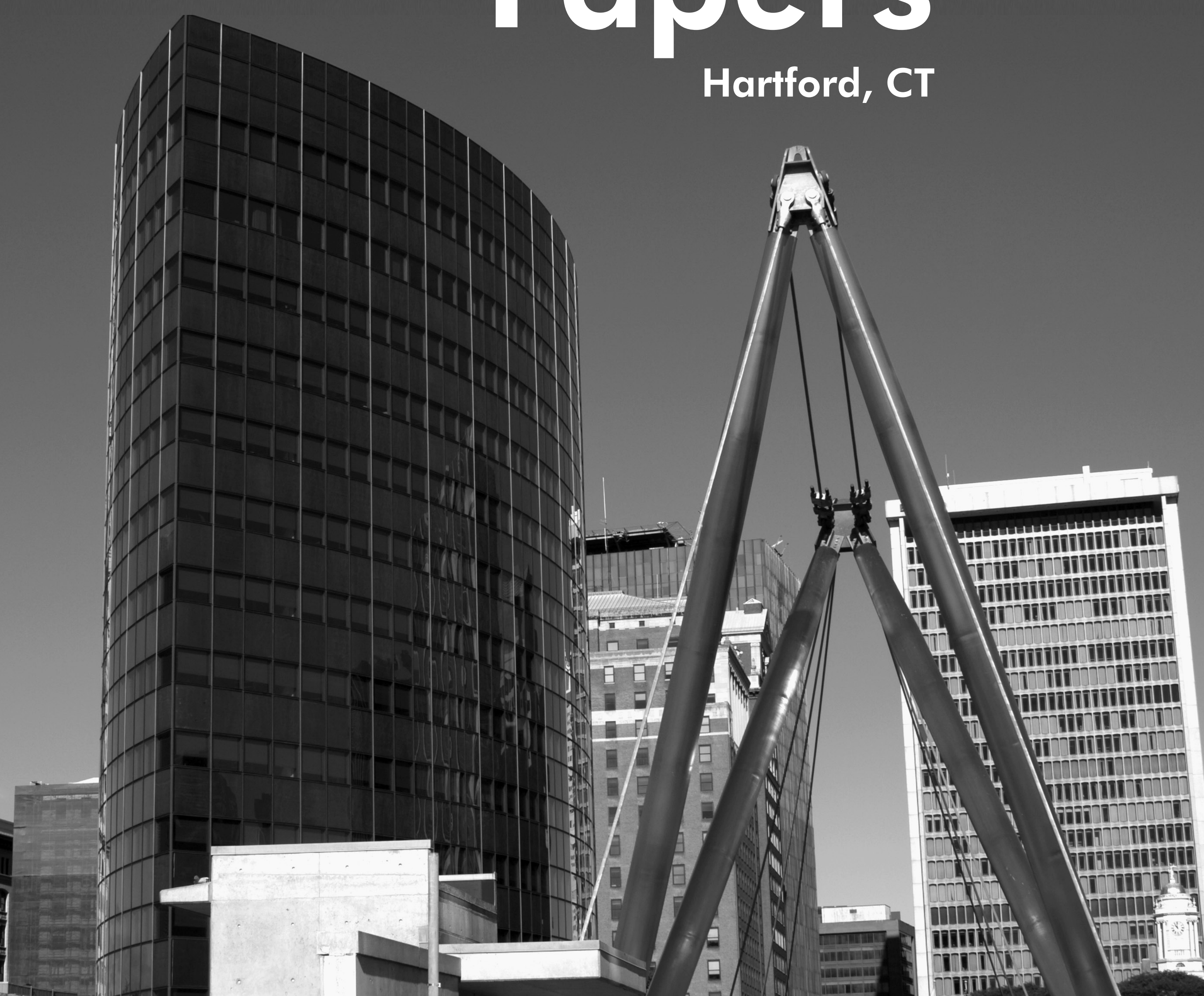
MAA MATHFEST

Jul 31-Aug 3, 2013

Abstracts of Papers

Hartford, CT



Abstracts of Papers Presented at

MathFest 2013

Hartford, Connecticut

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MAA

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Contents

Invited Paper Sessions

Open and Accessible Problems in Knot Theory

Thursday, August 1, 1:00–5:00 PM, Marriott Ballroom A

Colin Adams Williams College (cadams@williams.edu)

Turning Knots into Flowers

A triple crossing is a crossing in a projection of a knot that has three strands of the knot passing straight through it. We will show that every knot K has a triple crossing projection and then investigate $c_3(K)$, which is the minimum number of triple crossings in a projection of K. In joint work with students, we also consider extensions to $c_n(K)$. In particular we show that for any knot, there exists an n such that $c_n(K) = 1$. That is to say, every knot has a projection with only a single multi-crossing, and in fact that projection looks just like a daisy. This opens up a variety of directions for further research.

Lew Ludwig Denison Universtiy (ludwigl@denison.edu)

Knot Mosaics

In 2008, Lomonaco and Kauffman introduced knot mosaics, which have been shown to be equivalent to tame knot theory. This topic is easily accessible to undergraduate students as it is very hands-on. We explore several areas of this new approach to knot theory including the mosaic number (the smallest integer n required to fit a given knot on an $n \times n$ mosaic board). We also consider the mosaic number for different families of knots. Several open questions will be posed. This presentation is for a general mathematical audience.

Sandy Ganzell St. Mary's College of Maryland (sganzell@smcm.edu)

The Forbidden Number of a Knot

Virtual knot theory has a useful diagramatic approach: two diagrams represent the same knot if and only if they differ by a sequence of extended Reidemeister moves. But some moves are forbidden. So, like many forbidden things, we should do them, and find out what the consequences are. It turns out they're pretty interesting.

Elizabeth Denne Washington & Lee University (dennee@wlu.edu)

Folded Ribbon Knots in the Plane

Knots and links are modeled as folded ribbons lying flat in the plane. The ribbonlength of a knot is the length of a polygonal knot divided by the width of the ribbon around it. In this talk we'll give examples of folded ribbon knots and their ribbonlength. We'll also discuss the topology of folded ribbons and the problem of minimizing ribbonlength for a given knot type. There are a number of open questions related to this problem.

Joel Foisy SUNY Potsdam (foisyjs@potsdam.edu)

Graphs that are Intrinsically Linked with an Unused Vertex

We say that a graph is intrinsically linked with an unused vertex provided it contains, in every spatial embedding, a pair of non-split linked cycles, as well as a vertex that is not part of the non-split link. In this talk, we will discuss three operations that can be used to obtain graphs with this property, as well as several open questions about graphs with this property.

Laura Taalman James Madison University (laurataalman@gmail.com)

Sequences, Spiral Knots, and the Elephant in the Room

Spiral knots are generalizations of torus knots in which certain overcrossings in the standard braid representation are changed to undercrossings. Previous work with undergraduates about the determinants of various classes of spiral knots has uncovered a suprising menagerie of patterns, each of which hints at a still unknown pattern for a general formula. After exploring what is known about spiral knots we present three open problems that are accessible for future undergraduate research.

Louis Hirsch Kauffman University of Illinois at Chicago (kauffman@uic.edu)

Problems in Virtual Knot Theory

Virtual knot theory is a generalization of classical knot theory that has both a topological and a diagrammatic interpretation. On the topological side virtual knot theory studies embeddings of circles in thickened surfaces. On the combinatorial and diagrammatic side, virtual knots can be represented by diagrams similar to classical diagrams, but having an extra type of crossing called virtual. A generalization of the Reidemeister moves makes a complete diagrammatic theory for virtual knots and links. This talk will discuss a number of accessible research problems about virtual knots.

Developments in Commutative Algebra

Thursday, August 1, 2:00–4:50 PM, Marriott Ballroom B

Neil Epstein George Mason University (nepstein@uos.de)

Zero-divisor Graphs of Certain Semigroups Associated to Commutative Rings

When is a pair of ideals comaximal? When is the tensor product of a pair of finite modules zero? Which pairs of closed subspaces of a topological space have empty intersection? It turns out that for each of these questions, one can associate a semigroup with zero, and in turn a graph associated to it, where the vertices represent the "nonzero" objects in question (proper ideals, nonzero modules, nonempty closed subspaces). One draws an edge whenever the associated question has a positive answer. This is called the zero-divisor graph of the semigroup. It turns out that there is a strong connection between certain graph-theoretic invariants of this graph (e.g., chromatic number, diameter) and ring-theoretic information associated to the original data. We explore these connections. This work is joint with Peyman Nasehpour.

Leah Gold Cleveland State University (l.gold33@csuohio.edu)

An Introduction to Path Ideals

In the study of commutative algebra, graphs have provided a rich resource for the study of monomial ideals. The algebraic structure of such ideals are related to the combinatorial structure of the graphs used to generate them. There are multiple ways to construct an ideal from a graph. In this talk we will concentrate on path ideals. Given a graph, the path ideal for paths of length t is generated by monomials corresponding to the vertices of paths of length t in the graph. The combinatorial information of the graph is not enough to completely determine the minimal free resolution of the path ideal, however, there is still a rich structure that is determined by the underlying graph. We will discuss what is known about these ideals and current research questions.

Cameron Bishop Fairfield University (cameron.h.bishop@gmail.com)

Associated Primes of the Third Power of Cover Ideals

Given a polynomial ring, one can associate a graph to an ideal generated by quadratic, square-free monomials. The cover ideal of the resulting graph is of interest as the associated primes of its powers reveal properties of the graph itself, such as adjacencies and induced odd-cycles. We consider here the third power of the cover ideal and its associated primes.

Janet Striuli Fairfield University (jstriuli@fairfield.edu)

Totally Reflexive Modules

Totally reflexive modules were introduced by Auslander and have shown to be extremely useful in commutative algebra. It is known that if a ring admits a non trivial totally reflexive module, then it admits infinitely many. The construction of non isomorphic reflexive modules remains a difficult task. In this talk we will survey the latest results on this topic.

Branden Stone Bard College/Bard Prison Initiative (bstone@bard.edu)

Hilbert Series, h-vectors, and the Fibonacci Sequence

The Hilbert series of a module over a commutative ring is a generating function that captures many invarients of the module. In the case of zero dimensional standard graded rings, the Hilbert series is a polynomial and we call the coefficients the h-vector. As it turns out, the number of such h-vectors of length n is bounded above by the nth Fibonacci number. In this talk, we will define the Hilbert series and give some basic examples. We will also discuss the sequence defined by the number of h-vectors of a given length and its relation to the Fibonacci numbers and partitions.

Hans Schoutens City University of New York (hschoutens@citytech.cuny.edu)

Going to Great Lengths...

Commutative algebra is a generalization of linear algebra, in which a field gets replaced by a ring, and a vector space by a module; length then generalizes the notion of dimension. However, anything of infinite length gets thrown onto the same "does-not-apply" pile. I will argue that using a tool from logic, to wit, ordinals, we can nonetheless distinguish between them by means of "ordinal length". This invariant tells us quite a lot about the module. Looking at a few examples, one sees that one can no longer expect additivity (which itself is a generalization of the rank-nullity theorem), but a vestige remains, namely semi-additivity, which often suffices to calculate the ordinal length. I will give a brief overview of the theory and some applications.

Complex Geometry Research and Accessible Problems

Friday, August 2, 2:00–4:50 PM, Marriott Ballroom A

Michael Brilleslyper U. S. Air Force Academy (mike.brilleslyper@usafa.edu)

Locating and Counting the Zeros of the Polynomial $p(z) = z^n + z^k - 1$

The polynomial $q(z) = z^n - 1$ has n zeros symmetrically spaced on the unit circle in the complex plane. Adding a lower order term to obtain $p(z) = z^n + z^k - 1$ creates a more complicated situation for the location of the zeros of p. In this talk, we discuss how to determine the number of zeros that occur inside, on, or outside the unit circle in terms of n and k. In addition, we investigate the ratio of zeros outside the unit circle compared to n. We show this ratio has maximum of $3/4$, and that for fixed k, the ratio tends to $2/3$ as $n \to \infty$. Several conjectures, extensions, and open problems will be presented.

Jane McDougall Colorado College (jmcdougall@coloradocollege.edu)

Minimal Surfaces and Harmonic Mappings

When parameterized in isothermal coordinates, a simple theorem states that a surface will be harmonic if and only if it is a minimal surface. In this talk I will indicate problems resulting from the interplay between a minimal surface and the corresponding harmonic mappings, with the aim of drawing conclusions about related minimal surfaces. In particular we will look at the Schwarz D-surface, and the Scherk surfaces.

Christopher Hammond Connecticut College (cnham@conncoll.edu)

Composition Operators and the Geometry of the Unit Disk

This talk will provide an overview of the theory of composition operators, a particular type of linear transformation defined on certain spaces of analytic functions on the complex unit disk. We will discuss the connection between the geometry of the disk and the properties of these operators, with particular emphasis on results relating to eigenvalues and the operator norm. Several open problems will be discussed.

Erik Lundberg Purdue (elundber@math.purdue.edu)

Complex Variables and Gravitational Lensing by a Spiral Galaxy

The gravitational field of a distant collection of mass can act as a lens for (even more distant) background sources. A growing number of astronomical observations include lensing events that create multiple images of a single source.

Problems of interest to astronomers, such as the image-counting problem, run parallel to some current questions in the theory of complex variables. In this talk, I will discuss complex variable models for the lensing effect due to an elliptical galaxy and due to a spiral galaxy. Several open problems will be stated, including some questions prompted by a recent REU at Purdue.

Julia A Barnes Western Carolina University (jbarnes@email.wcu.edu)
Clinton P. Curry Huntingdon College (clintonc@clintoncurry.net)
Elizabeth D. Russell Western New England University (elizabeth.d.russell@gmail.com)
Lisbeth E. Schaubroeck United States Air Force Academy (beth.schaubroeck@usafa.edu)

Connecting Real and Imaginary Parts of Complex Quadratic Functions to Julia Sets

Julia sets for the family of complex functions $f_c(z) = z^2 + c$ have been well studied for years, and the images of these Julia sets are commonplace. There are also many activities related to these functions and their Julia sets which undergraduate students are able to explore. In this talk, we consider viewing the family, $f_c(z) = z^2 + c$, from a different perspective, beginning with the question of what the graphs of these functions look like. Since the graph of any function from the complex plane to the complex plane is four dimensional, we analyze the graphs of $\text{Re}(f^n)$ and $\text{Im}(f^n)$ where f^n is the nth iterate of f and find some interesting connections between these graphs and the Julia sets of the corresponding functions. We will look at how undergraduate students can approach these problems and mention some more general results that are known for complex rational maps.

Michael Dorff Brigham Young University (mdorff@math.byu.edu)

Complex Analysis and Soap Films

There are a lot of exciting and interesting unsolved problems in complex analysis that undergraduate students can research. In this talk we will discuss an area of complex analysis called geometric function theory. Specifically, we will talk about planar harmonic mappings which are a generalization of analytic 1-1 mappings in the complex plane. We will begin by giving a brief introduction to planar harmonic mappings. Then we will jump to a discussion of the mathematics behind soap films — well, actually we will talk about minimal surfaces (minimal surfaces can be modeled by soap films, and this is a great way to entice students into doing research in this area). Planar harmonic mappings and minimal surfaces are surprisingly related. We will discuss this relationship while showing examples of problems that undergraduate students can and have researched. Finally, we will let you know how you can find more information about this topic including examples of further problems for students to work on.

AMS-MAA Special Session: Coding Theory and ...

Friday, August 2, 2:00–4:50 PM, Marriott Ballroom B

Susan Loepp Williams College (sloepp@williams.edu)

Using Coding Theory for Quantum Cryptography

Coding theory is used extensively for Quantum Cryptography. In this talk, we describe the basics of Quantum Cryptography and how coding theory is used to correct errors caused by an eavesdropper. We continue by discussing privacy amplification for Quantum Cryptography. These topics will be put in the context of an interdisciplinary course on Protecting Information that the speaker team-teaches at Williams College with William Wootters, a leading Physicist.

David Clark University of Minnesota (dcclark@umn.edu)

Coding Theory, Designs, and Finite Geometries

Combinatorial designs are highly structured set systems with close links to error-correcting codes. In particular, finite geometry designs produce classical examples of codes which admit majority-logic decoding. Hamada conjectured that, among all designs with the same parameters, designs obtained from finite geometries are the unique designs whose incidence matrices have the minimum p-rank — a property especially desirable in majority-logic decodable codes. This conjecture is false, but the counterexamples have often provided insights into the structure of finite geometries and their codes. Almost all known counterexamples to Hamada's conjecture were obtained by first identifying a linear code with specific properties, and then extracting the design supported by this code. In this talk, we will describe certain classes of designs which have highly geometric properties, including minimum p-rank, but which are not finite geometry designs. We will examine these pseudo-geometric designs in terms of the properties of the codes which support them, and identify specific properties which allow such a design to have a minimum p-rank.

Justin D. Peachey Davidson College (jupeachey@davidson.edu)

Coding Theory and Elementary Number Theory

Elementary number theory is a beautiful branch of mathematics which explores various relationships between the integers. Common topics in this field include modular arithmetic, primitive roots, and quadratic reciprocity. While beautiful in its own right, techniques from elementary number theory may also be applied to prove more complicated results in coding theory. In this talk, we explore several methods in which coding theory was integrated into a recent elementary number theory course at Davidson College. Topics from elementary number theory provided students the opportunity to learn about Reed-Solomon codes as well as to prove more recent results on some algebraic geometric codes arising from the Suzuki curve.

Nora Youngs University of Nebraska, Lincoln (s-nyoungs1@math.unl.edu)

Coding Theory and Neuroscience

Neurons in the brain represent stimuli via neural codes—binary vectors which represent firing patterns. One important problem confronted by the brain is to infer properties of represented stimulus spaces using only the intrinsic structure of neural codes. How does the brain do this? To address this question we define the neural ring, an algebraic object that encodes the full combinatorial data of a code, and show how it can be used to extract relevant features.

Elizabeth Weaver Indiana University Southeast (elizweav@ius.edu)

Coding Theory and Graph Search Algorithms

Graphs and shortest path algorithms, such as Dijkstra's algorithm, comprise part of the curriculum of many undergraduate discrete mathematics courses. Many of these concepts naturally lead to applications in the area of coding theory. For example, linear block codes can be represented by graphs called trellises. Additionally, Dijkstras algorithm as well as other search algorithms like the Viterbi algorithm, can be used in maximum likelihood decoding. This is a rich area of discussion for students, as these algorithms are used in satellite transmissions, digital television, and automatic speech recognition software. In this talk, we will discuss how these applications can be incorporated into existing undergraduate lessons.

Jonathan I Hall Michigan State University (jhall@math.msu.edu)

Coding Theory and Instrumentation

The most typical applications of coding theory are error-correction (through the addition of redundancy) and compression (through the subtraction of redundancy). Here we discuss the use of coding correlation properties (a measure of resonance) in instrumentation, particularly for synchronization and tuning.

Recent Developments in Mathematical Finance

Saturday, August 3, 1:00–4:45 PM, Marriott Ballroom B

Scott Philip Robertson Carnegie Mellon University (scottrob@andrew.cmu.edu)

Static Fund Separation of Long Term Investments

In this talk we will prove a class of static fund separation theorems, valid for investors with a long horizon and constant relative risk aversion, and with stochastic investment opportunities. An optimal portfolio decomposes as a constant mix of a few preference-free funds, which are common to all investors. The weight in each fund is a constant that may depend on an investor's risk aversion, but not on the state variable, which changes over time. Vice versa, the composition of each fund may depend on the state, but not on the risk aversion, since a fund appears in the portfolios of different investors. We prove these results for two classes of models with a single state variable, and several assets with constant correlations with the state. In the linear class, the state is an Ornstein-Uhlenbeck process, risk premia are affine in the state, while volatilities and the interest rate are constant. In the square root class, the state follows a square root diffusion, expected returns and the interest rate are affine in the state, while volatilities are linear in the square root of state.

Hongzhong Zhang Columbia University (hz2244@columbia.edu)

Occupation times, drawdowns, and drawups for one-dimensional regular diffusions

The drawdown process $Y = \bar{X} - X$ of an one-dimensional regular diffusion process X is given by X reflected at its running maximum $\bar{X}$. The drawdown process $\hat{Y} = X - \underline{X}$ is given by X reflected at its running minimum $\underline{X}$. In this paper we calculate the probability of the event $\{\sigma_a < \hat{\sigma}_b \wedge \mathbf{e}_q\}$ for $a, b > 0$, where σ_a and $\hat{\sigma}_b$ respectively denote the first passage times of Y and $\hat{Y}$ to a and b, and $\mathbf{e}_q$ is an exponential random variable which is independent of X. We then explicitly express the law of the occupation times of X below y until the first exit. Using this result we compute the laws of occupation times of X, its drawdown Y and drawup $\hat{Y}$, at σ_a or $\mathbf{e}_q$. These results are applied to address problems in risk analysis and for option pricing of the drawdown process. Finally, we present explicit formulas of the main results for the cases of Brownian motion with drift and three-dimensional Bessel process, where we prove an identity in law between an occupation time and the first passage time σ_a.

Stephan Sturm WPI (ssturm@wpi.edu)

Volatility — A Key Concept in Mathematical Finance

Volatility has emerged as a key concept in mathematical finance. The parsimonious assumption of constant volatility pioneered by the Samuelson, Black, Scholes and Merton has turned out to be unrealistic and gave way to different modeling alternatives as local and stochastic volatility models. Innovative financial products introduced the possibility to trade directly in volatility and indices were created to monitor it. The aim of the present talk is to provide an introduction and overview about volatility modeling and the interconnected notions of implied, local and stochastic volatility. We will present the main mathematical ideas that help to shed light on these relationships and will finally present a counterexample to a conjecture popular among practitioners.

Gerard P Brunick University of California, Santa Barbara (gbrunick@gmail.com)
Mihai Sirbu The University of Texas at Austin (sirbu@math.utexas.edu)
Karel Janecek RSJ Algorithmic Trading (Karel.Janecek@rsj.cz)

Optimal Investment in the Presence of High-water Mark Fees

In this talk, we will consider the problem of optimal asset allocation for an agent who may invest in a money market fund, a stock, and a hedge fund. We model the risky assets as correlated geometric Brownian motions and we assume that our investor maximizes discounted CRRA utility from consumption on an infinite horizon. We further suppose that the investment in the hedge fund is subject to a proportional performance fee that is assessed each time the cumulative profit-to-date derived from the investment in the hedge fund reaches a new running maximum. We will see that this problem reduces to the optimal control of a reflected diffusion. We will examine the regularity of the associated Hamilton-Jacobi-Bellman equation and show the existence of optimal controls. Finally, we will examine some qualitative properties of the optimal investment strategy.

Michael Carlisle Baruch College, City University of New York (michael.carlisle@baruch.cuny.edu)
Olympia Hadjiliadis Brooklyn College and the Graduate Center of C.U.N.Y. (ohadjiliadis@brooklyn.cuny.edu)
Ioannis Stamos Hunter College (istamos@hunter.cuny.edu)

Trends and Trades

In this work, we build a trend following algorithm based on the sequential statistical rule known as the cumulative sum (CUSUM), which has traditionally been used for the online detection of abrupt changes in the distribution of sequences of observations. We draw connections between these statistics and the problem of online statistical surveillance and quality control, which dates back to the 1930s. We build a trading strategy based on the CUSUM stopping rule and apply it to high-frequency tick data from 5-year and 30-year US Treasury notes sold at auction. We analyze the performance of the proposed trend following strategy in detail. In particular, it is seen that the proposed trading rule is most profitable during times of market instability and long trends. We further calculate in closed form the expected value of the gain of the proposed strategy for a class of random walk models. Not surprisingly, it is seen that the suggested strategy is most profitable in biased random walks but is indifferent to the direction of the bias. We also examine the performance of the proposed strategy in simulated data from a variety of random walk models and analyze this behavior in relation to the analytical results and the results of the performance of the strategy on the actual data. We finally discuss other statistics of interest, such as the speed of reaction of the CUSUM statistic, and the way in which they can help us improve the performance of our proposed algorithm.

Climate and Geophysical Modeling

Saturday, August 3, 2:00–3:50 PM, Marriott Ballroom A

Lewis Mitchell The University of Vermont (lewis.mitchell@uvm.edu)

Improving Climate Models Using Non-Global Data Assimilation and Parameter Estimation

A major challenge in climate modeling is tuning the parameters to produce the most realistic climate. This is not generally done in a systematic manner, instead relying upon expert knowledge to match the output of a relatively small number of model runs with historical data. An alternative and emerging approach uses data assimilation parameter estimation methods, which systematically find the optimal parameters to match both model forecasts and observational data. We will explore the realistic problem of estimating parameters which vary in both space and time, which requires the use of a local data assimilation technique such as the Local Ensemble Transform Kalman Filter (LETKF). We will use the LETKF to estimate non-global parameters for a toy atmospheric model exhibiting nonlinear chaotic dynamics, as well as for an intermediate-complexity global circulation model.

Stephen G Penny University of Maryland (stevepenny@hotmail.com)

A Hybrid Ensemble Kalman Filter / Variational Method for Data Assimilation of the Ocean

The stability of a standard Ensemble Kalman Filter (EnKF) is dependent on the number of ensemble members that can be used. Operational centers are constrained by the costs of computing facilities and physical observing systems. With operational-scale models, it is usually impractical to use over O(10) members. Further, the ocean has historically been poorly observed. By combining a simple variational method with an EnKF, I have recovered stability in the EnKF for small ensemble sizes and low observation coverage when applied to a simple Lorenz-96 model. This method is currently being developed for application to the Global Ocean Data Assimilation System (GODAS) of the National Centers for Environmental Prediction (NCEP) at the National Oceanic and Atmospheric Administration (NOAA).

Clement Alo Montclair State University (aloc@mail.montclair.edu)

Numerical Modeling of Vegetation-Climate Feedbacks: An Example over Western Africa

A number of previous studies have shown that a changing climate due to increasing anthropogenic greenhouse gas concentrations over the coming decades may substantially affect the world's ecosystems. However, most regional climate modeling studies on the impacts of greenhouse warming do not consider the impact on/of vegetation dynamics. This is especially so for West Africa, a region where climate is highly sensitive to land surface conditions and feedback from vegetation dynamics has been demonstrated to substantially influence climate. This paper describes an approach employing global and regional climate models, and a dynamic vegetation model for incorporating effects of vegetation dynamics in numerical simulations of radiatively-forced future climate change over western Africa. The focus here is on the response of vegetation to climate changes and the effects of feedback from such vegetation changes on surface hydrology and climate in this region.

Raj Saha Mathematics and Climate Research Network, and Bowdoin College (raj.saha80@gmail.com)

Quasi-Periodic Fluctuations in Climate Due to Sea Ice

During the last glacial period, several quasi-periodic abrupt warming events took place, known as Dansgaard-Oeschger (D-O) events. Their effects were felt globally, although the North Atlantic experienced the largest temperature anomaly. Geophysical observations indicate that some of the fluctuations occurred in packets, preceded by massive glacial meltwater releases (Heinrich events). In this study a simple dynamical model of an overturning oceanic circulation and sea-ice is developed. Under appropriate temperature and salinity forcing the model exhibits self-sustained oscillations in the circulation strength. Physically the relaxation oscillations are produced due to the insulating effect of sea ice in cutting off heat exchange between the polar ocean and atmosphere. Upon melting the stored heat is released quickly, triggering rapid cooling and convective sinking, and with it an increase in poleward advective and heat flux. When the model is subjected to freshwater scenarios mimicking Heinrich events, the periodic circulation strength is modulated, much like the observed patterns in paleoclimate records. The period of oscillations is shown to be directly tied to the physical size of the heat reservoir, i.e., the volume of the polar ocean covered by sea ice.

Contributed Paper Sessions

Best Practices for Teaching Online Courses

Thursday, August 1, 1:00–4:55 PM

Amy Wheeler Hondros College (amywheeler@fuse.net)

Bridging the Digital Divide: Building a Sense of Community and Improving Student Engagement

The transition to online mathematics and statistic courses has created some challenges for non-math majors. Student engagement is one of the most essential components to a successful online experience. This presentation will discuss approaches to developing a sense of community in the online class and its impact on student engagement and learning outcomes.

Magdalena Luca MCPHS University (magdalena.luca@mcphs.edu)

Collaboration and Assessment Strategies for Teaching Online Undergraduate vs. Graduate Courses

During the past academic year, I developed two completely online courses: an undergraduate Algebra course and a graduate Biostatistics course. The two online courses were very different in content delivery and student audience, and therefore each one presented its own challenges. This presentation will focus on the best strategies for content delivery, collaboration and assessment methods for teaching online undergraduate vs. graduate courses. Feedback and experience show that whats best for teaching online undergraduate courses is not necessarily best in graduate ones, and vice versa.

Jacci White Saint Leo University (jacci.white@saintleo.edu)
Scott White St. Petersburg College (scott.white@saintleo.edu)

Fostering online Discussion in Introductory Statistics

In this session, participants will see examples of open ended questions, questions with multiple response threads, team activities, social justice questions, values and critical thinking for effective problem solving, and writing across the curriculum discussion opportunities that encourage student-student as well as student-instructor interaction. This session will emphasize the importance of having a variety of response opportunities with no single correct answer, the need to post plagiarism information on your syllabus or course site, and how a collection of discussion options gives students the opportunity to shine and adjuncts the opportunity to enhance their class. Many of these discussion techniques can be used in other courses with small variations. The presenter first developed an online Statistics course in 1999 and will talk about improvements and changes that have been made, leading up to the most recent redevelopment in 2013.

Xinlong Weng University of Bridgeport (gatewaytochina@hotmail.com)

Teaching Online Courses to Overseas Students

During the past 5 years, we have been teaching online courses to Chinese students who are spread out across several campuses in China. While we still have some challenges, the benefits well overcome the shortcomings. Students and faculty started those online courses with reluctance due to the distance, but became increasingly engaged throughout the term. The best outcome of the online courses was "group study". While "group study" may enhance the traditional classroom teaching, it is essential for the success of online teaching and learning.

Carol Hannahs Kaplan University (karyle@windstream.net)

Getting Started in MY Online Math Class

Many schools have a standardized structure and platform for delivering online classes. However, this does not mean the students taking your online class really know what to do day 1 of a new term. By providing guidance in the form of a video, podcast or text based document you can get them off to a great start with tips and tricks for getting engaged in the course quickly and being successful in your online class. Ideas for implementing this at various levels of math will be shared as it is important to consider your student body when creating your video.

Elizabeth Miller The Ohio State University (elizmiller@math.osu.edu)

Teaching Online and Face-to-Face Students in the Same Class

In June of 2012, The Ohio State University switched from the quarter system to the semester system. This created many interesting logistical problems, including a large group of about 1000 students who had taken Calculus 1 under quarters and would need a special short course on basic integration to be able to take the Calculus 2 course under semesters. In order to meet the needs of these students, especially over the summer semester, the math department developed its first fully online course. Hybrid sessions of this course were also offered; another first for the math department! This presentation will talk about how that course was set up, what tools were used, what went well, what lessons were learned. Also, we will talk about how this experience has led us to design a new kind of course format which allows online and face-to-face students to participate in the same class sessions. We will be piloting this new course format in Autumn 2013. Tools which will be discussed in this presentation include: Smart Podium, Camtasia, Adobe Connect, OneNote, Samsung Slate, MyMathLab, Desire2Learn, and Articulate Storyline.

Cornelius P. Nelan Quinnipiac University (cornelius.nelan@quinnipiac.edu)

Creating a Community Within an On-line Class

Taking an on-line mathematics course generally means no one-on-one interaction with the professor and fellow students, except for communicating via e-mail or discussion board. At Quinnipiac University we teach 'Introduction to Calculus' and 'Liberal Arts Mathematics' on-line during the Summer term. These classes are lower level terminal classes and many of the students exhibit the classical symptoms of 'math anxiety.'

The individuals in these classes can really use moral support as well as technical support. As it turns out, your best resource for moral support is the rest of the students in the class. Acknowledging this is important, and it is also important to experiment with community building exercises. I have found that setting up cooperative groups to work collectively on quizzes is both incredibly frustrating and eventually very rewarding. The benefits of having students cooperate on quizzes is that it forces them to discuss the material intelligently and also makes them answerable to their classmates.

Donna Flint South Dakota State University (donna.flint@sdstate.edu)
Becky Diischer South Dakota State University (rebecca.diischer@sdstate.edu)

Teaching an Activities Based Course Online

Quantitative Literacy (Math 103) is a new course at South Dakota State University which was designed to satisfy the General Education Requirement for Mathematics. Our goal was to create a course that would convince students that mathematics is truly relevant in their life, while maintaining the same level of academic rigor as the standard College Algebra course. We wanted to accomplish this with an activities based class, but one purpose of the new Math 103 course was to help students across the state who had stalled in the completion of their degree because of the mathematics requirement. Therefore, an online course was desirable. While some students may feel that an online course should be a stripped down version of the on-campus class, we believe that online students should have a similar experience to those taking the same course face-to-face. We will discuss how we accomplished implementing an activities based course in an online format, using various technological tools, while still maintaining academic rigor and security.

John C Miller The City College of The City University of New York (xyalgebra@mindspring.com)

Raising Standards for Math Practice Software

Math practice software should emulate an instructor working directly with a student. So it should:

- Allow solution entry one step at a time.
- Detect immediately and respond constructively to any incorrect step.
- Suggest an appropriate next step at any stage, regardless of solution method being used.

Iterating this "Suggest an appropriate next step" capability allows such a program to complete, as needed, any partial correct solution, regardless of the student's initial method.

A review of basic algebra programs from major publishers reveals that most do not accept entry of intermediate steps, and thus satisfy none of these three criteria. Much of our profession appears to have concluded that such materials are "good enough."

Examples will also include a basic algebra program embodying these principles and available at no cost via the Internet.

The presenter is interested in discussions with potential collaborators.

Lea Rosenberry Kaplan University (lea.rosenberry@gmail.com)
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Michelle Lis Kaplan University (mlis@kaplan.edu)

Living it Up with Live Binders: Organizing Faculty Shared Web 2.0 Resources

With the rise in popularity of Web 2.0 classroom tools such as YouTube, Screencast, VoiceThread, Prezi, Wordle, Animoto, Dropbox, etc., online educators have more ways than ever to deliver information to students dynamically. The sheer volume of resources available makes them difficult to locate and use within the classroom, and even more difficult across classrooms. Live Binders can be used to organize Web 2.0 resources so they are easier for faculty to share and present to students, and for students to use to get the help and support they need. This presentation will demonstrate how Live Binders were implemented within online courses to better organize faculty created and shared Web 2.0 resources.

Live Binders are virtual 3-ring binders that can be used for organizing, sharing, and collaborating. When using Live Binders, resources developed by faculty in multiple formats can be compiled into one place. Students don't have to click on separate links that take them to the different sites where the Web 2.0 presentations are housed; they can view them from within the Live Binder. Separate Live Binders were created and placed within each unit of a course to make topic specific resources quickly and easily available for students.

The positive impact of the implementation of these Live Binders will be shared, as well as new ways Live Binders are being utilized for faculty training within the department. An interactive tour of livebinders.com and a demonstration of how to build a Live Binder will also be included. Participants will be able to create and use their own Live Binders after attending the presentation.

Tamara Eyster Kaplan University (TEyster@kaplan.edu)
Lea Rosenberry Kaplan University (lea.rosenberry@gmail.com)

Using Digital Game-Based Learning in Online Math Courses

In an effort to increase student motivation and find more engaging ways for students to learn, the use of simulation and digital game-based learning are on the rise. Digital games are being used by early childhood and elementary educators, secondary educators, college and adult educators, as well as business and industry trainers. In addition to the wide range of educational settings, a wide range of disciplines have offerings in virtually every subject area.

Digital games range from traditional drill and practice to inquiry based learning, and everything in between. When evaluating digital games, determining the game type is as important as determining the learning outcome students are to master, since some learning outcomes work better with certain game types and designs than with others. While a wide variety of ready-to-play games can be found online, seldom is a game a perfect fit for the concept the instructor would like students to master. When a good fit is not readily available, there are tools instructors can use to create their own digital games.

We will discuss what digital game-based learning is, how to find ready-to-play games online, how to evaluate digital games for student use, and how to get started creating your own digital games, focusing on topics from an online math course.

Eric Ruggieri College of the Holy Cross (ruge337@stny,rr.com)

Teaching Statistics Online Using Blackboard Collaborate

This past January, Blackboard Collaborate was installed as the new medium for delivering online course content and was piloted by a small number of faculty members. In this talk I will describe my experiences using Blackboard Collaborate as a platform to deliver several online lectures to both an introductory and major level statistics course. Specifically, I will discuss the preparation required to deliver an online lecture versus a traditional face-to-face lecture, interactive features of Collaborate (including utilizing the chat and microphone features), efforts to engage students in an online lecture, problems encountered, and student reaction to the online lecture format.

History and Philosophy of Mathematics: Euler's Mathematics

Thursday, August 1, 9:00–10:30 AM, Room 27

Robert E. Bradley Adelphi University (bradley@adelphi.edu)

Leonhard Euler's Mathematical Correspondence — The Early Berlin Years

Leonhard Euler spent the early years of his professional career at the St. Petersburg Academy. His reputation as a mathematician increased throughout this period and eventually earned him a senior appointment to the Berlin Academy in 1741. During his early Berlin years, he engaged a number of colleagues in fruitful mathematical correspondence. We examine this period of expansion of his scientific community, paying particular attention to his correspondence with Nicholas (I) Bernoulli and Gabriel Cramer.

Stacy Langton University of San Diego (stacy.g.langton@gmail.com)

Vector Calculus in Euler's Fluid Mechanics

The basic differential operators of "vector calculus" are the gradient, divergence, and curl. Like other vector operations, such as dot products and cross products, they developed out of Hamilton's work on quaternions. (The "∇" symbol was introduced by Hamilton in 1846.)

Nevertheless, all three vector calculus operations occur in Euler's first major paper on fluid mechanics, the "Principia motus fluidorum" (E258) of 1752. This paper is divided into two parts. The first part, in which Euler shows that, for an incompressible fluid, the velocity field must have divergence ~ 0, has been discussed by Ed Sandifer in his "How Euler did it" column for September 2008. This talk will focus mostly on the second part, in which Euler writes down the general equations for fluid motion—which involve the gradient of the internal pressure—and in particular will discuss what Euler does with the curl. Some of the ideas here had occurred previously in the work of d'Alembert.

Michael P. Saclolo St. Edward's University (mikeps@stedwards.edu)

Euler's Method for a Plentiful Harvest

Euler's deep scientific curiosity is reflected by the wide variety of topics he studied. In a document delivered to the Free Economic Society of Russia, he takes on agriculture and economics as he reports on a farming method for grain that he claims to increase the crop yield tenfold. In this talk we shall look at the steps Euler outlines from sowing to harvest as well as his estimates of the potential yield. We shall also try to situate the original document, first published in Russian with a German version appearing a few years later, within the context of Euler's life.

History and Philosophy of Mathematics: Seventeenth and Eighteenth Centuries

Thursday, August 1, 1:00–5:30 PM, Room 27

Christopher Baltus Baltus SUNY Oswego (christopher.baltus@oswego.edu)

Conics in the 17th Century: Claude Mydorge and After

Claude Mydorge (1585 - 1647), friend and collaborator of Descartes, published *Prodromi catoptricorum et dioptricorum: sive conicorum operis* . . . , in two books, in 1631, with two more books to follow in 1639. His was the first in a burst of interest in the conic sections; later authors included Desargues, St. Vincent, de Witt, Wallis, and de la Hire. The subject was still just footnotes to the Conics of Apollonius in the 1620's; Mydorge wanted to introduce and simplify the study for his readers. With Mydorge, as with work that followed, we see: adherence to the structure of Apollonius, interest in mechanical production of the conics, and preference for synthetic methods in an era of enthusiasm for the analytic. Mydorge's influence seems small, although Oughtred's unpublished reworking and translation was the basis for an addition to Jonas Moore's Arithmetic, 1688.

John Bukowski Juniata College (bukowski@juniata.edu)

Christiaan Huygens's Work on the Catenary, 1690-1691

In 1646 the young Christiaan Huygens proved that the shape of the hanging chain, or catenary, was not a parabola. Forty-four years later, no one had yet described the actual shape of the catenary, so Jakob Bernoulli posed the problem publicly in *Acta Eruditorum* in 1690. Huygens then studied several aspects of the curve, such as arclength, radius of curvature, and the evolute. He published these characteristics of the catenary in a short article one year later in the *Acta*, along with articles by Leibniz and Johann Bernoulli. We will examine some of the background work in Huygens's Notebook G that led to his published results.

Maria Zack Point Loma Nazarene University (mzack@pointloma.edu)

The Geometric Algebra of John Wallis

This talk discusses at Wallis' interest in the quadrature of curves using material from his books *Arithmetica Infinitorum* (1656) and *De Cycloide* (1659). The talk also considers how Wallis' work connected with the work of other mathematicians including Roberval, Wren and Newton.

Troy Larry Goodsell Brigham Young University, Idaho (goodsellt@byui.edu)

Newton's Writings on the Calculus

In this talk we will look at the history of Sir Isaac Newton's publications on the calculus. We will consider the audience and context of the different publications and discuss how these affected the content, organization, and style of his writings.

David Richard Bellhouse University of Western Ontario (bellhouse@stats.uwo.ca)

Aprés 1713: Bernoulli, Montmort et Waldegrave

The fifth section of the second edition of Pierre Rémond de Montmort's *Essay d'analyse sur les jeux de hazard* published in 1713 contains correspondence on probability problems between Montmort and Nicolaus Bernoulli. This correspondence begins in 1710. The last published letter, dated November 15, 1713, is from Montmort to Nicolaus Bernoulli. There is some discussion of the strategy of play in the card game Le Her and a bit of news that Montmort's friend Waldegrave in Paris was going to take care of the printing of the book. From earlier correspondence between Bernoulli and Montmort, it is apparent that Waldegrave had also analyzed Le Her and had come up with a randomized strategy as a solution. He had also suggested working on the problem of the pool, or what is often called Waldegrave's problem. The Universitätsbibliothek Basel contains an additional forty-two letters between Bernoulli and Montmort written after 1713, as well as two letters between Bernoulli and Waldegrave. The letters are all in French. The trio continued to discuss probability problems, particularly Le Her which was still under discussion when the *Essay d'analyse* went to print. We describe the probability content of this body of correspondence and put it in its historical context. We also provide a proper identification of Waldegrave based on manuscripts in the Archives nationales de France in Paris.

Theodore J. Crackel Papers of George Washington (crackel2@yahoo.com)
V. Frederick Rickey West Point (fred.rickey@me.com)
Joel Silverberg Roger Williams University (joel.silverberg@alumni.brown.edu)

George Washington's Use of Trigonometry and Logarithms

You will probably remember from your grade school education that George Washington spent several of his youthful years as a professional surveyor. But how much mathematics did he know and how did he use it as a surveyor? Thanks to two "cyphering books" he compiled as a teenager, we are able to show what he learned of trigonometry and surveying. His combined use of these subjects is very perplexing to the modern reader, so we shall illustrate and explain the methods he used. Finally, in contrast to what one would expect, we will argue that he did not use trigonometry in surveying.

Scott Guthery Docent Press (sbg@acw.com)

Mathematics as Practiced in Colonial and Post-Colonial America

In their 1934 Carus monograph, *A History of Mathematics in America Before 1900*, David Smith and Jekuthiel Ginsburg confined their attention to the mathematics as found in scholarly texts. As a compliment to their work, this presentation considers mathematics as it was put to work in the field, on the street and at sea in colonial and post-colonial America. Books, tracts, periodicals and patents readily accessed in public libraries that describe in do-it-yourself terms how harness mathematics in various practical contexts form the primary source material for the study. In contrast to Smith and Ginsburg who found very little scholarly mathematics of note immediately after the revolution, we find mathematics of considerable maturity, creativity and insight being used in building the new nation. The presentation includes illustrative vignettes from the engineering of water works and the construction of flour mills.

Florence Fasanelli AAAS (ffasanel@verizon.net)

Images of Andrew Ellicott (1754–1820)

Andrew Ellicott defined our country with federal commissions to survey its borders, west, south and north, and lay the boundaries of the Federal District in the late 17th and early 18th centuries. This talk will share information about his images as they fit into the history of his noteworthy life as an astronomer and mathematics educator who provided generations of surveyors with the skills to mark out the land.

Duncan J Melville St. Lawrence University (dmelville@stlawu.edu)

How Brook Taylor Got Joshua Kirby a Position

In 1748, Joshua Kirby was a provincial coach-painter in Ipswich, Suffolk. By 1755 he was tutor in perspective to the Prince of Wales (the future George III). In between, he published *Dr. Brook Taylor's Method of Perspective Made Easy*, a book that aimed to explain Brook Taylor's notoriously difficult *Linear Perspective*. Using the subscription lists of the three works he published during this period, we trace how Kirby's expanding social networks brought him to the notice of those in power.

History and Philosophy of Mathematics: Nineteenth Century

Friday, August 2, 8:30–10:00 PM, Room 26

Ezra A Brown Virginia Tech (ezbrown@math.vt.edu)

Origins of Block Designs, Normed Algebras, and Finite Geometries: 1835 to 1892

In this talk, we give a brief tour of the birth and early development of finite geometries, combinatorial designs, and normed algebras. Arthur Cayley (1845), Jakob Steiner (1853) and Gino Fano (1892) are credited with the creation of (respectively) the 8-dimensional real normed algebra, certain block designs with block size 3, and the first finite geometry. During our tour, we learn about the truly ground-breaking work of Julius Plücker, John Graves, Wesley Woolhouse and Thomas Kirkman, work that anticipated Cayley, Steiner and Fano by one, 18, and 57 years, respectively. Even better—from the speaker's point of view—the tour begins with elliptic curves and ends with the (7,3,1) block design.

Salvatore John Petrilli Adelphi University (petrilli@adelphi.edu)

Monsieur François-Joseph Servois: His Life and Mathematical Contributions

Who was the mathematician François-Joseph Servois (1767–1847)? To the extent that his name is known at all, it is for introducing the words "distributive" and "commutative" to mathematics. Servois was ordained a priest near the beginning of the French Revolution. Had it not been for the revolution, it seems likely he would have remained a priest and become a successful mathematician. With the outbreak of the revolutionary wars, he joined the armed forces and followed a military career while also pursuing mathematics during his leisure time. His mathematical career flourished once he was appointed professor of mathematics at the French artillery schools. This talk will present a survey of the life and subtle mathematical contributions of Servois.

Shigeru Masuda Kyoto Univ (hj9s-msd@asahi-net.or.jp)

The Definite Integral by Euler, Lagrange and Laplace from the Viewpoint of Poisson

Since 1806, Poisson issued many papers on the integral of mixed equations of difference and differential, transcendental functions, and remarked on the necessity of careful handling to the transition from real to imaginary numbers. Poisson owes his knowledge to Euler, Lagrange and Laplace, and builds up his principle of integral, consulting Lacroix, Legendre, and others. Poisson is not in agreement with

Euler's or Laplace's diversion from real to imaginary. Of interest is Euler's paper on the origin of the gamma function or the Euler's second law of integral written in 1781 (this paper is not edited now in the Leonhardi Euleri Opera Omnia.) Euler's method for discovering the integral formula is due to the passage of real to the transcendental function of the problem of seeking a convergence point of a spiral. The papers by Laplace, which show the origins of the Laplace transform, are on the integral method using the passage from real to imaginary, and its application to the problem of Euler's spiral. Poisson wrote a series of papers (1806–1823) which criticize the methods used by Euler and Laplace.

History and Philosophy of Mathematics: Twentieth Century, Part 1

Friday, August 2, 8:30–11:30 AM and 2:30–3:00 PM, Room 27

David Orenstein Toronto District School Board (david.orenstein@utoronto.ca)

Statistics at the 1924 Toronto IMC and BAAS

In August 1924 the University of Toronto hosted both the International Mathematical Congress (IMC) and the British Association for the Advancement of Science (BAAS). Statistics played a significant, though contrasting role in both.

At the IMC, Section V, STATISTICS, ACTUARIAL SCIENCE AND ECONOMICS, included such respectably mathematical topics as probability theory, analysis of variance and quadrature under life curves. By contrast, Section F, ECONOMIC SCIENCE AND STATISTICS, at the BAAS included statistical papers but these were compilations of descriptive statistics.

R. A. Fisher, notably, presented papers to both the IMC and the BAAS. At the IMC's Section V, he summarised the analysis of variance in experimental design that formed the experimental background for his renowned STATISTICAL METHODS FOR RESEARCH WORKERS (1925). At the BAAS, Fisher eschewed Section F, presenting instead to Section M, AGRICULTURE, his work on the "Incidence of Rainfall in Relation to the Wheat Crop", similar to a paper he had published that year by the Royal Society.

This talk further investigates whether this fleeting concentration of statistical expertise left a lasting legacy, whether at the University of Toronto's Mathematics Department, the Ontario Agricultural College, or the Dominion Bureau of Statistics.

Matthew Clemens Keene State College (matt.eby.clemens@gmail.com)

Fictionalism and mathematical practice

In a prominent critique of mathematical fictionalism, John Burgess has argued that there is no version of the view that can preserve the desideratum that a philosophy of mathematics be philosophically modest, i.e., non-revisionary with respect to mathematical practice. Several advocates of mathematical fictionalism have recently offered defenses of their views against this critique from Burgess. In this paper, I consider a number of such defenses of fictionalism, and argue that none are compelling solutions for the philosopher of mathematics who aims to respect mathematical practice. By contrast, I suggest that given a significant broadening of the definition of mathematical fictionalism, a fictionalist view might be articulated which is genuinely non-revisionary with respect to mathematical practice. Such a view retains the fictionalist analogy between the mathematical and the fictional, but maintains that the entities of such realms exist as abstract artifacts; call this 'artifactual fictionalism'. As this new view departs radically from traditional fictionalism, I offer some remarks relating artifactual fictionalism to traditional versions of mathematical fictionalism.

Janet Beery University of Redlands (janet_beery@redlands.edu)

Who's That Mathematician? No, Really, Who Is She (or He)?

This presentation will feature photographs taken by mathematician Paul Halmos (1916–2006) during the 1950s through the 1980s, along with tales told by photo subjects more recently. The speaker will highlight women mathematicians from the era. She also will discuss the pleasures and perils of online research on a deadline.

Robert H C Moir Western University (robert@moir.net)

Rational Discovery of the Natural World: An Algebraic and Geometric Answer to Steiner

Steiner (1998) argues that the mathematical methods used to discover successful quantum theories are anthropocentric because they are "Pythagorean", i.e., rely essentially on structural analogies, or "formalist", i.e., rely entirely on syntactic analogies, and thus are inconsistent with naturalism. His argument, however, ignores the empirical content encoded in the algebraic form and geometric interpretation of physical theories. By arguing that quantum phenomena are forms of behavior, not things, I argue that developing a theory capable of describing them requires an interpretive framework broad enough to include geometric structures capable of representing the forms, which set theory provides, and strategies of algebraic manipulation that can locate the required structures. The methods that Steiner finds so suspect or mysterious are entirely reasonable given two facts: (1) discovering new theories requires algebraic and structural variation of old theories in order to access new forms of behavior; and (2) recovering the (algebraic and geometric) form of the prior theories is necessary to retain their empirical support. Accordingly, I argue that the methods used to discover quantum theory are both rational and consistent with naturalism.

Jonathan Seldin University of Lethbridge (jonathan.seldin@uleth.ca)

Mathematical Logic and the History of Computers

Many mathematicians and computer scientists are aware that mathematical logicians played a significant role in the original development of electronic computers, but as time has passed some of the details of this role seem to be on the verge of being forgotten. Since I was a student of H. B. Curry, who was one of the logicians involved, I know some aspects of this role which have not, to my knowledge, been published, especially some of Curry's personal recollections. In this talk, I intend to discuss my knowledge of this role, including the personal recollections, in the hope that awareness of this important role of mathematical logicians will be revived. The talk will include: 1) a discussion of the role λ-calculus and combinatory logic played in the development of recursive function theory and the development of programming languages, and 2) a discussion of he early work of Curry and von Neumann on the ENIAC and Curry's theory of programming which arose from that.

Jean-Pierre Marquis Université de Montréal (Jean-Pierre.Marquis@umontreal.ca)

Canonical maps: where do they come from and why do they matter?

The term "canonical" is now common in mathematics and the term "canonical map" finds its way in various mathematical contexts. However, there is no definition of what a canonical map is in general. In this talk, I want to sketch some of the roots of the terminology and explore why canonical maps are important mathematically and philosophically. I will focus on its progression in the literature and how this progression is intimately linked to the growth of category *theory.*

Glen Van Brummelen Quest University (gvb@questu.ca)

Tools of the Table Crackers: Quantitative Methods in the History of Numerical Tables

The application of quantitative methods as a research tool in the history of the exact sciences in recent decades has been powerful, tempting, and fraught with danger. Given their structure, historical numerical tables provide a proving ground for quantitative analysis and a potential for insights concerning historical treatises, authors, and users; but these methods may be applied only extreme caution and vigilance. We shall survey attempts to "crack" historical numerical tables, attempting to classify the various goals of researchers, elucidating their methods, and exploring the historiographic implications. This paper represents joint work with Clemency Montelle and Matthieu Husson.

History and Philosophy of Mathematics: Twentieth Century, Part 2

Friday, August 2, 2:30–4:00 PM, Room 26

Roger Godard RMC (godard-r@rmc.ca)

On the Chebychev Quadrature

In 1873, Charles Hermite studied a numerical solution to the integral $\int_{-1}^{+1} \frac{f(x)}{\sqrt{1-x^2}}\,dx$. Then in 1874, following him, Chebychev published in Liouville's journal an important article on quadratures. Chebychev assumed that, given a function $F(x)$, he searched to approximate integrals of the type $\int_{-1}^{+1} F(x)\phi(x)\,dx$ for any function $\phi(x)$ by the formula $k[\phi(x_1)+\phi(x_2)+\cdots+\phi(x_n)]$ where $x_1, x_2, \ldots, x_n$ are the Chebychev nodes. This formula differed from the Gaussian quadrature by the fact that all values of $\phi(x)$ have the same weight k.

In this present work, we analyze Chebychev's article and we follow the progress of numerical quadratures in France, Germany and Russia from 1874 to 1936. Finally, we comment the following diagram which represents the frequency of numbers of published articles about all methods of numerical integration from 1816 to 1960.

Charlotte Simmons University of Central Oklahoma (cksimmons@uco.edu)

Felix Hausdorff: We Wish for You Better Times

According to a German television broadcast on April 30, 1967, entitled *Die Wissenschaftler im Exil* (Scientists in Exile), the percentage loss of scholars suffered by the German universities in 1933 was greatest for mathematicians. As many as 144 German-speaking mathematicians can be listed who had to leave their positions and homes after 1933. As Michael Goolomb put it, "Most of them emigrated, but some of them lost their lives." Felix Hausdorff, credited as one of the founders of topology, is amongst these. In this talk, we will explore the life of this great mathematician, astronomer, and litterateur, who wished for his friends that they would "experience better times."

History and Philosophy of Mathematics: Using History and Philosophy in Teaching Mathematics

Friday, August 2, 3:00–5:30 PM, Room 27

Martin E Flashman Humboldt State University (flashman@humboldt.edu)

Logic is Not Epistemology: Should Philosophy Play a Larger Role in Learning about Proofs?

Many transition to proof courses start with a review or introduction to what is often described as "logic". The author suggests that students might be better served with an alternative approach that connects notions of proof with philosophical discussions related to ontology and epistemology. Examples will be offered to illustrate some possible changes in focus.

Xinlong Weng University of Bridgeport (gatewaytochina@hotmail.com)

Teaching Mathematical Ideas by the History of from Quadratic to Quartic Equations

Almost all of today's textbooks in mathematics have solutions in the back of the book. What students need to do is to find those answers by using the methods learned from the class. Long time practice from elementary schools to colleges, students believed that all questions have answers. We don't teach the history of mathematics on our campus, but I teach some history beginning with the quadratic equation continuing on with the quartic equation and ending with Galois Theory. I don't teach students the techniques involved, but the history. More importantly, I am teaching the principle of mathematical research which is that we really don't know in advance what is the right answer or if there is an answer at all. Students are excited to learn the possibility that there is not an answer, but they also learn that in order to claim that there is no answer, they have to prove that the problem has no answer. My experience has shown this piece of math history not just suitable for upper levels of engineering/science/math students, but also fits quite well into developmental mathematics courses.

J. Lyn Miller Slippery Rock University (lyn.miller@sru.edu)

Playful History: A Generalizable Mesolabium for Geometer's Sketchpad

In the history of attempts to duplicate the cube, the mechanical device called a mesolabium plays a role. During a recent sabbatical to study history of mathematics, the presenter engaged in a little mathematical recreation by constructing a generalizable mesolabium using the commonly accessible software package Geometer's Sketchpad. This talk will review the background of this historical device, then demonstrate some variations of the object as simulated in Sketchpad. The process lends itself well to student projects combining history of mathematics with educational technology.

Diana White University of Colorado Denver (diana.white@ucdenver.edu)

Historical Accuracy, Popular Books, and Videos: Three Components of a History of Math Class

There are a myriad of approaches to teaching the history of mathematics, including which content to address and the instructional approaches to take to facilitate student learning. Over the past five years, the speaker has tried a variety of different approaches to both the content and implementation. In this talk, we discuss three of these implementation approaches: a historical accuracy activity, popular book assignments, and videos. For each of these, we discuss the rationale, implementation, assessment, student feedback, and instructor conclusions related to student learning. While these activities were implemented by the speaker in a course dedicated solely to the history of mathematics, they are also suitable for implementation in a variety of other courses that incorporate the history of mathematics in some capacity.

The speaker hopes that the audience will be interested in modifying some of these activities for their own use, in providing ideas and feedback to further push the speaker's thinking related to the activities, and in contributing to a discussion throughout the conference on how to use the history of mathematics to contribute to undergraduate mathematics education.

Santhosh Mathew Regis College (santhosh.mathew@regiscollege.edu)

The Use of History of Mathematics as a Tool in Teaching Mathematics

Mathematics, in general, has the dubious distinction of being abstract and it often instills a widespread sense of concern for many students. Mathematics education, the foundation of all sciences, has lately become a political issue with some experts arguing for revolutionary changes in the curriculum by suggesting the replacement of traditional algebra with the elements of quantitative reasoning. Although the assimilation of technology has played a significant part in the continuing effort to improve the teaching of mathematics many challenges still remain and these need to be addressed. This paper investigates the possible role of the history of mathematics in undergraduate mathematics courses. Specifically, the paper explores how the history of mathematics can be integrated in the classrooms through well designed lesson plans that can relate to the learning community at large. The goal is to generate a positive attitude towards mathematics by encompassing historical aspects of the subject and to use such components to inspire students. This ongoing research focuses on identifying the segments from both ancient and modern history of mathematics that are suitable to incorporate in current programs in order to achieve the recommended outcomes. The paper also presents a case study that demonstrates the feasibility and a possible framework towards the implementation of history of mathematics as an active tool of learning and teaching. The research will continue to analyze the opportunities and limitations of this approach in the context of teaching mathematics.

History and Philosophy of Mathematics: The Arc of Time

Saturday, August 3, 8:30–10:30 AM, Room 27

Charlie Smith Park University (charlie.smith@park.edu)

Euclid's Treatment of the Golden Ratio

"The division of a finite straight line in extreme and mean ratio" originated in the Pythagorean School, and not surprisingly appears several times in The Elements. This talk will discuss some of Euclid's significant propositions which feature the golden ratio and the golden triangle.

Examples will include Book 2, Prop 11, the area formulation of the golden ratio; Book 4, Prop 10, the construction of the golden triangle; Book 4, Prop 11, the construction of a regular pentagon in a given circle; Book 6, Def 3, the proportion definition of the golden ratio; Book 6, Prop 30, the division of a line segment in the golden ratio; and Book 13, Prop 8, the diagonals of a regular pentagon divide each other in the golden ratio.

Elaine Landry University of California, Davis (emlandry@ucdavis.edu)

Plato was Not a Mathematical Platonist

I will argue that Plato was not a mathematical Platonist. My arguments will be based primarily on the evidence found in the Republic's Divided Line metaphor. Typically, the mathematical Platonist story is told on the basis of two realist components: a) that mathematical objects, like Platonic Forms, exist independently of us in some metaphysical realm and "the way things are" in this realm fixes the truth of mathematical statements, so that b) we come to know such truths by, somehow or other, "recollecting" how things are in the metaphysical realm. Against b), I have demonstrated, in another paper (Landry [2012]), that recollection, in the Meno, is not offered as a method for mathematical knowledge. What is offered as the mathematician's method for knowledge, in the Meno and in the Republic, is the hypothetical method. My aim in this paper will be to argue against a) by showing, especially given that recollection is not used at all in the Republic, that since both the method and the faculty used by the mathematician are distinct from those of the philosopher, then so too must be their objects. Again, taking my evidence from the Divided Line metaphor, I argue that mathematical objects are not Forms, and so do not either exist independently of us in some metaphysical realm or fix the truth of mathematical statements. Against the standard interpretation of mathematical objects as Forms, my argument will be that from both an epistemic and ontic perspective, mathematical objects and Forms must be distinct.

George P.H. Styan McGill University (geostyan@gmail.com)

Some illustrated comments on selected "magical squares with magical parts"

Following the classic book "Mathematische Mussestunden" by the mathematician Hermann Schubert (1848–1911), first published in 1898, we say that the 8x8 classic magic matrix A defines a "magical square with magical parts" whenever the four corner 4x4 submatrices are all fully-magic each with the same magic sum. In the 1845 booklet "A New Method of Ascertaining Interest and Discount", Deacon Israel Newton (1763–1856) noted that such a magic square satisfies what we call the "Newton-shuffle": the subsquares "may be cut apart, shuffled up together and placed together in a square form as it may happen, and it will form a [magic] square retaining all [the same properties that A has]". The earliest such "magical square with magical parts" seems to be by Thakkura Pheru (fl. 12911323), who wrote encyclopedic books on coins and gems in Delhi. We have found examples published much more recently by the polymath Benjamin Franklin (1706–1790), the French physician and botanist Jacques Barbeu-Dubourg (1709–1779), the American conchologist and malacologist Lorraine Screven Frierson (1861–1938), and by the Belgian mathematician Maurice Kraïtchik (1882–1957). We illustrate our findings with Anderson graphs (Bragdon's "magic lines" diagrams, Moran's "sequence designs") and whenever feasible with images of postage stamps or other philatelic items. With this talk we are pleased to celebrate the International Year of Statistics 2013 and the special year for Mathematics of Planet Earth 2013. [Joint research with Ka Lok Chu (Montreal) and Reijo Sund (Helsinki).]

Amy Shell-Gellasch Hood College (shell-gellasch@hood.edu)
Amy Ackerberg-Hastings NMAH/UMUC (aackerbe@verizon.net)

Mathematical Devices at the Smithsonian: Ideas for using digital collections in the classroom

Most museums have collections of mathematical and scientific objects. In the past, these collections have often been rarely known or seen. However, many institutions around the world are now in the process of digitizing their collections and making them available on-line. The Smithsonian museums in Washington DC have an extensive collection of mathematical and scientific devices and models housed throughout many of its 19 museums. Those items are currently being digitized and made available at institutional websites including http://americanhistory.si.edu/collections/object-groups/ and http://collections.si.edu. In this talk we will present an overview of the Smithsonian's online collections and suggest ideas for using these collections in the mathematics classroom.

Interactions Between History and Philosophy of Mathematics

Part 1: Saturday, August 3, 10:30–11:30 AM, Room 27

Thomas Drucker University of Wisconsin, Whitewater (druckert@uww.edu)

Zeno Will Rise Again

The adage that history is written by the victors has been as true in mathematics as elsewhere. When one looks at texts in the history of mathematics, there is more attention paid to the developments of the past that can be construed as leading to what mathematicians do today than to avenues that have proved to be dead ends. It is not surprising that mathematicians are interested in the roots of what they do, and the Whig interpretation of history cuts across many disciplines.

Texts in the philosophy of mathematics are more catholic in their accounts of the past. This may be the result of the sense that no philosophical position, however unfashionable, is incapable of resuscitation by later hands and arguments.

Mathematicians are willing to relegate pieces of the past to a footnote, while philosophers do not readily inter those pieces. When one looks at the history of the philosophy of mathematics, it looks more like a spiral than a chronicle of progress. This talk will look at particular examples of the revival of philosophical positions and the difference in attitude toward the past between historians and philosophers.

Amy Ackerberg-Hastings NMAH/UMUC (aackerbe@verizon.net)

Analysis and Synthesis in Geometry Textbooks: Who Cares?

Thirteen years ago, I completed a history of technology and science degree by writing a dissertation on how early 19th-century college teaching in the United States was shaped in part by two ubiquitous terms, analysis and synthesis, and three distinct but interrelated definitions for the terms: as mathematical styles, as directions of proof, and as educational approaches. To the best of my knowledge, however, the hardy few who read the dissertation were more interested in my biographies of Jeremiah Day, John Farrar, and Charles Davies than in the claims I made about the interactions between mathematics, philosophy, and pedagogy in these men's cultural context.

Now, I am rewriting the dissertation, rearticulating these intellectual connections, and, ultimately, reaffirming their historical significance. This talk will report on this process of rethinking in order to highlight the importance of philosophy in intellectual and cultural approaches to history. I will also discuss how an awareness of this interplay between philosophy and history can positively influence how we present mathematics to students.

Interactions Between History and Philosophy of Mathematics

Part 2: Saturday, August 3, 2:30–5:00 PM, Room 27

Robert S D Thomas University of Manitoba (thomas@cc.umanitoba.ca)

Assimilation in Mathematics and Beyond

'Assimilation' is my term for the operation of assigning something to a class, whether others would do so or not, and for the formation of classes in that way. This is an ordinary-language phenomenon; one sees a chipmunk and recognizes it as a chipmunk. One has available one's personal class of chipmunks based on acquaintance with past chipmunks and what one knows of mammalian species or just pictures. This operation has an interesting relation to mathematics. Poincaré goes so far as to say "Mathematics is the art of giving the same name to different things." It has been done successfully, and it has failed. It is avoided, and it can be done well (formation and representation of equivalence classes). But there is not even a standard term for it. It is the method of my essay, "Extreme Science: Mathematics as the Science of Relations as such" in the Gold/Simons MAA anthology, where I assimilate mathematics to the sciences. In the paper, I discuss assimilation in a historical way.

Lawrence D'Antonio Ramapo College (ldant@ramapo.edu)

Euler and the Enlightenment

The Swiss mathematician and scientist Leonhard Euler is also a key figure in the philosophical discourse of the Enlightenment. In this talk we will take a detailed look at Euler's contributions to the metaphysics of his era. For example, the theory of causality found itself under attack from the skepticism of Hume and also from philosophers who tried to reconcile Newtonian physics with role of God in the universe. The primary theories of causality in the early 18th century were that of pre-established harmony as put forth by Leibniz and Wolff and the theory of occasionalism as supported by the Cartesians. Against these theories, Euler in his *Letters to a German Princess*, argued for the interaction of substances known as the theory of physical influx. Euler's theories of causality, the nature of forces, the divisibility of space, and the general nature of space and time, are important influences on the work of Immanuel Kant.

Maryam Vulis NCC and York College CUNY (maryam@vulis.net)

Persecution of Nikolai Luzin

This presentation will discuss the life and work of the Russian mathematician Nikolai Luzin, who was denounced by the Soviet Government over his adverse views on the philosophy of mathematics. Luzin was involved in the early 20th century crisis of philosophical foundations of mathematics. He built on L. E. J. Brouwer's intuitionist work. In particular, their rejection of the Law of Excluded Middle was condemned as contrary to Marxist dogma that every problem is solvable. Luzin was accused of following the traditions of the Tsar Mathematical School which among other transgressions promoted religion. Many important details of Luzin's case came to light only recently. Even his famous students, Kolmogorov, Aleksandrov, and Pontryagin joined the vicious campaign, however despite the danger he faced, Luzin never renounced his position.

Roger Auguste Petry Luther College at the University of Regina (roger.petry@uregina.ca)

Philosophy Etched in Stone: The Geometry of Jerusalem's 'Absalom Pillar'

Built in the first century CE, the "Absalom Pillar" is an impressive 20 meter monument in Jerusalem's Kidron Valley noted for its unusual archaeological and geometric features. Over many years scholars have debated the meaning and function of the pillar, especially what portions serve as a sepulchral monument and what (if any) as a tomb. This paper makes use of a practical philosophical approach employed mathematically to identify external geometric features of the pillar and from these features derive principles that seem to inform its construction. In doing so, the paper draws upon (and constrains itself) to geometric knowledge available to builders in the first century CE. A complex geometry seems to underlie the monument's construction with seeming allusions to Archimedes' works "Measurement of a Circle" and "On the Sphere and the Cylinder". Possible philosophical interpretations of these geometric findings are also explored through the writings of the Jewish philosopher, Philo of Alexandria (20 BCE – 50 CE). The Pillar's geometry is shown to be readily intelligible through Philo's symbolic interpretations of mathematics including numeric symbolism he draws from Hebrew Scriptures. The paper concludes that the upper portion of the Pillar is likely a tomb marker and the lower portion a tomb on the basis of a possible geometric allusion to Archimedes' famous tomb marker in Syracuse.

Jeff Buechner Rutgers University and Saul Kripke Center, CUNY GC (buechner@rci.rutgers.edu)

Understanding the Interplay between the History and the Philosophy of Mathematics in Proof Mining

What is the nature of the relationship between the history of mathematics and the philosophy of mathematics? We conjecture one particular aspect of this relationship (which we take to be a necessary condition) contextualized to the field of proof mining: understanding issues in the philosophy of mathematics is needed to properly understand episodes and developments in the history of mathematics, and episodes and developments in the history of mathematics are needed to properly understand issues in the philosophy of mathematics.

Hilbert's program which is a precursor of proof mining cannot be properly understood without understanding the philosophical problem of theoretical terms, their explanatory role in mathematics, their role in questions of mathematical realism, the crisis in the foundations of mathematics, the change from classical to modern mathematics, and the nature of mathematical understanding. Some philosophers misunderstand Hilbert's epistemology because they neglect the history of mathematics and some historians misunderstand Hilbert's program because they neglect the philosophy of mathematics.

We illustrate the symmetrical relation between the philosophy of mathematics and the history of mathematics in Hilbert's original formulation of his program, how Gödel's second incompleteness theorem eliminated certain aspects of Hilbert's program and motivated the revision of other aspects, Kreisel's re-interpretation of the program in terms of proof transformations needed to extract information from proofs such as effective bounds and algorithms for computing witnesses to ineffectively specified existential formulas, Kreisel's no-counterexample interpretation, Kriesel's notion of unwinding proofs, Gödel's Dialectica (functional) interpretation, and some of Kohlenbach's recent work in proof mining.

My Favorite Geometry Proof

Friday, August 2, 1:00–4:55 PM, Room 14

Michael Nathanson Saint Mary's College of California (man6@stmarys-ca.edu)

Pizzas, Calzones, and Crusts: Using symmetry to slice up a circle

My love affair with the so-called Pizza Theorem is now entering its third decade and continues to provide new joys and unexpected twists, both for me and my students at both the high school and college levels. The original theorem shows how to fairly divide the area of a circle in half by drawing four coincident lines meeting at an arbitrary point on the interior. I will explain and prove this result using a basic property of perpendicular chords. I will then show a new dissection proof of how to split a circle fairly among many people and indicate how this proof provides new insights into the fair splitting of an annulus, a sphere, and a cantelope. Whatever that is.

The motivation to share this with students is two-fold: To share beautiful mathematical results and ideas and to communicate the process by which instructive proofs spawn new questions for students to ponder.

Daniel E. Otero Xavier University (otero@xavier.edu)

Heron's Formula: a Proof Without Words

My current favorite theorem in geometry is a "proof without words" for Heron's formula. This ancient proposition gives the area of a triangle in the Euclidean plane in terms of the side lengths: a triangle with side lengths a, b, c has area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a + b + c)$ is its semi-perimeter. Traditional proofs involve a good deal of algebra and/or trigonometry, or the evaluation of a matrix determinant. The one presented here is based on the existence of an incircle and a dissection of some simple figures, so it has a more appropriate geometric "feel" to it.

Diana White University of Colorado Denver (diana.white@ucdenver.edu)

Heron's Formula for the Area of a Triangle

This beautiful result states that the area of a triangle is the square root of $s(s-a)(s-b)(s-c)$, where s is the semiperimeter (half the sum of the side lengths a, b, and c). It thus provides a formula for the area of a triangle in terms of just the side lengths. As such, it has both practical applicability and mathematical purity. It first appeared in Heron's *Metrica*, which was perhaps written around 75 C.E.

Heron's proof, couched in modern notation and language, requires only basic knowledge of Euclidean geometry, relying heavily on the use of similar triangles. Yet the proof is a classic example of "deep" mathematical arguments that require minimal prerequisite knowledge, reminding students that sophisticated arguments need not require extensive mathematical background.

Additionally, the proof brings out many key ideas in geometry that are often not intuitive to those newer to the study of geometry; for example, inscribing circles in triangles and drawing auxiliary lines.

Moreover, in this proof students can readily get "lost" chasing the symbols in the algebraic steps, seemingly wandering aimlessly through a series of non-motivated equations that they are just following line-by-line. By focusing on the drawing, and pointing to it to track the various steps, students greatly enhances the understanding of the proof. We discuss how the author guides her students to use this approach to gain a deeper understanding of both the proof itself as well as when to use diagrams as a primary focus, and algebraic steps for the formal recordkeeping.

Jeff Johannes SUNY Geneseo (johannes@geneseo.edu)

Spherical Triangle Area and Angle Sum

In teaching my geometry class, I spend the first month looking forward to deriving spherical triangle area. Before we get there, students regularly ask me if they can assume the sum of the angles is a straight angle, and I just happily anticipate revealing the magic to them. Over the years, I have enjoyed explaining this argument to several different audiences, ranging from a general audience with no background to those with a sophisticated undergraduate training. In this talk we will assume a familiarity with great circles as straight lines on the sphere to derive a formula for the area of spherical triangles and infer from it the range of possible angle sums for triangles on the sphere.

Marshall Whittlesey California State University San Marcos (mwhittle@csusm.edu)

The Angle Sum Theorem for Triangles on the Sphere

The angle sum theorem on the sphere states that the sum of the measures of the angles in a spherical triangle is greater than 180 degrees (π radians). There are a number of ways of proving this theorem. One standard approach involves the use of tetrahedra inside the sphere, and another involves showing that the area of a triangle is proportional to the spherical excess. Here we present a proof which relies on more basic results of intrinsic spherical geometry rather than on the extrinsic properties of 3-dimensional space. The proof resembles a similar standard proof for the angle sum theorem in hyperbolic geometry.

Stephen Andrilli La Salle University (andrilli@lasalle.edu)

The Existence of the Nine-Point Circle for a Given Triangle

Every triangle has an associated "nine-point circle" which contains the following nine points: the midpoints of the three sides of the triangle, the feet of the three altitudes to the triangle, and the midpoints of the three segments from the vertices of the triangle to the orthocenter of the triangle. (In special cases, some of these points may coincide.) In this talk, a proof of this classic result will be given. The "nine-point circle" for a given triangle can be constructed in a straightforward manner using compass and straightedge. A corresponding lab that the author created using The Geometer's Sketchpad program that constructs the "nine-point circle" for a given triangle will be introduced.

Pat Touhey Misericordia University (ptouhey@misericordia.edu)

Ptolemy's Theorem

A well known proof of a classic result that is not as well known as it should be to undergraduate students. We then utilize Ptolemy's Theorem and some basic geometry of the circle as a prelude to some trigonometry.

Mary Platt Salem State University (mplatt@salemstate.edu)

When is the Inversion of Circle C over Circle k Orthogonal to Circle k?

My favorite undergraduate geometry theorem resulted from an Aha moment when I first understood how the Poincare model for the hyperbolic plane is put together. The theorem that tells how to construct the arc orthogonal to the Poincare disk is the following: "Let P be any point that does not lie on circle k and that does not coincide with the center O of circle k, and let C be a circle through P. Then C cuts k orthogonally if and only if C passes through the inverse point P' of P with respect to k."

There are several reasons why this theorem makes a good lesson for undergraduate students. The Poincare model of the hyperbolic plane uses Euclidean techniques to study hyperbolic geometry which gives students an example where one part of mathematics appears in another part of mathematics. Also, this theorem allow for a discussion of a generalization of the reflection across a line in the Euclidean plane to a reflection across a circle (inversion). In addition, the proof uses the power of a point with respect to a circle making the power of a point more relevant from a students perspective. Finally, this theorem can be generalized using a dilation of the plane where this theorem is a special case.

Braxton Carrigan Southern CT State University (carriganb1@southernct.edu)

Convex Quadrilaterals

Finding a way to constantly review proof techniques and concepts previously covered in class for students who have fallen behind but continually moving forward in Neutral Geometry provides a challenge for any instructor. This talk will highlight how one theorem can be used to revisit common misconceptions while still exploring new concepts. Showing that a quadrilateral $\square ABCD$ is convex if and only if $\overline{AC} \cap \overline{BD} = \varnothing$ can introduce the idea of convex quadrilaterals while revisiting the relationship between segments, rays, and lines. As a bonus this theorem can uses symmetry or "without loss of generality" which is often misused by students.

Clark P Wells Grand Valley State University (wellsc@gvsu.edu)

Quadrature, the Geometric Mean, Hinged Dissections, and the Purpose of Proof

Mathematicians will generally agree that proofs are a good thing (why else would we be talking about our favorites?) and that rigor is important. But as educators, what is the purpose of proof? I would argue that in a perfect world a proof should not only verify the truth of a proposition, but should give insight into the proposition itself. A sad fact is that proofs often do not give insight, and worse, they can sometimes seem to students as if they were written to deliberately obscure insight. Sometimes, though, you can have both rigor and insight.

Among my favorite geometry proofs are quadrature proofs, which I typically discuss in our senior capstone course, The Nature of Modern Mathematics. The idea of quadrature is to create (typically by compass and straight-edge construction) a square that is "the same size" as a given geometrical object.

My very favorite geometry proof is the quadrature of the rectangle for several reasons. One is that the side length of the square obtained is the geometric mean of the side lengths of the rectangle, another is that it can be proven using hinged dissections and then animated using GeoGebra, as I will show in my talk, which leads to insight about what quadrature and the geometric mean really are. Furthermore, by taking a theorem due to the ancient Greeks and proving it using modern technology, I can emphasize the connectedness of mathematical ideas across centuries.

Martin E Flashman Humboldt State University (flashman@humboldt.edu)

A Simple Proof of the Classification of Conics by the Discriminant

The discriminant of a conic curve in the real cartesian plane determined by a non-degenerate quadratic equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is given by $B^2 - 4AC$. I will present a simple proof of the classification of the conic (ellipse, parabola, or hyperbola) by the discriminant. Many proofs of this result in calculus and pre-calculus textbooks are developed using planar rotations. The proof I will present (without rotations) is based on elementary concepts of homogeneous coordinates in real projective algebraic geometry and 3 dimensional coordinate geometry. It is suitable for an upper division geometry course that discusses the interaction of synthetic and algebraic geometry.

Thomas Q Sibley St. John's University (tsibley@csbsju.edu)

It's Not Hyperbole: A Transforming Proof

Nineteenth century geometry transformed our thinking about geometry, mathematics as a whole and how we see the world—and not just once, but several times. Hyperbolic, projective and transformational geometries each reshaped mathematics. In turn a geometry course building on these topics can open and engage students' minds. The proof of the theorem characterizing hyperbolic isometries in the Klein model provides a capstone for these three areas of geometry. In addition, this result links this material to spherical geometry and even to Einstein's theory of relativity.

Inquiry-Based Learning Best Practices

Part 1: Friday, August 2, 3:20–5:15 PM, Room 16

Brian Loft Sam Houston State University (loft@shsu.edu)

Course Notes for Differential Calculus

A complete set of course notes for differential calculus has been written, tested, and recently accepted for publication. The content, use, and development of the notes will be discussed with particular attention to suggestions to the audience towards writing their own notes for later calculus courses. In addition, the success in integral calculus of those students that used these course notes will be examined.

Tim Boester Wright State University (timothy.boester@wright.edu)

Using Inquiry-Based Leaning to Define Continuity

Because mathematical definitions need to be precise and well-constructed, it may seem that they would be a poor fit for Inquiry-Based Learning. But definitions should still make sense, in terms of their necessity (why we need a definition for a particular thing) and what they define (the details in the definition). This talk will explore how the definition of continuity at a point is constructed through a class discussion of student-produced examples of discontinuous functions in a conceptually-focused IBL Calculus course for future middle school math teachers.

Ellie Kennedy Northern Arizona University (ellie.kennedy@nau.edu)

A Flipped Classroom Study in Second Semester Calculus

In Spring 2013, a study was done with 175 second-semester calculus students. Half of these students were taught via direct instruction and half were taught using a flipped classroom approach, where videos were utilized as lectures and classroom time was devoted to students working on problems in small groups and asking questions. Through survey data (MSLQ and SALG-M) and exam scores, the two different teaching methods were compared. In particular, learning gains, student perception of the effectiveness of the instructional approach, and student attitude towards learning mathematics were analyzed. This talk will discuss the details of the instructional methods and the results of the data analysis.

Janice Rech Univeristy of Nebraska at Omaha (jrech@unomaha.edu)
Angie Hodge University of Nebraska at Omaha (amhodge@unomaha.edu)

Calculus — The IBL Way!

Inquiry-Based Learning (IBL) is often thought to be a type of teaching/learning that takes place in upper division courses such as real analysis and number theory where classes are small and students are engaged in proof writing. However, having students engaged in sense making activities and exploring what it is like to think like a mathematician is something that could/should also occur in lower division coursework. In this session, we will share techniques for how we have began to include IBL techniques into the calculus series where classes are larger and content coverage is a concern. Student presentations, group work, independent thinking, and multiple modes of assessment will be discussed. Although the talk has a focus on calculus, instructors at all levels should be able to take something from this talk.

Daniel Shifflet Clarion University of Pennsylvania (dshifflet@clarion.edu)

Calculus Group Projects to Motivate Sequences and Series by Major

I have always been intrigued by the near paradigm shift that occurs in most Calculus II classes when we convert from teaching about integration techniques to introducing sequences and series. As instructors of the course, we see how these topics are interconnected and guide the students towards this same level of insight. But, for a few days at least, it must seem to our pupils that they are no longer sitting in a Calculus classroom. To combat this disconnect and help motivate the transition from integrals to series, I have created a collection of discovery-based group projects. Their topics range from fractals to computer algorithms to probability, giving each of the most popular majors of Calculus II a chance to see sequences applied within their own field. In this talk I will discuss the content of the projects, the process by which I incorporated them during the semester, and review the results of their inclusion.

Olympia Nicodemi SUNY Geneseo (nicodemi@geneseo.edu)

IBL in the Time of MOOCs

A recent article (4/30/2013) in the NY Times asserts, “Traditional teaching will be disappearing in five to seven years as more professors come to realize that lectures are not the best route to student engagement.” But it was not IBL but rather the MOOC that replaces the lecture. Lets gather to garner and perhaps answer questions about IBL vis a vis MOOCs. How, why, and for whom will IBL survive and/or co-exist in a MOOC culture?

Inquiry-Based Learning Best Practices

Part 2: Saturday, August 3, 8:30–11:25 AM, Room 16

Susan Ruff MIT (ruff@math.mit.edu)

MathDL Mathematical Communication: Resources for engaging students in communicating about mathematics

I will present highlights from a collection of online resources for engaging students in communicating about mathematics. With more than 90 pages, 80 attachments, and 400 external links, this collection addresses assignment design, assessment, teamwork, peer critique, research on inquiry-based learning, characteristics of effective writing and presentations, and more. The collection originated in the communication-intensive courses at MIT. With the support of an NSF grant, the collection has been expanded with diverse online resources and literature from other institutions, and is now housed in the MAA's Mathematical Digital Sciences Library. Further contribution of materials is welcome. Presented highlights will include resources for teamwork and presentations.

Martha Allen Georgia College (martha.allen@gcsu.edu)
Blair Dietrich Georgia Military College (bdietrich@gmc.cc.ga.us)

Discovery on "Number Theory Island"

We will examine how a number theory course has evolved from a "theorem-proof-lecture" format to a setting where the students are guided to discover mathematics and are then responsible for presenting their results to the class. To help facilitate this discovery-style approach, a storyline was created in which all the students were active participants on "Number Theory Island," where they encountered numerous mathematical adventures and challenges. In this session, we will describe the structure of the student-centered course as well as look at some specific examples of the discoveries made on the island.

Lauren Rose Bard College (rose@bard.edu)

Inquiry Based Learning in a Number Theory Course for Non-Majors

Since 1999, I have a taught course at Bard College entitled "Exploration in Number Theory," geared toward non-math students, most of whom take the course to fulfull their math requirement. The course involves exploratory and group activities both in class and for homework, and students develop a surprising level of problem solving skills and mathematical creativity. Over time, I have honed these activities to the point where I am now using similar activities in most of my courses that I teach. I will report on sample activities from the class, student responses, and how to use such courses as a pathway into the mathematics major.

David Richeson Dickinson College (richesod@dickinson.edu)

A Collaborative, Student-Written Textbook in a Writing Intensive, IBL Discrete Mathematics Course

Discrete Math is the introduction to proof-writing course at Dickinson College. It is officially designated writing intensive. This semester I taught the class using an IBL approach. I gave the students the skeleton of a textbook, in the form of a workbook; it contained definitions, statements of theorems, and exercises. I put the LaTeX source code on ShareLaTeX, an online LaTeX site, and added the students as collaborators. The students hand-wrote their proofs and solutions in the workbook, then presented them in class. Each day one student (the "secretary") typed the day's work into the ever-growing online textbook. The book went through several rounds of editing before being printed and bound at the end of the semester. In this talk I will describe the mechanics of this course and reflect on what went well and what changes I plan to make in the future.

Rachel Schwell Central Connecticut State University (schwellrac@ccsu.edu)

From Cookbook to Toolbox: Modified Moore Method in Discrete Math and Abstract Algebra

Over the past few years I (with a colleague) have developed Moore Method course notes for discrete math and abstract algebra, which we collectively have taught a total of seven times via a modified Moore Method approach. We have found that while students may have less of a fingertip usage of certain "standard" concepts through this method, they have traded up for the higher-order skill of critical reasoning to solve a much broader range of problems (of which I will give examples). Our notes paid careful attention to developing this skill via open-ended questions, definitions provided for the first time within a problem, and directives to research concepts on their own; I will show samples of such questions from within the notes. I will also share some projects we created for abstract algebra involving GAP. Lastly, I will discuss the course logistics for those seeking nuts-and-bolts suggestions, as well as outcomes, both measured and anecdotal.

Julianna Stockton Sacred Heart University (stocktonj@sacredheart.edu)
Nicole Trommelen Sacred Heart University (trommelenn@mail.sacredheart.edu)
Jennifer Robillard Sacred Heart University (robillardj@mail.sacredheart.edu)
Cole Matthew Sacred Heart University (colem182@mail.sacredheart.edu)
Bowers Jonathan Sacred Heart University (bowersj@mail.sacredheart.edu)

An IBL Proofs Course: Student Perspectives

A panel of students from an Introduction to Proof course taught using Inquiry Based Learning in Spring 2013 will share their perspectives on the experience. Students will describe the general design of the course including depiction of a typical class session, and will discuss what they each perceive to be the primary benefits and challenges of being a student in an IBL course. Time will be reserved at the end of the session for an extended Q&A discussion, so that faculty members considering using IBL in future courses can solicit student input on any questions you may have about course logistics or pedagogical methods.

Theron James Hitchman University of Northern Iowa (theron.hitchman@uni.edu)

Assessment in an IBL Geometry Course

Inquiry based course structures require a different way of thinking about how students learn, and about how to support that learning. They also require that we rethink how we assess student progress, and how we communicate with students about that progress. I will describe how I have tried some ideas from Standards Based Assessment in an IBL geometry course.

Brian Winkel United States Military Academy (BrianWinkel@hvc.rr.com)

SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

We introduce SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations. We will describe the purpose, operation, and possibilities of a web-based community of teachers and learners of differential equations through modeling and use of technology throughout the learning process. SIMIODE offers an NSF developed HUB environment for the community of learners with rich and complete (all material textual, scenarios, mathematical, models, data sets, activities, assessments, evaluations, projects, etc. at SIMIODE HUB) resources for teaching modeling and technology based differential equations courses. Students will register for $45 for lifetime registration and not need to purchase an expensive text book in this area ever. We will engage in modeling and being a member of the SIMIODE community at this session. In one sense modeling could be said to be the ultimate inquiry-based learning experience and we plan, through engagement, to richly illustrate the power of community in SIMIODE.

Jason Grout Drake University (jason.grout@drake.edu)

Explorations using Sage

Sage (http://sagemath.org) is a free comprehensive open-source mathematics software system with an online interface (no installation required, only a modern web browser is needed). Sage calculations and links to calculations are easy to embed into online or paper books/notes, blogs, web pages, Webwork problems, and more using the Sage Cell server (http://sagecell.sagemath.org).

We will describe how Sage activities have been used in several IBL-style courses or activities, such as multivariable calculus and linear algebra, to encourage student exploration, augment and check student understanding, and introduce students to computational issues and techniques. We will describe how you can easily embed Sage calculations into your activities using the Sage Cell server. We will also share some insights into using technology in an IBL course.

Inquiry-Based Learning Best Practices

Part 3: Saturday, August 3, 1:00–4:55 PM, Room 16

Robert Sachs George Mason University (rsachs@gmu.edu)

Computer environments promoting student inquiry

Advances in computing provide new opportunities for creating student inquiry. Beyond plotting a given function, students can explore categories of examples and be asked to conjecture behavior. Some examples from multivariable calculus will be shown and student initial responses described.

Jacqueline Jensen-Vallin Slippery Rock University (jacqueline.jensen@sru.edu)

A Student-Centered Approach to Intermediate Algebra

In the Spring 2013 semester, my course load included intermediate algebra for the first time since graduate school. Back then, my approach to teaching was very much “the sage on the stage.” Now I use a much more student-centered approach, which led to a conundrum how to cover basic algebra skills in a student-centered classroom, while making sure that these students had the support and knowledge to help them succeed in this and subsequent courses. In this talk, we will discuss resolving this issue, as well as discussing student performance in that course. Examples of student responses to this method, via course evaluations, will be given.

Robert W Vallin Slippery Rock University (robert.vallin@sru.edu)

Presentation Fridays in Advanced Calculus

One issue with many potential users of Inquiry-Based Learning is the desire to not completely convert the class over, but instead make part of the class IBL. In 2011 I inherited the second semester of our Advanced Calculus sequence. Since the students had spent the previous semester in a Lecture/Book/Homework format and I did not want the transition to my style to be too abrupt. So I developed our Presentation Fridays. Monday and Wednesday I lectured, as they were used to, and we followed the text. On Friday the students themselves did the presenting, solving exercises designed to expand their quiver of good examples and extend the topics beyond just what one found in the book. In this talk I will introduce the circumstances of the class, show a wide selection of the problems/examples from class, and share some students comments on Presentation Fridays.

Brian Katz Augustana College (briankatz@augustana.edu)

A Bridge between IBL and Student Inquiry

In order to prepare our students for their Senior Inquiry capstone experiences, my department has added a course, called Tools of Inquiry, in which the students learn to ask and explore their own mathematical questions. In this talk, I will describe the course structures that have become important and effective and discuss the ways that they crossed over from my IBL teaching experience. Teaching this course has also helped me reflect more explicitly on my IBL techniques, so I will conclude by sharing some of the ways that practices have crossed back from Tools of Inquiry to my IBL courses.

Elizabeth Thoren University of California, Santa Barbara (ethoren@math.ucsb.edu)

IBL Classroom Activities Beyond Student Presentation

The IBL classroom activity paradigm is student presentation to the whole class, but there are a wide variety of classroom activities that can support and enhance IBL learning. Many of these activities have students collaborating and communicating in small groups; and keeping this work productive presents some challenges. This talk will focus on some examples of such activities, motivation for including them in an IBL classroom, and suggestions for structuring and managing them successfully.

Erin Moss Millersville University (erin.moss@millersville.edu)

Strategies For Implementing Inquiry-Based Learning in the College Mathematics Classroom

When I first heard of inquiry-based learning (IBL) approaches to mathematics pedagogy, I was completely committed to the idea in theory. To me, it seemed obvious that students who were engaged in problem-solving, discovery, and the construction of mathematical knowledge would understand and retain more. However, I struggled a great deal to infuse IBL into my teaching because of my own experiences as a student in traditional mathematics classrooms and my inability to envision how a different type of teaching might look.

In this talk I discuss pedagogical strategies for integrating principles of IBL into your teaching. In particular, I discuss the matters of task posing and the instructors role in facilitating classroom interactions (including management of small-group work and whole-class discussions). Along the way, I share my experiences teaching mathematics via IBL to prospective teachers. I describe students' initial response to my pedagogy, ways to overcome student resistance, and finally the transformations I witness in my students over the course of each semester. This talk aims to motivate instructors to try IBL, and to provide practical strategies for implementing this in your classroom while getting students on board with your goals for their learning.

Timothy Whittemore University of Michigan (timwhitt@umich.edu)
Vilma Mesa University of Michigan (vmesa@umich.com)

IBL Teachers' Perspectives on Gettting Students to Work Together, Present, and Critique

Students in inquiry-based learning [IBL] courses are asked to work together in small groups, present their work to the class, and critique their peers presentations. These skills are quite different from those expected of students in lecture-based classrooms. We present responses and preferences offered by 28 IBL instructors as to how they motivate their students to take these unusual actions. These instructors ranged from beginners to expert IBL instructors and taught courses ranging from those for future teachers to lower division courses and upper division courses. Six of these instructors have been interviewed twice and offer examples of how these motivations can change with experience and time. The remaining 22 instructors offer a wide range of the experiences instructors can have as they use this method of teaching. The qualitative nature of this study offers a starting point for those teachers concerned with how to teach with IBL methods. By summarizing and analyzing these interviews, we present advice for other IBL instructors and study how these responses develop as instructors become more familiar with teaching with the method.

Christine von Renesse Westfield State University (cvonrenesse@westfield.ma.edu)
Volker Ecke Westfield State University (vecke@westfield.ma.edu)

Asking Good Questions to Promote Inquiry and Mathematical Conversations.

In this interactive workshop participants will be using actual conversations from a classroom to find and discuss different kinds of questions a professor could ask. Good questions promote deeper thinking, clarify student's reasoning, reveal contradictions, or stimulate participation and discussion among students.

Conversations can take place as a whole class, in a smaller group or just between the professor and the student. Each situation requires slightly different skill sets. Is it possible to "classify" the different kinds of questions one can ask, so that we can create a library of questions from which to choose when we want to support mathematical conversations and inquiry?

Our work on the use of questions has grown out of broader effort on promoting student inquiry. At Westfield State University we successfully use inquiry-based materials and techniques to engage students in mathematics. In our project "Discovering the Art of Mathematics" (http://artofmathematics.westfield.ma.edu), we are now developing teacher materials and offering workshops making our best practices explicit through vignettes, videos and reflections on our own teaching.

Cheryll Crowe Eastern Kentucky University (cheryll.crowe@eku.edu)

Teachers Teaching: An Inquiry-Based Approach to Math Education

Two years ago the capstone course for middle grades mathematics education at a university was changed to an inquiry-based class. A key component of the course is now problem solving where pre-service teachers actually teach problems to their classmates at the beginning of class without any feedback from the instructor. Following each class, the teachers reflect on the experience, including the mathematical and teaching, in a journal. Anecdotal evidence as well as student feedback on course surveys indicates this approach has been successful in preparing teachers. This presentation will highlight best practices gained over the past two years from this inquiry-based approach.

Cynthia L. Stenger University of North Alabama (clstenger@una.edu)
James A. Jerkins University of North Alabama (jajerkins@una.edu)

Using Computer Programming to Push Students to Build Mental Frameworks for Abstraction and Generalization

As colleagues in a Mathematics/Computer Science department, we found that many of our undergraduates were not able to participate successfully in the full range of STEM course offerings. In response to this need, we developed a strategy for explicit instruction in mathematical reasoning and computational thinking. Our instructional design is grounded in a theory of mathematical learning that uses computer programming to induce students to build the mental frameworks needed for a wide range of advanced math concepts. In our inquiry-based learning approach, students explore conceptual underpinnings through a series of strategically designed mini programs. As they form conjectures and explore them, they are taught to write general expressions and develop convincing arguments concerning the conjectures. Rather than teach a series of steps to solve a particular problem, we use programming to push students to build mental frameworks that are associated with the problem being explored. In many instances, students have no prior experience with programming and are taught to program in the context of the problem being explored.

We have tested our Instructional Treatment (IT) in professional development for high school math teachers and regional specialists, in high school math classrooms and in our math and CS undergraduate classes. We have recently been awarded a Math/Science Partnership grant through the Alabama State Department of Education in conjunction with the Alabama Math Science and Technology Initiative to support implementation in the high math school classroom. We will share results from classroom implementations both at the college and high school level.

Randall E Cone VMI (conere10@vmi.edu)

Creating an IBL Summer Mathematics Institute

Following a request from a county public school system in Virginia, we have established the week-long 2013 Summer Mathematics Institute (SMI) in IBL for in-service teachers, grades six through twelve. In this paper we carefully examine and discuss: the general appropriateness for inclusion of IBL in mathematics curricula at the secondary level, the challenges faced in creating such a summer mathematics institute in IBL, as well as the successes and shortcomings of the 2013 SMI.

Xinlong Weng University of Bridgeport (gatewaytochina@hotmail.com)

Tile Flooring and Recursive Relation

The class starts with a very simple question such as "How many ways can we tile flooring using tiles avalible at Home Depot?" Students work on simple cases first, then go on until they feel it is too complicated to solve the problem by hand. Well, then it is the time to introduce the concept of "recursive sequence" and follow the method to solve linear recursive relations. This kind of teaching approach really attracts students' attention and interests. As a result, we produced a few student research papers in this area.

Undergraduate Research Activities in Mathematical and Computational Biology

Thursday, August 1, 1:00–3:55 PM, Room 15

Edwin Tecarro University of Houston-Downtown (tecarroe@uhd.edu)
Jeong-Mi Yoon University of Houston-Downtown (yoonj@uhd.edu)
Youn-Sha Chan University of Houston-Downtown (chany@uhd.edu)
Akif Uzman University of Houston-Downtown (uzmana@uhd.edu)

UBM Program at University of Houston-Downtown: Experiences and the Challenge to Sustain It

Since 2007, the University of Houston-Downtown (UHD) has been running an NSF-supported Undergraduates in Biology and Mathematics (UBM) research program that aims to provide students with long-term interdisciplinary research experiences. In this talk, we will highlight the different research projects supported by the program. The program also promoted changes in the undergraduate curriculum. The most prominent among these changes is the development of a two-semester Calculus sequence for Life Sciences that is now required for most Biology majors. Over 50 Biology and Mathematics research students have been supported since inception of the program. These students have made over 50 oral and poster presentations in local and national conferences. The program has initiated and sustained strong interactions between Biology and Mathematics faculty at UHD and has spawned collaborations with Houston-area institutions with graduate programs. It also promoted annual student conferences in Mathematics and Biology that have been attended by undergraduate research students from Texas and Louisiana. As current funding for the program is coming to a close, the program is facing the challenge to sustain itself via efforts to seek other government and private funding and UBM alumni support.

Pam Ryan Truman State University (pjryan@truman.edu)

Undergraduate Mathematical Biology Research at Truman State University

Truman State University in Kirksville, MO is a primarily undergraduate state, liberal arts university with approximately 6,000 students. We have been fortunate to have received two UBM grants from the National Science Foundation for training teams of biology and mathematics students to work together on problems at the interface of Mathematical Biology. I will give an overview of our program and discuss some of the specific projects students have worked on.

Jo Anthony Ellis-Monaghan Saint Michael's College (jellis-monaghan@smcvt.edu)

Graph Theory in DNA Self-Assembly: an Early Entry Point for Interdisciplinary Student Research

For several years we have run an undergraduate research program, both summers and AY, using graph theoretical techniques to provide design strategies for DNA self-assembly. A unique aspect of this program is that we involve students from all STEM disciplines, particularly biology, and not just math majors. A working team frequently includes students from mathematics, computer science, and biology, as the project genuinely needs expertise from these different fields, thus requiring students from these different disciplines to use their backgrounds collaboratively. Furthermore, we accept students quite early in their careers, often the summer between their first and second year, with a side effect that often students choose to pursue minors or double majors as a result of this experience. We will describe how we initiated and maintain this project, our project outcomes, integration of different fields, and how we achieve pragmatically useful results with students who may join the group with very little mathematical background.

Natalie Stanley Dickinson College (stanleyn@dickinson.edu)

Using Bioinformatic Approaches to Predict Gene Expression Based on Promoter Structure in Acute Myeloid Leukemia

Acute Myeloid Leukemia (AML) is a hematopoietic cancer, characterized by the rapid growth of aberrant white blood cells in the bone marrow. We model this cancer using the HL-60 cell line, derived from a patient with AML. Through the addition of a chemical agent (PMA) we are able to force these cells to behave like normal macrophages in culture. As the cells differentiate into this macrophage-like state, there are numerous changes in gene expression. To quantify these changes, we employed gene microarray analysis. The differentially expressed genes were then clustered, based on temporal expression patterns. Upon obtaining the expression clusters, we were interested in understanding the mechanisms involved in transcriptional regulation that allow some genes to behave similarly. To investigate this phenomenon, we are using a Naive Bayes Model to predict expression patterns, based on over-represented subsequences (motifs), in the gene promoters.

Jacob Liddle Houghton College (Jacob.Liddle14@houghton.edu)
Nicholas Fuller Houghton College (Nicholas.Fuller13@houghton.edu)
Junkoo Park Houghton College (jun-koo.park@houghton.edu)

Analysis of refined Gaussian Network Model for HIV-1 protease

Houghton College has established an undergraduate Summer Research Institute which brings together motivated physics, chemistry, biology, math and computer science students into a more intense research environment. In Summer 2013, I and two students majoring in math and physics/computer science will be researching on the analysis of the refined Gaussian Network Model (GNM) for HIV-1 protease, a biologically interesting structure. Proteins are an important class of biomolecules, and they are fundamental research subjects of life sciences. While a protein folds to a unique structure, it makes dynamic fluctuations. These movements may correspond to certain functions and provide valuable insights for drug design. Therefore, studying the structural fluctuation is essential in protein modeling. The structural fluctuations can be analyzed theoretically for a given structure. Elastic Network Model (ENM) has been proposed recently and proved to be reasonably accurate. More recently, the refined Gaussian Network Model (GNM), based on atomic interaction potential, has been proposed and shown relatively more accurate than ENM. The HIV-1 protease plays an essential role in the life cycle of HIV. The essential motions of HIV-1 protease have recently been studied. Accurate predictions on its structural fluctuations may provide great insights into how the dynamics of the structure relate to their functions. The goal of this study is to refine the non-homogeneous Gaussian Network model and apply it to the HIV-1 protease and analyze the results in detail, and further improve our predictions than the refined GNM.

Anita Kummamuri Rao Texas Academy of Math & Science, Denton, TX (anteater44@gmail.com)

A Mathematical Model of Sleep Regulation

We present a physiologically based mathematical model that is fitted to EEG data obtained from sleeping humans. The project goal was to develop a simplified model of sleep regulation which would account for the three main drivers of the human sleep: the circadian rhythm, the homeostatic drive and the ultradian alternation. All-night polysomnographic (PSG) records with no identifying information other than gender were scored by a registered sleep technologist before the data was imported into MATLAB for data reduction. Each of the sleep stages, Wake (W), REM(R) and NREM (N), were characterized by activation or inhibition of different neural populations identified in the Ascending Arousal System of the brain. The mathematical model implemented in MATLAB accounted for the interaction of W, N and R states with their respective neurotransmitters E(norEphinephrine), G(GABA) and A(Acetylcholine. The reduced model showed the expected interaction between wake, NREM and REM and the sleep homeostat. The average durations of the REM bouts was calculated and the duration for each successive REM period increased for first four episodes, while the final episode was reduced.

Ted Theodosopoulos Saint Ann's School (ttheodosopoulos@saintannsny.org)
Patricia Theodosopoulos Saint Ann's School (ptheodosopoulos@saintannsny.org)

A Cognitive Neuroscience Modeling Experiment

We describe a laboratory experiment undertaken by six advanced high schoolers under our supervision, as part of our Math of Life class, in an effort to investigate the interference between attention, memory and consciousness among conflicting tasks. This group project culminated our study of theoretical neuroscience, which began at the physiology of the action potential and synaptic transduction, and proceeded through a series of modelling exercises of neuronal assemblies and their network organization in the cortex. The laboratory experiment involved a series of nine computer-based visual tracking tasks, undertaken by 23 student volunteers. The resulting data analysis highlighted several surprises, from unexpected grouping strategies to lingering learning effects and stronger than expected interference predictability. Aside from its primary goal as a capstone for this segment of our class, this experiment proved to be a uniquely effective way for our students to gain experience in scientific research, from iteratively refining their experimental protocol, to the logistics of performing experiments involving human subjects, and the challenges in separating data analysis and description of results from their more contentious and open-ended interpretation.

Anthony DeLegge Benedictine University (adelegge@ben.edu)

Undergraduate Research in Epidemic Modeling

For the past three summers, I have worked with student researchers on two epidemic modeling projects. These projects involve the construction, analysis, and interpretation of mathematical models for disease spread under given conditions, with the ultimate goal being to see how to suppress, or completely eliminate, the disease from the population. They are great introductions to mathematical research for students because they are interesting and relevant applications as well as models that students do not need to know a large amount of technical biology to work with, so even freshmen who have passed Calculus II can participate.

In this talk, I will describe the two projects I have worked with students on, some of the results we have obtained, and some general lessons I have learned while mentoring students on these projects.

Timothy Lucas Pepperdine University (timothy.lucas@pepperdine.edu)

Undergraduate Research in Modeling the Response of Chaparral Shrubs to Wildfires

This talk will focus on a recent collaboration with an ecologist that has led to an undergraduate modeling project. For the past several years we have been studying how the recent increase in fire frequency in the Santa Monica Mountains (SMM) has drastically impacted the surrounding vegetation. Chaparral, the dominant vegetation in the SMM, can be divided into three life history types according to their response to wildfires. Nonsprouters are killed by fire, but reproduce by seeds that germinate in response to fire cues. Obligate sprouters survive by resprouting because their seeds are destroyed by fire. Facultative sprouters both resprout and reproduce by seeds post-fire. The undergraduate students developed discrete-time models of species survivorship under varying fire frequencies as well as a spatial simulation that models the growth, reproduction and resprouting behavior of individual shrubs that interact in an environment similar to our study site adjacent to Pepperdine University. The students have validated these models using over 27 years of local data on species survivorship, density, growth, rainfall and wildfires. Besides the research itself, I will discuss the recruitment and training of mathematics undergraduates and their contributions to the data collection, statistics, modeling and programming involved in the project.

Research in Mathematics for High School and Community College Students

Friday, August 2, 1:00–2:55 PM, Room 16

Daniel J. Teague NC School of Science and Mathematics (teague@ncssm.edu)

Good Problems are the Key to Building a High School Research Program

Finding good problems for research students is the most difficult and most important task in building a research program. Good problems should have a low threshold of background knowledge, some "low hanging fruit" so students can have some immediate success, yield to a variety of approaches, and offer an array of extensions on which the work can progress. In this session, I will share examples and strategies for finding good problems for high school research.

Gyo Taek Jin Dept. of Mathematical Science, KAIST (jingyotaek@kaist.ac.kr)
Hun Kim Korea Science Academy of KAIST (HunKim@kaist.ac.kr)

Studying Knot Theory with High School Students

Korea Science Academy of KAIST was designed as a school for scientifically gifted students by the Korea Government to nurture creative global scientists. One of the most important education programs is Creative Research Activities, called "R&E (Research and Education)."

Through R&E Program, we have been studying knot theory with high school students since 2003. In each year, students studied basic knot theory and they researched about arc indices, lattice edge numbers, or semi-lattice knots.

In this talk, we will introduce R&E program and mathematical results of our studies.

Shenglan Yuan LaGuardia Community College, CUNY (syuan@lagcc.cuny.edu)

Undergraduate Math Research with Games and Puzzles

Mathematicians deal in proofs. The careful delineation of an argument that is ultimately irrefutable is an absolutely essential part of how mathematic concepts and techniques come to be used, understood, and also built on each other. And yet college students rarely take proof-based math courses. Students at community colleges, in particular, are not exposed to proofs, and they know little of what me might call "mathematical thinking."

Most college math courses are taught in recipe style. That is, students are taught step-by-step procedures for solving problems. Rarely are they encouraged to test out a mathematical idea and prove or disprove its validity. To give interested students some experience with mathematical research, we propose to incorporate some classical games and puzzles that are elaborate or complicated enough to go beyond what is found in the classroom, but that at the same time are limited enough that students can understand the problem without having had advanced math courses. Using topics from areas such as number theory, graph theory, and combinatorics the projects we've developed will give invited students a chance to get their hands dirty with the kind of conceptual experimentation done by mathematicians.

Christine E. Belledin NC School of Science and Mathematics (belledin@ncssm.edu)

Structuring a Research in Mathematics Program for High School or Community College Students

The North Carolina School of Science and Mathematics has offered opportunities for research to advanced high school mathematics students for several years. Students have a variety of research options, including an intensive two-week course, a full semester course, and an extended project with a mentor from a local university. In this session I will share information about how we structure our research program and recruit students, as well as strategies relevant to those interested in developing a research program at their school.

Jean Marie Marie Linhart Texas A&M University (jmlinhart@math.tamu.edu)

Research with Zombies

In this talk, I will tell you how you can take advantage of the allure of the undead and other movie monsters to introduce students to mathematical modeling and the fundamentals of research. Models for an outbreak of zombie infection are done with standard epidemiological modeling techniques, and are accessible to students familiar with differential equations. You can also use this problem as an introduction to discrete stochastic models. It is easy to get students started with access to any computer algebra system such as Mathematica, Matlab or Maple. Students have fun talking about zombies while learning mathematics with real biological applications.

Victor Piercey Ferris State University (piercev1@ferris.edu)

Using the Gini Coefficient as a Research Project in Precalculus

The Gini Coefficient, an index that measures income inequality, offers an opportunity to conduct research to students in Precalculus using fairly elementary mathematics. In addition to being a research opportunity, the computations involve some basic Riemann sums. This offers students a chance to develop intuition about integration as part of their preparation for calculus.

The Mathematics of Planet Earth in Research

Friday, August 2, 1:00–2:15 PM, Room 15

Roger William Johnson South Dakota School of Mines & Technology (roger.johnson@sdsmt.edu)

Modeling the Size of Raindrops

Knowledge of raindrop size distribution is useful in a number of fields including meteorology (weather prediction models), hydrology (total volume of water falling), agriculture (soil erosion) and telecommunications (signal loss). In many cases raindrop size may be well-modeled by a two-parameter gamma distribution for which a method of moments estimation procedure has been traditionally used. Unfortunately, while these estimates are simple to implement, they are subject to substantial bias and large variability. Instead, we consider the use of other estimation procedures such as maximum likelihood to produce parameter estimates. Because disdrometers, the instruments that record drop size, may fail to record raindrop sizes outside a given range of values (truncation) or may only record counts of drops within specified bins (binning) some care needs to be taken to appropriately adjust the estimation procedures in these cases. Among other results, we show the improvement in performance of maximum likelihood over method of moments for disdrometers which may truncate and/or bin raindrop size.

David Coulliette Asbury University (david.coulliette@asbury.edu)
Kenneth Rietz Asbury University (kenneth.rietz@gmail.com)

Rate-Limited Sorption Modeling in Contaminant Transport

Computational models of contaminant transport are used regularly for designing subsurface environmental remediation systems. These models predict the movement of the contaminant ‘plume’ through a porous media containing groundwater. In many soils, the contaminant sorbs to the solid matrix in the porous media. As a result, the rate at which this contaminant may be removed by traditional pump-and-treat flushing is much slower than that of the contaminant in the fluid portion of the media. This phenomenon is called rate-limited sorption (RLS) and it is particularly problematic in cases where the contaminant has been in place for a long period of time. Although RLS has been noted in the academic literature for years, production models used for field work have failed to incorporate the issue. This work presents preliminary results of an attempt to model RLS in a production contaminant transport code.

Christina Selby Rose-Hulman Institute of Technology (selby@rose-hulman.edu)

Using Photometric Instruments to Observe and Model the South Atlantic Anomaly

Society relies on satellite technology for a wide variety of purposes. GPS navigation, cellular telephone communication, and television broadcasting are just a few of the technologies many use daily. Therefore, engineering satellites that properly function in the extremely harsh environment outside of Earth’s protective atmosphere is very important. In particular, radiation can cause a great amount of damage to a satellite. The South Atlantic Anomaly (SAA) is a region where the Earth’s inner Van Allen radiation belt is closest to the surface of Earth. Satellites passing through this region are exposed to higher than average amounts of radiation. The intensity of the South Atlantic Anomaly is not constant, and its location and size are also variable. It is of interest to study these changes in order to understand the impacts of the SAA on satellite functionality, as well as gain a greater understanding of our planet. A spherical harmonics model has been developed to characterize the SAA in low earth orbit using data from photometric instruments. This presentation will discuss the development of this model.

Amir Y. Ahmadi Purdue University - Agricultural Economics (ahmadia@purdue.edu)
Xin Zhao Purdue University - Agricultural Economics (zhao269@purdue.edu)
Daniel Ghambi Purdue University - Agricultural Economics (dghambi@purdue.edu)

The Impact of Temperature on Chinese Coal Demand

China’s domestic coal consumption has more than doubled over the past decade. Over the same period, China experienced high and low temperature extremes. This study applies an econometric model to empirically investigate the relation between Chinas coal demand and temperature values at the provincial-level for years 2001-2006. Results provide estimation of the effect of temperature variation on coal demand across China. This may assist future energy policy respond to climate change.

The Mathematics of Planet Earth in the College Mathematics Curriculum

Friday, August 2, 3:05–3:55 PM, Room 15

Jessica M. Libertini University of Rhode Island (jessica@jhu.edu)

Motivating a Gen-Ed Math Modeling Course with Food Policy Issues - A Follow-up Report

At the JMM, we presented the process of developing a new general education math modeling course centered completely around issues of food policy and sustainability. Using mathematical tools such as discrete dynamical systems, curve fitting, and network analysis, our students explored questions ranging from the sustainability of the tuna population to the robustness of the university's food distribution network. In this talk, we will present an overview of the course content and its unique delivery, as well as the surprising student transformations we observed.

Carrie Elizabeth Diaz Eaton Unity College (ceaton@unity.edu)

Exploring the conversion of alternative energy

As we explore harnessing of alternative and renewable energies for power production, cost-benefit and power capacity must be well understood. In general terms, power curves represent the conversion of incoming alternative sources (*e.g.*, sunlight angle or wind velocity), by a specific product (*e.g.*, wind turbine) to power. Many manufacturers of energy generation equipment include power curve data and/or functions in their sales documentation. To explore the best alternative energy options for a given application, students can collect data, locally or published, as inputs for these power generation curves. This data is typically represented as a function of time or as a distribution of input which motivates a variety of approaches to calculating cumulative power to meet application requirements. Such projects would therefore be applicable to a variety of courses, including applied calculus, probability and statistics, or introductory modeling. I will introduce resources for a guided project using wind energy data collected in Unity, Maine and also discuss alternatives for more open-ended research projects.

Marc Laforest Ecole Polytechnique de Montreal (marc.laforest@polymtl.ca)

Hurricanes: Engines of Destruction

We describe two physical processes hidden behind the Hurricanes using only linear algebra and several variable differential calculus. The presentation begins with a description of the force of Coriolis. This force induces a rotation of the winds centered around an isolated tropical depression, thus explaining in part of the particular circular structure of hurricanes. Secondly, we present a model of Bister and Emanuel describing the thermodynamics of energy use within a hurricane. Based on an analogy with the Carnot engine, we deduce an explicit relation between the wind speed in a Hurricane and the temperature differential at the ocean surface.

Recreational Mathematics: New Problems and New Solutions

Part 1: Thursday, August 1, 1:00–4:15 PM, Room 10

Colm Mulcahy Spelman College (colm@spelman.edu)

Fitch Cheney's Five Card Trick for Four or Three Cards

William Fitch Cheney, Jnr, served many years in the Department of Mathematics at the University of Hartford, CT. His classic Five Card Trick from the late 1940s is a masterpiece of mathematical communication. Aodh gives a deck of cards to a spectator, asking for five randomly chosen cards. Aodh glances at the these and hands one back to the spectator, who hides it. Aodh places the remaining four cards in a face-up row on the table. Bea, who has witnessed nothing prior to this display, enters the room, glances at the cards on the table, and promptly reveals the identity of the hidden card. A common variation of this is to decks of size greater than 52.

We take a different approach, and suggest ways to pull off the "Given any n cards from a deck, have one hidden and reveal its identity with the help of the rest" effect for $n = 4$ and even for $n = 3$.

Robert W Vallin Slippery Rock University (robert.vallin@sru.edu)

Continued Fractions from a Magic Trick, a preliminary report

In 1958 a mathematics major at UCLA named Norman Gilbreath published a new card trick based on what is now known as the Gilbreath Principle. The generalized form of the principle is related to a particular rearrangement of the numbers $\{1, 2, 3, \ldots, N\}$ called a Gilbreath Permutation. This leads in a natural way to a finite simple continued fraction in the unit interval. We look at the set of these continued fractions and two different ways to turn this finite continued fraction into an infinite one and their place in $[0, 1]$.

Duk-Hyung Lee Asbury University (duk.lee@asbury.edu)

Pop-Guitar-Music and Mathematics

When a music class talks about a golden ratio, it usually refers to where the climax occurs in a classical music piece; If a music consists of 1000 measures, its climax may hit somewhere around the 620th measure. This talk will explore if the golden ratio appears in pop and guitar music, including other connections to non-classical music and math.

Kevin Ferland Bloomsburg University (kferland@bloomu.edu)

The Easiest Possible NY Times Crossword Puzzle

What is the maximum number of clues possible for a 15×15 daily NY Times crossword puzzle? We find this number as well as all possible grids achieving this maximum. A puzzle on such a grid was submitted to the NY Times, but it was considered too easy for publication there.

Michael Mulligan PuzzMill (mike@puzzmill.com)

Thinking Outside of the Box: The Mathematics of Swirldoku

Sudoku puzzles offer many opportunities for exploring mathematical concepts such as symmetry, equivalence, and enumeration, to name a few. A new variation, Swirldoku, opens up additional avenues for exploration. In contrast with the standard 81-cell grid, these puzzles can be constructed with 27*N cells, where N is any positive integer. The rules are the same but the grids are different, with rows and columns intersecting on spiral arcs. This talk explores various aspects of these new puzzles, from their construction, to counting the number of distinct solutions, to open problems such as the minimum number of clues necessary to guarantee a unique solution.

John Perry University of Southern Mississippi (john.perry@usm.edu)

Nim$^\infty$

When teaching modern algebra, I begin with a simple game that illustrates some of my favorite algebraic properties. This talk describes the game, how it generalizes Nim in a new way, and how a slightly harder variant works as a “solitaire Nim” that illustrates one of the most important algorithms of the latter 20th century.

Blane Hollingsworth Middle Georgia State College (blanehollingsworth@gmail.com)

Utilizing Information “Perfectly” in a Logic Puzzle

We first pose and solve an innocuous recreational logic problem (involving “perfectly rational” characters who deduce what color balls they have in a mislabled box). We then argue that there is some subtle issue with the behavior of the “perfectly rational” players; we find that it generates lots of interesting complications. In particular, we find that very little information was necessary, which hopefully helps us reflect on the nature of properly utilizing what we know to solve problems.

Jeff Johannes SUNY Geneseo (johannes@geneseo.edu)

Tinkering with a Mathematical Goldmine

While pursuing a renewed passion for the old standby of Rubik’s cubes, I stumbled across the sequel of Rubik’s magic. And, then in searching for information on-line I discovered a vast resource. Jürgen Köller has assembled an immense collection of pages in recreational mathematics entitled “Mathematische Basteleien” — mathematical tinkering. In the talk we will overview this collection of 234 separate pages of recreational mathematics ranging from paperfolding to geometric puzzles, from number games to four-dimensional puzzles. We will also examine a couple of the pages in detail, and leave the audience with the invitation to explore this amazing site for themselves.

Eleanor Farrington Massachusetts Maritime Academy (efarrington@maritime.edu)

Parametic Equations Go to the Circus: Trochoids in Poi Flower Patterns

Poi spinning is a performance art, related to juggling, involving two weights on the ends of short chains, which are swung making visually interesting patterns. In this talk we will be concentrating on a certain class of technical moves for poi, where the patterns created are centered trochoids, which are closely related to the cycloid. Like all curves in the cycloid family, they are best expressed using parametric equations. Based on the calculus of the curves, we can easily determine that there are just a few places in these patterns where one pattern can be smoothly transformed into another. We will classify the patterns of these types performable with two poi using combinatorics, and the possible smooth transitions between such patterns.

Mike Pinter Belmont University (mike.pinter@belmont.edu)

Randomly generating a Dekaaz poetry form

Rachel Bagby developed a new poetic form called Dekaaz. Dekaaz uses a 3-line format, with the first line containing exactly two syllables, the second line containing exactly three syllables, and the third line containing exactly five syllables (compared with the haiku 3-line form, which requires five, seven and five syllables, respectively). We will discuss two different ways to count the number of possible Dekaaz variations, including one using a binary framework, and then we will randomly generate a few of the Dekaaz variations and try our hands at writing (as time permits).

Recreational Mathematics: New Problems and New Solutions

Part 2: Friday, August 2, 8:30–10:45 AM, Room 16

Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

A Brief Study of Abundant Numbers Not Divisible by Any of the First n Primes

Positive integers are classified as either abundant, deficient or perfect. An abundant number is a positive integer such that the sum of its divisors exceeds twice the number. For example, 12 is the initial abundant number while 945 is the first odd abundant number. We consider the ratio of the sum of the divisors of an integer to the integer itself known as the abundancy of the integer. When this ratio is an integer, we classify such integers as multiply perfect. For example, the perfect numbers 6 and 28 are 2-perfect while the integers 120 and 30240 are 3-perfect and 4-perfect respectively. With the aid of MATHEMATICA, we secure the initial square-free abundant numbers not divisible by each of the first one hundred primes and the initial abundant numbers not divisble by the each of the first several primes which is a much more difficult problem.

Roger Bilisoly Central Connecticut State University (bilisolyr@ccsu.edu)

Discovery of Unusual Patterns of Squares Modulo an Odd Prime

Analogous to integer squares, the same definition can be used modulo p, an odd prime. However, unlike Z, exactly half of the non-zero elements of Z_p are squares. This talk explores what patterns of squares and non-squares are possible and sets out to find examples of these where feasible. For example, starting at 1, how many squares in a row can there be? Using quadratic reciprocity and the Chinese remainder theorem, we will show how to find primes with as many initial squares as desired. For instance, modulo 795228332459531065284600120787338361, all the numbers from 1 through 100 are squares. This prime is larger than 2^{100}, and if squares were random, their probability would be 1/2, so it is not surprising that a run of 100 can be found for such a large number. Another example comes from statistics: the runs test is used to check if binary data is random or not, which can be applied to squares by counting the number of transitions, call this n, from a square to a non-square or vice versa. For random data of a fixed length, n has an approximate normal distribution, but for Z_p, $n = (p-1)/2$ exactly. We will see in this talk that the pattern of squares in Z_p has both random and deterministic aspects, which will be illustrated with some interesting numerical examples.

Aihua Li Montclair State University (lia@mail.montclair.edu)

New Roles of an Old Puzzle: the Magic Square Problem

The mystery, historical, and entertaining feature of the magic square puzzle has amazed and challenged many people – from all ages and with various background. It is also a great topic to be integrated in undergraduate classrooms and to be used for enriched activities outside the classroom. A more difficult related problem is the construction of magic squares of squares (all entries are perfect squares) and the existence of such squares with all entries distinct (still open). These puzzle-like problems always attract students' attention quickly. When teaching number theory and abstract algebra, the author used the magic squares of squares over certain finite fields to help students' understanding of the abstract concepts and to draw students attention. The concepts involved include modular arithmetic, quadratic residue, Legendre symbol, group structure of the finite field, polynomial equations over finite fields, and many others. Students love to build magic squares of squares and to identify/test properties of the squares using the knowledge they have just learned. The author also developed student research projects from the unsolved open questions on magic squares of squares.

Ethan Brown Phillips Academy Andover (ethan@ethanmath.com)

Solving the World's Hardest Magic Square

People around the world look to the magic square (a 4x4 grid with all rows and columns summing to the same number) as a mathematical challenge that is both intellectual and entertaining. Many performers restrict the spectators freedom to the choice of just one grand total number, giving them the opportunity to fix squares in advance. The speaker will demonstrate and explain how to quickly create a 4-by-4 magic square when the audience gets to choose the total as well as any three numbers to put in any three squares.

Thomas Q Sibley St. John's University (tsibley@csbsju.edu)

Getting Hyper from Painting Cubes

In a children's puzzle, each of 8 small cubes is painted one of four colors. The cubes are stacked to make a bigger cube. The goal is to arrange the colored cubes so that each face of the larger cube has all four colors. One known generalization uses n^3 cubes and n^2 colors and includes interior planes. What happens when we also increase the number of dimensions?

Bruce Torrence Randolph-Macon College (btorrenc@rmc.edu)

Rubber Sheet Photography

Advances in digital photography have made it possible to stitch together a seamless panorama from several overlapping photographs. Taking this idea to its extreme (something that comes naturally to mathematicians), one can create a full "spherical" panorama of a scene: A sphere with imagery painted on it showing the scene in every direction from a single vantage point—the center of the sphere. The mathematical photographer can now apply a mathematical projection to the spherical image to produce a flat photograph with an impossibly wide field of view, or replicate the visual experiments of M.C. Escher, or pioneer new ways of seeing.

In this talk I will explain the basic mathematics, as well as provide some practical advice, for creating such photographs. Photos from several well-known mathematical photographers, as well as some of my own, will be shown.

David Jacob Wildstrom University of Louisville (dwildstr@erdos.math.louisville.edu)

Classification of Polyominoes by Spinal Character

For certain construction or design methodologies, it can be useful to build physical models of mathematical objects via accretion of material around a central "spine". This work will discuss how such constructions can be engineered for polyominoes, specifically raising questions relevant to the enumeration of certain classes of polyominoes and finding minimal polyomino examples requiring certain levels of spinal complexity.

Curriculum Development to support first year mathematics students

Part 1: Saturday, August 3, 8:30–11:25 AM, Room 14

James Anthony Mendoza Epperson The University of Texas at Arlington (epperson@uta.edu)
Julie Marie Skinner Sutton The University of Texas at Arlington (julie.sutton@mavs.uta.edu)

Effectively Supporting First-year Students in Precalculus and Calculus Via the Arlington-Emerging Scholars Program

In a recent study conducted by the Mathematical Association of America in the United States, participating college and universities reported an average 27% D-F-withdrawal rate in calculus (Bressoud, et al., 2012). As part of a Science Talent Expansion Program (STEP) project (NSF DUE #0856796) partially funded by the National Science Foundation in the United States, we adapt the Emerging Scholars or Treisman-style Program model as an intervention program to reach a broad population of students at risk of failure in precalculus or calculus. The Arlington-Emerging Scholars Program (A-ESP) incorporates increased class time, cooperative learning, and challenging work in addition to regular lectures and labs. Data collected over several semesters shows higher pass rates for the A-ESP students when compared both to non-A-ESP students in the same lecture sections and also to those in other sections. Findings suggest that A-ESP students with no prior exposure to high school calculus perform at levels comparable, if not better than, their counterparts and that A-ESP structure may be most effective for students who were initially unsuccessful in their first college calculus course.

Fei Xue University of Hartford (xue@hartford.edu)

Flipping Calculus: A departmental project of the University of Hartford

In this session we will introduce the departmental project of flipping Calculus I at the University of Hartford in which the primary delivery of content is moved outside of class via videos and the homework such as problem sets is shifted into the classroom. Some technologies that are used in the project, including videos, iPads, and WeBWork, will be introduced. Brief statistical results and lessons we learned from our one year exploratory study and assessment will also be discussed.

Brandy Wiegers National Association of Math Circles (brandy@msri.org)
Addie Evans San Francisco State University, CSME (adde@sfsu.edu)
Emiliano Gomez University of California, Berkeley (emgomez@berkeley.edu)

Math Workshop for Accelerated Pathway to Calculus

The AMG Engineering Pathway brought together a cohort of first year community college students to study intermediate algebra, geometry, advanced algebra, and trigonometry as one set of courses over one year (rather than the more typical 2–3 year range of coursework). This cohort model was successful through a combination of joint faculty planning, student service support, engineering and other STEM faculty support, and bi-weekly math content workshops that discussed broader mathematical problem solving topics. This workshop provided the glue to bring together the sets of coursework while also creating foundations for the students to be prepared for success in Calculus. Topics included scientific notation, working with big numbers, understanding the bigger picture of linear relationships, and applied mathematics applications. Results of the project will be discussed in the context of the cohort pre- and post- survey that evaluated student mathematical background and mathematical attitudes.

Alison Ahlgren-Reddy University of Illinois (aahlgren@illinois.edu)
Marc Harper UCLA (marcharper@ucla.edu)

Precalculus Redesign: The Influence of a Placement Program and the Power of a Name

The University of Illinois redesigned its Precalculus course using information from its placement program and the name was changed to "Preparation for Calculus" to accurately represent the purpose of the course. Entry and exit student data will be shown. The data indicates the mathematical strengths and weakness of the students entering and exiting the Precalculus course. The entry data was used to tweak curricular emphasis in the course. For example students enter the course with a strong foundation in polynomial functions and a weak foundation in exponential and logarithmic functions. Further, the data is compared to student data of those who enter Calculus I directly their first year. We are able to show that the redesigned course is preparing students to compete with those who enter Calculus I directly. The name was changed to address campus wide misconceptions of the course; many students thought that Precalculus was a review of all mathematics prior to calculus and not a course within itself and thus it was viewed as a review and terminal course.

Caitlin Phifer University of Rhode Island (caitlin@math.uri.edu)
Jessica M. Libertini University of Rhode Island (jessica@jhu.edu)

The Precalculus Competency Exam: A Remediation Program for Calculus

As with many institutions, URI's math program is faced with the challenge of teaching mathematics to students who lack the necessary background to succeed. In addition, each student comes to class with individual strengths and weaknesses, minimizing the efficacy and desirability of course-wide co-remediation. In an effort to help students and instructors identify and target those individual weaknesses, URI's Calculus I course has piloted a competency-based precalculus remediation program. The goals of this program are to help students see the value of the prerequisite material and to offer learning and assessment opportunities to demonstrate proficiency. We will present the early evolution of the program, data on its efficacy, and our ideas for further development.

Jill Jordan Houghton College (jill.jordan@houghton.edu)

Variations on the Theme of Calculus Support

In an attempt to increase student success in Calculus I over the past three semesters, our department has implemented several versions of a "calculus support lab." In this talk, I will describe each implementation and explain the rationale behind it. Then, using instructor impressions, student evaluations, and student performance as evidence, I will explore the strengths and weaknesses of each approach, and suggest ways to maximize the efficacy of calculus support labs.

Aminul Huq University of Minnesota Rochester (ahuq@umn.edu)

Developing an Integrated Mathematics Curriculum in a Health Sciences Program.

The Center for Learning Innovation (CLI) at the University of Minnesota Rochester is a novel interdisciplinary center that started with the inauguration of the campus in 2009 with just two degree programs, namely, Bachelor of Science in Health Science(BSHS) and Bachelor of Science in Health Profession(BSHP). Developing an integrated mathematics program in an interdisciplinary center like this poses its own challenges. All students are required to take three math courses for their degree requirement and two of them are in their first year starting with an introductory statistics course. In this talk I'll talk about our attempt to design a mathematics curriculum focused toward students in health sciences, the challenges we are facing and future development plans to meet the vision of CLI to foster education that is evidence based, assessment driven, technology enhanced, writing enriched, learner centered, concept based and integrated across discipline.

Melinda Schulteis Concordia University, Irvine (melinda.schulteis@cui.edu)

Great Ideas in Mathematics and Interdisciplinary Connections Restructuring Core Content to Engage and Retain Students

In 2010, we were forced to reorganize the content of our introductory math course what resulted was a new course that exceeded our expectations. It is a class that is extremely satisfying to teach, it engages students both majors and non-majors in an exciting new way, and it has contributed to increased retention rates at the university. Three years ago, Concordia University Irvine instituted a new Core Curriculum program. Our course, Nature of Mathematics, became a required course for every incoming freshman. The course is paired with a philosophy course, where a learning community of students usually takes the classes back to back on the same day with occasional jointly led lectures. A focus of the overall Core is to equip students to think critically, to develop intellectual habits of mind, and to explore interdisciplinary connections while reading Great Works across disciplines. Within the math class, we restructured our curriculum to tackle influential books (Euclid's Elements, Fibonacci's Liber Abaci), great ideas and theorems (Cantor's Theorem on sizes of infinity, the Incompleteness Theorem, Non-Euclidean geometries, Arrow's Impossibility Theorem), and connections with philosophy (Plato's views on mathematics, Pythagorean influences, Pascal's Wager). The class is enjoyable to teach with its focus on some of the greatest ideas in mathematics and how it relates to the world around us. This talk will cover basic features of the class, describe an accompanying Oxford model tutorial system with specialized Core Peer Tutors, and provide data to support positive student perceptions of the class/mathematics and increased retention.

Jacqueline Dewar Loyola Marymount University (jdewar@lmu.edu)
Suzanne Larson Loyola Marymount University (slarson@lmu.edu)
Thomas Zachariah Loyola Marymount University (tzachari@lmu.edu)

Increasing Math Majors' Skills, Confidence, Community and Retention with a 1st Year Course

Two decades ago the Loyola Marymount University (LMU) mathematics department, motivated by the desire to increase retention, initiated a year-long course for beginning mathematics majors. It aimed to improve problem solving and writing skills, increase students' confidence, provide information about career opportunities and the relevance of mathematics to society, and create a community of scholars. Assessment data indicated the 1992–93 pilot was a success and the course continues to play a significant role in the major curriculum to this day. Other LMU STEM departments have since developed similar courses. We will describe the course structure and learning outcomes and present data on retention, skill development and student/faculty perceptions of the course.

Curriculum Development to support first year mathematics students

Part 2: Saturday, August 3, 1:00–4:55 PM, Room 14

Brian Hollenbeck Emporia State University (bhollenb@emporia.edu)

A Re-redesign of College Algebra: Maximizing Flexibility and Consistency

About ten years ago, Emporia State University replaced traditional instruction for College Algebra with a computer-assisted format using online homework and video lectures. Although this had the desired effect of achieving consistency across the various sections, student feedback was generally negative. After surveying different approaches used at other universities in the region, a new redesign of College Algebra was implemented in 2010. Shifting from a "Coordinator" model to a "Mentor" model allowed for multiple formats to be offered each semester, thus offering more flexibility for both student and instructor, while ensuring the desired level of consistency. We will discuss the details of these changes as well as evaluate the success of such an approach.

Malissa Peery University of Tennessee (mpeery@utk.edu)
Jennifer Fowler University of Tennessee (jcfowler@utk.edu)
Charles Collins University of Tennessee (collins@utk.edu)

College Algebra Delivered Online: An Autopsy of an Unsuccessful Initiative

In this presentation we give a description of our online College Algebra course, the influences for choosing this method of delivery, and the outcomes. Statistics comparing different methods of delivery will be presented and possible reasons for the outcomes will be discussed. What went wrong, and what did we learn that we can use to make improvements for the future?

Suzanne Ingrid Doree Augsburg College, Minneapolis (doree@augsburg.edu)

Just Enough Algebra — A Successful Approach to Preparing College Students.

Too often students placed in developmental mathematics courses in the first year have a negative experience and question continuing in college. In 1994, my colleagues at Augsburg College and I had a vision for a new course to replace our intermediate algebra course to turn that trend around. We wanted a college level course that would serve primarily as preparation for quantitative courses across the curriculum. The framing question that led to our curricular adventure of the past nearly two decades was: What algebra do college students need to know, and how can we make it relevant to their future studies, their lives as citizens, and their everyday life? From these questions "Just Enough Algebra" was born. We are pleased with our high pass rates and student success in subsequent courses.

Everything we do in the course is set in some applied context. Our choice to focus primarily on linear and exponential models; to emphasize verbal, numerical, and graphical interpretation of functions; and to include only the most essential symbolic techniques align well with curricular guides from the MAA and AMATYC. More importantly, it works. Students learn a lot in this course. They are ready for what comes next. And, they enjoy it.

In this talk I will outline the process used for constructing the course; describe the course curriculum, pedagogy, and supporting instructional materials; and show a couple of examples to illustrate the approach.

Daniel Cole SUNY Maritime College (dcole@sunymaritime.edu)

Realigning A Service Mathematics Curriculum To Better Serve The Major Department

SUNY Maritime is a four-year college with an emphasis on preparing students to work in the maritime industry. Its mathematics faculty functions entirely as a service department, with the majority of its courses focused on preparing students for engineering majors. Recently the mathematics faculty have been working with the Engineering Department to streamline remedial coursework and realign the calculus sequence to better prepare students for the introductory engineering courses. The ultimate goal of this project is to help reduce average graduation time for Maritime's engineering majors, the majority of whom take an unusually heavy course load due to Coast Guard licensing requirements. Current and future efforts to meet these goals and preliminary results from our curriculum realignment will be discussed.

Darry Andrews The Ohio State University (dandrews@math.osu.edu)
Elizabeth Miller The Ohio State University (elizmiller@math.osu.edu)

Supporting Large-First Year Courses with a Mathematics and Statistics Learning Center

This talk will explain how the Ohio State University has supported and enhanced its first-year, large-scale courses from developmental math to calculus with a Mathematics and Statistics Learning Center (MSLC). The MSLC consists of five full-time staff members, almost 300 undergraduate and graduate tutors, and averages about 50,000 student tutoring hours a year. The MSLC also works closely with course coordinators, provides supplemental workshops and exam reviews, and supports the department's effort to utilize instructional technology in large courses. This talk will talk about how the MSLC was founded, how it is set up, how it has expanded overtime, and some current initiatives the MSLC is involved in.

Karen Santoro Central Connecticut State University (santoroka@ccsu.edu)

Uniting to Support First-Year Success: A Collaboration between State Universities in Connecticut

Developmental mathematics courses are often considered 'gate-keepers' that are obstacles to college graduation and retention. In this talk I will present the coordinated efforts that began this year between representatives from all four of Connecticut's state universities (CSUs), through a College Access Challenge Grant, to address this issue. I will discuss how we incorporated the CCSS as we aligned our algebra curricula, I will describe our long-term vision for providing our students with free, shared, online support tailored to the common CSU algebra outcomes, and I will share a sample of the curricular packages we are developing.

Stepan Paul UC Santa Barbara (spaul@math.ucsb.edu)
Michael Yoshizawa UC Santa Barbara (myoshi@math.ucsb.edu)

Creating an Online Math Lab

In order to cope with ballooning class sizes at UC Santa Barbara, our department received an Instructional Improvement Program grant to develop an Online Math Lab to support our lower-division classes. We built a website containing a database of pre-screened links to existing online videos, textbooks, and interactive applications. The links are organized by class and topic, and we also include resources for learning pre-requisite material for students needing extra help. To bring students to the site, we have integrated links to the database into every WeBWorK homework problem. For example, an online calculus problem on "product rule" will display at the bottom a link to a list of resources on product rule including sites like Khan Academy and Paul's Notes. The website was made available to all lower division classes this spring, and we are in the process of evaluating student use and the site's perceived effectiveness as a resource for students.

Donna Flint South Dakota State University (donna.flint@sdstate.edu)
Charles Wesley Bingen South Dakota State University (charlesw.bingen@gmail.com)

Implementing a Mastery-Based Format for Remedial Mathematics Courses—an Iterative Approach

Over the last 15 years South Dakota State University has experienced a significant increase in the number of students who are required to take remedial Mathematics courses followed closely on its heels by a growing failure rate in both remedial and early credit bearing math courses. Therefore, we decided to investigate a new method of teaching and learning remedial mathematics with the goal of increasing the success rate in our courses. We decided to implement a mastery-based, emporium style format for our remedial mathematics course. In preparation, we participated in several workshops where campuses eagerly reported great success and student enthusiasm for this style of learning, supported with powerful data demonstrating drastically improved students success. With great excitement we set out to create a course of our own. We will tell you about our path to "success" along with all the things we wished someone had told us before we started.

Awilda Delgado Broward College (adelga1@broward.edu)

Improved Success Rates in Developmental Math through Acceleration, Collaboration, and Technology

Since Fall 2009, Broward College has offered Math Redesign courses through 8-week sessions allowing students to complete two sequential math courses in a single semester. Not only can math redesign students complete their remedial coursework faster, but they also complete their college-level mathematics courses with greater success.

Collaborative learning is an integral component of our math redesign efforts. In redesigned sections, instructors begin class with video mini-lectures then facilitate in-class group learning worksheets that allow students to immediately apply what they learned. These are collected at the end of class, graded, and returned the following day to provide the students with immediate feedback. This higher level of engagement allows instructors to quickly identify individual student needs and intervene with appropriate strategies.

Computer advances have made mastery learning obtainable to today's learner through interactive, adaptive, cognitive, artificially intelligent programs such as ALEKS (Assessment and LEarning in Knowledge Spaces). ALEKS continuously makes adjustments to each student's learning pathway from the time the student completes an initial assessment until the student reaches the goal completion assessment which cumulatively assesses mastery of all course topics.

The Math Redesign initiative has expanded from 17 sections in its first semester to more than 100 sections in the Spring of 2013 in Developmental Math I and II and Intermediate Algebra. Students in Math Redesign courses have achieved significantly higher success rates (as much as 18% higher) and significantly lower attrition rates (as much as 17% lower) in the redesigned classes when compared to traditional sections.

Mary D Shepherd Northwest Missouri State University (msheprd@nwmissouri.edu)

Math Skills, An Emporium Model Modified: What We learned from the Pilot Year

During the Fall 2011 trimester, the Mathematics department began a pilot implementation of the newly redesigned Math Skills course. Students with less that a 21 Math ACT subscore were advised to take the course, but the course was not required. The course was redesigned from a lecture course into a course involving 10 modules that the students could complete somewhat at their own speed. Many students took the course but completed fewer than 10 modules during the pilot period. This is a report on these "pilot" students as to their "success" in their first college credit bearing math course and on the students who chose not to take the redesigned Math Skills course, their level of success also in a first college credit bearing math course. In addition there will be discussion of some of the many tweaks that have occurred during this first full year of implementation with mandatory placement.

Kerry Luse Trinity Washington University (lusek@trinitydc.edu)
Joseph Sheridan Trinity Washington University (Sheridanj@trinitydc.edu)

Serving the Under-Resourced Student in a University setting through Mathematics

Statistics show that open-access schools enroll many students with little or no numerical competency. A report referenced in the President's State of the Union address on January 25, 2012 states that in "many minority institutions of higher education, 80-95% of entering freshman are required to take remedial math, reading, and English. Even in these courses, the dropout rate is estimated to be around 70-80%." Studies have shown that of the students who place into pre-foundational math courses nearly 75% either will not pass the pre-foundational course (or sequence) or, if they do pass, will fail at the next sequential credit-bearing course. At Trinity, not only did we completely redesign our pre-foundational courses, but we also limited the pre-foundational sequence to one semester, added a one-credit student-centered supplemental laboratory, incorporated a rigorous adaptation of MyMathLab pedagogy in conjunction with Visual, Auditory and Kinesthetic (VAK) styled classroom lectures, and raised the standards of the course. The curriculum redesign objectives were to (1) maintain math course content integrity, (2) develop students' self-efficacy, and (3) improve retention. Results after several semesters have shown that, for students who complete the course, the pass rate is 80% and that pre-foundational math students' retention rate is 84%. Furthermore, for students who successfully complete the pre-foundational math course and move on to the sequential credit-bearing course, the pass rate is now approaching 80%. Our presentation will include a description of our standardized first year math courses, how we implemented them, and data collected over the last few semesters.

Stephen Hardin Fast Limestone College (sfast@limestone.edu)

Year One Results from Developmental Course Redesign

This paper presents first year data on students' learning outcomes using redesigned developmental courses incorporating a learning software system. The data come from pretest/posttest scores measuring basic number sense, arithmetic, and algebraic skills. The setting is a small liberal arts college with a significant percentage of incoming students needing developmental mathematics. Formats include onsite and online classes. Discussion of the process of redesign and faculty adjustment is also included.

Math Circles: Best Practices

Thursday, August 1, 1:00–4:15 PM, Room 17

James Tanton MAA (tanton.math@gmail.com)

A Sampler of Math Circle Problems

Finding a topic to provide multiple hours of mathematical intrigue and exploration for students is the continual challenge for math circle instructors. In this talk we outline a sampler of delightful tidbits that can inspire rich challenge and mystery. Devise a divisibility rule for the number seven in base one-and-a-half. When do coloring schemes of checkerboards exist that match given lopsided motions? Which cells in checkerboard regions are possible "cheat spots" to make impossible tilings almost possible? What properties are to be discovered in a "ring" version of Pascal's triangle? And more! We'll explore the issue of learning how to find and create interesting Math Circle topics on your own.

Cheryll Crowe Eastern Kentucky University (cheryll.crowe@eku.edu)

Math (Circles) Magic!

The Eastern Kentucky Math Teachers' Circle was established in 2011. One of our most successful and interesting circle meetings was a night of math magic. The group explored the "magic" behind several algebra problems that are common in middle grades. In this presentation, participants will learn about the problems addressed during the meeting and the creative methods for engaging teachers in the activities.

Elgin Johnston Iowa State University (ehjohnst@iastate.edu)

Two Circle Projects

A brief discussion of two Math Circle Projects I have used, both inspired by articles in MAA journals. One project is centered around the existence and construction of Golomb rules. The other will either involve the algebra behind repeating decimals or a problem in marble testing.

Paul Andrew Zeitz University of San Francisco (zeitzp@usfca.edu)

Circle in a Plane: Can Math Circle Activities be done with Tablets?

I have spent part of my sabbatical year learning to write iOS apps in order to grapple with the issue of how best to use tablets, such as the iPad, to engage students and teachers in math circle activities. I am not at all convinced that this is a good thing, but I am sure that it is worth exploring. This talk will demonstrate some of my initial forays into this world, and hopefully will generate an interesting discussion on what directions, if any, "circles in a plane" should take.

Edward C Keppelmann University of Nevada Reno (keppelma@unr.edu)

More Games for Little Wranglers

At MathFest 2012 in Madison the concept of a Math Wrangle for little kids K–6 was presented. While not the traditional Wrangle of Proofs and Critique this format involves games with twists and surprises where kids can be creative, tricky and of course (in a friendly way) combative. In this talk we will continue in that spirit with 5 new ideas for games including — knotty number designs, arithmetic battleship, Dice Solitare, and playing tag on graphs.

Brandy Wiegers National Association of Math Circles (brandy@msri.org)

"I need a drink of water!": 10 Things to Think About When Working with Elementary Math Circle Students

SFMC is a weekly program for teachers and students centering around a community of students who want to work together on intriguing and challenging mathematical problems. We are excited to have added a new program to San Francisco Math Circle (SFMC) aimed at students in grades 2, 3, 4 and 5. The program, SFMC Elementary, is designed to develop a positive attitude towards mathematics by introducing young children to elements of mathematical culture. It has the same mission and philosophy as the SFMC. Through this program SFMC has learned a lot about best practices in working with elementary students to keep everyone safe. This session will share some of these lessons learned to help you make the most of your elementary Math Circle program.

Gili Rusak Albany Area Math Circle (rusakgili@gmail.com)

Albany Area Math Circle: Building a Mathematical Community

What started as just a gathering of a few inspired students in 2001 by Dr. Mary O'Keeffe, the Albany Area Math Circle (AAMC) has become a phenomenal platform for enthusiastic math students. Over the years, it has grown in numbers to fifty high school mathletes today. Not only is this a place for students to excel their knowledge in mathematics by solving challenging math problems, it is also a place where communities are built. The older students outreach to younger students, mentoring them like they had been encouraged when they were younger. Last year, I, Gili Rusak, a member of the AAMC for the past five years, started creating outreach programs for local inner city students, especially encouraging middle school girls and aiming to instill the love of mathematics in them. These programs offer a platform for non-competitive collaboration between the students. They create a magical atmosphere for everyone involved. The objective of these events is to build the confidence of the less experienced students so they become more involved in mathematics whether it is in the form of a competition or any other venue. These programs are unique and truly inspire the students to pursue mathematics. Over the past year, I have created three such events, each with a slightly different goal; each inspiring a diverse community of students. Thus, the AAMC, through these outreach programs, takes the gathering for math and problem solving one step farther and turns the students into a community by involving and inspiring everyone.

Lauren Rose Bard College (rose@bard.edu)
Beth Goldberg Linden Avenue Middle School, Red Hook, NY (bethcgoldberg@gmail.com)
Joy Sebesta Bard College (joys5@msn.com)

Developing Collaborative Lesson Plans for Math Enrichment

In this presentation, we will focus on the Bard Math Circle's ongoing activities in developing enrichment materials for middle school students and teachers. This is a combined effort that includes undergraduate math majors, mathematics professors, and middle school teachers in the Mid-Hudson Valley region of New York state. In particular, we will give an example of a problem that arose in a Bard College undergraduate class on Problem Solving, was turned into an activity for a Math Teacher's Circle workshop, and then became a lesson plan for a middle school mathematics classroom. We have found that working together with mathematics students and educators at different levels has strengthened and enhanced our Math Circle.

Gulden Karakok University of Northern Colorado (gulden.karakok@unco.edu)
Katherine Morrison University of Northern Colorado (katherine.morrison@unco.edu)
Cathleen Craviotto University of Northern Colorado (cathleen.craviotto@unco.edu)

Northern Colorado Math Teachers' Circle

Northern Colorado Math Teachers' Circle was formed in the summer of 2011 with the goal of engaging local middle school mathematics teachers in problem solving tasks. The leadership team consists of two middle school math teachers, a district math coach and three School of Mathematical Sciences faculty members at University of Northern Colorado. During the 2012–2013 academic year we held 6 evening problem solving sessions each of which was three hours long with free dinner, and a summer workshop with 26 middle school teachers. In this presentation we will share the problems used at our summer workshop, the structure of the workshop and our recruitment process. We will also discuss some of the obstacles we faced starting our circle, and the strategies we implemented to overcome some of the issues.

Diana White University of Colorado Denver (diana.white@ucdenver.edu)

What happens in the classroom of Math Teachers' Circle participants?

Math Teachers' Circles (MTCs) have been around since 2006, and at this point, a large volume of anecdotal data points to their value as a professional development opportunity. Yet we still have minimal information on their actual effect on teachers' classroom teaching. Through a National Science Foundation DRK12 grant, a team of investigators is systematically researching what is happening in classrooms of MTC participants. In this talk, we report on the process, our framework for conducing the analysis, and the results from the first round of classroom observations.

General Contributed Paper Sessions

GCPS Session 1: History and Philosophy of Mathematics

Thursday, August 1, 8:30–10:10 PM, Room 14

Shigeru Masuda Kyoto University (hj9s-msd@asahi-net.or.jp)

The Fourier's Fecundity of Analytic Method or Application

Fourier's motto: my analysis is deduced from fecund method. In 1806, Fourier began with studying communication of heat motion. In 1878, Freeman published the first English-translation of 2nd version without seeing the manuscript of 1807. In 1888–90, Darboux edited also Fourier's 2nd version of 1822. We know the Fourier's manuscript by Grattan-Guinness in 1972, who speculates that Darboux had seemed to have seen the manuscript at the time of editing the 2nd version in 1888. Why?

To Fourier's main work: "The analytical theory of heat" in 1822, and to the relating papers, Poisson points the diversion applying the algebraic theorem of De Gua to transcendental equation. Fourier extended the heat theory to the fluid dynamics, however, this aim was not appeared until 1833 13 years after its submission in 1820, even after getting the permanent secretary in the Academy of Science after Delamble passing away, in 1822. Darboux explains Fourier's shrewdness by putting some examples. If Fourier would publish it, he could done it by utilizing his position. Why?

After Fourier returned from Egypt in 1815, he is occupied with the algebraic analysis. Navier edits and publishes the first part of this sort of works after Fourier's death, and emphasizes the preceding works of algebraic analysis by Fourier since 1797, when he graduated from École polytechnique. Why?

We discuss Fourier's motivations of works, which based on the fecundity for the applications or method. Dirichlet says also so as a student of Fourier.

Kenneth Rietz Asbury University (kenneth.rietz@gmail.com)

Beyond Euclid

Euclidean geometry is famous for its three unsolvable problems: trisecting an angle, doubling a cube, and squaring a circle. However, Archimedes and other ancient Greek mathematicians were able to trisect any angle using a marked straightedge (a straightedge with two fixed points on its edge) to make a neusis construction. That same construction technique is also able to double a cube and construct the roots of any polynomial of fourth degree or less, given the coefficients, even allowing complex values for all of them.

Philip Blau Shawnee State University (pblau@shawnee.edu)

Galois and His Theory

This talk will provide an overview of Galois Theory as he set forth in his "Memoir on the Conditions for Solvability of Equations by Radicals." Some biographical facts about Galois will also be presented.

Andrew Perry Springfield College (perryand@gmail.com)

Mathematics in the Book of Michael of Rhodes, A Fifteenth-Century Maritime Manuscript

In the fifteenth century, a Venetian mariner, Michael of Rhodes, wrote and illustrated a text describing his experiences in the Venetian merchant and military fleets. We will consider some of the mathematics in this book and draw comparisons to early American school mathematics.

Patricia Giurgescu MAA (pgiurgescu@gmail.com)

Mathesis Universalis

A discussion of quest for unifying mathematical ideas through the contributions of R. Descartes, G. Leibniz and J. Wallis

Timothy Sipka Alma College (sipka@alma.edu)

Kempe's Flawed Proof That Four Colors Suffice

The Four Color Theorem is a simple and believable statement: at most four colors are needed to color any map drawn in the plane or on a sphere so that no two regions sharing a boundary receive the same color. It might be surprising to find out that mathematicians searched for a proof of this statement for over a century until finally finding one in 1976. In this talk we'll consider the "proof" given by Alfred Kempe, a proof published in 1879 and thought to be correct until a flaw was found in 1890. You'll be invited to look carefully at Kempe's proof and see if you can do what many 19th century mathematicians could not do—find the flaw.

Chris Christensen Northern Kentucky University (christensen@nku.edu)

Recruiting and Training Mathematicians as Codebreakers

As World War II loomed, the US Navy needed to increase the number of codebreakers in Naval Communications. Through "friends" on college campuses, the Navy sought out mathematics faculty members who were interested in taking a correspondence course in elementary cryptanalysis. Successful "students" in the correspondence course were commissioned into the Navy when war broke out. In this presentation, we will explore the recruitment, training, and work done by the US Navy cryptologic mathematicians.

GCPS Session 2: Research in Graph Theory or Combinatorics

Thursday, August 1, 8:30–10:10 AM, Room 15

Steven J. Tedford Misericordia University (stedford@misericordia.edu)

Fuzzy Greedoids - Structure and Invariants

Introduced in 1965, fuzzy sets have been used in situations where it is appropriate to use varying levels of membership. By the 1980's the approach of fuzzification had been applied to a variety of combinatorial objects—fuzzy greedoids being one such application. In this talk, we will give a feasible set definition of a fuzzy greedoid and consider which greedoid structures and invariants can be extended to fuzzy greedoids. In particular, we will look at rank functions, closures, and polynomial invariants.

Francis Edward Su Harvey Mudd College (su@math.hmc.edu)

A Combinatorial Proof of the Poincare-Miranda Theorem

The Poincare Miranda Theorem is a generalization of the Intermediate Value Theorem to functions from the n-cube to R^n. We provide a new combinatorial proof of this theorem using the Polytopal Sperner Lemma of DeLoera-Peterson-Su (2002). This is joint work with undergraduate Connor Ahlbach.

Chris Spicer Morningside College (spicer@morningside.edu)

2-Color Rado Numbers for $\sum_{i=1}^{m-1} x_i + c = x_m$

Rado numbers are a branch of Combinatorics and are closely related to Ramsey numbers. In this talk, after discussing some of the historical work done on this topic, we will completely determine the 2-color Rado numbers for equations of the form $\sum_{i=1}^{m-1} x_i + c = x_m$.

Natacha Fontes-Merz Westminster College (fontesnc@westminster.edu)
Jeffrey Boerner University of Wisconsin, Stout (boernerj@uwstout.edu)
James Anthony Westminster College (anthonj@westminster.edu)

Harmonious Chromatically Critical Graphs

A harmonious coloring of a graph is a proper vertex coloring in which each pair of colors appear on at most one pair of adjacent vertices. The minimum number of colors needed to harmoniously color a graph G is called the harmonious chromatic number of G. In this talk we will characterize graphs which have the property that the removal of any edge reduces the harmonious chromatic number of the graph.

Michael Barrus Brigham Young University (barrus@math.byu.edu)

Adjacency Relationships Forced by Graph Degree Sequences

Given the degree sequence of a graph, there are usually several graphs having that degree sequence. Vertices that are adjacent in one of these realizations may not be adjacent in another. In some cases, however, it is possible to conclude that two vertices with certain degrees must be adjacent (or non-adjacent) in every realization of the degree sequence. This is certainly true for the threshold graphs, where every adjacency relationship is uniquely determined by the degree sequence. We describe general conditions leading to forced adjacencies and non-adjacencies among realizations of a degree sequence.

Jason Molitierno Sacred Heart University (molitiernoj@sacredheart.edu)

The Algebraic Connectivity of Planar Graphs

The Laplacian matrix for a graph on n vertices labeled $1, \ldots n$ is the $n \times n$ matrix $L = \ell_{i,j}$ in which ℓ_{ii} is the degree of vertex i and ℓ_{ij} is -1 if vertices i and j are adjacent and 0 otherwise. Since L is positive semidefinite and singular, we can order the eigenvalues $0 \leq \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. The eigenvalue λ_2 is known as the algebraic connectivity of a graph as it gives a measure of how connected the graph is. In this talk, we derive an upper bound on the algebraic connectivity of planar graphs and determine all planar graphs in which the upper bound is achieved. If time permits, we will extend these results in two directions.

First, we will derive smaller upper bounds on the algebraic connectivity of planar graphs with a large number of vertices. Secondly, we will consider upper bounds on the algebraic connectivity of a graph as a function of its genus.

Michael Fraboni Moravian College (mfraboni@moravian.edu)

The Birank Number of Ladder Graphs

A k-biranking on a graph G is a function $f : V(G) \to \mathbb{N}$ such that for all $u, v \in V(G)$, if $f(u) = f(v)$, then each $u - v$ path contains an x with $f(x) > f(u)$ and a y with $f(y) < f(u)$. The minimum k for which G has a valid k-biranking is the birank number of G. We compute the birank number for ladder graphs and provide an algorithm for obtaining an optimal birank on these graphs.

GCPS Session 3: Probability or Statistics

Thursday, August 1, 8:30–9:40 AM, Room 16

Leo Chosid NYC College of Technology (elico@elico.com)
Jonathan Natov NYC College of Technology (JNatov@citytech.cuny.edu)

In Quest of Fairness, Randomness and Independence

Quantum theory sees a photon as both particle and wave. Our physical knowledge of the real world is limited by Plank's constant and by Heisenberg's Uncertainty Principle. The Copenhagen agreement allows for both states to exist simultaneously but not in observation. Probability has become the path to account for what appear conflicting phenomena. The lack of mathematical completeness in physics forces us to view the world with only rational numbers. What we aim to demonstrate is that the non-existence of real numbers leads to a proof that randomness and fair coins cannot exist.

The deep mathematical proofs in probability, especially the Central Limit Theorem, depend heavily on continuity. The measure of rationals is zero but in practice "randomly" selected numbers always appear rational. We argue that a non-standard characterization of independence is more practical when dealing with discrete data such as the rationals.

Krishna K Saha Central CT State University (sahakrk@ccsu.edu)

Goodness of Fit of Some Well-Known Over-Dispersed Models to Proportion Data

In many biological and similar investigations, data in the form of proportions show extra variability that predicted by a simple proportion model like binomial. A number of over-dispersed proportion models have been proposed by introducing an extra parameter in the models, which can take into account the extra variability shown in the proportion data. In this talk, we aim to discuss the properties of these models, along with the estimation of the model parameters. Finally, we will discuss the goodness-of-fit of these models based on the several goodness-of-fit approaches using a set of real-life data.

Aminul Huq University of Minnesota Rochester (ahuq@umn.edu)
Wei Wei Metropolitan State University (wei.wei@metrostate.edu)
Heidi Hulsizer Hampden-Sydney College (hhulsizer@hsc.edu)

Re-sequencing Hypothesis Testing in an Introductory Statistics Course with Active Learning

The process of hypothesis testing is essential in understanding the logic of data analysis. We tested whether the introduction of the concept of hypothesis testing early in the semester in an active learning classroom significantly increases student understanding of the topics surrounding it. We tested our students on various aspects of the learning process: inquiry process, formulation, algorithm, double-negative, and contextualization with regards to hypothesis testing. In this paper we discuss two comparisons, one between an active learning class and interactive lecture based class, and the second between introducing hypothesis testing at the beginning and middle of the semester. In this paper we describe our findings that the introduction of hypothesis testing in an active learning environment had a greater influence on student success than did the timing of the hypothesis introduction or an active group learning method alone.

Azar Khosravani Columbia College Chicago (akhosravani@colum.edu)
Constantin Rasinariu Columbia College Chicago (crasinariu@colum.edu)

A New Class of Benford Random Variables

A random variable is Benford distributed if the occurrence frequency of its most significant digit d is $P(d) = log(1 + 1/d)$. This law is observed by a wide variety of empirical data sets to different degrees of accuracy. We explore the use of the mod 1 map in studying the digit distribution of random variables and construct a new class of "Benford-like" distributed random variables that have a logarithmic distribution for the first n-digits. We will present how Mathematica can be used to experiment with such random variables.

David DiMarco Neumann University (dimarcod@neumann.edu)
Ryan Savitz Neumann University (savitzr@neumann.edu)
Fred Savitz Neumann University (fsavitz@neumann.edu)

The M-tile Means, A New Class of Measures of Central Tendency

Assessment. Indeed, the watchword for institution of higher education accountability in the twenty-first century is assessment. It is the mechanism that holds educators accountable to their pupils and colleges and universities accountable to their governing bodies. Data collection at both the professorial level and the institutional level involves the administration of numerous and frequent assessments. Consequently, the accumulation of information will consist of large sets of data.

It is here where assessment can bog down, large amounts of data may contain troublesome characteristics, characteristics that may predispose useful interpretations of results with unintended consequences. Measures of central tendency enable educators and administrators to acquire rough global views of classroom or institutional performance. However, they are subject to distortion as a result of outliers, and it is important to militate against undue influences meted by outliers. Here is where our m-tile mean measures of central tendency can promote accurate assessment.

A new measure of central tendency, the quartile mean, is introduced and is generalized to a class of measures, called m-tile means. These new measures were motivated by the authors desire to find a measure of central tendency that was not only resistant to outliers, but also resistant to undue changes incurred by altering a small number of data values, regardless of whether or not they would be considered outliers. The new class of measures introduced in this paper not only meets the aforementioned criteria but compares favorably to extant measures of central tendency in regards to other properties as well.

GCPS Session 4: Teaching Advanced Mathematics

Part 1: Thursday, August 1, 8:30–10;10 AM, Room 26

Leon Kaganovskiy Touro College Brooklyn Campus (leonkag@gmail.com)

Applications of Maxima to Calculus and Differential Equations

In this presentation we would like to explore using freely available Maxima CAS to create codes which significantly enhance students' learning of more complicated Differential Equations concepts. Among the topics considered are the creating slope fields and phase portraits, population dynamics, object cooling, RK methods, and Ecological models.

Nermine El-Sissi The American University in Cairo (nelsissi@aucegypt.edu)

Students' Learning Journey in Linear Algebra

How can one engage students in a course that they "must take for no reason," as many of them stated in a survey administered at the beginning of the semester? Students are taken into a writing journey to explore the theory and applications of linear algebra via weekly journals, mini-projects and a final project. The benefits and drawbacks of this approach will be discussed. In addition, students' feedback on this method of teaching will be presented. This teaching approach is inspired by last year's contributed session on "Incorporating Writing and Editing into Mathematics Classes," at the MAA MathFest in Madison, WI.

Anders O.F. Hendrickson Saint Norbert College (anders.hendrickson@snc.edu)

Teaching Determinants by Rook-Arrangements

Among ten teachers of linear algebra will be found at least twelve opinions about how or whether to teach determinants. This talk describes a visual approach to determinants, using arrangements of non-attacking rooks on a chessboard, which makes proofs of the basic lemmas far more accessible than typical double-subscript-laden proofs. In fact, we give many such lemmas as discovery-learning exercises in a proof-based linear algebra course.

Michael David Smith Lycoming College (smithm@lycoming.edu)

Bulls-Eye Jenga

This paper presents a classroom activity for an abstract algebra class which is designed to introduce students to the axioms of an abelian group. Through completing the activity, students discover for themselves seven "first examples" of abelian groups. The finished product of this activity is a colorful display of a bulls-eye which contains information about which of the abelian group axioms hold for a variety of sets under different operations.

Sarah L. Mabrouk Framingham State University (smabrouk@framingham.edu)

A Sweet Way to Explore Statistics

Important components of any introductory statistics course are data collection and analysis. Having the opportunity to experience from where and how data is obtained, managing and cataloging data, and dealing with unexpected responses and data values are important for those who may use data analysis and statistics in other courses as well as in their future careers. Experiencing this process, even on a small scale, is useful and adding a bit of whimsy is helpful for students who have mathematics anxiety as well as for those who may have had negative experiences in mathematics and/or statistics courses in the past. In this presentation, I will discuss my use of 1.69-ounce bags of Milk Chocolate M&M's in my introductory statistics courses as a source for student collected data and to motivate the collection, analysis, presentation, and interpretation of data and statistics, in particular, frequency and relative frequency distributions, graphs, summary statistics, confidence intervals, and hypothesis testing.

Pete Johnson Eastern Connecticut State University (johnsonp@easternct.edu)
Marsha Davis Eastern Connecticut State University (davisma@easternct.edu)

An Advanced, Applied Statistics Course for Mathematics Majors

The College Report from Guidelines for Assessment and Instruction in Statistics Education (GAISE, 2012) suggests that college statistics courses "emphasize statistical literacy," "stress conceptual understanding," and "use real data"—in other words, that statistics courses stress applications. At Eastern Connecticut State University, we have developed a first statistics course for mathematics majors with an advanced but applied focus. The course also serves as one of the University's writing-intensive courses. In this session, we share some suggestions for an advanced, applied statistics course based on ten years of our experiences. Among the innovations we have implemented are extensive use of Minitab, Maple, and the TI graphing calculator for labs and guided discovery; along with the use of a large scale, national database as a resource for the major written project in the course.

Andrew Lazowski Sacred Heart University (lazowskia@sacredheart.edu)

Curriculum Infusion of Alcohol Prevention In Probability and Statistics Courses

Alcohol usage is a common topic at college campuses across America. Plenty of research has been done on how alcohol and other drugs affect student's lives. In an effort to educate students on this topic and learn real world applications of probability and statistics, curriculum infusion (the process of integrating alcohol prevention content into courses across the curriculum) is a great practice for elementary probability and statistics classes. This presentation will discuss methods used in the previous year and future ideas for project enhancement.

GCPS Session 5: Research in Linear Algebra or Geometry

Thursday, August 1, 1:00–4:25 PM, Room 26

Keivan Hassani Monfared University of Wyoming (k1monfared@gmail.com)

An Structured Inverse Eigenvalue Problem

A result of Duarte asserts that for real $\lambda_1, \ldots, \lambda_n, \mu_1, \ldots, \mu_{n-1}$ with $\lambda_1 < \mu_1 < \lambda_2 < \mu_2 < \cdots < \mu_{n-1} < \lambda_n$, and each tree T on n vertices there exists an $n \times n$, real symmetric matrix A whose graph is T such that A has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ and the principal submatrix obtained from A by deleting its last row and column has eigenvalues $\mu_1, \ldots, \mu_{n-1}$. We extend the result in two different ways including and extension to connected graphs through the use of the implicit function theorem. This is a joint work with Bryan L. Shader.

Bruce Atkinson Samford University (bwatkins@samford.edu)
Braxton Carrigan Southern CT State University (carriganb1@southernct.edu)

Golden Triangulations

An isosceles triangle with the property that the ratio between the longer side and the shorter side is the golden ratio (τ) is known as a golden triangle. Every golden triangle is either acute with degree angle measures (36, 72, 72) or obtuse with degree angle measures (36, 36, 108). One interesting fact about golden triangles is that each can be "split" into two triangles where one is an acute golden triangle and the other is an obtuse golden triangle. It is thus natural to explore triangulations of other regions that only use golden triangles, which we will call golden triangulations. We will present the following results of golden triangulations of connected regions. If there exists a triangle in the triangulation with one side length equaling a power of τ, then every side length of every triangle in the triangulation equals a power of τ. Similarly, if there exists a triangle in the triangulation with area equaling a power of τ, then the area of every triangle in the triangulation is a power of τ and thus the area of the triangulated region is a τ-rational number. We will also classify all convex polygons that can be triangulated with golden triangles and discuss methods of refining such triangulations to create finer golden triangulations.

Jonathan Sondow New York City (jsondow@alumni.princeton.edu)

A Property of the Tangent Rectangle of the Parbelos: My Proof Compared with Tsukerman's

The arbelos is a classical geometric shape bounded by three mutually tangent semicircles with collinear diameters. In a paper to appear in the Monthly (a preprint is at http://arxiv.org/abs/1210.2279), I introduce a parabolic analog, the parbelos, formed by replacing the semicircles with arcs of parabolas. It is easy to see that the tangents to the parbelos at its cusps enclose a rectangle. I will "sketch" (pun intended) two proofs that a diagonal of the tangent rectangle is also tangent to the parbelos. The first proof is short, but uses analytic geometry. The second uses only Euclidean geometry, but is longer. It is due to Emmanuel Tsukerman, an undergraduate at Stanford University. His proof introduces a converse to the theorem of Lambert that states: the circumcircle of any triangle of tangents to a parabola passes through the focus.

William Roger Fuller Ohio Northern University (w-fuller@onu.edu)
Lauren Cassell Ohio Northern University (l-cassell@onu.edu)

Guarding a Koch Fractal Art Gallery

This paper presents a generalization of the standard art gallery problem to the case where the sides of the gallery are continuous curves which are limits of polygonal arcs. The allowable limiting processes for such generalized art galleries are defined. We construct an art gallery in which one side is the Koch fractal and the other sides are three sides of a rectangle. The appropriate measure of coverage by guards is not the total number of guards but, rather, the guards-to-side ratio. We compute this ratio for the cases of shallow and deep versions of the Koch fractal art gallery.

Genghmun Eng Self (geng001@socal.rr.com)

Hidden Equilateral Triangles Inside Circles on Square Hyperbolas

Let $\{X, Y\}$ be orthogonal axes with origin O, and $XY = -K$ being a square hyperbola. Any straight line through O that intersects the hyperbola defines two opposing points, R and S, with $OR = OS$. Let S be the center of new circle passing through R, with radius $SR = 2OR$. In general, this circle intersects the hyperbola at 3 new points, which can be connected together to give a perfect equilateral triangle.

This theorem can be proven using elementary algebra and trigonometry, but its motivation is more subtle. Although this construction makes no reference to angle trisection, the surprise occurrence of a hidden equilateral triangle makes it a cousin to Morley's Trisection Theorem (1909). It is a graphic illustration of this special symmetry between an angle (3θ) and its trisector (θ):

$$[1] \quad \frac{\sin(3\theta)}{\sin(\theta)} - \frac{\cos(3\theta)}{\cos(\theta)} = 2.$$

Since all square hyperbolas having a common origin O are self-similar, OR functions as a unit distance. Choosing S as the center of a new circle doubles OR to match the r.h.s. of Eq. [1].

Judith Ann Silver Marshall University (silver@marshall.edu)

Mathematics and Art on the Sphere

Artist Dick Termes paints on a sphere and also gives workshops on perspective, on spherical drawing, and on the platonic solids. In a recent workshop the artist posed two questions: Is a sphere 2-colorable when it is covered by circles? Can a known circumference be used to inscribe an icosahedron onto the surface of a sphere? We answer the first question in the affirmative, and extend it to n-dimensions. The second question can also be answered analytically and by use of computer software. In fact, edge lengths can be determined for all of the Platonic solids inscribed on a sphere.

Alexander Louis Garron Sand Box Geometry LLC (alexander@sandboxgeometry.com)

Using a Curved Space Division Assembly, Two Plane Geometry Curves, for Partition of Linear Magnitude.

The first construction I experienced was Euclid's Perpendicular Divisor. I will expand his construction into two plane geometry curves, manufactured with help of a Computer Algebra System, to provide means for exact division of magnitude. Traditional square space number line division, using integer as parts, if not an exact fit, will have a remainder. I will show plane geometry curved space division is never encumbered with remainder. All plane geometry curved space analysis begins with Euclid's Perpendicular Divisor. Half his magnitude, (the part with Cartesian center), becomes the radius of the discovery curve circle. With its radius I construct the magnitude of considered linear definition curve parabola. The parabola focal radius latus rectum, at slope ($m = 1$), defines congruent meter of curves and lines meeting the positive number line latus rectum focal radius intercept with structured parabola loci at considered linear magnitude to be divided by curved space. The CSDA assembly operates division using system center to build a square space perimeter [(y axis as integer number) * (number line as considered base magnitude)] with attendant traditional multiplication table diagonal. Magnitude, slope, and number of division diagonals, are all derived and constructed from the dependent definition parabola curve using high school level math.

Brian Kelly Fisher College (bkelly@fisher.edu)

Identifying The Right Recursion

In Dragon Curve fractals constructions, two distinct recursive geometric constructions are often treated as equivalent. After illustrating the similar results produced by these procedures, physical models of the "fold" and the "sweep" processes are used to highlight the essential differences. This then leads to a complete characterization of when results of the procedures agree to show why the distinction was originally overlooked.

Margaret Symington Mercer University (symington_mf@mercer.edu)

Klein's Hypercycles in 3D

The Klein model of hyperbolic geometry can be cumbersome to work with because even though lines are simple (chords in a circle), angle measurements are not intuitive. However, in the Klein disk a hypercycle (the set of points equidistant from a hyperbolic line) is beautifully simple, namely an ellipse. In this talk I will discuss two applications of Klein hypercycles—first to understanding distance in the Klein disk and second to drawing images of cones and their bases. Three-dimensional thinking figures in both applications.

Damiano Fulghesu Minnesota State University, Moorhead (fulghesu@mnstate.edu)
Ishan Subedi Minnesota State University, Moorhead (subediis@mnstate.edu)

Möbius Transformations Fixing Finite Sets of Points

Let S be a finite set of points in $\mathbb{P}^1(\mathbb{C})$. We call $\mathrm{Aut}(S)$ the group of Möbius transformations which permute the points in S. We provide a complete classification of $\mathrm{Aut}(S)$ when $|S| = 5$, and we give some general results for larger number of points.

Sylvester Reese (sylreese@yahoo.com)

Some Not-So-Well-Known Constants Associated with the Conic Sedtions.

Pi is well-known constant associated with the quadrature and rectification of a semicircle. We give some similar constants associated with some Latus Rectum Cousins of the semicircle. See the "Universal Parabolic Constant" for a Preview or Trailer.

Marcia R Pinheiro RGMIA (needanacademicpositionintheusa@gmail.com)

Minima Domain Intervals, Dimensions, and How to Extend the Class 'Convex Functions'

In this paper, we expose the conclusions of our studies on the current analytical definitions of the phenomenon S-convexity (Hudzik and Maligranda's). We have made use of all the foundations of Mathematics to refine the current proposal to maximum level of refinement in terms of preserving the main characteristics of it. Our conclusion is that even so we do not have a proper extension of the phenomenon Convexity. We are then left with a very well designed plan for future work that should finally lead to a proper extension.

Ryan Trelford University of Calgary (rgtrelfo@ucalgary.ca)

The Equivalence of the Illumination and Covering Conjectures

Let K be a convex body in E^d, and let v be any non-zero vector (referred to as a direction). A point P on the boundary of K is said to be illuminated by v if the ray emanating from P with direction v intersects the interior of K. One can ask what is the smallest positive integer n such that there exists a set of distinct directions $\{v_1, \ldots, v_n\}$ whereby every boundary point of K is illuminated by at least one of the v_i's. The illumination conjecture (formulated by I. Gohberg and A. Markus) states that n is at most 2^d. Surprisingly, 2^d is also the conjectured maximum number of smaller homothetic copies of K that are required to cover K (conjectured by H. Hadwiger and V. Boltyanski). In this talk, I will outline the proof that the Illumination Conjecture and the Covering Conjecture are indeed equivalent.

Sam Northshield SUNY-Plattsburgh (northssw@plattsburgh.edu)

The Complex Descartes Circle Theorem

In 1643, Descartes showed that the curvatures (reciprocals of radii) of four mutually tangent circles satisfy the equation $(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$. It was only in 2001 that a complex version of this result was discovered and proved; the equation is still satisfied by replacing each curvature by that curvature times its respective circle's center (as a complex number). We present a thoroughly elementary proof of this fact.

GCPS Session 6: Assessment, Mentoring, or Outreach

Friday, August 2, 8:30–11:25 AM, Room 14

Fariba Nowrouzi Kashan KYSU (fariba.kashan@kysu.edu)

Assessment and Curving Grades

Some teachers and instructors use different methodologies to change the average grades of a course. They may shift all grades up vertically, or change a distribution of a course grades, or .... We will discuss different formulas used in this matter, and then will direct our talk to fairness or unfairness of this idea.

Carrie Muir University of Colorado, Boulder (carrie.muir@colorado.edu)

Getting at the (Grade) Point of Grading

When I began teaching, I adopted the grading scales and policies that I had seen as a student and saw being used by my colleagues. However, over time I grew to have practical and philosophical objections to those practices. Consequently, I decided to switch all of my grading to a grade-point scale. In this session, I will discuss the process of implementing this grading change, how different classes have reacted to an unexpected grading scale, and how making a change in scale affected the way I think about grading.

M. Leigh Lunsford Longwood University (lunsfordml@longwood.edu)
Phillip L. Poplin Longwood University (poplinpl@longwood.edu)

The Scarlet Letter: Assessment with a Purpose

We will discuss a multi-year program of assessment and intervention with the purpose of improving student success in non-calculus introductory statistics at Longwood University. Following an action research model we used assessment results from one study to inform the implementation of an early intervention program in a second study. In our first study, conducted during the 2006–2008 academic years we found, among other things, that students who have poor basic mathematics skills are less likely to succeed in the course. Those results lead to a second study, started in 2011, in which we implemented an early intervention program (required tutoring) for these students. We will discuss the results from both studies as well as how we believe assessment can actually be used for a purpose and not be the bane of our existence.

David C. Wilson SUNY, Buffalo State (wilsondc@buffalostate.edu)
Chaitali Ghosh SUNY, Buffalo State (ghoshc@buffalostate.edu)

Placement Tests: Are Students Getting the Course They Need?

Students are regularly asked to take a placement test at colleges and universities around the country. In many cases, the test results determine the course that the student is eligible to take. What would happen if students took a placement test and then were allowed to enroll in the course that they wanted? Data will be shared that explores this scenario over several semesters at Buffalo State. Students that registered for calculus were given a placement test and advised based on their score. With a minimal number of students making the decision to take a different course, they provided an opportunity to examine the level of success in calculus and whether the placement test served as a reasonable predictor of that level. Surprisingly, the placement test scores proved to be a rather poor indicator of student success accounting for only 20% of the variance in students' course gpa. The placement test recommends the test be used as part of the decision process when determining course placement, but at many colleges and universities, that poses problems as other data may not be readily accessible to those making the determination. Additional analyses explored the inclusion of high school gpa and SAT scores to supplement the placement test score, but results revealed that even the combination of factors was only a moderate predictor of the level of success.

Carey Childers Clarion University (cchilders@clarion.edu)

High School Mathematics Competition — Females versus Males

How have the females scored at the Clarion University High School Mathematics Competition versus the males. Nineteen years of data will be analyzed to see the differences in scores. What are potential causes of these differences? What kind of trends has the country seen over the past years? What can be done going forward to address the differences?

Frank Wang LaGuardia Community College, CUNY (fwang@lagcc.cuny.edu)

Teaching Faculty How to Improve Students' Quantitative Skills through Cognitive Illusions

Cognitive scientists have documented many biases and misconceptions when ordinary people make intuitive predictions and judgment. We will report our empirical findings revealing that even in a math test environment, where students are hypothesized to be more alert and cautious, there are still systematic errors in performing quantitative tasks, e.g., judging fuel efficiency and estimating probability. Based on our findings, we have developed a learning unit for an online Numeracy Infusion Course for Higher Education (NICHE, sponsored by an NSF grant) to introduce educators across disciplines ways to improve students quantitative reasoning skills. The overarching conceptual framework for this unit is the dual-process theory of human thinking, recently popularized by Daniel Kahneman's book *Thinking, Fast and Slow*. This theory explains why some problems trigger intuitive but incorrect responses. Knowledge on how the mind works should empower educators to motivate students to be more alert, more intellectually active, less willing to be satisfied with superficially plausible answers, and more skeptical about their intuitive. We will share materials for educators to motivate students to develop the disposition to resist reporting the response that springs impulsively to mind. We will also disseminate resources produced during the NICHE project.

Eoin Gill Maths Week Ireland (egill@wit.ie)

Maths Week Ireland: Lessons from a Small Island?

What can Americas math community learn from a math festival on a small island the far side of the Atlantic? Promoting an interest in math is a concern across most of the developed world and Maths Week Ireland may have some interesting ideas applicable in the US.

Maths Week Ireland takes place across the Island of Ireland every October and has grown to be the biggest festival of its kind in the world. Ireland is a small island covering an area almost as large as Connecticut, Massachusetts, New Hampshire and Vermont combined and has a population of 6.3 m, just slightly less than Massachusetts, yet over 150,000 people participated in Maths Week 2012. The festival earns wide media coverage, enjoys widespread recognition and is firmly established in the education calendar.

Maths Week Ireland is a partnership of universities, colleges, public bodies, mathematics groups, visitor centers, industry and schools cooperating together to promote awareness, appreciation and understanding of mathematics for all.

This paper will recount the story of the establishment of Maths Week in 2007: how getting people to work together made a big impact and started an unstoppable movement harnessing ideas, enthusiasm and goodwill among over 50 different groups who have chosen to work together for the common good. Novel activities will be described such as Maths in the City where mathematicians take to city streets and engage citizens and events at the interface of maths and music, history and art. Challenges met and lessons learned will be shared. www.mathsweek.ie

Matthew Haines Augsburg College (haines@augsburg.edu)

Outreach with Grades K–8 Teachers Impacting Pre-Service Mathematics Courses

Outreach in working with K–8 teachers to deepen their mathematical understanding has impacted the teaching of pre-service teachers. This presentation will reflect on the presenter's connections with K-8 teachers and subsequent resulting changes to college courses that prepare students to become teachers of mathematics.

Rebecca Etnyre Cal State Fullerton (beccake@csu.fullerton.edu)
Christina Tran California State University, Fullerton Mathematical Circle (cnt_tran@csu.fullerton.edu)

Training Gifted Students: The Fullerton Mathematical Circle Experience

Mathematical Circles are a form of outreach that bring mathematicians together with middle school and high school students. The Fullerton Mathematical Circle was established on September 24, 2011 and meetings have been held on numerous Saturdays since at California State University, Fullertons campus. The format of our sessions is inspired from the experience of similar events developed around the monthly publication Gazeta matematica, a journal established in 1895. We are describing our program's experience and present a series of sample problems discussed in our sessions.

Andrew deLong Martin Kentucky State University (andrew.martin@kysu.edu)

Professor Abian Teaches a Lesson from Kelley's "General Topology"

My PhD adviser at Iowa State University was the mathematical artist Alexander Abian. When we first met he gave me his copy of Kelley's "General Topology", and told me to absorb the proof therein of the equivalence of Tychonoff's Theorem and the Axiom of Choice. This was the first of many tricks he pulled on me as my mentor, as he knew something about that proof...

In this talk I'll present Kelley's proof and explain what Abian knew.

Brian Arthur Christopher University of Northern Colorado (brian.christopher@unco.edu)
Gulden Karakok University of Northern Colorado (gulden.karakok@unco.edu)

The National Research Experience for Undergraduates Programs' (NREUP) Influence on Minority Students

The National Research Experience for Undergraduate Programs (NREUP) is a summer intensive mathematics research program for underrepresented undergraduates supported by the Mathematical Association of America and its Strengthening Unrepresented Minority Mathematics

Achievement program with funding being provided by the NSF and the NSA. The program piloted in the summer of 2003 with the goal of increasing underrepresented minorities interest in obtaining advanced degrees and careers in mathematics or closely related field. Since then, 410 undergraduate students have participated in the program with about 46% of them being female. Of the 410 participants, 62% were African American, 21% were Hispanic Americans and 11% were Latino Americans with the remaining 6% comprised of American Indians, Pacific Islanders and Asian Americans. Each participant was requested to evaluate their program at the end of the summer they participated, and at one, two and four years after their participation. We will share the results of the interviews and online surveys as well as our future plans to explore other aspects of the NREUP.

Jenna P. Carpenter Louisiana Tech University (jenna@latech.edu)

Professional Development Training for Graduate Students: A Different Kind of Seminar

Most graduate programs focus almost exclusively on providing technical training for students, assuming that graduate coordinators and research advisors will answer other questions that students may have and prepare them for "life after graduate school." But most faculty are too busy to address these non-technical skills and consequently, most students receive little mentoring on how to navigate as a professional after graduation. Almost five years ago we began a twice monthly graduate seminar designed to provide professional development training for graduate students, ranging from how to submit a poster and give a good presentation to basic negotiation skills to how to apply and interview for a job. While the seminar has existed in several different formats and is now a requirement for all first-year PhD students, the goal has largely remained the same: to provide graduate students with training on the non-technical skills needed to succeed as a professional.

GCPS Session 7: Teaching Calculus

Part 1: Friday, August 2, 8:30–10:25 AM, Room 15

Douglas B Meade University of South Carolina (meade@math.sc.edu)
Philip B Yasskin Texas A&M University (yasskin@math.tamu.edu)
Raymond E Patenaude University of South Carolina (raypatenaude@comporium.net)
Robert Petrulis EPRE Consulting LLC (robert.petrulis@epreconsulting.com)

Assessing Maplets for Calculus: Best Practices for Instructors and Software Developers

The Maplets for Calculus (M4C) is an award-winning and NSF-funded project to create a collection of computer applets designed to enhance student learning of precalculus and calculus. This talk will describe two separate assessments that have been implemented and how their results have impacted the project.

Student attitudes towards mathematics, calculus, and technology are measured with surveys administered at the beginning and end of each semester. The surveys, several years of results, and the use of the results will be discussed. Of particular interest is how student attitudes evolve during the first two semesters of calculus.

More recently we have started a second assessment thrust with an emphasis on measuring actual student learning and the specific features of the M4C that are most effective. The primary data for this assessment is digital recordings of students including their verbal expression of their thought process as they work through a collection of three applets relating to limits.

Philip B Yasskin Texas A&M University (yasskin@math.tamu.edu)
Douglas B Meade University of South Carolina (meade@math.sc.edu)
Matthew James Barry Texas A&M University (komputerwiz.matt@gmail.com)

Maplets for Calculus Expands Offerings in Precalculus, Calculus and Differential Equations

The Maplets for Calculus v.1.3 is an award-winning, and NSF supported, collection of 129 applets that tutor students in precalculus and calculus. A new release, v.1.4, for fall of 2013 will have over 50 new or revised applets, including 13 new applets on precalculus (including 4 on complex numbers), 11 on limits, 9 on derivatives, 3 on integrals, 1 on series, 8 on geometry and vectors, 2 on partial derivatives, 4 on differential equations. Some of these will be presented in the talk. Most of these were initially designed by students in honors calculus classes at Texas A&M University. Work is also proceeding on converting the Maplets into the Java-independent MYMathApps Lessons which will work in any browser including those on mobile devices.

Amanda Harsy Ramsay IUPUI (Indianapolis University Purdue University Indianapolis) (aharsy@iupui.edu)

Using Programming to Understand Limits in a Calculus II Class

Students in calculus often struggle with understanding the precise definition of the limit of a function, sequence, or series. This talk will discuss the interesting aspects of a Calculus II programming assignment that was used to help students understand this concept. Students were asked to write a program that would first help them determine the limits of some specific sequences and series. Then they were asked to program tests that would give good justification supporting their claims. This justification required them to use an "adjusted" precise definition of a convergent sequence or a sequence which diverges to infinity. Finally, the students were asked some response questions about the project. For example, they were asked to discuss the benefits and shortcomings of using a program to "prove" the existence of a limit. As a bonus, this project required students to be aware of some interesting programming issues like variable overflow. This exercise allows students, especially computer science majors, to connect calculus and programming ideas.

Derek Thompson Trine University (theycallmedt@gmail.com)

Video Games and Calculus

Video games are now a huge part of American culture, and are ripe with examples for calculus. In this talk, I will describe video game projects for Calculus I students involving tangent lines, linear approximation, optimization problems, and related rates. I will focus on the iOS capabilities of the iPad and iPhone and how to use them with PC/Mac software packages Reflector and Geogebra to create your own examples for your students. My examples include modern games like Angry Birds as well as classics such as Final Fantasy.

Mako Haruta University of Hartford (haruta@hartford.edu)

iPads in the Classroom: A Departmental Project at the University of Hartford

This talk will outline our mathematics department's one-year experience introducing classroom sets of iPads into all sections of Calculus I, as well as trial units in Precalculus and Calculus II. Departmental iPad use has been primarily in group work settings in conjunction with the Flipping Calculus project, and in tandem with WeBWorK and the TI-89 graphing calculator. Tablet technology along with Apple TV and short throw projectors are being used to facilitate classroom discussion, enhance group work projects, create and share class notes, and monitor student iPad use. Effective, interactive apps for both student and instructor use will be presented.

Kristen Sellke Saint Mary's University of Minnesota (ksellke@smumn.edu)
Janel Schultz Saint Mary's University of Minnesota (jschultz@smumn.edu)

Implementing the Flipped Classroom in a First-Year Pre-Calculus/Calculus Course

As an attempt to improve the completion and pass percentages for students in a two-semester combined pre-calculus and calculus course, two members of the department began experimenting with the flipped classroom pedagogy in the fall of 2012 and continued into the spring semester. This presentation will briefly describe how the flip changed the daily class structure, look at student response to the flip through surveys, investigate the impact on student grades in the courses and discuss the departmental response to the experiment.

Lori Dunlop-Pyle University of Central Florida (lori.dunlop-pyle@ucf.edu)
Ivan Garibay University of Central Florida (igaribay@ucf.edu)
Ozlem Garibay University of Central Florida (ozlem@ucf.edu)
Amanda Koontz Anthony University of Central Florida (Amanda.Anthony@ucf.edu)

SONET-MATH: Using Social Networks to Learn Mathematics

In the summer of 2012 our group created a pilot program using a social network called "Discuzz" to enhance STEM education in a Calculus II class. Our goal in this ongoing project is to facilitate learning among the current generation of digital natives who compose our calculus classes. Our pilot course used social media as a tool to extend the classroom beyond the physical time and space that we met. In our report we detail the course structure we developed to maximize the benefit of using social media in the classroom.

Over the semester we studied how students used the social media to increase learning. Students chose study groups at the beginning of the semester and were given weekly collaborative assignments. The sociologist on our team surveyed students in our pilot course (our experimental group) as well as the students enrolled in the other summer sections of Calculus II (our control group) three times throughout the semester. Through these surveys we learned about students' perspectives on the roles that they and the course structure play in their learning. Additionally, we studied the learning communities within the class and how students used social media to facilitate learning.

Our findings were a wealth of information. We learned how students use social media and the features that a social media site needs when used for STEM education. Our assessments showed a 42% increase in student learning based on grades and also a 42% increase in student learning based on a Pre-Test and Post-Test assessment given to the experimental and control groups. As this is an ongoing project, our first pilot helped refine questions we want our research to answer and goals we wish to accomplish when we run our next pilot and develop the social media program we will use in future classes.

Elizabeth Miller The Ohio State University (elizmiller@math.osu.edu)

Technology Enhanced Large Calculus Lectures

In order to improve student experience in large calculus lectures at The Ohio State University, the Mathematics Department began a project in 2008 to develop and implement a better calculus experience for students. The result is our Technology Enhanced Calculus Lectures, which were first piloted in Autumn of 2010 and have been continuously improved since then. This style of lecture includes teaching using a computer, writing in digital ink, demonstrating concepts using interactive online figures, and posting pdfs and videos of each lecture online for students after class. Some instructors have also experimented with using clickers to make the class more interactive. A challenge of this program has been to recruit and train new lecturers to teach in this method each semester, and has led to the creation of a technology enhanced lecturer coordinator role in the department. This type of teaching has been shown to increase student satisfaction with the calculus course and to at least maintain student achievement. The implementation of these technology enhanced lectures has also led to a higher visibility for the uses of instructional technology in the department, more experiments with instructional technology, and the formation of a department eLearning committee. This talk will explain and demonstrate various ways these technology enhanced lectures are being taught in our department.

GCPS Session 8: Other/Research in Applied Mathematics

Friday, August 2, 8:30–11:30 AM, Room 17

Narayan Thapa Minot State University (narayan.thapa@minotstateu.edu)

Numerical Solution of sine-Gordon Equation by Spectral Method

In this paper we have presented spectral method to solve one dimensional nonlinear sine-Gordon equation. The results of numerical experiments are presented and are compared with analytical solution. We established error bounds for finite spectral approximations $u_N(t)$. Numerical results are presented.

Corban Harwood George Fox University (rharwood@georgefox.edu)

Stonger Numerical Stability for Nonlinear PDEs

This paper presents conditions on numerical methods for nonlinear PDEs which forces them to be oscillation-free as well as stable. Stability conditions can allow infeasible results and should not be tolerated. Positive eigenvalues are a sufficient condition to provide oscillation-free behavior, but an optimal upper bound on the necessary condition would allow maximal freedom for the step sizes. This necessary condition is found for various nonlinear PDEs by correlating the balance between positive and negative eigenvalues for linear problems with ensuring numerical monotonicity for certain nonlinear problems. Our primary focus is on reaction-diffusion equations.

Terry Jo Leiterman St. Norbert College (terryjo.leiterman@snc.edu)

An Exploration in Differential Equations for Modeling Population Growth

We study a system of coupled, linear, constant-coefficient difference equations. The system of equations were built to model particle population dynamics influenced by piece-wise constant growth and settling in a fluid system arranged as vertically stacked horizontal layers. We transform the set of difference equations to a set of differential equations and compute exact solutions under limiting conditions on the coefficients. These conditions model idealized physical scenarios of particle growth and settling. We focus on the mathematical approaches required to compute and analyze the model solutions. We offer time-varying predictions for the dynamics of specific particle populations associated with fixed parameter conditions.

Raymond Puzio PlanetMath.org (puzio1@excite.com)

A Theory of Formal Mathematical Reasoning

This presentation will give an overview of a theory of mathematical reasoning which is being developed for use in automated proof checking and representation of mathematical knowledge. It is organized in three levels:

The lowest level is that of expressions and statements. These are represented as a certain type of network which form an algebraic system under juxtaposition and contraction.

The next level is that of inference. Deductions take the form of substitution of subnetworks. The totality of admissible modes of inference may be described by a relational system or by consequence operators.

The third level is that of relation between theories. The goal here is to understand how statements in one theory translate to statements in other theories. This is accomplished by formalizing what it means for one theory to be a metatheory of another theory.

The goal of this work is to produce a mathematical model of the process of mathematical reasoning which hopefully is an accurate reflection of the way mathematicians think about their subject and so will be of use in applying techniques of artificial intelligence to mathematics.

Brooke Andersen Assumption College (brandersen@assumption.edu)

Comparing Reducibilities on Computably Enumerable Sets

We will look at various strengthening of Turing reducibility and see how these reducibilities compare when restricted to computably enumerable sets. Particular attention will be paid to truth-table reducibility and we will see how strengthening or weakening this reducibility will give rise to new reducibilities.

Oksana Bihun Concordia College at Moorhead, MN (obihun@cord.edu)

Solvable and/or Integrable Many-body Models on a Circle

We will consider several many-body models that describe N points confined to move on a plane circle. Their Newtonian equations of motion are integrable: they allow the explicit exhibition of N constants of motion in terms of the dependent variables and their time-derivatives. Some of these models are moreover solvable by purely algebraic operations, by explicitly performable quadratures and functional inversions. The models are constructed using the techniques of generalized Lagrange interpolation or by reinterpreting known models. This is a joint work with Francesco Calogero.

Becky Hall Western Connecticut State University (hallb@wcsu.edu)

An Assignment that Promotes a Symbiotic Relationship Between Math Pre-Service Teachers and High School Students

It can be difficult to provide pre-service teachers with real teaching opportunities prior to student teaching. In this talk I will discuss an assignment given to pre-service secondary mathematics teachers that provides such an opportunity. This assignment allows pre-service teachers to teach actual high school students and gives them an opportunity to reflect upon and adjust their lesson. Additionally, the assignment benefits the local high school students who participate. Details of the assignment and how it is implemented will be discussed.

Pari Ford University of Nebraska at Kearney (fordpl@unk.edu)

Flipping a Math Content Course for Elementary School Teachers

Learn math in your pajamas. The flipped class format involves students receiving instruction outside the classroom and working on "homework" inside the classroom. In this talk I will share my experience with flipping a math content course for pre-service elementary teachers in Fall 2012 at the University of Nebraska at Kearney. I prepared video lessons for my students to view using various tools; video camera, iMovie, and iPad apps like Educreations and Doceri. A class session would begin with me leading the class through a warm up problem to provide me with feedback on questions they still had related to the material. The remainder of class was spent with the students working in groups on exercises and I observed their work, addressing questions as needed. Each group submitted one write up of their solutions at the end of class to be graded. The focus will be on this first experience with the particular audience of pre-service teachers, but this year I also flipped my college algebra, trigonometry, and applied calculus classes (all general studies courses). I will share comments from students as well as components of this teaching method that worked well for my students and components that need re-examined for their effectiveness.

Ekaterina Lioutikova Univeristy of Saint Joseph (Connecticut) (elioutikova@usj.edu)
Barbara Henriques University of Saint Joseph (bhenriques@usj.edu)

Integrating Content, Pedagogy, and Cognitive Coaching to Support K–8 Teachers' Implementation of Common Core

Acceptance of the Common Core Mathematics Standards by the majority of states leads to a renewed need for professional development efforts to support practicing teachers of mathematics. In this talk, we will share highlights from the Innovative Mathematics Academy Advancing Learning and Leadership, a three-year program for teacher leaders developed at the University of Saint Joseph in partnership with the Consolidated School District of New Britain (Connecticut) and supported by a state MSP grant. Results indicate a positive impact of the program on the participants' mathematical content knowledge for teaching, on their classroom practices, and on their confidence in leadership.

Martha Ellen Waggoner Simpson College (murphy.waggoner@simpson.edu)

Using Doodling to Teach the Mathematics of Art

This talk describes activities used in a course called Mathematics of Art. This general education course satisfies both quantitative reasoning and arts requirements and has no prerequisites.

To qualify as an arts general education course, the students must learn to express themselves through art. I use doodling as a medium that enables students with no prior experience to produce art from the beginning of class using simple materials. Another reason for doodling is that it can be used to illustrate several mathematical concepts.

In this presentation I will describe activities where doodling was used to introduce the mathematics of knots and links, spirals, 2-colorable maps, Euler's formula for polyhedra and other concepts. Examples of student work, their reflections on the artistic process and their responses to the course will be included.

David Nacin William Paterson University (nacind@wpunj.edu)

A Complex Calcudoku Classification

We present a short proof of the conditions that a class of Calcudoku puzzles over the complex numbers must meet in order for solutions to exist and be unique.

GCPS Session 9: Teaching Introductory Mathematics

Part 1: Friday, August 2, 1:00–3:55 PM, Room 17

Christopher Schroeder Morehead State University (c.schroeder@moreheadstate.edu)

College Algebra in the High Schools

Over the last couple of years, Morehead State University has embarked on an aggressive campaign to make college classes available to high school students in our region. The Early College Program offers dual-credit in numerous subject areas, including mathematics. By far the largest class offered is College Algebra. We will look at some of the techniques involved in coordinating multiple sections at different schools including homework, testing, and technology. Also, we will evaluate some of the results of this effort and comment on the future of the program.

Dale Bachman University of Central Missouri (dbachman@ucmo.edu)
Nicholas Baeth University of Central Missouri (baeth@ucmo.edu)

Honors College Algebra at the University of Central Missouri

This presentation gives an overview of a College Algebra course designed for Honors students that has been offered at the University of Central Missouri once each year for the past six years. The purpose of the course is twofold: to provide honors students with a standard toolbox of mathematical skills, and to help the students learn to see the world from a mathematical viewpoint. We employ a mixed strategy involving

- integrated technology in the form of graphing calculators and computer-aided learning, and
- a homework model using video lectures and computer-based exercises.

In this presentation we will detail our approach to the course and communicate student learning results based on scores on standardized tests as well as anecdotal results gathered during semi-formal interviews.

Alexander G. Atwood Suffolk County Community College (atwooda@sunysuffolk.edu)

Using Algebra in the Classroom to Understand the Way in which Automobiles Collide

The concepts of Intermediate Algebra can provide surprising qualitative and quantitative insight into the way in which automobiles crash into fixed barriers and into other automobiles. Using linear equations in one variable and quadratic functions, and some elementary physical concepts, one can analyze and understand what really happens when a car crashes.

Curtis Card Black Hills State University (Curtis.Card@bhsu.edu)
Daluss Siewert Black Hills State University (Daluss.Siewert@bhsu.edu)

Developmental Math as a Gateway, not a Gatekeeper.

Black Hills State University, like so many other universities, has struggled with the problem of students failing Basic Algebra and Intermediate Algebra. Many of these students subsequently failed to graduate. The mathematics department made structural changes in the way these courses were offered. To address this issue, the mathematics department transformed the way these courses were offered. In this presentation, we will give a brief historical overview of this transformation, its impact on student success, and how it led to receiving a National Science Foundation TUES (Transforming Undergraduate Education in Science) grant. In addition to structural changes several instructional changes were implemented. This presentation discusses these implementations and the effect they had on the pass rates of students taking Basic Algebra and Intermediate Algebra.

Supported by NSF Grant #1141334: The Predictability of Student Attributes and Instructional Milieu on Success in Developmental Math Courses, College Algebra, the CAAP exam, and Matriculation to Degree. L. Pearce, PI; C. Card, K. Pearce, and D. Siewert, Co-PIs.

Daluss Siewert Black Hills State University (Daluss.Siewert@bhsu.edu)
Curtis Card Black Hills State University (Curtis.Card@bhsu.edu)

Transforming Developmental Mathematics Classes

(NOTE: This talk expands on the work discussed in our earlier session titled "Developmental Math as Gateway not a Gate Keeper")

This spring we completed the fourth year of our developmental math project. The initial results from these changes have motivated us to develop a multi-tiered system of support to identify and assist at-risk students as early as possible. We will discuss the systems of support used and how they were implemented, as well as a discussion of how these students performed in their subsequent college level classes and how much this transformation cost the university. Lastly, opportunities to be involved with this project by replicating our work at your institution will be discussed as we are planning to apply for an NSF Type II TUES grant.

Supported by NSF Grant #1141334: The Predictability of Student Attributes and Instructional Milieu on Success in Developmental Math Courses, College Algebra, the CAAP exam, and Matriculation to Degree. L. Pearce, PI; C. Card, K. Pearce, and D. Siewert, Co-PIs.

Pangyen Weng Metropolitan State University (pangyen.weng@metrostate.edu)

Preparing Students for College Math: A Successful Model of One-Semester Developmental Math

At Metropolitan State University, students are required to pass a one-semester developmental math course (instead of several) before entering college-level math courses. Of the three parallel courses at Metro State, MATH 98 is offered most often and usually taken by students with least preparation.

The author describes the design and methodology of MATH 98, as well as how it is coordinated and implemented in 25-plus sections a year amongst 10-plus adjunct instructors. The author will then assess the effectiveness of MATH 98 by examining data of student performances in their next college-level math classes after MATH 98.

Solomon Abogunde Iyekekpolor Federal University, Wukari, PMB 1020, Wukari-Nigeria (driyekekpolor@gmail.com)

Improving Secondary School Students' Mathematics Achievement in Nigeria through the use of Tutorial Computer-Aided Instruction

The study examined the influence Tutorial Computer-Aided Instruction (CAI) in collaboration with conventional teaching method would have on male and female secondary school students' geometry and trigonometry achievement. A sample of 173 students made up of 125 boys and 48 girls drawn from two federal unity secondary schools in North-east geopolitical zone of Nigeria was investigated. The sample was divided into two groups the experimental and control. The two groups were taught geometry and trigonometry for six weeks using conventional instruction for the control group and supplement of Tutorial-CAI programmed packages for the experimental group. Pre-test and Post-test were administered to all the research subjects before and after the experiment respectively to determine differences in achievement if any, between the two groups and between the genders in the experimental group. After data analysis using means and Analysis of Covariance (ANCOVA) statistics, the findings showed that: (i) Tutorial-CAI produced higher geometry and trigonometry achievement in students than the conventional instruction (ii) with Tutorial CAI, girls performed better than boys. It was recommended among others that Government and well-meaning individuals and corporate organizations should make provision for the introduction, implementation and funding of Tutorial CAI in mathematics in Nigerian schools to remedy the current students' dismal achievement level in the subject.

M Tip E Phaovibul University of Illinois: Urbana-Champaign (phaovib1@illinois.edu)

Learning Geometry via Manipulate

In this talk, I will discuss how to use manipulate such as Magformer to help students visualize several concepts in geometry such as Tessellation, Solid Geometry, and Symmetry Group.

Emek Kose St. Mary's College of Maryland (ekose@smcm.edu)

Linking "Women in Mathematics" and Middle School Girls Through Mentoring

The mathematics survey course "Women in Mathematics," studies the lives and mathematical contributions of women mathematicians throughout history, the current gender equity issues in education and mathematical careers. The students of the class mentored 20 middle school girls throughout the semester. The MAA Tensor Women and Mathematics Grant made the design and teaching of this class possible. We will present a typical lesson on Emmny Noether and symmetry groups, sample activities for the middle schoolers and the results of attitude change towards mathematics and equity both by the students and the middle schoolers.

George F McNulty University of South Carolina (mcnulty@math.sc.edu)
Nieves F McNulty Columbia College (nmcnulty@columbiasc.edu)
Douglas B Meade University of South Carolina (meade@math.sc.edu)
Diana White University of Colorado Denver (diana.white@ucdenver.edu)

South Carolina High Energy Mathematics Teachers' Circle: A First Year Experience—Playing It By Ear

We launched our Circle in the summer of 2012 with a summer immersion workshop. During the academic year we met in four-hour sessions one Saturday almost every month, finishing up in June with an emergence workshop. We have begun to recruit more members to our circle for the coming year and plan to hold a second immersion workshop in July. We will report on how our Circle has evolved and how we have adapted our circle sessions to the desires of our teachers; for example including connections to the Common Core. We will also report on the incorporation of new members into the existing circle, the growing role that computer technology may play, and the assessment of our efforts during the first year.

Ted Theodosopoulos Saint Ann's School (ttheodosopoulos@saintannsny.org)

From Problem Solving to Research

We describe a math circle environment used to bridge advanced high schoolers from exploring problem solving techniques to formulating and investigating research questions. We discuss examples of resulting research projects, from Fourier analysis and linear algebra to generating functions, the probabilistic method and foundational issues of formal undecidability. In each case, the original inspiration arose from group exploratory analysis of strategies to approach problems in graph colorings, recursions, cooperating games and functional equations. After a solution to the original problem is arrived at, the students pursued generalizations and extensions to push the envelope of applicability of the requisite technique, ultimately arriving at conjectures, which form the seeds of year-long research projects. Throughout the process, we emphasize the importance of recasting old questions in the light of new representations, and communicating our evolving, if partial, understanding of the problem landscape to our peers. Ultimately it is this iterative reshaping of our language, to better capture more facets of our object of study, that leads to fruitful research projects.

Victor Piercey Ferris State University (piercev1@ferris.edu)

Using Projects to Support Quantitative Literacy

Projects can be used to allow students to conduct sophisticated analysis with elementary mathematics. As such, the use of projects fits very well within a quantitative literacy course. In this talk, I will discuss examples of both short and long term projects and the observed impact on student engagement.

Sarah Ultan UW-BC (sarah.bennett@uwc.edu)

Doing SoTL (Scholarship of Teaching and Learning) Projects

For this interactive discussion I will begin by describing a few different kinds of SoTL projects, summarize a few that I have done, and give some explanation of the ndings. This includes two lesson studies with all four math instructors on our campus and a three semester pre and post survey of students, all from College Algebra courses. I will share some resources for getting started in doing SoTL research. And, I will to open the discussion up so others can share SoTL work they have done to inspire and motivate new ideas for the group. We will talk about what some valuable and interesting projects might be that participants are interested in considering.

GCPS Session 10: Research in Algebra or Topology

Saturday, August 3, 8:30–9:55 AM, Room 15

Sean Corrigan Saint Louis University (sean_corrigan@yahoo.com)

Best Representations and Intervals of Uncertainty in a Weakened Topology for the Integers

We will investigate a particular metrizable group topology for the integers in which the sequence of factorials converges to zero. In this topology, determining an integer's metric distance from zero comes down to writing the integer as a particular combination of factorials. This combination is known as a best representation, and we will find some explicit best representations around intervals in the integers where previous work was inconclusive. Furthermore, we will show that in this topology, each integer has a unique best representation.

Aaron Heap SUNY Geneseo (heap@geneseo.edu)

Understanding the Johnson Filtration of the Mapping Class Group via Geometric Topology

The mapping class group of a surface (with one boundary component) is the group of orientation-preserving homeomorphisms of the surface. When one studies the Torelli group and Johnson filtration of the mapping class group, it is natural to use the tools of algebra and study their corresponding subgroups of the automorphism group of a free group. While this is quite useful for understanding the algebraic structure of these groups, it hides some of the topological aspects of them. As surface homeomorphisms, the elements of the mapping class group are certainly topological objects as well as algebraic objects. We explore how a mix of geometric topology and algebraic topology notions can be used to understand their algebraic structure from a more topological point of view.

Heather Dye McKendree University (hadyle@mckendree.edu)

Bounds on Mosaic Knots

We investigate relationships between bounds on the crossing number and the mosaic number of mosaic knots.

RICHARD K. OLIVER Missoula, Montana (rko15@att.net)

On the Parity of a Permutation

One of the most useful ideas in combinatorial reasoning is that of a group action. I will present a proof based on this idea of the fact that the parity map $\varepsilon : S_n \to \{-1, 1\}$ is an epimorphism. This follows my note: On the parity of a permutation, *Amer. Math. Monthly* 118 (2011) 734–735. I will then discuss several other proofs, starting with the one given by Cauchy in 1815.

B Ntatin Austin Peay State University (ntatinb@apsu.edu)

Semi-Simple Lie Groups Acting on Flag Manifolds

A Lie group acting on a manifold normally gives rise to an induced action on the parameter space of certain geometric objects related to the manifolds in question. For instance, from the canonical representation of $SL_n(R)$ on C^n, one obtains actions on the parameter spaces of linear subspaces of C^n, that is, Grassmannians $Gr_k(C^n)$ of k-dimensional subspaces of C^n, flag manifolds $F_{1,2,3,\ldots,k}(C^n)$ and so on. In this talk we will give a characterization of the parameter space of certain nonlinear geometric objects (cycles) in the natural setting of group action of real forms of semi-simple Lie groups on flag manifolds. We will consider low-dimensional examples.

GCPS Session 11: Teaching Calculus

Part 2: Saturday, August 3, 8:30–10:40 AM, Room 21

Jose Giraldo Texas A&M University Corpus Christi (jose.giraldo@tamucc.edu)

Can The Beauty of Limits Be Recovered in Calculus?

It is not a mystery that the discussion of limits in calculus is avoided as much as possible due to the difficulty of the concept. Most of the time the conversation on this subject is reduced to calculating limits. Even more it is reduced to "plugging in" to obtain a number or infinity. Experience shows that students difficulty start with simple understanding of convergence of real numbers. In this approach the intuition of limits is introduced early attempting to use the knowledge of basic functions to move to discuss them through sequences. Convergence of sequences is discussed to later use it to make claims about limits of functions for which the students have already made conjectures. The conversation about limits is maintained throughout the course to allow the student to be exposed as much as possible to the concept and its uses. Introduction of new notation to treat limits has shown to help students distinguished when a limit is reached and when it is not, which completely changes the tone of the conversation for students.

Relevant examples about how understanding of limits of basic functions help to manipulate more complicated ones are to be presented. Even more, several limits that normally are done using L'Hoppital's' rule can be done without using derivatives.

Tim Boester Wright State University (timothy.boester@wright.edu)

Deconstructing the Formal Definition of Limit at a Point

Different theories of how students come to conceptualize the formal definition of limit at a point have all broken apart the monolithic definition into conceptual pieces. This talk will briefly explore those different dissections, along with a real-world problem that helps students unpack the definition. Evidence from interview questions will be presented to give insight as to what conceptual pieces are most difficult for students, and which pieces necessarily proceed others.

David Dwyer University of Evansville (dd4@evansville.edu)
Mark Gruenwald University of Evansville (mg3@evansville.edu)
Mike Axtell University of St. Thomas (axte2004@stthomas.edu)
Ken Luther Valparaiso University (ken.luther@valpo.edu)
Joe Stickles Millikin University (jstickles@mail.millikin.edu)
Nicholas Baeth University of Central Missouri (baeth@ucmo.edu)

Resequencing Calculus with an Early Multivariate Approach

The Resequencing Calculus project is redesigning the traditional calculus sequence with an early multivariate approach. Topics are ordered so that material prerequisite for upper-level STEM courses is front-loaded into the first two semesters and so that there is a natural progression of difficulty throughout the 3-course sequence. This is accomplished by introducing multivariate calculus in Calculus 2 and postponing infinite series, arguably the most daunting topic in calculus, until Calculus 3. As a consequence, Calculus 1 and 2 form a strong two-course sequence for students in the life sciences, economics, and chemistry, all of whom are likely to encounter multivariable models in later courses within their disciplines. Moreover, students successful in Calculus 2 may enter directly into not only Calculus 3, but also differential equations, linear algebra, or calculus-based probability. The restructuring eases time pressure in Calculus 3, thus facilitating a thorough treatment of vector calculus through Stokes Theorem and the Divergence Theorem while maintaining a rigorous treatment of the material. We will discuss the progress and assessment of the project and the supporting text to date, plans for pilots at multiple institutions in the fall 2013 semester, and approaches to contending with challenges posed by course transfers and AP credit. This project is supported by NSF Grants DUE 1225566 and 0836676. Details can be found at http://www.resequencingcalculus.com.

Melissa Stoner Salisbury University (mastoner@salisbury.edu)

Rigorous Calculus I Course for Biology Majors

Based on reports released in 2010, mathematics is quickly becoming important for science majors as their career fields change and adapt to handle the worlds newest problems. With an emphasis on exposing students to the skills they may need within the context of their field, we strive to better equip our biology majors to handle changes in their field and develop critical thinking skills. At Salisbury University we have created a course, MATH 198, which is designed to be a rigorous differential calculus course with a biological focus. This talk will focus on meeting all the demands of a standard calculus 1 course for math majors while providing a meaningful experience for students. We will discuss the differences in the target population, challenges, successes and potential change as the course evolves.

Bradley James Paynter University of Central Oklahoma (bpaynter@uco.edu)

Convincing Students That Old Dogs Can Learn New Tricks

Non-math majors are often apprehensive about taking a math course. They bring with them significant baggage from earlier classes that has convinced them that they are no good at math and cannot learn. Following the work of C. S. Dweck, I have developed an idea to try and change this mindset in my Business Calculus students using my own poor basketball skills as an analogy. In this talk I will present the materials developed (including video evidence of the aforementioned terrible basketball) and student reactions.

Charlotte Ann Knotts-Zides Wofford College (knottszidesca@wofford.edu)

Teaching Calculus to Students who have already seen Calculus

The math department at Wofford College decided to resequence the topics in Calculus I and II to address the concern that an increased number of students in our classes have already had calculus in high school. This gave us an opportunity to create a calculus class where derivatives and integrals are introduced almost simultaneously. For example, shortly after we discuss the product rule for a derivative, we introduce integration by parts as its partner for antiderivatives. The result is a course that is refreshingly new to students who had calculus in high school and that demonstrates applications of both derivatives and antiderivatives even to students who only take one semester of calculus. In this talk, I'll discuss our new sequence of topics in Calculus I and II and talk about its impact on our students.

Darja Kalajdzievska University of Manitoba (kalajdzi@cc.umanitoba.ca)

Taking Math Students from 'Blah' to 'Aha!'; What Can We Do?

Students coming from high school entering university level mathematics courses are not always prepared for the level of content, assessments, and teaching environment that they face. This talk will present some research and strategies which may help to bridge this gap by re-thinking some of our teaching practices at the university level. Examples including some non-traditional approaches to assignments, exam review sessions, and test questions in Introductory Calculus and Differential Equations classrooms will be highlighted, as well as student responses and feedback.

Paul Sisson Louisiana State University in Shreveport (paul.sisson@lsus.edu)
Tibor Szarvas Louisiana State University Shreveport (tibor.szarvas@lsus.edu)

Teaching Calculus through History, Intuition, Exploration, and Development (HIED)

Calculus is still too often presented as a collection of tools and theorems devoid of human connections and relationships to other topics. This tendency is understandable, given the sheer amount of material many departments try to cram into their calculus sequence, but learning usually suffers as a result. Drs. Sisson and Szarvas, both of whom have many years of experience as professors of mathematics and as university administrators, describe their approach in the classroom and in their textbook as teaching calculus through the use of History, Intuition, Exploration, and Development (HIED). These four themes allow students to learn calculus by making connections with what they already know, what they suspect to be true, what they discover through the use of technology, and what they logically develop with the guidance of the professor and each other.

Jeffrey William Clark Elon University (clarkj@elon.edu)

Unit Acceleration Vectors

In Multivariable Calculus, a good deal of the analysis of differentiable curves begins with the unit tangent vector, or equivalently, with parametrizing the curve so that the velocity vector has unit length. This talk will look the analysis of differentiable curves by parametrizing them so that the acceleration, while possibly changing direction, will always have unit length. (This can be interpreted as traversing the curve in such a way that a passenger in a vehicle traveling the curve will be subjected to a force of constant magnitude.) High curvature will require slow speeds, and this talk will examine the influence of curvature on the time required to traverse the curve.

GCPS Session 12: Teaching Introductory Mathematics

Part 2: Saturday, August 3, 8:30–10:25 AM, Room 22

Russell Coe Suffolk County Community College (coer@sunysuffolk.edu)

Behind the Scene: What the Brain Thinks the Eyes Are Seeing

Consider the expression $p + (q + r)$. To apply the associative property $[p + (q + r) = p + (q + r)]$, the brain must think it sees three objects; but to apply the commutative property $[p + (q + r) = (q + r) + p]$ the brain must think it sees only two. Making students consciously aware of this type mental-visual coordination is important for their advancement in mathematical ability; yet it is not pointed out in a typical math text. If a student needs to apply the associative property to an expression of the form $(p + q) + (r + s)$ without having the proper mental-visual coordination, the student will not see this expression as an operation of three objects, p, q, and $r + s$, and will thus get stuck. There are many other problems, both simple problems and more challenging problems in higher levels of mathematics, requiring sophisticated uses of this brain-eye technique. Without learning this technique students may not be able to handle these problems or be able to use the technique innovatively on their own. I wish to discuss this and other mental-visual teamwork situations, such as coping with distracting detail and avoiding over-dependence on visual form, which would highly benefit students were they to be made more consciously aware of them. Overall, students need to be both more flexible and also more cautious in determining what their eyes are showing them.

James Fulton Suffolk County Community College (fultonj@sunysuffolk.edu)

A New Approach for the Liberal Arts Mathematics Courses

The liberal arts mathematics course is one of the most difficult courses to teach. Two approaches to these courses have emerged. The first takes a modeling approach and tries to teach students the utility of mathematics through various predictive models. The problem with this approach is that not all students are equipped to apply the mathematics at this level, so many students become lost or disinterested. The second is to present the students with a hodge-podge of mathematical topics from numbers to sets, logic, probability, statistics, etc. This approach is totally unsatisfying, and leaves the students with the impression that mathematics is just an odd array of disconnected concepts and ideas, with not real impact on their lives.

In this talk we propose an alternative liberal arts approach. The courses have a unifying theme with one course focusing on showing mathematics as a true liberal art with a connection to philosophy, history, psychology, education, cognition, epistemology, science, and critical thinking in general. The other presents the mathematics of patterns—exploring random, geometric, and numerical patterns, ending with the connection to symmetry and its language—group theory. Liberal arts students appreciate the courses, and have commented that its their favorite course, or years later have contacted the instructor stating how the course has impacted them. Textbooks were written to accommodate this new approach, and the courses have become a very satisfying and rewarding experience, as we can now take the students on a fascinating Mathematical Mystery Tour.

Xinlong Weng University of Bridgeport (gatewaytochina@hotmail.com)

Belended Developmental Mathematics Courses

We may all agree that it is essential to do homework in order to understand mathematics. But, the matter of fact is that very few students have done homeworks regularly before we introduced this belended math course. Beside some other benefits, one obvious advantage is that almost all students do homework online regularly because of the instant feeback and the chance to re-try again.

Hari Naraayan Upadhyaya Scholars Home Academy (hnupadhyaya@gmail.com)

Helping Students Learn Geometry Using the Teacher made Manipulative

The first lesson of mathematics to every child begins from geometry. The child enters the world of learning communicating with the geometric shapes of some kind where learning becomes touching, feeling and experiencing and learning becomes fun and excitement to the child unless this learning sequence is no broken. Unfortunately our approach to geometry teaching becomes too abstract reasoning as a sudden jump from concrete world to the very formal one that kills their interest to learn. Therefore, my approach in this connection is to help students learn mathematics, geometry in particular, where they they can touch, feel and experience the geometrical proofs playing with the teacher made resources in the beginning. Later on it becomes the self motivation to find and seek the geometric reasoning in the materials made by themselves. Most of the school level lessons could be designed this way, but for the purpose of the simplicity of ideas, I have taken an example of geometric proof of the relation between the different angles when a pair of parallel lines are intersected by a transversal. My approach to these is to help students prepare materials working together with me, identify different kinds of angles, visualize the relations between them, justify the relation (prove the theorems) and use/apply them in proving the most advance cases and solve problems related to these generalizations.

Edmund A Lamagna University of Rhode Island (eal@cs.uri.edu)

Puzzles + Games = Mathematical Thinking

Puzzles and games form the basis of a freshman seminar designed to develop mathematical and computational problem solving skills. The desired learning outcomes include: 1) helping students transition from high school to college, 2) motivating and creating excitement for the further study of mathematics, computer science, and other STEM disciplines, 3) allowing students to work in small groups on fun, interesting problems, 4) introducing mathematical and computational problem solving strategies not typically encountered in high school or college, 5) instilling the confidence and persistence needed to solve complex, difficult problems, and 6) encouraging "out of the box" thinking and applying alternative problem solving strategies.

Each session begins by introducing a set of puzzles of a particular type or a game illustrating a particular principle. Students spend most of the period in small groups solving the puzzles or playing the game. Toward the end of class, students present and discuss their solutions with guidance from the instructor.

Topics include sequential movement puzzles, figurative numbers, proofs without words, probability, logic, number systems, algorithms, recursion, and graphs. Among the games played are Clue, Mastermind, and Ticket to Ride.

Brian Heinold Mount St. Mary's University (heinold@msmary.edu)

Some Different Applications of Logarithms

Over the last year or so I have been collecting a variety of applications of logarithms for use in teaching. Many, if not most, should be different from the ones you may be familiar with. The applications are both interesting and useful for any class that introduces logarithms.

Gargi Bhattacharyya University of Baltimore (gargibhattacharyya@gmail.com)

Case study: Student with Dyscalculia Offered History of Mathematics Course to Satisfy General Education

The talk will discuss the case study of a student with Mathematics disability dyscalculia—a learning disability involving innate difficulty in learning or comprehending arithmetic. The student had appealed for waiver from the general education mathematics requirement but was denied initially. There was an appeal again and the issue dragged for two years. Eventually a history of mathematics course was developed for the student that satisfied the state general education requirement.

Tracey McGrail Marist College (tracey.mcgrail@marist.edu)

The Challenges of Designing a Mathematics Course for Liberal Arts in a Former Soviet Republic

Last year, I had the exciting opportunity to visit and teach at the American University of Central Asia, an American-style liberal arts college in Bishkek, Kyrgyzstan, the "Paris of Central Asia." In this talk I will discuss some of the pedagogical challenges to designing and teaching an engaging yet rigorous mathematics course for liberal arts students in a former Soviet republic.

GCPS Session 13: Modeling/Applications

Saturday, August 3, 8:30–10:45 AM, Room 26

Brian Winkel United States Military Academy (BrianWinkel@hvc.rr.com)

Modeling Opportunities with Differential Equations in the Classroom

We demonstrate a number of mathematical modeling activities for motivating and learning differential equations using modeling and technology. We shall engage those present in at least one modeling activity and demonstrate others. These include population modeling with M&Ms (simple and logistic growth), sublimation of carbon dioxide, Torricelli's Law, Tuned Mass Dampers, oil slick spread, coffee filters falling, spring mass systems.

Pavel Belik Augsburg College (belik@augsburg.edu)
Doug Dokken University of St. Thomas (dpdokken@stthomas.edu)
Kurt Scholz University of St. Thomas (k9scholz@stthomas.edu)
Mikhail Shvartsman University of St. Thomas (mmshvartsman@stthomas.edu)

Fractal powers in Serrin's swirling vortex solutions

We consider a modification of the fluid flow model for a tornado-like swirling vortex developed by J. Serrin, where velocity decreases as the reciprocal of the distance from the vortex axis. Recent studies, based on radar data of selected severe weather events, indicate that the angular momentum in a tornado may not be constant with the radius, and thus suggest a different scaling of the velocity/radial distance dependence.

Motivated by this suggestion, we consider Serrin's approach with the assumption that the velocity decreases as the reciprocal of the distance from the vortex axis to the power b with a general $b > 0$. This leads to a boundary-value problem for a system of nonlinear differential equations. We analyze this problem for particular cases, both with nonzero and zero viscosity, discuss the question of existence of solutions, and use numerical techniques to describe those solutions that we cannot obtain analytically.

Edward Aboufadel Grand Valley State University (aboufade@gvsu.edu)
Beth Bjorkman Grand Valley State University (bjorkmab@mail.gvsu.edu)

Fighting Fires in Siberia

In this talk, we will describe a prize contest organized jointly by the Societe de Calcul Mathematique SA and the French Federation of Mathematical Games. Inspired by stories of Siberian fires in 2012, the proposers challenged contestants to place fire-fighters in a mathematical model of Siberia, so as to minimize fire damage costs and fire-fighting expenditures. We will describe the contest and present our solution to the challenge.

Jean Marie Marie Linhart Texas A&M University (jmlinhart@math.tamu.edu)

Mathematical Models of a Zombie Outbreak

Students love zombies, and zombies are a great way to get students interested in learning more mathematics. A basic model of a zombie outbreak involves standard techniques of epidemiological modeling using ordinary differential equations. These models can also be used to introduce discrete stochastic models. I will give an overview of the projects involving zombies that I've mentored in my mathematical modeling class.

Yan Hao Hobart and William Smith Colleges (hao@hws.edu)

Generosity Without Reciprocity: Computation Models of Need-Based Transfers and Risk-Pooling

Throughout our evolution, humans have faced uncertainty due to drought, disease, theft and other unavoidable risks. One way to manage risk is to pool it. Existing mechanisms of cooperation (e.g., kin based-altruism and reciprocity) do not appear to be designed for the pooling of risk. Effectively solving this adaptive problem may be best accomplished through alternative mechanisms such as a system of need-based sharing. Ethnographers have documented many such systems throughout the world. To test the viability of such a system in ecologically realistic conditions, we modeled the basic characteristics of the need-based sharing system (known as osotua) of the Maasai of East Africa. Our first model showed that this need-based sharing system promotes herd survival through a limited pooling of risk between partners. Next, we modeled osotua on a network. More connected and heterogeneous social networks lead to higher rates of survival. Further, individuals who selectively asked their wealthiest partners for resources survived longer than those who asked partners at random. We are now comparing the performance of need-based transfer rules to reciprocity/account-keeping rules. Our preliminary results suggest that, under ecologically realistic conditions, reciprocity does not fare as well as a system of need-based sharing.

Mohammed Yahdi Ursinus College (myahdi@ursinus.edu)

Stochastic Differential Equation Models of the Nosocomial Infection VRE

The nosocomial infections caused by Vancomycin-Resistant Enterococci (VRE) in intensive care units (ICU) are of great concern. Several stochastic modeling techniques, including differential equations as well as continuous and discrete-time Markov chains approaches are used to illustrate the VRE dynamics and the roles of the parameters, and ultimately to better prevention strategies. In particular, critical health conditions are linked to the effectiveness of the level of chlorhexidine baths in reducing VRE infections, mortality rates and the basic reproduction number. The models encompass the uncertainty of the parameter values and transitions, the distinction between VRE colonization and infection, as well as the use of special preventive measures and VRE treatments.

Sally Cockburn Hamilton College (scockbur@hamilton.edu)

Modeling Preferntial Admissoins at Elite Liberal Arts Colleges

This talk presents the results of a model that simulates the effects of varying preferential admissions policies on the academic profile of a set of 35 small liberal arts colleges. An underlying assumption is that all schools in the set use the same ratio of preferential to non-preferential admissions. The model predicts that even drastic changes in this ratio have little effect on these institutions' comparative academic profile, as measured by the mean SAT score of the incoming class. When preferential admissions rates are altered, admissions numbers must be altered as well to achieve a desired class size; the model predicts that these changes effectively cancel each other out.

Brian Harris Nathanson OptiStatim, LLC (brian.h.nathanson@att.net)

Ranking the Academic Output of Medical Schools in the United States Using Data Envelopment Analysis

One mission of every medical school in the United States is to conduct research. Because medical schools in the United States receive public funding from institutions like the National Institutes of Health (NIH), there is broad interest in evaluating medical schools' research programs. Unfortunately, identifying which medical schools are performing best relative to their peers is controversial.

U.S News & World Report publishes the most widely read rankings of medical school research programs. Their rankings are based on a simple formula that uses fixed weights for key metrics like the faculty to student ratio and the amount of NIH funding received. Critics have correctly pointed out that this naive methodology lacks flexibility and does not focus on productivity.

To overcome these flaws, we have used data envelopment analysis (DEA) to rank the academic output of US medical schools. DEA is a new methodology that uses fractional linear programming and the simplex algorithm to derive optimal (ie, custom) weights for each medical school to create a non-parametric production frontier of the best schools based on their relative efficiency. The analysis used variables that measured each school's resources (ie, inputs) such as NIH funding and faculty size and each school's academic output which are the number of publications and their quality assessed by metrics like the journal impact factor and number of citations. The presentation will explain our results and give a worked example of DEA.

Marcus Pendergrass Hampden-Sydney College (mpendergrass@hsc.edu)

Timbral Partial Orders

Timbre refers to the gestalt of audible qualities, aside from pitch and intensity, that constitute a musical sound. It is a complex phenomenon, with physical, psychological, and cultural dimensions. Musicians tend to speak of it in metaphorical terms: the trumpet is a "bright" instrument, as is the oboe, while the horn has a more "warm" or "mellow" sound. In this talk we develop partial orders for making comparisons between sounds in terms of qualities like brightness or warmth. We will show that these models yield intuitively satisfying results by applying them to recorded samples of familiar orchestral wind instruments. We also show how these models can be used to solve certain sound design problems.

Matthew Fury Penn State Abington (maf44@psu.edu)

Numerical Estimates for the Regularization of Nonautonomous Ill-Posed Problems

The regularization of ill-posed problems has become a useful tool in studying initial value problems that do not adhere to certain desired properties such as continuous dependence of solutions on initial data. Because direct computation of the solution becomes difficult in this situation, many authors have alternatively approximated the solution by the solution of a closely-defined well-posed problem. In this paper, we demonstrate this process of regularization for the backwards heat equation with a time-dependent diffusion coefficient, among other nonautonomous ill-posed problems. In the process, we provide two different approximate well-posed models and numerically compare convergence rates of their solutions to a known solution of the original ill-posed problem.

GCPS Session 14: Teaching Advanced Mathematics

Part 2: Saturday, August 3, 1:00–4:10 PM, Room 21

Blane Hollingsworth Middle Georgia State College (blanehollingsworth@gmail.com)

A Simple Explanation of Stochastic Differential Equations

The subject of stochastic differential equations, while quite interesting and extremely applicable, is difficult due to its heavy dependence on deep calculus/probability ideas. Nevertheless, we can give an intuitive explanation of how they work, sacrificing technicality for basic understanding. Only elementary knowledge of calculus and probability is needed.

Brian Sutton Randolph-Macon College (bsutton@rmc.edu)

Differential equations without derivatives

We advocate a focus on integral equations in place of differential equations in parts of the undergraduate curriculum. Differential operators are dangerous because they are typically unbounded. This concept traditionally belongs to functional analysis, but the danger is one that undergraduate students and practicing engineers should avoid. We will demonstrate how to convert differential equations to integral equations, explain some of the benefits of integral equations, and lay the groundwork for functional analysis without encumbering undergraduates with graduate-level jargon. The resulting methods are natural, efficient, and reliable, and they are appropriate for courses in differential equations and numerical analysis.

L. Felipe Martins Cleveland State University (luizfelipe.martins@gmail.com)
Barbara Margolius Cleveland State University (b.margolius@csuohio.edu)

Essay-style Problems in Differential Equations with WeBWorK

In this presentation, we discuss a style of WeBWorK problems that allows you to ask open-ended questions. We use Flash-enhanced problems, now part of the Open Problem Library, to illustrate how we are using these problems in introductory Differential Equation courses.

Anna Davis Ohio Dominican University (davisa@ohiodominican.edu)

I want it all, and I want it now! (or, may I please graduate on time?)

The Mathematics Department at Ohio Dominican University (Columbus, OH) is pioneering a One-Room-Schoolhouse (ORS) model for its upper-level mathematics courses. The ORS setting utilizes the flipped classroom approach to allow a single instructor to conduct multiple courses in the same classroom at the same time. This allows a small liberal arts college to offer a great variety of specialized, low-enrollment courses every semester at no additional cost. Thus, students can take the courses they need, when they need them. In this presentation we will consider the following aspects of ORS: scheduling, physical space, classroom management, and instructional materials.

Christopher Sass Young Harris College (ctsass@yhc.edu)

Teaching an Honors Seminar on Fractals for Non-Majors

I taught a one credit-hour seminar on fractals for the honors program at a small liberal arts college in spring 2013. We explored fractals in nature, art, music, architecture, and math; we also studied related areas such as chaos and cellular automata. My approach throughout the course was to allow students to actively explore phenomena through computer programs and hands-on activities. In this talk I will outline the course content, share the resources used in the course, describe the class activities, and survey the final projects submitted by the students. I will conclude that this sort of course provides a wonderful opportunity to introduce non-majors to some beautiful mathematical ideas.

Vera Cherepinsky Post University (vcherepinsky@post.edu)

Mathematics of Origami Honors Seminar—Successes and Lessons Learned

Most schools with an undergraduate mathematics major offer an Honors Seminar class, in some form. The topic of the seminar is typically selected by the instructor. Whether the course is successful depends in large part on the topic, in particular on its level relative to the students' preparation.

This paper describes the use of "Mathematics of Origami" as a topic in such a course, along with the logistics of running the course, the resulting successes, and the lessons learned, based on the Honors Seminar class taught by the author at Fairfield University during the Fall 2009 semester.

Maryam Vulis NCC and York College CUNY (maryam@vulis.net)

Teachable Math in Cryptocurrency Phenomenon

Many virtual currencies based on cryptographic protocols have been proposed since e-Gold in 1990s. Bitcoins are the most popular example gathering much mass media attention. This presentation will discuss several mathematical aspects of virtual currencies, and of Bitcoins in particular.

Mindy Capaldi Valparaiso University (mindy.capaldi@valpo.edu)

The 2-Column Method: a better way to teach proofs?

Many math students have difficulty with proving theorems. Often, this can be because they do not know what exactly is expected of them when it comes to proof format and rigorousness. This presentation will discuss the pros and cons of requiring students to write proofs using two columns and the highest level of detail possible. One column lists the logical progression of the proof, while the second column justifies each step. This practice was implemented in an abstract algebra course. Descriptions of how and why two-column proofs were used will be presented. Also, the success of this teaching practice, in terms of student work and student preference, will be reviewed.

Antonia Cardwell Millersville University of Pennsylvania (antonia.cardwell@millersville.edu)

"Where have I seen this before?"–Encouraging undergraduate students to see connections.

Throughout an undergraduate mathematics program, a student often sees the same concept introduced in different contexts, and encounters objects in different fields that share the same name. To encourage the student to consider why this occurs, and how the topics across the courses are related, I introduced a weekly "Connections Journal", first in a senior-level introductory real analysis course, and subsequently in a sophomore-level introductory proofs course. I will present a description of the project, provide examples of student submissions, and present some student feedback.

Kristi Karber University of Central Oklahoma (kkarber1@uco.edu)

Transformative Learning in an Analysis Course: A Tactile Approach

An introductory analysis course often begins with a study of sequences of real numbers. In doing so, students are quickly given the "official" definition of a convergent sequence. This may be the first time a student is introduced to the verbage "for each real number $\epsilon > 0$, there exists a positive integer N such that ...". The initial exposure to this type of definition can be intimidating. In this talk, I describe a tactile approach using party favors to help students wrap their minds around this definition and other concepts relating to sequences.

Marian Anton Central Connecticut State University (anton@ccsu.edu)

The Constant of Integration

We discuss the symmetries of a differential equation in the context of a senior seminar project for undergraduates. In particular, the constant of integration turns out to be a special type of symmetry that sends a solution to another solution by a translation of the real line. Are there other symmetries? To answer this question we follow a method by Sophus Lie.

Violeta Vasilevska Utah Valley University (Violeta.Vasilevska@uvu.edu)

Addapted Sequence/Function Project

This Project is appropriate for Intro to Analysis classes. It requires each student to choose a sequence/function, do research and write a paper about the properties and any other relevant information about the chosen sequence/function. In this talk, the requirements involved in the project, the benefits of the project to students, and the students' feedback on this written assignment will be discussed.

Charles Funkhouser California State University Fullerton (cfunkhouser@fullerton.edu)
Miles R Pfahl Turtle Mountain Community College (mrpfahl@tm.edu)

Native American-based Mathematics Materials for Integration into Undergraduate Courses

This project develops and researchers undergraduate mathematics materials based in the culture and mathematics of Native American Peoples for integration into undergraduate courses. Mathematics topics include probability, number theory, transformational geometry, and preservice elementary and secondary education-related content. These materials—both paper and electronic—are classroom ready, and are developed and piloted in consultation with Tribes in the the Rocky Mountains, the Plains, the Pacific Northwest, and the Southwest. This work is an NSF DUE TUES Type 2 funded project.

GCPS Session 15: Research in Number Theory

Saturday, August 3, 1:00–2:25 PM, Room 22

Mark Bauer University of Calgary (bauerm@ucalgary.ca)
Richard Guy University of Calgary (rkg@cpsc.ucalgary.ca)
Michael Katsuris Wanless University of Calgary (mwanless@gmail.com)
Colin Weir University of Calgary (cjweir@ucalgary.ca)

Class Numbers and Continued Fraction Expansions

D. Zagier proved an interesting result that provides an explicit formula for the class number of $\mathbb{Q}(\sqrt{-p})$ given certain conditions on p and $\mathbb{Q}(\sqrt{p})$. Remarkably, this formula is based almost entirely on the negative continued fraction expansion of $\sqrt{p}$. It turns out that this result is actually just a special case of a more general, but much more abstract theorem. We will explore this general theorem and discuss other concrete versions for families of non-prime radicands.

Paul Spiegelhalter University of Illinois at Urbana Champaign (spiegel3@uiuc.edu)

Distributions of Sequences Modulo 1: The Good, the Bad, and the Ugly

The theory of distribution of sequences modulo 1 originated in the early 20th century with work by Weyl, Fejer, and van der Corput. It has since grown to an important branch of number theory with applications to many other areas such as numerical analysis and probability. The focus of nearly all of this development has been on the case of "nice" or "uniform" distribution, which roughly means that each subinterval of $[0, 1]$ contains its "proper" share of the terms of the sequence reduced modulo 1. In this survey we investigate the opposite case, that of sequences that are, in some sense, as far from uniformly distributed as possible, and we develop a cohesive theory of "bad" distribution. We consider several natural notions of "bad" distribution, each obtained by taking a well-known characterization of uniform distribution and turning it on its head, and study the relations between these notions. We tie together the few scattered and little known results that exist in the literature on the subject of bad distribution, produce a unified treatment, and round out the picture with some new results.

Rosemary Sullivan West Chester University of PA (rsullivan@wcupa.edu)

Independent Divisibility Pairs on the Set of Integers from 1 to N

Let $\mathbb{S}_N$ denote the set of integers from 1 up to N and A_i be the event that a number selected from $\mathbb{S}_N$ is divisible by i. For the sample space $\mathbb{S}_N$, with the uniform probability measure, consider the question of the independence of the events A_i and A_j, $i \neq j$. We determine a characterization in terms of N, i and j. Using this we consider various situations and supplementary questions.

John Pesek University of Delaware (pesek@udel.edu)

Equality of Cardinality of Sets of Subsets with Cardinality Congruent to Values Modulo k

An old result is that the number of subsets of a non-empty finite set with an odd number of elements is equal to the number of subsets with an even number of elements. If we consider the numbers of subsets with number of elements congruent to 0, 1 or 2 modulo 3, then they cannot all be equal but surprisingly they differ by at most one. Here we generalize these results by considering multisets. For instance suppose that each element of a set is allowed to belong zero, one or two times. Then the numbers of subsets whose cardinality is congruent to zero, one or two modulo three respectively are equal. Furthermore if we consider the numbers of subsets whose cardinality is even or odd, then these numbers disagree by at most one. A similar result holds when we consider congruence classes modulo four.

Jay Lawrence Schiffman Rowan University (schiffman@rowan.edu)

A Delightful Interconnection Between Pythagorean Triples and Fibonacci-Like Sequences

A delightful connection exists between Pythagorean triples and the Fibonacci sequence. Consider four consecutive terms in the Fibonacci sequence. First form the product of the first and fourth terms. Next double the product of the second and third terms. Finally form the sum of the squares of the second and third terms. Applying these three steps leads to the formulation of a Pythagorean triple. (We note that this is true in any Fibonacci-like sequence where the initial two terms are different, but the recursion rule in the Fibonacci sequence is followed for subsequent terms. In this paper, we determine all primitive Pythagorean triples (PPT's) (x, y, z) obtained by the standard formula $x = m^2 - n^2$, $y = 2mn$, $z = x^2 + y^2$ where m is greater than n, m and n are relatively prime, of opposite parity and less than one hundred. In addition, we explore another popular Fibonacci-like sequence; namely the Lucas sequence and explore the PPT's generated in this fashion. Finally, we delve further into some number theoretic morsels including determining those PPT's where one of the legs and the hypotenuse are both prime numbers.

Frederick Donald Chichester Montclair Tutoring Center (fdchichester@gmail.com)

Squares and Pythagorean Triples II

This paper, an extension of work reported at the 2012 Mathfest, presents development of equations for mentally calculating squares of integers between 100 and 200. These squares are then used to identify 16 additional primitive Pythagorean triples which represent the sides of Pythagorean triangles each of which has a length less than or equal to 200.

GCPS Session 16: Mathematics and Technology/ Research in Analysis

Saturday, August 3, 1:00–3:25 PM, Room 26

Joseph Manthey University of Saint Joseph, West Hartford, CT (jmanthey@usj.edu)

Are You Ready for R

R is an extremely powerful tool for statistical computing used by many universities and prominent companies including Facebook, Google and the New York Times. This presentation will provide an overview of using R in an educational setting and will begin with a brief summary of R's capabilities and limitations followed by a comparison of several commonly used graphical user interfaces such as RStudio and Deducer. Several interesting applications of R including the creation of animations, maps and the automatic generation of reports and quizzes will also be discussed.

Barbara Margolius Cleveland State University (b.margolius@csuohio.edu)

Applets embedded in WeBWorK homework problems

This is a report on our NSF-CCLI grant. For this grant we developed a library of Flash applets embedded in WeBWorK homework assignments for entry level university mathematics courses including calculus, pre-calculus and differential equations. Applets are self-contained programs that play within the user's web browser. These applets are designed to interact with theWeBWorK online homework system to facilitate the creation of a richer array of assessment and enhancement of student learning than is presently offered by most online homework resources. WeBWorK is a free Perl-based system for delivering individualized homework problems over the web. WeBWorK is partially supported by the DUE Division of NSF and is now sponsored by the Mathematical Association of America. It is used extensively in the US and Canada and to a lesser extent throughout the world.

Nathan Carter Bentley University (ncarter@bentley.edu)
Kenneth G. Monks University of Scranton (monks@scranton.edu)

Using *Lurch* in an Introduction to Proofs Course

Lurch is a mathematical word processor that can check your reasoning, aimed at a first course in proofs. It is a free and open-source project available for Mac, Windows, and Linux. In this talk, the author will briefly introduce the software's capabilities and then report on the experiences of his collaborator, who integrated the software throughout an introduction to proofs course at a liberal arts university, during the Spring 2013 semester.

Helmut Knaust The University of Texas at El Paso (hknaust@utep.edu)

Technology in the Mathematics Classroom

We report our experiences with teaching a graduate course for current and future teachers that exposes students to software useful for their own Mathematics classroom. We will include a presentation of the results of a pre-course survey about the level of technological expertise of the participating students.

Ulrich Hoensch Rocky Mountain College (hoenschu@rocky.edu)

Creating and Analyzing Chaotic Attractors Using Mathematica

We demonstrate how two-dimensional chaotic attractors Ω can be easily created and analyzed using Mathematica. Specifically, we demonstrate sensitive dependence on initial conditions and calculate the box-counting dimension of Ω. Examples include the forced damped pendulum and the experiment of Moon and Holmes. The tools presented here were developed for and used in an undergraduate Differential Equations course.

Xiao-Xiong Gan Morgan State University (xiao-xiong.gan@morgan.edu)

An Introduction to Formal Laurent Series

Several kinds of formal Laurent series have been introduced with some restrictions since the establishment of the space of formal power series. Given two series

$$f(z) = \cdots + a_{-2}z^{-2} + a_{-1}z^{-1} + a_0 + a_1 z + a_2 z^2 + \cdots \quad \text{and} \quad g(z) = \cdots + b_{-2}z^{-2} + b_{-1}z^{-1} + b_0 + b_1 z + b_2 z^2 + \cdots .$$

What is the product of f and g if f and g are not analytic on an annulus region? What could be the composition of g and f in any sense? We are going to answer those questions by introducing the space of formal Laurent series, denoted by $\mathbb{L}$.

References

[1] X. Gan and D. Bugajewski, On Formal Laurent Series, *Bull. Brazilian Math. Soc.*, 42(3), 415–437, 2011.

[2] X. Gan and N. Knox, On Composition of formal power series, *Int. J. Math. and Math. Sci.* 30 (12), 761–770, 2002.

Douglas Daniel Presbyterian College (ddaniel@presby.edu)

Classifying Rational Points in Generalized Cantor Sets and Cantor Like Sets

It is well known that 1/4 is a rational point contained in the famous Cantor set. Other rational points have been discovered that are also contained in this set. The question arose as to what sorts of rational points would exist in Generalized Cantor sets. This led to first considering Cantor-like sets and then Generalized Cantor sets. In this talk, I consider the types of rational points to be found in these sets and what form they might take depending upon how the set is created.

Sergei Artamoshin CCSU (sartamoshin@gmail.com)

Geometric approach to the computation of certain definite integrals.

In this presentation we are going to use elementary geometry to proof the following integral identity

$$\int_0^{2\pi} \frac{d\theta}{(a - b\cos\theta)^p} = (a^2 - b^2)^{1/2-p} \int_0^{2\pi} (a - b\cos\theta)^{p-1} d\theta \, ,$$

where p can be any complex number and $a > b > 0$. It is clear that this identity allows us to compute the left-hand side integral for $p = 1,2,$.

Joseph A. Iaia University of North Texas (iaia@unt.edu)

Traveling Wave Solutions of the Porous Medium Equation

In this talk we look for traveling wave solutions of the two dimensional porous medium equation: $u_t = \Delta(u^m)$. After substituting $v = \frac{m}{m-1}u^{m-1}$ this gives: $v_t = (m-1)v\Delta v + |\nabla v|^2$. We then look for solutions in the special form: $v = rF(\theta)$ where $r = \sqrt{x^2 + (y - ct)^2}$ and $\theta = \tan^{-1}(\frac{y-ct}{x})$. This results in the following ordinary differential equation for F:

$$F'^2 - c\sin(\theta)F' + F^2 + c\cos(\theta)F + (m-1)F(F'' + F) = 0$$

along with:

$$F(\pi) = a > 0, F'(\pi) = 0.$$

We then seek solutions so that $F(0) = 0$ which then allows us to extend F to be 2π-periodic.

Kristine Roinestad Georgetown College (kristine_roinestad@georgetowncollege.edu)

Geometry of Fractal Squares

This talk will examine analogues of Cantor Sets, called fractal squares, and some of the geometric ways in which they trigger issues not raised by Cantor Sets. Also discussed will be the technique using directed graphs to prove bilipschitz equivalence of two fractal squares.

Graduate Student Session

Great Talks for a General Audience: Coached Presentations by Graduate Students

Saturday, August 3, 1:00–5:30 PM, Connecticut Convention Center, Room 11

Maximillano Liprandi University of Calgary (mliprand@rcalgary.ca)
Richard Guy University of Calgary (rkg@cpsc.ucalgary.ca)

The game of BASIC MANCALA

Mancala is the name for a family of "sowing" games played in many regions of the world. One of its versions, Kalah, is played on a board that consists of two rows of six holes or pits, as well as two stores, one to each side of the rows. Here, the game starts with m seeds in each pit. A move in the game is to "sow" the seeds from one of the pits. In the end, the player with the most seeds in his store wins the game. Based on this game, we can design a combinatorial version of it, with similar mechanics, but a different winning condition: the player to make the last available move is the winner. We call this game BASIC MANCALA. At the start of the game there are m seeds in each of the $2n$ pits. The cases $n = 1, 2$ are games which are of interest in both the partizan and impartial (both players have access to either side of the board) versions.

John Asplund Auburn University (jsa0011@auburn.edu)
Chris Rodger Auburn University (rodgec1@auburn.edu)

Decomposition of Graphs using Blocks!?

How do you ensure that you can pair up an entire class room of students day after day without repeating partners? If there are v people at a conference, the conference lasts $v - 1/2$ nights and there are t round tables of the same size, is it possible to seat v people at t tables with the same number of chairs at each table so that each person sits next to every other person exactly once. These questions and more will be answered during this talk! We will also discuss tools in Design Theory that can assist us in solving these problems and other similar problems.

Joni Schneider Texas State University, San Marcos (js1824@txstate.edu)
Xingde Jia Texas State University, San Marcos (jia@txstate.edu)

Extremal Cayley Digraphs

Let Γ be a finite group with m elements. Let A be a nonempty subset of Γ. The $Cayleydigraph$ of Γ generated by A, denoted by $Cay(\Gamma, A)$, is the digraph with vertex set Γ and arc set $\{uv \mid u^{-1}v \in A\}$.

A Cayley digraph can be considered as a graphical representation of a finite group by its generating set. Cayley digraphs of finite abelian groups are often used to model communication networks. Because of their complex algebraic structure and their applications in network theory, Cayley digraphs have been studied extensively in recent years. In this presentation, we focus on some optimization problems about Cayley digraphs. In particular, we study how large the number of vertices a Cayley digraph can have for a given diameter and degree. This is one of the central problems in the study of extremal Cayley digraphs.

We will introduce a geometric representation of $\mathbb{Z}_m$ with respect to a generating set A. This representation was first introduced by C. K. Wong and Don Coppersmith in 1974. This geometric representation of $\mathbb{Z}_m$ is very useful in establishing upper bounds for the extremal function $m(d, k)$. We will discuss some properties of the A-representation of $\mathbb{Z}_m$ in two and three dimensional cases. We will also use this method to prove upper bounds for $m(d, 2)$. Some other related extremal functions will also be studied in this presentation.

Adam Giambrone Michigan State University (ag9018@gmail.com)
Efstratia Kalfagianni Michigan State University (kalfagia@math.msu.edu)

A Beautiful Connection Between Graphs and Knots

Graphs and knots are two very important objects of study. They appear not only in mathematical fields like algebraic topology and combinatorics, but also in other fields like biology, chemistry, and physics. Visually, a graph consists of a collection of points connected by lines (which may be curved). A knot, on the other hand, is a "nice" 1-dimensional knotted loop sitting in 3-dimensional space. While these two mathematical objects may seem quite different, it turns out that there is actually a classical connection between them. This connection provides an example of how beautifully interwoven mathematics can be. In this talk, we will see exactly what knots are graphs are, what they look like, and precisely how we can translate back and forth between them.

James Monroe Hammer Auburn University (jmh0036@auburn.edu)

Introduction to Mutually Orthogonal Latin Squares

A Latin square of order n is an array where each of the n numbers appears once in each row and column. After centuries of studies, Latin squares are still investigated to this day. Sudoku is an example of a Latin square. Even further, we can construct mutually orthogonal Latin squares (MOLS) from Latin squares. MOLS have been studied extensively for several centuries, even by Leonard Euler who notably studied the existence of two MOLS of order 6. Uses of MOLS can be seen within designs of experiments as well as the construction of semi-magic squares.

Christa Moreno San Francisco State University (morenocrista@gmail.com)
Mariel Vazquez San Francisco State University (mariel@sfsu.edu)

Modeling Changes in DNA Topology using the Tangle Method

In the 1980's, researchers applied knot theory to the study of enzymes called site-specific recombinases, which change the topology of DNA. In the topological approach to enzymology, substrates and products of recombination were studied experimentally, and mathematical tools were developed to help uncover the enzymatic mechanism. This talk will begin with an introduction to knot theory and tangle theory. A tangle is a 3-dimensional ball with two arcs, which may intertwine in nontrivial ways. I will provide the necessary background from tangle calculus and explain how tangle calculus is used to model enzymatic actions on circular DNA. Given the observed knot types of the DNA substrates and products of site-specific recombination, the tangle method constructs a system of tangle equations. I will present several examples of the model with a Java Applet called TangleSolve. Finally, I will consider questions on how to improve this model, and discuss applications to the Xer site-specific recombination system.

Rachel Webb Brigham Young University (1oldkingcoel@gmail.com)
Tyler Jarvis Brigham Young University (jarvis@math.byu.edu)

What Your Professor Didn't Tell You About Critical Points

A central idea of calculus is that when a function's derivative is zero (a critical point), something interesting is happening. For example, local extrema occur will occur at critical points and this information can be used to solve optimization problems. This talk will generalize the concept of a critical point to a multivariable function and will introduce the exciting new area of math known as mirror symmetry. We will explore the fascinating relationship between critical points and the levels sets of the function. In fact, these critical points signal drastic changes in the appearance of the level sets, such as one circle splitting into two. We will also investigate symmetries of these level sets, which can be used to "glue up" the level set in interesting ways. Together, functions with critical points and the symmetries of their level sets give rise to extraordinarily beautiful objects that exhibit surprising relationships.

Stepan Paul UC Santa Barbara (spaul@math.ucsb.edu)
David R. Morrison UC Santa Barbara (drm@math.ucsb.edu)

A Block-Stacking Game With Applications to Veronese Varieties

We introduce a Tetris-like block-stacking procedure, which we will use to construct a certain class of simplicial complexes. By flipping our Tetris board upside-down, we obtain a duality theorem for the reduced homology groups of these complexes, which we illustrate with examples. Due to a result from Bernd Sturmfels, the ranks of these homology groups are the multigraded Betti numbers for Veronese modules.

PosterFest 2013

Saturday, August 2, 3:30–5:00 PM, Connecticut Convention Center, Pre-function area

Jonathan Godbout University of Vermont (jgodbout@uvm.edu)

A weighted bijection from colored permutations to rook placements

The Robinson-Schensted correspondence gives a bijection between permutation and pairs of standard Young tableau of the same shape. The Bruhat partial-order describes the containment of the closure of $GL(V)$ orbits of flags over $\mathbb{C}$. It is known that permutations index these orbits while coloured permutations give an analogous ordering in a larger space. In hopes of understanding this larger flag space, Roman Travkin expanded on Robinson-Schensted correspondence with the Mirabolic RSK algorithm, giving a bijection between colored permutations and augmented tableau. Permutations have an obvious bijection between rook placements of size n on an $n \times n$ chessboard and permutations. Using a weighting induced by the Mirabolic RSK algorithm, we give a weighted bijection between rook placements of any size on an $n \times n$ chessboard and coloured permutations of length n.

Ellen Ziliak Benedictine University (eziliak@ben.edu)

Geometry of Quasigroups and Groups

In this poster we will discuss the results of three years of collaborations with undergraduate research students in the field of abstract algebra. I will discuss how one can study the differences between quasigroups and groups from a graphical setting. A quasigroup is an algebraic object where the multiplication table forms a Latin square. So the only axiom it has in common with a group is closure. One of my goals is to be able to visualize the associativity or the non-associativity of an algebraic structure by looking at the Cayley graphs or other directed graphs that describe the multiplication tables of these structures. This work was originally motivated by a question of building group extensions. However, we have since found several applications to this material in the field of cryptography.

Adam Giambrone Michigan State University (ag9018@gmail.com)

A Combinatorial Approach to Volume Bounds for Knot Complements

Recent research in knot theory and 3-manifold topology has been concerned with relating combinatorial information with geometric information about objects called knots (embedded loops in 3-space). From the combinatorial side, we can associate a graph to a given projection diagram of a "nice" knot. Furthermore, the complexity of this graph is intimately related to a polynomial invariant that can be associated to the knot. On the geometric side, it has been shown that the complement of "most" knots in 3-space are hyperbolic. As a result, the volume of such a knot complement is actually an invariant of the knot. As a step towards relating these two ideas, Futer, Kalfagianni, and Purcell have created a framework that uses "nice" surfaces in the knot complement to connect the combinatorial and geometric information. In this poster, I will display some families of knots for which I was able to find volume bounds from the combinatorics of the knot diagram.

Thomas Alden Gassert University of Massachusetts Amherst (thomas.alden.gassert@gmail.com)

On Chebyshev Radical Extensions

Let $T_d(x)$ denote the Chebyshev polynomial of degree d. Consider the polynomial $\Phi(x) := T_q(x) - t$ where $q = \ell^n$ for some odd prime ℓ, and $t \in \mathbb{Z}$ such that $T_q(x) - t$ is irreducible. If θ is a root of $\Phi(x)$, we say that θ is a Chebyshev radical, and we call $\mathbb{Q}(\theta)$ a Chebyshev radical extension. Much can be said about these number fields. In particular, we give a formula for the discriminant of $\mathbb{Q}(\theta)$, a generating set for its ring of integers, and the decomposition of all but finitely many primes in this extension.

Hilary Smallwood Colorado State University (smallwood_h@hotmail.com)

Counting Abelian Surfaces with Real Multiplication over $\mathbb{F}_q$.

Given an simple abelian surface, $A/\mathbb{F}_q$, the endomorphism algebra, $\mathrm{End}(A) \otimes \mathbb{Q}$ contains a real quadratic subfield. For squarefree $d > 0$, we estimate the number of principally polarized abelian surfaces $A/\mathbb{F}_q$ such that $\mathbb{Q}(\sqrt{d}) \subset \mathrm{End}(A) \otimes \mathbb{Q}$, and show that this quantity is approximately $q^{5/2}$.

Christina Tran California State University, Fullerton Mathematical Circle (cnt_tran@csu.fullerton.edu)

Training Gifted Students: The Fullerton Mathematical Circle Experience

In our poster, we will describe the concept and structure of our Fullerton Mathematical Circle which began in September 2011. Currently, over 70 students from Orange County, Riverside County, and Los Angeles County attend this program. We will demonstrate how we attract our gifted students with problems from American Mathematical Competitions, *Gazeta Matematica* (a Romanian monthly publication established in 1895), and the Abacus International Challenge (a program inspired from the experience of the Hungarian School of Mathematics). Additionally, we will have copies of *Gazeta Matematica* available where the solutions elaborated by our students have been recognized by the editor.

James Emil-Maxum Grenier Radford University (jgrenier@radford.edu)

An Expository Proof of Bezout's Theorem

Bezout's Theorem gives a relation between the degrees of two curves and the number of intersection points in projective space. In this paper we briefly outline the history leading to the discovery and proof of the theorem; we explain the importance of Bezout's Theorem and illustrate this with several applications; then we present an expository proof of the theorem designed to be comprehensible at the undergraduate level. The proof is based on the outline given by Rational Points on an Elliptic Curve by Silverman and Tate.

Crista Moreno San Francisco State University (morenocrista@gmail.com)

Applying the Tangle Method to DNA topology

In the 1980's, researchers appliedknot theoryto the study of enzymes called site-specific recombinases, which change the topology of DNA. In thetopological approach to enzymology, substrates and products of recombination were studied experimentally, and mathematical tools were developed to help unveil the enzymatic mechanism. This presentation will begin with an introduction to knot theory and tangle theory.A tangle is a 3-dimensional ball with two arcs, which may intertwine in nontrivial ways. I will provide the necessary background fromtanglecalculus and explain how tangle calculus is used to model enzymatic actions on circular DNA. Given the observed knot types of the DNA substrates and products of site-specific recombination, the tangle method constructs a system of tangle equations. I will give several examples of the model generated bya Java Applet called TangleSolve. Finally, I will consider questions on how to improve this model, and discuss applications to the Xer site-specific recombination system.

Timothy Goldberg Lenoir-Rhyne University (timothy.goldberg@gmail.com)

Projectivizing SET

It is well-known that the card game SET can be used as a model of the finite affine geometric space of degree 3 and dimension 4. By choosing a particular card as a zero vector, one can projectivize SET to obtain a model of a finite projective space of dimension 3. Although this can be done by choosing coordinates, the main ideas of the geometric models and the projectivization process can also be explained in a coordinate-free fashion using only the basic properties of the card game. In this project, the authors explored this approach, with the goal of providing a tangible and accessible introduction to these geometric concepts. This work developed as part of an undergraduate senior research project.

Stepan Paul UC Santa Barbara (spaul@math.ucsb.edu)

Special linear systems based at fat points in $\mathbb{P}^n$

We show that the existence of a certain kind of partition of the set of degree-d monomials in $n + 1$ variables implies that the linear system of degree-d divisors in $\mathbb{P}^n$ passing through each of r general points with multiplicity m is non-special—i.e. has the expected dimension. We use a Mathematica implementation of an algorithm which searches for such partitions to demonstrate the technique, which is applied to give approximations to Nagata's Conjecture, and to give a new proof of certain cases of its higher-dimensional analogue.

Mark Radosevich Hobart & William Smith Colleges (radosevich@hws.edu)

Spin Concave Fillings of Contact 3-Manifolds

Every contact 3-manifold admits a weak concave symplectic filling, and every spin 3-manifold can be expressed as the boundary of a 4-manifold such that the spin structure on the boundary is the restriction of a spin structure on the interior. This poster will show that these fillings may be one and the same: any spin contact 3-manifold admits a weak concave symplectic filling by a spin 4-manifold. This presentation will also demonstrate a visual and accessible approach to low-dimensional topology.

Rachel Webb Brigham Young University (1oldkingcoel@gmail.com)
Tyler Jarvis Brigham Young University

The Frobenius Manifold Structure of the Landau-Ginzburg A-model for Sums of A_n and D_n Polynomials

Mirror symmetry is a branch of mathematics that explores isomorphisms between dual pairs of objects with multiple layers of structure. Landau-Ginzburg (LG) mirror symmetry is one conjectured version of mirror symmetry that is broadly applicable. On the other hand, K3 surfaces exhibit many kinds of mirror symmetry, including LG mirror symmetry, and hence are important examples. Previously, only a handful of LG A-model constructions were known. I have computed structure constants determining the Frobenius manifold structure of the LG A-model for polynomials that are arbitrary sums of A_n and D_n polynomials, paired with their maximal symmetry group. In particular this applies to a large class of K3 surfaces. This is a valuable step in proving LG mirror symmetry.

Corrine Previte Colorado State University (previte@math.colostate.edu)

Homology of the D-Neighborhood Complex on Graphs

Let G be a connected simple graph. Let the distance between vertices v_i and v_j, written $d(v_i, v_j)$, be the length of the shortest path between v_i and v_j. Define the diameter of G, $diam(G)$, to be the maximum of the distances between all pairs of vertices. Let D be a subset of the set of graph distances $\{0, 1, \ldots, diam(G)\}$. For each vertex v_i, there is an associated square-free monomial given by taking the product of all vertices v_j such that $d(v_i, v_j) \in D$. From these square-free monomials, build a simplicial complex, denoted as the D-Neighborhood Complex. This simplicial complex has an associated chain complex. We relate some properties of G to properties of the D-Neighborhood complex and the homology of the associated chain complex.

Matthew Hoffman Rochester Institute of Technology (mjhsma@rit.edu)
Steven Scorse Rochester Institute of Technology
Elizabeth Cherry Rochester Institute of Technology

Data Assimilation and Prediction of Cardiac Electrical Dynamics

Data assimilation techniques, commonly used in weather forecasting, are used to infer the three-dimensional electrical dynamics of cardiac tissue. Our work will aim to identify the conditions under which data assimilation can be used and its limitations in the context of both perfect-knowledge (mathematical model) and imperfect-model (experimental data) systems and to use the forecasting tools developed to study the stability of electrical wave sources during fibrillation events. Preliminary results with a simplified model are presented here.

Ashley Rand Bethany Lutheran College (rand@math.utk.edu)
Carrie Elizabeth Diaz Eaton Unity College
Eleanor Abernethy University of Tennessee

Changing attitudes about teaching through a STEM GTA teaching and mentoring program, a case study

Institutions of higher education which have PhD programs tend to have programs with a research emphasis. Often graduate students at these institutions are required to teach as part of their program funding. But there is a lack of encouragement to improve their teaching or to pursue careers where teaching is emphasized. With a goal of not only improved teaching but also building a community of teachers, we implemented a GTA teaching and mentoring program at a large research 1 department. We consider whether or not this program was successful in changing attitudes about teaching.

Jason Miller The Ohio State University (millerj@math.osu.edu)

A correspondence between Borel orbits and unions of faces of the Newton Polytope for wonderful group compactifications

Given a projective toric variety T, there is an associated polytope P which encodes information about the variety. In particular there is a correspondence between the torus orbits and faces of P that allows one to determine certain properties of each orbit closure from the associated face. This rich correspondence between projective toric varieties and polytopes can be extended to the class of projective spherical varieties. These are varieties equipped with an action of a reductive group G where the induced action of any Borel subgroup has a dense open orbit. A similar correspondence exists between the G orbits and certain faces of an associated polytope P. For the class of spherical varieties, one would also like to have information about the Borel orbit closures since they play an important role in the geometry of the variety. My project achieves this goal for the class of wonderful group compactifications. Given a wonderful group compactification X, I demonstrate a correspondence between the Borel orbit closures of X and certain unions of faces of a polytope $\Delta(X)$. This correspondence extends the correspondence that already exists for G orbits in wonderful group compactifications. I show the correspondence enjoys similar properties as in the toric case.

Hernando Bermudez Emory University (hbermud@emory.edu)

Algebraic Groups and Linear Preserver Problems

Given a group G which acts on a vector space V, a polynomial f on V is said to be a invariant of the action if the values of f remain unchanged under the action of G. In this situation the linear preserver problem is the problem of determining the set of all endomorphisms of V which leave f invariant. We discuss the solution of a set of linear preserver problems using methods from the theory of linear algebraic groups.

Jonathan Weisbrod Burlington County College (jweisbro@bcc.edu)

Berggren Tree of Pythagorean Triples Using Dickson's Formulas

We introduce a method of producing the Berggren Tree of primitive Pythagorean Triples by using functions of two parameters rather than linear transformations. The two parameters, α and β, which define each primitive Pythagorean triple by the formulas $a = \beta + \sqrt{2\alpha\beta}$, $b = \alpha + \sqrt{2\alpha\beta}$, and $c = \alpha + \beta + \sqrt{2\alpha\beta}$, are mapped to three pairs of integers which in turn produce a unique pair of new parameters. By continuing this process infinitely, we generate all primitive Pythagorean triples.

Noriko Tanaka (tanaka-nagoya@y5.dion.ne.jp)

Plane developments of a polyhedron with margin

To make a polyhedron, plane developments of a polyhedron with margin (NORISHIRO in Japanese) was considered. In this poster, "Beautiful NORISHIRO" was defined about the margin of the edges alternately. The polyhedron with "Beautiful NORISHIRO" was searched.

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Prime Labelings of Graphs

A *prime labeling* of a graph G is a labeling of the vertices with the integers $1, 2, \ldots, v$ (where v is the number of vertices) such that any two adjacent vertices have labels that are relatively prime. We call a labeling of G a *coprime labeling* if the labeling set comes from the integers $1, 2, \ldots, m$ for some $m \geq v$. We will discuss the existence of prime or coprime labelings for various classes of graphs including ladders and complete bipartite graphs.

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2-Fusion Systems with Standard Components of Small Rank

We study two problems in the area of fusion systems which are designed to mimic, simplify, and generalize parts of the Classification of Finite Simple Groups. In the Classification, the simple groups are split between those of characteristic 2-type and those of component type. Following a major program of research laid out by Aschbacher, we work toward a classification of fusion systems of "component type" in order to establish a new proof of the Classification for groups of component type. In general, a finite simple group G is determined to a great extent by the structure and "conjugacy pattern" of a Sylow 2-subgroup. A 2-fusion system considers only a 2-group S equipped with a family of injective homomorphisms (called fusion maps) on subgroups of S without reference to an ambient group G. The general framework of fusion systems also arises naturally in the study of modular representations and classifying spaces; and so results proved for fusion systems have potential ramifications beyond the realm of finite group theory. One problem in this area is to determine S or, whenever possible, the entire 2-fusion system only from the knowledge of certain subgroups and fusion maps between these subgroups. We consider two such problems: where S contains subgroups and fusion maps that arise in the Classification with standard components of type $SL_2(q)$ and $PSL_2(q)$.

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A Classification of Non-degenerate Quasihomogeneous Polynomials

Classification theorems in mathematics are often very powerful ways to understand objects of interest. Many such classification theorems have been developed over the years, such as the classification of finite simple groups and the classification of matrices by various canonical or normal forms.

I will be discussing the classification of certain polynomials that we use in FJRW-theory. FJRW-theory is a recent advancement in singularity theory arising from physics. The construction of FJRW rings involves a polynomial as well as an associated group. Each polynomial must be quasihomogeneous, and as a result has a weight system attached to the variables. It is known that two different polynomials with the same weight system will have isomorphic FJRW rings as long as the associated groups are the same. I will explain some recent progress we've made in developing a classification system for weight systems and associated polynomials.

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