

Paul R. Halmos - Lester R. Ford Awards

Daniel Ullman and Daniel Velleman

“Differences of bijections,” *The American Mathematical Monthly*, 126(3), 199–216.
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The sequence 1, 2, 3 can be written as a difference of permutations of the integers modulo 3: for example, 3, 2, 1 and 2, 3, 1 are permutations of the integers modulo 3, and their difference, taken entry-wise and modulo 3, is 1, 2, 3. But the sequence 1, 2, 3, 4 can't be written as a difference of permutations of the integers modulo 4. The article explains why this is and what it has to do with juggling. More generally, which functions from an abelian group to itself can be written as the difference of two bijections? This is pursued in increasing generality, for finite and then infinite groups, and then with injections and surjections instead of bijections, and finally beyond the realm of group theory, leading to transversals in Latin squares.

Response

Our collaboration on this project began with a suggestion from Dan Kalman that we find a topic on which we could write a “three Dan” paper. Unfortunately, Dr. Kalman didn't have time to work on the project, but we thank him for discussions leading to our “two Dan” paper. We were led to our topic by a question posed by Louis Funar in Richard Guy's “Unsolved Problems” column in the *Monthly* in 1986: Given an arbitrary function f from the reals to the reals, do there exist functions g and h , the first one bijective and the second one injective, such that $f = g + h$? Our research eventually led us to discover that the problem was actually not unsolved; the solution could be found in a 1958 paper (in German) by László Fuchs, building on work of Marshall Hall, Jr. from 1952. We enjoyed discovering and extending this literature, and we are very pleased and honored that the MAA has chosen to recognize our work with this award.

Biographical Sketch

Daniel H. Ullman received his AB degree from Harvard University in 1979 and his PhD from the University of California, Berkeley in 1985. Since then he has been a member of the faculty at George Washington University. He served as Department Chair from 2001 to 2006 and as Associate Dean for Undergraduate Studies from 2011 to 2015. He is an author of *Fractional Graph Theory* (with Ed Scheinerman) and *The Mathematics of Politics* (with Robbie Robinson). In 1991 and 1992, he was Deputy Leader (Cecil Rousseau was Leader) of the US delegation to the International Mathematical Olympiad. He served as the American Mathematical Society AAAS Science Policy Congressional Fellow from 2006 to 2007. He is presently the director of the William Lowell Putnam Mathematical Competition and he also serves as an editor of the Problems and Solutions column of the *Monthly*.

Daniel J. Velleman received his BA from Dartmouth College in 1976 and his PhD from the University of Wisconsin–Madison in 1980. He was an instructor at the University of Texas before joining the faculty of Amherst College, where he taught from 1983 to 2017. He has also taught at Erindale College, Middlebury College, St. Michael's College, and he is now an adjunct professor at the University of Vermont. He is the author of *How To Prove It, Which Way Did the Bicycle Go?* (with Joe Konhauser and Stan Wagon), *Philosophies of Mathematics* (with Alexander George), *Calculus: A Rigorous First Course*, and the forthcoming *Bicycle or Unicycle?* (with Stan Wagon). He was the editor of the *American Mathematical Monthly* from 2007 through 2011 and he currently serves as an editor of the Problems and Solutions column of the *Monthly*.