

## Paul R. Halmos - Lester R. Ford Awards

### Zhaodong Cai, Matthew Faust, A. J. Hildebrand, Junxian Li and Yuan Zhang

“The Surprising Accuracy of Benford’s Law in Mathematics,” *The American Mathematical Monthly*, 127:3, 217–237. 10.1080/00029890.2020.1690387

This inspiring article finds new mysteries in sequences as familiar as the powers of 2. The leading decimal digits of these powers are known to be distributed following Benford’s law, but numerical data from the first billion powers of 2 indicates that the predictions from Benford’s law can be surprisingly accurate—perhaps suspiciously so. Could the observed accuracy for these first billion powers be a mirage, perhaps one caused by Guy’s “strong law of small numbers”?

To explain these small errors, the authors go further and actually investigate their distributions. They give elementary explanations for some cases, then use the theory of Diophantine approximation to go further. The hardest cases are those in which the distribution appears to be normal. In these cases, even cutting edge theory falls short, leading to interesting and unexpected conjectures.

#### Response

We are thrilled, honored and humbled to receive this award. Our article is the outgrowth of a multi-year research adventure that started out as an undergraduate research project at the University of Illinois in spring 2016 and continued for the better part of the following two years—an adventure filled with unexpected twists and turns and the joys and frustrations of mathematical research and discovery. Much of our work was carried out in weekly meetings in historic Altgeld Hall, the home of the mathematics department at the University of Illinois and the very same building in which Paul Halmos had spent much of his formative years while he was a student here during the 1930s.

We thank the mathematics department of the University of Illinois for providing a wonderfully supportive environment for undergraduate research through its Illinois Geometry Lab and the University of Illinois for providing computational resources for our work through its Illinois Campus Cluster Program. We also thank the editor and referees of the *Monthly* whose comments and suggestions helped improve the presentation of our article.

#### Biographical Sketches

**Zhaodong Cai** began his college education at the University of Illinois at Urbana-Champaign, where he spent many happy winters thinking about mathematics. He graduated with a BS degree in 2017, receiving summa cum laude honors. He went on to become a PhD student at the University of Pennsylvania, where he is currently studying arithmetic geometry.

**Matthew Faust** received BS degrees in mathematics and computer engineering from the University of Illinois in 2018. Currently he is working towards a PhD in mathematics at Texas A&M University under the supervision of Dr. Frank Sottile. His current research interests are in combinatorial algebraic geometry and spectral theory. In his free time, he enjoys playing strategy games, preferably with Yuan Zhang.

**A.J. Hildebrand** received his PhD in mathematics from the University of Freiburg and has been on the faculty of the University of Illinois since 1986, becoming professor emeritus in 2012. His research interests are in the areas of number theory, probability and statistics, and combinatorics. Since retiring from the University of Illinois he has supervised over one hundred undergraduates on research projects in pure and applied mathematics and allied areas.

**Junxian Li** received her PhD from the University of Illinois in 2018. She then spent a year at the University of Göttingen and is currently at the Max Planck Institute for Mathematics as a postdoc. Her research interests lie in analytic number theory, additive combinatorics and automorphic forms. During her PhD studies, she enjoyed working with enthusiastic undergraduates at the Illinois Geometry Lab on various research projects.

**Yuan Zhang** began his college education at the University of Illinois as a major in natural resources and environment sciences. After his freshman year, he switched his major to mathematics, receiving his BS degree in 2018. He is currently pursuing a PhD in mathematics at the University of Virginia under the supervision of Dr. Peter Abramenko. His current research interests are in Tits buildings and are mostly focused on their algebra aspects. In his free time, he enjoys playing RPGs and strategy games, preferably with Matthew Faust.