MAA Session #21 – Thursday, August 5, 2010 – 8:30-10:25 am – Three Rivers Room

8:30-8:45

Partition Regularity for The Jordan Canonical Form and Nilpotent Operators Liam Solus, Oberlin College

It is well known that any real matrix is partition regular if and only if it satisfies the columns condition. If a square, real matrix A satisfies the columns condition with some partition \mathcal{I} , then A^k also satisfies the columns condition with partition \mathcal{I} . Here, we show that a matrix A need not be partition regular for A^k to be partition regular for some natural number k. We use the Jordan Canonical Form to generate nontrivial partition regular matrices that are powers of nilpotent matrices, which themselves are not partition regular. For any such partition regular matrix we determine the set of columns condition numbers. In doing so, we define a measure of -closeness" to partition regularity for any real matrix, called the *progress*. We establish some properties of the progress of a matrix, and we apply the progress measure to matrices having a real spectrum containing 0 to produce partition regular matrices with a special form.

8:50-9:05

Generators of the Symmetric Group Kelsev Watson, Valparaiso University

A permutation is an arrangement of n objects in a way such that order matters. The collection of all permutations of length n forms the symmetric group, Sn. Composition of permutations generates a subgroup of Sn, which can be represented graphically using Cayley graphs. In my research, I am studying when two permutations generate Sn.

9:10-9:25

(open)

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9:30-9:45

A Study on Tails of Bivariate Distributions

Steven Collazos, State University of New York at Binghamton, and Diego Canales, University of Texas at El Paso

There are disciplines that model some stochastic processes as heavy-tailed. Whether a distribution is heavy-tailed or not depends, of course, on the behavior of the tails of the distribution under study. Although tail behavior of statistical distributions in one dimension has been studied by several authors, this is not the case for distributions in higher dimensions. In this paper, we explore the concept of heavy-tailedness in two dimensions by constructing bivariate distributions via copulas and linear transformations, projecting the data generated from such distributions to different axes, and observing the extreme spacings of the data projected onto these axes through the use of the test statistic

$$T = -\frac{\ln(F_n(\ln(X_{(n)}+1)))}{\ln(X_{(n)}+1)}(X_{(n)}-X_{(n-1)}),$$

where X is a vector whose entries are the samples generated after performing a projection and \bar{F}_n is the empirical survival function.

We observe that principal component projections, which provide perhaps the most intuitive approach to studying the problem, do not always capture the tail behavior of the bivariate distributions we constructed and we provide estimates for the probability for type I error and power of the test statistic T. There are other observations pending. Finally, we discuss our results and future possible approaches for studying this problem.

9:50-10:05

Analysis of Use it or Lose it Trees Thomas James Murphy, Marist College

Consider a tree where each vertex is labelled with an integer from $\{0, 1, ..., n\}$. We perform iterations on this tree, first described by Bahls, Knox, and McClure, by randomly selecting ("using") a vertex with a positive label and attaching m vertices to it. We label those m + 1 vertices with n and decrease the remaining positive labels by 1; once a vertex has label 0 it can no longer be used so we say it is dead. Such trees are called *Use it or Lose it Trees (ULTs)*. We give a proof of the *Leaf Probability Conjecture* which states that, after a large number of iterations, the probability that a new random vertex dies as a leaf is equal to $\frac{m}{m+1}$, independent of n. We also prove several other theorems involving ULTs.

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10:10-10:25

Equitent Problems

Katelynn Kochalski, Canisius College

Equitent problems ask what are the surface area minimizing figures that have fixed boundary (extent) and fixed volume (content). We use metacalibration, a new optimization technique, to prove results about the equitent problem in the plane. Specifically, we investigate connecting the vertices of a regular polygon while also enclosing a given area.

Additional Sessions appear on the pages to follow.

MAA Session #23 – Thursday, August 5, 2010 - 8:30-10:25 am – Bob Hope Room

8:30-8:45

The Graph of Equivalence Classes of Zero-Divisors Blake Allen, Utah Valley University

We will examine properties of the graph of equivalence classes of the zero-divisors of a commutative ring.

8:50-9:05

Establishing Further Properties With Partition Regular Matrices Ryan Wormald, Mount Holyoke College REU 2010 and Kara Finley, University of Rochester

Rado characterized all rational partition regular matrices as those matrices which satisfy the columns condition. Recent work by Hogben and McLeod examined the columns condition form a liner algebraic perspective. Here we explore further, certain linear algebraic properties of partition regular matrices. In particular, we determine which binary operations preserve partition regularity as well as examine how binary operations impact the cells of any partition satisfying the columns condition. Additionally, we use knowledge of the rank of a PR matrix to answer questions about the columns condition numbers that matrix can attain.

9:10-9:25

A stochastic game based on the gambler's ruin scenario for a cookie random walk Timothy Pluta, Iowa State University; Brian Wu, Iowa State University; and Matthew Temba, Iowa State University

We study a stochastic game between a "buyer" and a "seller," whose major component is a reinforced random walk performed by the -buyer" on the integer lattice. We assume a Gambler's Ruin scenario, where in contrast to the classical version the walker ("buyer") has the option of consuming a "cookie" which increases the probability of moving in the desired direction for one step. The cookies are supplied to the walker by the "seller." For several modifications of the game we have determined the players' optimal strategies. Our initial motivation is provided by a popular model of "cookie" or "excited" random walks. We aim to measure the gain of the walker from exploiting the reinforcing cookies. The questions we investigate are partially inspired by their counterparts in stochastic control theory involving a random walk controlled by a reinforcing mechanism ("seller") that leads to termination, but is interested in keeping the walker in the "game" as long as possible.

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9:30-9:45

Ciliary Dynamics with Boundary Wall Correction

Michael Casey, Mount Holyoke College REU; Faisal Ahmad, Mt. Holyoke College REU

We present a model for the motion of cilia using the Stokes flow approximation for low Reynolds number flows of an incompressible fluid, that is, the viscous forces of the fluid flow dominate in this regime. Applying our model to the motion of cilia attached to a wall, we address the necessary boundary wall correction to the fluid flow using an integral equation formulation. Provided data on the motion of actual cilia attached to a coverslip, we determine the internal stresses associated to the motor-proteins in the cilia. This work was done at the Mount Holyoke College REU, summer 2010.

9:50-10:05

Solution Theory for Bilinear Systems of Equations Dian Yang, College of William and Mary

For $A_1, \ldots, A_m \in M_{p,q}(\mathbf{F})$ and $g \in \mathbf{F}^m$, any system of equations of the form $y^T A_i x = g_i$, $i = 1, \ldots, m$, with y varying over \mathbf{F}^p and x varying over \mathbf{F}^q is called bilinear. A solution theory for complete systems (m = pq) is given by Johnson and Link.[1] Given here is a general solution theory for bilinear system of equations. To obtain this result we use a reduction from bilinear system to linear system.

10:10-10:25

Tree Generation using the Use-it-or-Lose-it algorithm Chris Kirkland, Mercer University

In this talk, we present a two-fold investigation of a particular random tree generation algorithm known as the *use-it-or-lose-it* algorithm first introduced in a 2009 paper by Bahls, Knox, and McClure. First, we discuss structural properties related to internal degree measure of the generic algorithm. Second, we extend a technique of Bahls, Knox, and McClure which we refer to as *multiplicative decremental tag systems* to explore probabilistic aspects of a modified, unrestricted version of the algorithm. We close with some suggestions of future work.

Additional Sessions appear on the pages to follow.

MAA Session #22 – Friday, August 6, 2010 – 8:30-11:45 am – Three Rivers Room

8:30-8:45

An upper bound of conjugacy classes of a group with its Sylow p-groups Cong Tuan Son Van, College of Saint Benedict and Saint John's University

Problem 14.74 of the Kourovka Notebook of Unsolved Problems in Group Theory (submitted by L. Pyber) states: "Let k(H) denote the number of conjugacy classes of a group H, and G be a finite group with Sylow p-groups P_1, \dots, P_n . Prove or disprove: $k(G) \leq k(P_1) \cdots k(P_n)$ ". We will discuss some families of group for which this result holds.

8:50-9:05

Equivalence Classes of Zero Divisor Graphs

Kaylee Joy Kooiman, Calvin College and Cathryn Holm, St. Olaf College

We will discuss the realizability of equivalence classes of zero divisor graphs and what they can tell us about the ring.

9:10-9:25

Equivalence classes of zero divisor graphs and zero divisor graphs Dane Skabelund, Brigham Young University, and Eric New, The College of New Jersey

We will discuss some properties of equivalence classes of zero divisor graphs and how some of the current results for zero divisor graphs correspond.

9:30-9:45

Measure-Preserving Actions with Various Mixing Properties Ran Bi, Williams College

In this presentation, I will introduce measure-preserving actions and several notions of mixing, such as mixing, weak mixing and mild mixing. Then I will provide examples with different mixing properties.

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9:50-10:05

Mathematical Modeling of Delamination Growth with Time Luis Flores, Gainesville State College

Using various methods of mathematical modeling, the topic of the Asymptotic approach to the constant velocity of hydrogen delamination growth was further explored. Within this topic, the issue of Hydrogen embrittlement was focused on, where the equation of the delamination opening equation was investigated. Furthermore, through this equation the equation of volume was obtained by simple integration. Next the gas pressure inside the crack was investigated and with the use of the volume and mass equations, the kinetic equation for the delamination radius, a(t), was obtained for ideal gas conditions. This integral equation was reduced to a differential equation. The differential equation was solved, and an analytical solution for the dependence of the delamination radius on time was reached. Finally, the same procedure was conducted, and in place of ideal gas conditions, real gas conditions were used. After solving the differential equation, a formula for the radius of the delamination on time was also obtained. Because of the complexity of the calculations, it was possible to get the analytical solution only in implicit form. Two graphs, both for large and ideal gas pressures, for the dependence of demalination radius on time were produced for comparison on the same ordinate system. This allowed us to find the critical pressures starting from which the ideal gas equation should be taken into account.

10:10-10:25

Investigating the Higher-order Sheffer Polynomial Sequences Jeffrey Scavo, Penn State Behrend

In 1939 I.M. Sheffer completely analyzed a special type of generating function, which he entitled Type 0, and also discovered all of the orthogonal polynomial sequences that came out of it, which are amongst the most studied and most applicable. Moreover, Sheffer also developed a more general generating function than Type 0 (entitled Type k). Dr. Daniel J. Galiffa (Penn State Erie) analyzed a special case of the Type 1 (k=1) class and showed that no new OPS arise from this structure by developing a new procedure, which relied on the computer algebra system Mathematica 7. Our current project focuses on implementing this method to generalize the analysis completed on this special case of the Type 1 class and also conducting a preliminary analysis of the Type 2 class. The research will give insights into these higher-order Sheffer classes and also demonstrate the applicability of the method developed in the previous research.

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10:30-10:45

Asymptotic Connectivity of Hyperbolic Planar Tilings: Independence of Basepoint Robin Neumayer, University of South Carolina

Asymptotic connectivity is a measure of overall connectivity of infinite graphs, computed by finding the average connectivity of a ball centered at a basepoint and looking at the behavior of the connectivity in the limit. It is unclear in the general case if asymptotic connectivity is independent of the choice of basepoint. We will study infinite graphs comprising half-planes with *d*-regular hyperbolic tilings of squares. In particular, we will prove that the asymptotic connectivity of a graph G formed by joining arbitrarily many half-planes of regular hyperbolic tilings of varying degrees of squares is independent of basepoint.

10:50-11:05

The Effect of Coalitions on the Banzhaf Power Index Amanda Ketner, Elon University

In politics, the Banzhaf power index offers a way to quantitatively measure power among voters. However, this index is not always accurate because it assumes that every possible coalition between voters is equally likely to occur. In practice, certain coalitions are unlikely to form due to differing political ideologies. For example, liberal and conservative voters are typically not going to create a coalition without including moderates. We study the effects of restricted coalitions by examining the convex Banzhaf index, a modification of the traditional power index.

11:10-11:25

k-Independence Polynomials

Nicole Gin, Spring Arbor University

A subset, S, of the vertices of a graph is called a k-independent set if the length of the shortest path between each distinct pair of vertices in S is greater than or equal to k+1. In this talk, we generalize the well-known independence polynomial to a k-independence polynomial, which enumerates the k-independent sets in a graph. A method of computing the k-independence polynomial of a graph will be explained. It will be followed by a discussion of properties, including unimodality and logarithmic concavity, of the k-independence polynomials of various families of graphs.

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11:30-11:45

Families of Hyperbolic Tilings

Katie Izzo, Keuka College

We consider a method of construction for regular tilings of heptagons of a closed surface with genus at least 2. These special tilings have associated graphs and the constructions can be iterated to create a family of graphs. We then analyze the properties of these families such as the number of vertices and edges, independence number and connectivity. Using our findings, we describe the infinite graphs that are the limits of these iterations.