Talk for Math Fest August 6 – 9 2025

They told me to open with a joke.

My dog and I went on a 17-mile overnight hike. He thought we were only going out to the canyon where he could run around, chase things and splash in the water.

They told me to write a problem using factorials. I was staring at a blank piece of paper trying to come up with an idea. Nothing came to mind. Finally, I said to myself, "Do something." So, I wrote 10! – (7!) ^2. Well, I did something, now let's see what that equals to. Factor out 7! We have 10*9*8-7! = 720-5040. Wait a minute, what is 6!? The rest is easy.

2024AMC 10B problem 2.

What is $10! - 7! \cdot 6!$

$\textbf{(A)}\ -120 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 600 \qquad \textbf{(E)}\ 720$

2024 10A Problem 11

How many ordered pairs of integers (m, n) satisfy $\sqrt{n^2 - 49} = m$?

$(\mathbf{A})\ 1 \qquad (\mathbf{B})\ 2 \qquad (\mathbf{C})\ 3 \qquad (\mathbf{D})\ 4 \qquad (\mathbf{E})\ \text{infinitely many}$

The key here is to know the difference between any two perfect squares is a series of consecutive positive odd integers. 49 is the 25th odd number so 25^2 – 49 = 24^2. Are there more? Well (+-7)^2 = 49, thus *m* = 0. So, the solutions are (0, 7), (0, -7), (24, 25), (24, -25). But there are no other series greater than 1 of consecutive odd numbers whose sum is 49.

2024 10A Problem 3

What is the sum of the digits of the smallest prime that can be written as a sum of 5 distinct primes?

$(\mathbf{A})\ 5 \qquad (\mathbf{B})\ 7 \qquad (\mathbf{C})\ 8 \qquad (\mathbf{D})\ 10 \qquad (\mathbf{E})\ 13$

(Get rid of the 13)

2012 10B Problem 7

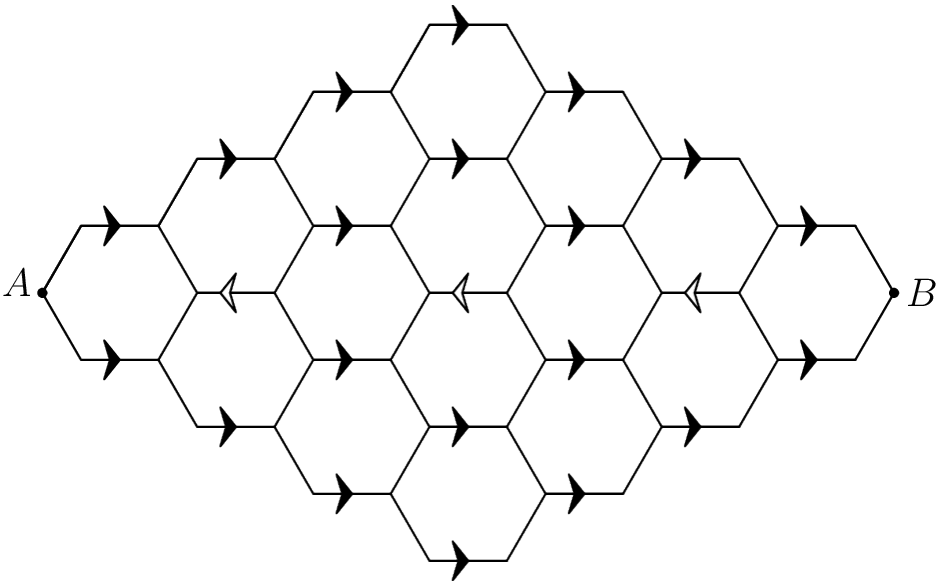
For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

$(\mathbf{A})\ 30 \qquad (\mathbf{B})\ 36 \qquad (\mathbf{C})\ 42 \qquad (\mathbf{D})\ 48 \qquad (\mathbf{E})\ 54$

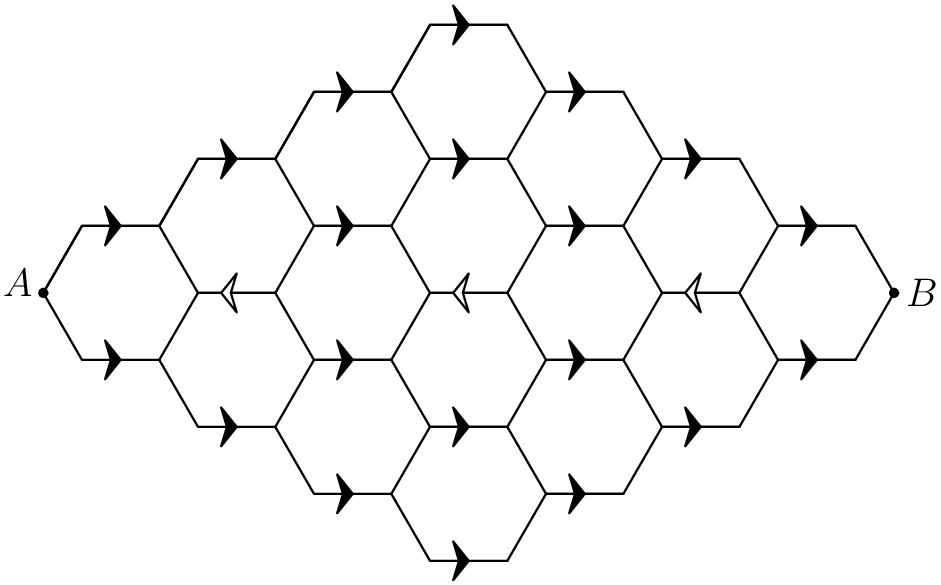
3c = 4(s - 4); 3c= 4s – 16, or 16=4s – 3c. Make a table so that 4s=3c and find when the difference is 4; s=3, c=4; s=6, c=8; s=9, c=12; s=12, c=16. 16-12=4

2012 10B Problem 25

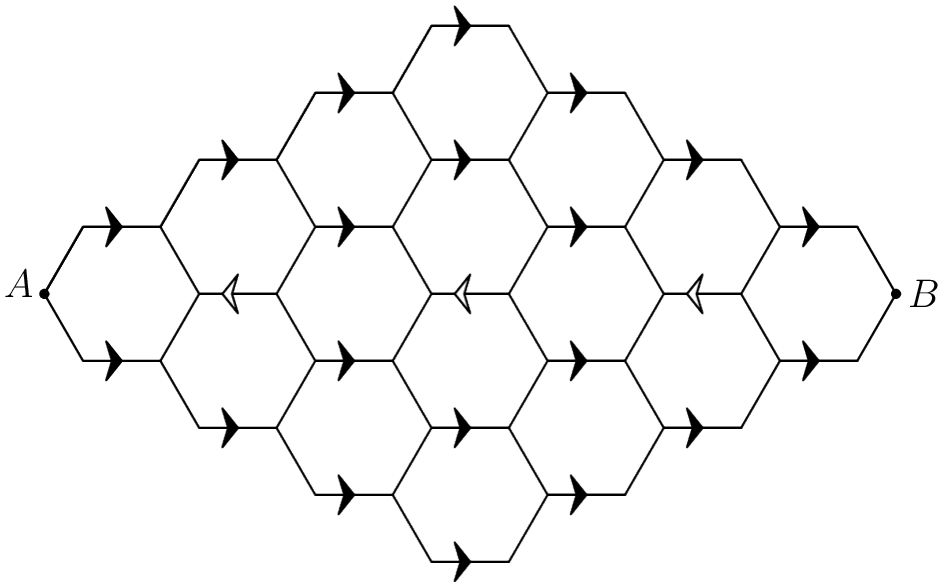
A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there? (The white backwards paths are labeled S1, S2, S3, in order left to right.)



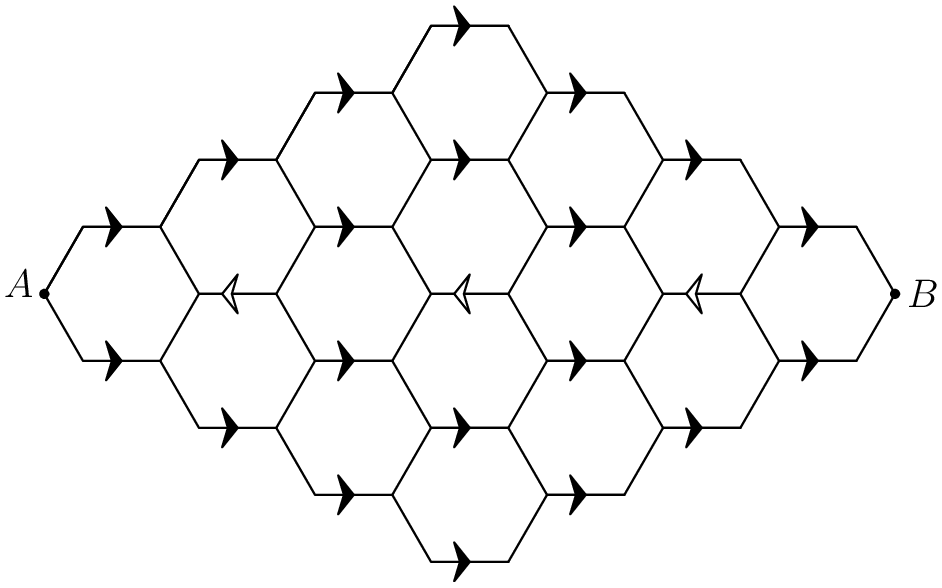
None2*2*4*4*4*2*2=2^10



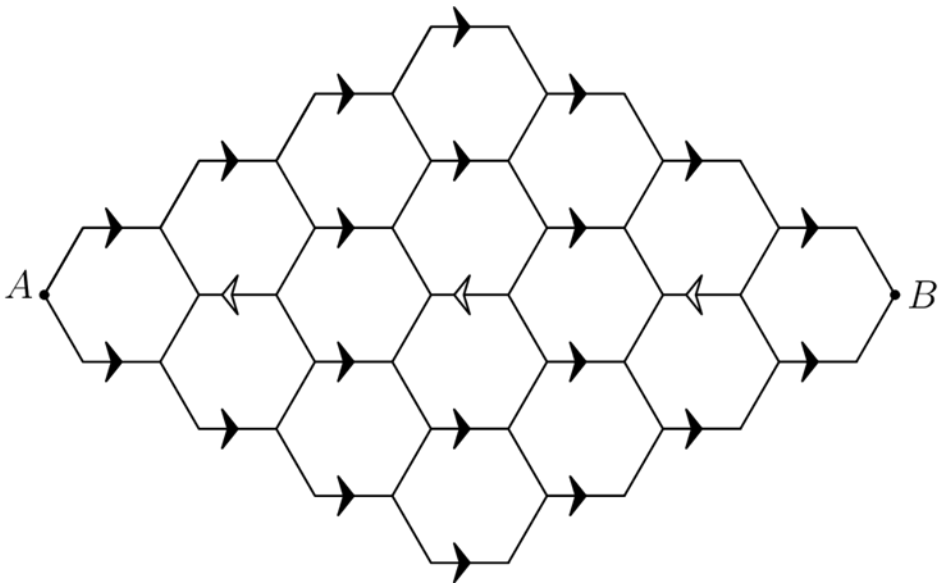
S1 2*1*2*4*4*2*2=2^8



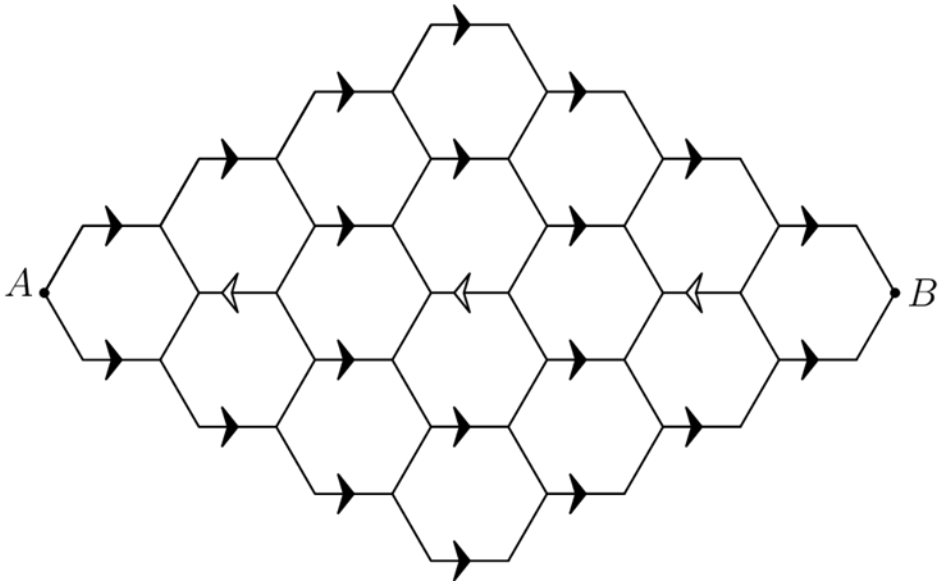
S2 2*2*4*(2*2)*2*2*2=2^9



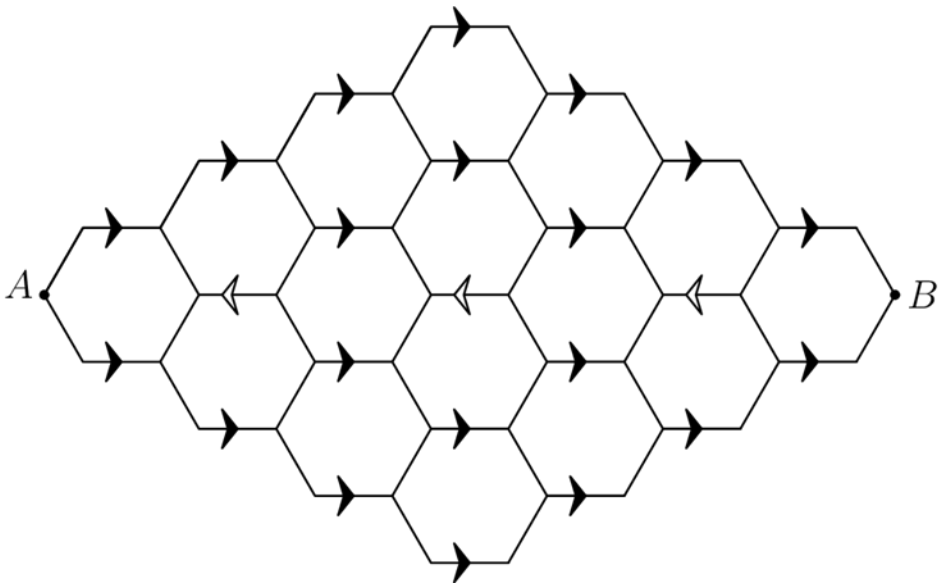
S3 2*2*4*4*4*1*1=2^8



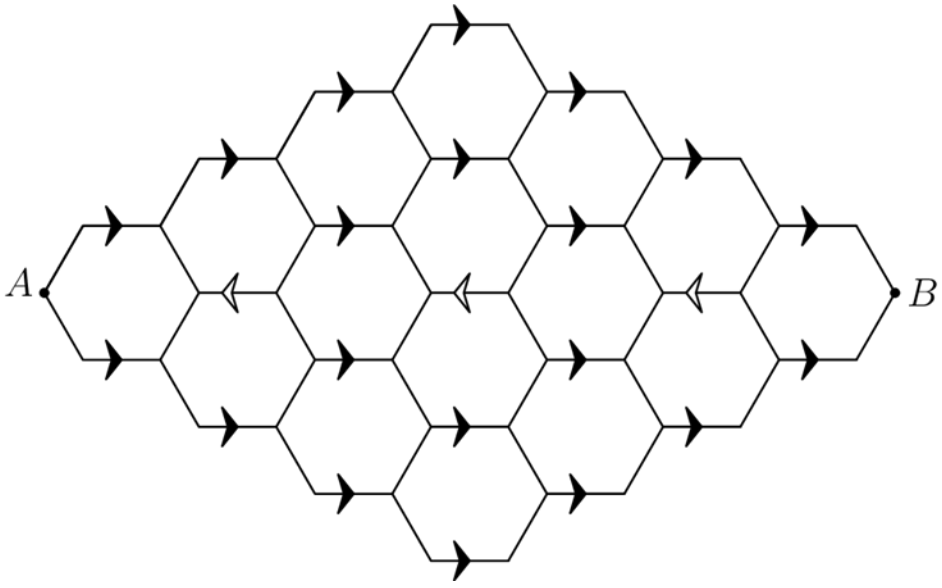
S1, S2 2*1*2*(2*2)*2*2*2=2*7



S1 S3= 2*1*2*4*4*1*1= 2^6;



S2 S3= 2*2*4*(2*2)*2*1*1=2^7



S1 S2 S3= 2*1*2*(2*2)*2*1*1=2^5

The paths the bug can take are 2^10+2*2^8+2^6+2^9+2*2^7+2^5=2400

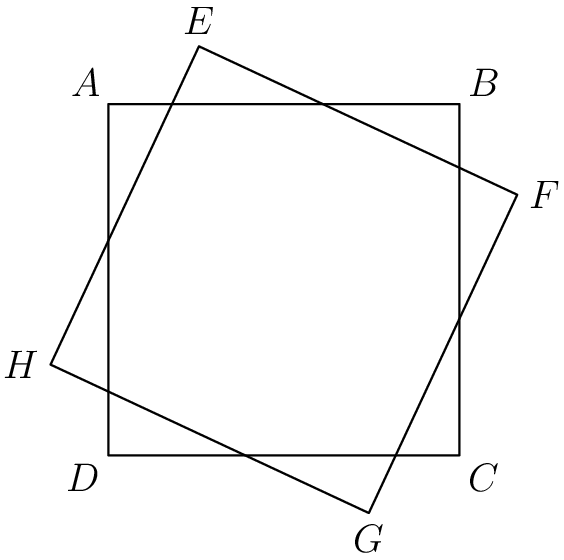
2023 AMC 10A Problem 3

How many positive perfect squares less than 2023 are divisible by 5?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

2023 AMC 10B Problem 7

Square $ABCD$ is rotated 20° clockwise about its center to obtain square $EFGH$, as shown below. What is the degree measure of $\angle EAB$?



$\textbf{(A)}\ 24^\circ \qquad \textbf{(B)}\ 35^\circ \qquad \textbf{(C)}\ 30^\circ \qquad \textbf{(D)}\ 32^\circ \qquad \textbf{(E)}\ 20^\circ$

2022 Problem 13

The positive difference between a pair of primes is equal to 2, and the positive difference between the cubes of the two primes is 31106. What is the sum of the digits of the least prime that is greater than those two primes?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 11 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 16$

We have a-b = 2 and $a^3 - b^3 = 31106$ or $(x+2)^3 - x^3 = 31106$, or $x^3 + 6x^2 + 12x + 8 – x^3 = 31106$. Thus $3x^2 + 6x + 4 = 15{,}553$. Or $3x^2+6x=15549$ or $x^2 + 2x = 5183$, or $x^2 + 2x + 1 = 5184$. Or $x + 1 = 72$. So, the twin primes are 71 and 73. The next largest prime is 79. The sum of the digits is 16.

2010 12A Problem 23

The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 52 \qquad \textbf{(E)}\ 68$

90! has 21 5's and 45 + 22 + 11 + 5 + 2 + 1 = 86 2's. Let AB= 90!(where A consists of all the factors relatively prime to 5 and B consists of all the factors that are divisible by 5.) $90!=10^{21}N$ where N is an integer not divisible by 10, and N~=n (mod100). Now 1*2*3*4 = 24~=(-1) mod 25, which is the same for any 4 numbers between a pair of successive 5's. So (-1)^18~=1 mod 25. The multiples of 5 in 90! are (5*10*15*20)*(30*35*40*45)*(55*60*65*70)*(80*85*90)*(25*50*75). Divide by 5^21 yields (1*2*3*4)*(6*7*8*9)*(11*12*13*14)*(16*17*18)*(1*2*3)~=24^3*(-9*-8*-7*6)= $(-1)^3$*1=-1mod 25, (9*8*7*6=3024), so $2^{21}=2*(1024)^2$~=$2*(-1)^2$~=2 mod 25, so $13*2^{21}$~=13*2~=1(mod25). Therefore N~=$(13*2^{21})$N~=13*(90!/5^{21})=13*A*B/5^{21}=13*1*(-1) mod 25~=-13~=12mod 25. Thus n = 12, 37, 62, or 87. Only 12 is divisible by 4.

2022 AMC 10A Problems

On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternators arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.

"Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.

"Are you an alternater?" The principal gave a piece of candy to each of the 15 children who answered yes.

"Are you a liar?" The principal gave a piece of candy to each of the 9 children who answered yes.

How many pieces of candy in all did the principal give to the children who always tell the truth?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 21 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 31$

(Note:This problem is broken in an interesting way that helps the test-taker. Since the true answers alternate for alternaters, you can still get the correct answer if you misinterpret the problem as "alternaters alternate their yes/no answers, but not their truth values". Suppose that A alternaters answer yes-no-yes, and A' alternaters answer no-yes-no. You still get $A = A' = 9$.

In addition, you can even get the correct answer with the misinterpretation "alternaters have two types: alternaters of the same type give the same arbitrary answers to the three questions, and alternaters of different types give the opposite arbitrary answers to the three questions".

It's also notable that the misinterpretation makes the problem harder, so that the solution actually relies on all the information. This suggests that the **question-writer** may have been mistaken but got lucky.)

2013 10B Problem 15

A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{a}{b}$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{\sqrt{6}}{2} \qquad \textbf{(C)}\ \sqrt{3} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{3\sqrt{2}}{2}$

Math for little, middle school kids A horse named Sparky.

2012 AIME I Problem 4

The middle school kids in the secret math clubhouse were puzzling over a problem about a horse named Sparky. The problem read as such: Butch and Sundance need to get out of Dodge. To travel as quickly as possible, each alternates walking and riding their only horse, Sparky, as follows. Butch begins by walking while Sundance rides. When Sundance reaches the first of the hitching posts that are conveniently located at one-mile intervals along the route, he ties Sparky to the post and begins walking. When Butch reaches Sparky, he rides until he passes Sundance, then leaves Sparky at the next hitching post and resumes walking, and they continue in this manner. Sparky, Butch and Sundance walk at 6, 4, and 2.5 miles per hour, respectively. The first time Butch and Sundance meet at a milepost, they are *n* miles from Dodge, and they have been traveling for *t* minutes. Find *n* + *t*.

“Whose idea was this problem,” wondered Harry Happenstance as he read all the words to the problem.

“It was my idea,” said Professor Starkey as he passed by on his daily amble. “I named the horse Sparky based on the horse character Spark Plug in the old Barney Google and Snuffy Smith comics. By the way Charles Schultz was also nicknamed Sparky based on the same cartoon character.” Professor Starkey was never at a loss for interesting things to say, which made him more interesting because what was he doing at the middle school kids secret math clubhouse?

“Try making a table,” Professor Starkey continued. “That’s what I did when I first worked on this problem, and then it occurred to me for all the miles they covered either one person was riding while the other person was hiking or vice-versa. That’s why they call it hitch-hike, because you hitch the horse and then you hike. Whooee, those boys must’ve had fun back in those days,” Professor Starkey finished as he left the kids in the secret math clubhouse.

The kids looked after Professor Starkey in amazement. “Someone really ought to look out for the poor guy. I think he lost his mind, thinking it’s better to travel by horse than modern electrical cars,” remarked Sammy Sandstone who couldn’t help from watching Professor Starkey walking down the path.

“Also a modern electrical car can drive by itself which a horse couldn’t always do,” Harry chipped in.

“Well never mind that,” said Winfield Jones, “we got work to do to figure out this problem. Professor Starkey said we should make a table so let’s start with that.”

The kids made a table. They used minutes for the variable and figured out how many minutes it took each person/horse to walk one mile. Since Sparky, the horse can walk 6 miles per hour then it takes him 10 minutes to walk one mile. Butch can walk 4 miles per hour so he walks one mile in fifteen minutes and Sundance walks 2.5 mph so he can walk one mile in 24 minutes.

Elizabeth Springfield wrote the numbers on the white board as such:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
15	10	15	15	10	15	15	15	10	15	15	15	10	15	15	15	10	15
15	25	40	55	65	80	95	110	120	135	150	165	175	190	205	220	230	245
10	24	10	10	24	10	10	10	24	10	10	10	24	10	10	10	24	10
10	34	44	54	78	88	98	108	132	142	152	162	186	196	206	216	240	250

“The top row,” explained Elizabeth “is the mile posts. The next row is when Butch is walking or riding and the row after that is the total time for Butch.”

“I see,” said Paula Periwinkle. “The first milepost for Butch is 15 minutes because he has to walk there and the next milepost is 10 minutes because he rides Sparky to that milepost.”

“That’s right,” said Jeffery Pine. “So in the third row it takes Butch 15 minutes total time to get to milepost 1 and then 25 min total time to get to mile post 2.”

“And the last two rows are for Sundance,” finished Aloe Vera. “He rides to the first milepost which takes 10 minutes, hitches Sparky, and then walks to the next milepost in 24 minutes. So his total time is 34 minutes to milepost 2, as indicated in row 5. Since Butch was able to get to milepost 2 in 25 minutes, then he would have tied up Sparky and continued walking, leaving the horse for Sundance when he arrives at that milepost.”

“So if we continue to fill in all the values we’ll be able to find the milepost when the total time is the same for both Sundance and Butch,” concluded Winfield.

The kids filled in all the values in the table but the whiteboard only had room for 18 mileposts. Can you finish this problem and find the distance and time when both Sundance and Butch will be at the same milepost at the same time?

At the 19th milepost, Butch will arrive in 260 minutes walking the last mile and Sundance riding Sparky the last mile will also arrive at 260 minutes. They will all be together at milepost 19 after 260 minutes. Thus *n* + *t* is 279.

Now for each mile either Sundance rides and Butch walks or Butch rides and Sundance walks. Therefore let x be the number of miles Butch walks and y be the number of miles Butch rides. Then 15x + 10y = 10x + 24y. So $\frac{x}{y} = \frac{14}{5}$ and the smallest values that solve this equation are x = 14 and y = 5 which is 19 miles and the total time is 15·14 + 10·5 = 260 minutes. 19 + 260 = 279.

Number Theory

- Find the least number that when divided by 16, 18, and 20 leaves a remainder of 4 in each case, and is also completely divisible by 7.

Geometry

- How many cubic centimeters are in the volume of a rectangular prism whose faces have areas 8 sq. cm, 10 sq. cm, and 12 sq. cm? Express your answer in simplest radical form.

Counting and Probability

- Seventy-five bingo balls, each with a different positive integer from 1 through 75, are placed in a cage. A random ball is selected, its number is announced, and the ball is returned to the cage. This process occurs a total of 20 times. What is the probability that at least one ball is selected more than once? Express your answer as a decimal to the nearest hundredth.

Mixed Word Problems

- Abhinav, Bella, and Chandra were all born after 2000. Each of them was born in a year after 2000 that is divisible by exactly one of the prime numbers 2, 3, or 5. Each of these primes is a divisor of one of the birth years. What is the least possible sum of their birth years?

Final Answers:

1. Number Theory:

Final Answer: $\boxed{2884}$ (Yep, you nailed it!) ✅

2. **Geometry**: Let's break this down step-by-step:

- Step 1: Let the dimensions be a, b, and c. We have $ab = 8$, $ac = 10$, and $bc = 12$.

- Step 2: Multiplying these equations gives $(abc)^2 = 8 \times 10 \times 12 = 960$.

- Step 3: So, the volume $V = abc = \sqrt{960} = \sqrt{64 \times 15} = 8\sqrt{15}$ cubic cm.

- Final Answer: $\boxed{8\sqrt{15}}$ ✅

3. **Counting and Probability:**

Final Answer: Yep, your approach is spot on! The probability is $1 - \frac{{75}P{20}}{75^{20}} = 1 - \frac{\frac{75!}{(75-20)!}}{75^{20}}$. Nice work!

✅

4. Mixed Word Problems:

Final Answer: $\boxed{6008}$ (You're absolutely right again!)

✅

Here are some problems for you to try.

1. As in the example above suppose Sparky can walk 6 mph, Butch can walk 5 mph and Sundance can walk 2.5 mph. Again let Butch walk first while Sundance rides Sparky and they tie up Sparky at the next milepost whenever they pass each other, then how many minutes and how far will they be when they all meet at a milepost?

   A) 7 miles, 104 minutes B) 8 miles, 94 minutes C) 9 miles, 108 minutes D) 10 miles, 118 minutes E) 11 miles, 128 minutes

2. What is the sum of 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 – 6 – 5 – 4 – 3 – 2 – 1 – 0?

   A) 0 B) 1 C) 2 D) 6 E) 7

3. What is the next number in this sequence: 10, 20, 40, 80, 160, …?

   A) 240 B) 320 C) 400 D) 480 E) 640

4. Paula is drawing stars with colors. Her first star is colored red, her second star is colored green, her third star is colored blue and her fourth star is colored violet. Then she begins over coloring her stars red, green, blue and violet. What color will be her fourteenth star?

   A) Red B) Green C) Blue D) Violet E) cannot be determined

5. In one of the following pictures exactly three quarters of all objects are hearts. Which one is it?

   A) 

B) 

C) 

D) 

E) 

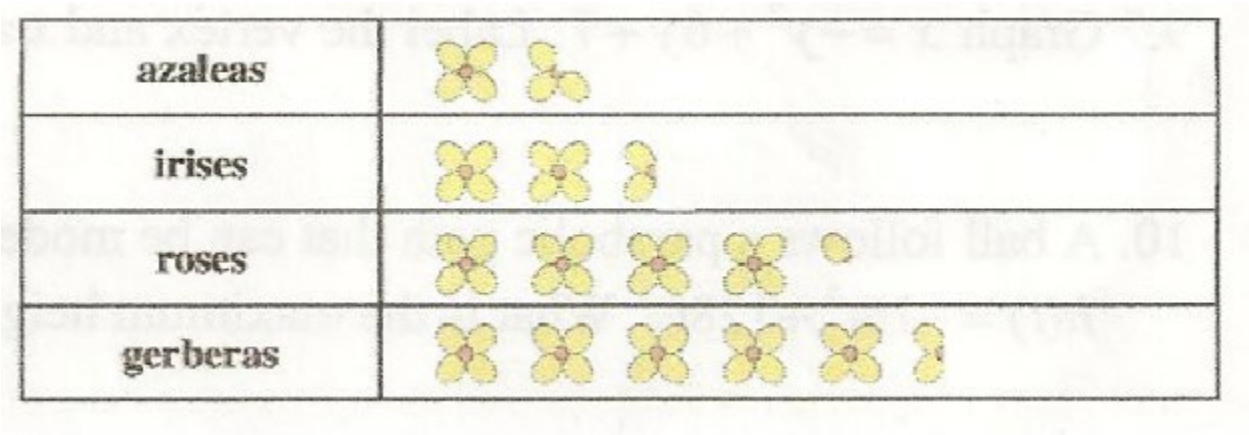
6. On this line the distance AC = 12 inches, BD = 10 inches and AD = 18 inches.

Find the distance BC.

A B C D

A) 1 inch B) 2 inches C) 3 inches D) 4 inches E) 5 inches

7. The table below shows the quantity of different types of flowers in the Botanical garden. Winfield asked the gardener and found out, that there were 35 azaleas, 50 irises and 85 roses in the garden. What is the number of gerberas growing in the garden?



A) 95 B) 100 C) 105 D) 110 E) 115

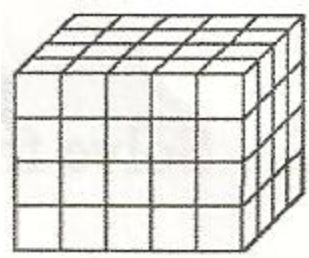
8. In the faculty lounge there are 4 tables with 4 chairs each, 3 tables with 3 chairs each and 2 tables with 2 chairs each. How many chairs are in the faculty lounge?

A) 25 B) 29 C) 36 D) 50 E) 81

9. Harry thought to himself if he had twice as many pencils as he now has he would have 16 more pencils. How many pencils does he now have?

A) 8 B) 12 C) 16 D) 24 E) 32

10. A 5 x 5 x 5 cube is dipped in a red can of paint and separated into 125 small cubes. How many cubes do not have red painted on any of their faces?



A) 27 B) 32 C) 48 D) 64 E) 125

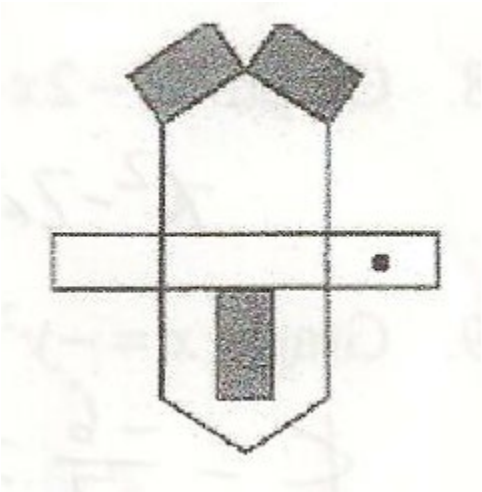
11. Joey jumped 50 dm 50 cm and 50 mm on his first jump. His second jump was 123 cm longer. How long was his second jump?

A) 5 m 55 cm B) 5 m 73 cm C) 6 m 78 cm D) 7 m 28 cm E) 11 m 28 cm

12. A 24-hour digital clock will display 9:37 PM as 21:37. The sum of the digits of 21:37 is 13. What is the largest possible sum of the digits of a 24-hour digital clock display?

A) 19 B) 23 C) 24 D) 25 E) 36

13. The picture below has been drawn on paper and cut out to make a house. Which of the houses did it become?

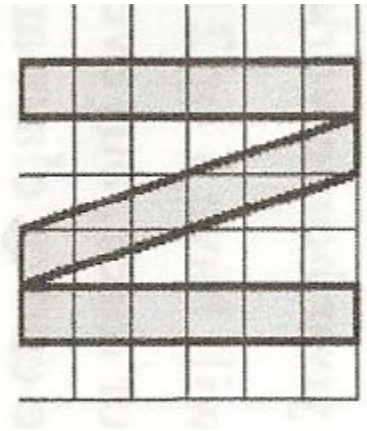


14. Sally fell asleep at 9:15 PM and woke up at 6:15 AM the next morning. Her brother Harry had been sleeping 1 hour 15 minutes longer. How many hours had Harry been sleeping?

A) 9 hours B) 9 hours 15 minutes C) 9 hours 30 minutes

D) 10 hours 15 minutes E) 10 hours 45 minutes

15. If the length of the side of each little square is 1 cm, what is the area of the letter Z?



A) 14 B) 15 C) 16 D) 17 E) 18

16. In a class of 22 students, 12 students have a brother and 16 students have a sister. Three students do not have a brother or a sister. How many students have both a brother and a sister?

A) 9 B) 10 C) 11 D) 12 E) 16

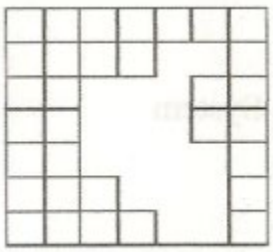
17. The number of digits used to write all the page numbers in a book is 35. How many pages are in the book?

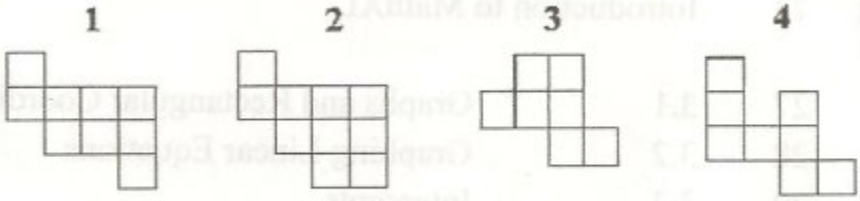
A) 20 B) 21 C) 22 D) 23 E) 24

18. A bar code is formed using 17 black and white bars (the first and the last bars are black). The black bars are two types: wide and narrow. The number of white bars is 2 more than the number of wide black bars. What is the number of narrow black bars?

A) 2 B) 3 C) 4 D) 5 E) 6

19. Which two of these figures can be used to completely fill in the empty area?





A) 1 and 3 B) 2 and 4 C) 2 and 3 D) 3 and 4 E 1 and 4

20. Suppose the bar code had 21 black and white bars. The black bars are narrow and thick. The first and last black bars are both thick. The number of thick black bars is three more than the number of narrow black bars. How many narrow black bars are in the bar code?

A) 2 B) 3 C) 4 D) 5 E) 7

Solutions: Competitive Math problems levels 3, 4 2003

1. B

2. E

3. B

4. B

5. D

6. D

7. D

8. B

9. C

10. A

11. C

12. C

13. A

14. D

15. E

16. A

17. C

18. B

19. C

20. C