**Abstracts of Papers** 

**Presented at** 

## MathFest 2011

Lexington, Kentucky August 4–August 6, 2011



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## **Invited Addresses**

## **Earl Raymond Hedrick Lecture Series**

## Manjul Bhargava Princeton University

## The World of Algebraic Curves (and the Special Role that Elliptic Curves Play)

**Biography** Manjul Bhargava received his A.B. degree from Harvard University and a Ph.D. from Princeton University in 2001. After spending a year each at Harvard and the Institute for Advanced Study (on a Clay Research Fellowship), he joined the Princeton faculty as Professor of Mathematics in 2003. Bhargava's research interests include algebraic number theory, representation theory, and combinatorics, though he also spends much of his time on algebraic geometry, linguistics, mathematics education, and Indian classical music. A versatile speaker, he has given numerous seminars, colloquia, invited addresses, and public lectures at colleges and universities across North America and Europe.

Bhargava's honors include the AMS-MAA-SIAM Frank and Brennie Morgan Prize in 1997, the MAA Merten Hasse Prize for mathematical exposition in 2003, the AMS Blumenthal Award for the Advancement of Research in Pure Mathematics in 2004, a Packard Foundation Fellowship in 2004, the Clay Research Award in 2005, the SASTRA Ramanujan Prize in 2005, and the AMS Cole Prize in 2008. In 2002, Bhargava was named to *Popular Science* magazine's first list of "Brilliant 10," an annual celebration of ten scientists who are shaking up their fields.

## Lecture 1: From Right-angled Triangles to Algebraic Curves

## Main Lecture Hall, Thursday, August 4, 10:30-11:20 am

The classical and ancient problem of finding right-angled triangles with integer side-lengths has a natural and beautiful solution in terms of the geometry of conics. In this lecture, we describe this method, and how it can be adapted to find all rational points—i.e., points having rational coordinates—on any conic (ellipse, parabola, or hyperbola) in the plane. The method immediately leads to questions about curves in the plane of higher degree, and in particular, to elliptic curves.

## Lecture 2: The Special Role of Elliptic Curves Main Lecture Hall, Friday, August 5, 9:30–10:20 am

Unlike conics, which are curves in the plane defined by equations of degree two, elliptic curves are defined by equations of degree three. This slight change already makes elliptic curves very different from conics, and it also turns out that the behavior of elliptic curves is drastically different than that of curves of degree 4 and higher. What really sets elliptic curves apart is that the set of rational points on an elliptic curve has a natural group structure. In this lecture, we describe this group structure, and explain how it can be used to answer many questions about elliptic curves (and other objects) that would be difficult to answer otherwise. We also describe many open questions, as well as some recent progress on their answers.

## Lecture 3: How Many Points Are Needed, on Average, to Generate All Rational Points on an Elliptic Curve? Main Lecture Hall, Saturday, August 6, 9:30–10:20 am

A rational elliptic curve may be viewed as the set of solutions to an equation of the form  $y^2 = x^3 + Ax + B$ , where A and B are rational numbers. The set of rational points on this curve possesses a natural abelian group structure, and the Mordell-Weil theorem states that this group is always finitely generated. The rank of a rational elliptic curve measures how many rational points are needed to generate all the rational points on the curve. There is a standard conjecture—originating in work of Goldfeld and Katz-Sarnak—that states that the average rank of all elliptic curves should be 1/2; however, it has not previously been known that the average rank is even finite! In this lecture, we describe recent work that shows that the average rank is finite; in fact, we show that the average rank is less than 1! It follows that many—indeed, we show at least 10% of—elliptic curves have no rational points. This is joint work with Arul Shankar.

## MAA-AMS Joint Invited Address

## Main Lecture Hall, Thursday, August 4, 8:30–9:20 am

## Laura DeMarco University of Illinois at Chicago

## **Polynomial Dynamics: Conjugacy and Combinatorics**

In the study of any collection of dynamical systems, one of the main goals is a classification of possibilities. Ideally, for each equivalence class of systems, we can find a simple model that captures all of the important long-term information about the system.

In this talk, I will introduce this problem of classification in the setting of complex one-dimensional polynomials. I will describe some of the combinatorial tools that have been used to address this problem, with the aim of presenting recent work (joint with Kevin Pilgrim) about the conjugacy classes and the geometry of the moduli space of polynomials.

**Biography** Laura DeMarco works in dynamical systems and complex analysis. Her research is focused on the moduli spaces of polynomials or rational functions of one complex variable. She is currently an Associate Professor at the University of Illinois at Chicago. She is the recipient of an NSF Career Award and a Sloan Foundation Fellowship. She earned her PhD at Harvard University in 2002.

## MAA Invited Addresses

## Main Lecture Hall, Thursday, August 4, 9:30–10:20 am

## Edward Burger Williams College

## Planting your roots in the natural numbers: A rational and irrational look at 1, 2, 3, 4, ...

Some people see magical features in the famous Fibonacci numbers and the alluring golden ratio phi. But what if you replace the phamous phi with your phavorite obscure quadratic irrational real number? Is the magic still there? Here in 1 hour, 2 examples, 3 theorems, and 4 acts we'll consider these questions, highlight their history, and share some recent insights that will transfigure the magic into mathematics. Revealing any more here would be simply absurd.

**Biography** Edward Burger is Professor of Mathematics and Lissack Professor for Social Responsibility and Personal Ethics at Williams College and Vice Provost for Strategic Educational Initiatives at Baylor University. He is the author of over 35 research articles, 12 books, and 15 video series. Burger was awarded the 2000 Northeastern Section of the Mathematical Association of America (MAA) Award for Distinguished Teaching and 2001 MAA Deborah and Franklin Tepper Haimo National Award for Distinguished Teaching of Mathematics. The MAA also named him their 2001–2003 Polya Lecturer. He was awarded the 2003 Residence Life Teaching Award from the University of Colorado at Boulder. In 2004 he was awarded Mathematical Association of America's Chauvenet Prize and in 2006 he was a recipient of the Lester R. Ford Prize. In 2007 and 2008 he received two awards for his video work. In 2007 Williams College awarded him the Nelson Bushnell Prize for Scholarship and Teaching. Burger is associate editor of the *American Mathematical Monthly* and *Math Horizons* and serves as a Trustee of the Kenan Institute for the Arts at the University of North Carolina School of the Arts. In 2006, Reader's Digest listed Burger in their annual "100 Best of America" as America's Best Math Teacher. In 2010 he was named the winner of the 2010 Robert Foster Cherry Award for Great Teaching—the largest prize in higher education teaching across all disciplines in the English-speaking world. Also in 2010 he appeared on a mathematics segment for NBC-TV on the Today Show and throughout the 2010 Winter Olympic coverage. Most recently The Huffington Post named him one of their 2010 Game Changers; "HuffPost's Game Changers salutes 100 innovators, visionaries, mavericks, and leaders who are reshaping their fields and changing the world."

## Main Lecture Hall, Saturday, August 6, 10:30–11:20 am

## Annalisa Crannell Franklin & Marshall College

## In the Shadow of Desargues

Those of us who teach projective geometry often nod to perspective art as the spark from which projective geometry caught fire and grew. This talk looks directly at projective geometry as a tool to illuminate the workings of perspective artists. We will particularly shine the light on at Desargues' triangle theorem (which says that any pair of triangles that is perspective from a point is perspective from a line), together with an even simpler theorem (you have to see it to believe it!). Given any convoluted, complicated polygonal object, these theorems allow us to draw that object together with something that is related to it — its shadow, reflection, or other rigid symmetries—and we'll show how this works. If you enjoy doodling or sketching, bring your pencil, a good eraser, and a straightedge.

**Biography** Annalisa Crannell is a Professor of Mathematics at Franklin & Marshall College, a past Governor of the MAA-EPADEL section, and a recipient of the MAA's 2008 Deborah and Franklin Tepper Haimo Award. Together with Marc Frantz, she co-authored the book, *Viewpoints: Mathematical Perspective and Fractal Geometry in Art*. She has worked extensively with students and other teachers on writing in mathematics, and with recent doctorates on employment in mathematics. In her copious spare time, she adopts children, cans her own salsa and jam, and attempts to play the banjo. She especially enjoys talking to nonmathematicians who haven't (yet) learned where the most beautiful aspects of the subject lie.

## Main Lecture Hall, Saturday, August 6, 8:30–10:20 am

## Lauren Ancel Meyers University of Texas - Austin

## Mathematical Approaches to Infectious Disease Prediction and Control

Mathematics has long been an important tool for understanding and controlling the spread of infectious diseases. I will present an overview of compartmental models, the traditional approach to modeling infectious disease dynamics, and then introduce contact network epidemiology, a relatively new approach that applies bond percolation on random graphs to model the spread of infectious disease through heterogeneous populations. As I will illustrate, these methods can be used to address public health challenges and have recently been coupled with powerful computational methods to optimize epidemic control strategies.

**Biography** Lauren Ancel Meyers received her B.A. degree in Mathematics and Philosophy from Harvard University in 1996 and her Ph.D. from the department of Biological Sciences at Stanford University in 2000. After completing a postdoctoral fellowship supported by the National Science Foundation and the Santa Fe Institute, she joined the faculty of Integrative Biology at the University of Texas at Austin in 2003 where she was promoted to Full Professor with tenure in just three years and served as the first Associate Director of the new campus-wide Division for Statistics and Scientific Computation. She has also been an active member of the external faculty of the Santa Fe Institute since 2003.

Lauren's research lies at the interface of evolutionary biology and epidemiology. She has developed new network-based mathematical methods to study the interplay between disease transmission dynamics, human behavior, and the evolution of pathogens including those

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responsible for epidemic meningitis, influenza, walking pneumonia, and SARS. During the 2009 H1N1 influenza pandemic, Lauren and her research group worked closely with public health officials around the globe to characterize the new strain and develop optimal strategies for controlling its spread.

Lauren's research has been funded by research grants from National Institutes of Health (MIDAS), the National Science Foundation, Canadian Institutes of Health Research, and the James S. McDonnell Foundation and has resulted in over 40 publications in top scientific journals. The Wall Street Journal, Newsweek, the BBC, and other news sources have highlighted her work; and a number of government agencies have sought Lauren's expertise, including the Centers for Disease Control and Prevention (CDC), the Biomedical Advanced Research and Development Authority (BARDA), the British Columbia Centre for Disease Control (BCCDC), the US National Intelligence Council, Los Alamos National Labs, Lawrence Livermore National Labs, and the Texas Department of State Health Services. In 2004, the MIT Technology Review named Lauren as one of the top 100 global innovators under age 35.

## AMW-MAA Etta. Z. Falconer Lecture

## Main Lecture Hall, Friday, August 5, 8:30–9:20 am

## Dawn Lott Delaware State University

## Mathematical Interventions for Aneurysm Treatment

Aneurysm is a vascular disease characterized by a weak or thin spot on a blood vessel that balloons out and fills with blood. Cerebral aneurysm can occur anywhere in the brain, but most are located along a loop of arteries that run between the underside of the brain and the base of the skull (Circle of Willis). Cerebral aneurysms are particularly dangerous because of their difficulty to treat and their high mortality and morbidity rate. There are several risk factors for cerebral aneurysm, among which are hypertension, heavy alcoholic consumption and cigarette smoking. Rupture of cerebral aneurysm (subarachnoid hemorrhage) can cause stroke. Understanding the mathematical relationships and the hemodynamic factors predisposing aneurysms to further growth and rupture will enable neurosurgeons to determine which aneurysms have a high likelihood of rupture and should thus be treated. In addition, such knowledge may also help predict which aneurysms will never rupture. In making such predictions, only patients with a risk of aneurysmal rupture will thus be exposed to the risk of surgery. This talk will discuss mathematical and biomechanical interventions for aneurysm treatment and will address societal effects of this vascular disease.

**Biography** Dr. Dawn Alisha Lott is an Associate Professor of Applied Mathematics at Delaware State University in Dover, Delaware. She earned her Ph.D. in Engineering Sciences and Applied Mathematics at Northwestern University in Evanston, Illinois in 1994. Her major research interest is the numerical study of solutions of partial differential equations that model physical phenomena in nonlinear solid and fluid mechanics, biomechanics and physiology; in particular, research in the development and implementation of effective numerical methods for equations that exhibit localized regions of rapid variations and/or large deformations. Lott's research is divided into two areas: (i) methods for solutions equations that exhibit complex nonlinear waves and mathematical predictions for aneurysm treatment.

She is a member of the Applied Mathematics Research Center (AMRC) and the Center for Research and Education in Optical Science and Applications (CREOSA) at DSU. She is the director of the DSU Honors Program and she serves on the advisory board of the Ronald E. McNair Post-baccalaureate Program.

Currently, Lott is a member of the Association for Women in Mathematics (AWM), the Mathematical Association of America (MAA), the Society for Industrial and Applied Mathematics (SIAM), the American Mathematical Society (AMS), the Biomedical Engineering Society (BMES), and she is Vice President of the National Association of Mathematicians (NAM). She lives in Dover, DE with her husband Kenneth Green and her two children, Samuel and Carilyn.

## James R. Leitzel Lecture

## Main Lecture Hall, Friday, August 5, 10:30–11:20 am

## Philip Kutzko University of Iowa

## Just Walk Away, René: Cultural Issues in Broadening Participation in Mathematics

Science, as we know it today, developed in a particular time and place for reasons that have never fully been explained. The concept of a function—a concept that underlies all of modern science—first appears in Descartes' *La Géometrie* in 1637; within a generation, Newton and Leibniz had developed the calculus and Newton had laid the foundation for modern physics. Similar transformative advances occurred shortly thereafter in chemistry, biology and medicine. This is the context in which we do science today; a West European, Cartesian context in an increasingly non-European nation. The Western approach to science embodies certain cultural values, among them skepticism, objectivity, secularism and a belief in progress as an unmixed virtue. These values are by no means universally accepted, either internationally or within our own country. Further, they have sometimes been used to justify aggression and sometimes worse by Europeans and their descendants in the Americas against other ethnic groups and even against groups within European society. This, it would seem, is reason enough for underrepresented minority groups and other Americans who have not historically been invited to the table to steer clear of European science.

Any approach toward broadening participation in science that fails to take into account this cultural context can only go so far. Examples are afforded by standardized testing and affirmative action each of which is ultimately motivated by the same goal: to remove impediments to access caused by overt ethnic and class discrimination (standardized tests) and by the consequences of such discrimination (affirmative action). Both have been valuable in extending access to ethnic and national groups who have found Western science culturally appealing as well as to individuals with similar proclivities from underrepresented groups; indeed, the use of standardized testing transformed the populations doing science during the *Sputnik* era while affirmative action has been responsible for similar transformations in more recent times. However, these and other strategies that have focused largely on removing barriers to inclusion may be nearing the limit of their utility.

One of the distinctive features of our math department's initiative to broaden participation in our graduate program is the awareness we have developed of the cultural context in which our effort takes place. I will discuss this cultural context in my talk and argue that an understanding of this context can lead to new strategies, strategies which, in our case, have transformed a traditional mathematics department in an ethnically homogeneous state into what some have called a model for what an American math department should look like in the twenty-first century.

**Biography** Phil Kutzko was born and raised in New York City and is a product of the New York City public schools. He attended the City College of New York and received his M.S. and Ph.D. degrees at the University of Wisconsin. He joined the University of Iowa mathematics faculty in 1974. Kutzko's research is in the area of pure mathematics known as *the representation theory of p-adic groups*, an area with applications to the theory of numbers. He is the author, with Colin Bushnell, of a monograph in the Annals of Mathematics Studies (Princeton) and has lectured widely on his work. He is presently a University of Iowa Collegiate Fellow. Kutzko is honored to have played a part in the Department of Mathematics' activities in minority graduate education and in the extension of these activities to other departments at the three Iowa Regents universities. In this context, he directs the departmental Sloan Foundation minority fellowship program as well as the National Alliance for Doctoral Studies in the Mathematical Sciences, an NSF funded project which involves mathematics science departments at a variety of colleges and universities including the three Iowa Regents universities and whose goal is to increase the number of doctoral degrees in the mathematical sciences awarded to underrepresented US minority students. Kutzko was recently honored with the 2008 Presidential Award for Excellence in Science, Mathematics and Engineering Mentoring. This award was presented to him by President Obama in a White House ceremony in January, 2010.

## Pi Mu Epsilon J. Sutherland Frame Lecture

## Main Lecture Hall, Friday, August 5, 8:00-8:50 am

#### Margaret Wright Courant Institute of Mathematical Sciences

#### You Can't Top This: Making Things Better with Mathematics

Many problems in science, engineering, medicine, and life involve choosing the best way (or at least a better way) to do something. Mathematical optimization can often provide the answers we want; the speaker will describe when, why, and how this happens, along with a few examples.

**Biography** Margaret H. Wright is Silver Professor of Computer Science and Mathematics in the Courant Institute of Mathematical Sciences, New York University. She received her B.S. (Mathematics) and M.S. and Ph.D. (Computer Science) from Stanford University. Her research interests include optimization, scientific computing, and optimization in real-world applications. Prior to joining NYU, she worked at Bell Laboratories (Lucent Technologies), where she was named as a Bell Labs Fellow. She was elected to the National Academy of Engineering (1997), the American Academy of Arts and Sciences (2001), and the National Academy of Sciences (2005). During 1995–1996 she served as president of the Society for Industrial and Applied Mathematics (SIAM), and she has chaired advisory committees for several mathematical sciences institutes and government agencies. In 2000, she received an honorary doctorate in Mathematics from the University of Waterloo, and she was named as an honorary doctor of technology by the Swedish Royal Institute of Technology in 2008.

## NAM David Blackwell Lecture

## Main Lecture Hall, Friday, August 5, 1:00–1:50 pm

### Farrah Jackson Chandler Elizabeth City State University

#### Using E-mentoring to Prepare the Next Generation of Mathematics Teachers

It has been well documented that the mathematics and science scores of United States students are lagging behind those of their international peers. A study conducted by the National Commission on Mathematics and Science Teaching for the 21st century suggests that providing more training and professional development opportunities for teachers is the clearest way to increase the achievement level of students in both mathematics and science. In this talk, I will present an overview of work that I have been involved in to prepare and train mathematics teachers through ementoring.

**Biography** Born in Prince George's County Maryland, Dr. Chandler received her Bachelor of Science degree in Mathematics Education from North Carolina Agricultural and Technical State University. She received her M.S. and Ph.D. degree in Mathematics from North Carolina

#### Invited Addresses

State University where she studied symmetric spaces and the classification of involutions of linear algebraic groups. As a graduate, Dr. Chandler was a David and Lucile Packard Fellow and received several awards for her outstanding teaching. Upon graduation Dr. Chandler worked as an Assistant Professor in the Department of Mathematics and Statistics at the University of North Carolina Wilmington. In 2007 she joined the faculty at Elizabeth City State University where she currently serves as the Chair of the Department of Mathematics and Computer Science.

Dr. Chandler currently serves as the Principal Investigator on the National Science Foundation Robert Noyce Teacher Scholarship Grant which seeks to encourage talented STEM majors and professionals to become mathematics and science teachers. Her main research interest is on mentoring pre-service and beginning secondary mathematic teachers, with a specific focus on virtual mentoring. Dr. Chandler has been involved in various programs aimed at increasing the number of women and minorities pursuing graduate degrees in the mathematical sciences. She believes her involvement in training and retaining quality minority teachers will play a key role in producing more underrepresented minorities who pursue Ph.Ds.

## MAA Lecture for Students

## Main Lecture Hall, Thursday, August 4, 1:00 pm

Roger Nelsen Lewis & Clark College

#### Math Icons

An *icon* (from the Greek for "image") is defined as a picture that is universally recognized to be representative of something. The world is full of distinctive icons. Flags and shields represent countries, graphic designs represent commercial enterprises, and computer icons are essential tools for working with a variety of electronic devices from desktop computers to cell phones.

What are the icons of mathematics? Numerals? Symbols? Equations? After many years of working with visual proofs, I believe that certain geometric diagrams play a crucial role in visualizing mathematical proofs. In this talk I'll present several of these diagrams, which I call *math icons*, and explore the mathematics that lies within, and that can be created with them.

**Biography** Roger Nelsen is Professor Emeritus of Mathematics at Lewis & Clark College in Portland, Oregon, where he taught for 40 years.

His research interests lie in the area of mathematical statistics, where he uses copulas (multivariate distribution functions with uniform margins) to study dependence among random variables. He is also interested in the process of visualization in mathematics, specifically how mathematical drawings help students understand mathematical ideas, arguments, and proofs.

He has authored over 100 research and expository papers, and six books. The most recent, with co-author Claudi Alsina, is *Charming Proofs: A Journey into Elegant Mathematics*.

He enjoys travel, photography, and sailing on the Willamette River in Portland.

## **Presentations by Alder Awards Winners**

## Main Lecture Hall, Friday, August 5, 2:00-3:30 pm

Alissa Crans Loyola Marymount University

## Count Me In!

## 2:00 pm

Building and sustaining an inclusive mathematical community both locally and nationally has involved, and continues to require, the efforts of many all working toward this common goal. It includes, among many other things, devoted mentoring, inspiring and engaging students at all levels, and increasing students' appreciation and enthusiasm for the beauty, creativity, and excitement of mathematics. While many challenges still remain, we will discuss my small efforts at "paying it forward" in my classes and in our community in gratitude for the support, advice, and encouragement I have received from the numerous mathematicians and professional organizations dedicated to increasing the number of women in the mathematical sciences.

## Sarah Eichhorn University of California Irvine

## A Course in Computational Applied Mathematics Undergraduate Research: Game Theory for Fun and Profit 2:30 pm

iCAMP is UC Irvine's new program for first and second year undergraduates to get early research experience in computational applied mathematics (funded by NSF-PRISM grant). In this talk, we will discuss one of the new courses on Game Theory developed as part of the iCAMP program. This course is designed to prepare students with just a Calculus background to be able to do original research by the end of the course. Through a combination of instruction in traditional game theory and computer adaptive learning, students were taught how to formulate conjectures and rigorously analyze various games with associated open problems.

This presentation will include information about the course content, sample assignments, sources for research project topics and samples of undergraduate student research results. There will also be discussion about the hurdles and successes to transitioning students from classroom learners to independent researchers.

## Sam Vandervelde St Lawrence University

## Greater Than the Sum of Its Parts

## 3:00 pm

To the uninitiated it may seem that the ideal mathematics position would involve teaching smart kids at a prestigious institution, preferably with a generous complement of graders and well-equipped classrooms. Fortunately a productive, rewarding teaching career is largely independent of all these criteria. To illustrate the sorts of factors that have actually made a significant impact upon my quality of life as a teacher, we will consider Mathematical Outings, IF-AT movies, and the case of Chris. For it is in the fabulous subject we teach, the new ideas we encounter and implement, and the students whom we inspire and by whom we are challenged that teaching exceeds our expectations.

## **Invited Paper Sessions**

## **Connections to Complex Dynamics**

## Session 1: Thursday, August 4, 1:00-4:30 pm in Heritage 2

**Rodrigo Perez** Indiana University - Purdue University Indianapolis (rperez@math.iupui.edu)

Roland Roeder Indiana University - Purdue University Indianapolis (rroeder@math.iupui.edu)

This session will complement the Joint AMS-MAA Invited Address by Laura DeMarco. The speakers represent many perspectives on Complex Dynamics in one and several variables. Many of the talks will connect Complex Dynamics with other areas, including Topology, Algebraic Geometry, and Mathematical Physics.

## Jeffrey Diller Notre Dame (diller.1@nd.edu)

## **Complex Dynamics in Two Variables: an Example**

## 1:00-1:30 pm

There's been a lot of recent work devoted to understanding dynamics of polynomial and rational maps of two or more variables. My goal in this talk is to discuss a very particular example in order to give an idea of what's at stake. I'll rely on lots of pictures, a little combinatorics, and a little bit of algebraic geometry, repeatedly arriving at the Fibonacci sequence.

Roland Roeder Indiana University - Purdue University Indianapolis (rroeder@math.iupui.edu)

## Lee-Yang-Fisher Zeros and Rational Dynamics

1:45-2:15 pm

A classical approach to understanding the Ising model of magnets is to study the limiting distributions of certain zeros of polynomials. In the 1950s, Lee and Yang considered the distribution of these zeros as a function of an external magnetic field. Later in the 1960s, Fisher considered similar distributions of zeros in terms of the complex temperature of the system. I will describe both of these settings and how they can be combined to form the Lee-Yang-Fisher zeros, a union of algebraic curves in two-dimensional complex space. I will conclude by describing a specific lattice for which study of the limiting distribution of Lee-Yang-Fisher zeros becomes manageable through the use of complex dynamics (in 2 variables). This is joint work with Pavel Bleher and Misha Lyubich.

## Joshua Bowman SUNY Stony Brook (joshua.bowman@gmail.com)

## **Rock-Paper-Scissors in the Complex World**

2:30-3:00 pm

This will be a report on a previously unexplored family of polynomial maps on  $P^2$ . They have a geometric characterization that shows their origins in the field of mathematical biology, and exhibit much of the splendid complexity often found in such dynamical systems.

## Jan-Li Lin Indiana University (janlin@indiana.edu)

## **Three-Dimensional Monomial Maps**

## 3:15-3:45 pm

Two-dimensional monomial maps are completely classified by Favre, Jonsson, and Wulcan. In this talk, I will present my work on classifying three-dimensional monomial maps by their dynamical behavior.

## Paul Blanchard Boston University (paul@bu.edu)

## **Checkerboard Julia Sets**

4:00-4:30 pm

In this talk, we discuss joint work with Figen Çilingir, Daniel Cuzzocreo, Robert L. Devaney, Daniel M. Look, and Elizabeth D. Russell on the family of rational maps

$$F_{\lambda}(z) = z^n + \frac{\lambda}{z^d},$$

where  $n \ge 2$ ,  $d \ge 1$ , and  $\lambda \in \mathbb{C}$ . We consider the case where  $\lambda$  lies in the main cardioid of one of the n-1 principal Mandelbrot sets in these families. We show that the Julia sets of these maps are always homeomorphic. However, two such maps  $F_{\lambda}$  and  $F_{\mu}$  are conjugate on these Julia sets only if the parameters at the centers of the given cardioids satisfy  $\mu = \nu^{j(d+1)}\lambda$  or  $\mu = \nu^{j(d+1)}\overline{\lambda}$ , where  $j \in \mathbb{Z}$  and  $\nu$  is an (n-1)st root of unity. We define a dynamical invariant, which we call the minimal rotation number. It determines which of these maps are conjugate on their Julia sets, and we obtain an exact count of the number of distinct conjugacy classes of maps drawn from these main cardioids.

## Session 2: Friday, August 5, 1:00-4:30 pm in Heritage 2

## Araceli Bonifant University of Rhode Island (bonifant@math.uri.edu)

## The Period *p*-Curve for Cubic Polynomials

1:00–1:30 pm

In this talk I will describe the parameter space for monic centered cubic polynomials with a marked critical point of period *p*, and discuss its topology.

## Rodrigo Perez Indiana University - Purdue University Indianapolis (rperez@math.iupui.edu)

## A Brief But Historic Article of Siegel

#### 1:45-2:15 pm

One of the most influential papers of the 20th century is just six pages long. Make no mistake, despite its brevity, Siegel's 1942 result is very profound and inspired much work in celestial mechanics and dynamical systems (including the KAM Theorem). The main goal of this talk is to convince you that a motivated undergraduate student can understand the entire proof. With this in mind, I will explain the problem of small denominators, and guide you through the basic steps of Siegel's argument.

## **Bob Devaney** Boston University (bob@bu.edu)

## Julia Sets of Rational Maps Converging to Filled Julia Sets of Quadratic Polynomials

2:30-3:00 pm

In this lecture we show that the Julia set of the rational map  $z^2 + C/z^2$  converges to the filled Julia set of  $z^2$  (i.e., the unit disk) as C tends to 0. This is interesting because Julia sets can never contain open sets unless these sets are the entire complex plane. We also give several other examples where similar things occur in other families of rational maps. And then we show that this never occurs for the family of maps  $z^n + C/z^n$  when n > 2.

## Clinton Curry SUNY Stony Brook (clintonc@math.sunysb.edu)

## Wandering Vertices and Condensity in Julia Sets

3:15-3:45 pm

Let P be a polynomial with connected Julia set J. A point  $z_0 \in J$  is called a wandering vertex if three conditions hold: First,  $J \setminus \{z_0\}$  has at least three components; second,  $z_0$  has an infinite orbit; and third, no critical point of P is in the orbit of  $z_0$ . Thurston proved in the 80's that quadratic polynomials have no wandering vertices, whereas Alexander Blokh and Lex Oversteegen found examples of cubic polynomials whose Julia sets are locally connected and which have wandering vertices.

In this talk, we discuss a more flexible construction developed jointly with Blokh and Oversteegen. We find a set of polynomials *P*, dense in the appropriate parameter space, with locally connected Julia sets and wandering vertices. Even more, the wandering vertex itself intersects every continuum in the Julia set, a property we call condensity. If time allows, we will also discuss ramifications of having a condense orbit.

## **Phil Mummert** Taylor University (phmummert@taylor.edu)

### Algorithms for Finding the Julia Sets of Hénon Maps

## 4:00-4:30 pm

We present some algorithmic procedures that naturally impose a symbolic dynamics on the repelling periodic points of the quadratic complex Hénon map. We introduce a system of differential equations due to Biham and Wenzel, a pseudo-orbit correcting operator for implicit shadowing from the anti-integrable limit due to ideas of Sterling and Meiss, and a topological path-lifting scheme related to iterated monodromy groups. All these procedures can be used to generate pictures of some interesting four-dimensional fractals.

## **Offering Students Lessons Beyond Mathematics, Through Mathematics**

## Session 1: Thursday, August 4, 2:00-4:00 pm in Heritage 3

## **Edward Burger** Williams College (Edward.B.Burger@williams.edu)

Here we explore what mathematicians can offer the world long after that world forgot how to solve for 'x' and how to graph a cubic. Speakers will offer innovations in reaching non-science students through various means including unconventional campus-wide projects, leading faculty development workshops, and reaching beyond to the general public. Here we offer a wide range of ideas and perspectives on how to offer people—at all levels and backgrounds–lessons through math that will transform lives (for the better).

## Michael Starbird The University of Texas - Austin (starbird@math.utexas.edu)

#### Teaching Lessons That Last a Lifetime

2:00-2:20 pm

Two issues of instruction are: What to teach and how to teach it. What to teach: Teach creative problem-solving, curiosity, independent thinking, willingness to make mistakes, and the ability to distinguish correct from flawed reasoning. How to teach it: Use Inquiry-Based

Learning methods that focus on what the students are doing. When students are asked to figure out concepts on their own, they often learn general thinking attitudes and skills that last a lifetime.

#### Harry Lucas, Jr. The Educational Advancement Foundation (hlucas@edu-adv-foundation.org)

## An Entrepreneur's Appreciation of the Moore Method: How Lessons Learned from R.L. Moore's Classes Helped me Succeed in Life and in Business

## 2:30-2:50 pm

My lifelong learning started with two and a half years in R.L. Moore's sequence at The University of Texas at Austin. Having never pursued research mathematics beyond Dr. Moore's senior level "Foundations of Mathematics" class, I spent my career in business. Here I will share the story of lessons learned through mathematics and the positive effect they have had on my life. That early educational experience has moved me to support similar inquiry-based learning initiatives through the non-profit foundation I established, The Educational Advancement Foundation, as well as our annual Legacy of RL Moore Conference (legacyrlmoore.org/events.html)

## Deborah Bergstrand Swarthmore College (dbergst1@swarthmore.edu)

## Thinking, Writing, and Life-Long Learning

3:00-3:20 pm

Two courses with very different audiences and formats encompass very similar goals: exploration of mathematics as a way to enhance thinking, to sharpen skills, and to broaden views of the world. A course for non-science undergraduates emphasizes writing skills, incorporating mathematical thinking into writing assignments that often accommodate students' other intellectual interests. A non-credit seminar for adults invites life-long learners to explore and discuss profound mathematical ideas. In addition to describing these two courses, we will discuss how the focus and standards of the undergraduate course have evolved as well as the rewards of working with adult learners.

## Lew Ludwig Denison University (Denison University)

## When Am I Ever Going to Use This? Developing the Life Skill of Thinking Mathematically

## 3:30–3:50 pm

According to data from the Conference Board of Mathematical Science, roughly 2.5% of college students take a math course beyond the calculus sequence. So for 97.5% of our students, calculus (or some precursor) is their last math class. But how many times will the average person use calculus in his or her daily life? Here we will discuss two courses (one for non-majors, the other for majors) I have developed in which students explore a range of mathematics beyond calculus without completing the calculus sequence. While we discuss enticing ideas like the size of infinity or if two people in New York City have the same hair count, the main point of these courses is not the mathematical content, but the process of thinking like a mathematician—a skill that can be used on a daily basis throughout their lives.

## Session 2: Friday, August 5, 2:30–4:00 pm in Heritage 3

## Christina Carter Buffalo State College (cartercl@buffalostate.edu)

## Changing Instructor Perception of Mathematics and its Role in the Intellectual Development of Liberal Arts Students 2:30–2:50 pm

Supported by a five year Title III grant and our college's unwavering commitment to the principles of a liberal arts education, we have successfully designed a LAS math course that both surprises students and motivates them to think in ways they never imagined they could. I will talk specifically about the long term professional development necessary to win the confidence and enthusiasm of our large team of mostly part time instructors. The college's commitment to support faculty development workshops, accommodate changes in pedagogical practice and operate a student run tutoring lab has been critical to the success of our course. As evidence of that success I will share some of the work our students have done.

## "Card Colm" Mulcahy Spelman College (colm@spelman.edu)

#### Where's the Magic?

## 3:00-3:20 pm

Mathematics, like literature, music and art, is a magical, thriving, dynamic area of human creativity. It has many sparkling gems and open questions, both classical and modern, that are accessible to educated adults with essentially no specialized training. Little, if any, evidence of this can be found in either the courses most of our students take, or in the mainstream media. On the contrary, it often seems that the way we feel we must teach crushes curiosity and stifles the imagination, while concealing the organic process of mathematical discovery and the joy of thinking. Even worse, decades later, too many people who were once students are happy to brag about how little mathematics they remember. What are the consequences of all of this for our curriculum, our profession, and for humankind in general? Is it time to inject a little magic into all of our courses?

## Candice H. Dance Onondaga Community College (SUNY) (dancec@sunyocc.edu)

## The Nature of Mathematics: Projects from Inspired Students

## 3:00-3:20 pm

Each student in our liberal arts math class creates a project of his/her choosing that will connect "math" to a personal love—art, music, sports, gardening, jewelry making - to name just a few possibilities. The finished projects have generated so much excitement on campus that they have become a featured highlight of our annual Math Appreciation Week. This presentation will explore how to engage students in math through a project-based course and further show how the enthusiasm from such projects can reverberate campus wide, shattering preconceived "I can't do math" notions. Sample projects will be included.

## Polyhedra are Everywhere!

## Friday, August 5, 2:00–5:00 pm in Thoroughbred 1

## **Benjamin Braun** University of Kentucky ()

The study of polyhedra began in antiquity, yet they play a prominent role throughout modern mathematical research. This session will focus on the ubiquity of polyhedra in mathematics by introducing various contexts in which they arise, including number theory, applied analysis, computational mathematics, algebra, combinatorics, and math education.

## **Matthias Beck** San Francisco State University (beck@math.sfsu.edu)

## Polyhedra in Number Theory: Integer Partitions from a Geometric Viewpoint

## 2:00-2:20 pm

The study of partitions and compositions (i.e., ordered partitions) of integers goes back centuries and has applications in various areas within and outside of mathematics. Partition analysis is full of beautiful—and sometimes surprising—identities, starting with Euler's classic theorem that the number of partitions of an integer k into odd parts equals the number of partitions into distinct parts.

Motivated by work of G. Andrews et al from the last 1 1/2 decades, we will show how one can shed light to certain classes of partition identities by interpreting partitions as integer points in polyhedra. Our approach yields both "short" proofs of known results and new theorems. This is joint work with Ben Braun, Ira Gessel, Nguyen Le, Sunyoung Lee, and Carla Savage.

This is joint work with Ben Braun, ira Gessel, Nguyen Le, Sunyoung Lee, and Carla Savage

## Jesus De Loera University of California - Davis (deloera@math.ucdavis.edu)

## Polyhedra in Optimization: Why Calculus Methods Fail to Really Integrate Over a Polyhedron and How to Actually Do It! 2:30–2:50 pm

The volumes of and integrals over polyhedra are perhaps the most fundamental and basic concept in the history of mathematics. Already ancient civilizations worried about it (e.g., Egyptian, Babylonian) and we teach formulas for volumes of pyramids and cubes to K-6 students. Yet, volumes and integrals of convex polytopes are properties to be computed, from algebraic geometry to computer graphics, from combinatorics to probability and statistics.

So, how does one go about actually computing an integral over a convex polytope if one cares to compute the number exactly? In this talk we survey why exact integral computation is relevant in everyone's life, why calculus techniques fail miserably for the goal, what is currently known about efficient computation of integrals of polynomials over convex polytopes. Software may be used in this talk.

## Carl Lee University of Kentucky (lee@ms.uky.edu)

## Polyhedra in Math Education

3:00-3:20 pm

We will discuss examples of where polyhedra have appeared in the K–16 curriculum, and then consider some ideas for additional topics and activities involving polyhedra that might be propitiously incorporated into the curriculum.

## Caroline Klivans The University of Chicago (cjk@math.uchicago.edu)

#### Polyhedra in Geometric Combinatorics: The Characteristic Polynomial Strikes Again

## 3:30-3:50 pm

Motivated by the problem of p-value calculations in statistical hypothesis testing, we study the projection volumes of polyhedral cones. Namely, given a polyhedral cone C in n-dimensional real space, which fraction of the unit sphere is occupied by points x such that the orthogonal projection of x onto C lies in the interior of a k-dimensional face of C?

For cones arising from reflection arrangements, the answer is given by the coefficients of the characteristic polynomial associated to the hyperplanes which bound the cone. The proof of this result is achieved by considering the geometry and combinatorics of angle sums of zonotopes, a particularly nice class of polyhedra.

(No knowledge of any of the above terms will be assumed for the talk.)

Joint work with Mathias Drton and Ed Swartz.

## **Jeffrey Blanchard** Grinnell College and University of Edinburgh (jeff@math.grinnell.edu)

#### Polyhedra in Compressed Sensing

#### 4:00-4:20 pm

Classically, a typical measured signal will be sparse after an appropriate transformation, meaning that most of its components will be very close to zero. After transforming a signal, one compresses it by setting all of these small values to zero. In Compressed Sensing, a recent twist in the theory of digital signal processing, the opposite approach is taken. It is assumed that the signal being measured is sparse, and that far fewer measurements are taken than the number of components in the signal. This leads to an underdetermined system of linear equations, which can have infinitely many solutions, so recovering the actual signal we measured among this infinite set of solutions seems like a lost cause.

Surprisingly, we can use the sparsity assumption to our advantage, and can develop tractable algorithms to recover our original signal more often that you might think. Further, the techniques of convex optimization used for finding this sparest solution lead to the study of the neighborliness of a polytope associated with the problem. In this talk, we will discuss this relationship and similar relationships to related problems.

Margaret Readdy University of Kentucky and the Institute for Advanced Study (readdy@math.ias.edu)

## Polyhedra in Algebraic Combinatorics: An Eulerian Relation for the Semisuspension

4:30-4:50 pm

The flag vector contains all the face incidence data of a polytope, and in the poset setting, the chain enumerative data. It is a classical result due to Bayer and Klapper that for face lattices of polytopes, and more generally, Eulerian graded posets, the flag vector can be written as a cd-index, a non-commutative polynomial which removes all the linear redundancies among the flag vector entries. This result holds for regular CW complexes. We relax the regularity conditions to show the cd-index exists for non-regular CW complexes and extend the notion of a graded poset to that of a quasi-graded poset.

This is work-in-progress with Richard Ehrenborg and Mark Goresky.

## **Projective Geometry Applied to Perspective Art**

## Saturday, August 6, 1:00-4:00 pm in Heritage 2

Annalisa Crannell Franklin & Marshall College (annalisa.crannell@fandm.edu)

**Marc Frantz** Indiana University (mfrantz@indiana.edu)

The rich and beautiful subject of projective geometry can be enjoyed as an abstract, axiomatic subject. But in this session, we examine projective geometry as an applied subject, paying especial attention to the creation and analysis of perspective art.

**Don Row** University of Tasmania (DonRow@matroid.net)

### One-point, Two-point and Three-point Perspective Drawings and How to View Them

#### 1:00-1:20 pm

We begin with the idea of perspective drawing that was familiar to Albrecht Dürer and Leonardo da Vinci, and satisfy ourselves that extended Euclidean space is the appropriate environment for our discussion, classification, and creation of such drawings.

Da Vinci's words guide our approach to anamorphic art and the apparent distortion of perspective drawings.

We conclude by establishing necessary and sufficient conditions, based only on evidence internal to the drawing itself, for a point to be a viewpoint of any given perspective drawing.

## Dick Termes Artist (termes@blackhills.com)

#### The Geometry of Visual Space

#### 1:30-1:50 pm

Dick Termes is a professional artist whose creations have been featured in many mathematics publications. This talk describes the mathematics behind his eponymous *Termespheres*. Imagine you are inside St. Denis Cathedral in northern Paris. You are inside of a transparent sphere with a full set of paints and you begin to paint on the inside of the sphere, copying what you see outside the sphere — north, east, south west, up and down. Now you move to the outside of the sphere to look at what you were inside of. With many visual examples, Termes will show how perfectly these images fit on the sphere and why this technique involves six-point perspective. He will talk about the importance of six-point perspective and why it involves the understanding of three-dimensional geometries.

## Talmadge James Reid University of Mississippi (mmreid@gmail.com)

## **Scene Analysis**

#### 2:00-2:30 pm

A scene is a perspective drawing of a non-planar figure. We investigate when a planar figure is a scene. We present a sufficient condition that ensures that a given planar set of lines is a perspective drawing of the pairwise intersection of planes. Using this condition and others, we consider scenes representing various figures such as the non-planar Desarges figure. This work is based on a chapter of a forthcoming book by Don Row and Talmage James Reid entitled *Geometry, Perspective Drawings, and Mechanisms* to be published by World Scientific Publishing CO.

## Norman J. Wildberger University of New South Wales (n.wildberger@unsw.edu.au)

## Straightening out Escher: Projective Geometry as a Gateway to Visual Hyperbolic Geometry

## 2:30-2:50 pm

The well-known Dutch artist M. Escher was a long time friend of the geometer D. Coxeter, who explained to him the Beltrami-Poincaré model of hyperbolic geometry, involving arcs of perpendicular circles as geodesics inside a disk, with reflections given by Mobius inversions in those circles. What would Escher have done with the Beltrami-Klein model, in which the geodesics are actually straight lines inside the disk?

In this talk we present a new approach to hyperbolic geometry which is based on projective geometry, and so extends the Beltrami-Klein model to include points and lines outside the disk, in fact also the points and line at infinity. The basic symmetries become projective reflections. By measuring not the transcendental quantities distance and angle but rather algebraic replacements called quadrance and spread coming from Rational Trigonometry, we can compute faster and more accurately, everywhere in the model. And the basic trigonometric relations become polynomial!

So a fresh visual world of hyperbolic geometry emerges, with beautiful new theorems, and novel opportunities for artists and graphics illustrators to merge art and mathematics to create new patterns and designs. This talk will provide an introduction to the speaker's YouTube series "Universal Hyperbolic Geometry" on the subject (which provides a lot more additional information), and will be mostly pictures.

## Tomás García-Salgado UNAM, Mexico (tgsalgado@perspectivegeometry.com)

## Modular Perspective vs. Traditional Methods

## 3:00-3:20 pm

To introduce the principles of "Modular Perspective" (MP) I will explain how its own model works. This model comprises three planes: the Plane of Symmetry X (PLSX) and the Plane of Symmetry Y (PLSY) that together (PLSX/Y) conform a spatial system of coordinates; and a third plane called the Perspective Plane (PPL), in which the perspective of the object is projected. To bring forwards the main difference between MP and traditional methods, I have prepared two questions, which might sound somewhat tricky, but they are not. Then I briefly describe the five cases of punctual projection through which you can draw any point on the PPL. And finally, I will show some application examples.

## Marc Frantz Indiana University (mfrantz@indiana.edu)

## The Most Underrated Theorem in Projective Geometry

#### 3:30-3:50 pm

Rather than dare to name such a theorem, we discuss three possible candidates for the most underrated theorem in projective geometry. Each theorem establishes a numerical projective invariant having both practical and pedagogical value. We demonstrate all three theorems using photographs of a local discount department store.

## Cultural and Philosophic Underpinnings of Western Science: Implications for American Mathematics in the Twenty-First Century

## Saturday, August 6, 1:00–4:00 pm in Heritage 3

## Philip Kutzko University of Iowa

We will explore the role played by unexamined cultural assumptions in the practice of modern mathematics in the persistently low number of mathematicians and statisticians from US ethnic groups that are traditionally underrepresented in these fields.

## Edray Herber Goins Purdue University

## Transforming Undergraduates into Researchers: Best Practices from an Afrocentric Perspective

Many research experiences for undergraduates (REUs) cater to just the most prodigious students, subscribing to the notion that "a mathematician's best work is done by age 40." There are some programs, however, that focus on building a community through a support network, subscribing to the notion that "it takes a village to raise a child." By examining several cohorts of underrepresented minority undergraduates who experienced summer research, we observe some best practices which have made young mathematicians of color successful.

## Douglas Mupasiri University of Northern Iowa

## The Outsider's Challenge in Learning Science: culture and/or orientation to scientific thinking and practice

Efforts to broaden participation to groups that have been traditionally underrepresented in in STEM disciplines all too often encounter significant challenges. One school of thought sees the problem as having a cultural basis and another sees the problem simply as one of orientation to the methods of scientific practice. Whatever the case, the solution seems to call for a different paradigm of teaching, which incidentally holds the promise of benefiting majority students as well.

## David Manderscheid University of Nebraska

#### Cultural Aspects of Finding and Keeping Graduate Students in the Mathematical Sciences

I will discuss proven methods to recruit and retain graduate students, especially students from underrepresented groups. I will emphasize methods that are scalable and their cultural underpinning. I will frame the discussion using my experiences as a student, a faculty member, a graduate director, a department chair and a dean.

## Paulette Willis University of Houston

## Cultural issues in transitioning to graduate school in mathematics: a personal perspective

I will discuss cultural ideas and expectations of various minority groups that affect successfully transitioning to graduate school.

## **Contributed Paper Sessions**

## Fostering, Supporting and Propagating Math Circles for Students and Teachers

## Session 1: Thursday, August 4, 8:30–10:30 am in Elkhorn A

Debra Geddings	University of South Carolina (geddings@math.sc.edu)
Nieves McNulty	Columbia College (nmcnulty@columbiasc.edu)
Douglas Meade	University of South Carolina (meade@math.sc.edu)

## The Beginnings of a Math Teachers' Circle

Earlier this summer a team of faculty from the University of South Carolina and Columbia College and middle school mathematics teachers attended the How To Run a Math Teachers' Circle workshop hosted by AIM in Palo Alto, CA. In addition to reporting on our experiences at the workshop, we will discuss our plans for our first meetings later this year and our current vision for this Math Teachers' Circle. In addition to being a resource for middle school mathematics teachers throughout South Carolina, we want to coordinate our efforts with recently created local Math Circles and provide opportunities for pre-service teachers at USC and Columbia College to begin to integrate into their future profession by developing professional relationships with current middle school mathematics teachers and by serving as mentors for the Math Circles.

This presentation will be particularly appropriate for anyone interested in starting a new Math Teachers' Circle and thinking about attending any of the AIM-sponsored workshops.

## Japheth Wood Bard College (jwood@bard.edu)

## Math Circles along the Hudson River: from New York City to Albany

Math Circles, including the New York Math Circle, the Bard Math Circle and the Albany Area Math Circle, have established themselves in cities along the Hudson River over the last decade. This talk is a description of how these efforts have interacted and cooperated to provide high quality math enrichment to students and teachers. Some ideas on future directions will also be presented.

## David Patrick Art of Problem Solving (patrick@artofproblemsolving.com)

## San Diego Math Circle: History, Structure, and Curriculum

We will discuss the history, organization, and mathematical content of the San Diego Math Circle (SDMC). SDMC serves students in grades 5–12 with three groups: Fermat (grades 5–7), Euler (grades 7–9), and Gauss (grades 9–12). The mathematical curriculum of SDMC can be thought of as three main components: (1) self-contained one-session topics, (2) a longitudinal sequence of classes used throughout the year (with the Euler group), and (3) contest-related activities including SDMC's ARML team. We will discuss aspects of these three main curricular components, along with some suggested resources for each. We also discuss the interaction between the different stakeholders involved with SDMC, including students, parents, teachers, UC San Diego, Art of Problem Solving, and corporate sponsors.

**Diana White** University of Colorado Denver (diana.white@ucdenver.edu)

## Math Teachers' Circles—A Descriptive Analysis of Final Evaluations from Summer Workshops

The Math Teachers' Circle Program, developed through the American Institute of Mathematics in 2006, is a national network of professional development programs designed to establish the foundation for a culture of problem solving among middle school math teachers in the United States. There are approximately 23 local chapters across the country, with 10-12 more expected to launch in Summer 2011. By offering teachers the opportunity to engage in open-ended mathematical problems in sessions facilitated by mathematicians, the program strives to better equip teachers to initiate more student-centered, inquiry-based pedagogies in their mathematical classrooms. To begin to investigate this, we analyzed teacher comments from the final evaluations of three immersion workshops from Summer 2010. In this talk, we report on our preliminary findings from that work, as well as discuss future directions of the research.

## Francis Edward Su Harvey Mudd College (su@math.hmc.edu)

## AMS Epsilon: Funding for Summer Math Camps

In 1999 the American Mathematical Society started the Epsilon Fund to provide partial funding to support summer programs for mathematically talented middle and high school students, especially replicable and innovative ones. I will explain the Epsilon Awards Program, and discuss my own rewarding experiences in working with a program that has received such funding: MathPath.

## Brandy Wiegers Mathematical Sciences Research Institute/ San Francisco State University (brandy@msri.org)

#### Always Be Prepared: Tips for Safe Outreach Programs

Math Circles should be fun and engaging. To keep it this way it is important to be prepared with a box of tricks and some quick plans for the worst case scenario. In this session we'll briefly discuss what we keep in our Math Circle Box of Supplies, important legal aspects of working with minors including adult to student ratios and the buddy system. We'll conclude the session discussing participant waivers and plans for emergencies. With a little bit of work we can all be more prepared to ensure that we never need to use our emergency plan. This way we can get back to math and everyone can have fun!

## Session 2: Friday, August 5, 8:30–10:30 am in Elkhorn A

**Pari Ford** University of Nebraska at Kearney (fordpl@unk.edu)

## Central Nebraska Math Teachers' Circle and Sticky Note Mathematics

The Central Nebraska Math Teachers' Circle was established in the fall of 2008. The group meets 5 times during the year, connecting teacher candidates and faculty from the University of Nebraska at Kearney and practicing middle school teachers, some of which travel more than an hour for our meetings. In addition to providing more information about our mission to create a network of confident problem solvers in rural Nebraska, this talk will also include our most recent activity, Sticky Note Mathematics. One goal of the session was to create a large square using a given number of 3-inch by 3-inch sticky notes without any gaps or overlaps. When the number of sticky notes is not a perfect square, the process may involve constructing a shape that is almost a square. The problem solving challenges participants' number sense, mental estimation, and flexibility with fractions. An indirect outcome of the session was a connection to mental computation of squares of fractions.

## **Robert Sachs** George Mason University (rsachs@gmu.edu)

## Polynomial Interpolation (Newton) and Related Activities

A session or more of my math student circle will soon try (for the first time) some activities that hopefully enable them to discover Newton's interpolation for polynomials. I will present the results in this session and seek feedback and improvements. The start will be students having created a fairly simple polynomial of known maximal degree for the others to identify using function values. Stage 2 is aiming for the minimum number of values needed. Stage 3 is to identify the algorithm(s). Beyond that, it is hoped to play a bit more. The interpolation polynomial is highly symmetric but the standard algorithm is not. One can also view it as a recursive problem. Evaluation of polynomials by nested multiplication should hopefully emerge as well.

## Tatiana Shubin San Jose State University (tatiana.shubin@sjsu.edu)

## Geometric Combinatorics — a Treasure Trove of Math Circle Problems

In this talk we will discuss several sessions of math circles—students' and teachers' ones—inspired by solved and unsolved problems in geometric combinatorics, such as integer point sets or chromatic numbers. An abundance of problems will be provided to challenge and delight you and your circlers!

#### Harold Reiter University of North Carolina Charlotte (HBREITER@uncc.edu)

#### Using Modified KenKen puzzles in Math Circles

This talk examines several extensions of the popular puzzle KenKen. In some extensions, the set of allowed entries (usually the digits 1 to n) is replaced by a set of n primes. In another, the target numbers are given but the operations are not. In others, the distribution of digits is given but the cages must be determined. In yet another, the operations are not the usual multiplication, division, addition, and subtraction.

## James Tanton St. Mark's School (jamestanton@stmarksschool.org)

#### Fibonacci Surprises

In this talk we utilize the power of a picture to generate a host of surprising appearances of the Fibonacci numbers. The creativity of play will astound and delight, and provide fodder for deep mathematical thinking for you and your students. Mathematical poetry on full display!

## **Discussion Session**

10:10-10:30 am

## Mathematical Modeling Projects that Matter

## Session 1: Thursday, August 4, 8:30–10:30 am in Elkhorn B

## Christopher Thron Texas A&M University - Central Texas (thron@ct.tamus.edu)

## Student Mathematical Modeling Projects with Interactive Excel Spreadsheets

Two semester-long student projects are described. Both involved the students constructing interactive spreadsheets that modeled practical situations. One student modeled the effect of different grading schemes on high school math students' semester grades. Another student modeled projected auto repair cost distributions based on the auto's repair history (using data from multiple vehicles). Students created posters and gave presentations at two separate student research conferences. The projects were valuable learning experiences for the students on several levels.

## **Eileen Fernandez** Montclair State University (fernandeze@mail.montclair.edu)

## Modeling Loan Payments With Algebra and Spreadsheets

This proposal's lessons were developed for a 100-level undergraduate course on modeling with spreadsheets. The lessons use the formula Savings =  $P \cdot (1 + \frac{r}{n})^{nt}$  to derive the complicated Loan Payment Equation whose solution gives the periodic payment needed to pay back a loan with an annual rate r, compounded n times a year, for t years. The derivation reveals how the Savings formula can be interpreted to model a lending institution's thinking and the (present) value the institution places on each payment. Synthesizing information from the resulting algebraic expressions generates the Loan Payment Equation. After the equation is solved, a spreadsheet design demonstrates how to build on this derivation to model the loan's amortization, the payment breakdown to the bank and to reducing the loan, and consequences to the lender and buyer of changing conditions on the loan.

In my presentation, I describe features of deriving the equation and designing the spreadsheet as exercises for students in modeling different features of the loan process. I also discuss how the resulting algebraic equation and spreadsheet design give students opportunities to exploit patterns, make and test predictions, and interpret variables in multiple settings (in an equation and in technology)—all important aspects of the modeling process. Finally, the comprehensive nature of this exercise, from derivation to solution to technology application, builds students' appreciation for the connections between these phases and how essential they are in ensuring a valid use of the model in students' lives.

## John H. Doty University of Dayton (john.doty@notes.udayton.edu)

## Mathematical Analysis of a Simple In-Class Experiment to Emphasize Use of Statistical Tools in Learning

A simple rubber band launcher system was developed and experimental data gathered for launch distance with the purpose of illustrating mathematical analyses of experimental data. The goal of the lab session is to apply basic statistical concepts using several software tools to demonstrate simple experimental analysis techniques. Specific objectives of the lab are to develop descriptive statistics of experimental data such as mean, variance, range, etc. as well as to characterize the data via a univariate regression equation. The final objective is to validate the prediction of the launch distance using the regression equation with follow-on experiments.

The data set obtained in this experiment as well as the analyses are developed in a manner that supports the learning process. The lab is very hands-on and can be performed in a typical 45–50 minute class period.

Learning Objectives:

- 1) Apply basic engineering design concepts to build a quantifiable launcher system.
- 2) Develop univariate experimental test matrix suitable for statistical analysis.
- 3) Apply basic data acquisition methods and quantify data precision requirements.
- 4) Be aware of data integrity issues such as roundoff, truncation, and interface errors.
- 5) Apply basic statistical analyses to infer meaning from experimental data (histograms, descriptive statistics, regression equation).
- 6) Develop data presentation skills for visualizing experimental results.
- 7) Improve writing skills by presenting results in professional report.
- 8) Creatively apply minimal resources to solve a problem of interest.
- 9) Enjoy the process!

## Jerry Dwyer Texas Tech University (jerry.dwyer@ttu.edu)

Patty Schovanec Texas Tech University (patty.schovanec@ttu.edu)

## Mathematical Modeling for Pre-Service Middle School Teachers

Texas Tech University offers a series of specialized mathematics courses for pre-service middle school teachers. Some deficiencies in students' conceptual understanding, math modeling, and the use of technology have appeared over several years. A new course on Mathematical Modeling has been developed. The course introduces students to linear models, curve fitting, exponential growth, population models, and logistic growth models. Technology such as Maple programming and Excel spreadsheets are used. Applications from business and science are included. Students have appreciated the new course and have developed an understanding not found in previous related courses. This course may provide a model for other institutions as they develop courses for middle school teachers.

## David Arney United States Military Academy (david.arney@usma.edu)

## Modeling Cooperative Systems

The Cooperative Systems course connects various modeling methodologies and perspectives across mathematics and other disciplines to build a framework for humanistic/social problem solving involving cooperative entities. This presentation describes the elements of a new undergraduate mathematics course that involves innovative study to understand the utility and effectiveness of adaptive intelligent (cooperative) systems through system modeling. This course looks at dynamic systems and networks that replace centralized organization or control with distributed cooperation (component collaboration) through the development of structures and processes for communicating, adapting, learning, reasoning, governing, organizing, and decision-making by people in the system. Most of the material is learned through modeling and solving current, complex, capacious social problems and public issues. The major objectives of the course are to understand the mathematics and science of problem solving through modeling cooperation, learning, pattern analysis, decision-making, and cognition.

## Lee Gibson University of Louisville (lrgibs01@louisville.edu)

## Student Multimedia Projects Connect the Real World to Model Visualization in Multivariable Calculus

Come and see student video projects resulting from a Calculus 3 class effort to use critical thinking and digital media tools to connect geometric model visualization to the real world. Hear the students' and instructor's responses to the project and about the supporting role of the university quality enhancement plan. 3-D glasses will be distributed for viewing clips from some videos.

## Session 2: Thursday, August 4, 1:00-4:00 pm in Elkhorn B

**Robert Allen** University of Wisconsin - La Crosse (allen.rob3@uwlax.edu)

## Surviving an Outbreak of Zombiism: Mathematical Modeling Meets Pop Culture

In this talk I will discuss a series of models for the spread of a zombie infection through a healthy population. These models are based on the standard SIR model, modified to introduce various coping techniques as well as dynamical behavior. Implementation of the models have currently been done in Mathematica, but this can easily be changed to software such as Excel, MATLAB, and Maple. These models can be used in courses from precalculus to differential equations, and I will discuss how to use these models in the various courses. Although an outbreak of zombiism is fictional, students will gain an understanding of how mathematical models are used to study the spread of an infectious disease, and the work can be directly translated to studying real diseases such as H1N1.

## Melanie Pivarski Roosevelt University (mpivarski@roosevelt.edu)

## Expandable Combinatorial Lottery Modeling Projects

In an upper level combinatorics course, students created a model for a complicated lottery question. As described in a 2011 Wired article, Mohan Srivastava used frequency analysis to determine, with a high probability of success, whether a particular type of scratch lottery ticket was a winner. Tickets had eight tic-tac-toe games on them along with a list of 'winning' and 'losing' numbers. In this project, students reverse engineered the process by developing a set of reasonable assumptions on how the tickets were created, and modeled the chance of a successful win. This was a three week long project, and each student independently developed a different set of assumptions to model the situation. This could easily be expanded into a larger project. In this talk we will discuss the project, the assumptions made, combinatorial ideas used in the models, and suggestions for expanding this to a larger project.

Gabriella Pinter University of Wisconsin Milwaukee (gapinter@uwm.edu)

**Istvan Lauko** University of Wisconsin Milwaukee (iglauko@uwm.edu)

## From Modeling Projects to Original Research - An Undergraduate Course in Modeling

In this talk we describe a year-long modeling course that involves the development of programming skills, hands-on data collection and the critical reading and reconstruction of primary literature. The course develops discrete and continuous models in parallel, together with the corresponding mathematical analysis techniques. Both deterministic and stochastic models are addressed. We present an example from primary literature that illustrates particularly well how different techniques can be used to investigate a question, and how mathematics, simulation and hands-on data collection strengthen each other, and let the students extend the original research problems. Sample student work will be shown. We found that the ability and experience to reconstruct mathematical models from biological research papers are particularly empowering to our students, especially those who initially do not have a lot of confidence in their mathematical skills. The students gain strong motivation, and satisfaction from being able to check results, and from devoting a substantial amount of time to understand the authors' train of thought. This in turn lets them ask new questions, investigate problems further, and develop a keen sense of the scientific endeavor.

## Michael A. Karls Ball State University (mkarls@bsu.edu)

Brenda M. Skoczelas Ball State University (bmskoczelas@bsu.edu)

## Modeling a Diving Board

The beam equation is a classic partial differential equation that one may encounter in an introductory course on boundary value problems or mathematical physics, which can be used to describe the vertical displacement of a vibrating beam. A diving board can be thought of as a

cantilever beam, which is a bar with one end fixed and the other free to move. Using a video camera and physics demonstration software to record displacement data from a vibrating cantilever beam, we verify a modified version of the beam equation that incorporates damping and a forcing term.

#### Mark lan Nelson University of Wollongong (mnelson@uow.edu.au)

#### How Does the Size of Your Stomach Change During a Meal?

Consider the humble ham and cheese sandwich. Chutney is optional. Nice. (Discard the ham if you are of the vegetarian persuasion).

What happens within the human stomach after this sandwich has been digested? Many things happen. Following ingestion of the food, the pH within the stomach changes. Initially the pH increases to accommodate the activities of the salivary amylase and gastric lipase. After digestion takes place, the pH of the stomach falls back to its original value due to continuous secretion of the acidic gastric juices. The time variation of the pH is known as the pH profile of the stomach.

The pH within the stomach is a very strong indicator of its condition. Small changes in the pH profile can affect the the disintegration of coating materials and capsules, the interaction of a drug with gastric secretions, drug absorption and dosage performance. Therefore there is a need to develop models, both experimental and mathematical, to describe the change in pH of a 'normal' stomach following digestion of a meal.

The first step in modelling the pH profile is to develop a model for how the 'size' of the stomach changes during the course of a meal. This model is developed by considering the processes that cause the stomach to increase or decrease its size. This leads to an ordinary differential equation in which the RHS consists of three Heaviside functions. To plot the solution we need appropriate values for experimental parameters. At this point, the plot thickens...

#### Brian Hollenbeck Emporia State University (bhollenb@emporia.edu)

#### Modeling Water Flow from a Clepsydra in Calculus II

A clepsydra, or water clock, is a clear container from which water flows out of a hole at the bottom. Markings on the side of the container indicate intervals of time. It is well known that Torricelli's Law and calculus can be used to find the optimal shape of a radially symmetric clepsydra so that equally-spaced markings correspond to equal time intervals. We will discuss a lab that asks students to model and predict water flow for a cylindrical clepsydra, as well as other extensions. This lab expects students to develop an approach, collect data using a limited amount of water and a two-liter bottle, formulate a mathematical model, and then solve the model using integration. In the next class period, students then verify their solution. Extensions include considering a non-cylindrical clepsydra that is a solid of revolution, as well as analyzing a multi-level clepsydra with the help of direction fields. This lab provides opportunities for students to use a graphing calculator, spreadsheet, or computer algebra system.

#### Kimberly Kendricks Central State University (kkendricks@centralstate.edu)

#### Interdisciplinary Connections: Applications of Differential Equations in Water Quality, Biomechanics, and Robotics

Through collaborative efforts between three STEM departments (Mathematics & Computer Science, Manufacturing Engineering, and Water Resources Management), the course curriculum for Differential Equations & Discrete Dynamical Systems was enhanced through three unique modeling projects in Water Quality, Biomechanics, and Robotics. Led by the Department of Mathematics and Computer Science, the course structure and curriculum were redesigned to deepen student understanding while engaging students in research. The four week, group projects were created and team taught (and supervised) by faculty members in the above departments. Students studied the Streeter Phelps Equation for measuring stream quality, applied Groebner Basis Theory to inverse problems in human movement, and modeled a spring loaded and doubly-damped mass vibrating without force excitation. Additional resources such as MATLAB, Maple, and Microsoft Excel were used to support each project. This presentation will provide an overview of the course structure and curriculum, strategies for collaborating with faculty across disciplines, as well as an in depth discussion of each modeling project including student outcomes and course successes.

#### **David Coulliette** Asbury University (david.coulliette@asbury.edu)

#### Sparse Matrix Solution in Muscle Contraction Modeling

Linear system solution is a very common topic in an introductory undergraduate numerical methods course. Most presentations divide the algorithms into direct and indirect (or iterative) methods. Iterative methods are particularly helpful in solving sparse linear systems. Muscle contraction modeling is a rich source of such systems. In contrast to traditional differential equation applications that are used to motivate the study of sparse symmetric systems, muscle fiber modeling requires only basic background information in linear spring physics. These models generate symmetric matrices that are 'almost' tridiagonal.

This presentation will summarize a classroom application that grew from a collaborative research effort with the physiology department at the University of Kentucky. This application was used subsequently to a standard presentation on tridiagonal techniques (Gauss-Jacobi, Gauss-Seidel and Successive-Over-Relaxation) and iterative refinement. It provides the opportunity for students to explore a new application with numerical tools that they have recently acquired.

## **Melvin Royer** Indiana Wesleyan University (melvin.royer@indwes.edu)

## Thinking About Thinking: Projects with Neural Network Models

Studies of the human nervous system at various biological and cognitive levels have motivated the late 20th century development of neural network models spanning several mathematical subdisciplines. Here I consider several optimization and vector classification projects to be implemented with well-known perception and delta-rule neural network models. Simple Maple code is provided which students can modify to explore the effects of the model parameters and the methods of mathematically representing the data. The code also generates graphs which are helpful in understanding how the neural network is 'learning' the input patterns. I have used the projects in a mathematical modeling course, but they involve algorithms, linear algebra, and multivariate calculus, and could potentially be used as example applications in any such classes.

## Trends in Undergraduate Mathematical Biology Education

## Thursday, August 4, 1:00–2:40 pm in Elkhorn C

## Timothy D. Comar Benedictine University (tcomar@ben.edu)

#### Preparing Students to Engage in Research Through a Biocalculus Course

One of the goals of the biocalculus courses at Benedictine University is to prepare students to engage in research in mathematical biology, mathematics, or related areas upon completion of the course. This presentation will focus on course activities that help prepare students to begin research work and will highlight several student course projects. We conclude with a discussion of student feedback and student research involvement after completion of the course sequence.

**Rohan Attele** Chicago State University (attele@ameritech.com)

Dan Hrozencik Chicago State University (dhro@att.net)

## An Undergraduate Algebra Curriculum Motivated by Recent Advances in Mathematical Biology

With the increase of applications of higher mathematics in biology, much work has been done in revising the undergraduate curriculum to reflect the convergence of these two disciplines. In particular, proposals for new biomathematics courses tend to focus on biocalclus or applications of differential equations to biology. However, techniques from algebraic geometry and commutative algebra have also led to advances in biology, yet there seems to be little attention to how these advances should affect the undergraduate algebra curriculum. In particular, a typical abstract algebra course has not changed to reflect these advances. The authors propose a comprehensive undergraduate algebra track to better prepare both math and biology students for the convergence of biology and algebra. This is joint work with Rohan Attele and Victor Akatsa (both Chicago State University).

Istvan Lauko University of Wisconsin Milwaukee (iglauko@uwm.edu) Gabriella Pinter University of Wisconsin Milwaukee (gapinter@uwm.edu)

## A Close-Knit Group at a Large Urban University - Long Term Involvement in Undergraduate Research

The University of Wisconsin Milwaukee is a typical, large urban university with over 30,000 students. Three years ago we started an initiative with internal and NSF support to involve groups of Biology and Math majors in a long-term undergraduate research experience in aquatic sciences mixed with mathematical modeling. Each cohort of students stays with the program for two and a half years, including an intensive summer field program. Results after three years are very encouraging. The research experience brought students and faculty closer, significantly enhanced the educational experience and motivation of the participants, and stimulated new collaborations. In this talk we describe the backbone of the program - our first year mathematical modeling course and our research seminar. The key role of primary literature, programming skills and first-hand data collection will be highlighted in the design and implementation of these activities. We discuss strategies that gradually enable the groups to define their own original research problem which they work on throughout their second year under the close mentoring of interdisciplinary faculty. Some assessment and recruitment issues will also be addressed.

## Andrew Lazowski Sacred Heart University (lazowskia@sacredheart.edu)

#### Using Elementary Probability and Statistics to Understand the Florida Panther Population

In a recent edition of "Science" magazine, an article appeared describing the population problem with Florida Panthers. Due to a decline in the number of Florida panthers, panthers from Texas were introduced to hopefully mate and revamp a depleting gene pool. In an effort to incorporate applications to biology into an introductory probability and statistics course, members from the biology and mathematics departments teamed up to form case studies and experiments. The Florida panther was one project that was created. This presentation will discuss how the project was put together, how it is administered, and what was learned from the first semester of using it. Dan Hrozencik Chicago State University (dhro@att.net)

## Undergraduate Research Projects Focusing on the Comparison of Boolean and Continuous Dynamics of Gene Networks

An active area of current research centers on the development of mathematical models used to test the workings of gene networks. The standard models are either continuous (differential equations) or discrete (Boolean). Recent research indicates that these different models lead to vastly different conclusions regarding the operation of n-gene networks, where n = 2 or 3. The authors outline several projects they have designed to introduce undergraduate students to these models and to compare their results. This work is coauthored with Tim Comar (Benedictine University) and Graham Atkinson.

## First Year Seminar / First Year Experience Mathematics Courses

## Thursday, August 4, 1:00–5:20 pm in Elkhorn D

## Susan Jane Colley Oberlin College (sjcolley@math.oberlin.edu)

## A First Year Seminar: What Is Mathematics and Why Won't It Go Away?

We describe a first-year seminar with low prerequisites that intersperses various topics in mathematics (e.g., elementary number theory, proof by induction, some geometry and topology) among discussions about more humanistic and cultural aspects of the discipline. The student audience to date has consisted both of those without any significant affinity for mathematics, as well as those having a background in calculus and an interest in the sciences. As a result, class time has been very lively. Reading and video lists and sample assignments will be presented and the challenges and rewards of teaching such a seminar will be discussed.

## Alessandra Pantano University of California - Irvine (apantano@uci.edu)

## Engaging Students in Mathematical Conversations: From Book Clubs to Freshmen Seminars

This talk will survey recent experiences and future plans for engaging small groups of students in creative mathematical thinking. The emphasis will be placed on identifying common principles in interacting with two very different groups of students: freshmen (from all disciplines) and upper-division mathematics majors. A successful book club was offered in AY 2010-11 for three consecutive quarters to an audience of approximately 5–8 math majors, mostly juniors and seniors. The two books adopted were *The Adventures of Numbers* by Giles Godefroy and *The Book of Numbers* by Conway and Guy. A report of the experience, incorporating students' feedback, will be provided. In AY 2011–12, a Freshmen Seminar will be offered with a similar format, adopting the book *Problem Solving Through Recreational Mathematics*, by Averbach and Chein. Selection criteria for successful books in these two very different courses will be discussed in detail. In particular, I will strive to identify common features that characterize a 'successful book' for this interactive teaching format, regardless of the mathematical maturity of the audience.

## Peter Floodstrand Blanchard University of Iowa (peter-blanchard@uiowa.edu)

## Paul Erdős, Ramsey Theory, and the Question of Beauty in Mathematics

The goals of our course is to give first-year students a perspective on what motivates mathematicians, and to give students an idea of what it might be like to be a creative mathematician. Two principal sources serve as subject matter for our mostly Socratic discussions. The first is a biography of Paul Erdős which is assigned reading throughout the semester. The second is a list of easily understood combinatorial questions whose status is not stated, i.e., some of the questions are easily solved, some have difficult solutions, and others are open questions of mathematics like the x, y, x + y, xy analogue of Schur's theorem or the union closed sets conjecture. Response to the course was positive. This course is a joint effort with Victor Camillo of the University of Iowa.

Teri MurphyNorthern Kentucky University (murphytj1@nku.edu)Lisa HoldenNorthern Kentucky University (holdenl@nku.edu)

#### A Learning Community Partnering Pre-Calculus with Introduction to STEM Degrees and Careers

As part of a larger effort to support student efforts to persist in science, technology, engineering, or mathematics (STEM) majors, Northern Kentucky University has created a learning community that partners one section of pre-calculus with a new one-hour seminar-type course titled, Introduction to STEM Degrees and Careers. The learning community is designed for first-semester freshmen who are pursuing a STEM major, with a goal of engaging students in discovery and discussion about the nature of the disciplines and opportunities within them. The designated section of pre-calculus will parallel the other sections of the same course in terms of content, timing, and number of students enrolled. The Intro-STEM course will take a three-pronged approach: (1) an underlying emphasis on issues such as career paths available to STEM majors and why it is worth persisting in the face of frustration, (2) an emphasis on modeling that makes explicit some of the importance of mathematics for STEM majors, and (3) an emphasis on study habits that correlate with success in STEM majors. The initial offering of the learning community will be in Fall 2011. This work is the result of a collaboration between NKU's Department of Mathematics & Statistics,

Center for Integrative Natural Science and Applied Mathematics, and Office of First Year Programs. The work is funded by a National Science Foundation STEP Type 1A grant (award DUE-0960280). Any opinions, findings, and conclusions or recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF.

## **R. Duane Skaggs** Morehead State University (d.skaggs@morehead-st.edu)

## Calculating Truth, Beauty, Justice, and Fairness

We will discuss the First Year Seminar at Morehead State University. We will focus on one section which considers quantifying the concepts of truth, beauty, justice, and fairness in different contexts. Can one thing be 'more true' than another? If you think you are telling the truth but have incorrect information, are you lying? How can a piece of cake be cut so that one person gets half and another gets more than half? What should be done if two people inherit one house? Why is Shakespeare considered great while Marlowe is not? Why is Citizen Kane near the top of every list of great movies while Plan 9 from Outer Space is near the top of every list of terrible movies? What are the features of great music or art? How are 50 Most Beautiful People lists determined? What makes one building beautiful while another is simply an eyesore? Does symmetry equal beauty or can there be too much symmetry? How can we tell whether a person on trial for murder should be convicted? How do we recognize 'half-truths' and outright false statements in advertising or political campaigns? Can we quantify our beliefs?

This course considers these questions and others throughout the semester. The underlying theme is to determine which concepts can be quantified and measured in an objective way. Are truth, beauty, justice, and fairness subjective, or can they be determined objectively?

#### **Ximena Catepillan** Millersville University (ximena.catepillan@millersville.edu)

#### Culture, Science, and Mathematics in the Pre-Columbian Americas

In 2006 I developed the Freshman Experience Seminar "Culture, Science, and Mathematics in the Pre-Columbian Americas" for the School of Science and Mathematics at Millersville University of Pennsylvania. Seminar Description: An introduction to the study of the Pre-Columbian Americas, part of the broad interdisciplinary field of Native American Studies. The emphasis will be on the role that science and mathematics played in the culture of these indigenous groups (including the Aztec, Inca, Maya and other Native American groups). The course will explore the Pre-Columbian world through the eyes of our ancestors, as well as through our classmates. Special attention to the science of archaeoastronomy and mathematics in which all of the great cultures of antiquity have left a mark. In this talk, the syllabus will be discussed as well as classroom activities used in the seminar.

## Agnes Margaret Rash Saint Joseph's University (arash@sju.edu)

## Great Mathematics Discoveries — Elucidated through Reading and Writing

This newly designed course is a First Year Experience, which requires students to read and to write extensively. The courses that fulfill this requirement can be in any discipline as long as the learning objectives are met.

Students read articles or chapters from a reading list relevant to a variety of topics, including but not limited to historic information, modern applications of mathematics, biographies, unsolved problems, careers using mathematics, and other topics depending on the level of preparation of the students. Each week there are assigned readings and students prepare to discuss them in class. Students also submit a written summary and prepare a personal statement concerning what the person learned and enjoyed, or a critique of why he/she did not appreciate the article. While time consuming to read, the summaries have shown that students gain an expanded horizon in the subject and a broader appreciation of mathematics and mathematicians.

- A complete course description, reading list and outcomes will be discussed in the presentation.
- By the end of the course students will
- 1. Improve their ability to read texts on unfamiliar topics and be able to summarize the content;
- 2. Improve their ability to research an unfamiliar topic using library and internet sources;
- 3. Expand their knowledge of how mathematics and mathematicians play an important role in society;
- 4. Improve their ability to speak confidently in front of an audience; and
- 5. Improve their ability to write using precise mathematical language.

## John B. Little College of the Holy Cross (little@mathcs.holycross.edu)

## Mathematics Courses in the Montserrat Program at the College of the Holy Cross

In their first year, all students at the College of the Holy Cross take a sequence of two linked small-enrollment seminar courses. The first year program, known as Montserrat, is comprised of five permanent, thematically linked 'clusters' ('The Divine,' 'The Self,' 'The Natural World,' 'Global Society,' and 'Core Human Questions'). All students in a cluster live together in the same residence, and seminars in the same cluster share readings and common co-curricular events. All Montserrat seminars aim to develop students' skills in writing, speaking, and analytical thinking. Residential and co-curricular components seek to foster a vital learning atmosphere outside of the classroom and to connect students to the Chaplains' Office, the Library, and other important resources for their academic and personal development. All departments are expected to offer Montserrat seminars and the Mathematics and Computer Science department has done so in each of the first four years of the program. In this talk, I will describe the courses that members of our department have offered, indicate what has and what has not worked, and discuss many of the challenges involved in developing mathematics courses that can reasonably 'fit' into a tightly-designed program like this one, a program whose goals appear, in fact, to be partially orthogonal to those of much of our traditional curriculum.

## Helmut Knaust The University of Texas at El Paso (hknaust@utep.edu)

## Two First Year Seminar Course Designs for Science and Mathematics Majors

At the University of Texas at El Paso, first year seminars combine the teaching of university survival skills with an academic topic chosen by the instructor. We report on two designs for such a course. The first one, "Scientific Revolutions", is directed at science students who are simultaneously enrolled in a developmental mathematics course. Topics include the switch from a geocentric to a heliocentric cosmology, the birth of classical mechanics, and the scientific revolutions in evolution and genetics in the nineteenth century. Accompanying mathematical topics are polynomial functions, difference equations, and elementary probability theory. The second course "Boolean Algebras" is aimed at mathematics and computer science majors who are ready for college level mathematics classes. It gives an introduction to set theory and logic, exposes the students to mathematical reasoning with axioms and definitions, and illustrates applications to digital computing.

## **Jon L. Johnson** Elmhurst College (jonj@elmhurst.edu)

## **Numbers and Native American Mathematics**

Elmhurst College's First Year Seminar (FYS) course is part of a new General Education program. With the help of an internal grant, I developed an FYS course on Native American mathematics. The academic content of the course was meant to answer: What are numbers and what is mathematics? To do so, the course traced the development of numbers and mathematics from ancient civilizations in the Americas to the current use of mathematics in a computer-driven society. Along the way, we investigated ancient and current number systems, patterns in Native American art, how language and culture influence number representation, and the impact of mathematics in today's world. In the two years that I have taught the course, all FYS courses were to include: (1) explaining and modeling Elmhurst College's liberal arts education, (2) incorporating a service learning project, (3) discussing a common reading, (4) having students attend three cultural events, (5) introducing information literacy, and (6) the faculty member as an academic advisor of his or her FYS students. In addition, the class is expected to enhance the students' study skills, ease their transition from high school to college, and model a multi-disciplinary approach to the academic topic. I'll discuss the structure of the course and the challenges and rewards.

## Keith E. Mellinger University of Mary Washington (kmelling@umw.edu)

## **Cryptology and Problem Solving with No Prerequisites**

The first-year seminar program at the University of Mary Washington requires all first-year students to take a process-driven seminar style course. In the last several years, I have developed two courses as part of the program. My first course, titled "Pirates, Liars, and Pigeons" is a course about the art of problem solving with absolutely no prerequisite apart from solid high school algebra skills. The second course, titled "Cryptology," examines the broader field of cryptology including crypt-analysis, enciphering algorithms, and the historical development of the field. Both courses have an emphasis on speaking and writing assignments, a tricky topic for mathematics courses, and the discussion-based format makes them an exciting challenge for the instructor. I hope to share my experiences, content, techniques, and outcomes during my talk.

## Alison Marr Southwestern University (marra@southwestern.edu)

#### Wheels and Deals: An FYS on Television Game Shows

For the past two years, I have taught an FYS on television game shows. This talk will discuss some of the mathematical concepts we study in this course related to game shows. In addition, I will mention some of the other non-mathematical ideas we explore to make learning about game shows a true liberal arts experience.

Jacqueline M. DewarLoyola Marymount University (jdewar@lmu.edu)Suzanne LarsonLoyola Marymount University (slarson@lmu.edu)Thomas ZachariahLoyola Marymount University (tzachari@lmu.edu)

#### Extrapolating from a First Year Experience for Mathematics Majors

A two-semester "workshop course" that engages students in problem solving and expressing mathematical reasoning was originally developed to address a serious problem with student retention in the math major at our institution. We offer this course as a model for fashioning a first-year experience for students with a broad range of mathematical backgrounds. After the introduction of the workshop course, the dropout rate for beginning mathematics majors taking both semesters of the course was nearly halved. Pre/post tests, surveys and interviews indicated other effects of the workshop course include improvement in problem solving skills, study skills, and providing more complete mathematical arguments as well as increased motivation to pursue a career in mathematics. Surprisingly, students also reported carrying writing skills gained in the workshop course to other courses, even to nontechnical courses. Since the course, especially in the first semester, must accommodate students with varying mathematical backgrounds, it uses non-standard problems accessible to students enrolled in precalculus but still challenging to students who have entered college with AP credit and are taking third-semester calculus or differential equations. In addition to describing the course design and outcomes, we will provide sample activities, assignments and assessments, and suggest how it might be redesigned to serve as a first year experience for a broader audience.

## The History of Mathematics and its Uses in the Classroom

## Session 1: Thursday, August 4, 3:30–5:30 pm in Elkhorn A

## Andrew Leahy Knox College (aleahy@knox.edu)

## An Early Approach to Finding Surfaces of Revolution

Surfaces of revolution are a common part of almost any calculus curriculum. Christiaan Huygens is generally regarded as the first person to seriously consider the question of how to find the surface area of a solid of revolution. But the problem of finding surface areas of revolution goes back to Archimedes, who showed that the ratio between the surface of the sphere to the surface of a circumscribing cylinder was 2:3. In our presentation we will discuss the work of Huygens' contemporary James Gregory, who likewise developed a general technique for dealing with surfaces of revolution using classical Greek techniques and Cavalieri's method of indivisibles.

## Thomas Drucker University of Wisconsin - Whitewater (druckert@uww.edu)

## Infinity-Past, Present, and Possible

Ideas connected with infinity have occupied mathematicians and philosophers for millennia. In order for students to understand some of the issues connected with foundational debates in the early part of the twentieth century, it can help to appreciate a historical perspective from Zeno to Lewis Carroll. In order to give them a sense of what was strange about Cantor's reasoning, it is useful to point out some of the challenges of talking about infinity from Aristotle onwards. It is not as though even the way in which infinity enters into basic mathematics such as the evaluation of infinite decimals is a matter of agreement. Some of the livelier points that come out of student discussion of infinity will be used as evidence for the value of the topic in encouraging students of history to appreciate philosophy and vice versa.

## Wayne R. LundbergPhysicist (wayne29rl@gmail.com)

## Modern Developments in Hilbert's Program - and Problem 6

Mathematicians, particularly David Hilbert, have sought a formal foundation of mathematics which is self-consistent. This presentation will discuss nomenclature which minimizes the set of symbols defined for use in arithmetic and algebra, to ensure they are fully distinct from those defined for use in set theory. A fundamental ternary operator is defined in which the usual arithmetic symbols are an extension of a unique underlying formalism. This approach resolves one issue of self-consistency by defining all symbols of both formal axiomatic systems uniquely. By eliminating several symbols from Peano arithmetic, one finds a reduced instruction set algebra that is well-suited to high-speed mathematical computing (although cumbersome for human manipulation). The fundamental ternary operator can be used to construct extremely large finite numbers with great ease. New areas of mathematical exploration are exposed to further enquiry. This leads to the concept of 'finity', defined as a bound to self-consistent mathematics. A self-consistent physics must also be finitely-bounded. Observational and theoretical consistencies also require finite-bounded empirical and axiomatic constructs. A fundamental theorem of physical preons in 4-space is proposed. Generators of particle physics are constructed directly and shown to be 1-to-1 with the well-established empirical formulation of Dirac and Gell-Mann. The Langland-like relation between geometric representation and non-commutative matrix algebra of the theory is explained. The resulting non-disproven particle theory is Causal. It is also shown to inherently include an oscillating neutrino.

## Nathan Moyer Whitworth University (nmoyer@whitworth.edu)

## Introducing Mathematical Philosophy in the Classroom

Is mathematical truth invented or discovered? Is math an artifact of the structure of the universe or a product of the human mind? Undergraduate mathematics majors will often complete their four year degrees without once being exposed to the philosophical assumptions and underpinnings of mathematics. They focus on the question of *how* to do mathematics and *why* mathematics works. Yet rarely do they have an opportunity in the classroom to ask the simple question: *what* is mathematics? This talk describes a class project that helps students wrestle with the deep philosophical questions of mathematics in a relevant way. Through article readings and group discussions, various historical views of mathematical philosophy are explored. The four main philosophies of logicism, formalism, intuitionism, and Platonism are examined and critiqued. The goal of the project is to help students establish their own responses to philosophical questions that are grounded upon their personal worldviews.

## Ezra Brown Virginia Tech (ezbrown@math.vt.edu)

## The Ball of Cork: Using History in Calculus, Abstract Algebra, and Dynamical Systems

A ball of cork of known density is floating in water; determine how far the bottom of the ball sinks below the surface. The problem dates back over 2000 years. The way to its solution passes through the principle of buoyancy, applications of integration, solutions of cubic equations and chaotic dynamical systems, and involves Archimedes, Heron, Fibonacci, Cardano, Newton, Poincare, and many others. This talk is about how the speaker has used the Ball of Cork as a way for students in integral calculus, abstract algebra, differential calculus, and dynamical systems classes to discover that mathematics is not merely a box of tools that fell from the sky.

## Jerry Lodder New Mexico State University (jlodder@nmsu.edu)

## The Notion of Tree: Problems from the Past

The development of the concept of a 'tree' in graph theory is explored from various primary historical sources, beginning with Arthur Cayley's 1857 publication "On The Theory of Analytical Forms Called Trees," to Otakar Borůvka's 1926 paper "On a Certain Minimal Problem." Borůvka's work today is viewed as one of the most efficient algorithms for finding a 'minimal spanning tree,' which is a tree of least total edge length over the domain of all possible trees on *n* fixed vertices. From Heinz Prüfer's 1918 publication "A New Proof of a Theorem about Permutations," we know that the total number of trees on *n*-labeled (fixed) vertices is  $n^{n-2}$ , which clarifies one of Cayley's previous results. Thus, Borůvka's algorithm identifies the minimal spanning tree(s) from a set of  $n^{n-2}$  possibilities. Neither Prüfer nor Borůvka use the word 'tree' in their publications, but motivate their work via rather applied problems, to be discussed in this presentation.

## Session 2: Saturday, August 6, 8:30–11:30 am in Elkhorn A

## **Jim Fulmer** University of Arkansas at Little Rock (jrfulmer@ualr.edu)

Thomas McMillan University of Arkansas at Little Rock (tcmcmillan@ualr.edu)

## A Look at Five Interesting Triangles: Pascal, Chinese, Harmonic, Euler, and Prime.

The Chinese knew Pascal's triangle some 400 years before Pascal lived. Patterns continue to be discovered. This session will focus on five interesting triangles: Pascal, Chinese, Harmonic, Euler, and Prime. The history, similarities, differences, and patterns of each triangle will be discussed. The objectives of the presentation are: 1) To acquaint participants with each of the five interesting triangles: Pascal, Chinese, Harmonic, Euler, and Prime, 2) To discuss a brief history of each of the five triangles, 3) To explore similarities and differences among the five triangles, 4) To discover various patterns on each of the five triangles, 5) To look at the Chinese version of Pascal's triangle and the stick numeral system used, and 6) To connect the five triangles to number sets, algebra, geometry, probability, number sense, and calculus.

## Lisa D. Elliott Austin Peay State University (lelliott1@my.apsu.edu)

## **Pedagogical Methods of Ancient Chinese Mathematics**

The ancient Chinese counting boards, counting rods, and the Nine Chapters of the Mathematical Arts signify an origin for advanced mathematical operations. This presentation illustrates the ancient procedures for multiplying large numbers quickly, solving systems of linear equations, and presents a few of the pedagogical advantages of introducing word problems earlier math education to enhance the level of mathematical literacy.

## Wojciech K. Kosek Colorado Technical University (wkosek@coloradotech.edu)

## Bringing Historical Context to First Year Mathematics Courses.

A typical college algebra or calculus course does not devote much attention to the history of mathematics and the importance of some of the famous theorems. Instead, we tend to focus on the techniques our students need to master. However, for many students, mathematics may seem like a dead discipline. When one is first learning about a new concept or technique, it feels good to know that even some of the greatest minds in the history of mathematics were at times having the same difficulties as most of us. What is the significance of the Pythagorean theorem? How were irrational numbers discovered? What can novices learn from Galois about the solutions polynomial equations, even if it is their first math class at the college? In this talk, we will look at a few examples of how Thales, Pythagoras, Cardano, Galois and Leibnitz can help students put things in perspective.

## Brian J. Lunday U.S. Military Academy (brian.lunday@usma.edu)

## Bringing Students into the Fold: Engagement through Mathematics History

We present our use of mathematics history in the classroom of a first-year differential calculus course. We detail our use of such history for three purposes: to engage with and reduce math anxiety for the less-mathematically inclined students who are taking a required mathematics course, to increase students' identification with the mathematically gifted alumni who preceded them in our educational institution, and to introduce talented students to offerings within our department prior to their declaration of an intended major. Based on a one-year pilot study, we present empirical instructor observations and student feedback on the efficacy of these techniques.

## Daniel John Curtin Northern Kentucky University (curtin@nku.edu)

## Cardano: A Minus Times A Minus Is Minus !?

Throughout his *Ars Magna* Gerolamo Cardano multiplies minus by minus to get plus. However, in his appended booklet *De regula aliza* he states that minus times minus is, in fact, minus. He gives several arguments for this, and explores the consequences. This presentation is a brief introduction to his discussion. It also includes suggestions for how this oddity may be used, without necessarily expending much time, in any class in which the nature of number, or of numerical operations is discussed.

## Patricia Baggett New Mexico State University (baggett@nmsu.edu) Andrzej Ehrenfeucht University of Colorado (andrzej.ehrenfeucht@colorado.edu)

## **Negative Numbers in School Mathematics**

Negative numbers were discovered and rejected many times. This rocky past was due not to computational (algorithmic) difficulties, but to the lack of an established and accepted meaning of such numbers. Finally negative numbers found their place in eighteenth and nineteenth century algebra. They became a part of arithmetic during the first half of the twentieth century, and at that time they penetrated school mathematics. They became a staple of the middle school curricula during the second half of the twentieth century (after the New Math).

But even now negative numbers are not well integrated with the rest of school arithmetic because of a lack of clarity about their meaning, which is presented to students in several rather unsatisfactory ways. (For example: Points on a number line to the left of zero. Debt being measured in negative dollars.)

In the talk we will present some historical background based mainly on the use of negative numbers in math textbooks between 1750– 1950. We also describe a conceptual interpretation of negative numbers (as a change of a quantity), which may better integrate them with the rest of arithmetic than the interpretations currently used in classrooms. And finally we will comment on the need and possibility of moving this concept from middle to elementary grades.

## Barbara Shipman University of Texas at Arlington (bshipman@uta.edu)

#### Historical Quotes in Finding the Meaning and Language of Limits

This is a view of some of my classroom activities that integrate quotes from historical writings on limits to help students uncover the meaning of the concept of limit and analyze the language used in describing limits. The quotes are juxtaposed with targeted questions to instigate critical thinking and debate among the students. Supported in part by NSF grant #0837810.

## Amy Osborne-Wheeler AIOHC (awheeler@aii.edu)

## Mathematics and the Arts: A Historical Perspective

Teaching mathematics to art students can be challenging at best. This presentation will cover topics in mathematics taught from a historical perspective (eg elementary number theory), which allow students to understand the cultural impact. Selected lessons will be presented with examples of student work.

## Troy Goodsell Brigham Young University - Idaho (goodsellt@byui.edu)

## **Quadratures of Parabolas: Integrating before Newton**

We will look at various methods through the ages of what we now call integration. This will begin with Archimedes and proceed through Torricelli and his book Opera Geometrica of 1644 which summarized the different methods known to that point.

## Geometry Topics That Engage Student

## Session 1: Friday, August 5, 8:30–10:30 am in Elkhorn B

Deane E. Arganbright Divine Word University (Papua New Guinea) (dearganbright@yahoo.com)

## **Engaging Explorations in Geometry using Excel**

The spreadsheet is an excellent and engaging tool for facilitating student investigations of a wide range of concepts from geometry. This presentation presents illustrative examples of the analytic geometric use of Excel in algebra, geometry, calculus, linear algebra, and other classes. We present eye-catching animated models for the construction of pedal and inverse curves, evolutes, and similar topics, as well as the use of geometry in the classes listed above, and in the creation of attractive drawings for cultural designs and alphabet books. In the process of creating these models, students learn the underlying geometrical concepts through their implementations.

## William Schellhorn Simpson College (william.schellhorn@simpson.edu)

#### Using GeoGebra to Improve Understanding of Proofs in Geometry

In this presentation, I will discuss how I use the free software GeoGebra to help students understand constructive proofs in geometry. My examples will include proofs in both Euclidean geometry and hyperbolic geometry. I will also discuss ways that GeoGebra can be used to help students form conjectures about properties of geometric objects.

## Christopher Thron Texas A&M University - Central Texas (thron@ct.tamus.edu)

## Voila! Proofs With Iteratively Inscribed Triangles

Designs with iteratively inscribed triangles can easily be created using dynamic software programs such as Geometer's Sketchpad. Besides their visual appeal, these designs can be used in serious mathematical proofs that combine elements of classical geometry with the concept of limit. We demonstrate how to use iterated inscribed triangles to create a simple *Voila!* proof of the Euler line property. We will also show how iterated inscribed triangles can be used to characterize and locate any triangle's Brocard points, and to characterize these points as points of concurrence of a special set of logarithmic spirals. These proofs are amenable to the discovery approach, and may be used in the classroom. All our demonstrations utilize 'Compass and Ruler', a freely-available Java applet that enables dynamic, iterative geometrical constructions.

## Margaret L. Morrow SUNY Plattsburgh (morrowml@plattsburgh.edu)

## Proofs That Explain: An Example.

Researchers in Mathematics Education often draw a distinction between a proof that convinces and a proof that explains. In this talk we will discuss a quintessential experience of this distinction for a result in the elementary geometry of the circle. Dissatisfaction with a proof that convinced, but failed to explain to the author's satisfaction, led to an exploration on Geometer's sketchpad. This provided insight which enabled construction of a different proof which the author believes has more explanatory power. We will discuss the process and the proof.

## **Cheryll Crowe** Eastern Kentucky University (cheryll.crowe@eku.edu)

## Using Tangrams to Engage Students in Geometry

This talk describes a project implemented in a geometry class for elementary and middle school pre-service teachers. The purpose of the project was to use tangrams to explore deep mathematical connections related to area, perimeter, and the Pythagorean Theorem. Through the project, students also expanded their knowledge of regular polygons and relationships among quadrilaterals.

The project commenced with an overview of tangrams and creation of quadrilaterals using tangram pieces. Pre-service teachers also created polygons from tangrams and determined if the polygons were regular. Following this task, students calculated the area and perimeter of each tangram piece, utilizing various methods including the Pythagorean Theorem. At the conclusion, pre-service teachers explored an activity from the National Library of Virtual Manipulatives (NLVM) to further explore the mathematics behind tangram puzzles.

In this talk, the presenter will discuss how the project engaged students in geometry, show samples of student work, and demonstrate the NLVM activity exploring tangrams.

## Ivona Grzegorczyk California State University Channel Islands (ivona.grze@csuci.edu)

#### Visualizing Algebraic Surfaces

We will show student projects involving classification of quadratic and cubic surfaces using singularities, symmetry groups, lines and curves on them as well as other geometric invariants. The visualization of these surfaces using modern software produce beautiful and unexpected images that serve as a motivation for students to work on semester long projects.

## Session 2: Friday, August 5, 1:00–4:00 pm in Elkhorn B

Judith SilverMarshall University (silver@marshall.edu)Jonathan CoxMarshall University (coxj@marshall.edu)

#### A Geometry Based Math/Art Course with a Studio Component

Marshall University offers a four hour freshman honors seminar in mathematics and art, with emphasis on geometry. The beauty and usefulness of mathematics is enhanced by a studio component taught by an art professor. Topics covered include perspective, symmetry, mathematical themes in art, and studio skills.

## Ockle Johnson Keene State College (ojohnson@keene.edu)

#### Geometry in an Historical Frame

In this talk I will describe the course I developed that teaches geometry within an historical context. This approach has many positive features. Students read the first book of Euclid's Elements to review some basic Euclidean geometry. Students learn that important contributions were made by mathematicians from various civilizations. Students discover that geometry has developed over the years and that every development in mathematics, e.g., algebra, calculus, and abstract algebra, has provided new tools and raised new questions for geometry. Students realize that geometry is much broader than they thought and remains an area of active mathematical research and development.

## **Emma Smith Zbarsky** Wentworth Institute of Technology (emmasz@gmail.com)

## Analyzing Floor Plans: A Geometry Lab

I will describe a project that I developed for my geometry course. I collaborated with an architecture professor to select a number of floor plans designed by architecture students. Then I let the math students loose to prepare a lease space analysis applying basic concepts from two and three dimensional geometry.

## Penelope Dunham Muhlenberg College (pdunham@muhlenberg.edu)

## Symmetry and Shape: Geometry for Non-majors

What should a geometry course for non-majors look like? In particular, what topics will convey the beauty of geometry and, at the same time, attract students from the humanities who only want to satisfy their general reasoning requirement? My solution is "Symmetry and Shape," a 100-level course that examines geometric concepts as it engages students with hands-on explorations and examples from art and nature. Although I originally designed the course to appeal to students from the arts, it has also been a popular choice for preservice elementary teachers and majors in biology, theatre, and history. This talk will address issues in designing the course, including topic selection and assessment options. I'll list the major topics covered and give examples of innovative assignments, in-class explorations, technology-based labs, and available resources. I'll also describe assessment components, including two portfolio projects: one focused on examples of symmetry from students' environment, and another featuring original student art based on concepts studied in the course (culminating in a display for the campus Arts Week).

## Marian Anton Centre College (marianfanton9@gmail.com)

#### Geometry via Modeling

According to Euclid, if two points are taken at random on the circumference of a circle, then the straight line joining the points falls within the circle. Starting from this example, we show how topological data analysis could reshape the teaching of geometry. In particular, we outline a course in elementary geometry aiming to engage students in learning via modeling.

## Kristen Sellke Saint Mary's University of Minnesota (ksellke@smumn.edu)

## Using 'Arts and Crafts' to Reinforce Geometric Concepts

My geometry course is a whirlwind tour through different geometries such as Euclidean, hyperbolic, analytic, finite, and transformational. Geometry is offered every two years so the students enter the course with a very diverse mathematical background. This presentation will examine a series of hands-on activities done throughout the course including constructions, paper folding and the creation of hyperbolic paper . We will discuss how my goals of the activities: to develop visualization skills for all students, to motivate proof-writing for second-year majors , and to give pre-service teachers examples of activities they can use in their future classes were met and look at student responses to the activities.

## Carroll G. Wells Lipscomb University (carroll.wells@lipscomb.edu)

## Using Paper Folding to Explore Euclidean Geometry

Paper folding activities will be used to explore over 90 terms or results from Euclidean Geometry. One of the results will be the creation of an icosahedron with a look at the use of this Platonic Solid in the biological sciences and how a writing activity can be centered around the icosahedron. When used by young participants, some of the activities can be extended to include computation of areas as a fractional part of one of the polygons formed. Also volume and area are emphasized during the creation of boxes.

These activities are appropriate for use in geometry classrooms at all levels from elementary through college by the proper selection of the terms and results discussed. The activities teach and review geometric concepts while also engaging the participants.

The presentation is appropriate for teachers of geometry, pre-service teachers, and anyone interested in a hands-on look at geometric concepts. The presenter has used all of the activities in professional development institutes for middle grades and high school teachers and in summer programs for 7-10th grade students and is currently using them in college geometry courses for mathematics education students. The audience will be asked to participate in the activities.

## Todd D. Oberg Illinois College (toberg@ic.edu)

#### Kinesthetically Experiencing Geometry

With the increased emphasis on Transformational Geometry in the PreK-12 curriculum, and the continued need to study Euclidean Geometry, a rethinking of Geometry courses for preservice teachers may be necessary. One way of preparing future teachers, well as current teachers, is to combine Euclidean and Transformational Geometries into a single study rather than treating each as a separate topic. In this presentation, I will share some paper folding and Patty Paper activities that invite preservice students to more actively engage in the study of Geometry and also provide these students with opportunities to explore both Transformational and Euclidean techniques for creating proofs. In addition, some of these activities can be extended to explore ideas in Non-Euclidean Geometries.

## Thomas Q. Sibley St. John's University (tsibley@csbsju.edu)

## All Hands on Deck: In Praise of Toys

Geometry students benefit from 'playing' with geometrical objects. I have students use mirrors, basketballs, approximations of hyperbolic planes, both knitted and plastic, and other toys. These experiences help them develop valuable geometrical intuition and make conjectures. I will discuss how I have used hands on experiences to help students develop geometrical approaches to proofs and understand mathematical ideas.

## Novel Ways to Incorporate Writing into Mathematics Classes

## Session 1: Friday, August 5, 8:30–11:30 am in Elkhorn C

## Maritza Branker Niagara University (mbranker@niagara.edu)

## Assigning a Mathematical Book Review in a Capstone Course

In the math senior seminar at Niagara University we discuss Godel's theorem. In the past students failed to appreciated the significance of this result so to provide context and historical background the graphic novel Logicomix: An epic search for truth has been added as a supplementary textbook. Students read the book before we discuss Godel's theorem and write a book review of the novel. After we discuss Godel's theorem they also write a reflection paper on the topic of the nature of mathematics and the need for the foundations of mathematics to be exact, consistent and complete. In this talk details of the writing assignments will be provided as well as examples of student feedback on the experience of being assigned a book report in a mathematics course.

## **Thomas Q. Sibley** St. John's University (tsibley@csbsju.edu)

## **Expository Papers: Windows into the World of Mathematics**

Students' experience of mathematics is often confined to the standard course syllabi, missing a wider world of topics. But students aren't standardized. I have used expository papers in my Geometry and Capstone courses for majors and in my liberal arts mathematics course. Expository papers provide a good opportunity to pursue a topic of interest. I will discuss topics and the way I structure the paper assignment.

## Susan Schwartz Wildstrom Walt Whitman High School (susan@wildstrom.com)

## Journals and Reading and Websites, Oh My!

Students are accustomed to their math classes being so curriculum driven that they rarely have time to stop and admire the beauty of mathematics - either that which they are learning or topics in mathematics that they have heard about but not had time to explore. The presenter has developed several assignments that give students opportunities to read about, research about, and talk about mathematics. Feedback from students over many years has supported her belief that these assignments have a profound affect on many students in encouraging them to pursue mathematics beyond high school. This paper will give the details of these assignments which include student/teacher conversations in the form of biweekly email journals, student research on topics relevant to course materials in the form of finding websites that expand on classroom topics, and a reading assignment in which students may read anything mathematical that interests them (and then write a brief summary of their reading).

## Sarah L. Mabrouk Framingham State University (smabrouk@framingham.edu)

## Mathematics and Me: Reflections on Liberal Arts Mathematics

Framingham State University's MATH 119 Liberal Arts Mathematics is a Quantitative General Education course taken by students whose major is in Art, Communication Arts, English, or History or by students who have not, as of yet, chosen a major. In general, students taking this course have poor quantitative skills and little, if any, appreciation of mathematics. Through topics such as problem solving, pattern recognition, number systems, set theory, counting, probability, logic, and consumer mathematics, students explore the beauty and effectiveness of mathematics as well as the application of mathematics in examining, analyzing and solving a variety of interesting problems. In this presentation, I will discuss my "Mathematics and M" assignment for which students write about, among other things, how at least two course topics can be useful to them in their course of study, future career, and/or in life as well as reflect on the topics and methods examined throughout the course. In addition, student reaction to the assignment and student attitudes toward taking the course will be discussed.

## **Benjamin James Braun** University of Kentucky (benjamin.braun@uky.edu)

## Personal, Expository, Critical, and Creative: Using Writing in Mathematics Courses

Developing problem-solving and theorem-proving skills are often the primary focus of mathematics courses. However, to foster life-long learning and commitment to mathematical habits of mind, other basic student needs must be addressed in coursework; writing assignments provide an excellent opportunity for meeting these needs.

Through writing assignments, I hope to cultivate in students several habits: to engage their emotional self in mathematical life, to participate in the three activities of exposition, critical analysis, and creative work, and to consistently leave open to revision their understanding of mathematics. In pursuit of these goals, I have used various combinations of personal reflections, critical analysis of reading assignments, large expository projects, and cover letter writing for homework and/or course portfolios in my courses.

In this talk, I will discuss my successes and failures using writing assignments in three contexts: junior/senior level History of Mathematics courses, sophomore/junior level Matrix Algebra courses, and graduate courses in combinatorics. I will share some of the materials that I have developed, along with some of the resources that have informed and inspired my use of writing in the classroom.

## Chris Oehrlein Oklahoma City Community College (coehrlein@occc.edu)

## **Reflecting and Connecting through Journals and Essays**

Students majoring outside the STEM fields are usually required to fulfill a mathematics or quantitative analysis core requirement for completion of a degree. Many of these students are more inclined towards the humanities and social sciences, disciplines that from their perspective give them more freedom of expression. They read, listen and watch, and then they present their own thoughts and opinions about their observations — what they like or do not like, positions with which they agree or disagree, where they do or do not see patterns emerging, etc. In their required mathematics courses, even those designed around the aptitudes of non-STEM majors, they are still required to perform a lot of tasks and skills, and the "reflection" they are asked to do tends to be about the effectiveness of algorithms and procedures and tends to have a "more" or "most correct" answer.

In both his Contemporary Mathematics and Introduction to Statistics courses, the instructor expanded the writing component from one capstone essay or project to a series of journals and portfolios that gave students opportunities to reflect on the topics being studied and the work they were doing in class, and to share true thoughts, opinions and observations about issues without a necessarily correct or best response. The instructor will share a sample of the assignments and prompts used for this writing component.

### **Mike Pinter** Belmont University (mike.pinter@belmont.edu)

#### Seeing the Unseen: A Metaphor for Mathematics

For general education mathematical reasoning courses that I teach, I use a metaphor at the beginning of the course to help provide an overrall frame for the course. In particular, I suggest to students that "seeing the unseen" is a skill that the course will help them develop, both in a literal sense and in their mind's eye. The idea for the metaphor initially came from various authors, including Keith Devlin, who indicate that mathematics helps to make the invisible visible. The specific metaphor was inspired by a Harold Edgerton photography exhibit entitled "Seeing the Unseen." There are a variety of specific places during the course where students are reminded of the framing metaphor and asked to respond to it in some way. The final exam includes a writing prompt for a take-home portion in which the students are invited to revisit and explore "seeing the unseen" at the completion of the course.

## Marion Deutsche Cohen Arcadia University (mathwoman199436@aol.com)

## Truth and Beauty: A Course in Mathematics in Llterature

Mathematics in Literature is the title of a course which I developed at Arcadia University in Glenside, PA. This course is meant for mathmajors as well as non-math majors; it satisfies both a math and a writing requirement. We read literature (prose and poetry) that is inspired by math in some way. In class, and also for written homework, we investigate "non-math questions" that do not involve the actual doing of math. I believe that an important reason for reading literature of all kinds is to find meaning in it that relates to one's one life; many of the questions are designed with that in mind (though students are also encouraged to add their own questions to the list). I also teach some of the math that inspired the particular reading, and thus "math questions" are also part of the homework. Two projects are due at the end of the term: a term paper (which often turns out to be a piece of their own math fiction, or a paper about what this course has meant to them) and a poem (ungraded), There is also a math final, which so far has counted 15%.

## Linda McGuire Muhlenberg College (lmcguire@muhlenberg.edu)

#### **The Mathematics Correspondent**

This talk would outline an approach to help students in a 100-level, general mathematics course at a liberal arts college to acquire, develop and deepen problem-solving, research, and mathematical communication skills.

In these group work modules, students act as 'mathematics reporters' to introduce and inform their classmates about new material in the course. Their presentations and written materials are modeled after the structure of segments on the PBS Newshour. Typically a group of three to four students is assigned to introduce a new concept, provide historical context for the topic, to create worksheets that highlight preliminary examples, and/or to lead a class exploration with the rest of the group. There are clear expectations with regard to the oral and written aspects of these assignments.

This approach has proven to be very successful in generating student excitement, encouraging student focus, developing communication skills, teaching them how to anticipate questions, and improving individual as well as collective output.

During the presentation the underlying structure of the activities would be introduced and defined. Sample problems and presentations from course would be discussed as well as several examples of student work. Assessment tools used to measure efficacy and student reactions upon course completion would also be addressed.

## Session 2: Saturday, August 6, 8:30–11:30 am in Elkhorn C

Jason Molitierno Sacred Heart University (molitiernoj@sacredheart.edu)

#### Do the Students Really Understand What They're Doing?

Often times when we teach a math course, we teach them how to do the calculations and we may verbally explain why we use the steps that we do. But when doing a problem, do the students truly understand why they are using the process that is required for that particular problem? In this talk, I give several examples of essay questions that I have given on tests and as out-of-class assignments for my lower level classes such as precalculus, calculus, and statistics. I show how these essay questions can motivate student-understanding so that when they do a problem, they understand the reasoning behind the process they are using. Of course, students in these classes are surprised that there is writing in a math class, so I give them a motivation in the beginning of the semester that is comical and something they truly remember!

# Irina A. Chernikova<br/>Michael JohanyakThe University of Akron, Ohio (irina@uakron.edu)Michael Johanyak<br/>Sheldon WriceThe University of Akron (mfjohan@uakron.edu)Sheldon WriceThe University of Akron (swrice1@uakron.edu)

#### Curriculum Crossroads: Where the Writer Meets the Mode

Educators know that first-year student success and retention depend on successful outcomes in general mathematics courses. Most of these courses, designed for both science, technology, engineering, and mathematics (STEM) and the humanity paths, depend heavily on students' ability to read and write well. Many institutions have long realized that writing should be an integral part of all subject areas, so calls for "Writing Across the Curriculum" are heard across most if not all campuses. In a response to our own university's efforts to improve first-year student success and retention, we have proposed a reverse approach: incorporating mathematics (and particularly statistics to start) into a general education writing course as a first step towards 'mathematizing' the general education curriculum.

In our session, we will describe a process of collaboration between administration and faculty in mathematics, engineering, and English to develop and implement a data analysis component in a course called Technical Report Writing (TRW), along with our plans to further mathematize TRW and, ultimately, to integrate mathematics into other humanities courses. This multi-disciplinary approach involves three stages, two of which have been completed and a third one (currently being implemented) that will include a discussion of some unexpected 'ripple effects' that are merging literacy with quantitative and qualitative reasoning. We have found that our collaborative, cross-disciplinary efforts to mathematize a general education writing course - in a seamless fashion - are creating a novel, engaging and authentic first-year learning experience that helps students reach a deeper, more holistic understanding of mathematics and writing.

## Joe DeMaio Kennesaw State University (jdemaio@kennesaw.edu) Kennesaw State University (ahillen@kennesaw.edu)

#### Incorporating Writing Into an Introductory Statistics Course Focused on Sports and Games

The ability to reason statistically is critical to becoming an informed citizen and intelligent consumer (Aliaga, et al., 2010). Despite its importance, research indicates that students struggle to reason statistically and that students typically enter introductory statistics courses "under sufferance, with levels of resistance related to their beliefs of the relevance of statistics" (Howley, 2008).

One approach for helping students make sense of complex mathematical ideas is to situate the ideas in real-world contexts that have meaning for students. We contend that the context of sports and games might be particularly promising, because: 1) it is a rich site for studying statistical ideas (e.g., probabilities in card and board games; correlation of player and/or team statistics); and 2) it is likely of interest to a wide range of students. In this talk, we describe writing assignments used in a general education elementary statistics course that focused on sports and games, and discuss student reflections on the efficacy of such assignments.

## Dan Schultz-Ela Mesa St. College (dschultz@mesastate.edu)

### Instruction in Mathematical Communication through Problem-Solving Explanations

Communication skills are essential for college graduates, including those in mathematics. Learning to communicate mathematical concepts and methods effectively cannot be accomplished simply by exhorting students to 'explain better,' which also engenders an implicit cultural bias that the students share the teacher's assumptions. These issues are particularly critical for my students, who are future elementary-school teachers. They learn mathematics content, but in the past received little explicit guidance in how to explain mathematics. Creation of a detailed rubric for effective communication and brief instruction in its content markedly improved my students' abilities. This instruction accompanies practice in problem solving, requiring a minimal net investment of classroom time. Rather than simply having students submit their answers to major problems, they produce two to three carefully crafted reports that are also scored for effectiveness of mathematical communication.

In the pilot class to assess improvement, the students devised the rubric-based set of criteria for effective mathematical explanations and then evaluated their peers' and their own explanations to problem-solving exercises. Results of tasks representing a pretest and posttest were compared using descriptive statistics and a Wilcoxon signed-rank test for matched pairs of ordinal (rubric) data. All measures showed significant and rapid improvement in mathematical explanation abilities during the instructional unit. Scores on oral presentations consistently exceeded scores of written explanations for the same problem. Thus, improvements in more efficiently graded written work suggest at least similar improvement in the oral communication abilities required of future teachers and mathematics graduates.
# **Joshua Holden** Rose-Hulman Insitute of Technology (holden@rose-hulman.edu)

# Laboratory-Based Writing Activities in an Engineering Statistics Course

The use of laboratory assignments with written reports has become fairly common in college precalculus, calculus, and differential equations courses. Statistics courses also frequently use laboratories, as they are in many ways ideal to illustrate for students how to collect and analyze data. When I decided to introduce laboratory activities into my sections of Engineering Statistics, however, I found that while several of my colleagues had developed interesting and exciting activities for students, none were requiring the students to turn in full-fledged written reports as opposed to more of a 'worksheet' format. In this talk I will describe how I adapted my colleagues' activities into assignments with written reports and the rubrics I developed to grade those reports.

# **Eric Ruggieri** Duquesne University (ruggierie@duq.edu)

#### Statistics in the World Around Us

The Department of Mathematics and Computer Science at Duquesne University offers two introductory Statistics courses to its students, one geared towards science majors and the other to its Liberal Arts majors. In this talk, I will describe writing projects that I have used in each of these courses to help students appreciate the Statistics they will encounter outside of the classroom.

In my Biostatistics course, I ask students to find a research article that interests them, investigate the statistical methods used, and then both present to their classmates and write a report about what they have discovered. Topics from this past semester ranged from Plyometric training for basketball players to studies analyzing treatments for children with uncontrolled asthma.

In my Statistics course for Liberal Arts majors, I take a different approach and ask students to conduct a statistical survey about a topic of their choice, analyze their data, and then write a report about their findings. Here, students investigated topics that ranged from the number of people who push the Handicap button to open a door to a comparison of GPA and smoking habits.

In both courses, I offer feedback throughout the writing process as the goal is to help students become more comfortable speaking and writing about Statistics.

### **Jacqueline Jensen** SRU/SHSU (jacqueline.jensen@sru.edu)

#### **Teaching Minitab by Student Memo Writing**

Elementary Statistics students at Slippery Rock University are expected to demonstrate mastery not only of basic statistical principles, but also a working knowledge of Minitab. Much of their knowledge of Minitab is gained from out-of-class projects. To make these more relevant to their future work experiences, we have students present their Minitab analyses in a professional memo. This response to open-ended questions requires not only the use of Minitab but also asks students to demonstrate (and improve upon) their writing skills. Examples of assignments will be given, as will as samples of completed student projects, both good and bad.

#### Lanee Young Fort Hays State University (lyoung@fhsu.edu)

#### Using the New York Times in Statistics Class

Many times students ask "when are we going to use this?" We know there are examples all around, the trick is getting your students to join the search for these applications of mathematics. At Fort Hays State University students and faculty are encouraged to creatively use the campus subscription to the New York Times. In this project, students are required to find statistics in the newspaper which match the mathematics currently covered in class. Students learn how to properly cite an article, discuss the statistics being learned, and critically think about the process used to obtain these statistics.

# Therese Shelton Southwestern University (shelton@southwestern.edu)

#### Writing Math from Intro Stat to Capstone

Writing assignments can truly enable students to communicate mathematics well, which solidifies their own understanding. Students in introductory statistics created group technical reports with coherent descriptions of various concepts relating to sampling distributions and the Central Limit Theorem. Aided by software, the students generated samples from various parent distributions, collected summary statistics, and produced graphs. Then they engaged in conversations about how to write up their findings. In multivariate calculus, students wrote a rationale for matching symbolic function definitions with 3D graphs and level curves. In the capstone course, students practiced a variety of writing styles, from a short article on polynomials to a paper about their mathematical modeling project. The assignments and guidance given to the students will be shared, along with a rubric used for the capstone paper.

# Session 3: Saturday, August 6, 1:00-4:00 pm in Elkhorn C

#### Chad Awtrey Elon University (cawtrey@elon.edu)

#### **Combining Problem Solving and Writing in Single Variable Calculus Courses**

We discuss a program, currently being implemented in Elon University's single variable calculus courses, that incorporates daily writing activities, introduces basic proof techniques, and assigns several major writing projects. We describe this program in detail, including specific projects on the intermediate and mean value theorems, Newton's method, volumes by revolution, and sequences and series. For each project, we present sample student work and grading rubrics.

# Teena Carroll St. Norbert College (teena.carroll@snc.edu)

#### Using Two Phased Writing Projects and Rough Draft Meetings For Calculus Writing Projects

St. Norbert College has a writing across the curriculum requirement, where all general education courses must have a writing component. The writing requirement is follows a developmental model; students are given an opportunity to revise and improve their work. When I first began assigning writing projects in my calculus and precalculus courses, I would spend laborious hours writing comments on rough drafts, only to have many of them ignored on the final drafts. I found that if I give feedback in person they were much more likely to improve their papers. Additionally, I found the final drafts much easier and more fulfilling to grade.

Typically I assign a major group project, but I have recently been using a short individual project as a means of choosing groups. I put students together in writing groups whose papers have similar qualities, for example, attention to detail, overall creativity, or similar mathematical errors. Students have reported high levels of satisfaction with groups chosen this way. The first phase allows students to get used to being graded with a rubric, and practice takes away some of the anxiety of having to write a math paper, and allows me to build on the skills they acquire by doing a small project first.

The overall quality of the papers that I have received has consistently risen using these two methods. Students come in dreading having to write a math class, but often self report that the writing project was their favorite aspect of the course.

#### J. Christopher Tweddle University of Evansville (ct55@evansville.edu)

# Writing Projects for Calculus and Liberal Arts Mathematics

Inspired by Gavin LaRose (and the book *Writing Projects for Mathematics Courses* with co-authors Crannell, Ratliff and Rykken), I have incorporated short projects that involve both using mathematics to solve pseudo real-world problems and providing written explanation of the solution. In this talk, I will share of some samples of projects that I have used and some examples of student work. I will also discuss my experiences with using writing projects in the classroom. Samples of the projects can be found on the "Assignments and Announcements" pages for the appropriate classes at my website: http://faculty.evansville.edu/ct55/

# **Donna LaLonde** Washburn University (donna.lalonde@washburn.edu)

Jennifer D. Wagner Washburn University (jennifer.wagner1@washburn.edu)

#### Students Writing for One Another on Course Wikis

We use wikis in a variety of our courses as a platform for students to write for one another about mathematics. The public format of the wiki shifts the responsibility for expertise and accuracy from the instructor to the student. This allows each student to be the 'class expert' on some definition, theorem, or other topic, and share his or her expertise with the rest of the class. For some students this is their first experience writing for an audience rather than for the 'teacher.' The students receive feedback directly from their peers, which encourages them to think about their audience when they write, and promotes a sense of community. We will review our successes and struggles, as well as our preliminary understanding of the impact. We will also draw on our experiences to offer suggestions for using wikis to help incorporate writing into both upper and lower division classes.

# Rebecca Vandiver St. Olaf College (vandiver.rebecca@gmail.com)

#### Using Wikis to Break the Language Barrier between Mathematics and Biology

With an increase in interdisciplinary endeavors, it is becoming increasingly important for students to be able to communicate mathematics to people in other disciplines. To facilitate this goal, I have required students to write Wiki pages that would be understandable to undergraduate biology majors. Other classmates can critique the pages and then the students have time for revisions. These pages are stored in the MathBio Wiki (co-created with Sarah Hews) as part of a database of mathematical terms commonly used in biological research. In this talk I will introduce the MathBio Wiki and discuss how it has been used to incorporate writing assignments into the classroom.

#### **Brian Katz** Augustana University (briankatz@augustana.edu)

Elizabeth Thoren University of California - Santa Barbara (ethoren@math.ucsb.edu)

#### Wiki Technology Supports Inquiry

Inquiry-based learning creates new challenges for students and faculty. For example, some students struggle with the lack of a textbook resource and others are anxious about the heavy emphasis on public speaking. We believe that student-generated wikis offer support for these challenges and additional benefits by mirroring the IBL classroom experience in writing. We will share some of our experiences using wikis in a wide variety of courses from an Honors Calculus III class at a large university to Calculus I and Modern Geometry at a small college.

# Katie Spurrier Quertermous James Madison University (querteks@jmu.edu)

#### Using Writing to Reveal Students' Thinking and Learning

In this talk, I will describe two types of writing assignments, documented problem solving and reflective essays, that I use in my calculus courses to enable students to analyze their problem-solving strategies and their learning in the course. Documented problem-solving assignments ask students to explain their thought processes in writing as they solve a given problem. Reflective essays are used in my courses to

encourage students to explore their goals for their learning at the beginning to the semester and to assess the progress that they have made at the end of the semester. Although these two types of assignments are quite different, they both require that students consider their thinking and produce a written record of the process. These documents allow both the students and the instructor to recognize patterns of successful and unsuccessful strategies, and I have found, at least anecdotally, that students will share more about their learning on paper than they will share in a conversation.

#### Yu-Ju Kuo Indiana University of Pennsylvania (yjkuo@iup.edu)

#### What I Wish I Knew the First Time I Taught a Writing Intensive Class

Topics in Computational Mathematics is designed to be a writing intensive course for seniors in Applied Mathematics and graduate students in the Master of Science in Applied Mathematics program at Indiana University of Pennsylvania. It is also one of the capstone courses for undergraduates in Applied Mathematics. Course content is subject to change but must focus on using computational methods to solve mathematical problems.

In this course, the assignments include analysis of numerical methods, report of computer simulations and experimentations, incorporation of scientific visualization, construction of computer programs in an appropriate programming language to solve computational mathematics problems, summary of journal articles, proposal of a research topic, and a final report and oral presentation of a research project.

This course was offered for the first time in Spring 2011. The author chose portfolio optimization and option pricing as the main themes throughout the semester. All mathematical content discussed in the course encompassed these two applications. In portfolio optimization, we discussed various nonlinear optimization models and numerical methods. For option pricing, binomial models, stochastic differential equations, Black-Sholes Equations, and finite difference methods were introduced.

In this presentation, the author will share the writing assignments, topics of students' research projects, and ideas for future improvements.

#### **Eric Aurand** Kaplan University (eaurand@kaplan.edu)

Laurel Clifford Mohave Community College (lclifford@mohave.edu)

#### Using Writing to Motivate General Education Math Students

Many students come to their general education courses unmotivated about mathematics. A key tool in helping students to get motivated is through their writing. Key strategies that the presenters have used to motivate students through writing will be presented as well as a brief literature review of writing to learn in mathematics. Participants will also be asked to share strategies that they have used to motivate students to learn mathematics through writing.

# What I Wish I Knew the First (or Second or ... or *n*th) Time I Taught Statistics

# Friday, August 5, 1:00–3:00 pm in in Elkhorn A

Talithia Williams Harvey Mudd College (twilliams@hmc.edu)

# Uncertain about Uncertainty: Motivating Statistical Thinking in Introductory Statistics Courses

The field of mathematics is often the launching pad for students of statistics. However, the teaching and learning of statistics is fundamentally different from mathematical thinking. Statistics is a science of uncertainty and as such, must be approached from an entirely different perspective; one in which you rarely know the truth. In this talk, I will share the viewpoint I use to motivate science, engineering and mathematics students to think like a statistician. I will share what I wish I had known when I first taught statistics, including rethinking the central limit theorem as the course climax and the importance of connecting with students using real world 'messy' data that they are passionate about. We will conclude with tangible techniques that will make you a better teacher of statistics.

#### **Richard Cleary** Bentley University (rcleary@bentley.edu)

#### Statistical Concepts in Everyday Language

Everyday speech is loaded with sentences that can be related to probability and statistics concepts. For example, "This sandwich is pretty good for a fast food place," suggests that quality is a function of restaurant type, with some random variation from an expected value. This presentation includes examples of classroom activities, brief assignments and longer projects that aim to help students see the statistical concepts inherent in their world.

#### **Jason Samuels** CUNY (jsamuels@bmcc.cuny.edu)

#### Better Graphs, Better Formulas, and a Little Technology — Making Statistics a More Unified Course

Statistics seems to many students like a collection of isolated topics and formulas. The speaker served on a textbook search committee for Introductory Statistics and reviewed multiple textbooks and software packages. Based on this review and experience teaching Statistics, the speaker organized the material in an Introduction to Statistics class with a conceptual elegance and coherence which is missing from most textbooks and presentations, and which makes the material easier to learn.

In this approach, improved formulas with simplified notation help students see the connections across topics in statistics. Improved graphs with uniform implementation organize much of the work in inferential statistics into a single template. One piece of technology

used in the course which complements this integrated approach has a graphical user interface for finding probabilities and critical values in various distributions. Another interactive program can be used to build an intuitive understanding of correlation and regression. This modified approach with user-friendly tools has allowed students to design, execute and submit their own statistical projects. The instructional innovations and examples of student work will be presented.

Annela Kelly Roger Williams University (adamannika@cox.net)

#### **Technology in Basic Statistics Class**

Interested in presenting problems with real world data? Do you want to introduce the skills of data analysis that they will use in their future career? You need to use technology. This talk will compare several forms of technology for Basic Statistics class. I will discuss the advantages and differences in using TI-84(TI-83) calculators compared to Excel and Minitab. We will briefly discuss SAS and Fathom. The knowledge about the capabilities and strengths of different technologies will aid the new instructor to make a good choice that fits her teaching style.

Susan D'Agostino Southern New Hampshire University (s.dagostino@snhu.edu)

#### Helping Business Students Acclimate to the Statistics Classroom

All students majoring in business at our mid-sized, regional university are required to take a business statistics course. In the past, many of these business majors approached the abundance of statistical tables, notation and formulas in this required class with anxiety. This talk will address practical steps we have taken to help business students acclimate to the statistics classroom, including a user-friendly formula sheet that was generated with ample student input, color-coded statistical tables that promote early organization and incentives for business students to address deficits that pose roadblocks in the statistics classroom. These easy-to-implement efforts help business students acquire a facility with the notation, formulas and tables of statistics early in the semester, leaving more time to discuss the big ideas.

# Jason Price Nichols College (jason.price@nichols.edu)

#### Let n = 1: The First Time Teaching Statistics

I am teaching Statistics for the first time this semester. The class is taught once a week in the evening to a group of adult business students. I last took a course in statistics in 2001 as an undergraduate and I have only taught traditional day classes in the past. In this talk I will discuss the initial shock and eventual joy that I experienced throughout this experience. I will outline the differences I found in planning this course versus planning other mathematics courses. I will also discuss the challenges that the course presented and the methods of instruction and assessment I eventually found to be the most effective.

# Quantitative Reasoning and Literacy: Pedagogical Strategies

# Friday, August 5, 1:00–3:20 pm in Elkhorn C

Stuart BoersmaCentral Washington University (boersmas@cwu.edu)Caren DiefenderferHollins University (cdiefenderfer@hollins.edu)Shannon DingmanUniversity of Arkansas (sdingman@uark.edu)Bernard MadisonUniversity of Arkansas (bmadison@uark.edu)

# Keeping Quantitative Reasoning Courses Fresh and Relevant to Contemporary Society

For the past few years, the authors have been developing similar courses in quantitative reasoning at their universities, and they have authored a book of case studies of media articles as a principal resource for the courses. The last two authors described the course at the University of Arkansas and some of the results of research surrounding the course in two articles in the July 2010 issue of *Numeracy*, the journal of the National Numeracy Network. One of the challenges of quantitative reasoning courses that rely on media articles as source material is keeping the course fresh. This presentation will include some ways that this has been done in the courses under consideration. A grant from the National Science Foundation (DUE-0715039), Quantitative Reasoning in the Contemporary World, supported this work.

Stuart BoersmaCentral Washington University (boersmas@cwu.edu)Caren DiefenderferHollins University (cdiefenderfer@hollins.edu)Shannon DingmanUniversity of Arkansas (sdingman@uark.edu)Bernard MadisonUniversity of Arkansas (bmadison@uark.edu)

#### Using Rubrics for Assessment, Guiding Student Thinking, and Designing Course Materials for Quantitative Reasoning

Using a modification of the quantitative literacy rubric developed by the Association of American Colleges and Universities, the authors have mapped the study questions in their quantitative reasoning casebook to the six core competencies in the rubric and showed that the rubric can be consistently applied by different readers. The authors concluded after this study that the rubric is very valuable in helping to guide students' quantitative reasoning thought processes and in improving the preparation of course materials. The results of this study are scheduled to appear in *Numeracy*, the journal of the National Numeracy Network. A grant from the National Science Foundation (DUE-0715039), Quantitative Reasoning in the Contemporary World, supported this work.

# Maura MastUniversity of Massachusetts - Boston (maura.mast@umb.edu)Ethan BolkerUniversity of Massachusetts - Boston (eb@cs.umb.edu)

#### Common Sense: A Ten Year Plan for Quantitative Literacy

In this talk we give an overview of a new approach to teaching quantitative reasoning. This work, which is now supported by the National Science Foundation, started when we asked ourselves the question "what do we want our students to remember ten years from now?" rather than "what should the syllabus cover?" Asking that question dramatically changed both what and how we teach. As a result, we have developed both a course and text that focuses on helping students bring common sense, common knowledge and appropriate useful mathematics to bear when facing genuine questions that require them to make sense of numbers. The pedagogy is student-centered and the course is honestly real world and connected to issues relevant to the students. Our philosophy is that in our increasingly data-rich society, students who cannot apply common sense when quantitative reasoning is called for, or who do not have the confidence to use their mathematical knowledge to solve problems, are at a disadvantage. This course is one of the last opportunities to give students that knowledge and that confidence.

# Jodie Miller Mary Baldwin College (jmiller@mbc.edu)

#### The State of Quantitative Literacy at Small Colleges and Universities

This paper examines the current status of quantitative literacy programs and courses at small colleges and universities. Small institutions (those with undergraduate enrollments less than two thousand students) may face unique challenges in implementing QL programs, in terms of limited faculty resources and competing demands resulting from serving both the general undergraduate population and a program for mathematics majors. The researcher examined course descriptions and demographic information for four hundred sixty-four small institutions in the U.S that also offer a major in mathematics. The paper presents summary findings of the research in terms of the types of general education courses and programs currently being offered, institutional demographics, and the interaction of the two.

# Martha Ellen Waggoner Simpson College (murphy.waggoner@simpson.edu)

#### **Quantitative Reasoning Taught Across the Curriculum**

Simpson College recently developed a new general education program called the Engaged Citizenship Curriculum, which includes a skillsacross-the-curriculum program. Quantitative Reasoning (QR) is one of those skills, and students take two QR courses for graduation. In this criteria-based curriculum, departments propose courses that satisfy specific course characteristics and learning objectives. A review panel, composed of members from three different departments, approves the proposals. Departments have access to all approved proposals for examples, and faculty development workshops help departments incorporate QR into their courses. Formative and summative assessment will be developed based on the criteria to assess the college's effectiveness in helping students develop QR skills.

Currently there are 16 QR courses in the curriculum, from 11 of our 25 departments including economics, biology, political science and music. We expect both the number of courses and participating departments to increase, and anticipate courses to be added from sociology, communication studies, and sports science and health education.

This talk will include the rationale for developing a skills-across-the-curriculum program, an outline of the Engaged Citizenship Curriculum, the criteria for QR courses, examples of QR courses from departments other than mathematics and how they meet the criteria. This talk is given from the perspective of one of the original authors of the criteria, who is the chair of the Educational Policy and Curriculum Committee (which has oversight on the development of general education courses) and a teacher of QR courses.

# **Allison Henrich** Seattle University (allisonhenrich@gmail.com)

#### Service Learning in a Quantitative Reasoning Course

In most college-level math courses, the goals of the course revolve around the acquisition of knowledge in a particular area of math. In quantitative reasoning courses, however, it is not uncommon for one of the primary course goals to be reducing the level of math phobia of the students. Many students who take quantitative reasoning courses are students who have decided at some point that they are bad at math and that it is okay to be bad at reasoning quantitatively because this skill is probably not useful for their lives. Incorporating service learning into a QR course can help to address these issues. A quantitative reasoning course with service learning can help students gain confidence in their abilities, overcome their fears of math, become more proficient at reasoning quantitatively and see how this skill is useful in their lives. We will discuss the benefits to students of QR courses in general and, in particular, QR courses with a service learning component that requires students to tutor children in math.

#### Ben Galluzzo Shippensburg University (bjgalluzzo@ship.edu)

#### Ladders Don't Slide

Applied or business calculus is a terminal course for many students at many universities. However, sliding ladders and changes in widget production fail to capture true real world connections. In an attempt to encourage students to discover that concepts encountered in calculus exist in real life, application-based problem sets were added to the curriculum of two Applied Calculus courses at Shippensburg University. In this talk, we examine students' solutions to some of the problems and will discuss how this conceptual approach translates to: student growth in quantitative literacy, student achievement in the classroom, and student understanding of standard calculus calculations.

# **Recreational Mathematics: New Problems and New Solutions**

# Session 1: Friday, August 5, 1:00–4:40 pm in Elkhorn D

# Michael A Jones Mathematical Reviews (maj@ams.org)

#### Utility Theory and Deal or No Deal

*Deal or No Deal* was a prime time game show on NBC in which a Contestant selects one of 26 suitcases. Inside each suitcase is a different monetary amount; the amounts are known beforehand. In a series of rounds, the Contestant is asked to "deal" (in which she accepts a monetary offer from a Banker) or to "no deal" (in which she has to open a specified number of suitcases, thereby revealing the dollar amounts inside the suitcases). The game ends when either she accepts an offer or, after opening all of the suitcases except the one she selected initially, she receives the amount in her selected suitcase.

In this talk, I will introduce the basics of utility theory and will explain how the Banker could use a utility function to determine the offer. Such a utility function describes the Contestant's value for money and incorporates the Contestant's view toward the risk of participating in the lottery over the monetary amounts in the remaining suitcases. The Banker makes an offer so that the Contestant is indifferent between accepting the Banker's offer and continuing to play the game. I will demonstrate how data from televised episodes may be used to recover the utility function. Further, I will examine a paradoxical offer from NBC's online version of the game.

#### Anthony DeLegge Benedictine University (adelegge@ben.edu)

#### "Come On Down!" Mathematics Behind The Price is Right

*The Price is Right* is the longest-running, and one of the most popular, game shows on American television. For almost 40 years, contestants have "Come On Down" and have won over \$2 billion worth of prizes. Many of these prizes are won by playing one of the show's various pricing games. Although every game has an element of pricing something in it, many games also have an element of luck to them. Some games, in fact, can be won with little or no pricing knowledge at all.

In this talk, we will consider some of the show's pricing games from a probabilistic point of view. Specifically, we will look at the odds of winning a game assuming no knowledge of prices and then how much the odds of winning improve if some pricing knowledge is assumed.

#### **David Strong** Pepperdine University (david.strong@pepperdine.edu)

#### Wicked: a problem in counting and probability

Every evening before *Wicked* was performed at the Pantages Theater in Hollywood, a pre-show lottery drawing was held. Thirteen lucky persons would have their names drawn, and be able to buy front row seats at a very low price. Each person could buy two tickets, which means that if more than half of a group's members had their names drawn, they would have access to more lottery tickets than they need. Thus two groups of patrons could 'team up' with each other with the agreement that if either group gets more tickets than needed, they would allow the other group to buy those extra lottery tickets. The question then is what the optimal size group is with which to team up? The answer isn't terribly surprising, but it is mathematically very interesting.

#### **Jeff Johannes** SUNY Geneseo (johannes@member.ams.org)

#### **Probability and Strategy in Farkle**

Farkle is a popular dice game played with six dice. In this talk we will describe the rules and present probability computations which lead to analysis of different strategies. Nothing more than elementary probability will be used.

#### Jonathan Needleman Le Moyne College (needlejs@lemoyne.edu)

#### Words Searched: The math of BOGGLE logic puzzles

Boggle ©Hasbro Inc, is a popular word game where players try to find as many words as they can in a grid of letters. In this talk we will take the opposite point of view. Instead of a grid, you are given a list of words and try to construct a board with these words. It is not clear under what conditions this is possible and unique. Under some simplifying conditions we will discuss both the minimum number of words needed to guarantee a unique solution, as well as the minimum number of words (carefully chosen) that yields a unique board.

#### **Robert Vallin** Slippery Rock University (robert.vallin@sru.edu)

#### **Diagonal Tricks for KenKen Puzzles**

KenKen (also called KenDoku) is an arithmetic and logic puzzle invented by Tetsuya Miyamoto in 2004. KenKen Puzzles have a little more mathematics behind them than the wildly popular Sudoku puzzles. In this talk, we introduce the puzzles and look for some patterns to the solutions, hoping to find an edge in finishing one.

# Andrew deLong Martin Kentucky State University (andrew.martin@kysu.edu)

#### Pro Prob. Problem: Expected Number of Wins vs Expectation of Winning Two-in-a-Row

You will play three tennis games against your father and the club champion, alternating the games between them. The probability of winning against your father is high, but that against the club champion is low. If a special prize will go to you if you win two games in a row, will you

have a better chance of winning that prize playing father-champ-father or champ-father-champ? This is Problem 2 of Frederick Mosteller's classic "Fifty Challenging Problems in Probability with Solutions". I will present the solution. In addition, I will pursue the question of how many times greater is the likelihood of winning if you make the correct choice? Can it be twice as likely? Three times as likely? Is the sky the limit?

#### **Andrew Simoson** King College (ajsimoso@king.edu)

#### The Ben-Hur Staircase Climb

Suppose we wish to exercise the right and left legs alike when climbing a staircase. Such a climb is called a Ben-Hur climb after a fictional character aboard a slave galley who requested that he be allowed to switch rowing positions periodically between the left and right side of the ship so as to develop both sides of his body. In particular, some cardiologists prescribe staircase climbing as therapy for their recovering patients: take one or two stairs at a time - as more than that is far too strenuous - alternating between the left and right leg so that the number of times one stair or two stairs are taken with the left equals the same number for the right, respectively. If the staircase has 2n stairs, how

many different Ben-Hur staircase climbs are there? We show that the answer is  $\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} {\binom{n-i}{i}}^2$  when  $n \ge 1$  and sketch how to find the

generating function as well.

#### David Jacob Wildstrom University of Louisville (dwildstr@erdos.math.louisville.edu)

# Tracking Fugitives in the Hotel Yao: Organizational Schemes for Efficient Lookup

From a population of m people, n check into a hotel with n rooms. Our confederate at the front desk arranges the lodgings to make it easy for us to determine whether any particular individual (the 'fugitive') is staying there. A 1981 result of A. Yao presents an organizational scheme for  $m \leq 2n-2$  so that, regardless of who the fugitive is, raiding a single room is sufficient to determine whether they are at the hotel; Yao also proves that there is no such strategy for m > 2n - 2. We explored several variants of this problem, specifically considering both constraints placed on the set of n people that can arrive, and considering the improvements on the m = 2n - 2 bound when we can investigate two rooms instead of just one. This presentation is based on a collaborative project with Paul Horn, Adam Jobson, and Andre Kézdy.

#### **Susa Stonedahl** Northwestern University (susa@u.northwestern.edu)

**Forrest Stonedahl** Northwestern University (forrest@u.northwestern.edu)

#### Think-Tac-Toe: When are puzzles solvable?

We present a new pen-and-paper logic puzzle that we call Think-Tac-Toe (based on a surface similarity to Tic-Tac-Toe, although the puzzle mechanics more closely resemble the computer game Minesweeper). In Think-Tac-Toe, you are given a rectangular grid of numbers and asked to logically deduce which cells contain X's and which O's. The number in each cell represents the number of X's in that cell's neighborhood, where a cell's neighborhood is composed of the cell itself and any adjacent cells (including diagonals). Even small grids (e.g. 3x3) can provide a reasonable challenge of deductive logic, while larger grids can encode hidden pictures for the puzzler to discover. These puzzles were well received in a middle school math enrichment program that we teach. However, more interestingly, only certain grid sizes are guaranteed solvable (i.e. possess a unique solution for any configuration of X's and O's). We show that a grid size is guaranteed solvable if and only if the adjacency matrix of the graph associated with the connections among grid cells is invertible. For any given grid size, this can be found using basic linear algebra. We further prove that a grid size is guaranteed solvable if and only if both dimensions are not congruent to 2 (modulo 3). However, it is an open question to characterize when a puzzle will be guaranteed solvable when played on an arbitrary graph (rather than a rectangular grid). We will distribute example Think-Tac-Toe puzzles to the audience, and discuss the underlying mathematics.

#### Hollie Lee Buchanan West Liberty University (hbuchanan@westliberty.edu)

#### Minimal Modified Slitherlink Puzzles

A popular logic puzzle involves reconstructing a predetermined circuit on a grid by using the number of edges of the circuit that are incident with certain grid regions. Although published puzzles typically involve large grids and some redundant information, we consider the minimal set of information from which a puzzle can be solved. We also discuss a modification made to facilitate counting such puzzles. Some discussion of enumerating circuits in grid graphs is included.

# Susan Goldstone Ellie Baker

#### Building a Better Bracelet: Wallpaper Patterns in Bead Crochet

Jewelry fashioned from bead crochet ropes has caught the imagination of crafters and trendsetters every few decades for the past century, and bead crochet bracelets are in the midst of a surge in popularity. A bracelet consists of a narrow tube of crocheted seed beads sewn together to form an uninterrupted hollow band. Creating symmetric color patterns on bead crochet bracelets is challenging because the beads form a continuous spiral along the length of the bracelet, making it difficult to align design motifs uniformly.

We have devised a new technique for translating bracelet patterns into plane tilings and used this to create new bracelets based on wallpaper group symmetries to stunning effect. This talk will give a classification of the wallpaper groups that can be adapted in bracelet form and exhibit designs substantially more intricate than those available in current pattern books.

# Session 2: Saturday, August 6, 8:30–11:30 am in Elkhorn D

Richard Douglas ChathamMorehead State University (d.chatham@moreheadstate.edu)Maureen DoyleNorthern Kentucky University (doylem3@nku.edu)Robert JeffersNorthern Kentucky University (Robert.I.Jeffers@gmail.com)Walter A. KostersUniversiteit Leiden (kosters@liacs.nl)R. Duane SkaggsMorehead State University (d.skaggs@moreheadstate.edu)Jeffrey WardNorthern Kentucky University (wardj1@nku.edu)

## Centrosymmetric Solutions to the N + k Queens Problem

For integers  $n \ge 4$  and  $k \ge 0$ , the N + k Queens Problem calls for the placement of n + k queens and k pawns on an  $n \times n$  square chessboard so that any two queens on the same row, column, or diagonal have at least one pawn between them. In this talk we consider solutions that are unchanged by a 180-degree rotation ('centrosymmetric') and those that are unchanged by a 90-degree rotation ('doubly centrosymmetric'). We also consider solutions for which the symmetry of the queens differs from that of the pawns.

# **Greg N. Frederickson** Purdue University (gnf@cs.purdue.edu)

### The Proof is in the Pizza

You pick up your (circular) pizza from the pizzeria, but the guy was sloppy when he made cuts along 4 chords. Although he made all the cuts go through a single point, and the cuts are every  $45^{\circ}$ , the common point of intersection is not at the center of the pizza. Even so, you and your friend can each get equal shares of pizza, if the two of you alternate taking slices in clockwise order. This is the Pizza Theorem, and it holds for any even number of cuts,  $k \ge 4$ , with angles of  $180^{\circ}/k$ . For any such k, we give a dissection-based proof, whose correctness relies on a healthy helping of mirror symmetry. If you instead order a perfectly spherical calzone, and the appropriate set of cuts is again off-center, there is still a dissection-based proof that each of you can get equal shares, as we shall show.

#### **Pam Warton** The University of Findlay (warton@findlay.edu)

## When Two Worlds Collide

What happens when a mathematician also has a passion for classical music? Group theory meets Mozart when some of his most famous themes are transformed by different permutation groups. Do even permutations result in a more pleasing melody than odd permutations? How do permutations of different orders affect the music? I will address these questions as well as how permuting notes affects harmony.

Shenglan Yuan LaGuardia Community College, CUNY (syuan@lagcc.cuny.edu)

#### **Fun Puzzles Using Modular Arithmetic**

Modular Arithmetic is a foundation for many branches of mathematics and it has been used extensively to discover profound properties of numbers. Here we'll show how we can use modular arithmetic to create fun puzzles that can be enjoyed by both non-mathematicians and mathematicians alike.

# Jonathan Stadler Capital University (jstadler@capital.edu)

# **Juggling Sequences and Restrictions**

The mathematics of juggling is an engaging application that can be shared in a variety of undergraduate courses. Juggling sequences, which describe how long objects are in the air, are an especially interesting addition to the discrete mathematics curriculum. We present a novel way to generate juggling sequences and discuss various restrictions that are possible. In particular, we describe a method for counting juggling patterns with bounded heights.

**Patrick X. Rault** SUNY Geneseo (rault@geneseo.edu)

#### From Doodles to Induction: Recreational Research in Office Hours

Some colleges are attempting to bridge the gap between research and classroom instruction. A natural avenue is to bring recreational inquiry into the classroom. For example, a standard induction question about tri-ominoes covering  $2^m \times 2^m$  boards leads to natural questions about covering general  $n \times n$  boards. Once students realize the value of generalizing homework questions, they begin coming to office hours with projects: one number theory student developed a pattern of doodles which (unbeknownst to her) involved deep properties of the arithmetic group  $Z_p$ . Another student questioning the length of repeating decimal expansions rediscovered a result of Gauss. These recreational pursuits provide a taste of mathematics research while simultaneously motivating success in the classroom. We will discuss the solutions of some of these problems and provide food for thought on others.

#### Marc Renault Shippensburg University (msrenault@ship.edu)

#### Describing all Gibonacci Sequences, Modulo m

A Gibonacci sequence (generalized Fibonacci sequence) G is defined by integers  $G_0$ ,  $G_1$ , and the recurrence  $G_n = G_{n-1} + G_{n-2}$  for all integers n. In this talk we examine properties of these sequences modulo m. How many are there? What are their periods? For a given modulus, how might we naturally partition the set of all these sequences? The results of this talk came from a very successful undergraduate research project conducted with student Josh Ide at Shippensburg University.

# Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

#### Exploring The Home Prime Conjecture in Bases Duodecimal and Hexadecimal

The Home Prime Conjecture is a fascinating problem that is easily posed but is ripe for further research. Consider any composite integer and resolve the integer into its prime factorization in conanical form. Concatenate the resulting integer and repeat the process always expressing the new prime factors in increasing order of magnitude. The Home Prime Conjecture asserts that after finitely many iterations, a prime will be obtained known as the Home Prime of the original composite integer. To cite a simple example, we consider the composite integer 10 in base ten. The sequence of iterations can be compactly represented as follows: 10 = 2\*5 - 25 = 5\*5 - 55 = 5\*11 - 511 = 7\*73 - 773 a prime which is obtained in four steps and is the Home Prime of 10. While many composite integers have their Home Primes resolved in a few steps, the search for the Home Prime for the integer 49 is stalled after more than one hundred iterations. The paradoxical nature of the primes is in focus here. In each iteration, a larger integer is formed. The likelihood that a randomly generated large integer is prime decreases as we go further out in the sequence of natural numbers. On the other hand, the number of positive primes is infinite so that a Home Prime for each composite integer must happen! We will review some results in base ten, but will focus on results I have obtained for one and two digit base twelve Home Primes as well as my current work on Home Primes in the hexadecimal base.

## Hossein Behforooz Utica College (hbehforooz@utica.edu)

#### Weighted Magic Squares

This talk is dedicated to Martin Gardner, a great non-mathematician who contributed so many mysterious mathematical puzzles to the world of recreational mathematics. Yes MATH is FUN. In this talk, for the first time in the history of magic squares, you will be introduced the weighted magic squares, obtained by changing the numbers to weights in each cell. This idea opens many questions and we will discuss about the center of mass (equilibrium point) of these kinds of weighted magic squares. Is there any imbalanced weighted magic square?

# Teaching High School Mathematics: Beautiful Lessons found on the Scenic Route

# Saturday, August 6, 1:00–3:00 pm in Elkhorn D

Daniel Jones Teague NC School of Science and Mathematics (teague@ncssm.edu)

## **Exploring Families of Curves**

Consider the family of lines Ax + By = C where A, B, and C are in arithmetic progression. If we graph simultaneously members of this family, what will we see? How about a geometric progression? or the condition that A \* B = C? Suppose instead of linear equations, we consider curves of the form  $Ax^n + By^m = C$  with similar restrictions on A, B, and C. This lesson involves students making connections between the algebraic structure imposed on the equations and the geometric presentations of their graphs. It makes a great first creative investigation for students in precalculus and illustrates the importance of simple proofs of simple conjectures.

# **Peter Joyce** CCBC at Catonsville (pjoyce@ccbcmd.edu)

### Graphing Rational Functions and Solving Rational Inequalities Without a Calculator

Many textbooks that solve the problem of graphing rational functions or solving rational inequalities lack a nice way to unify their steps to create a simple method to solve problems.

With standard rules of end behavior of rational functions based on the degree of the numerator and the degree of the denominator and the sign of leading coefficients of the numerator and the denominator together with the multiplicity of the zeros and vertical asymptotes we can solve the problems in record time. Also, given a graph of a rational function we can approximate the actual rational function very easily.

Solving rational inequalities and graphing rational functions based on solid mathematical theory is fun.

#### Kristine Hoffman West Platte R-II Jr/Sr High School (hoffmank@wprii.k12.mo.us)

#### **Outdoors with Mathematics in the National Parks**

In my 4 years of teaching, I've been reflective about how my 'math' mind functions differently from my students' minds. To help them to begin thinking more mathematically, I created a project that explores mathematics using the National Parks as a background. I will discuss how a roadtrip to the National Parks of the southwest captured math in its natural states, and how photos and data collected from the trip were used to create a math poster series and curriculum. For example, natural arches are presented as parabolas and quadratic equations, and canyons likened to the negative set of numbers. Also to be discussed is how students will develop individual "Math and Nature" education portfolios to include their own photographs of math in nature and written reflections, following the development of both their math skills and critical thinking skills over the course of a year.

This project organically brings several disciplines together through a focus on math and nature and captures how present math is in our everyday lives. My hope is that nature will be a non-intimidating way to experience math and will give students a positive experience with math. I want students' confidences to be built, opening their minds to the idea that they can do math and math is important in their lives. The

best way to develop their math literacy and their math vocabulary is to help them 'see' math in the world around them. This project uses the scenic National Parks to do just that.

# Susan Schwartz Wildstrom Walt Whitman High School (susan@wildstrom.com)

#### Proofs by Mathematical Induction for High School Students

Proof of any sort, but especially inductive proofs can be very difficult for students to construct and even harder for them to 'believe'. The presenter has developed a method for teaching students to construct inductive proofs according to a pattern that is both easy to remember and straightforward to use. To enhance the method, she teaches the topic at the beginning of a unit in which it is relevant and then assigns one to three proofs each day over the course of a couple of weeks. By giving this topic an extended period in which students learn and practice it, the presenter has had success in students' ability to understand the power of inductive proofs.

#### **Robert Sachs** George Mason University (rsachs@gmu.edu)

#### Size of n! (Stirling Approximation) in Calculus: Important, Useful, and Fun

For large n, n! grows like  $\sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ . In my university honors calculus and as a visiting speaker (for one hour) in an AP class, I love to show how to obtain this approximately, except for the constant, which is estimated. The derivation ends up using ideas of numerical integration, a touch of integration by parts, and logarithms, along with the idea of the integral test run backwards. When discussing Taylor series, this approximation helps clarify which functions are likely to have finite radii of convergence versus infinite radii of convergence.

#### Susan Schwartz Wildstrom Walt Whitman High School (susan@wildstrom.com)

#### Using Explorations to Discover Derivatives

Students learn best those things which they either discover themselves or which they see as evolving from logical underpinnings. Students' easy access to technology has made it possible for teachers to show students what the graphs of the derivatives of elementary functions actually look like. Seeing the features of the graphs of the original function and its derivative may reinforce a particular derivative (for example the derivative of the sine is the cosine), or it may help students see where additional elements influence the derivative (how the derivatives of composite functions involve a 'chain' rule). In addition, looking at values for a derivative for certain values of the variable, might lead a student to see a pattern (the derivative of the natural log function is the reciprocal of the variable; the derivative of an exponential function is also an exponential function but with a constant coefficient whose value might be discovered). Even groupwork brainstorming can be an effective way to discover a derivative rule - for example the product and quotient rules can be developed through clever experimentation. A handout of a number of explorations suitable for use in AB or BC Calculus courses will be available and one of the explorations will be highlighted in the presentation.

# Know More, Teach Better? Content Knowledge for Secondary Teaching and Above

# Saturday, August 6, 1:00–3:40 pm in Elkhorn B

**Diana White** University of Colorado Denver (diana.white@ucdenver.edu)

#### Number and Operation for Preservice Secondary Math Teachers

Mathematical knowledge for teaching can be described simply as the "mathematical knowledge needed to carry out the work of teaching mathematics" (Hill, Rowan, and Ball, 2005). This knowledge is different from the knowledge developed in the course of a traditional math major. Recognizing this, there have been numerous recommendations put forth. In particular, the Conference Board of the Mathematical Sciences (2001) recommends dedicated mathematical coursework for preservice teachers at both the middle and high school level.

In this talk, we discuss an innovative course in number and operation for pre-service secondary (grades 6–12) math teachers. This course is dedicated to unpacking and deepening knowledge of the material in number and operation from approximately grades 3–8, with a particular focus on rational numbers. In this course, pre-service teachers focus on explaining terms and concepts, analyzing student solutions, using precise mathematical language, using rigorous mathematical reasoning from age-appropriate definitions, and learning multiple representations of various concepts. We discuss the set-up, goals, and outcomes of the course, with a focus on results from student surveys and from a pre-post error analysis instrument.

Elizabeth BremiganBall State University (egbremigan@bsu.edu)Ralph BremiganBall State University (bremigan@bsu.edu)John LorchBall State University (jlorch@bsu.edu)

#### Pre-college Mathematics from an Advanced Viewpoint

We discuss a mathematics content course for prospective secondary school teachers offered at Ball State University for the past eight years, as well as a related NSF-funded curriculum project that has resulted in a textbook recently published by the MAA. Students investigate topics of central importance in the secondary school curriculum using tools from college-level mathematics, including functions, polynomials,

trigonometry, exponential and logarithmic functions, number and operation, and measurement. Beyond the obvious goals of conceptual understanding and computational fluency, students are invited to devise mathematical explanations and arguments, create examples and visual representations, remediate typical student errors and misconceptions, and analyze student work.

Angela BowzerWestminster College (Angela.Bowzer@westminster-mo.edu)Angela HodgeNorth Dakota State University (ang.hodge@gmail.com)

# The Impact of K-12 Policy on 14-16+ Mathematics Education

Developed as college and career readiness expectations, the Common Core State Standards (CCSS) are the closest our nation has come to a common curricula for our nation's K-12 students. Over 40 states have adopted the CCSS to serve as the framework for their students' mathematical development. As we consider the development of a common core of mathematical knowledge for our K-12 students, we also need to consider the potential impact that shifting the knowledge base in our high schools has on the mathematical preparation of future secondary mathematics teachers. In this session, we provide background on the CCSS and discuss potential implications for teacher education, especially as it relates to preservice teachers' mathematical needs. Specifically, we will address the mathematical content knowledge future mathematics teachers will need to successfully implement the CCSS in their classrooms.

#### Justin Hill Texas A&M University - Central Texas (justin.hill@ct.tamus.edu) Christopher Thron Texas A&M University - Central Texas (thron@ct.tamus.edu)

# An Abstract Algebra Class for Secondary Mathematics Teachers

Almost all math majors at Texas A&M - Central Texas are prospective secondary mathematics teachers. This paper describes our redesign of our Abstract Algebra class to serve these student's needs. This involved both writing a textbook that would target the learning modes of typical non-mathematics students, as well as changing the style of content delivery in class. Our new text (based on open-source material) had separate chapters on complex numbers, modular arithmetic, symmetries of geometrical figures, and permutations; these were presented as prototypical algebraic systems. Students were given numerous examples and directed exercises to illuminate and prove these systems' algebraic properties. Building upon this knowledge, students were then able to prove general group properties based on group definitions. Students' proving skills were carefully developed through explicit instruction, examples and exercises, taking nothing for granted. Our modified classroom approach deemphasized lecture in favor of individual and cooperative work on carefully selected examples and exercises. Guided reading assignments and brief lectures were used to communicate and test the students' understanding of foundational concepts from the day's text. The bulk of class time was highly interactive, as the students dialogued with each other and the instructor as they worked on exercises. Concepts and skills were re-taught as the need arose. We evaluated the effectiveness of our approach through a student survey. Student scores on the algebra class to student scores on the algebra class to be monitored.

# Todd D. Oberg Illinois College (toberg@ic.edu)

# Capstone as a Transition

At Illinois College, the Capstone for Secondary Education serves as a capstone experience for Math majors seeking Secondary Certification and as a transition experience to the secondary classroom. As part of this experience, teacher candidates revisit topics from the secondary curriculum, re-examining them in light of their undergraduate coursework. Teacher candidates focus on developing a deeper understanding of some of the mathematics that they may teach in a high school setting. This talk aims to give a brief overview of the Capstone course offered at Illinois College, and then focus on one of the key components of the course - the extended problem analysis. In an extended problem analysis, a teacher candidate starts with a simple problem - typically coming from the high school mathematics curriculum - and is guided through an analysis of its solution(s) and its variant(s) to gain a deeper understanding of the problem itself, the underlying mathematics in the problem, and connections associated with that problem.

# **Joyati Debnath** Winona State University (jdebnath@winona.edu)

# Blending Mathematical Content with Pedagogy in Upper-Level Mathematics Classes

It has been a topic of discussion for at least last two decades how concepts of mathematical content is to be blended with the pedagogical concepts in teaching of mathematics. It is very important to understand how a teacher connects these two ideas as they cannot be separated. In this presentation, different topics of higher-level mathematics courses will be discussed that can blend these concepts.

# Jessica de la Cruz Assumption College (jdelacruz@assumption.edu)

# Increasing Prospective Secondary Teachers' Content and Pedagogical Knowledge Through Problem Solving

Research has shown that secondary teachers need to have more than strong content and pedagogical knowledge to be effective teachers. They also must possess deep pedagogical-content knowledge (i.e. best practices for teaching the subject) and knowledge of their students' thinking (i.e. ways students are likely to solve a problem, typical misconceptions). One way to blend instruction on content and pedagogy, as well as increase knowledge of students' thinking, is to use a problem solving approach. By posing carefully selected problems, the instructor can address common misconceptions, increase mathematical communication and explanation abilities, and, as a result, deepen content knowledge. This presentation will provide examples of such problems and discuss the ways that they can be used to deepen pedagogical content knowledge.

# Taliesin Sutton University of Arizona (tsutton@email.arizona.edu)

#### Teachers Analyzing Math in the Classroom: Using Video to Assess, Train, and Prepare Future Teachers

Due to increasing interest and scrutiny in the quality of teachers and their preparation, there is a growing need to provide tools to measure the knowledge teachers implement in their classrooms. I will discuss how teaching pre-service teachers to analyze video clips of math classrooms is an effective means to train and prepare them for their future careers as well as provides a means to assess their preparedness to engage in mathematics in the classroom setting. To do this, I will draw upon examples from an assessment that has proven to be promising in measuring teacher effectiveness and discuss how this robust format for assessing teachers can be implemented in teacher preparation programs. This form of assessment is modeled after the work done in the Capturing Teacher Knowledge grant funded by the Institute Education Sciences.

# **General Contributed Paper Sessions**

# GCPS #1: History and Philosophy

# Thursday, August 4, 8:30–10:15 am in Elkhorn C

Julianna Connelly Stockton Sacred Heart University (stocktonj@sacredheart.edu)

# **Designing a New History of Mathematics Course**

When it comes to designing a new course, there are a number of questions to be answered: Which content will be covered? What textbook will be used, if any? What are the desired learning outcomes for students? How will these goals be assessed? And perhaps more importantly, what is the heart of the course? History? Mathematics? Something else? We will address these questions in the context of a new history of mathematics honors course, for majors and non-majors, being offered in Fall 2011. In designing this course from scratch, it proved helpful to examine available offerings at other schools as well as current research on using history to teach mathematics and incorporation of primary historical sources in mathematics courses. Through discussion of this design process, we hope to provide a more general framework that could be adapted to a variety of circumstances.

#### Jeff Johannes SUNY Geneseo (johannes@member.ams.org)

#### A Mile Wide and an Inch Deep - 4000 BCE to 1950

After being warned not to, and seriously questioning it, I taught history of mathematics and started at the beginning and went to the end. We discussed many cultures and learned a lot of mathematics and history along the way. Unlike the other times I have taught this class - this was a history class more than a mathematics class. They wrote essay exams comparing the topics, and I was more interested in them demonstrating some learning than any particular learning. The experiment was enjoyable and seemed surprisingly successful.

# Mary B. Walkins Lee University (leolawalkins@yahoo.com)

#### History of Mathematics: An Exercise in Strengths

As a leader in strengths-based education, Lee University encourages each new student, since fall 2003, to take the Gallup StrengthsFinder to determine their top 5 signature themes (out of a possible 34). At Lee, the syllabus for the History of Mathematics course calls for students to write a paper on a mathematician. As an added dimension, students were asked to critically think about and incorporate the strengths they believe that mathematician may have. Each student was required to compare and contrast his or her strengths with those of the mathematician. This was done with the hope that, as aspiring mathematicians, they may be inspired to persevere to make their mark in the history of mathematics, since math is still evolving. In this presentation, through an exercise in strengths, I will share examples of how students were inspired by each mathematician.

Linda Becerra University of Houston-Downtown (becerral@uhd.edu) Ron Barnes University of Houston-Downtown (barnes@dt.uh.edu)

#### Key Contributors in the Evolving Role of Women in Mathematics

The role of women and their contributions in mathematics have evolved over time. This talk considers five key women contributors, each of whom had significant contributions to mathematics and broke new ground in this evolution. The five women considered are Hypatia, Sophie Germain, Ada Lovelace, Sonya Kovalevsky and Emmy Noether. The talk begins with a short sketch of the significance of each of these individual's mathematical contributions. This is followed by a biographical sketch of the individual with the challenges she addressed, and how in meeting these challenges, she paved the way for other women in mathematics. In addition, the talk explores some of the myths and legends that have accrued around these individuals. The talk concludes with a profile of the role of women in mathematics today, utilizing recent statistical data. Some tentative conjectures will be made on the future of the role of women in mathematics, in light of this profile.

# Dale Lewis McIntyre Grove City College (dlmcintyre@gcc.edu)

# Ten Mathematicians Who Proclaimed the Greatness of God

British author, mathematician, and philosopher Bertrand Russell (1872-1970) once observed that, "Religion is something left over from the infancy of our intelligence. It will fade away as we adopt reason and science as our guidelines." Fortunately such beliefs were not embraced by Newton, Leibniz, Euler, Cauchy, and other profound thinkers who believed God had commissioned and equipped them to honor Him in their pursuit of truth through mathematics. In this paper, ten of history's most influential mathematicians express such convictions in their own words; the author does not wish to summarize or paraphrase the eloquent and moving statements of these gifted and godly practitioners of the art of mathematics. It is hoped that we modern mathematicians will learn from or be reminded by our illustrious forebears of an all-wise and benevolent God who enables us to learn and apply mathematics for His glory and man's betterment.

# **Doy Hollman** Lipscomb University (doy.hollman@lipscomb.edu)

#### The Mathematics of the Calendar

Geometry has been thought as the math of the earth, astronomy the math of the sky. The calendar can certainly be thought of as the math of time-its beginning, length, cycles, and measurement. This session is a look at the astronomy and mathematics of different cultures, including the Egyptian, Babylonian, Islamic, European, and especially the Maya culture and its calendar, as man continued his search for that true and accurate interval of time called 'year.'

#### **David Nacin** William Paterson (nacind@wpunj.edu)

#### Assessment Methods for Teaching the History of Mathematics Online

In this presentation I will discuss the course design, grading criteria and assessment methods I used in my online courses, focusing on what worked and what didn't.

An online class is a very different environment than the traditional classroom, and the History of Mathematics is very different than the traditional math course taken by majors. Much care must be taken to avoid common pitfalls and keep the students engaged without a face to face connection. My methods of addressing these problems included checkpoints for long range assignments, discussion board topics, peer feedback and both timed and untimed quizzes in blackboard.

# GCPS #2: Teaching and Learning

# Thursday, August 4, 8:30–10:15 am in Elkhorn D

Alexander Atwood Suffolk County Community College (atwooda@sunysuffolk.edu)

#### Bringing Neuroscience into the Mathematics Classroom

Two concepts in neuroscience help inform how students learn mathematics: working memory and the society of mind. Working memory, conceptually formulated by Alan Baddeley and Graham Hitch in 1974, is composed of four major components in the human brain: a central executive that controls conscious decision making, a visuospatial sketchpad for the storage and manipulation of visual information, a phonological loop for the silent repetition of phrases, and a short term memory buffer for the storage of small amounts of information. Working memory is intimately tied to fluid intelligence, the type of intelligence that enables humans to solve novel mathematical problems. The society of mind, conceptually formulated by Marvin Minsky and Seymour Papert in 1986, is a way to understand how conceptual connections and complex intelligence in the human brain emerge from the interaction of relatively simple cognitive agents. The construction of connections between mathematical concepts in the human mind is enabled by the society of mind. Students can benefit greatly from understanding how working memory and the society of mind enable them to create important linkages between mathematical concepts and to solve mathematical problems.

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Kathryn A Brenneman North Carolina State University (kabrenne@ncsu.edu)
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**Janine Haugh** UNC Asheville (jhaugh@unca.edu)

Sarah Wright College of the Holy Cross (swright@holycross.edu)

# Empowering Students through Peer Mentoring in Early College Mathematics Courses

We will discuss a course-centered Peer Mentoring Project designed to be implemented at mid-semester to engage successful students in tutoring their struggling peers. Originally conceived as a project for a 28-student Precalculus class at NC State, and spread by word of mouth and collaboration to two sections of Calculus I at UNC Asheville and two sections of Calculus II at Holy Cross. We will give the motivation for offering the project in each of the three courses, the results, and student feedback.

**Elizabeth Burroughs** Montana State University (burroughs@math.montana.edu)

#### Examining How Teachers' Mathematics Content Knowledge Explains Teaching Practice

This study investigates how mathematics knowledge for teaching explains facets of teacher practice. It uses a large sample of teachers from 24 school districts across seven states to explore relationships between data from a commonly used, valid and reliable multiple-choice measure

of teacher mathematics knowledge for teaching and data from a commonly used, valid and reliable classroom observation protocol. The analysis explores the question of how data about teacher mathematics knowledge is related to mathematics content, classroom culture, and teaching in standards-based mathematics classrooms. This analysis also examines predicting differences in teaching practice among low and high scoring teachers.

**David Easdown** University of Sydney (de@maths.usyd.edu.au)

#### **Excursions to and from Semantic Oblivion**

This report offers some recent experiences from the author's teaching that highlight the fragility of language and mathematical formalism in communicating ideas and concepts. Syntactic reasoning is at the superficial end of the spectrum, "skating on the surface", and involves formal manipulation of symbols, simple rules and substitutions. Semantic reasoning is deeper, "diving down towards the seabed", drawing conclusions from underlying meanings and heuristics. The author argues that illuminating the tension and interplay between these complementary modes of reasoning, and creating heightened awareness, may enhance approaches to successful learning, improve morale and attitudes, and lead to more robust outcomes and willingness to engage in challenging mathematical activities.

#### Sue R. Beck Morehead State University (s.beck@moreheadstate.edu)

#### Mastery Learning: Meeting the Developmental Math Needs of a Variety of Students

Our student population is composed of new high school graduates, high school seniors, non-traditional students, and displaced adult workers returning to higher education. We have found that this diverse population requires different teaching methods. In this session there will be discussion of each of the methods we use to elevate our students to college readiness.

# Holly Zullo Carroll College (hzullo@carroll.edu)

#### Understanding Classroom Voting

Classroom voting is a student-discussion oriented approach to teaching in which students learn by discussing and responding, via clickers or another voting mechanism, to multiple-choice questions presented in class. The NSF-funded Project MathVote involves a collaboration of faculty from six schools, including two community colleges, who are studying the nature and effectiveness of various strategies for implementing classroom voting. We will present preliminary results from this study. Questions being addressed include: What types of questions lead to better small group and whole class discussions? What instructional decisions lead to better small group and whole class discussions? In what ways do student conceptions of mathematics evolve while using classroom voting? In what ways does student confidence evolve while using classroom voting?

# **Gregory Kelsey** University of Illinois at Urbana-Champaign (gkelsey2@illinois.edu)

#### Social Choice in a Liberal-Arts Mathematics Course

We present a unit on voting theory and social choice taught as the beginning of a liberal-arts mathematics course for undergraduates. In this unit, students vote on the course policies and content and then analyze the results using various voting systems. By deciding important aspects of the course collectively, we seek to empower our students and increase their motivation. The activities also serve as examples of how mathematics plays an important role in their lives.

# **GCPS #3: Applied Mathematics**

# Thursday, August 4, 8:30–10:30 am in Berea

**Champike Attanayake** Miami University, Middletown (attanac@muohio.edu)

#### Convergence of an Immersed Finite Element Method for Semilinear Parabolic Interface Problems

In this talk we analyze an immersed interface finite element method for second order semilinear parabolic interface problems. Convergence of the semi discrete solution to the exact solution is shown to be of the optimal order in  $L^2$  and energy norms. Further, fully discrete scheme based on backward Euler method is studied and optimal error in  $L^2$  norm is established.

# Venkata Chandra Dinavahi The University of Findlay (dinavahi@findlay.edu)

# Decomposition of a Complete Graph into Paths with no subsystems

Decomposing a complete graph into paths of any length was settled by Dr.Tarsi (1983). In this talk we will extend this into decomposing a complete graph into paths of any length with the additional property of containing no subsystems. A *G*-decomposition of a complete graph, T = (V, D) is said to contain no proper subsystem if there does not exist a G-decomposition, S = (V', D') of  $K_v$  where, V', V and D', D and |V| > |V'| > 1. In this talk we will explore the special case of when the length of path is odd. We will use some of Tarsis constructions along with so many new constructions.

# **Kevin Murphy** St. Norbert College (kevin.murphy@snc.edu)

#### Derivation of the Kinetic Component of the Gluon Four Momentum Operator

When studying gluons in point form quantum chromodynamics, the four momentum operator is found by integrating the stress energy tensor (written in terms of free gluon fields) over the forward hyperboloid. The four momentum can be broken into three parts: a kinetic term, a trilinear coupling term and a quartic term. This talk will verify that the kinetic term is equivalent to that of the free four momentum operator.

# **Nikolai Krivulin** St. Petersburg State University (nkk@math.spbu.ru)

#### Idempotent Algebra Solutions for Some Minimax Location Problems

Idempotent algebra, which deals with vector semimodules over idempotent semirings, finds expanding application as a promising modeling and solution tool in applied mathematics, computer science, and operations research. The progress in the area is mainly due to the fact that many complicated problems that are actually nonlinear in the ordinary sense become linear and so more tractable when translated into the language of the algebra. To illustrate, we examine both unconstrained and constrained multidimensional minimax single facility location problems with rectilinear and Chebyshev distances. We show how to represent the problems in terms of idempotent algebra, and then reduce them to evaluation of the eigenvalues and eigenvectors of an appropriate matrix.

#### **Narayan Thapa** Minot State University (narayan.thapa@minotstateu.edu)

#### Identification Problems for Nonlinear Hyperbolic PDEs, with Boundary Data.

In this paper, we consider the problem of determining parameters in second order nonlinear partial differential equation of hyperbolic type from boundary measurements. The existence, uniqueness, and continuous dependence of weak solution of second order nonlinear PDE of hyperbolic type are established. Model parameters are obtained by method of transposition. Computational algorithm and numerical results are presented.

#### Irinel Dragan University of Texas (dragan@uta.edu)

#### On the computation of Multiweighted Shapley Values.

An early dynamic algorithm for computing the Shapley Value is due to late M.Maschler (1982). In this paper we propose a similar algorithm for computing the Multiweighted Shapley Values of Cooperative Transferable Utilities games, that were introduced in an earlier work, Dragan (1994). The algorithm is illustrated by an example of a three person game.

## Wayne R. Lundberg Physicist (wayne29rl@gmail.com)

#### Quantum Chromo/electro-dynamic matrix algebra and its geometric dual

The simplified, reduced instruction set, nomenclature of algebra will be briefly introduced. A simple approach to unification of strong and weak algebraic representations is exposed through examination of  $3 \times 3 \times 3$  matrix algebra.  $3 \times 3 \times 3$  matrix algebra is a direct extension of familiar 3x3 matrix algebra. Following the approach taken by Dirac, a basic Rotation operator is defined. This operation, when employed on any  $3 \times 3$  section of  $3 \times 3 \times 3$  matrix, yields the well-known Rubik's cube algebra. This result can be taught to undergraduate math and physics students. Faces of the cube are identified according to the standard particle model's QCD charge-color symmetry diagram. Identifying cube corners with each colored quark and leptons defines a unified QC/ED representation algebra. Its geometric dual is presented. Complex operations are defined in a manner consistent with Feynman representation theory. The result is a simplification of string theory - which has standard particulate ground states. Strong (and weak) non-commutative matrix operations are exposed to yield an active engagement teaching technique. Demonstrations will be available for the audience, and arXiv references given. Research has further revealed that this approach yields a Causal particle theory, when the full representation geometry is quantified and analyzed via quantum gravity and M-brane mathematical tools. Analysis of geometric QCD objects leads to the no-boundary wave-function causal instanton-particles. NBWF is thus both well-founded and expanded to application in quantum mechanics. The QCD-restricted  $3 \times 3 \times 3$  matrix algebra has proven to be a key to unifying QM and General Relativity.

#### **Roummel Marcia** University of California, Merced (rmarcia@ucmerced.edu)

#### Sparse Signal Recovery Using Large-Scale Optimization

Traditionally, optical sensors have been designed to collect the most directly interpretable measurements possible. However, recent advances in image reconstruction and compressed sensing indicate that substantial performance gains may be possible in many context via computational methods. In this talk, we explore the potential of physically realizable systems for acquiring 'incoherent' measurements. Specifically, we describe how given a fixed size photodetector, compressive measurements with sophisticated optimization algorithms can significantly increase image quality and resolution.

# GCPS #4: Technology and Teaching

# Thursday, August 4, 1:00-4:45 pm in Berea

# Carol Vobach University of Houston Downtown (vobachc@uhd.edu)

# **3 Online Homework Systems for Statistics Service Courses**

Online homework systems have come into common usage in recent years for a variety of reasons: increased use of both online and hybrid courses; ease of access for students; various tutorial aids instantly accessible outside of the classroom; ease of grading and record-keeping for faculty; consistent problems for multiple sections of a course. Due to its popularity, many publishers are including online homework system access as part of their packages for university-level textbooks. This presentation gives a brief description of 3 systems (MyStatLab; Webassign; Connect) presenting common elements and contrasting differences among these systems. Pros and cons will be noted.

Megan Sawyer North Carolina State University (mesawyer@ncsu.edu)

Kathryn A. Brenneman North Carolina State University (kabrenne@ncsu.edu)

Brenda Burns-Williams North Carolina State University (bdburns@ncsu.edu)

# Enhancing the Classroom Experience with Tablet PC Technology

In this talk, we present ways to incorporate technology as a way of engaging students both inside and outside the classroom; techniques for both large ( $\tilde{2}50$  student) and small ( $\tilde{3}0$  student) classrooms will be discussed. Emphasis will be placed on the use of Tablet PCs in conjunction with Maple and GeoGebra within the lecture experience.

# Hieu Nguyen Rowan University (nguyen@rowan.edu)

# Experimental Mathematics and Data Mining: Extracting Identities from the Online Encyclopedia of Integer Sequences

This talk describes a data mining approach to discovering new mathematical identities by performing a computer-automated analysis of integer sequences derived from the Online Encyclopedia of Integer Sequences (OEIS). An implementation based on the computer algebra system Mathematica and MySQL database will be discussed, along with the challenges of detecting numerical patterns by experiment and developing fast and accurate sequence matching algorithms. Preliminary results will be presented.

Luiz Martins Cleveland State University (1.martins@csuohio.edu)

# A Flash Library for Ordinary Differential Equations

Flash&Math is a well-known ActionScript library for the development of Flash applets for the teaching and learning of mathematics ('mathlets'). In this talk, we describe an extension of Flash&Math for ordinary differential equations, which allows the dynamic display of solutions of ODEs. The structure and use of the library will be discussed, and sample applets using the library will be presented. Also, as part of the National Science Foundation DUE-0941388 grant, Flash applets for WeBWorK online homework system, we are developing a collection of Flash enabled online homework problems. We will present a sample of Flash-enabled problems on ordinary differential equations which will be made available in the National Problem Library.

Jerome Heaven Indiana Tech (jiheaven@indianatech.edu)

# Strategies for Using Computer Algebra Systems in Undergraduate Math Courses

I will present strategies for using Maple and MATLAB that have been successfully employed in Calculus I-III, Linear Algebra, Differential Equations and Numerical Analysis.

# Philip B Yasskin Texas A&M University (yasskin@math.tamu.edu)

# Student Programming Projects to Create Pedagogically Effective Calculus Applets

The speaker and Dr. Douglas Meade of the University of South Carolina have developed the Maplets for Calculus (M4C) collection of 129 Maple applets which help students to learn and teachers to teach precalculus and calculus. The applets are highly pedagogical. Most ask a question and provide step-by-step guidance through the solution, requiring correct answers to intermediate steps. Hints are available and when errors are detected, instructive feedback is provided. There is an endless supply of problems with significant variation. The maplets use symbolic, graphic (2D or 3D, some animated), numeric and verbal devices to investigate problems. Problems are algorithmically generated or entered by the student or instructor. The M4C can be used for self-study, in a computer lab, or as lecture demonstrations.

Between 2002 and 2009, the speaker taught 8 semesters of Honors Calculus at Texas A&M University in which students wrote preliminary versions of many of the M4C applets. In addition to learning to program in the Maple computer algebra system, students had to think through the algorithm for solving a generic version of each problem, had to decide what intermediate questions to ask and had to work backwards through the solution to be able to generate reasonable randomly generated problems which produce reasonable answers. The students' names are listed as assistant programmers for the respective applets in the M4C collection which is available at http://mymathapps.com/. The M4C are supported in part by NSF DUE CCLI Grants 0737209 (Meade) and 0737248 (Yasskin).

# **Gail Nord** Gonzaga University (nord@gonzaga.edu)

#### The Microsoft Word Free Mathematics Add-In and Microsoft Mathematics Beta 4.0

Abstract: This paper explores the use of free software with a powerful computer algebra system (CAS) for the K-12 and university science, technology, engineering and mathematics (STEM) classroom. Educational institutions are sorely in need of affordable technology. The financial reality facing many schools prohibits the allocation of limited resources for appropriate mathematical software. The free mathematics add-in offered to license holders of Microsoft Word 2007 and 2010 offers a prudent solution to this challenge. With particular strengths of graphing in two- and three-dimensions, Word is a viable choice. Microsoft Mathematics Beta 4.0 includes a full-featured graphing calculator that's designed to work just like a handheld calculator. This software is not limited to a license holder of Microsoft Word. Graphics and mathematics results can be inserted into a Word document. Additional math tools help users evaluate triangles, convert from one system of units to another, and solve systems of equations.

# Lisa Townsley University of Georgia (townsley@math.uga.edu)

#### Using Online Assessment Effectively in Large Courses

At University of Georgia, 2000 students take Precalculus each year and 90% of their course grade is earned through online work: homework, quizzes, and proctored tests. Our platform is WebAssign, with enriched content from the textbook publisher and home-coded questions as well. I present some of the interesting features of using online work to assess student understanding of function, and a personal view of what it is like to securely manage 50+ sections per year.

#### Kendra Schroeder Morehead State University (k.schroeder@moreheadstate.edu)

#### Using Technology to Manage and Teach Devlopmental Online Mathematics Courses

By using technology in our developmental mathematics program, we are easily able to deliver effective, meaningful, and secure online classes to our students at Morehead University and across the country. We will demonstrate tools and practices that best enable us to manage and teach our courses as well as the nuts and bolts of our assessment and delivery.

# Douglas Meade University of South Carolina (meade@math.sc.edu)

#### Using Translations as a Tool to Broaden Effectiveness of Maplets for Calculus Applets

The speaker and Dr. Philip Yasskin of the Texas A&M University have developed the Maplets for Calculus (M4C) collection of 129 Maple applets for students to learn and teachers to teach precalculus and calculus. The applets are highly pedagogical, and effective at improving both conceptual understanding and manipulative skills. Most ask a question and provide step-by-step guidance through the solution, requiring correct answers to intermediate steps. Hints are available and when errors are detected, instructive feedback is provided. There is an endless supply of algorithmically-generated problems that use symbolic, graphic (2D or 3D, some animated), numeric and verbal devices to investigate problems. The M4C can be used for self-study, in a computer lab, or as lecture demonstrations.

The mathematical feedback with M4C involves both technical details about the current problem (e.g., "Your equation must have exactly one dx and one dy. Give it another try.") and lighthearted encouragement (e.g., "Cool beans!" or "Radical dude!" or "Cowabunga!"). Both styles are easily understood by native English speakers. However, now that M4C is being used in more than 40 countries on 6 continents, the pedagogical effectiveness can be preserved only if each applet is translated into the user's native language.

At present, some or all of the M4C applets are available in ten additional languages. These translations must be done by a mathematically-knowledgeable speaker of the target language — not by an automatic translator.

This talk will focus on administrative and technical issues involved in translating educational mathematical resources into other languages.

#### **Jason Samuels** CUNY (jsamuels@bmcc.cuny.edu)

#### A New Way to Teach Calculus - with Visualization, Technology and Local Linearity

This talk will trace the development and justification for an innovative way to teach Calculus. The central idea is to reorganize instruction around the notion of local linearity, which can be expressed both intuitively and rigorously, and to delay formal limits until the end of the semester. The talk will present the redesigned first semester calculus curriculum with resequenced topics and computer labs, as well as the development and use of the local linearity mathlet and other technology tools.

## Aaron Wangberg Winona State University (awangberg@winona.edu)

#### **Constructing Geometric Concepts with Real, Tangible Surfaces**

Many beautiful ideas in differential geometry courses are based upon simple extensions of single variable calculus ideas. Too often, these beautiful geometric ideas are lost behind a myriad of parameterizations and coordinate-dependent formulas and the need to use technology in a multivariable calculus course. In this talk, I'll share how students explore and discover the fundamental concepts of Multivariable Calculus using surfaces cut on a CNC Machining Center. We'll show the surfaces and share three short activities which help students "get their hands on" the geometry of gradient vectors, the usefulness of directional derivatives, and the underlying geometry behind Lagrange multipliers - all in about 15 minutes of class time.

# Susan Schwartz Wildstrom Walt Whitman High School (susan@wildstrom.com)

### Homework Helpers in Multivariable Calculus

Using MAPLE, the presenter has adapted many worksheets to provide boiler plate code that will enable the user (students in her multivariable calculus classes) to enter functions from homework problems into the code so they can see the graphs of surfaces, calculate derivatives or integrals, or solve multistep problems. Not a substitute for doing the problems themselves by hand, these Homework Helpers are meant to allow students to check their work or use the computer algebra system capabilities to view all of the steps in a difficult solution.

# Denise LeGrand UALR (djlegrand@ualr.edu)

### Using M4C Maplets and Online Calculus at UALR

Technology has changed the way we teach Calculus. Students today expect immediate feedback, visualizations and step by step explanations when working through assignments. Many resources are available at little or no cost which enhance the Calculus experience and provide investigative methods and critical thinking avenues. An innovative tool that meets all of these criteria and more are the Maplets designed by Dr. Douglas Meade and Dr. Philip Yasskin. A Maplet is an "interactive graphical user interface" that leads a student through practice problems. The collection includes 129 Maplets for Calculus (M4C). In our online Calculus courses, some of the collection serve as required tutorials before a student begins the homework assignment. The student can work through a Maplet as many times as they choose. What is not required work can be assigned as optional practice or review. Most include hints and feedback with 'check' and 'show' buttons and also graphs and animations. Besides the benefit of practice, other benefits which we did not expect came from the use of Maplets. It is energizing to see the level of student participation rise with the introduction of this tool. We will present how M4C fit into our online Calculus courses, how they are graded, and student comments. Examples will be shown. We also use Maplets in our tutorial lab where students in traditional Calculus classes may use them for practice and exploration and instructors can use them for demonstrations. The M4C play a very valuable role in understanding and interpreting Calculus.

#### Barbara Margolius Cleveland State University (b.margolius@csuohio.edu)

# WeBWorK Online Homework Problems with Embedded Flash Coming Soon to the National Problem Library

As part of the National Science Foundation DUE-0941388 grant, Flash applets for WeBWorK online homework system, we are developing a collection of Flash enabled online homework problems. WeBWorK is an open source free online homework system for mathematics available through the MAA. The WeBWorK National Problem Library contains a collection of thousands of homework problems available for use in mathematics and other courses. In this brief talk, we demonstrate problems which will be available in the National Problem Library for use this fall which incorporate Flash. These problems allow for better visualization and for different types of student interaction with the content as compared to the standard online or textbook homework problems.

# **GCPS #5: Modeling and Applications**

# Friday, August 5, 8:15–11:30 am in Berea

Darcie DelzellWheaton College (darcie.delzell@wheaton.edu)Richard F. GunstSouthern Methodist University (rgunst@mail.smu.edu)William R. SchucanySouthern Methodist University (schucany@mail.smu.edu)Patrick S. CarmackUniversity of Central Arkansas (casmock@mac.com)Qihua LinUniv of TX Southwester Medical Center at Dallas (Catherine.Lin@UTSouthwestern.edu)Jeffrey SpenceUniv of TX Southwester Medical Center at Dallas (Robert.Haley@UTSouthwestern.edu)Univ of TX Southwester Medical Center at Dallas (Robert.Haley@UTSouthwestern.edu)

#### D-optimal Statistical Designs for fMRI Experiments with Application to Nonlinear Models

In order to properly formulate functional magnetic resonance imaging (fMRI) experiments involving interconnected mental activities, it is advantageous to permit great flexibility in the statistical components of the design of these studies. Issues that are fundamental to the selection of an appropriate design include considerations of both the statistical model to be used and the timing of stimuli. Major advances in understanding the implications of various statistical designs of fMRI experiments have taken place over the last decade. Nevertheless, new and increasingly difficult issues relating to the modeling of hemodynamic responses and the detection of activated brain regions continue to arise because of the increasing complexity of the experiments. In this article, a statistical D-optimality criterion is used for the selection of interstimulus intervals (ISIs) in event-related experimental designs where the hemodynamic response function (HRF) is modeled with a function that is nonlinear in the parameters. The model investigated in this paper is a two-gamma HRF, but the methods are applicable to any nonlinear model. A number of constant, cyclical, and probability-based ISI selection alternatives are investigated. The best designs are identified along with optimal design characteristics. In addition, D-optimality is shown to indicate better performing designs than the A-optimality criterion. Applications concentrate on a single-stimulus-type model, but the methods are readily extended to multiple stimulus types and to block designs.

# Atabong T. Agendia Madonna University Nigeria (agendia@yahoo.com)

# A Model for Psychiatric Patients Behaviors in Madonna University Teaching Hospital, Nigeria

Psychiatry deals with the prevention, the assessment of and diagnosis of mental illness, disorders and emotional or behavioral disorders, the rehabilitation of the mind and the treatment of these illnesses and disorders. The philosophy of those who practice psychiatry is to bring relief to those who suffer from the symptoms associated with mental disorders and to bring about some improvement in their mental wellbeing. Across Africa, most of these mentally ill individuals are been ridiculed due to their behavior which sometimes looks funny. We carved out a model to explain different behaviors of mentally ill individuals in Madonna teaching hospitals and simulated this model using different parameters. Our aim was to draw to the attention of the undergraduate students the significance of mathematical modeling in the understanding of a variety of concepts across disciplines. Very interestingly, the students saw in the simulation how the physical expressions of the illness comes from the interaction of the parameters.

# Michael MinerAmerican Public University System (jcmhs77@aol.com)Javad SeyedAmerican Public University System (JAVADSEYED@aol.com)

# A Quantitative Optimization Mathematical Model Analysis of Attributes Affecting Retention and Satisfaction Rates

Across the nation the appetite and demand for higher education has resulted in tremendous growth of enrollment in both traditional and non-traditional institutions of higher education. Online learning paradigm shifts and lack of accommodations and resources in traditional classrooms settings have resulted in a dynamic growth in the virtual classroom and online coursework delivery using interactive online classroom delivery systems. Traditional and non-traditional educational institutions are encountering many obstacles as they attempt to meet the growing demand for accredited and substantive online degree programs. The goal is to deliver a program of high quality, meets the highest standards of academic rigor, and is manageable for student, the instructor/facilitator, and the institution. In this study, the research team will address two key challenges in the delivery of online programs. The two challenges are the retention rates and the satisfaction rates of students currently enrolled in online programs. In order to address these challenges, a quantitative mathematical model for optimization will be conducted that provides a holistic analysis of the attributes affecting retention and satisfaction rates.

# Audey Shen Henry M. Gunn High School (audeyshen36@gmail.com)

# A Special Quadratic Function Unifies Two Fundamental Quantities in Physics

We present a modified classic quadratic function and obtain new findings in both mathematics and physics. First, we prove that the classic Vieta's formulas are special cases of our new formulas, and we find the conditions under which our results coincide with Vieta's formulas. Then, based on this modified quadratic function, we propose a novel "unified kinetic quantity" in modeling a unification of two fundamental quantities in physics: kinetic energy and linear momentum. We further show that there exists a linkage between this new unified quantity and Einstein's energy-mass equation  $E = mc^2$ . In this presentation, we also provide numerical examples and computer simulations to visualize and validate our analytical results.

# Alexander Louis Garron Sand Box Geometry LLC (alexander@sandboxgeometry.com)

# Plane Geometry Modeling of Gravity Field Space Time Curves.

Methods to use Euclidean Geometry objects to measure and study gravity field mechanics of both constant and changing acceleration curves. Mathematica graphing capability will be used to construct a parametric geometry standard model of a gravity field orbital. Two familiar fields will be structured; our earth/moon system and the first plane geometry "n" body presentation of our planet group about our sun.

# Leon Kaganovskiy Touro College (leonkag@gmail.com)

# Applications of MATLAB/CAS to Ecological Modeling.

We consider how MATLAB and CAS can be employed to elucidate classical Lottka-Volterra and Competition models as well as more complex Hollings extensions. The goal is to create programs which allow students to investigate any two dimensional non-linear dynamical system analytically as well as numerically. The computational methods of detection of limit cycles and their ecological importance is discussed.

Yun KangArizona State University (yun.kang@asu.edu)Rebecca ClarkArizona State University (rebecca.m.clark@asu.edu)Michael MakiyamaArizona State University (Michael.Makiyama@asu.edu)Jennifer FewellArizona State University (j.fewell@asu.edu)

# Mathematical Modeling of Obligate Mutualism Interactions: Leaf Cutter Ants and Fungus

We propose a simple mathematical model by applying Michaelis-Menton equations of enzyme kinetics to study the mutualistic interaction between the leaf cutter ant and its fungus garden at the early stage of colony expansion. We derive the sufficient conditions on the extinction and coexistence of these two species. In addition, we give a region of initial condition that leads to the extinction of two species when the model has an interior attractor. Our global analysis indicates that the division of labor by workers ants and initial conditions are two important factors that determine whether leaf cutter ants colonies and their fungus garden survive and grow can exist or not. We validate the model by doing the comparing between model simulations and data on fungal and ant colony growth rates under laboratory conditions. We perform sensitive analysis and parameter estimation of the model based on the experimental data to gain more biological insights on the ecological interactions between leaf cutter ants and their fungus garden. Finally, we give conclusions and discuss potential future work.

# Michael C. Sostarecz Monmouth College (msostarecz@monm.edu)

#### Mathematical Modeling with High-Speed Imagery

In this talk, we share how high-speed imagery can be used to illustrate mathematical concepts in the classroom and serve as the inspiration for student projects in mathematical modeling. Through comparisons with analytic solutions or numerical simulations of differential equations, we are able to see when a model is an accurate description of the experiment or if revisions to the model need to take place. Examples will be drawn from experiments in projectile motion, harmonic oscillators, and fluid dynamics.

#### **Mike Johnson** Meredith College (johnsomi@meredith.edu)

#### Queuing Model of Congested System Having Unknown Number of Servers, with Application to Highway Traffic

In a typical application of queuing theory, basic system characteristics such as server capacities are known, and one computes the resulting wait time or total system time as a function of offered load and server capacities. In some congested systems such as wireless networks, the server capacity decreases in response to increased offered load, so that not even an average capacity can be predicted. Queuing models for such systems have been previously described (Mathfest 2010) in which the unknown server capacity is solved as a nonlinear regression fit to observed total system times. In this presentation I summarize a more generalized queuing model for systems in which even the number of servers is unknown. For these models, obtaining an equivalent number of servers for the congested system as a fit to observed system times is inherently more difficult than solving for, say, equivalent server capacity in a single-server congested system. The model is applied to highway traffic in a short section of roadway, and compared with simulation results.

# **Darren Narayan** Rochester Institute of Technology (dansma@rit.edu) **Joy Lind** University of Sioux Falls (joy.lind@usiouxfalls.edu)

# Real World Applications of Mathematics

Traditional curricula seldom offer students concrete examples of cutting-edge, real world applications of higher mathematics. As a result students finish their undergraduate mathematics career asking themselves the question, "What else can I do with a mathematics degree besides teach?" To address this need, we launched the STEM Real World Applications Modules Project funded by an NSF-CCLI grant. Topics included in this talk will include applications of graph theory to telecommunication network design (National LambdaRail) and to reconstruction of three-dimensional images (Microsoft Research).

The goal of the STEM Real World Applications Modules Project is to better prepare faculty to answer the question, "What else can mathematics graduates do besides teach?" A student completing these modules will already know. Material presented in this talk was supported by the following grants: NSF #0536364, NSF #1019532, and NSF #1110939.

#### **Terry Jo Leiterman** St. Norbert College (terryjo.leiterman@snc.edu)

Audra Weaver Mathematical Association of America (aweaver@maa.org)

#### Settling Velocity and Sedimentation in Low Reynolds Number Fluid Systems

This talk seeks to formulate settling velocities for sparse particle populations of both spherical and slender shape with a focus on the interaction between the forces of drag, buoyancy, and gravity in a strong experimental capacity. The formulated models for settling velocity assume and exploit the low Reynolds nature of a system theoretically while concentrating on the process of sedimentation, in which particles separate from fluid suspensions. This talk is strongly guided by use of laboratory equipment, which includes a sedimentation tank and a collection of vertical cylinders that allow the characterization of particulate separation by sedimentation. The formulas generated for the settling velocities are fundamental components of quantifying and predicting the population dynamics of phytoplankton in freshwater, inland ecosystems. Phytoplankton are microscopic plants of diverse shape and, through photosynthesis, become responsible for much of the Earth's oxygen. The organic compounds they produce form the basis of many ocean and freshwater food webs. Moreover, through respiration, phytoplankton play an important role in the Earth's carbon cycle.

#### Paul von Dohlen William Paterson University (vondohlenp@wpunj.edu)

#### Tweaking the NFL's Quarterback Passer Rating for Better Results

In this talk, we will analyze the components of the current NFL quarterback passer rating, consider possible deficiencies in the rating and propose a couple of modifications. In doing so we will also address common criticisms of the rating system, based both on perception and calculation. While not the first or only analysis/modification of the rating, this is a simple and straightforward modification of the rating aimed at addressing what the author believes to be the two main deficiencies of its calculation. The primary features of the current quarterback passer rating system will be preserved, while the modifications produce desirable results based on calculation and historical perspectives.

Shinemin Lin Savannah State University (lins@savannahstate.edu)

#### **Rail Gun Bore Scan Data Analysis**

Mr. Christian James pointed out the difficulty to collect unbiased data and predicted that bore wear is a function of current, velocity, and voltage in his SSG Bore Scan Analysis research. This research project uses the statistical method to analyze scanned data, so that we can

estimate the locations of bore wear or deposit after each shot. The data are collected by Bore scanner whose scan head can do 4400 angular scans per step and is capable of  $0.005^{\circ}$  movement. Initially the scanner is at the center of the bore and scan clockwise starting at 9 o'clock direction.

Scanned data are processed in a customized MATLAB toolbox. The rotation effects of the scanner are also processed out. The output data file after converted by MATLAB toolbox is a text file. Each file is a 6400 by 4800 matrix. For convenience I will use EXCEL 2007 to do all data analysis.

# GCPS #6A: Assessment, Mentoring, and Outreach

# Friday, August 5, 1:00–3:30 pm in Berea

Brian Heinold Mount St. Mary's University (heinold@msmary.edu)

#### A 200-Question, Campus-Wide Math Contest

A few years ago at Mount St. Mary's, we held a campus-wide, 200-question math contest to celebrate the university's bicentennial. Each week throughout the school-year we posted seven new problems, ranging from the easy, "What math word can be split into two words that might describe a man after a long day at the beach?" to the more mathematical, "How many integers between 1 and 10,000 contain the number 3?" Problems were designed to be fun and solvable, at least in principle, by non-math majors. People from all parts of the university participated - students, faculty, staff, administrators, and seminarians. This talk will give an overview of the contest and suggestions on how to run something similar at your school.

#### Jenna Carpenter Louisiana Tech University (jenna@latech.edu)

#### A Model for Creating a Professional Development Program to Mentor Women Graduate Students

Louisiana Tech University is in the second year of an NSF ADVANCE grant to promote the success of women faculty in STEM fields, focusing on climate issues, retention and professional development/leadership. Women graduate students in STEM fields experience many of the same issues and challenges as tenure-track faculty. Utilizing a twice monthly lunchtime seminar, we have been able to re-package and deliver professional development modules developed for the ADVANCE project, as well as modules which target specific needs of graduate students. In addition, by partnering with ADVANCE initiatives which bring distinguished lecturers, workshops and training seminars to campus, we have created a robust professional development program for women graduate students. Other institutions can easily adapt this model, capitalizing on professional development opportunities unique to their campuses to mentor women graduate students from STEM fields and give them valuable training to help launch a successful career as a faculty member.

# Elizabeth Mathai Norwich University (emathai@norwich.edu)

#### An In-House Placement Test: Challenges and Solutions

Norwich University uses a math placement test designed and created by the mathematics department. The test has undergone many changes over the past few years. We present a brief history of challenges, solutions and plans for improvement.

#### Tara L. Smith University of Cincinnati (tararuns42@gmail.com)

#### AP Calculus: Concepts, Computation, and Communication

As more and more students enter college having had a first experience with calculus through an AP Calculus course in high school, it becomes increasingly important for college faculty to be familiar with the content of the AP Calculus curriculum and the way in which it is assessed on the AP Calculus exams. This presentation will focus on these items, emphasizing the major concepts and computational skills covered in the curriculum, and the expectations for communicating mathematical thinking, understanding of these concepts, and correct computational skills on the AP Calculus exam. The intent is to increase understanding and knowledge of the AP Calculus program among college faculty.

#### Stephen Kokoska Bloomsburg University (skokoska@bloomu.edu)

#### AP Calculus: Facts, Figures, and FAQs

The Advanced Placement Calculus Program has grown dramatically over the last decade. More students than ever are taking AP Courses and the exams, both nationally and internationally. Success in AP Calculus still serves as a measure of high academic achievement for students, teachers, and schools. The purpose of this presentation is to emphasize the depth and rigor of the AP Calculus curriculum, and the collaboration with college faculty in the grading process. Example questions, scoring standards, and the methodology for determining grades will be used to demonstrate the skills necessary to succeed in AP Calculus. The intent is to increase understanding and knowledge of the AP Calculus program and strengthen connections with high school and college faculty.

#### Senan Hayes Western Connecticut State University (hayess@wcsu.edu)

#### Helping Teachers and Parents Increase Persistence of Minority Students, and all Students, in Mathematics.

In this talk teaching resources and methods will be given and discussed to aid teachers and parents in their goal to improve the persistence of minority students, and all students, in mathematics related fields. In addition, a review of successful programs used in retaining minority students in the fields of science, technology, engineering and mathematics (STEM) will be given.

#### Marc Harper UCLA Bioinformatics (marcharper@ucla.edu)

Alison Ahlgren University of Illinois (aahlgren@illinois.edu)

#### Identifying Crucial Concepts and Skills for Success in College Algebra through Calculus

Data from three years of placement covering thousands of students at the University of Illinois on 182 concepts and skills has yielded unprecedented specificity in determining which items are indicative of student preparedness and success in College Algebra, Precalculus, and three forms of Calculus. Items are naturally partitioned by statistical analysis into a spectrum from basic to advanced and are traceable throughout the sequence of courses, clearly identifying particularly strong and weak students. These results are of use to educators, instructors, and placement program developers. We present examples and vivid visualizations of the results from this study.

# Gulden KarakokUniversity of Northern Colorado (gulden.karakok@unco.edu)Brian ChristopherUniversity of Northern Colorado (brian.christopher@unco.edu)

#### In-depth Look into National Research Experience for Undergraduates Programs (NREUP)

In this presentation, we will share some of the results of an on-going evaluation of the National Research Experience for Undergraduates Program (NREUP). NREUP is sponsored by the Mathematical Association of America (MAA) and its Strengthening Unrepresented Minority Mathematics Achievement (SUMMA) program, and funded by NSF and NSA. NREUP's main goals are to provide intensive summer research experience for underrepresented groups and increase their interest in obtaining advanced degrees and career in mathematics or a closely related field. The evaluation of NREUP is designed to evaluate the impact of this experience on participating students and is a longitudinal study in nature. The NREUP is initiated in 2003 and served more than 300 students 200 of whom volunteered to participate in the evaluation process. Students who participated in evaluation completed online pre- and post-program surveys, follow-up survey and interview. As we share results from our data collection from student participants, focusing on what students found more effective, we will also discuss our plans to explore the experience of the program directors and mentors.

#### Robin Lydiann Anderson Southwestern Illinois College (robin.anderson@swic.edu)

#### Math Chats: Engaging Students in Mathematics Outside the Classroom

As educators, many of us face the challenge of generating student interest in mathematics. To address this issue, Southwestern Illinois College created "Math Chats," a series of informal talks about various math topics and careers that use mathematics. Southwestern, a twoyear community college, currently has six chats a year with attendance averages of 130 students per talk. The students' ability levels range from developmental algebra to differential equations. Come hear their secrets!

#### **Elizabeth Yanik** Emporia State University (eyanik@emporia.edu)

#### Women Count, Everyone Counts

This talk will present a brief overview of the biennial Women Count conferences. The purpose of this program and its basic format will be described. Also the challenges to the continuation of this and similar programs will be discussed.

# GCPS #6B: Pure Mathematics 1

#### Friday, August 5, 3:30–5:00 pm in Berea

# Charlie Smith Park University (charlie.smith@park.edu)

#### **Three Irrational Rabbits and Meditating Transcendentals**

The content of this talk is based on an undergraduate independent study course, *Irrational and Transcendental Numbers*, which I taught to one student during Spring Semester 2011. We used two classic books by Ivan Niven, *Numbers: Rational and Irrational*, and *Irrational Numbers*. Beginning with simple irrationality proofs involving radical, trigonometric and logarithmic values, we progressed to advance irrationality

proofs which use the basic technique of Hermite, as refined by Hurwitz, Niven, and others.

The presentation will discuss major results from both books, along with outlines of selected proofs, with emphasis on Hermite's technique. These results will be highlighted: e is irrational; if r is a nonzero rational number, then  $\cos r$  is irrational;  $\pi$  is irrational; if r is a nonzero rational number, then  $e^r$  is irrational; e is transcendental;  $\pi$  is transcendental.

My experience with this topic has inspired me to research deeper into its history.

# Donald Leigh Hitzl Retired from the Lockheed Palo Alto Research Laboratory and Stanford University (domarkd@comcast.net)

#### Investigations of the Riemann Hypothesis

This is a work in Applied Mathematics, on a very famous problem in Number Theory. The paper includes both analytical developments, starting from page 17 of Edwards well-known book, together with computer results using Mathematica. Concentration is placed on the "Companion Function Xi(s)" to the Riemann Zeta Function  $\zeta(s)$ , where the complex variable  $s = \sigma + it$ . It is found that Xi(s) is a spiral, continually approaching the origin as t goes to infinity. Moreover, the function Xi(s) has a mirror symmetry for values of  $\sigma$  symmetrically located on each side of  $\sigma = 1/2$ , the critical value for the Riemann Hypothesis. We have confirmed, both numerically and analytically, that there is a hidden scaling of the Zeta Function on each side of  $\sigma = 1/2$ .

# Tilak de Alwis Southeastern Louisiana University (talwis@selu.edu)

# Riemann Zeta Function and Integral Representations of Apery's Constant

The Riemann Zeta function is well-known in connection with yet unsolved Riemann Hypothesis, one of the seven Millenium Problems of mathematics. The zeros of this function are also intimately related to the distribution of prime numbers. This paper discusses a special value of this function, namely the value of the Riemann Zeta function at z = 3. By definition, it is equal to the convergent infinite sum of the cubes of the reciprocals of the natural numbers. This mathematical constant is also known by the name Apery's Constant, and has an approximate value of 1.2026. The paper deals with several integral representations of this constant. Some of these integrals involve exponential, logarithm and trigonometric functions, while others include hyperbolic sine, cosine and tangent functions. All our derivations can be verified by a suitable computer algebra system such as Mathematica.

#### Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

#### Variations on Euclid [n], the Product of the First n Primes Plus One.

In Euclid's classical proof demonstrating the infinitude of the positive primes over two thousand years ago, one considers the product of the first n primes plus one (known as Euclid [n] in the CAS MATHEMATICA) and achieves a contradiction to the assumption that there are only finitely many prime numbers. This is the case whether or not Euclid [n] generates a prime output. In this paper, we consider variations on this theme. For example, we investigate the product of the first n factorials, Fibonacci, Lucas, triangular, square, pentagonal, and hexagonal numbers plus one. In addition, we investigate the product of the first n such numbers minus one to see if twin prime pairs are obtained. We finally consider the sum of the first n primes to discover prime outputs.

### Chad Awtrey Elon University (cawtrey@elon.edu)

#### Galois Group Computations Via Resolvents and Subfields

An important aspect of number theory is the study of arithmetic invariants of finite extensions of the rational and p-adic numbers. The arithmetic of these fields is reflected in the Galois groups of their field extensions of finite degree, these groups acting as symmetries of roots of polynomials defining the extensions. Consequently, there is great interest in developing computational methods for determining Galois groups of irreducible polynomials over arbitrary fields. Focusing on the rational and p-adic fields, we discuss the general method for computing Galois groups (resolvents) as well as newer methods (based on subfields) which are especially useful in the p-adic case.

# Jack Mealy Austin College (jmealy@austincollege.edu)

#### Asymptotic Regions in Snell Geometries

Further results in the category, Snell Geometry. (See various Snell Geometry abstracts from MathFests 2008 - 2010, and Joint Meeting 2011.) After a short overview of the general category and methodology of Snell Geometry (and how it contrasts with other categories in Geometry), for this talk we will focus on the construction of new asymptotic regions in various Snell systems. This continues our program to find specific geometries, and objects (typically limiting polygons and disks) within these geometries which feature asymptotic behavior that markedly contrasts with the better-known asymptotic objects in Hyperbolic geometry, Euclidean geometry, and Spherical geometry. We'll conclude with an overview of the success of the program so far, and of some of the open problems which remain.

# GCPS #7: Pure Mathematics 2

# Saturday, August 6, 8:30–10:30 am in Elkhorn B

# Melissa Dennison Baldwin-Wallace College (mdenniso@bw.edu)

#### A Sequence Related to the Stern Sequence

We define a two-parameter family of recursive sequences which we call the bow sequences. The general bow sequence is defined similarly to the Stern sequence, and many of the properties of the bow sequences are related to known properties of the Stern sequence. In particular, we will discuss the generating function for the general bow sequence, and give interpretations of the generating function for two basic cases. We will also discuss interesting properties of the bow sequences modulo 2.

#### **Daniel Joseph Galiffa** Penn State Erie, The Behrend College (djg34@psu.edu)

# Another Way to Obtain the Sheffer Type-0 Orthogonal Polynomial Sequences

The Type-0 orthogonal polynomial sequences were originally achieved by I.M. Sheffer in 1939, which include the Hermite and Laguerre polynomials. Sheffer developed a sophisticated method for obtaining these polynomials, which included various functional relationships amongst a wealth of interesting theorems. In this talk, we show how these same polynomials can be obtained via a strikingly simplistic and elegant approach that uses only the generating function that defines the Type-0 sequences and a three-term recurrence relation.

#### **Peter Joyce** CCBC at Catonsville (pjoyce@ccbcmd.edu)

#### Correct and Simple Proof that 'Product of Even Transpositions is not Product of Odd Transpositions'.

Author N. Ghosh and P. Joyce Theorem. The product of an even number of transpositions cannot be written as the product of an odd number of transpositions. Herstein [1, p. 67] points out that in proving the above theorem, it is standard to introduce the polynomial but this polynomial "seems extraneous to the matter in hand." Liebeck [2, p. 668] makes an incorrect supposition in his proof. He claims that if the identity permutation e is written in the form: e = (1, c1)(1, c2)...(1, cn) where the second entry in each transposition is not equal to 1 then there must be an even number of transpositions in the above expression of the form for all symbols cs where 0 < cs < n + 1. The permutation is mapping the first n positive integers onto itself. Clearly, e = (12)(13)(12)(13)(12)(13) is an easy counterexample. We present a simple proof based on standard properties of permutations. Reference 1. I. N. Herstein, Topics in Algebra, Blaisdell, New York, 1964. 2. Hans Liebeck, American Mathematical Monthly, v. 76, no 6.

Nick ScovilleUrsinus College (nscoville@ursinus.edu)Seth AaronsonUrsinus College (seaaronson@ursinus.edu)Laura StibichSt Francis University (Imsst8@mail.francis.edu)Mitch SmithUrsinus College (mismith@ursinus.edu)Marie MeyerCollege of St. Benedict - St. Joseph (MEMEYER@CSBSJU.EDU)

# Equivalence of Discrete Morse functions on graphs

Discrete Morse theory combines ideas in graph theory, algebra, and topology. A discrete Morse function on G assigns real numbers to each edge and vertex of G which gives us a sequence  $G(a_0), G(a_1), \ldots, G(a_n) = G$  where  $G(a_i)$  is a subgraph of  $G(a_{i+1})$  for all i and each  $a_i$  is called a critical element of G. These critical elements "detect" a change in topology. This is seen by recording the Betti numbers of the homology groups in dimensions 0 and 1. In the case of a graph, the Betti numbers of the homology groups of  $G(a_i)$  in dimensions 0 and 1 simply refer to the number of components and number of holes of  $G(a_i)$ , respectively. If we record the number of components and holes at each stage in the construction of G, we can associate to each discrete Morse function on G two sequences of numbers. In this talk, we will study different notions of equivalences of these sequences. This work is based on the work of four undergraduate students during an REU at Ursinus College.

# Papiya Bhattacharjee Penn State Erie (pxb39@psu.edu)

#### Frame Extensions

A frame is a complete lattice which satisfies the frame law, that is, finite infima distributes over arbitrary suprema. An M-frame is a frame which is algebraic and satisfies the finite intersection property. Given two M-frames L and M, we can describe several different extensions between them, such as, rigid extension, r-extension,  $and r^{\flat}$ -extension. These extensions between M-frames,  $L \leq M$ , gives results related to the spaces of minimal prime elements, Min(L) and Min(M), respectively.

In this talk the speaker will give basic definitions from the theory of frames and discuss various frame extensions between M-frames. Moreover, if time permits, the speaker will also discuss the application of these extensions to ring theory and lattice-ordered group theory.

# Robert Peter Schneider University of Kentucky (petsoundsstudio@msn.com)

# On a Golden Pair of Identities in the Theory of Numbers

We prove the pair of identities  $\phi = -\sum_{k=1}^{\infty} \frac{\varphi(k)}{k} \log\left(1 - \frac{1}{\phi^k}\right)$ ,  $\frac{1}{\phi} = -\sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log\left(1 - \frac{1}{\phi^k}\right)$  relating the golden ratio  $\phi$ , the Euler totient function  $\varphi$ , the Moebius function  $\mu$  and the natural logarithm, all central players in the theory of numbers. These identities display a connection between the golden ratio and the factorization of integers that is not obvious, as well as an inverse relationship between  $\varphi$  and  $\mu$  with respect to  $\phi$ .

# Jonathan Weisbrod Rowan University (weisbrodj1@verizon.net)

#### On Numbers that Can Be Expressed as the Sum of Two Positive Squares in Exactly n Distinct Ways

The primary objectives of this project are to quickly and easily determine the number of distinct ways a particular positive integer can be expressed as the sum of two positive squares and to generate sequences of positive integers that can be expressed as the sum of two positive squares in exactly n distinct ways. In order to do this, we only need elementary factoring and algebraic techniques. Pierre de Fermat and Leonard Euler are two of the famous mathematicians who have worked on this problem. Their theorems are used in this paper to prove the objectives above.

# Adam CoffmanIndiana University - Purdue University Fort Wayne (CoffmanA@ipfw.edu)Marc FrantzIndiana University Bloomington (mfrantz@indiana.edu)

#### Möbius Transformations and Ellipses

We consider the action of the group of Möbius transformations on ellipses in the plane. In particular, we find which Möbius transformations are symmetries of an ellipse, and which take one ellipse to another. We also survey some of the "special plane curves" which appear as inversive images of conic curves.

# GCPS #8: Teaching Introductory Mathematics

# Saturday, August 6, 8:30–10:00 am in Berea

Chris Schroeder Morehead State University (c.schroeder@gmail.com)

# College Algebra: Redesigned with Technology

Recently Morehead State University's College Algebra class has undergone a drastic redesign. Through a grant from the National Center for Academic Transformation (NCAT) we have modified our delivery of content, lecture time, student requirements and amount of student interaction. We will look at the details of how the course was redesigned, technology used, implementation issues and solutions, possible additional changes for the future, and early results showing the effectiveness of the changes.

#### **Susan Foege** Kentucky State University (susan.foege@kysu.edu)

#### Engaging Students Using Instant Feedback Assessment... is it Better than Winning the Lottery?

Engaging students in general education mathematics courses can be challenging. I have found that lecturing is not the approach that works for me. Several years ago, I was introduced to an interesting multiple choice answer sheet, called the IFAT. In this brief talk I will introduce my audience to the IFAT, how I use it in class, and why using the IFAT it is better than winning the lottery!

## **Jeffery D. Sykes** Ouachita Baptist University (sykesj@obu.edu)

#### Introductory Problem-Solving Skills: An Alternative to Intermediate Algebra for Liberal Arts Majors

Many college students now take general studies mathematics courses that focus on appreciation of mathematics and quantitative reasoning skills. Many of these courses maintain the level of rigor of a college algebra course, but choose to cover historical, applied, and discrete topics that do not require mastery of algebraic skills. The traditional developmental mathematics curriculum, on the other hand, is designed to echo the algebra-focused high school curriculum that prepares a student for college algebra and eventually calculus. Is this the best approach to developmental coursework for students bound for these newer breed of general studies courses?

The author has been involved in the design of a new developmental mathematics course at the level of intermediate algebra, but focusing on problem-solving techniques instead of the mastery of algebraic skills. This course is intended to prepare students for success in a quantitative reasoning course that does not require algebraic mastery. In this follow-up to a presentation at the 2010 Joint Meetings, the author will discuss further development of this course - including the current form of the course content, sample activities, and reference materials - and will provide information about the performance of students in this and subsequent courses.

Gabriela SchwabEl Paso Community College (gschwab@epcc.edu)Emil Daniel SchwabThe University of Texas - El Paso (eschwab@utep.edu)Helmut KnaustThe University of Texas - El Paso (hknaust@utep.edu)

#### Mandatory Supplemental Instruction in Precalculus Courses

We report on our cooperative project "Cross-institutional Implementation of Supplemental Instruction", funded by the Department of Education MSEIP program. Graduate students from the Department of Mathematical Sciences at the University of Texas at El Paso serve as supplemental instruction leaders for their undergraduate peers at El Paso Community College. The Supplemental Instruction workshops, held one hour per week in a small class setting, give students the opportunity to obtain a deeper understanding of the concepts introduced in the lecture and to gain additional practice in solving problems. We will describe our project and also report on the main project assessment data.

**Jose Giraldo** Texas A&M University Corpus Christi (jose.giraldo@tamucc.edu)

#### Math Courses in the Science First Year Learning Community at TAMUCC

Texas A&M University Corpus Christi (TAMUCC) through a STEP/NSF grant added math courses to the regular offering of the Science Learning Community along with the implementation of a special mentoring program. Students who enrolled in the STEP math courses (small sections) are also expected to enroll in the large sections of biology and chemistry, as well as the First Year Seminar and English. The data collected from fall 2007–spring 2011 show that although the course load of the STEP students is heavier than the non-STEP students, their performance in the math and science courses is better than non-STEP students, and the retention into the second year is higher than the TAMUCC general retention. Data on performance and attendance to mentoring will be shared.

# Deborah J Gougeon University of Scranton (gougeond1@scranton.edu)

# Introducing Joint and Conditional Probability in an Introductory Business Statistics Course

The importance of probability in an Introductory Business Statistics course cannot be overstated, however it is a topic which is often cited by students as the most confusing and difficult to understand. This presentation offers a classroom-tested outline that provides the student with a more structured way of looking at probability, specifically focusing on the concept of independence as it relates to marginal, joint, and conditional probabilities. This approach has invariably resulted in improved quiz and test grades on this somewhat complex topic.

# GCPS #9A: Pure Mathematics 3

# Saturday, August 6, 1:00-3:00 pm in Elkhorn A

John Polhill Bloomsburg University of Pennsylvania (jpolhill@bloomu.edu)

# A Brief Look at Graphs, Codes and Schemes Obtained from Difference Sets

Difference sets are a particular type of algebraic set with numerous relationships to combinatorial structures. While they were introduced about 75 years ago, there has been much activity in this area of mathematics in recent years. We will introduce both difference sets and some of their applications in combinatorics. Some recent developments will also be given.

Rus May Morehead State University (r.may@moreheadstate.edu)

# **Counting knot mosaics**

Knot mosaics are formed by arranging tiles, pictured with strands of rope, into a grid so that the strands jointly form a knot. These types of mosaics were first introduced as a basis for quantum knot theory, but turn out to be related to more traditional combinatorial objects, such as alternating sign matrices. We take a look at methods of counting knot mosaics, involving eigenvalues and determinants of large matrices, and consider the likelihood of an achieving an exact solution to this counting problem.

# Joshua Roberts Piedmont College (jroberts@piedmont.edu)

# Homological Generators and a Vanishing Conjecture

Given a finitely presented group G, Hopf's formula expresses the second integral homology of G in terms of generators and relators. We give an algorithm that exploits Hopf's formula to estimate  $H_2(G; k)$ , with coefficients in a finite field k. Moreover, we use these algorithms to give generating sets for this homology group. These calculations are motivated by a family of conjectures of Quillen regarding the homology of certain linear groups; using the algorithms we give new proofs of the conjectures at the primes 3 and 5, and new information at the prime 7.

# Matt Hudelson Washington State University (mhudelson@wsu.edu)

# Inverting a Class of Vast Matrices Using Their Anti-Transposes

Pascal's triangle is a canonical example of a Riordan array, loosely defined as an "infinite-by-infinite matrix" that catalogs coefficients of series expansions for powers of the same function. Here, we extend the usual definition of a Riordan array by allowing for negative powers of the variable and the function under consideration. This enables us to define the "flip" or "anti-transpose" of an extended Riordan array. We will demonstrate that this operation yields an array that is closely related to the right-sided inverse of the original array.

# Andrew deLong Martin Kentucky State University (andrew.martin@kysu.edu)

# On Infinite Subseries of the Harmonic Series.

From the Basel Series to Brun's Series of reciprocal twin primes, the Harmonic Series is parent to a cornucopia of fascinating series. In this talk will be presented several elementary (but new to the author) results to kindle or rekindle your love of this divergent sum par excellence.

**Jayantha Lanel Gan Hewage** University of Sri Jayewardenepura (jglanel@oakland.edu, lanel\_j@yahoo.co.uk)

# The Number Of Complex Roots Of A Univariate Polynomial Inside A Rectangle

Let f(z) be a nonzero complex univariate polynomial and let  $\mathbb{R}$  be a rectangle in the complex plane. The number of complex roots of f(z) inside  $\mathbb{R}$  is given by the winding number of f(z) on  $\mathbb{R}$  if f(z) has no roots on the boundary of  $\mathbb{R}$ . In this paper the result is extended to all rectangles  $\mathbb{R}$  even when f(z) has roots on the boundary of  $\mathbb{R}$  under the condition that f(z) is square-free. It can also be used to formulate an algorithm that isolates the complex roots of any polynomial.

**Duokui Yan** Chern Institute of Mathematics (duokuiyan@gmail.com)

# Existence of the Schubart Periodic Orbit with Arbitrary Masses

This paper gives an analytic existence proof of the Schubart periodic orbit with arbitrary masses, a periodic orbit with singularities in the collinear three-body problem. A "turning point" technique is introduced to estimate the shape of the orbit. This technique is new and fundamental to the existence proof.

## Benselamonyuy Ntatin Austin Peay State University (ntatinb@apsu.edu)

#### Some Geometric Aspects of the Finite Element Method for Elliptic PDEs.

It is well known that many physically significant problems under steady state conditions can be modeled in the form of partial differential equations. Typical examples of how such equations arise are found in models involving the heat, the wave or the Schrödinger equation. The finite element method has played an instrumental role in the solution of such equations. In this talk, we present the geometry involved in the implementation of the finite element method, and how the model equation is reduced to an equation of the form Ax = b where A is a positive definite symmetric, and possibly nonlinear operator. Illustrative examples in 1-dimension that model displacement of an elastic cord subjected to a force or heat flow across a conducting bar are also presented.

# GCPS #9B: Interdisciplinary Topics

# Saturday, August 6, 3:00–3:45 pm in Elkhorn A

# **Duk-Hyung Lee** Asbury University (duk.lee@asbury.edu)

#### Creating a Real Music with Mathematical Ideas

Since Pythagoras, Mathematics and Music have been closely related. Recently, as Mandelbrot's fractal geometry and its self-similarity concepts evolved and invaded into science, nature, and arts, musicians are also becoming interested in analyzing musical compositions in a new way, and they commend new languages like a self-similarity and fractals in music. Beautiful and fascinating arts are created by fractal ideas, and people ponder if beauty they find in art works has to do with the so-called 'self-similarity'. Mathematical procedures pertained in fractal geometry have been implemented in making music by many. Some of 'fractal music' sounds interesting, but mostly strange, especially to the Westerner's ear. This talk is a glimpse of an interdisciplinary project of creating an esthetically pleasing music, combining some components of mathematics and of music theory.

#### Gabriel Prajitura SUNY Brockport (gprajitu@brockport.edu)

#### Things to Do with a Broken Stick

We will discuss several geometric probability problems related to the classical broken stick problem: if a stick is broken at random in three pieces, what is the probability that the three pieces can be the sides of a triangle? This is joint work with Eugen Ionascu from Columbus State University, GA.

# Ryan Rahrig Ohio Northern University (r-rahrig@onu.edu)

# A Graph Theory Approach to an RNA Problem

In the study of RNA, an important task is finding the structural similarities and differences of two molecules. As the number of 3D structures available has been increasing dramatically in the past decade, it has become important and necessary for automated methods of RNA structural comparison to be developed. This talk will explore how elementary graph theory concepts which are accessible to undergraduate students have been applied to form the basis of a new method designed to compare RNA 3D structures.

# GCPS #10: Teaching Advanced Mathematics

# Saturday, August 6, 1:00–3:30 pm in Berea

# William R Green Eastern Illinois University (wrgreen2@eiu.edu)

#### Abel's theorem simplifies reduction of order

In a standard differential equations courses one studies the *n*th order homogeneous linear equation  $y^{(n)} + p_1 y^{(n-1)} + \cdots + p_{n-1} y' + p_n y = 0$ . Due to Newton's laws and the ease of generalization to higher order, the case of n = 2 is paid special attention.

In the method of "reduction of order" one uses the knowledge of one solution to a second order homogeneous equation to find a linearly independent solution. This method relies on the use of two substitutions. We propose an alternative method that relies on Abel's theorem which is quicker and cleaner to use. We also show that this method extends to higher order equations.

# Chris Oehrlein Oklahoma City Community College (coehrlein@occc.edu)

#### An Explore-First, Lecture-Later-and-Little Approach to Teaching Introduction to Differential Equations

Students typically take a course on Ordinary Differential Equations after they have successfully completed at least two calculus courses, one physics course and some other science and engineering courses. Our students have shown in these courses that they can learn and apply computational procedures, but they didn't learn them from our lectures. They actually learned both concepts and procedures from working through problems together— getting stuck and looking for help in their textbook or from online sources, some of which may have been provided by their professors. Students tell us that they rarely use their lecture notes as a reference when working homework problems or preparing for tests. So why do we, their instructors, think it is so important to lecture over every concept, theorem and computational details of a myriad examples? Would students who have shown the necessary aptitude and ability be better served by guidance through the terrain of the textbook and the concepts and procedures in a more active environment? That is exactly what this instructor did with his section of Introduction to Ordinary Differential Equations. The course was centered around guided reading assignments, in-class collaborative explorations and problem-solving, and limited, but targeted lecture time. Students were much more willing to ask questions during class and most came to office hours to discuss harder homework problems instead of conceptual or algorithmic basics. I will also discuss tests, grades and overall student perceptions.

# Kelly M. Bubp Ohio University (bubp@ohio.edu)

# From Procedures to Proofs: Some Promising Ways for Undergraduates to Transition to Upper-Level Mathematics Courses

While most freshman- and sophomore-level mathematics courses focus on developing procedural fluency, junior- and senior-level courses emphasize writing proofs. Undergraduates often struggle with the transition from procedures to proofs. Promising transitional options will be explored, each intended to ease this progression and lay a solid foundation for students to develop rigor and creativity in writing proofs.

Hortensia Soto-JohnsonUniversity of Northern Colorado (hortensia.soto@unco.edu)Sarah RoznerUniversity of Northern Colorado (sarah.rozner@unco.edu)Kristin NobletUniversity of Northern Colorado (kristin.noblet@unco.edu)Lee RobersonUniversity of Northern Colorado (lee.roberson@unco.edu)Michael OehrtmanUniversity of Northern Colorado (michael.oehrtman@unco.edu)

# Geometric Interpretations of Complex Variables and Complex-Valued Functions

Quantitative reasoning combined with gestures, visual representations, or mental images has been at the center of much research in the field of mathematics education. In this research we extend these studies to include the arithmetic of complex numbers and the analysis of complex valued functions. Our data consists of videotaped interviews with experts, including mathematicians, physicists, and graduate students. Microethnography and phenomenological methods were used to analyze and interpret the data. In this study we synthesize how our experts employed geometric representations, gestures, metaphor, diagrams, and models to describe their understanding of complex variables topics. These findings may serve as a foundation for creating teaching experiments to help students develop geometrical representations of the mathematics behind complex variables.

#### Leah Childers Pittsburg State University (leah.childers@gmail.com) TaraLee Mecham College of Mount St Joseph (TaraLee\_Mecham@mail.msj.edu)

#### **Proof Journals**

Teaching students to write good proofs is often a difficult task. We have introduced proof journals in a Discrete Structure course to help facilitate this process. The purpose of the proof journal is to provide a place where students can practice writing proofs, have a dialogue with their instructor as well as their peers, and get constructive feedback on their proofs. We will discuss the implementation and success of proof journals as well as students' reactions to the journals.

# **Douglas Andrew Lapp** Central Michigan University (lapp1da@cmich.edu)

#### Span, Basis, and Eigenvectors: Using Dynamically-Connected Representations to Reify Abstract Concepts in Algebra

This session describes the use of technology to promote mathematical exploration in upper-level undergraduate courses. The use of technology and approaches to teaching illustrated are viable alternatives to the typical lecture format used in undergraduate courses such as abstract and linear algebra. Experiences will be shared from research conducted on students' learning of linear algebra and approaches for infusing technology into this course to promote mathematical discourse will be discussed.

This talk illustrates a pedagogical shift from a teacher-centered toward student-centered environment using the TI-Nspire CAS' hand-held and software that combine a computer algebra system, dynamic geometry system, spreadsheet, data collection, and dynamic statistics system into one device where all representations are dynamically connected. Changes to any created object in any problem page of a document results in real-time changes in other connected objects on any page within the same problem. This has been a powerful tool in student exploration and promotes discourse among students. Research shows that it is this ability for the student to interact with multiple representations visible on the same screen and notice aspects of situations that remain invariant across representations that promotes reification of abstract concepts. Examples of curricular changes that promote this reification within the traditional linear and abstract algebra courses will be shared.

#### **Donna Beers** Simmons College (donna.beers@simmons.edu)

# Strengthening the Teaching and Learning of 'Function': Addressing the Classroom Challenges and Identifying Research Opportunities for Faculty

This presentation identifies areas where faculty research is needed to find engaging applications of functions which can be used to enhance undergraduate mathematics courses. We describe our efforts to address gaps and misconceptions in student understanding of 'function' and of the one-to-one and onto function properties. In particular, we report on recent research on image reconstruction (deblurring) and information retrieval (data mining) which we included in a sophomore linear algebra course to illustrate the power of function inverses. We look at how the function concept is currently developed throughout mathematics education, identify what is missing, and offer suggestions for and examples of remedies.

#### Debra Czarneski Simpson College (debra.czarneski@simpson.edu)

#### Teaching an Introduction to Mathematical Research Course for First-Year Students

In the fall semester of 2009, I taught an Introduction to Mathematical Research through Graph Theory course for incoming first-year students. Students learned how to ask appropriate questions, how to form conjectures, and how to present their findings both orally and in writing. This talk will discuss the course requirements, course format, topics covered, and course outcomes.

# Daniel Biles Belmont University (daniel.biles@belmont.edu)

#### Topics for Actuarial Exam P/1

The first actuarial exam requires detailed knowledge of a number of topics that are not always covered in every first course in probability. We will provide an overview of these topics for use by instructors of that course and students interested in preparing for Actuarial Exam P/1.

#### **David Austin** Grand Valley State University (austind@gvsu.edu)

# Using the JPEG Compression Algorithm in a First Linear Algebra Course

One of the challenges in teaching a first course in linear algebra lies in relating abstract principles to natural solutions of real world problems. In this talk, I will describe an activity, created for students in such a course, that uses the JPEG compression algorithm to motivate the concept of a basis for a vector space and the utility in changing bases to represent information more naturally and efficiently.

# **Great Talks for a General Audience**

# **Coached Presentations by Graduate Students**

# Saturday, August 6, 1:00-5:30 p.m. in Thoroughbred 2

Susan Crook North Carolina State University (sbcrook@ncsu.edu)

# Using Computers to Solve Jigsaw Puzzles

Have you ever considered how you approach putting together a jigsaw puzzle? Do you sort all the pieces first and create the edge? Do you pick a focal point in the image and start there? What knowledge do you collect about each piece before you look for its place? Now, imagine that you have a puzzle with no image on it. This is the situation we face when we use computers to solve a jigsaw puzzle. In this talk, we'll discuss how computers approach the problem and the tools they use to complete them. Other applications will also be discussed. This talk is accessible for undergraduate students.

# Scott Kaschner Indiana University Purdue University Indianapolis (skaschner@math.iupui.edu)

# An Introduction to the Mandelbrot Set and Related Recent Results

Dynamical Systems is a fascinating branch of modern mathematics that is not often part of a typical undergraduate experience. The main purpose of this talk is to display the topic to a brand new audience, so very little background is required. The Mandelbrot set will be used to show how simple ideas produce surprising and sometimes complicated behavior. To further illustrate this, some major research results and the set's basic properties will be highlighted. These will be presented in conjunction with related concepts such as Julia sets and complex quadratic polynomials. With definitions and properties discussed, conjectures and more modern research results will be presented, including local connectedness, Shishikura's result about the Mandelbrot set's boundary dimension, and Buff and Cheritat's result that there are Julia sets with positive area.

# Donald Sampson Brigham Young University (sampson.dcs@gmail.com)

# Bubbles and Boundary: An Invitation to Isoperimetric Research

This talk is for anyone who has ever played with bubbles. Soap films and soap bubbles have the extraordinary ability to form exotic surface area minimizing shapes. In this talk I will present some of the research that has been done in studying the mathematical properties of these shapes. This includes an introduction to a new class of isoperimetric problems with boundary, and information on how you can get started on your own research, even as undergraduates.

# Kristina Leifeste New Mexico State University (kleifest@nmsu.edu)

# "The Complete Measurer": A Window into 19th Century Geometry

Unlike today, geometry was not a required subject for school-age students in the early days of America. Geometric concepts were only needed for higher education and practical purposes, such as carpentry, masonry, and even gauging. William Hawney and Thomas Keith's 1817 London book "The Complete Measurer" gives us a glimpse into the geometry behind the professions of the past, and helps put a real-world spin on the mathematics that we are all familiar with today. Originally meant for bricklayers, painters, glaziers, brewers, surveyors, and the like, Keith transformed the text into a compendium of geometry worthy of use in the public schools. The book not only contains work with shapes and solids (even Heron's formula for the area of a triangle), but also outdated concepts such as weight and dimensions of balls and shells and surveying using Gunter's Chain. We are able to learn from it long lost units of measure, how to ullage a cask, and how to find the specific gravity of many materials obsolete today. Along with amazing woodcuts, "The Complete Measurer" is a sort of time-machine, which ultimately helps us to appreciate the ways in which we learn and use geometry today.

# Chris Mattingly University of Kentucky (cmatting@ms.uky.edu)

# Approximation by Rational Functions on Compact Nowhere Dense Sets in the Plane

This talk is intended to be an introduction to approximation in various norms by rational functions on compact nowhere dense sets in the plane. We will survey some of the results for the uniform norm and the  $L^p$  norm.

# Robert Edward Campbell University of California, Irvine (rcampbel@math.uci.edu)

# Algebraic Coding and the Reed Solomon Code

We will explore the basics of algebraic coding looking at a few definitions and bounds. As our primary example, we will be looking at the Reed-Solomon code. We will finish by discussing why the Reed-Solomon code is most commonly used for coding today.

Dan Roberts Auburn University (dpr0003@tigermail.auburn.edu)

# Packing Complete Graphs with k-stars

A *packing* of a graph G with another graph H is a partition of the edges of G into as many edge-disjoint copies of H as possible along with some left over edges called the *leave*. We investigate k-star packings of  $K_n$  and  $\lambda K_n$ .

# Young Hwan You Purdue University (yyou@purdue.edu)

#### The necessary geometric condition for the solvability of the inhomogeneous Cauchy Riemann equation.

One of the ways to find an analytic function is to solve the inhomogeneous Cauchy-Riemann equation. In this talk, we introduce the basic notions and discuss the necessary conditions to solve the inhomegeneous Cauchy Riemann equation in several complex variables.

# Laura Dykes Kent State University (ldykes@math.kent.edu)

#### **Matrix Decompositions**

Many undergraduates are often not made aware of the more applied and practical aspects of the Mathematics they study in their Calculus/Linear Algebra sequences. Linear Algebra is one of those areas that students are repeatedly told they need to know but sometimes there is not much motivation for this. In an introductory Linear Algebra course, students are not often exposed to the computational aspects of Numerical Linear Algebra. Mathematical techniques that are explored in Linear Algebra are often not of practical use when faced with large matrices from physical applications. I would like to discuss the basics of matrix factorizations such as LU, QR, SVD (Singular Value Decomposition) and why they are important and useful in Numerical Linear Algebra. (I would like to use Matlab to show some examples)

# Julian Poranee Khongkha University of Cincinnati Mathematics department (julianpk@mail.uc.edu)

# **Geometric Properties of Quasihyperbolic Distance**

The purpose of this presentation is to introduce the length and weighted length distances in Euclidean space. We live in Euclidean space and are familiar with Euclidean geometry. We will see in some situations the correct distance is not the Euclidean distance. Instead we will replace the Euclidean distance with the length distance. However, there is a distance, the weighted length distance, which describes the geometry of an area of interest better than the length distance. We will discuss the so-called quasihyperbolic distance as an example of a weighted length distance in the plane. Some basic geometric properties of quasihyperbolic distance will be demonstrated as well.

# Amanda Pascoe Streib Georgia Institute of Technology (apascoe3@math.gatech.edu)

#### Markov chains for Self-assembly: How to create something out of (almost) nothing.

Imagine trying to design a complex structure made up of nanoparticles (keep in mind that the ratio of one nanometer to a meter is the same as the ratio of a marble to the earth). It seems impossible to construct complex forms at such a small scale; however, biological systems successfully do this every day. Simple elements with very basic functions work together to form incredibly complex systems in living things. For example, amino acids assemble themselves into proteins according to simple rules prescribed in our DNA. Nanoscientists try to mimic this self-assembly process as follows. They design systems of particles in solution, where each particle floats around until reaching a cluster of other particles, which it can then attach to (and potentially detach from later). The process continues until eventually the system reaches a steady state, hopefully (and sometimes *necessarily*) in the desired shape. How long does it take for the system to reach an equilibrium? We can use the theory of Markov chains to approach this question. In this talk, I'll give some background on Markov chains and explain how to formulate this self-assembly process as a Markov chain. I will also explain how to apply new techniques for Markov chains to solve a special case.

# Rebecca Dorff BYU (beccadorff@gmail.com)

#### Unification and the Multiple Bubble Conjecturet

One of the difficulties in proving the Triple Bubble Conjecture in space is proving the weighted Triple Bubble Conjecture. This talk describes a new method called Unification that has been used to prove properties of weighted multiple bubbles.

#### Kathryn A Brenneman North Carolina State University (kabrenne@ncsu.edu)

#### Lifting Commuting Involutions

In the study of Lie Groups and Symmetric Spaces, the question arises of how to lift (or extend) a pair of commuting involutions,  $\theta$  and  $\sigma$ . Such lifting is not unique (meaning we can get different actions on the larger vector space with the same action on the subspace); specifically we are interested in the set of lifts that preserve the commutativity of  $\theta$  and  $\sigma$ . In this talk we will look at lifting involutions of  $SL_n$ .

# Mehdi Nikpour The university of Toledo (mehdi.nikpour@rockets.utoledo.edu)

#### Parametric Toeplitz Operators

From the matricial point of view, moving one step to the southeast, provides us a bounded operator-valued linear transformation on the  $C^*$ algebra of all bounded linear operators on the Hilbert-Hardy space to itself, which enables us first to answer partially a question raised by Paul R. Halmos, and second to embed Toeplitz operators in an extended setting, which we call it Parametric Toeplitz Operators. Some of the properties of parametric Toeplitz operators (PTO), on the Hardy-Hilbert space, are investigated and is shown that much of their behaviors is similar to that of the classical Toeplitz operators. We also study some other operator equations.

# **Poster Session: Celebrating AWM Student Chapters**

# Friday, August 5, 1:00-2:30 pm in Heritage Ballroom East, Level 1

Helen Parks University of California San Diego (hfparks@math.ucsd.edu)

# The UCSD AWM

In 2007, graduate students at the University of California-San Diego founded a student chapter of AWM. In the following three years, the chapter was awarded a TENSOR-MAA Women and Mathematics Grant which (along with matching funds from UCSD's Physical Sciences Division) has allowed them to organize a number of activities at UCSD. We will discuss these outreach activities as well as some of the lessons learned along the way

# Alexandra Ortan University of Minnesota (aortan@umn.edu)

# Reaching out to younger generations through fun mathematics

The poster will highlight some of the outreach activities members of the AWM chapter at the University of Minnesota have been involved in during the last year. Among others, this includes our participation as mentors in the 2011 Girls and Mathematics Summer Program organized at the Institute for Mathematics and its Applications at the University of Minnesota. The program is aimed at supporting the interest of middle school girls in mathematics and serves 50 students which are exposed to a variety of fun mathematical topics outside the classroom setting. The poster will focus on some of the educational material we have developed in connection with this involvement and on the teaching and mentoring experience gained by the undergraduate and graduate AWM members during the program.

# Jill Dunham Hood College (dunham@hood.edu)

# Hood College student chapter of the AWM

The AWM student chapter at Hood College put on three great activities during the past academic year that we will feature on our poster. The first was a seminar by Professor Jacqueline Jensen. Professor Jensen gave a colloquium on knot theory. Preceding the talk, students had the opportunity to meet her over lunch. Then in January, two student members traveled to the Nebraska Conference for Undergraduate Women in Mathematics. They met outstanding women in mathematics and learned about graduate school and career opportunities, as well as attending mathematics talks. The highlight of the year for our AWM chapter came in March, when we hosted our seventh Sonia Kovalevsky Day for high school girls. This year, we had participants from all 10 Frederick County high schools. Over 40 high school students participated, along with their teachers. They took part in workshops on math and music, decoding the Postnet bar code, and the geometry of the pantograph, which tied together a very old mechanical device for drawing figures with modern computer software. Students also learned about the life of Sonia Kovalevsky from a current Hood math major and about careers in mathematics from recent Hood graduates and two successful women from NSA.

# Maia Averett Mills College (maverett@mills.edu)

# Mills College AWM Student Chapter

Our poster will share the activities of the newly-formed Mills College AWM Student Chapter in its founding year.

# **Rachel Elizabeth Bachman** Clarkson University's AWM Student Chapter (bachmare@clarkson.edu)

# Clarkson University's AWM Student Chapter: Service to the Scientific, Campus, and Local Communities

We describe AWM student chapter activities at Clarkson University that center around service on campus, in the local community, and beyond. These include peer tutoring for the freshmen Calculus gateway exams, undergraduate research presentations open to the campus, graduate school panel discussions, sponsoring colloquium speakers, grading for the national MATHCOUNTS competition, and running a local essay contest for middle school students.

# **Cynthia Kramer** Murray State University (cynthia.puckett@murraystate.edu)

# Murray State University AWM Chapter

Abstract: In fall 2008, Murray State University organized a student chapter of the Association for Women in Mathematics. We will chronicle the activities our chapter has undertaken during the past three years. These include outreach activities, organizing Spring and Christmas Mathematics Banquets, a Women Engaged in STEM research colloquium, etc.

# The Early Career and Graduate Students PosterFest at MathFest

# Friday, August 5, 3:30-5:00 pm in Heritage Ballroom East, Level 1

# David W Cook II University of Kentucky (dcook@ms.uky.edu)

# Enumerations deciding the weak Lefschetz property

We introduce a natural correspondence between artinian monomial almost complete intersections in three variables and punctured hexagonal regions. We use this correspondence to investigate the algebras for the presence of the weak Lefschetz property. In particular, we relate the field characteristics in which such an algebra fails to have the weak Lefschetz property to the prime divisors of the enumeration of signed lozenge tilings of the associated punctured hexagonal region. On the one side this allows us to establish the weak Lefschetz property in many new cases. On the other side, we can determine some of the prime divisors of the enumerations by means of an algebraic argument. For numerous classes of punctured hexagonal regions we find closed formulae for the enumerations of signed lozenge tilings and, thus, the field characteristics in which the associated algebras fail to the have the weak Lefschetz property.

# Judith Canner California State University, Monterey Bay (jcanner@csumb.edu)

# Measuring the response of species interactions to climate change

The study of species interactions and climate change relies on an understanding of the transient and long-term responses of species to environmental factors, such as temperature. In particular, the shifting phenology of species may lead to the decoupling of species interactions with warming. Though climate manipulation experiments test cause-and-effect relationships, the transient dynamics observed in experiments may not reflect the long-term response of a species to climate change. We developed a model of the seed dispersal by ants to observe the possible responses of myrmecochory to warming and to assess how we use experiments to predict plant population dynamics under warming. We compared our model under gradual warming conditions to models based on simulated fixed warming experiments of ant and plant dynamics and found that the predictions of short-term press experiments do not necessarily track the response of myrmecochore populations to gradual warming, especially at high levels of warming. We are now developing new ways to interpret climate manipulation experiments, in order to create and parameterize useful models that predict the persistence of species interactions under climate change. To continue our study, we use the new experimental results from warming experiments to update our model and observe the possibilities of decoupling of the relationship between ants and fruiting plants under different climate change scenarios. We will present the current results of the experiment and the model analysis and we will examine the long-term effects that climate change will have on seed dispersal by ants in temperate forests.

# **Yvonne Kemper** UC Davis (ykemper@math.ucdavis.edu)

# Small Matroids and a Conjecture of Stanley

Matroids appear in a variety of contexts and have applications in many fields, including combinatorial optimization, graph theory, and theoretical computer science. In this poster, we will look at an (open!) conjecture of a very topological nature made by Richard Stanley in 1977. We discuss some of the classes of matroids for which the conjecture has been verified, as well as some of the experimental techniques which we used in our explorations of the problem.

# Michael James Joseph John Carroll University (mjoseph10@jcu.edu)

# The Lambert W Function

The Lambert W Function is a function W(z) which satisfies the relation  $W(z) * e^{W(z)} = W(z)$  for any complex number z. It is defined as an inverse function, similar to square roots, logarithms, and inverse trigonometric functions, as it is the inverse of the function  $ze^z$ . This function is useful in finding closed form solutions to many equations which contain both exponential and power functions. For example, an equation of the form  $x^x = k$  has solution  $x = e^{W(ln(k))}$ . In the case where  $x^x = 27$ , this solution is  $x = e^{W(ln(27))} = 3$ , since  $3^3 = 27$ , but for most real numbers k, this solution cannot be written in closed form without referencing the W function. Many other equations also have solutions which can only be written in closed form using the W function. My research explores this function both from a complex numbers and a real numbers standpoint, and about the different branches of the function has. In the real numbers case, there are two possible values for W(x) when -1/e < x < 0, and one possible value for W(x) when  $x \ge 0$ . In the complex case, there are infinitely many values for W(z)except when z = 0, where w = 0 is the only complex numbered solution to  $we^w = 0$ . So there are infinitely many branches of the Lambert W function in the complex plane, which is similar to the complex logarithm, except that there actually is one branch defined at z = 0. I also have studied other mathematical problems related to this function such as the infinite exponential.

# Kristina Leifeste New Mexico State University (kleifest@nmsu.edu)

# "The Complete Measurer": A Window into 19th Century Geometry

Unlike today, geometry was not a required subject for school-age students in the early days of America. Geometric concepts were only needed for higher education and practical purposes, such as carpentry, masonry, and even gauging. William Hawney and Thomas Keith's

1817 London book "The Complete Measurer" gives us a glimpse into the geometry behind the professions of the past, and helps put a realworld spin on the mathematics that we are all familiar with today. Originally meant for bricklayers, painters, glaziers, brewers, surveyors, and the like, Keith transformed the text into a compendium of geometry worthy of use in the public schools. The book not only contains work with shapes and solids (even Heron's formula for the area of a triangle), but also outdated concepts such as weight and dimensions of balls and shells and surveying using Gunter's Chain. We are able to learn from it long lost units of measure, how to ullage a cask, and how to find the specific gravity of many materials obsolete today. Along with amazing woodcuts, "The Complete Measurer" is a sort of time-machine, which ultimately helps us to appreciate the ways in which we learn and use geometry today.

# JiYoon Jung University of Kentucky (jjung0328@uky.edu)

## Descent pattern avoidance

We study the problem of finding the asymptotics of sums over all permutations where each term is a product of weights depending on each consecutive pattern of a fixed length. This extends the notion of consecutive pattern avoidance and our technique extends the spectral method of Ehrenborg, Kitaev and Perry. When the weight depends on the descent pattern we show how to find the equation determining the spectrum. We give two length 4 applications: First, we find the asymptotics of the number permutations with no triple ascents and no triple descents. Second we give the asymptotics of the number permutations with no isolated ascents or descents. Our next result 4 is a weighted pattern of length 3 where the associated operator only has one non-zero eigenvalue. Using generating functions we show that the error term in the asymptotic expression is the smallest possible. This is joint work with Richard Ehrenborg

# Jonathan Goldfarb Florida Institute of Technology (jgoldfar@my.fit.edu)

# Numerical Analysis of Interface Evolution for the Nonlinear Degenerate Diffusion-Convection Equation

The mathematical model of nonlinear diffusion phenomena arises in many applications including heat radiation, particle diffusion in a plasma, the spatial spread of biological populations, etc. Such a model may take the form  $u_t = \operatorname{div}(\nabla u^m + \gamma(u)) + c(u)$ , m > 0 the so called reaction-diffusion-convection equation; solutions to this equation exhibit new qualitative properties unlike those present in solutions to the classical heat equation, including the appearance of interfaces and a finite speed of propagation. The problem of determining the behavior of interfaces for the reaction-diffusion equation ( $\gamma = 0$ ) is known as the Barenblatt problem, and was first formulated in the 1950s. A full solution of the Barenblatt problem was given by Abdulla and King (SIAM J. Math. Anal., 2000). The problem is still open for the diffusionconvection equation, which would be the case when c = 0. This project performs a numerical analysis on the problem of interface evolution for the nonlinear diffusion-convection equation in one dimension. First, we analyze self-similar solutions for a relevant range of parameters. In the general case we also apply finite difference schemes and solve the discretized problem, allowing any specific combination of parameters to be compared for purposes of finding the short-term interface behavior. These results complement those for the reaction-diffusion equation, and in the future the methods employed here will allow us to study reaction-diffusion-convection equations, and others of a similar but more general form.

# Chad Awtrey Elon University (cawtrey@elon.edu)

# Solvability of irreducible quintic polynomials

The solution to the quadratic equation  $ax^2+bx+c=0$  was found more than 4000 years ago. Corresponding solutions to the cubic and quartic equations were found in the 16th century. However, mathematicians struggled for centuries to find formulas for the solutions to quintics and equations of higher degree, but no general solutions were found. It was a major result (Abel-Ruffini) in the early 19th century that proved some quintics were solvable by radicals (i.e., using only *n*-th roots and the four basic arithmetic operations) while others were not. We discuss a new method for determining when an irreducible quintic polynomial is solvable by radicals.

# Jonathan Weisbrod Rowan University (weisbrodj1@verizon.net)

# Numbers That Can Be Expressed as the Sum of Two Positive Squares in Exactly n Distinct Ways

The primary objectives of this project are to quickly and easily determine the number of distinct ways a particular positive integer can be expressed as the sum of two positive squares and to generate sequences of positive integers that can be expressed as the sum of two positive squares in exactly n distinct ways. In order to do this, we only need elementary factoring and algebraic techniques. Pierre de Fermat and Leonard Euler are two of the famous mathematicians who have worked on this problem. Their theorems are used in this paper to prove the objectives above.

# Laurie Zack High Point University (lzack@highpoint.edu)

# Applying a Modified Google Ranking System to Football

Google is famous for coming up with the best search results partly due to the algorithm they use to rank web pages called the PageRank Algorithm. This method was then modified to the GEM method by Govan and Meyer in 2008 to rank football teams. The work presented here, done collaboratively as an undergraduate research project, is a modification to the GEM method which includes using more statistics to produce a more accurate ranking to the 2009 NFL football season.

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