Presented by:** Megan Gunn, Randolph-Macon College

**Co-Authored by:** Casey Kent, Randolph-Macon College

**Faculty Advisor:** Brian Sutton, Randolph-Macon College

We explore the problem of missing or unknown entries in a matrix. We begin by developing methods for recovering and filling in a single missing entry in rank one and rank two matrices using the known entries. Truly-lower-rank matrices as well as matrices with added noise are each considered. We then note that the rank two method can also be applied to higher rank matrices. Finally, we extend the methods for filling in a single missing entry to matrices containing a block of missing entries.

**Wednesday, 11:20 am**

## Convolutional encoder decoder network for the removal of coherent seismic noise

**Presented by:** Yash Agarwal, Dougherty Valley High School

**Co-Authored by:** Sarah Greer, Massachusetts Institute of Technology (MIT)

**Faculty Advisor:** Laurent Demanet, Massachusetts Institute of Technology (MIT)

Seismologists often need to gather information about the subsurface structure of a location to determine if it is fit to be drilled for oil. In a seismic experiment, a wave propagates from a source location, interacts with the underlying discontinuities in the subsurface, and arrives back to the surface to be recorded by receivers. This data is used to produce an image of the subsurface, which aims to show geologic structure below the area of interest. However, there is often coherent electrical noise in these datasets which is most commonly removed by disregarding certain frequency bands of the data with the use of a notch filter. Instead, we look at using a convolutional encoder decoder network to remove such noise by training the network to take the noisy shot record produced by the receivers as input and to give the denoised or "clean" shot record as output. Our results reveal that the convolutional encoder decoder network structure works quite well, removing almost all the coherent noise while still retaining most of the characteristics of the shot record.

**Wednesday, 11:40 am**

## Exact and Approximate Optimization Techniques for Knapsack Problems

**Presented by:** Anthony Dellinger, Kutztown University

**Faculty Advisor:** Amy Lu, Kutztown University

This paper examines the efficacy of exact and approximate techniques for solving a variety of different types of knapsack problems. Knapsack problems are combinatorial optimization problems that require the selection of a subset from a set of items that maximizes an objective function without violating any constraints. Techniques used in this paper include population based metaheuristics, local search metaheuristics, and commercial mixed integer programming solvers. Tradeoffs between speed, solution quality, and quality guarantees are quantified and presented. The relative ease of implementation and adaptability of different techniques are also discussed.

# Wednesday, August 4 – Session 2 – Algebra

### Wednesday, 11:00 am

## Introducing Three Best Known Binary Goppa Codes

**Presented by:** Christopher Soto, Queens College of the City University of New York

**Co-Authored by:** Jan Carrasquillo-López, University of Puerto Rico at Cayey
Axel O. Gómez-Flores, University of Puerto Rico at Río Piedras
Fernando Piñero, University of Puerto Rico in Ponce

**Faculty Advisor:** Fernando Piñero, University of Puerto Rico in Ponce

The current best known [239, 21], [240, 21], and [241, 21] binary linear codes have minimum distance 98, 98, and 99 respectively. In our research, we introduce three binary Goppa codes with Goppa polynomials $(x^{17}+1)^6$, $(x^{16}+x)^6$, and $(x^{15}+1)^6$. The Goppa codes are [239, 21, 103], [240, 21, 104], and [241, 21, 104] binary linear codes respectively. These codes have greater minimum distance than the current best known codes with the respective length and dimension. In addition, with the techniques of puncturing, shortening, and extending, we find more derived codes with a better minimum distance than the current best known codes with the respective length and dimension. This research was conducted at The Puerto-Rico/East Tennessee REU in Combinatorics, Probability, and Algebraic Coding Theory in Summer 2020 and supported by NSF-DMS REU-1852171.

### Wednesday, 11:20 am

## Affine Hermitian Grassman Codes

**Presented by:** Doel Rivera, Pontifical Catholic University of Puerto Rico

**Faculty Advisor:** Fernando Piñero, University of Puerto Rico, Ponce

In our current day and age, we are constantly sending messages across channels in which errors may enter. Current standards demand quick and effective communication, which means we need to find a way to detect and correct these possible errors. This, among other problems, is what error correcting codes look to solve.

Linear error correcting codes are used to send messages efficiently while resisting errors. We consider a new class of linear codes, called Hermitian Affine Grassman Codes. These are closely related to the Grassman codes. Their structure is based on evaluating matrix functions. We determine the code's parameters and improve the classic "Affine Grassman Code". We also study other related codes. This work was conducted under the supervision of Prof. Fernando Piñero through the PR-LSAMP program.

# Wednesday, August 4 – Session 3 – Applied Mathematics

**Wednesday, 1:00 pm**

## Mathematical Properties of Quantum Error Correcting Codes for Large Spins

**Presented by: Aditya Sivakumar, California Institute of Technology**

**Faculty Advisor:** Victor Albert, University of Maryland

A quantum system can be represented as a Hilbert space, where the possible quantum states of the system are unit vectors in the Hilbert space. Quantum information can be encoded in finite-dimensional subspaces of the Hilbert space. A disadvantage of these systems is that the information can be easily damaged by noise sources. The errors caused by this noise are represented as linear endomorphisms on the given Hilbert space. Fortunately, there often exist subspaces of the Hilbert space within which any stored quantum information cannot be irrecoverably lost due to the action of these endomorphisms. It turns out that for the information to be fully recoverable, all that is required is for the restriction of any composition of two such endomorphisms to the subspace to be diagonalizable with all its eigenvalues equal to each other. In this paper, we studied a specific quantum system: a quantum rigid body suffering from noise that alters its spin state. We conjectured that a type of subspace known as a binomial code would protect any stored information from this noise. We then developed a rigorous analytical proof to support this claim and provided an exact formula for the subspace.

**Wednesday, 1:20 pm**

## A Quantum Resource Theory for One Way Information

**Presented by:** Jack Rausch, Creighton University

**Co-Authored by:** Randall Crist, Creighton University

**Faculty Advisor:** Randall Crist, Creighton University

In quantum information theory, there is a recently defined measure that can quantify the one-way information (OWI) of the joint evolution of a composite system utilizing the concept of conditional mutual information $I(\alpha : \gamma | \beta) = I(\alpha : \gamma\beta) - I(\alpha : \beta)$. A quantum resource theory (QRT) can be developed to examine OWI. A QRT provides a way to examine a problem under a set of limitations where certain operations are identified as free operations (can be used without limitations) and others are identified as resources or operations with limitations or costs. The goal of this project is to develop a QRT for OWI, that is, to determine what the free states, resource states, and free and restricted operations are.

**Wednesday, 1:40 pm**

## Relative Frame Potential

**Presented by:** McKenna Kaczanowski, Ball State University

**Co-Authored by:** Roza Aceska, Ball State University

**Faculty Advisor:** Roza Aceska, Ball State University

In finite-dimensional Hilbert space, frames are exactly the spanning sets of the space. They are a generalization of bases that allow more flexibility than bases while sharing useful properties. The vectors in a frame do not have to be linearly independent, allowing for redundancy, which helps protect against information loss when transmitting a signal. Like orthonormal bases, the coefficients of the expansion of any vector in the space in terms of a frame can be easily computed.

A frame $G$ for a space H is considered to be dual to a frame $F$ if we can write any vector $x$ in $H$ as a linear combination of the vectors in $F$, where the coefficients are the inner products of $x$ with the vectors in $G$. The canonical dual frame of a frame $F$ is the only dual to $F$ whose vectors can be computed by applying an invertible operator to the vectors in $F$.

We explore a function that is based off of a frame potential function developed by John Benedetto and Matthew Fickus in 2001 that takes one frame as an input. This function is minimized exactly when the input frame is a tight frame. Our relative frame potential function, taken with respect to a frame $F$ for $R^n$, takes another frame $G$ for $R^n$ as input. When the input frame $G$ is a dual frame for $F$, the function will be minimized exactly when $G$ is the canonical dual frame for $F$, and this minimum value will be the dimension $n$.

**Wednesday, 2:00 pm**

## A Coalition Game on Finite Groups

**Presented by:** Ebtihal Abdelaziz, Goshen College

**Faculty Advisor:** David Housman, Goshen College

This is an initial investigation into possible connections between the mathematical theories of groups and coalition games. An example of a group is the set of symmetries of a square: (0-, 90-, 180-, and 270-degree rotations and flips across the four lines of symmetry) with composition as the binary operation. A coalition game is a set of players and a numerical worth for each coalition (a nonempty subset of players), and an allocation divides the worth of the all-player coalition as payoffs to the players. Given an allocation, the excess of a coalition is the sum of their payoffs minus the worth of the coalition, which is one way to quantify how happy the coalition is with the allocation. The prenucleolus is an allocation that successively maximizes the smallest coalition excesses. One coalition game on a group uses the group elements as the players, and the worth of a coalition as the number of elements in the subgroup generated by the coalition. For any such coalition game, the prenucleous payoff is shown to be nonnegative for each player. The prenucleolus for the group of integers 0, 1, …, $n$ using addition modulo $n$ as the binary operation is determined for certain values of $n$.

**Wednesday, 2:20 pm**

## Application of the Golden Ratio in Art and Music

**Presented by:** Elaine Saunders, University of Texas at San Antonio
JC Rivera, University of Texas at San Antonio

**Faculty Advisor:** Su Liang, University of Texas at San Antonio

Mathematical concepts are present in many forms in the real world, and are especially prominent in different forms of art. One mathematical concept that is readily found throughout different forms of art is the golden ratio and its relationship to the Fibonacci sequence. The golden ratio is said to be used so often in works of art because it creates aesthetically pleasing proportions. We have gathered and explained examples of this in a painting, architecture and web design. Then we used the relationship between the golden ratio and the Fibonacci sequence to develop our own original piece of art. We made a variety of different songs incorporating the golden ratio into the melody and/or the movement of each piece. We create one that strictly uses the piano, one techo piece and one ringtone to showcase the range of styles this mathematical concept could be utilized in.

#### Wednesday, 2:40 pm

### Seeing the Degradation of Lithium-ion Batteries through Electrochemical Impedance Spectroscopy

**Presented by:** Blake Harris, Lee University

**Faculty Advisor:** Debra Gladden, Lee University

Today, practically every electronic device we use has a rechargeable battery, many are lithium-ion batteries. We discuss the role of Electrochemical Impedance Spectroscopy, or EIS, in measuring the health and degradation of lithium-ion batteries. EIS measures the amount of impedance, or opposition to the flow of electrons, present in an electrochemical system with an alternating current. This project was completed in cooperation with Dr. Robert Sacci from Oak Ridge National Labs. Dr. Sacci has created a physical model of a lithium-ion battery that simulates its degradation overtime. In our project, we analyze data from his model and build python programs to process this information. We add noise (Gaussian and uniform) to the data, allowing for the data to resemble more closely real-world conditions. We discuss the role of the impedance data, fast fourier transforms of the data, and both Nyquist plots and Bode plots from the Impedance data. This project is funded through the ACA Ledford Scholarship Program.

## Wednesday, August 4 – Session 4 – Topology

#### Wednesday, 1:00 pm

### Persistent Homology

**Presented by:** Jenna Royce, Creighton University

**Co-Authored by:** Nathan Pennington, Creighton University

**Faculty Advisor:** Nathan Pennington, Creighton University

Persistent homology is a tool used to study qualitative features of data over a variety of size scales, often with the goal of identifying noise within high dimensional data sets. It has various real-life applications, such as finding similarities in medical imaging methods, as well as areas such as global development and tourism.

Specifically, the persistent homology process applied to a data set returns a barcode. The goal of this project is to construct and analyze a variety of possible “distance” functions on the space of all barcodes, which can be then be interpreted as providing a “distance” between data sets.

#### Wednesday, 1:20 pm

### Topological Data Analysis of Pattern Formation in Heterogeneous Cell Populations

**Presented by:** William Zhang, Brown University

**Co-Authored by:** Dhananjay Bhaskar, Brown University

**Faculty Advisor:** Ian Wong, Brown University

Topological Data Analysis (TDA) is an emergent field in Data Science that characterizes the shape of point clouds by tracking the presence of connected components, loops, and higher dimensional holes at multiple spatial scales. This information is can be summarized in a barcode or a persistence diagram, obtained by computing the persistent homology of filtered simplicial complexes. This technique has many applications, including the analysis of collective cell migration and pattern formation in cell biology. In this context, the point cloud represents cell nuclei positions and its topological features quantify the spatial organization of the tissue.

We used a simple mathematical model to observe of pattern formation in heterogeneous cell populations at varying cell-cell adhesion strengths, generating patterns of striped, spotted and honeycomb configurations that are reminiscent of skin pigmentation in animals. Next, we demonstrated that features derived from TDA can automatically classify these patterns. Finally, I will discuss how this methodology can be applied more generally to study many other interesting phenomena in active matter.

**Wednesday, 1:40 pm**

## Towards the Homotopy Type of the Morse Complex

**Presented by:** Connor Donovan, Ursinus College

**Faculty Advisor:** Nicholas Scoville, Ursinus College

Let $K$ be a simplicial complex. Using the Cluster Lemma of J. Jonsson and P. Hersh, we investigate the homotopy type of the Morse complex *M(K),* the complex of all gradient vector fields on $K$. First, using this method, we extend a well-known result by Ayala et. al., thereby recovering their result as a corollary regarding the collapsibility of the pure Morse complex of trees. Secondly, we provide an alternate proof for computing the homotopy type for any path graph, as well as extend this to show a path graph, $P_t$, $t = 3n$, has a strongly collapsible Morse complex. Lastly, we further investigate instances in which the Morse complex of a simplicial complex has the homotopy type of an $n$-sphere. We hope to further use this method to compute or estimate the homotopy type of the Morse complex of other trees, cycles, and larger simplicial complexes.

**Wednesday, 2:00 pm**

## Mean squared linking number of uniform random polygons in confined spaces with applications to $K_6$

**Presented by:** Pedro Morales, University of Maryland, College Park

**Co-Authored by:** Yasmin Aguillon, Swarthmore College
Xingyu “Tiger” Cheng, Purdue University
Spencer Eddins, University of Kentucky

**Faculty Advisor:** Kenji Kozai, Rose-Hulman Institute of Technology

The behavior of DNA in confined spaces is of big interest in molecular biology. Due to the length of polymerase chains, like DNA, it is inevitable for entanglement to occur. Random polygon models, like the one introduced by Arsuaga et al. provide a mathematical model for studying the entanglements. Tools from knot theory can be used to study these entanglements through invariants like the linking number that measures the degree of linking between two closed loops. In the present work, we generalize the results of Arsuaga et al. to polygons with a random number of edges and discuss applications of these generalizations to the complete graph $K_6$. This project was completed as part of the 2020 Research Experience for Undergraduates at Rose-Hulman Institute of Technology.

#### Wednesday, 2:20 pm

## Generalized Algorithm for the Dihedral Linking Number of p-Colorable Knots

**Presented by:** Elise Catania, Smith College
Jack Kendrick, Smith College
Olivia Del Guercio, Rice University

**Co-Authored by:** Dr. Patricia Cahn, Smith College
Sara Chimgee, Smith College

**Faculty Advisor:** Patricia Cahn, Smith College

Knot invariants have been essential computational tools for tabulation. A linking number is used to describe the number of times two distinct knots wind around each other. For p prime, from a p-colored knot, we can build a new space called a branched cover, where the knot lifts to one knot of branching index 1 and (p-1)/2 knots of branching index 2. From any pair of these knots, we can compute a linking number, called a dihedral linking number. The set of all dihedral linking numbers associated to a p-colorable knot is the dihedral linking invariant. Perko created a combinatorial algorithm for computing the dihedral linking number for all 3-colorable knots. In this work, we generalize Perko's algorithm to all p-colorable knots.

#### Wednesday, 2:40 pm

## On the Delta-Unlinking Number

**Presented by:** Noe Reyes, Andrews University
Jeannelle Green, Andrews University
Moises Reyes, Andrews University
Gabriel Palacios, Andrews University

**Faculty Advisor:** Anthony Bosman, Andrews University

It is known that links with pairwise vanishing linking number can be transformed into the trivial link by a series of local moves on the link diagram called delta moves; we define the delta unlinking number to be the minimum number of such moves needed. This generalizes the notion of delta unknotting number, defined to be the minimum number of delta moves needed to move a knot into the unknot. While the delta unknotting number has been well-studied and calculated for prime knots, no prior such analysis has been conducted for the delta unlinking number. We prove a number of lower and upper bounds on the delta unlinking number, relating it to classical link invariants including unlinking number, 4-genus, signature, and Arf invariant. Using these bounds, we determine the precise value of the delta unlinking number for all prime links with vanishing pairwise linking number up to 9 crossings. This work arises from the National Research Experience for Undergraduates being conducted at Andrews University, Summer 2021.

# Thursday, August 5 – Session 5 – Biomathematics

**Thursday, 1:00 pm**

## The Genetics of Human Aging: Predicting Age and Age-Related Diseases by Deep Mining High Dimensional Biomarker Data

**Presented by:** Hannah Guan, Basis San Antonio Shavano

**Faculty Advisor:** Yu Zhang, Trinity University

Aging is traditionally thought to be caused by complex and interacting factors such as DNA methylation. The traditional formula of DNA methylation aging is based on linear models and few works have explored the effectiveness of neural networks (NN). DNA methylation data typically consist of hundreds of thousands of feature space and a much smaller number of biological samples. This leads to overfitting and poor generalization of neural networks. We developed the Correlation Pre-Filtered Neural Network (CPFNN) model that uses Spearman Correlation to pre-filter the input features before feeding them into neural networks. We compared CPFNN with statistical regressions, NN with LASSO regularization and elastic net regularization, and dropout NN. CPFNN outperformed these models by at least 1 year in terms of Mean Absolute Error (MAE), with a MAE of 2.7 years. I also tested for association between the epigenetic age with Schizophrenia and Down Syndrome ($p = 0:024$, $p < 0:001$, respectively). I discovered that for a large number of candidate features, a key factor in improving prediction accuracy is to appropriately weight features that are highly correlated with the outcome of interest.

**Thursday, 1:20 pm**

## Surviving Ragnarok: Modeling Humanity's Chance of Survival after a Major Disaster Event

**Presented by:** Stephen Creamer, Wofford College
Whitney Kitchen, South Carolina Governor's School for Science and Mathematics

**Co-Authored by:** Eric McKenzie
Shepard Hawj, South Carolina Governor's School for Science and Mathematics

**Faculty Advisor:** Rachel Grotheer, Wofford College

Events such as natural disasters, disease epidemics, and wars heighten our collective awareness of the fragility of human life on Earth. Therefore, it comes as no surprise that researchers have sought to create mathematical models to predict the behavior of population growth following these events. Our research models the growth of a population from an initial number of survivors following a catastrophic event by using Markov population chains and differential equations. With these models, we establish a viable range for the initial population that ensures a steady growth rate.

**Thursday, 1:40 pm**

## Constructing Semi-Directed Level-1 Phylogenetic Networks from Quarnets

**Presented by:** Jonathan Garcia, Hobart and William Smith Colleges
Ainsley Woods, Hobart and William Smith Colleges
Eesha Anjum Anjum, Hobart and William Smith Colleges

**Co-Authored by:** Sophia Huebler, University of Oklahoma
Rachel Morris, , North Carolina State University
Joseph Rusinko, Hoshubart and William Smith Colleges
Yifei Tao, Hobart and William Smith Colleges

**Faculty Advisor:** Joseph Rusinko, Hobart and William Smith Colleges

We introduce two algorithms for reconstructing semi-directed level-1 phylogenetic networks from their complete set of 4-leaf subnetworks, known as quarnets. The first algorithm, the sequential method, begins with a single quarnet and adds on one leaf at a time until all leaves have been placed. The second algorithm, the cherry-blob method, functions similarly to cherry-picking algorithms for phylogenetic trees by identifying exterior network structures from the quarnets. We generate and implement the two algorithms using the programming language, Julia.

**Thursday, 2:00 pm**

## Modeling Chytridiomycosis Disease and its Thermal Acclimation within a Frog Population

**Presented by:** Addesyn Marshall, Dixie State University
Rosa Flores, Dixie State University

**Faculty Advisor:** Vindho Chellamuthu, Dixie State University

*Batrachochytrium dendrobatidis* is a pathogenic fungus implicated in the global decline of amphibians. Several studies suggest that temperature plays a significant role in *Batrachochytrium dendrobatidis* and a frog's development. However, there are many questions unanswered regarding these interactions. To investigate the effect of temperature dependency on the disease dynamics, we developed a mathematical model with vital rates of both the fungus and frogs depending on temperature. We utilized parameter estimation to compute and fit the model to different regions and show the decimation impact of the frog population. Our simulation results corroborate with literature studies showing the correlation between temperature and chytridiomycosis disease. Moreover, Our results suggest that temperature plays a major role in the life history of frogs and in disease dynamics. Furthermore, our results suggest critical intervention strategies that can be utilized to mitigate the disease within frog populations.

**Thursday, 2:20 pm**

## A two-patch model of *Batrachochytrium dendrobatidis*: Analysis of conservation strategies in frog populations

**Presented by:** Lori Smith, Dixie State University

**Co-Authored by:** Brandon Payne, Dixie State University

**Faculty Advisor:** Vinodh Chellamuthu, Dixie State University

*Batrachochytrium dendrobatidis* (Bd) is a fungus that causes the infectious disease chytridiomycosis in amphibian populations, this disease has been linked to frog population declines in many species and is pushing some species to the verge of extinction. The recovery of frogs involves a human intervention to ensure that the environment is suitable for their expansion. However, humans can interact with another frog population, creating an opportunity for Bd to spread between two patches. We have developed a two-patch mathematical model that explores the potential effects of Bd on vulnerable frog populations. Our model incorporates direct and indirect transmission, through Zoospores, temperature-dependent death rates, and models how humans can spread Bd between patches. Our simulations highlight the importance of mitigating the spread of Bd and assess the effect of the disease spreading rate and the population persistence in the long term.

# Thursday, August 5 – Session 6 – Dynamical Systems & Differential Equations

**Thursday, 2:40 pm**

## Mathematical and numerical approaches to the dynamic contact between a Timoshenko beam and a nonlinear spring

**Presented by:** Nicholas Tate, Arkansas State University – Jonesboro

**Faculty Advisor:** Jeongho Ahn, Arkansas State University - Jonesboro

In this work, we provide mathematical and numerical analyses of a differential equation (DE) system, in order to explore the real-world applications of a seesaw. In the DE system, two partial differential equations (PDEs) and a nonlinear DE, which is an extended Duffing equation, describe the motion of a Timoshenko beam and a nonlinear viscoelastic spring, respectively. For mathematical reasons, only half of the board is considered, where one end of the beam is clamped, and the other end comes into contact with the spring attached to the land. In this mathematical model, the contact between a beam and a spring is understood by the normal compliance conditions, and when the spring is compressed and reaches the land, Signorini's contact condition is applied. We propose the fully discrete numerical schemes using time discretizations and finite element methods. Because the conditions created a nonlinear system, we utilize the Newton-Raphson method to compute each time-step numerical approximation from a recursive formula in the fully discrete case. We present numerical simulations with selected data and discuss numerical stability as well.

**Thursday, 3:00 pm**

## The importance of being discrete: dynamics of flow-kick disturbance models

**Presented by:** Horace Fusco, Carleton College
Collin Smith, Carleton College

**Faculty Advisor:** Kate Meyer, Carleton College

Disturbances such as rain events, fires, and harvests shape earth's ecosystems and are changing in response to human impacts such as climate change and natural resource management. Flow-kick models offer an alternative to traditional ODEs to capture the discrete flavor of such disturbances. In this talk we'll examine the connection between continuous disturbance models and their flow-kick counterparts. We'll ask which features of a continuous disturbance model persist in general to nearby discrete models, and what new behavior we might discover by going discrete.

**Thursday, 3:20 pm**

## Calculating Invariant Measures of Chaotic Systems using Cupolets

**Presented by:** Jack Lynam, Christopher Newport University

**Co-Authored by:** Matthew A. Morena, Christopher Newport University

**Faculty Advisor:** Matthew Morena, Christopher Newport University

Chaotic systems are a class of dynamical systems that are notoriously difficult to understand due to their incredible complexity. One way to analyze these systems is via the unstable periodic orbits (UPOs) that are typically found densely embedded around an associated attractor. From the set of UPOs, several invariant measures, such as the topological entropy, dimension, and maximal Lyapunov exponent, can be calculated to provide a sense of complexity for a chaotic system. While UPOs are insightful tools for analysis, they are computationally expensive to find.

Alternatively, cupolets (Chaotic, Unstable, Orbit-LETS) represent highly accurate approximations to UPOs that are computationally inexpensive to generate en masse. In this work, we compare how well cupolets approximate the invariant measures of chaotic systems, such as the Lorenz and double scroll systems, with the true measures computed through UPOs. We explore whether cupolets can provide as insightful a framework for analyzing chaotic systems as that which has been demonstrated with UPOs.

**Thursday, 3:40 pm**

### Numerically Solving the Equations of Planetary Motion with an Adaptive Runge-Kutta Method

**Presented by:**

Caleb Froelich, Walla Walla University

**Faculty Advisor:**

Ross Magi, Walla Walla University

By utilizing Newton's Law, the system of differential equations that describe planetary trajectories in our solar system is derived. Put into standard form, the problem is then solved numerically using an adaptive Runge-Kutta method implemented from scratch. Relevant mathematical theory underlying the basics of the numerical analysis necessary to implement the model is presented. Comparisons to MATLAB's built-in solver ode45 showed that the models built from scratch outperform MATLAB's models in terms of speed, function evaluations and computational efficiency. Results from this analysis and modeling of complex equations will benefit the development of other challenging numerical analysis applications and problems.

## Thursday, August 5 – Session 7 – Geometry

**Thursday, 1:00 pm**

### Affine Manifolds and the Universal Cover of the Punctured Plane

**Presented by:** Toby Aldape, University of Colorado at Boulder

**Faculty Advisor:** Jonathan Wise, University of Colorado at Boulder

We develop the basic theory of affine manifolds, prove that the compact open and product topologies are identical on the space of affine automorphisms of an affine manifold, and prove that the universal cover of GL(2, $R$) is the automorphism group of the universal cover of the punctured plane.

**Thursday, 1:20 pm**

### Fractal Dimension as an Indicator of Urbanization

**Presented by:** Junze Yin, Boston University

**Faculty Advisor:** Emma Previato, Boston University

The existing research about fractal analysis focuses on analyzing urbanization via the contour of cities. The cities' contour resembles a coastline, which does not have a regular shape. Fractal analysis is an indicator of urbanization as it measures the urban sprawl, which is the degree of fragmentation and transition from monocentric to polycentric structures. The fractal dimension is an index that describes the complexity of a geometric object: if when measuring the object we scale the unit by $\varepsilon$ and the measure gets scaled by $N$, then the fractal dimension $D$ is defined by $\log_\varepsilon N = -D = (\log N)/(\log \varepsilon)$.

My goal is to create a visual representation for the changes of Boston's metropolitan area and its growth rate, and to do so I analyze its fractal dimension. Two methods, box-counting (after Apostolos Lagarias) and radial analysis (after Joseph Riley) are utilized to estimate Boston's fractal dimension, based on census data. A longitudinal study is lacking. I estimate the fractal dimension of Boston over the past 20 years by a time series, and by non-linear regression I project the future extent of Boston's urbanization.

**Thursday, 1:40 pm**

## Synthetic Geometry in Simplices of Constant Curvature

**Presented by:** Andrew Clickard, Bloomsburg University of Pennsylvania

**Co-Authored by:** Barry Minemyer, Bloomsburg University of Pennsylvania

**Faculty Advisor:** Barry Minemyer, Bloomsburg University of Pennsylvania

Previous work has been done in metric geometry to find an intrinsic method by which the distance between two points in an $n$-dimensional Euclidean simplex can be calculated. The natural question, then, arises concerning simplices embedded in spaces with constant positive or negative curvature. We give a simple technique to compute the distance between two points in an $n$-dimensional simplex of constant curvature given only the points' barycentric coordinates and the edge lengths of that simplex, as well as a simple test to verify the legitimacy of a set of positive real numbers as edge lengths of an $n$-dimensional simplex of constant curvature. We also determine a method of finding the barycentric coordinates of the orthogonal projection of a point in a simplex of constant curvature.

**Thursday, 2:00 pm**

## Manifolds with bounded integral curvature and no positive eigenvalue lower bounds

**Presented by:** Connor Anderson, Worcester Polytechnic Institute
Kamryn Spinelli, Worcester Polytechnic Institute

**Co-Authored by:** Xavier Ramos Olivé, Worcester Polytechnic Institute

**Faculty Advisor:** Xavier Ramos Olivé, Worcester Polytechnic Institute

We provide an explicit construction of a sequence of closed surfaces with uniform bounds on the diameter and on $L^p$ norms of the curvature, but without a positive lower bound on the first non-zero eigenvalue of the Laplacian $\lambda_1$. This example shows that the assumption of smallness of the $L^p$ norm of the curvature is a necessary condition to derive Lichnerowicz and Zhong-Yang type estimates under integral curvature conditions.

**Thursday, 2:20 pm**

## Patterns Between the Number of Cevians or Caliians and the Regions That They Form in Polygons

**Presented by:** Calia Kugler, Binghamton University

**Faculty Advisor:** Robert Gerver, Institute of Creative Problem Solving

This paper is a description of an investigation into cevians and caliians (cevians in a figure other than a triangle). The formula for the number of regions formed in a triangle by drawing $n$ cevians from each vertex is obtained. Next, formulas for the number of regions formed in other simple polygons by drawing $n$ caliians are developed. A pattern is observed and a general formula for the number of regions in any polygon is developed. This formula is then generalized to show the number of regions formed in a simple polygon by drawing caliians not only to the opposite side of a vertex but to other sides as well. Formulas for the number of regions in some simple polygons formed by drawing any number of cevians or caliians from any vertex to any side are developed. The general formula is then proved using multivariable mathematical induction.

# Thursday, August 5 – Session 8 – Number Theory

**Thursday, 2:40 pm**

## Partitions and Quantum Modular Forms

**Presented by:** Anna Dietrich, Amherst College
Keane Ng, Amherst College
Chloe Stewart, Amherst College
Shixiong (Leo) Xu, Amherst College

**Co-Authored by:** Amanda Folsom, Amherst College

**Faculty Advisor:** Amanda Folsom, Amherst College

A partition of a positive integer *n* is any non-increasing sequence of positive integers which sum to *n*, and the partition function *p(n)* counts the number of partitions of *n*. A classical identity due to Euler gives rise to the beautiful and unexpected fact that the generating function for *p(n)* is essentially a modular form, a well-studied type of symmetric, complex analytic function. Here, we study other new combinatorial partition generating functions with the goal of establishing modularity properties, if any. In particular we study quantum modular properties of such combinatorial functions, where quantum modular forms are a new type of modular form defined by Zagier in 2010, and which have been shown to exhibit rich connections to diverse areas of mathematics, including combinatorics, number theory and more.

**Thursday, 3:00 pm**

## What You Have Leftover is Perfect: Enumerating the PSR Divisors

**Presented by:** Isaac Reiter, Kutztown University of Pennsylvania

**Co-Authored by:** Ju Zhou, Kutztown University of Pennsylvania

**Faculty Advisor:** Ju Zhou, Kutztown University of Pennsylvania

There are certain positive integers that, when divided into any perfect square, always yield a remainder that is also a perfect square. For example, when 12 is divided into any perfect square, the remainder is 0, 1, 4, or 9. We will refer to these numbers as PSR divisors, PSR standing for Perfect Square Remainder. In my research with Dr. Zhou, we asked ourselves two questions. First, how do we prove that a number is a PSR divisor? Second, can we completely characterize all PSR divisors among the positive integers?

In this talk, I will provide answers to both of these questions. I will begin by demonstrating how to prove that a particular number is a PSR divisor. The proof method is a beautiful application of the fact that every perfect square can be expressed as a sum of consecutive odd numbers. After laying this foundation, I will illuminate the central result of our research. Namely, there are only eight PSR divisors among the positive integers. The proof of this is an excellent example of finding a few "special tricks" that help all the mathematical rigor fall into place.

**Thursday, 3:20 pm**

## Sums of *k*th powers in ramified *p*-adic rings

**Presented by:** Chandra Copes, McDaniel College
Kevin Rabidou, McDaniel College

**Faculty Advisor:** Spencer Hamblen, McDaniel College

Generalization of Waring's Problem—that for every natural number $k$ there exists an integer $g(k)$ such that every natural number can be written as the sum of at most $g(k)$ $k$-th powers—have been studied in a variety of contexts from

algebraic number fields to non-commutative groups. We will examine values of *g(k)* for certain ramified extensions of $\mathbf{Z}_p$, with specific focus on when $p$ divides $k$ and on wildly ramified extensions.

**Thursday, 3:40 pm**

### Nim Type Game

**Presented by:** Eddy MacDonald, Saint John's University

**Co-Authored by:** Dr. Kris Nairn, CSBSJU

**Faculty Advisor:** Kris Nairn, CSBSJU

Suppose we have N piles with a specified number of tokens in each pile. You may remove as many tokens as you want from one of the end piles or you may remove tokens from each of the end piles. The difference between the number of tokens that you remove from the left pile and the right pile must be less than or equal to a specified number. Player 1 often has a winning strategy for two piles and I am currently working on cases with more than two piles.

## Friday, August 6 – Session 9 – Topology

**Friday, 10:00 am**

### Tiling the Double Branched Cover of the Sphere

**Presented by:** Liza Jacoby, Williams College

**Faculty Advisor:** Colin Adams, Williams College

A complete classification of edge-to-edge tilings of the 2-sphere by regular polygons of three or more sides is given by the Platonic solids, the Archimedean solids, the prisms and antiprisms, and twenty-five of the ninety-two Johnson solids, beginning in 360 BCE and culminating in 1966. In 2021, Adams et al. completed the classification of non-edge-to-edge tilings of the 2-sphere using regular polygons of three or more sides, introducing 33 new families of non-edge-to-edge tilings. We now extend the notion of spherical tilings to branched coverings of the 2-sphere. We determine tilings by regular polygons of three or more sides of the double branched cover of the sphere with two branch points.

**Friday, 10:20 am**

### Relations on the Mapping Class Monoids of Planar Surfaces

**Presented by:** Victoria Quijano, University of California, Davis

**Faculty Advisor:** Laura Starkston, University of California, Davis

The mapping class group of a surface is a set of functions which continuously bend and stretch the surface, called homeomorphisms. In the mapping class group, two elements are considered to be equivalent if they are isotopic to one another. Furthermore, homeomorphisms of the annulus that fix the boundary pointwise are called Dehn twists, and the mapping class group of any surface is generated by Dehn twists along a finite collection of simple closed curves on that surface. In this project, we consider the elements of the mapping class group that can be generated only by positive Dehn twists, which form the mapping class monoid. We define new relations on the mapping class monoids of genus 0 surfaces in order to determine when two elements are equivalent. This information can be used to analyze Lefschetz fibrations, which build 4-dimensional manifolds with boundary using surfaces together with products of positive Dehn twists around simple closed curves. In particular, the boundaries of two Lefschetz fibrations are homeomorphic if their associated products of Dehn twists are equivalent in the mapping class monoid.

### Friday, 10:40 am

## Cylinder Configurations on Flat Surfaces

**Presented by:** Juliet Aygun, Purdue University
Janet Barkdoll, Swarthmore College
Jenavie Lorman, Harvard University
Theodore Sandstrom, Yale University

**Faculty Advisor:** Aaron Calderon, Yale University

A major theme in low-dimensional geometry and topology is to understand when given combinatorial or topological data can be realized geometrically. For example, any non-self-intersecting curve on a torus is the core curve of an embedded Euclidean cylinder. For flat cone metrics on higher genus surfaces this isn't always the case, even if you allow deformations of the metric. In this talk, we discuss criteria for when collections of curves on a surface can jointly be realized as cylinders in some flat cone metric. This represents work done at the Yale SUMRY REU.

### Friday, 11:00 am

## Curve Configurations on Non-Orientable Surfaces

**Presented by:** Julia Shneidman, Rutgers University
Sarah Ruth Nicholls, Wake Forest University

**Co-Authored by:** Nancy Scherich, University of Toronto

**Faculty Advisor:** Nancy Scherich, University of Toronto

A well studied problem in topology is to find families of nontrivial curves in orientable surfaces with fixed intersection numbers. For example, for an orientable surface of genus $g > 2$, there are at most $3g - 3$ nonintersecting nontrivial curves. We can look for similar families of curves in non-orientable surfaces. Because non-orientable surfaces contain Möbius bands and torsion, curve behaviors are counterintuitive. We exhibit nontrivial curve configurations in non-orientable surfaces with fixed intersection properties. This research was performed as part of the School of Math REU at Georgia Institute of Technology.

### Friday, 11:20 am

## Filling Curves on Surfaces on a Genus 3 Surface

**Presented by:** Alice Ponte, Georgia Institute of Technology

**Co-Authored by:** Elyssa Cirillo, Christopher Newport University
JiaMin Li, Agnes Scott College

**Faculty Advisor:** Wade Bloomquist, Georgia Institute of Technology

We study configurations of closed curves on topological surfaces. Given any collection of curves on a surface, we can cut along all of them in order to obtain a new surface. A pair of curves is a minimal filling pair if the surface obtained by cutting along both is a single disk. The first examples of minimal filling pairs on a genus g surface were constructed through an inductive procedure by Aougab and Huang. They also explored the combinatorial content of minimal filling pairs, producing upper and lower bounds on the number of distinct examples. In this talk we will focus on extensions and specializations of the above work in the case of a genus 3 surface. We explicitly classify all minimal filling pairs on this surface, including examples that do not come from the above inductive procedure.

**Friday, 11:40 am**

### Automorphisms of the fine curve graph

**Presented by:** Adele Long, Smith College
Anna Pham, University of Texas at Dallas
Claudia Yao, University of Chicago

**Co-Authored by:** Dan Margalit, Georgia Institute of Technology
Yvon Verberne, Georgia Institute of Technology

**Faculty Advisor:** Dan Margalit, Georgia Institute of Technology

The curve graph of a surface is a graph whose vertices are isotopy classes of simple, closed, essential curves on the surface, and there exists an edge between two vertices if the two isotopy classes are disjoint. Ivanov [1] showed that the automorphism group of the curve graph is isomorphic to the extended mapping class group, the group of isotopy classes of all homeomorphisms of a surface.

We extend Ivanov's work by considering the fine curve graph of a surface (defined by Bowden, Hensel, and Webb [2]), where vertices of the graph are all curves on the surface, and there exists an edge between two vertices if the curves are disjoint. Similar to Ivanov's work, we show that the automorphism group of the fine curve graph is isomorphic to the extended mapping class group. This is joint work with Dr. Dan Margalit and Dr. Yvon Verberne. This project stems from work completed at the Georgia Tech REU.

[1] J. Bowden, S. Hensel, and R. Webb. Quasi-morphisms on surface diffeomorphism groups. arXiv:1909.07164, 2019. To appear in *J. Amer. Math. Soc.*

[2] Nikolai V. Ivanov. Automorphism of complexes of curves and of Teichmüller spaces. *Internat. Math. Res. Notices*, (14):651-666, 1997.

## Friday, August 6 – Session 10 – Applied Mathematics

**Friday, 10:00 am**

### Machine Learning Approaches Towards Option Pricing

**Presented by:** Thiha Aung, University of California Santa Barbara

**Faculty Advisor:** Tomoyuki Ichiba, University of California Santa Barbara

In recent years, the Longstaff-Schwartz method, Stochastic Volatility models were developed to overcome the shortcomings of the classic Black-Scholes PDE for Option Pricing. This paper explores option pricing models that utilize machine learning techniques. First, we trained a Multilayer Perceptron Neural Network to predict the European option prices given the same input parameters as the Black-Scholes formula using both real and simulated data. For American Option Prices, we explored pricing using the Q-learning approach, one of the main approaches in Reinforcement Learning. Since American Option Pricing is an optimal control problem, this can be reformulated via Markov Decision Process, treating option price as the Q-function. The model we will present is a modification of the QLBS model by Halperin (2020). We start with a replicating portfolio to develop an MDP framework. We will show the Q-Learner is guaranteed (by construction) to converge to the correct optimal hedge and is guaranteed to converge to the classical Black-Scholes results for option price and hedge ratio, given certain conditions. The main focus of this paper will be presenting numerical results.

**Friday, 10:20 am**

## Differential Privacy of a Randomized Learning Algorithm

**Presented by:** Julianne Nierwinski, Towson University

**Faculty Advisor:** Vincent Guingona, Towson University

Differential privacy is a rapidly developing area of machine learning. Differential privacy is a property of an algorithm where the results derived from datasets that differ by a single sample are almost indistinguishable. Companies and agencies including the Census Bureau, Apple, and Google utilize differential privacy in their algorithms to gain statistical insight into a dataset of users, while preserving the individuals' privacy. I will discuss new connections between two notions of differential privacy. I will show that the two notions are equivalent for pure differential privacy and one condition is strictly stronger in the case of $(\varepsilon, \delta)$-differential privacy.

**Friday, 10:40 am**

## Private Machine Learning Algorithm for the Dual Class of Cross-Cutting Equivalence Relations

**Presented by:** Avery Schweitzer, Towson University
Ramon Suris-Rodriguez, Towson University

**Faculty Advisor:** Alexei Kolesnikov, Towson University

A randomized learning algorithm inputs a set of data samples, produces a probability distribution over a concept class, and selects a concept according to the distribution. The algorithm is differentially private if, for any two data samples differing by at most one element, the algorithm produces two nearly indistinguishable probability distributions. The learning algorithm is Probably Approximately Correct (PAC) if it selects a concept that has low error with high probability. It is known that a concept class admits a differentially privately PAC-learning algorithm if and only if it has finite Littlestone dimension. In our talk, we will compute the Littlestone dimension of the concept class dual to the class of cross-cutting equivalence relations on an ambient set. We will also examine the sample complexity of a differentially private PAC-learning algorithm on the dual class. Our work was done as part of a Research Experience for Undergraduates (REU) at Towson University.

**Friday, 11:00 am**

## Comparing Measures For The Identification Of Partisan Gerrymandering

**Presented by:** Karthik Seetharaman, Massachusetts Academy of Mathematics and Science at WPI

**Faculty Advisor:** Kevin Crowthers, Worcester Polytechnic Institute

Only 2% of Americans believe that the elections are completely fair, and this is mainly due to partisan gerrymandering. However, there is currently no effective, standardized way to identify the presence of partisan gerrymandering. Identification is normally done through generating a large neutral ensemble of voting plans, then comparing the given voting plan against the ensemble through the use of a mathematical metric. In this paper, we survey the different models and algorithms used for this identification. First, we compare the Flip and ReCom algorithms for generating a neutral ensemble of voting plans by comparing the generated set of Democratic vote percentages and Democratic seat shares through each algorithm. We show that the ReCom algorithm is more effective at producing a neutral ensemble of voting plans. We then analyze the accuracies of seven different mathematical metrics commonly used to identify partisan gerrymandering. We show that the declination and relative efficiency gap are the most accurate metrics, while the partisan bias and mean-median score are the least. We do this by tracking the number of false results of the seven metrics across twelve elections.

**Friday, 11:20 am**

## Exploring the Potential for Gerrymandering Within Single And Multi Member Legislative Redistricting Plans

**Presented by:** Catherine Brennan, University of Colorado, Boulder

**Faculty Advisor:** Jeanne Clelland, University of Colorado, Boulder

This project explores ways of quantifying what constitutes a gerrymandered electoral districting plan, and compares the potential for gerrymandering between single member and multi member districts. I attempt to illustrate the benefits of using a Single Transferable Vote (STV) election system within multimember districts as opposed to a single member district winner-take-all election system. Using Markov Chain Monte Carlo methods, I have created large random samples of districting plans for Colorado election. The project uses Ensemble Analysis to analyze the distribution of districting plans and identify outliers.

**Friday, 11:40 am**

## Modelling Crime With Stochastic Processes

**Presented by:** Adedoyin Adegbuyi, Towson University
Ryan Budahazy, Towson University
Cody Stephenson, Towson University
Maggie Trimpin, Towson University

**Co-Authored by:** Dr. Christopher Cornwell, Towson University
Dr. Mathew Gluck, Towson University
Dr. Michael O'leary, Towson University

**Faculty Advisor:** Mathew Gluck, Towson University

Self-Exciting Poisson point processes are typically used in earthquake modelling to account for aftershocks, which are triggered from the initial occurrence of an earthquake. Following the work of Reinhart and Mohler, our premise is that the occurrence of crimes can be similarly modelled. That is, the occurrence of crimes can trigger the nearby occurrence of crimes shortly thereafter.

As a group of 4 students from Towson University, we use publicly available crime data from Baltimore City, as well as the Expectation Maximization algorithm, to fit the parameters of a Self-Exciting Poisson point process model. After fitting, we can predict how likely crimes are to occur in a spatial region, and given that a crime occurs, the likelihood of a triggered crime occurring. We can then explore what the model tells us about the crime triggering mechanism on our data.

We are actively pursuing ways to use our model beyond simply predicting crimes in Baltimore City. One of the possibilities that we are currently exploring includes how the triggering mechanism varies from city to city. Additionally, we are beginning to analyze the effect of the pandemic on the triggering of crimes.

# Friday, August 6 – Session 11 – Graph Theory

### Friday, 1:20 pm

## Determining the Winner in a Graph Theory Game

**Presented by:** Eric Burkholder, Valparaiso University

**Co-Authored by:** Christopher Barua, Valparaiso University
Gabe Fragoso, Valparaiso University

**Faculty Advisor:** Zsuzsanna Szaniszlo, Valparaiso University

We are investigating who has the winning strategy in a game in which two players take turns drawing arrows trying to complete cycle cells. The game boards are graphs, objects with dots and lines between them. A cycle cell looks like a polygon (triangle, square, pentagon, etc.). We examined game boards where the winning strategy was previously unknown. Starting with a pentagon and a heptagon glued by two sides, we worked to solve multiple classes of graphs involving stacked polygons. We also explored variations of the game where cycles, as defined in graph theory, are used in place of cycle cells, which opens the game up to non-planar graphs, such as complete graphs and gives the game a graph theory twist on top of topology. The original game was described by Francis Su in his book Mathematics for Human Flourishing.

### Friday, 1:40 pm

## Introducing an Induced Matching Game on Graphs

**Presented by:** Naima Nader, Vassar College
Michael J. L. Rivera Serrano, University of Puerto Rico Mayagüez Campus

**Co-Authored by:** Erika L. C. King, Hobart and William Smith Colleges

**Faculty Advisor:** Erika L. C. King, Hobart and William Smith Colleges

A *matching* is an independent set of edges in a graph $G$. By independent we mean that no two edges in the set are adjacent. An *induced matching* is a matching with an additional property that no two of its edges are joined by an edge in $G$. An induced matching $M$ in a graph $G$ is *maximal* if no other induced matching in $G$ contains $M$. We propose a game in which players alternate choosing edges on a graph while maintaining an induced matching. They continue until the matching is maximal and the last player to go wins. We will describe winning strategies for this game and prove results on several families of graphs. In addition, we will talk about some new families of graphs where the winner is pre-determined. These graphs are called well-indumatchable or parity graphs. This material is based upon work supported by the National Science Foundation under grant no. DMS 1757616.

### Friday, 2:00 pm

## Achromatic Vertex Distinguishing Edge Colorings

**Presented by:** Fiona Smith, College of St. Benedict / St. John's University

**Co-Authored by:** Anne Sinko, College of St. Benedict/ St. John's University

**Faculty Advisor:** Anne Sinko, College of St. Benedict/ St. John's University

Consider a simple graph $G$ and color the edges using $k$ colors. Define $C(v)$ to be the ordered $k$-tuple indicating the number of edges of each color incident with vertex $v$. If $C(v)$ is unique for each vertex in $G$, then the coloring is a vertex distinguishing edge coloring, or vdec. This talk will define the achromatic vertex distinguishing edge coloring number for graphs and will present results for several classes of graphs.

**Friday, 2:20 pm**

## Probabilities of single-component spanning trees for a family of graphs

**Presented by:** Isabel Trindade, Yale University
Natalie Jean-Michel, Williams College

**Co-Authored by:** Miles Mena, Lewis University
Jonathan Figueroa, CUNY Baruch

**Faculty Advisor:** Zhanar Berikkyzy, Fairfield University

Given a simple graph, we can use the forest-building process to identity a spanning forest, as follows: we sort the edges of the graph in some order, then, moving in the order of the list, we keep each edge only if it is incident to a vertex that is not already incident to any previous edge, which will result in a spanning forest. To identify all possible spanning forests, we can repeat this process with every possible permutation of the edges. Therefore, the probability of obtaining a spanning tree (a single-component spanning forest) can be calculated as a fraction of the number of single-component spanning forests divided by the total number of spanning forests.

In this talk, we derive the probability of obtaining a spanning tree for graphs of various shapes and arbitrary sizes, cycles of size $n$, paths of length $n_1$ attached to a star(s) of size $n_2$ at either one or both ends, and other families.

**Friday, 2:40 pm**

## K-component Probabilities of Spanning Forest Building Techniques

**Presented by:** Miles Mena, Lewis University
Jonathon Figueroa Reyes, Baruch

**Co-Authored by:** Isabel Trindade, Yale University
Natalie Jean-Michel, Williams College

**Faculty Advisor:** Zhanar Berikkyzy, Fairfield University

From a simple graph, a forest-building process can be applied to create a new graph from the spanning forest on the original graph. The process is summarized as follows: remove all the edges from the graph and enumerate them in any fashion, then, in order, add the edges if and only if the edge can be drawn from a vertex that has not already been visited by an edge.

This talk provides probabilities, among other things, of $k$-component graphs arising after the forest building process on a collection of graphs including path graphs. A generalization of these probabilities is given and we establish a connection to the peaks and valleys problem in a set of integers. Topic studied as part of Fairfield summer-REU.

# Friday, August 6 – Session 12 – Probability and Statistics

### Friday, 1:00 pm

## Subsums of Random Numbers

**Presented by:** Wei Wang, University of Illinois at Urbana-Champaign
Judy Chiang, University of Illinois at Urbana-Champaign

**Co-Authored by:** Yifan Zhang, University of Illinois at Urbana-Champaign

**Faculty Advisor:** A J Hildebrand, University of Illinois at Urbana-Champaign

If you pick $n$ random numbers in [0,1], what is the probability that their sum also falls into the interval [0,1]? The answer turns out to be $1/n!$, which can be seen using symmetry arguments or multidimensional integrals. We consider more generally the probability, $p(n, k)$, that, given $n$ random numbers in [0,1], there are $k$ of these numbers whose sum falls into the interval [0,1]. We show that these probabilities satisfy a generalized Pascal triangle type recurrence and we give an explicit formula for $p(n, k)$. In the special case $k = n - 1$, these probabilities can be expressed in terms of Stirling numbers of the second kind and also in terms of harmonic numbers.

### Friday, 1:20 pm

## Assessing Adherance to Benford's Law in Presidential Elections

**Presented by:** Brooks Groharing, William Jewell College

**Co-Authored by:** David McCune, William Jewell College

**Faculty Advisor:** David McCune, William Jewell College

Many naturally occurring datasets have been found to follow Benford's law, which predicts that the digits 1–9 appear as first digits with specific, decreasing frequencies. This pattern has been observed in a variety of contexts, such as town populations in a given state, physical constants, and accounting data. In the wake of the 2020 presidential election, a claim was made that because Biden's votes by county in Pennsylvania appeared to deviate more from a Benford distribution than Trump's, there was evidence of substantial voter fraud or election tampering. In this talk, we test the applicability of Benford's law to county level presidential election results in 32 states, for Republican and Democratic candidates, in each of the last seven election cycles. While the distributions of digits follow Benford's law the majority of the time, deviations were far too frequent to support claims of anomalies in any specific case.

### Friday, 1:40 pm

## Finding Your Way Back in a Random Forest: Debias Regression Predictors

**Presented by:** John Ryan, University of South Carolina

**Co-Authored by:** Gundeep Singh, University of Houston,
Hannah Wahl, Tulane University
Jessica Zhao, Colgate University
Lihua Chen, James Madison University
Prabhashi Withana Gamage, James Madison University

**Faculty Advisor:** Lihua Chen, James Madison University

It is well known that the random forest may reduce the variance of regression predictors produced by single trees through bagging while leaving the bias little changed. In general, the bias that arises from the random forest is not negligible. Therefore, this work focuses on evaluating several bias correction methods in bias reduction. The default

bias correction method implemented in the R package *randomForest* often does not work well. We develop and explore several bias correction methods as alternatives to the R default. Both simulated and real data sets are used to compare the performance of these methods. We offer general guidance and suggestions on the selection of different methods when the need to correct for the bias of random forest predictors arises.

This work is supported by an NSF-REU grant (# 1950370) at James Madison University in Summer 2021.

**Friday, 2:00 pm**

## Analysis of hydrologic and weather data to address drought in Ventura County

**Presented by:** Christopher Salas, California State University, Channel Islands
Norberto Huerta, California State University, Channel Islands
Jessica Tapia, California State University, Channel Islands

**Faculty Advisor:** Alona Kryshchenko, California State University, Channel Islands

Drought is a devastating phenomenon in terms of both our natural surroundings and of human endeavors. It is costly, both in financial and human-life contexts. Southern California is no stranger to drought; Ventura County specifically has seen particularly devastating wildfires in the last several years, associated with severe drought periods. In this project we analyze the behavior of variables related to droughts by seeking and combining different sources of information to explain any discernible patterns. We make use of over 50 years of rainfall and streamflow measurements of water stations, weather data (wind, wind direction, and temperature) for Ventura County, and historic Oceanic Niño Index values. Our exploratory data analysis findings of this historical data set are presented. Recognizing the multivariate nature of drought, we also explore potential models involving variables present in our data, for descriptive/predictive purposes. Such models can provide a developed understanding of the issue, which could inform decisions aimed at extenuating the effects of drought.

**Friday, 2:20 pm**

## Statistical Modeling to Predict the Trend in Lung Cancer Data in Utah using Joinpoint Regression Analysis

**Presented by:** Gregory Schmidt, Dixie State University
Riley Morgan, Dixie State University

**Faculty Advisor:** Md Sazib Hasan, Dixie State University
Vinodh Chellamuthu, Dixie State University

Lung cancer is one of the deadliest cancers in the state of Utah. According to the National Cancer Society's estimates in the United States, there were about 228,820 new cases of lung cancer and about 135,720 deaths from lung cancer in the United States in 2020. We obtained data from the Surveillance Epidemiology and End Results (SEER) database of the National Cancer Institute (NCI) and utilized JoinPoint Regression to predict the future trend. We developed a hybrid joinpoint regression model to describe incidence and mortality trends in Utah, assuming that the observed counts are probabilistically characterized by the Poisson distribution. We will compare our results with the other traditional statistical techniques. Using these methods, we can identify common trends that will allow us to come to a conclusion about lung cancer mortality in Utah. Our simulation results predict the rise of new deaths per year and future implications to help the policymakers in decision making.

**Friday, 2:40 pm**

### Does the tail wag the dog, after all? Obesity Clusters and their influence on the predatory location choice of new fast food chain franchisees

**Presented by:** Ryka Chopra, William Hopkins Junior High School

**Co-Authored by:** Suparna Chakraborty, University of San Francisco

**Faculty Advisor:** Suparna Chakraborty, University of San Francisco

According to NIH, obesity is currently the second leading cause of preventable death in the United States. Existing research suggests a positive correlation between obesity rates and fast-food access, but most studies focus on the demand side—consumers with easy access to fast-food branches substitute away from healthy options to fast foods resulting in obesity. But, what about the supply side? Do fast-food chains intentionally target "obesity clusters," regions with relatively high obesity rates, and open branches to entice these vulnerable populations, exhibiting a predatory location choice behavior? In this project, I focus on this unanswered question using fast-food branch and historical obesity data. Employing a fixed-effects regression model, I find that given a mean obesity rate of 28.7% in my sample, every 1% increase in obesity rate results in 2.7 new branches opening within a 2-mile radius of the cluster. The impact is inverted U-shaped, with the highest concentration of new branches in the 1.5–2 miles range of an obesity cluster. This suggests that fast-food chains tend to open branches within navigable ranges of obesity clusters, thus exhibiting predatory behavior.

## Saturday, August 7 – Session 13 – Applied Mathematics

**Saturday, 10:00 am**

### Agent based models of brain network communication

**Presented by:** Aidan O'Keeffe, University of Texas at Austin
Jack Reever, University of Connecticut

**Co-Authored by:** Yan Hao, Hobart and William Smith Colleges
Daniel Graham, Hobart and William Smith Colleges

**Faculty Advisor:** Yan Hao, Hobart and William Smith Colleges

Network neuroscientists have made strides using topology and graph theory to uncover the brain's elusive wiring diagram, dubbed the connectome. Yet the brain's information routing mechanisms remain, for the most part, poorly understood. Do messages meander from node to node, or are they dispersed at every possible junction? Nonlinear message interaction is especially mysterious, and important too, for what should happen when messages collide? And what topological motifs do brain networks possess to facilitate efficient and sparse yet reliable function? On high-precision, whole brain, whole cortex mammal connectomes (wiring diagrams), we run two agent-based routing protocols: (1) an unbiased random walk and (2) information spreading, or message flooding. Simulation results show that different routing designs have advantages and disadvantages relevant to whole-brain communication, and they suggest an important role for redundant signaling.

This work is supported by the REU program at Hobart and William Smith Colleges under NSF grant no. DMS 1757616.

**Saturday, 10:20 am**

## 3D Eulerian Periodic Motion Amplification

**Presented by:** Sam Kirk, Illinois State University
Seraiah Kutai, Ursinus College
Kristen Mosley, Spelman College

**Co-Authored by:** Tom Boccuto, Ursinus College
Chris Tralie, Ursinus College

**Faculty Advisor:** Christopher Tralie, Ursinus College

Ordinary videos capture a surprising amount of hidden, visually imperceptible information. For instance, videos of peoples' faces may capture color changes in the skin and artery motion from heartbeats, while videos of mechanical systems can capture subtle vibrations indicating imminent failure. Algorithms can extract and exaggerate these signals for visualization on top of the original videos. In particular, Eulerian magnification algorithms sidestep the need to track hidden motions directly and instead devise multiscale bandpass filters to amplify signals in local spatial regions. In this work, we extend these techniques beyond color videos to geometric video data captured by 3D depth cameras such as the Microsoft Kinect. In our framework, we can spatially amplify a "bulging of the neck" during a heartbeat or the expansion of a chest/abdomen during a breath. We then exaggerate and display these signals as evolving 3D shapes. We explore pipelines based both on implicit and parameterized surface representations, and we discuss the merits, drawbacks, and challenges of both representations compared to ordinary color videos.

This work is sponsored by an NSF REU at Ursinus College.

**Saturday, 10:40 am**

## Using Drone Imagery to Classify Land Cover on Coastal Sand Dunes

**Presented by:** Heleyna Tucker, Hope College
Ted Lockett, Hope College

**Faculty Advisor:** Darin Stephenson, Hope College

The Hope College Dune Group has been studying West Michigan sand dunes for over twenty years. The group's interests include observing the mechanisms and effects of sand transport as well as learning how sand movement and resident dune vegetation affect one another. One of the fundamental tasks of this group is to use machine learning algorithms to create accurate ground-surface and vegetation models from drone imagery in an automated way. A key step in this process is the identification of various types of surface coverage—such as sand, live grass, trees, and other vegetation—automatically from images. In this talk, we will report on our work on land type classification using a variety of image classification techniques. The scale of our images ranges from high-resolution photos taken with digital cameras to orthomosaics of entire dune complexes taken remotely from a height of around 120 meters. We produce detailed classifications of high-resolution, low-altitude images and use this as a template for creating similar classifications from high-altitude, lower-resolution drone imagery. An eventual goal of this work is automatic land cover classification at the complex-wide scale.

# Saturday, August 7 – Session 14 – Algebra

### Saturday, 10:00 am

## The Picard Group of A General Toric Variety in Higher Dimensions

**Presented by:** Xiaorun Wu, Princeton University

**Faculty Advisor:** Chenyang Xu, Princeton University

Let $k$ be a field. Let $N = \mathbb{Z}^r$, and we denote the $k$-torus on $N$ as $T_N$. Next, define a fan $\Delta$ to be a finite set of strongly convex polyhedral cones in $N \otimes R \cong \mathbb{R}^r$ such that for all $s$, $t$ in $\Delta$, every face of $s$ is in $\Delta$ and $s \cap t$ is a face of both $s$ and $t$. For each rational $\Delta$, we may associate a toric variety $T_N(\Delta)$, which we denote as $X$.

In this article, we studied how the combinatory properties of $\Delta$ relates to Pic($X$). Extending the results of Ford and Stimets in 2002, we study the case in when $\Delta$ is a complete fan in $\mathbb{R}^n$ such that every three-dimensional cone in $\Delta$ is non-simplicial for some $n \geq 4$.

### Saturday, 10:20 am

## Recursions, q-series and intertwining operators

**Presented by:** Zachary Couvillion, Dartmouth College
John Lin, Binghamton University
Patrick McCourt, Kent State University

**Co-Authored by:** Christopher Sadowski, Ursinus College

**Faculty Advisor:** Christopher Sadowski, Ursinus College

The notion of principal subspace of an irreducible highest weight module for an affine Lie algebra was introduced by Feigin and Stoyanovsky and later studied by many other authors. Of particular interest is the graded dimension of the principal subspace, which is written as a $q$-series and in some cases tied to integer partition identities. In our work, we examine a certain filtration of subspaces of the irreducible highest weight modules from the work of Feigin and Stoyanovsky and construct exact sequences using all modes of an intertwining operator which generalize those found by Capparelli, Lepowsky, and Milas among principal subspaces of basic $\widehat{\mathrm{sl}(2)}$-modules. We obtain recursions satisfied by these subspaces and examine corresponding $q$-series one can obtain from these recursions.

### Saturday, 10:40 am

## Straightening Identities in the Onsager Algebra for $sl_3$

**Presented by:** Deanna Clayton, Park University

**Faculty Advisor:** Samuel Chamberlin, Park University

The original Onsager algebra is a Lie algebra named after Nobel Laureate Lars Onsager. It has numerous applications in physics. The original Onsager algebra can be built from the Lie algebra $sl_2$.

On the other hand, straightening identities are useful for studying the representation theory of Lie algebras in positive characteristic. All necessary straightening identities for the original Onsager algebra were formulated and proven recently.

In this talk we will discuss some straightening identities we have formulated and proved in a generalization of the Onsager algebra built using the same construction, but starting with the Lie algebra $sl_3$.

# Saturday, August 7 – Session 15 – Graph Theory

**Saturday, 10:00 am**

## Reconstructing Rooted Trees From Their Strict Order Quasisymmetric Functions

**Presented by:** Jeremy Zhou, Phillips Academy

**Faculty Advisor:** Yongyi Chen, MIT

Determining whether two graphs are isomorphic is an important and difficult problem in graph theory. One way to make progress towards this problem is by finding and studying graph invariants that distinguish large classes of graphs. Stanley conjectured that his chromatic symmetric function distinguishes all trees, which has remained unresolved. Recently, Hasebe and Tsujie introduced an analogue of Stanley's function for posets, called the strict order quasisymmetric function, and proved that it distinguishes all rooted trees. In this paper, we devise a procedure to explicitly reconstruct a rooted tree from its strict order quasisymmetric function by sampling a finite number of terms. The procedure not only provides a combinatorial proof of the result of Hasebe and Tsujie, but also tracks down the representative terms of each rooted tree that distinguish it from other rooted trees. This new technique can be applied to other chromatic symmetric functions as well, opening new theoretical approaches and computational applications for other graph isomorphism problems.

**Saturday, 10:20 am**

## Algebraic and Combinatorial Properties of Down-Left Graphs

**Presented by:** Beth Anne Castellano, Lafayette College
Marcella Manivel, Carleton College

**Co-Authored by:** Jennifer Biermann, Hobart and William Smith Colleges

**Faculty Advisor:** Jennifer Biermann, Hobart and William Smith Colleges

Given a graph, we can construct its edge ideal, which introduces a useful correspondence between monomial ideals of a polynomial ring and graphs. We investigate the independence complexes and vertex decomposability of a class of graphs called down-left graphs. Furthermore, we use induced matching numbers to bound the regularity of the edge ideals generated by these down-left graphs. This material is based upon work completed at the 2021 Hobart and William Smith Colleges Mathematics REU, supported by the National Science Foundation under grant no. DMS 1757616.

**Saturday, 10:40 am**

## Spinors and Graph Theory

**Presented by:** Beata Casiday, Yale University
Thomas Meyer, Amherst College
Sabrina Mi, University of Chicago
Ethan Spingarn, Amherst College

**Faculty Advisor:** Ivan Contreras, Amherst College

The evolution of quantum mechanical systems can be described by the Schrödinger equation, and the Dirac equation for spinors (particles with spin). In this talk, we study the behavior of spinors on a graph, as predicted by the Dirac equations, using analytic and algebraic methods. Analytically, we investigate the evolution of quantum states using graph-theoretic versions of the Schrödinger and Dirac equations, and we study discrete models of the Laplacian and Dirac operators. Algebraically, we study spinors as linear representations of the spin group using graph Clifford algebras. We also give gluing results on the centers of graph Clifford algebras.

# Saturday, August 7 – Session 16 – Mathematics Education

**Saturday, 1:00 pm**

## Modeling Dance with Mathematics

**Presented by:** Nicolette Colona, Towson University

**Co-Authored by:** Melike Kara, Towson University
Kimberly Corum, Towson University

**Faculty Advisor:** Melike Kara, Towson University

Results from a research study that focused on students' reasoning on a mathematical modeling task will be shared. In the first phase of the study, a modeling task was designed to have students analyze different dance moves using geometric concepts and generalize their findings to develop a model that could be used for future dance analysis. Next, the task was implemented in two different virtual settings: one was in a university class with 40 students who are prospective elementary and secondary teachers and one was in a high school class with 6 students. The data consisted of students' written notes on their handouts and their work on the Google Jamboard slides. The data were analyzed using constant comparative analysis and the common themes were determined. The common themes in students' analysis across both implementations included: angles formed in the dance moves and shapes formed in the dance moves. These themes also occurred as the most common themes in students' models that were created to be used as analysis guide for future dance moves. All students were able to notice the mathematics of a real-life situation by applying their current knowledge of geometric concepts.

**Saturday, 1:20 pm**

## Investigating Number Sense in a Math II Course

**Presented by:** Maia Tice, Elon University

**Faculty Advisor:** Aaron Trocki, Elon University

The goal of this research study was to analyze and investigate students' flexibility in understanding numbers, the structure of numbers, mental mathematics, and the ability to relate quantities to real-world situations, which is called number sense. Thirty-two high school Math II students completed a fourth-grade number sense assessment at the beginning and at the halfway point of the school year. Data were analyzed to assess if number sense knowledge and student reasoning improved over the course of half a school year. Quantitative findings showed that students' number sense did not significantly increase as they progressed in their Math II class and analysis of student work revealed a lack of number sense in some areas. Currently, we are conducting research with fifth-grade students with the same number sense assessment for comparison purposes. These results will inform the larger goal of reframing mathematics education to promote number sense comprehension and maintenance.

**Saturday, 1:40 pm**

## Examining Pre-Service Teachers' Conceptions of Area

**Presented by:** Brianna Davis, Sacramento State University
Enrique Villela, Sacramento State University

**Co-Authored by:** Sayonita Ghosh Hajra, Sacramento State University

**Faculty Advisor:** Sayonita Ghosh Hajra, Sacramento State University

Area is the amount of two-dimensional space within a 2D figure and is measured by counting the number of squares that can cover the shape without gaps or overlaps. Area is an important visual tool which supports learning in dif-

ferent fields. However, studies over the last four decades have shown that students of all ages in the U.S. performed poorly on area measurement assessments, and lacked a conceptual understanding of area. Prospective teachers (PTs) face similar challenges, due to shared scholastic experiences. What knowledge PTs bring to their classes and how PTs (re)learn mathematical concepts is important to understand in order to prepare PTs to teach the area concept. We interviewed five PTs on three area related tasks that investigated PTs' understanding of area measurement. Two tasks concerned finding the area of polygonal regions using square and non-square units of measure; the third task asked the PTs to interpret a fictional child's thinking about the procedure of finding the area of a rectangle. In this presentation, we will discuss different levels of sophistication of PTs' understanding of area, recurring difficulties they have, and implications of this work.

**Saturday, 2:00 pm**

## 4D Hypercube Perspective Modeling Using 3D Printing for Educational Purposes

**Presented by:** Mackenzie Ray, Augsburg University

**Faculty Advisor:** Matthew Haines, Augsburg University

The four-dimensional (4D) hypercube has been a topic of exploration for decades. Various figures of 4D hypercubes have been created with computer programing and 3D printing, many of which use rotated projections of the hypercube into 3D space. In this project, we are creating a moveable 3D printed model of the 4D hypercube with the end goal of making the 4D concepts accessible to a broader range of students by implementing research-based modeling improvements. Adaptations to past figures of 4D hypercubes will be made with intentions to have the new model be moveable, representing rotations around planes and highlighting the perspective viewpoint that correlates to the angle of which the 4D space is viewed in 3D. Also, psychological concepts of perception and learning are utilized to create a model that supports higher levels of cognition. Final objectives include the designing and building of a 3D printed hypercube model that can be manipulated and guidelines for creating educationally beneficial visualizations.

Funded by: TRIO McNair Scholars at Augsburg University

**Saturday, 2:20 pm**

## Designing a IBL Curriculum on Renewable Energy for the Girls Talk Math Camps

**Presented by:** Victoria Mirecki, Worcester Polytechnic Institute (WPI)
Aunika Yasui, Worcester Polytechnic Institute (WPI)

**Co-Authored by:** Francesca Bernardi, Worcester Polytechnic Institute (WPI)

**Faculty Advisor:** Francesca Bernardi, Worcester Polytechnic Institute (WPI)

Girls Talk Math (GTM) is a free math and media camp hosted at the University of North Carolina at Chapel Hill, the University of Maryland College Park, and in 2022 at Worcester Polytechnic Institute. Its goal is to make advanced mathematics accessible to girls and non-binary high school students. We designed a curriculum for the GTM camps focused on renewable energy; the problem set is written within an inquiry based learning framework and will be publically available on GitHub. Campers learn about renewable vs nonrenewable energy options and make quantitative predictions of their need, cost, and effects based on real-world data. Students learn how to employ dimensional analysis to understand the relationship between units, quantities, and the derivation of physical laws governing solar and windmill energy power. Finally, campers build their own windmill using cardboard and littleBits circuits. Windmills are programmed to freely rotate and use the energy produced to power an LED light. By integrating mathematics and engineering concepts with real-world applications, students experience first-hand the versatility of mathematics and the world of possibilities available in STEM.

# Saturday, August 7 – Session 17 – Number Theory

### Saturday, 2:40 pm

## Monodromy of Compositions of Toroidal Belyi Maps

**Presented by:** Edmond Anderson, Morehouse College
Aurora Hiveley, Macalaster College
Cyna Nguyen, Cal State Long Beach
Daniel Tedeschi, Grinnell College

**Faculty Advisor:** Rachel Davis, University of Wisconsin at Madison

Say that $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a Dynamical Belyi map. Given any Toroidal Belyi map $\varphi : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$, the composition $\beta \circ \varphi$ i: $E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is also a Toroidal Belyi map. There is a group Mon($\beta$), the monodromy group, which contains information about the symmetries of a Belyi map $\beta$. It is well-known that, for any Toroidal Belyi map $\varphi$, (1) there is always a surjective group homomorphism Mon($\beta \circ \varphi$) $\twoheadrightarrow$ Mon($\beta$), and (2) the monodromy group Mon($\beta \circ \varphi$) is contained in the wreath product Mon($\varphi$) $\wr$ Mon($\beta$). In this project, we study how the three groups Mon($\beta$) and Mon($\beta \circ \varphi$) and Mon($\varphi$) $\wr$ Mon($\beta$) compare as we vary over Dynamical Belyi maps $\beta$. This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Security Agency (H98230-21-1-0015).

### Saturday, 3:00 pm

## Critical Points of Toroidal Belyi Maps

**Presented by:** Tesfa Asmara, Pomona College
Maria Maalouf, Cal State Long Beach
Isaac Robinson, Harvard University
Sharon Spaulding, University of Connecticut

**Faculty Advisor:** Edray Goins, Pomona College

There are many examples of Belyi maps $\beta : S \to \mathbb{P}^1(\mathbb{C})$ associated to elliptic curves $S = E(\mathbb{C})$; several can be found online at LMFDB. Given such a Toroidal Belyi map of degree $N$, we have the divisors $\mathrm{div}(\beta) = \sum_{P\in B} e_P\,(P) - \sum_{P\in F} e_P(P)$ and $\mathrm{div}(\beta - 1) = \sum_{P\in W} e_P\,(P) - \sum_{P\in F} e_P(P)$ where $B = \beta^{-1}(\{0\})$, $W = \beta^{-1}(\{1\})$, and $F = \beta^{-1}(\{\infty\})$. There exists a point $P_0 \in E(\mathbb{C})$ such that $[-N]\,P_0 = \bigoplus_{P\in B} [e_P]\,P = \bigoplus_{P\in W} [e_P]\,P = \bigoplus_{P\in F} [e_P]$ P. Denote $G = \{\, P \oplus P_0 \mid P \in B \cup W \cup F\}$ as a translation of the critical points of the Belyi map. In this project, we investigated when $G$ is a subgroup of $S = E(\mathbb{C})$. This work was done at Pomona College as part of PRiME (Pomona Research in Mathematics Experience); it was funded by the National Security Agency (H98230-21-1-0015).

### Saturday, 3:20 pm

## Elliptic curves with non-trivial isogeny

**Presented by:** Alyssa Brasse, Hunter College
Nevin Etter, Washington and Lee
Gustavo Flores, Carleton College
Andrew Miller, University of California Santa Barbara
Summer Soller, University of Utah

**Faculty Advisor:** Alexander Barrios, Carleton College

For a positive integer $N$, we say that an elliptic curve $E$ exhibits a degree $N$ isogeny if $E$ has a cyclic subgroup of order $N$. Such elliptic curves are parameterizable, and by studying the explicit equations corresponding to the modular curve $X_0(N)$, we prove results about the minimal discriminants of elliptic curves with a non-trivial isogeny over the rational numbers.

**Saturday, 3:40 pm**

### Density Of Elliptic Dedekind Sums Over Imaginary Quadratic Integer Rings

**Presented by:** Nicolas Berkopec, University of New Mexico
Jacob Branch, Fairmont State University
Caroline Nunn, University of Maryland
Rachel Heikkinen, Augustana College

**Co-Authored by:** Tian An Wong, University of Michigan Dearborn

**Faculty Advisor:** Tian An Wong, University of Michigan Dearborn

The classical Dedekind sum is a deeply studied number theoretic object with known applications to knot theory, modular forms, and algorithmic complexity. Elliptic Dedekind sums were introduced by Robert Sczech as a generalization of classical Dedekind sums to functions over lattices. Verifying density of these functions in the real line is an intriguing challenge. Recently Hiroshi Ito proved that normalized elliptic Dedekind sums defined on Euclidean imaginary quadratic rings are dense in the reals. We show that this density result holds for elliptic Dedekind sums defined over any imaginary quadratic field, thereby expanding the known set for which density holds from a finite collection to a countably infinite one. Our proof also generalizes methods from a recent paper by Winfried Kohnen which gave another proof of density for classical Dedekind sums. This research was funded by and conducted at the University of Michigan-Dearborn REU in Mathematical Analysis and Applications.

## Saturday, August 7 – Session 18 – Number Theory

**Saturday, 1:00 pm**

### Domains of Convergence for Polyhedral Circle Packings

**Presented by:** Nooria Ahmed, Swarthmore College
Jake Viscusi, Swarthmore College
Emilie Rivkin, Swarthmore College

**Co-Authored by:** Dylan Torrance, Swarthmore College
William Ball, Swarthmore College
Ellis Buckminster, Swarthmore College
Runze Wang, Swarthmore College
Gary Yang, Swarthmore College
Ian Whitehead, Swarthmore College

**Faculty Advisor:** Ian Whitehead, Swarthmore College

For any three dimensional polyhedron, one can construct a circle packing by turning the vertices into circles with tangency relations based on the edges of the polyhedron. One way of exploring these packings is through considering the domain of convergence of a generating function which is a sum over circle configurations within the packing. In calculating the domains of convergence for these polyhedral packings we are able to render a beautiful image of these n-dimensional regions. These regions turn out to have a similar geometric structure to the underlying polyedron for the packing. These are new examples of visualizations of Tits cones for indefinite Kac-Moody root systems.

### Saturday, 1:20 pm

## Fractal Dimensions of Polyhedral Packings

**Presented by:** Dylan Torrance, Swarthmore College
William Ball, Swarthmore College

**Co-Authored by:** Emmi Rivkin, Swarthmore College
Jake Viscusi, Swarthmore College
Nooria Ahmed, Swarthmore College
Ian Whitehead, Swarthmore College

**Faculty Advisor:** Ian Whitehead, Swarthmore College

Polyhedral circle packings are a class of fractals that generalize the well-known Apollonian packing. For any polyhedron, one can construct a collection of circles corresponding to faces with tangencies corresponding to edges. The limit set of the reflection group generated by these circles is the polyhedral packing. While the Hausdorff dimension of the Apollonian packing has been computed accurately, the dimensions of other polyhedral packings have not been studied systematically before. We implement three algorithms to compute these fractal dimensions and compare their accuracy. Our work yields new fractal dimension estimates for many interesting examples, including infinite families.

### Saturday, 1:40 pm

## Study of Legendre's Theorem

**Presented by:** Bruce McOsker, Texas A&M University, San Antonio

**Faculty Advisor:** Jingbo Liu, Texas A&M University, San Antonio

As part of our research into the representation theory of quadratic forms we take a look at the famous Legendre's Theorem in number theory regarding ternary quadratic forms. Namely that the Diophantine equation $ax^2 + by^2 + cz^2 = 0$ with $a$, $b$ and $c$ are nonzero, square-free and pairwise coprime integers, has non-trivial solutions if and only if the following criteria are met: $a$, $b$ and $c$ do not all have the same sign. $-bc$, $-ac$ and $-ab$ are quadratic residues of $a$, $b$ and $c$ respectively. In our talk we attempt an alternate proof of this theorem using Local-Global Principle, Hasse invariant and Jacobi symbol.

### Saturday, 2:00 pm

## p-adic Valuation Trees of Quadratic Polynomials

**Presented by:** Emily Hammett, Rowan University
Stephen Hu, Rutgers University
Rachel Snyder, Western Washington University

**Faculty Advisor:** Olena Kozhushkina, Ursinus College

For prime $p$, the $p$-adic valuation $\nu_p(n)$ of a nonzero integer $n$ is the highest power of $p$ that divides $n$. In this talk, we examine the behavior of the $p$-adic valuations of quadratic polynomials with integer coefficients for certain values of prime $p$. We explore this behavior through tree representations of these sequences. For instance, the 2-adic valuation tree for $f(n) = n^2+1$ is finite, while the 2-adic valuation tree for $g(n) = n^2 + 2n$ has two infinite branches. We consider some properties associated with $p$-adic valuation trees such as finiteness, symmetry, and number of levels, along with their relationship to the corresponding sequences. This work was part of the REU at Ursinus College in Summer 2021.

# Saturday, August 7 – Session 19 – Algebra

### Saturday, 2:20 pm

## The Hilbert Series of O(2)

**Presented by:** Lawton Walker, Rhodes College

**Faculty Advisor:** Christopher Seaton, Rhodes College

Let O(2) be the $2 \times 2$ orthogonal group, the group of reflections and rotations. Let $\tau$a denote the irreducible representation of O(2) induced from the circle representation with weight $a$. Define $V = \tau a_1 + \tau a_2 + \cdots + \tau a_n$ to be an arbitrary representation of O(2). When the $a_i$'s are distinct we say our representation is generic and degenerate otherwise. The Hilbert Series of the invariants and covariants of V counts the respective invariants and covariants of each degree after we expand our Hilbert Series at $t = 0$.

We use the Molien-Weyl theorem for an arbitrary representation V , to solve two complex integrals, one for the reflection and one for the rotations. The resulting Hilbert Series after we combine the solutions can be interpreted in terms of shifting a power series. Because of this, we implement a Mathematica algorithm that performs a partial fraction decomposition on our integral along with the power series interpretation to yield the Hilbert series for any given weight vector. Finally, we compute a similar integral, that involves the character of our representation, to determine the Hilbert Series of the covariants of O(2).

### Saturday, 2:40 pm

## On Sums of Polynomial-type Exceptional Units in $\mathbb{Z}/n\mathbb{Z}$

**Presented by:** Anand, Disha Delphi Public School

**Faculty Advisor:** Professor Ravindranathan Thangadurai, Harish Chandra Research Institute, Prayagraj, India

A unit $u$ in a commutative ring with unity $R$ is called *exceptional* if $u - 1$ is also a unit. In this article, we introduce the polynomial notion of exceptional units. For a given polynomial $f \in \mathbb{Z}[X]$, and a commutative ring $\mathbb{Z}/n\mathbb{Z}$ for some integer $n \geq 2$, we define an element $a \in \mathbb{Z}/n\mathbb{Z}$ which is called a *f -exunit* if $\gcd(x, f(a)) = 1$. We study the representation problem as follows:

For a given element $c \in \mathbb{Z}/n\mathbb{Z}$, how many elements $x$ and $y$ that are *f-exunits* such that $x + y = c$? We answer this question for any non-constant polynomial $f \in \mathbb{Z}[X]$. This also generalizes a result by Sander. More precisely, we show that number of such representations are

$$n\prod_{p|n}\left(1-\frac{N^f(p,c)}{p}\right)$$

Where $N^f(p, c) = \#\{l \pmod p \mid \text{either } f(l) \equiv 0 \pmod p \text{ or } f(c-l) \equiv 0 \pmod p\}$.

### Saturday, 3:00 pm

## Real Powers of Monomial Ideals

**Presented by:** Ethan Partida, University of Minnesota Twin Cities

**Co-Authored by:** Josiah Lim, Brown University
Ethan Roy, The University of Texas at Austin
Dylan Ruff, University of Toronto

**Faculty Advisor:** Alexandra Seceleanu, University of Nebraska-Lincoln

This talk will give a generalization of the exponentiation of monomial ideals researched during the Polymath 2020 REU. The typical operation only considers ideals to a natural power, we extend this to powers of positive real numbers. We do this by generalizing a connection between ideals and convex polytopes. Using the tools of convex geometry, we provide an algorithm to compute the real powers of monomial ideals. We also show that the exponentiation operation is a step function and explore what its jumping points are.

### Saturday, 3:20 pm

## An Introduction to Point Modules

**Presented by:** Maximilian Kaipel, University of Edinburgh

**Faculty Advisor:** Susan Sierra, University of Edinburgh

In the late 1980s Artin and Schelter discovered algebras which were too difficult to understand with the algebraic tools they had at the time. Properties that are simple for some algebras proved to be impossible to determine. Eventually, Artin, Tate and Van den Bergh developed an entirely new technique of connecting abstract algebra with projective geometry: Point modules.

Point modules allowed them to use the tools of algebraic geometry they knew to analyse the underlying structure of these algebras. This lead to the complete classification of a massive group of algebras. Unfortunately, while the point modules of some algebras are very neat, the ones of others can be extremely unpleasant and almost fractal-like.

In this presentation, we will give an introduction of the Artin-Tate-Van den Bergh technique of point modules and describe how to compute them based on simple examples. We will focus on the point modules of Sklyanin algebras, the class of algebras which originally proved such a great challenge. In particular we consider the ill-behaved degenerate Sklyanin algebras and count the truncated point modules of length n for all positive integers n.

### Saturday, 3:40 pm

## Fixing Sets of Finite Groups

**Presented by:** Ummul Aymen, Towson University
James Della-Giustina, Towson University
Jason Riley Jr., Towson University
Erin Stales, Towson University

**Co-Authored by:** Lindsey-Kay Lauderdale, Towson University

Jay Zimmerman, Towson University

**Faculty Advisor:** Lindsey-Kay Lauderdale, Towson University

We will examine a graph invariant called the fixing number of a graph which is the smallest number of vertices that when fixed, destroy all non-trivial automorphisms of the graph. This concept will then be extended to what is known as the fixing set of a finite group G that is, the set of all fixing numbers of graphs with an automorphism group isomorphic to G. In our talk, we will introduce new results on the fixing sets of the symmetric, alternating, and quasi-dihedral groups which were found as part of an REU project at Towson University.

# Saturday, August 7 – Session 20 – Biomathematics

**Saturday, 1:00 pm**

## Determining the Reproduction Number, R0 for COVID-19

**Presented by:** Kentale Morris, Fisk University
Elijah Asemota, Fisk University

**Co-Authored by:** Ifrah Khurram, Fisk University
Sanjukta Hota, Fisk University

**Faculty Advisor:** Sanjukta Hota, Math Department/Fisk University

Coronavirus Disease of 2019 (COVID-19) is a disease caused by the novel coronavirus SARS-CoV-2. It started in Wuhan, China in late 2019 and has since spread worldwide. The origin and the properties of SARS-CoV2 is still an open question. It is a new strain of virus that has not been previously identified in humans. These viruses belong to a large family of viruses that are known to cause illness ranging from the common cold to more severe diseases such as Severe Acute Respiratory syndrome (SARS) and Middle East Respiratory Syndrome (MERS). This project presents an ODE model of COVID-19 dynamics incorporating the impact of Social Distancing, Quarantine and Asymptomatic cases and derives an expression for the reproduction number R0, a measure of how many secondary infections can be caused by a single infected person in a completely susceptible population. The model was validated against the COVID data, March-June 2020 of New York. The model simulations were performed through Numpy Python.

Fisk NSF-HBCU UP TIP 2020 Summer Research Internship Work

**Saturday, 1:20 pm**

## A Mathematical Model of COVID-19: Efficacy of Vaccination with Heterogeneous Populations

**Presented by:** Brandon Payne, Dixie State University

**Co-Authored by:** Cesar Vasquez, Dixie State University

**Faculty Advisor:** Vinodh Chellamuthu, Dixie State University

The novel coronavirus disease 2019 (COVID-19) remain superfluous as the disease continues to spread profusely across the world. Currently, our goal is to explore scenarios for different levels of vaccine-effectiveness and varying proportions of vaccinated-populations in order to demonstrate the effectiveness of a vaccine in mitigating the spread of COVID-19. We developed a mathematical model to analyze the disease dynamics in relation to vaccine-effectiveness. We performed a data fitting algorithm to estimate parameters within the model to resemble current infection trends using data from the Center for Disease Control and Prevention (CDC). Incorporated a vaccinated population and demonstrate the mitigation of COVID-19 with the introduction of a vaccine. Our simulation results determine possible best-case scenarios at varying degrees of vaccine-effectiveness and proportions of vaccinations. To account for the disease's varying infection and mortality rates, we partition the population by age groups to determine which age groups are most vital to vaccinate. Our simulation also identifies the minimal required vaccine-efficacy for a given proportion of vaccinated individuals.

**Saturday, 1:40 pm**

## Age-Structured Models for COVID-19 Outbreaks and Public Health Interventions

**Presented by:** Averly Sheltraw, Oberlin College
Luz Melo, SUNY Geneseo
Ana Abreu, Bronx Community College
Caleb Mahlen, Regis University

**Co-Authored by:** Jonathan Forde, Hobart and William Smith Colleges

**Faculty Advisor:** Jonathan Forde, Hobart and William Smith Colleges

SARS-CoV-2 emerged in late 2019 and quickly reached pandemic scale, leading to over 175 million confirmed cases worldwide and 33 million in the United States as of June 2021. To control the spread of the disease, governments implemented public health measures such as pods, testing, quarantining, social distancing, and vaccination. However, the efficacy of public health strategies has varied widely. To explore the utility of these interventions, we develop age-structured models of the impact of selected public health measures on COVID-19 outbreaks. We investigate the behavior of these models and their predictions using sensitivity analysis and numerical simulations.

This material is based upon work supported by the National Science Foundation under grant no. DMS 1757616.

**Saturday, 2:00 pm**

## Assessing the Efficiency of Predator-Prey Control Strategies in the Persistence of Dengue with Wolbachia Transinfection

**Presented by:** Rosa Flores, Dixie State University
Christian Riordan, Dixie State University

**Faculty Advisor:** Vinodh Kumar Chellamuthu, Dixie State University

Dengue is a mosquito-borne viral infection found in tropical/subtropical regions causing millions of infections each year. Dengue is caused by any of the four serotypes of the dengue virus. Several studies have shown that secondary infections have increased morbidity rates. Dengue is mainly transmitted by the Aedes aegypti mosquitoes (AEM). The transinfection of AEM with the bacterium Wolbachia pipientis has emerged as a "successful" method to reduce dengue virus transmission while minimizing environmental harm. Despite this control method in place the persistence of dengue virus is continuing to infect individuals worldwide. Studies indicate that incorporating a natural predator of the AE larvae such as the Toxorhynchites Splendens (TxS) larvae can help control the population of the AEM and aid in the control of Dengue. We have developed a mathematical model by incorporating both primary and secondary infection of two serotypes of the virus and utilizing two control strategies: Wolbachia-carrying mosquitoes and the Introduction of TxS larvae. Furthermore, our simulations show intervention strategies that might be useful in controlling Dengue based on the temperature dependency.

**Saturday, 2:20 pm**

## Modeling the Impact of Wolbachia's Transinfection and its Effectiveness on Mitigating the Spread of Dengue

**Presented by:** Austin Harper, Dixie State University

**Faculty Advisor:** Vinodh Chellamuthu, Dixie State University

Dengue fever is one of the world's most common viral-borne diseases, its spread is most prominent in tropical and subtropical climates. The infection is caused by bites from mosquitoes carrying the disease and a secondary infection

can lead to severe dengue fever. The cycle of transmission can be broken by using Wolbachia, a bacterium shown to reduce levels of dengue virus in the mosquito, shorten the host mosquito's lifespan, and cause reproductive abnormalities in the host. We develop a mathematical model using a system of nonlinear ordinary differential equations that incorporate Wolbachia transinfection to study the epidemiologic trends of two viral dengue serotypes. Moreover, our model considers temporary cross-immunity within the two serotypes and the impact of temperature dependency on mosquitoes' vital rates. Our simulation results suggest the introduction of Wolbachia carrying mosquitoes at optimal times can decrease the number of dengue transmitting vectors and, consequently, reduce the frequency and prevalence of outbreaks.

## Saturday, August 7 – Session 21 – Combinatorics

**Saturday, 2:40 pm**

### Counting Divisions of a 2 × *n* Rectangular Grid

**Presented by:** Jacob Brown, Ithaca College

**Faculty Advisor:** Emilie Wiesner, Ithaca College

Consider a $2 \times n$ rectangular grid composed of $1 \times 1$ squares. Cutting only along the edges between squares, how many ways are there to divide the board into $k$ pieces? Building off the work of Durham and Richmond, who found the closed-form solutions for the number of divisions into 2 and 3 pieces, we prove a recursive relationship that counts the number of divisions of the board into $k$ pieces. Using this recursion, we obtain closed-form solutions for the number of divisions for $k=4$ and $k=5$ using fitting techniques on data generated from the recursion. Furthermore, we show that the closed-form solution for any fixed $k$ must be a polynomial on $n$ with degree $2k-2$.

**Saturday, 3:00 pm**

### Time to steady state for box-ball systems using RSK

**Presented by:** Marisa Cofie, University of Maryland
David Zeng, Yale University

**Co-Authored by:** Olivia Fugikawa, Yale University
Madelyn Stewart, Yale University

**Faculty Advisor:** Emily Gunawan, University of Oklahoma

The Robinson–Schensted–Knuth (RSK) insertion algorithm is a bijection between the symmetric group and pairs (P,Q) of standard Young tableaux (arrays of positive integers with increasing rows and columns) of the same shape. The tableau Q is called the recording tableau of the corresponding permutation.

A box-ball system is a discrete dynamical system which can be thought of as a collection of time states. At each state, we have a collection of boxes with each integer from 1 to n assigned to a unique box. A box-ball system will reach a steady state after a finite number of steps. From any steady state, we can construct a tableau (not necessarily standard) called the soliton decomposition of the box-ball system. The minimum number of time steps needed to go from a permutation w to a steady state in a box-ball system is called the steady-state time of w.

We use the pair (P,Q) of a permutation w to study the box-ball system containing w. We prove the that the steady-state time of a permutation is determined by its recording tableau Q. For a family of tableaux, we give a formula for computing its corresponding steady-state time.

This work is part of SUMRY 2021 at Yale University.

### Saturday, 3:20 pm

## The (i,j) in Team: Optimizing Project Partners

**Presented by:** Ronan Kudzin, Hamilton College

**Faculty Advisor:** Sally Cockburn, Hamilton College

A class of 29 students had to be partitioned into groups of 2 or 3 for final oral presentations. Each student was asked to name four students with whom they would be willing to work. Ideally, each student would be in a group with at least one student they named and one student who named them in return. This can be modeled as an integer program whose objective is to maximize the number of mutually chosen teammates.

However, the number $N$ of potential teammates was chosen arbitrarily to be 4, and this begs the question: how many potential partners should each student choose to ensure that a feasible solution exists? To answer this, we again use integer programming to generate a sample of reasonable choice digraphs with various values of $N$ and determine the proportion with feasible solutions.

### Saturday, 3:40 pm

## Markov chain on edge-colorings of bipartite graphs

**Presented by:** Letong (Carina) Hong, Massachusetts Institute of Technology

**Co-Authored by:** Istvan Miklos, Alfred Renyi Institute of Mathematics, Institute for Computer Science and Control (SZTAKI), Eotvos Lorand Research Network

**Faculty Advisor:** Istvan Miklos, Alfred Renyi Institute of Mathematics

The Latin squares $L_n$ are $n \times n$ grids such that each row and column consist of numbers 1 to $n$. It is a family of important combinatorial objects but with no known easily computable formula for its quantity. A Latin rectangle is an $m \times n$ ($m < n$) grid with each row consisting 1 to $n$ and each column has no repeat. One may see that there is a natural bijection between all possible Latin square completions of Latin rectangles and the proper edge colorings of a regular equi-bipartite graph.

Counting and sampling are related problems. Motivated by the above, we exhibit an irreducible Markov chain $M$ on the edge $k$-colorings of bipartite graphs based on certain properties of the solution space. We show that diameter of this Markov chain grows linearly with the number of edges in the graph. We also prove a polynomial upper bound on the inverse of acceptance ratio of the Metropolis-Hastings algorithm when the algorithm is applied on $M$ with the uniform distribution of all possible edge $k$-colorings of $G$.


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