Exemplary Programs in Introductory College Mathematics

MAA Notes #47

INnovative Programs Using Technology

The Mathematical Association of America

Susan Lenker, Editor

Exemplary Programs in Introductory College Mathematics

Exemplary Programs in Introductory College Mathematics

Prepared by the
Central Michigan University Project Team
for the Innovative Programs Using Technology
(INPUT) Project

Susan S. Lenker, Project Director
Department of Mathematics
Central Michigan University

Co-Directors:

Nancy Cooley
Keith Nelson
Richard St. Andre

Department of Mathematics
Project Team Members:

Mitzi Chaffer
Robert Chaffer
Richard Fleming
Carl Lee
Terry Lenker
Debra Packer
Dennis St. John
Linda Smoke
Charles VonderEmbse

Published by
The Mathematical Association of America

Special thanks to the following for providing funding for the INPUT Project:
The Annenberg/CPB Project
Central Michigan University
National Science Foundation

Special thanks to the following faculty and graduate students from the Mathematics Department of Central Michigan University who helped to review projects submitted for the INPUT Project:
Robert Chaffer, Rodney Elmore, Nicholas Eugene,
Richard Fleming, Carl Lee, Robin Lewis, Ruomei Li

Library of Congress Catalog Card Number 98-88602

ISBN 0-88385-159-8

Printed in the United States of America

Current Printing (last digit)
10 9 8 7 6 5 4 3 2 1

The MAA Notes Series, started in 1982, addresses a broad range of topics and themes of interest to all who are involved with undergraduate mathematics. The volumes in this series are readable, informative, and useful, and help the mathematical community keep up with developments of importance to mathematics.

MAA Notes

11. Keys to Improved Instruction by Teaching Assistants and Part-Time Instructors, *Committee on Teaching Assistants and Part-Time Instructors, Bettye Anne Case,* Editor.

13. Reshaping College Mathematics, *Committee on the Undergraduate Program in Mathematics, Lynn A. Steen,* Editor.

14. Mathematical Writing, *by Donald E. Knuth, Tracy Larrabee, and Paul M. Roberts.*

16. Using Writing to Teach Mathematics, *Andrew Sterrett,* Editor.

17. Priming the Calculus Pump: Innovations and Resources, *Committee on Calculus Reform and the First Two Years,* a subcomittee of the Committee on the Undergraduate Program in Mathematics, *Thomas W. Tucker,* Editor.

18. Models for Undergraduate Research in Mathematics, *Lester Senechal*, Editor.

19. Visualization in Teaching and Learning Mathematics, *Committee on Computers in Mathematics Education, Steve Cunningham and Walter S. Zimmermann,* Editors.

20. The Laboratory Approach to Teaching Calculus, *L. Carl Leinbach et al.,* Editors.

21. Perspectives on Contemporary Statistics, *David C. Hoaglin and David S. Moore,* Editors.

22. Heeding the Call for Change: Suggestions for Curricular Action, *Lynn A. Steen,* Editor.

24. Symbolic Computation in Undergraduate Mathematics Education, *Zaven A. Karian*, Editor.

25. The Concept of Function: Aspects of Epistemology and Pedagogy, *Guershon Harel and Ed Dubinsky,* Editors.

26. Statistics for the Twenty-First Century, *Florence and Sheldon Gordon*, Editors.

27. Resources for Calculus Collection, Volume 1: Learning by Discovery: A Lab Manual for Calculus, *Anita E. Solow,* Editor.

28. Resources for Calculus Collection, Volume 2: Calculus Problems for a New Century, *Robert Fraga,* Editor.

29. Resources for Calculus Collection, Volume 3: Applications of Calculus, *Philip Straffin* Editor.

30. Resources for Calculus Collection, Volume 4: Problems for Student Investigation, *Michael B. Jackson and John R. Ramsay,* Editors.

31. Resources for Calculus Collection, Volume 5: Readings for Calculus, *Underwood Dudley,* Editor.

32. Essays in Humanistic Mathematics, *Alvin White,* Editor.

33. Research Issues in Undergraduate Mathematics Learning: Preliminary Analyses and Results, *James J. Kaput and Ed Dubinsky,* Editors.

34. In Eves' Circles, *Joby Milo Anthony,* Editor.

35. You're the Professor, What Next? Ideas and Resources for Preparing College Teachers, *The Committee on Preparation for College Teaching, Bettye Anne Case,* Editor.

36. Preparing for a New Calculus: Conference Proceedings, *Anita E. Solow,* Editor.

37. A Practical Guide to Cooperative Learning in Collegiate Mathematics, *Nancy L. Hagelgans, Barbara E. Reynolds, SDS, Keith Schwingendorf, Draga Vidakovic, Ed Dubinsky, Mazen Shahin, G. Joseph Wimbish, Jr.*

38. Models That Work: Case Studies in Effective Undergraduate Mathematics Programs, *Alan C. Tucker,* Editor.

39. Calculus: The Dynamics of Change, *CUPM Subcommittee on Calculus Reform and the First Two Years, A. Wayne Roberts,* Editor.

40. Vita Mathematica: Historical Research and Integration with Teaching, *Ronald Calinger,* Editor.

41. Geometry Turned On: Dynamic Software in Learning, Teaching, and Research, *James R. King and Doris Schattschneider,* Editors.

42. Resources for Teaching Linear Algebra, *David Carlson, Charles R. Johnson, David C. Lay, A. Duane Porter, Ann E. Watkins, William Watkins,* Editors.

43. Student Assessment in Calculus: A Report of the NSF Working Group on Assessment in Calculus, *Alan Schoenfeld,* Editor.

44. Readings in Cooperative Learning for Undergraduate Mathematics, *Ed Dubinsky, David Mathews, and Barbara E. Reynolds,* Editors.

45. Confronting the Core Curriculum: Considering Change in the Undergraduate Mathematics Major, *John A. Dossey,* Editor.

46. Women in Mathematics: Scaling the Heights, *Deborah Nolan,* Editor.

47. Exemplary Programs in Introductory College Mathematics: Innovative Programs Using Technology, *Susan Lenker,* Editor.

48. Writing in the Teaching and Learning of Mathematics, *John Meier and Thomas Rishel.*

These volumes can be ordered from:
MAA Service Center
P.O. Box 91112
Washington, DC 20090-1112
800-331-1MAA FAX: 301-206-9789

Contents

Tier III Projects

Project Listing by Category

Business Mathematics

Developmental Mathematics

Precalculus

Quantitative Literacy/Special Topics

Statistics

Introduction

This handbook contains a collection of the winning entries in the first INPUT Competition, part of the INPUT (**In**novative **P**rograms **U**sing **T**echnology) Project. The INPUT Project was designed to improve instruction by recognizing and rewarding college instructors who have rethought the mathematical content of their introductory mathematics courses to take advantage of current technology. The targeted "introductory" mathematics courses were developmental mathematics, precalculus, business mathematics, quantitative literacy, and introductory statistics.

> Advances in knowledge, new connections between disciplines, changes in instructional technology and a new generation of students have made the classical approach to teaching and learning unattractive, unexciting, and simply inappropriate for today's needs. New directions in education show that teaching is highly interactive. It engages students actively in making sense of ideas and applying what they learn to solve personally and socially relevant problems. Learning is a dynamic process by which meaning is developed by searching for relationships. Knowledge must be conceptual and inter-connected. Students will attain more 'meaningful learning' if they obtain it through several routes and in an integrated form. The key to meaningful learning is to tie new information to other things students are learning or to the concerns of their personal lives. Our students have grown up with many learning tools: they are part of the television age; they quickly learn new computer games; they program VCRs; they use words not yet in dictionaries because things are created or discovered at a faster pace than can be recorded. They are constantly seeing events unfold; and yet, we continue to teach them in the same way.
>
> —Martha Aliaga

This quote from Martha Aliaga of the University of Michigan, one of the top prize winners in the first INPUT Competition, reflects the challenge that we as mathematics educators currently face. Many teachers of K–12 mathematics and calculus have already accepted the challenge. However, many introductory mathematics courses are still taught using traditional methods.

Introductory College Mathematics Courses

With the possible exception of English composition courses, the courses with the largest enrollment at most institutions are mathematics "service courses," or "introductory" college mathematics courses. These courses serve more than 70% of the students studying mathematics in college. Because introductory college mathematics plays a critical role for many professions, improving instruction at this level is essential. For the non-math major, these courses present new applications and increased career options.

While much had been written describing the development and assessment of calculus reform, Cohen [26] noted that no group had yet attempted to establish standards for "introductory" mathematics programs. In 1995 the American Mathematical Association of Two-Year Colleges (AMATYC), with assistance from other professional mathematics organizations, issued *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* [26]. That document established three sets of standards that addressed goals for student intellectual development and the mathematics content and pedagogy necessary to support them.

Standards for Intellectual Development:

I-1 Problem Solving
I-2 Modeling
I-3 Reasoning
I-4 Connecting With Other Disciplines
I-5 Communicating
I-6 Using Technology
I-7 Developing Mathematical Power

Standards for Content:

C-1 Number Sense
C-2 Symbolism and Algebra
C-3 Geometry
C-4 Functions
C-5 Discrete Mathematics
C-6 Probability and Statistics
C-7 Deductive Proof

Standards for Pedagogy:

P-1 Teaching with Technology
P-2 Interactive and Cooperative Learning
P-3 Connecting with Other Experiences
P-4 Multiple Approaches
P-5 Experiencing Mathematics

The standards were intended as a call to action, a catalyst for national discussion, and an inspiration for creative thinking and innovation. It was anticipated that the document would help faculty revitalize the mathematics curriculum preceding calculus and stimulate changes in instructional methods so that students would be engaged as active learners in worthwhile mathematical tasks.

Since AMATYC released its standards document, a small but growing number of faculty have been working hard to revitalize the introductory mathematics curriculum. The use of technology has allowed many of them to approach instruction in ways that simply were not possible before, and it has freed both the faculty and the students from many rote-drill tasks associated with the traditional paradigm. Like faculty working on calculus reform, the work of faculty who are restructuring introductory college mathematics courses is not often promoted or rewarded.

The calls for reform, the potential of technology to support such reform, and the successes of a number of faculty who have pioneered reform have not changed introductory college mathematics courses as rapidly as one would expect. Change has been slow to occur in these courses for the following reasons:

1. Due to the large number of students who must take mathematics, the primary learning environment for these undergraduate students is usually a lecture-discussion, large-group format. A survey by the Mathematical Association of America [168] reported that 87% of mathematics discussion sessions were taught by graduate assistants or temporary faculty. These instructors generally do not have the influence to lead a reform effort.

2. Some instructors are not convinced that new approaches to teaching will work better than traditional methods. The old approach is what they know and it worked for them. Others may be unaware of alternative teaching methods or may not have access to innovative course materials.

3. Many instructors feel they will not be able to cover "all the topics" included in the traditional course if they use reform methods. These instructors believe that more students will continue on in mathematics than actually do. Another award winner, Sheldon Gordon, has indicated that approximately 700,000 students annually take some variety of precalculus, yet only 15–20% ever continue with calculus.

4. Although reform is often motivated by new technologies, introductory college mathematics instructors are often either inexperienced in the use of technology or are not given adequate support or time to retrain. Many colleges and universities are not devoting adequate financial support to these courses which generate substantial student credit hours.

5. Instructors are often promoted on the basis of research publications, not on time invested in teaching. Thomas Tucker, who chaired the CUPM subcommittee on calculus reform, noted that changing what happens in the classroom is hard work—it requires talent, dedication, and enthusiasm that for the most part goes unrecognized.

Purpose of the INPUT Project

The reports and documents of the past decade [81,26,122,169,125,127] have called for a paradigm shift from emphasis on teaching mathematics with a rule-oriented format to an emphasis on problem solving in a realistic context. This paradigm shift has prompted a change from rote drill exercises and short, artificial problems to applications of realistic problems. Consistent with the new paradigm, some innovative mathematics instructors have begun to focus on enhancing students' skills in critical thinking, pattern detection, and collaborative problem solving. Students are more frequently experiencing mathematics as a set of realistic problems that require independent thinking and sustained effort.

Why aren't more mathematics faculty using technology to support changes consistent with the mathematics paradigm shift? Guskin [70] ponts out that college administrators generically assert that "we need to restructure what faculty do" and "we need to introduce new information technologies," but fail to give any examples of what is meant, how this might be done, or how it relates to what we know about student learning.

The INPUT Project, funded by the Annenberg/CPB Project, the National Science Foundation, and Central Michigan University established several goals to hasten the pace of change in mathematics service courses:

1. to discover important changes in curriculum, pedagogy, and assessment inspired by the use of technology,
2. to reward instructors who took advantage of technology to rethink and improve the mathematical content of their courses,
3. to disseminate detailed examples of reform in mathematics service courses,
4. to encourage more faculty to reevaluate their course content by incorporating existing technologies into their curriculum.

To meet these goals the INPUT Project conducted an international competition to identify and reward college instructors who rethought the mathematical content of their introductory mathematics courses with innovative uses of technology. This volume, intended as a handbook, a World Wide Web page and a videotape were developed to inform instructors about successful projects and to motivate other faculty to follow the paths of these pioneers.

Purpose of the INPUT Project Handbook

Priming the Calculus Pump: Innovations and Resources [179] was designed to disseminate and promote the efforts of the many individuals and institutions trying to reform calculus instruction. This book included detailed descriptions of ten featured calculus reform projects, abstracts of more than sixty other projects, and a collection of reference materials and resources. It presented mathematics faculty with actual examples that they could use in their own classes to change calculus instruction.

The INPUT Project Handbook, *Exemplary Introductory College Mathematics Programs*, was designed to do for introductory college mathematics what *Priming the Calculus Pump* did for calculus reform; that is, to provide concrete, detailed descriptions of what some innovative instructors are doing, how they are doing it, what technologies they are using, and how these instructors can be contacted. This handbook contains descriptions of the top 20 submitted projects that best exemplified the goals of the competition. The included reference materials in this handbook are not exhaustive, but they provide a wide variety of resources for the faculty member contemplating a new approach to teaching introductory college mathematics.

It should be noted that the projects featured in this handbook are not the only examples of introductory college mathematics reform. They were selected from the applications received for the INPUT Competition. Many more institutions seriously committed to mathematics reform do not appear in this handbook because they did not submit applications. Several promising projects were not selected because they were not yet fully developed or did not contain evaluation results. Because many of these projects are in a formative stage, it is not yet possible to assess the long-term impact of these reform efforts. It is hoped that this INPUT Handbook will inspire others to experiment with their ideas and collaborate with peers in designing their own innovative courses.

Some preliminary analyses of the projects submitted suggest the following themes:

1. The focus is more on realistic applications, often derived from real-world data.
2. The restructured courses and programs represent an improvement over traditional instruction in their impact on student attitudes. Even though the results on standardized tests are not significantly different, course withdrawal rates have been significantly reduced, and students have gained confidence in their ability to solve real problems utilizing mathematics.
3. The use of technology has led to rethinking the mathematical content of these courses. Students have gained access to concepts and problems that would have been difficult or impossible to pursue without the technology. There is more emphasis on conceptual understanding not just rote manipulation.
4. There have been a number of obstacles for instructors to overcome before successful implementation. The obstacles include a lack of administrative, technical, financial, and peer support as well as a reluctance from students themselves to embrace change.
5. Innovation has occurred most readily within a context of support and reward.
6. Successful implementation of reform has rejuvenated the content, the classrooms, and the faculty involved. More importantly, it has expanded the base of students who are turned on by mathematics.

Crossroads in Mathematics [26] correctly predicted the implementation period for reform will be chaotic and uncertain, but it has been exciting, challenging, and professionally fulfilling. Faculty are beginning to understand their capacity to foster change. Although the instructors whose projects are featured in this handbook have won cash awards, the true winners are the students who have been provided with a more exciting learning experience.

About the First INPUT Competition

The first step in establishing the INPUT competition was to create an advisory board. The board, selected for its expertise in various disciplines and technology, assisted the CMU INPUT project team in all aspects of the project. The advisory board included:

- Bostwick F. Wyman, PhD Professor and Vice-Chair, Department of Mathematics, The Ohio State University—Chair of the Advisory Board
  Dr. Wyman is the Vice-Chair for Instruction in the Mathematics Department at The Ohio State University. He has been involved with numerical and symbolic computation for many years in conjunction with his research in control theory and algebraic system theory. For four years he directed summer workshops in pedagogy and technology with support from the Undergraduate Faculty Enhancement Program of the National Science Foundation.

- Lyn Allen, PhD, Education Manager, Ameritech
  Dr. Allen has been a teacher of mathematics and computer science and an educational technology consultant for schools, the Michigan Department of Education, and for Apple Computer. Currently she is coordinating educational efforts for Ameritech.
- Randy Bass, Ph.D., Assistant Professor of English, Georgetown University
  Dr. Bass is the director of Project FUTURE: Faculty Using Technology: Understanding Rewards and Evaluation.
- Stephen C. Ehrmann, Ph.D., Senior Program Officer, The Annenberg/CPB Project
  As an author and funding monitor/evaluator, Dr. Ehrmann has analyzed pioneering educational applications of computing, video and technology since the late 1970's.
- Joseph Fiedler, Ph.D., Associate Professor of Mathematics, California State University, Bakersfield
  Dr. Fiedler has been active in exploring the use of Computer Algebra Systems in calculus, business calculus, precalculus, and developmental mathematics. He has trained other faculty in the use of Computer Algebra Systems, and since 1990 has given many presentations and faculty workshops. He currently serves as Chair of the MAA subcommittee on Service Courses.
- Gregory Foley, PhD, Associate Professor of Mathematics, Sam Houston State University
  Dr. Foley is a mathematics educator, consultant, and textbook author who is interested in the appropriate use of technology in mathematics teaching and learning in Grades 6-14. He has directed three federally supported calculus projects and has contributed his services to AMATYC, CBAMN, MAA, MSEB, and NCTM.
- Catherine Fulford, PhD, Associate Professor, Department of Educational Technology, University of Hawaii at Manoa
  Dr. Fulford teaches courses in instructional design, distance education, media, and video. Her research focuses on interaction in distance education and cognitive speed.
- Barbara Jur, EdD, Associate Dean for Mathematics and Coordinator of Special Projects, Macomb Community College (MI)
  Dr. Jur is Chair of the Mathematics Department at Macomb CC. She is past Vice-President for Two Year Colleges of the Michigan Section of the MAA, and past Chair of the MAA Subcommittee on Service Courses which produced the "Report on Service Courses for Business."
- Richard Scheaffer, PhD, Professor of Statistics, University of Florida
  Dr. Scheaffer has led numerous statistics education projects directed at curriculum development and teacher enhancement within K-12 and undergraduate settings. He is now the Chief Faculty Consultant for AP Statistics.

In November, 1995, flyers about the competition were distributed at the AMATYC Conference in Little Rock and at the ICTCM Conference in Houston. They were also distributed, in January, 1996, at the AMS/MAA Joint Conference in Orlando. In addition, mailings were sent to mathematics departments throughout the world. Applicants for the competition were required to submit a 20-page application that addressed the following categories: methodology and soundness, measures of effectiveness, use of technology, and willingness to disseminate. Methodology and soundness was evaluated on the extent to which the applicant's project was grounded in the reform literature and met AMATYC and NCTM standards as well as the following criteria:

IN-1 Instructor, student and administrative feedback
IN-2 Appropriate support of client departments
IN-3 Potential for adoption at other institutions
IN-4 Extended accessibility for non-traditional students
IN-5 Improved efficiency (cost effectiveness)
IN-6 Revitalized course content through the use of technology
IN-7 Coordination of content with sequel or related courses
IN-8 Collaboration with other disciplines, colleges, business and extracurricular agencies

The project plan stated that 20 awards would be made: Tier I—the top five applications, including the grand prize of $5,000; Tier II—the next ten applications; and Tier III—five notable efforts. A 100-point rating scale and rating form were developed from the four categories of information requested in the application. The CMU project team included pairs of faculty and graduate students who reviewed all applications and assigned points to them using the rating form. Then the entire project team engaged in extended discussions that eventually led to consensus regarding the top 20 projects. The total points assigned to each application served as a starting point for separating those 20 applications into the three tiers. In addition to the criteria on the rating sheets, tier assignments were influenced by two factors: 1) the extent to which an applicant had met the criteria (e.g., the extent of the documentation of measures of effectiveness), and 2) the extent to which an applicant furthered the overall project goal of encouraging more faculty to rethink their course content and incorporate technology into the curriculum. Additional discussion led to consensus regarding the tier to which each application was assigned.

In September, 1996, the advisory board conducted a review of the top 20 projects identified by the CMU project team. Names and all references to institutions were eliminated from the applications before they were sent to the advisors for a blind review. The board made independent judgments regarding their assignment of the projects to the three tiers, and then met with the CMU project director to compare their tier placements with those of the CMU project team. With only one exception, the tier placements of the two groups were in agreement. The disagreement was resolved by selecting a Tier I winner from each of the 5 targeted introductory mathematics courses.

The top five award winners were featured on the program of the American Mathematics Society/Mathematical Association of America (AMS/MAA) 1997 annual conference in San Diego. The winners were also presented with cash awards ranging from $500 to $5000.

Tier I

This section includes comprehensive descriptions of the top five award-winning projects

INNOVATIVE
PROGRAMS
USING
TECHNOLOGY

Introduction to Statistics: Enhancements Through the Use of the Graphing Calculator

Martha Aliaga

Service Course Area

Statistics

Institutional Data

The University of Michigan was founded in 1817 and permanently located in Ann Arbor in 1837. With a faculty of over 3,000 and a student body of about 36,000 (of which 13,000 are graduate students) the Ann Arbor campus of the University of Michigan is able to provide instruction and carry out research in a wide range of disciplines.

The Department of Statistics at the University of Michigan was formed in 1969; it is part of the College of Literature, Science, and the Arts, which administers its undergraduate programs, and of the Rackham Graduate School, which administers its graduate programs. The Department offers graduate programs leading to the Master of Arts degree and the Ph.D. degree, combining courses in theoretical and applied statistics with experience in consulting and computing. In 1995–96, 22 faculty members were affiliated with the Department. Of these, ten held full-time appointments here, nine hold joint appointments with other departments (including Economics, Industrial & Operations Engineering, Psychology, and the Center for Statistical Consultation & Research), and there are three visiting or adjunct faculty. In the Fall term of 1995–96, 21 courses at the graduate level were offered. In addition to regular courses there are typically two or three special topics seminars operating each week as well as a colloquium. There are other groups of statisticians at the University. These groups are located in the Mathematics Department, the Biostatistics Department, the Engineering College, the Business School and the Institute for Social Research. Many courses offered in statistics and probability are cross-listed between these units and the Department of Statistics.

The Masters programs are designed primarily to equip students for careers in business, industry, or government as professional statisticians. The Ph.D. program provides students with the additional training to engage in research and to teach at the university level. The formal training includes courses in the Department, seminars, and guided reading. There is a great deal of flexibility and students may tailor programs to their own special interests.

Abstract

Statistics 100 at the University of Michigan is the Introductory Statistics class which many undergraduate students take to fulfill a quantitative graduation requirement. Three years ago Dr. Martha Aliaga introduced the use of the TI82 graphing calculator, to teach this large class. She combined it with highly interactive, cooperative learning and used relevant data from real life as a source of problems. Since that time she has co-authored a text, *Interactive Statistics* [3], with Dr. Brenda Gunderson, which states that the students are "co-authors in the sense that they have to 'write' in ideas...to make the text complete."

The use of the TI82 (upgraded to the TI83 in Fall, 1996) has the tremendous advantage of providing immediate feedback to inquiries. This allows students to spend more classroom time applying and extending skills than was previously possible when time consuming calculations were required. Instead of spending time calculating, students spend time thinking. This technology has transformed the classroom from a place where students come to listen to a lecture to a place where students come to learn and perform. A change has taken place from a focus on statistical methods to a focus on statistical thinking.

The goals of Statistics 100 are:

To use statistics in order to gather evidence to make conjectures, to formulate models, to invent counterexamples, and to build sound arguments.

To enhance students' attitudes toward mathematics and statistics.

To develop students' confidence in statistical skills and to incorporate these skills into everyday life.

THE PROJECT

Motivation for project

More and more college faculty are recognizing the need for change. Advances in knowledge, new connections between disciplines, changes in instructional technology and a new generation of students have made the classical approach to teaching and learning unattractive, unexciting, and simply inappropriate for today's needs.

New directions in education involves teaching that is highly interactive, and which engages students in making sense of ideas and applying what they learn to solve personally and socially relevant problems. Learning is a dynamic process by which meaning is developed by searching for relationships. Knowledge must be conceptual and interconnected. Students will attain more "meaningful learning" if they obtain it through several routes and in an integrated form. The key to meaningful learning is to tie new information to other things students are learning or to the concerns of their personal lives.

Our students have grown up with many learning tools: they are part of the television age; they quickly learn computer games; they own VCR's, they use words not yet in dictionaries because things are created or discovered at a faster pace than can be recorded; they are constantly seeing events unfold; and yet, we continue to teach them in the same way.

Teachers should avoid routine and repetitive tasks, and emphasize the power of communication, problem solving, and inventiveness, in order to make the students persistent and curious.

My experience in more than 25 years of teaching has led to a significantly different type of education. It has been my goal in the classroom to teach the students how to THINK. I strive to propose difficult problems to my students and let them design the solutions. I like to offer questions and problems which provide sufficient flexibility for the students to display their knowledge and their abilities. If I force them to solve the problem "my way," I curtail their potential to learn.

Students should be prepared to construct a world that we can not imagine, because it does not yet exist. Furthermore, they should learn how to solve problems and create the new world both individually and in groups. The new standards in education stress communication, cooperation, and group performance.

To value statistics, students' experiences must bring them to believe that statistics is relevant to their lives. Students must learn how to gather evidence to make conjectures, to formulate models, to invent counterexamples, and to build sound arguments. They have to learn how to read, write and speak about statistical topics as a strategy for understanding. A wide variety of problems must be presented that vary in context, length, difficulty, and solutions.

Activities for the students should be active, not passive. Emphasis should be on analysis and comprehension of the analysis, not on a single correct answer. Real data and hands-on experiences should be used whenever possible. Sample problems must be selected to build intuition, not to learn how to manipulate the data and deceive.

Modeling Teaching

In the keynote address delivered by Dr. D. J. Lewis, Chair of the Mathematics Department at the University of Michigan on July 29, 1993 at the Mathematics Education Reform Workshop in Ann Arbor, he placed calculus reform within the bigger picture of the tri-part academic mission of research, teaching, and service to the community.

Mathematics and Statistics play an essential role at all levels of the educational process, particularly in science and engineering. We must have excellence at the top in order to have excellence down the line. Research mathematicians train graduate students who go on to teach in the nation's colleges. Mathematics and statistics researchers receive their training from active researchers in university research departments; so too, do most college teachers, whether or not they become researchers. U.S. post secondary mathematics education and mathematics research are interdependent, and the university department is their nexus.

College teachers, in turn, train the next generation of primary and secondary school mathematics teachers. This process is producing too few teachers who are qualified to teach mathematics. Yet it is at the primary and secondary school levels that students often decide that they can or cannot undertake careers in science or engineering.

It is our obligation as teachers to strive to teach to the best of our abilities. We must constantly try to improve; we cannot repeat each semester the "routine" we have comfortably developed over the years. We cannot let teaching take a back seat to research or publication, or even to service. Try to imagine a system where our compensation came directly from the students, and they paid us each day based on our ability to teach, our demonstration of caring, and our commitment to their futures. If, then, you can imagine your behavior in the classroom "improved" from its current state, you have an obligation to achieve that higher order of teaching.

Today mathematics is too often a barrier that discourages students from making ambitious career choices. This is particularly true for minorities and women students. Major initiatives in mathematics and statistics of this kind will play a crucial role in strengthening mathematics and statistical education at all levels, and hence in assuring that more students will succeed in these two careers.

Statistics 100, Introduction to Statistical Reasoning, is designed to provide an overview of the field of statistics. Course topics include methods of analyzing and summarizing data, statistical reasoning as a means of learning from observations (experimental or sample), and techniques for dealing with uncertainties in drawing conclusions from collected data. The chief tenet supporting the use of the graphing calculator in the course is the motto: "learn by discovery." Research shows that once the students visualize the problem with a graph, the problem is easier to solve. Students can be led to many "discoveries" on the spot. The ability to change the setting of the problem, and have immediate results and feedback, allows the students to think through many new ideas in one class period. These new ideas will be timely and relevant, and take advantage of the "moment of readiness."

The use of graphing calculators eliminates much time spent on the manipulation of the formulae, and frees up time for teaching concepts and problem solving. It also alleviates the agony of many of the students who are intimidated by mathematics and calculations. With this technology every one of the 150 students per lecture can have the calculator in his or her hands during the instructor's lecture. The lecturer uses a big screen simultaneously. The use of the calculator also accommodates working in groups. Compared to computers, the use of calculators is cost effective and flexible. Some of the Statistics 100 students have already purchased the calculator, since it is required for Calculus I and II in the Mathematics Department; others have it from high school math courses.

Innovations

The use of the graphing calculator in Statistics was "borrowed" from calculus reform. Its application in the University of Michigan's Math Department is fluid; it is a "natural fit" and has impacted the teaching style in the department profoundly.

The tremendous advantage of using the graphing calculator is the immediate feedback to inquiries. This time maximization allows students to spend more classroom time applying and extending the skills than was previously possible when time consuming calculations were required. Instead of spending time calculating, students spend time thinking. This allows students the opportunity to problem solve with immediate results, and then either review or expand the thinking required to solve the problem. It becomes more apparent that there may be more than one way to solve a problem, and there may be more than one answer that is correct.

The text used in the course is *Interactive Statistics* [3] by Martha Aliaga and Brenda Gunderson. It incorporates the use of the graphing calculator, and is written in an interactive way. The students are co-authors in the sense that they have to "write" in ideas, thoughts and answers in order to make the text complete.

Pedagogical Principles

I use the following principles in teaching, some of which are from the literature, and some of which I have developed after years of teaching.

- The most effective time for learning, retention, and ability to utilize material later is that developmental "moment of readiness." Therefore, that is the time to respond to questions, solve problems, straighten out mistakes in learning (S18).

- Students must be tested on material while still at the beginning of the learning curve, so that errors can be noted immediately, and corrected before they become an erroneous part of the statistical thinking and understanding.
- All information about statistics is disseminated in sequences and sizes that mirror students' learning ability. As their incremental knowledge increases, one can move faster, and also, in larger conceptual blocks.
- Problems should include content which is relevant to real life.
- Quizzes test on all previous material, with emphasis on the most current. This approach avoids the feeling that all previous assumptions about statistics are to be erased, that class "starts over" with this instructor. (Remove obsolete and not useful paradigms about statistics.)
- The students work in small group settings. Although there is much advice in the literature about how to form groups, I prefer to do it at random. During the first day of class numbers are placed on the desks; the students pick a number at random at the door as they enter, and sit at the corresponding seat. The students then introduce themselves to the group through exercises, and share phone numbers. The following day students are invited to change seats if they prefer, although the majority choose to continue with the original group. They are also encouraged to work together on homework. The environment is conducive to learning as well as enjoyment.

  The students debate with and question each other to arrive at correct solutions. The group must share, discuss and agree on their combined answers, and then this is presented as the group's proposed response to the problem. This means that the group must review the statistical principles involved over and over, thus learning them more fully, as they try to convince the other group members about their suggested answers. If questions come up from the group or any individual student, the instructor redirects the questions back to the rest of the class. There needs to be a variety of methods to enable students to depend on their own understandings and abilities to move all the way to a solution. Also, it is worth using the class to expand the student's awareness of resources other than the teacher. Should no one have the answer, the instructor would of course provide it or provide a source to find it. The students are encouraged to participate, to talk, to debate, to disagree, to argue.
- Literature indicates that when the teaching objective of learning for understanding involves higher order thinking, task arrangements and instruction that constrain and routinize interaction will be less productive than arrangements and instruction that foster maximum interaction, mutual exchange, and elaborated discussions [26].
- *Believe* that *students* can achieve, and that you can teach them successfully.

PROJECT REPORT

Getting Started

The reform in the Statistics 100 course followed the reform in calculus in the Mathematics Department, in which I also participated. The support of the University of Michigan Department of Statistics was critical in the acceptance and implementation of the use of technology in teaching. The two large classes in Statistics are Statistics 100 (for non-majors) and Statistics 402 (required for majors and research degrees). Each senior faculty is required to teach either course. Those that teach Statistics 100 are required to use *Interactive Statistics* which incorporates the graphing calculator.

Pedagogy

Using the graphing calculator affects pedagogy. Students are able to *see* the function in a graph. This visualization allows them to more readily comprehend the scope and purpose of the function. They can prove, predict and verify statements. The calculator can also produce the simple statistical plots that help the students learn about analyzing data. While a computer can do all of these things, it is not financially or logistically practical for students to have access to a computer during large lecture classes.

I include an innovative strategy of using two overhead projectors when presenting problems. (one of these projectors shows the stated problem, while the other shows the solution as it is being solved by either a student or instructor). Pages from the book are projected so students are well oriented, and time is not wasted copying the problem. Students spend their class time on solutions. The pages of the book are formatted to accommodate this projected use.

Instructors. Statistics faculty rotate through the instructor assignments of teaching the largest classes, Statistics 100 and Statistics 402 with approximately 150 students in each class. Many of these faculty are established, tenured professors who are used to teaching in traditional methods, and many are greatly involved in research. Some of these instructors were reluctant to use my new methods of teaching statistics, and underwent training with skepticism. However, these faculty have "converted" to using the graphing calculator and cooperative learning. They have endorsed the idea of having students use "real life" data gathered from current newspaper articles, and data gathered from the class.

The Graduate Student Instructors (GSIs) meet once per week with the students in labs. They facilitate the hands-on experience of the students, solving projects and performing experiments, using both the TI calculator and Macintosh computers (SPSS).

Technology

The TI82 calculator has been used in the University of Michigan Mathematics Department since 1991. It started as a pilot project funded by NSF. The instructors found the calculator classes much more interesting to teach than the standard calculus class. The transfer to statistics was a logical extension and my familiarity with it through teaching mathematics provided the background to train the Statistics Department in its use. Many students who enrolled in Statistics 100 already had the calculator in their possession.

While the faculty acceptance of the use of the graphing calculator has been great, there have been a few members who have been reluctant to use it. One instructor preferred using the computer, and assigned the TA to use the calculator with the students. Some faculty who do much research are more reliant on the computer, and find the transition a bit annoying.

There are no problems supplying nor maintaining the calculators. One set of batteries lasts approximately one semester.

One other aspect of technology in the course is the use of the videotape series *Against All Odds: Inside Statistics*, produced by the Annenberg/Corporation for Public Broadcasting, 1989. Portions of the series are used intermittently to give an overview and to motivate students by showing the application of statistical techniques in real life.

Day to Day Mechanics

Many real problems are selected from the Chance Program available on the Internet. Election years are rich in topics for statistical discussions.

Appraisal

Internal measures of effectiveness. Perhaps the most revealing endorsement of internal success of Statistics 100 is the increased enrollment. The enrollment has increased substantially, from 210 in Fall, 1991 to 540 in Winter, 1996. Moreover, although exact data are not available, the percentage of students who have enrolled into Statistics 402 from Statistics 100 has increased.

When asked to evaluate the text in the question, "The textbook is easy to read and understand," 128 students rated it 4.10 on a scale of 5.0. In terms of teaching strategy, the highly interactive nature of the class accommodated a great deal of communication between teacher and students, making the students feel more engaged in discussions and more comfortable asking questions. This is reflected in the student response (58 students) to the question: "Students feel comfortable asking questions." This question was scored 4.05 on a scale of 5.0.

Some typical student responses follow:

"I really dreaded this course but it turned out to be one of my favorites."

"I really enjoyed the class, I think I will take another stats class."

External measures of effectiveness. The described course is being piloted at Garrett Community College in Maryland; Northeastern Illinois University, Mathematics Department in Chicago; and Turtle Mountain Community College for American Indians in North Dakota.

Cost benefit. Clearly the enrollment of Statistics 100 is large, averaging 150 per class. Cooperative, interactive learning and the use of the graphing calculators are compatible with this size class. Group work gives the opportunity for all students to be involved in problem solving. Experience teaching the course has found that groupings of three to four students were ideal when the class size was 20–25, and groupings of two were ideal when the class is large, over 100. The groups are easy to form and, if needed, can easily be combined to form larger groups. Interestingly, students prefer to work in a group of four students in a class of size 150 than to work individually in a small class of 20.

The calculators used in class are portable, user-friendly and affordable. They are cheap enough to expect students to purchase their own. While students can buy previously-used calculators, there are not many available since the students want to keep them.

Future Plans

Already several colleges and universities have consulted with me and are using the graphing calculator to teach statistics. A great deal of publicity through conferences and workshops has been accomplished to date both inside and outside of the country. Future plans include training high school teachers to use this method.

A Web Page is being created to describe the course, its implementation of technology and its assessment (http://www.stat.lsa.umich.edu/~aliaga/).

SAMPLE MATERIALS

Text Used: Aliaga, Martha and Brenda Gunderson. *Interactive Statistics*, 1998, Prentice Hall.

Bulletin Description

Statistics 100. Introduction to Statistical Reasoning. This course is designed to provide an overview of the field of statistics. Course topics include methods of analyzing and summarizing data, statistical reasoning as a means of learning from observations (experimental or sample), and techniques for dealing with uncertainties in drawing conclusions from collected data. Basic fallacies in common statistical analyses and reasoning are discussed and proper methods indicated. Alternative approaches to statistical inference are also discussed. The course emphasis is on presenting basic underlying concepts rather than on covering a wide variety of different methodologies. Technology is used in class with a graphing calculator (TI 82/83); there is a weekly computer lab. The text emphasizes cooperative and interactive learning. Course evaluation is based on a combination of a Thursday evening midterm examination, a final examination, four quizzes, 12 homework assignments, and teaching assistant input. The course format includes three lectures and a laboratory (1 hour per week).

Requirements. There are no prerequisites in calculus for this course. Some algebra is necessary. This is a first class in statistics. It may be taken by both statistics majors and non-majors. It is one of the courses that fills the quantitative graduation requirement of the University of Michigan.

Course Syllabus

TOPICS

Prelude
Overview — Decision Making
Producing Data — Sampling Techniques
Observational Studies
Experimentation
Confounding Variables
Control Group-Randomization-Blinding
Ethics
Summarizing Data Graphically
Line Plot- Stem and Leaf Plots
Histograms
Box Plots
Time Plots
Summarizing Data Numerically
Measuring Center
Measuring Variation
Models
Normal Distribution
Uniform Distribution
Discrete Distributions
Relationships Between Variables
Positive/Negative Association
Scatter Plots
Linear Relationship
Linear Regression
Outliers/Influential Points
Residual Plots
Correlation Coefficient
Coefficient of Determination
Contingency Tables
Marginal Distributions
Conditional Distributions
Probability by Simulation
Basic Ideas of Probability
Geometric Probability
Random Variables
Sampling Distributions
Sampling Distributions of a Sample Proportion
Sampling Distributions of a Sample Mean
Bias and Variability
One-Sample Inference
Testing Hypothesis about a Population Proportion
Testing Hypothesis about a Population Mean
Normal Distribution/t-Distribution

Course Assignments/Example of Tests

1. One principle of the justice system is that the defendant in a trial should be considered innocent until proven guilty. What would the null and alternative hypotheses be in the context of a criminal trial?
   Ho:
   H1:

What would be the trial equivalent of a Type I error and a Type II error?

Type I error :

Type II error:

2. The following table summarizes the hypotheses and results for three different studies.

	Null Hypothesis	Alternative Hypothesis	p-value
Study A	The true average lifetime is ≥ 54 months	The true average lifetime is < 54 months	0.0251
Study B	The average time to relief for Treatment I is equal to the average time to relief for Treatment II	The average time to relief for Treatment I is not equal to the average time to relief for Treatment II	0.0018
Study C	The true proportion of adults who work two jobs is ≤ 0.33	The true proportion of adults who work two jobs is > 0.33	0.3590

(a) For which study do the results show the most support for the null hypothesis? Explain.

(b) Suppose Study A concluded that the data supported the alternative hypothesis that the true average lifetime is less than 54 months, but in fact the true average lifetime is greater than or equal to 54 months. In our statistical language, would this be called a Type I error or a Type II error?

(c) If the results of Study C are "not statistically significant", which hypothesis would you accept?

3. Population A is divided in STRATA 1 and STRATA 2

STRATA 1 contains all the women
STRATA 2 contains all the men
The variable of interest is income

(a) STRATA 2 contains more people than STRATA 1

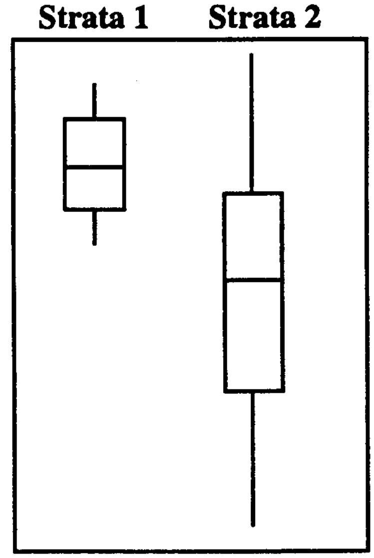
Circle one: True False Can't tell
Explain

(b) The distribution of Variable Income in Strata 1 is symmetric

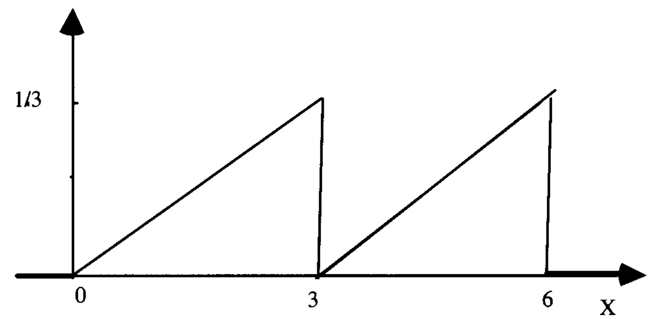
Circle one: True False Can't tell
Explain

Population A

Strata 1 **Strata 2**



4. The density function for a continuous variable X is given below:



(a) Is this distribution symmetric?

Circle one: Yes No
Explain

(b) What is the numerical value of the ***median*** of this distribution?

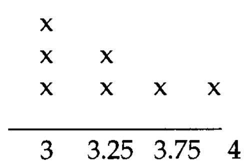
Circle one: Larger than 3 Equal to 3 Less than 3
Explain

5. Let X , Y , and L be three random variables whose distributions are:

X is N(3, 4), (4 represents the variance)

Y is U(3, 4),

L is



Calculate:

Mean of X =
Mean of Y =
Mean of L =

The Interquartile Range of X =
The Interquartile Range of Y =
The Interquartile Range of L =

The standard deviation of X =
The standard deviation of L =

Prob(X > 3) = ________ Prob(Y > 3) = ________ Prob(X = 3) = ________

Prob(2< X < 3) = ________ Prob(L = 3) = ________

6. THE THREE DOORS

This is a game in a TV show. There are three doors. Behind one door is a car. Behind the other two doors is a goat. Once you select a door, you will receive the prize that is behind that door.

Monty Hall, the game host, knows what is behind each door. At this point you, as contestant, choose a door. Before opening it, Monty reveals a goat behind *another* door.
You have the following options: (1) Stay with the door you originally selected.
(2) Switch to the other remaining closed door.

Which option should you choose? You want to win the car.
Let's carry out a simulation with three cards, two ones (doors with a goat) and one five (door with the car).

You will begin to take turns playing the game, and as you do, keep a record sheet, list your strategy (stay or switch) and the outcome (car or goat)

Strategy = STAY

Win Car	Win Goat

Strategy = SWITCH

Win Car	Win Goat

Which strategy is better? The more you repeat the game, the better your estimate will be.

Switch strategy:

Count the # of times the strategy was to switch, and of those, how many times you won the car.
Write this as a fraction and convert to a decimal.

Stay strategy:

Count the # of times the strategy was to stay, and of those, how many times you won the car.
Write this as a fraction and convert to a decimal.

Which strategy seems to be the best to use?

7. How does the Correlation Coefficient between two variables X and Y change if there are linear transformations of the variables X and Y?
Using your graphing calculator investigate the following cases.
Consider the following distributions for X and Y

X	2	3	4	5	5	6
Y	1	2	2	3	4	5

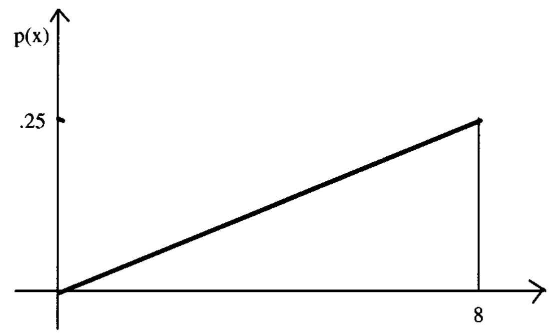
Calculate r (X, Y) =

Define $M = -3X - 1$ and $N = -Y - 1$ Define $T = 4X + 5$ Define $L = 7Y + 2$

Calculate r(M, N) = Calculate r(X , N) = Calculate r(L, T) =

Explain what you have observed.

8. The distribution (called a triangular distribution) for a continuous random variable X is given below.



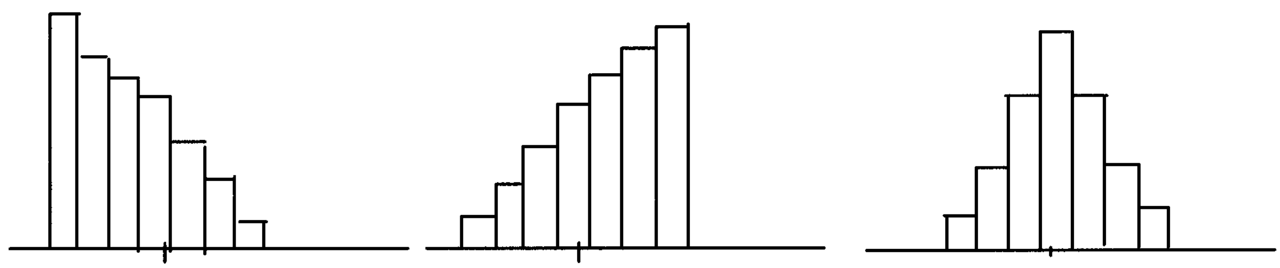
(a) Based on this distribution, is the median for X equal to 4? Explain your answer.

(b) Is the median of this distribution equal to its mean? Circle your answer : Yes No

If Yes, explain why. If No, state which value is larger and explain why.

(c) Suppose we are going to take a simple random sample of size 500 from this distribution.

Which of the following histograms best portrays the sampling distribution of the X bar (Sample mean)? Explain.



Instructions to Teaching Staff

Instructors and Special Training
Coordinator: Aliaga in Winter 1996

The following functions are performed by the Course Coordinator:
Preparing syllabus and computer labs, using other faculty input.
Preparing for and facilitating the Weekly Instructor meetings.
Organizing the duties of the instructors: preparing homework, proofing drafts of solutions, preparing exam questions.
Organizing exam and review times, handling special cases for students with special needs.
Overseeing the overall operation of this coordinated course.
The instructors (four instructors and six GSIs) meet once per week with the Coordinator. Among other things, the Coordinator instructs them on the use and integration of the TI82/83 calculator. New programs, in addition to those that came with the TI82/83, are added for statistical inference sections. (In Fall 1996 TI83 was introduced; this calculator comes with many more programs.) The past week's lessons are reviewed in the weekly session, and plans for the upcoming lessons are discussed.

Material for this article was submitted by Martha Aliaga.
Portions of this article are reprinted with permission from Prentice Hall Publishing.

Functioning in the Real World: A Precalculus Experience

Sheldon Gordon, Florence Gordon,
B. A. Fusaro, Martha Siegel, Alan Tucker

Sheldon Gordon

Florence Gordon

Martha Siegel

Alan Tucker

Service Course Area

Precalculus/College Algebra and Trigonometry

Institutional Data

Suffolk Community College is a large comprehensive two year college located on Long Island, about 50 miles east of New York City. The three campuses together enroll over 21,000 students, the majority of whom are part time. The "typical" student is about 28 years old and female.

Most instructors require their students to have graphing calculators in all math courses from college algebra on up; some instructors require these calculators in their statistics and finite mathematics courses. The math department provides access to computers for students in its Math Learning Center. The college provides a number of open computer labs for students in central Computer Laboratories housed in each campus' library.

The other members of the project working group are Florence S. Gordon of New York Institute of Technology, B. A. Fusaro of Florida State University, Martha J. Siegel of Towson State University, and Alan C. Tucker of the State University of New York at Stony Brook, with advice from Judy Broadwin of Jericho High School in New York.

The project materials have also been class-tested at about 75 colleges, universities and high schools around the country. The technologies used at these institutions vary dramatically. Minimally, students in the different versions of the project course have used graphing calculators. At many of the class-test sites, the institutions provide dedicated computer labs with Derive, Maple, Mathematica, TEMath, spreadsheets, etc.

Abstract

The Math Modeling/Precalculus Reform Project has developed, tested, and widely implemented an alternative to traditional precalculus/college algebra and trigonometry courses that is in the spirit of the calculus reform movement. It is based on the applications of mathematics from the point of view of mathematical modeling. The course features innovative emphasis on mathematical topics such as families of functions, data analysis and fitting functions to data, difference equa-

tions models, modeling periodic phenomena, matrix algebra and its applications, and probability models. All are introduced to study and analyze a wide variety of applications from many different fields. In the process, students are encouraged to view mathematics and its applications from graphical, numerical, and symbolic perspectives with a focus on conceptual understanding. Student project assignments, writing, use of technology, and collaborative learning are all emphasized throughout. All of these are major themes that permeate both the NCTM Standards and the AMATYC Standards for Introductory College Mathematics Courses. The project text, *Functioning in the Real World: A Precalculus Experience* [63], is published by Addison-Wesley and has been class-tested by approximately 200 instructors involving approximately 5000 students at universities, liberal arts colleges, engineering schools, two year colleges and high schools around the country.

THE PROJECT

Motivation for the Project

The calculus reform movement has made great strides in changing the emphasis in calculus to emphasize graphical and numerical ideas in addition to symbolic manipulations. Some of the major themes in the reform movement include focusing on student projects, realistic applications, the use of technology, and collaborative learning. In the MAA's ACRE (*Assessing Calculus Reform Efforts*) report, Tucker and Leitzel [180] indicate that about two-thirds of all colleges and universities have implemented moderate or large scale reform efforts. The results will hopefully transform calculus into a pump, not a filter.

The next step is to address the problem of how we "fill the tank": how we increase the numbers of students who proceed on into the calculus portion of the mathematics pipeline. Each year approximately 700,000 college students take some variety of precalculus course; yet only about 15% to 20% of them ever go on to start calculus. Most of those who do go on to calculus display a singular lack of retention of the material they were taught and often cannot successfully complete calculus. The standard precalculus courses neither motivate the students to go on in mathematics nor adequately prepare them if they do continue. These issues are the subject of the MAA Notes volume *Preparing for a New Calculus* [167].

However, as the reform calculus courses are more widely adopted and implemented, it becomes evident that we need to change the preparatory courses that lead toward calculus. The new calculus courses place very different expectations on the students who are required to do very different things in very different ways. Consequently, the preparation for calculus should emphasize the same themes.

The traditional courses at the precalculus and college algebra level are designed primarily to develop algebraic skills that once were essential for success in later courses. The wide availability of technology and the changing requirements, especially in the client disciplines, requires a rethinking of this paradigm. For the results of a series of interviews with leading educators in the client areas, see Gordon [60].

Currently, students in upper division courses in engineering and the sciences do relatively little with pencil and paper mathematics; instead, they focus on developing mathematical models to describe real-world phenomena. These models typically involve differential equations, matrix methods, or often probabilistic simulations. The students examine the behavior of the solutions, particularly as the parameters underlying the phenomena change.

Simultaneously, students in business, the social sciences, and the biological sciences are expected to recognize trends from sets of data, construct appropriate mathematical models to fit the data, and make corresponding predictions based on the models developed. This is actually remarkably similar to what students in lab courses have been doing for centuries; the difference is that the students in the business and social science courses typically use spreadsheets for the analysis rather than hand-drawn graphs.

Therefore the primary emphasis on algebraic manipulation in traditional preparatory mathematics classes does not provide the foundation that students now need for all of these disciplines, nor does it adequately prepare them for the new calculus. Instead, a broader preparation is needed, one that better reflects the practice of mathematics. Students must learn to:

1. Identify the mathematical components of a situation (i.e., model it).
2. Select the right tool (paper-and-pencil, graphing calculator, CAS package, spreadsheet, etc.) to solve the problem.
3. Interpret the solution in terms of the original situation and, if necessary, change the assumptions used (i.e., introduce additional factors) in the model.
4. Communicate the solution to an individual who likely knows less mathematics, but who pays the salary.

From this point of view, it is clear that

> No college graduate will be paid $30,000 per year to solve problems whose solutions were memorized in high school or college mathematics courses.

For that matter, given the existing technology (such as the TI-92 calculator),

> No college graduate will be paid $30,000 per year to be nothing more than a poor imitation of a $150 calculator! Yet, no matter how much we drill our students, we will never make them as fast or as accurate as that calculator.

It is essential that we aim to produce something far more valuable than an incomplete organic clone of a calculator or computer program.

Project Data

The Math Modeling/Precalculus Reform Project, with major funding from the NSF's Division of Undergraduate Education, addressed this challenge by developing, testing, implementing and disseminating information about a dramatically different alternative to standard precalculus and college algebra/trigonometry courses. The project was under the direction of Sheldon P. Gordon and B. A. Fusaro; Gordon is the principal author of the project materials, *Functioning in the Real World: A Precalculus Experience* [63]. As mentioned above, the members of the Project's working team included Florence S. Gordon, Martha J. Siegel, Alan C. Tucker, and Judy Broadwin.

The *Functioning in the Real World* course extends the common themes in most of the calculus reform projects, as well as the principles expressed in both the NCTM Standards and the AMATYC Standards. The course focuses on mathematical concepts, provides students with an appreciation of the importance of mathematics in a scientifically oriented society, provides students with the skills and knowledge they will need for subsequent mathematics and other quantitative courses, makes appropriate use of technology, and fosters the development of communication skills. Our goal is to emphasize the qualitative, geometric and computational aspects of mathematics within a framework of mathematical modeling at a level appropriate to precalculus students.

We capitalize on the fact that most students are more interested in the applications of mathematics than in the mathematics itself, so that the applications drive the mathematical development. All the mathematical knowledge and skills that students will need for calculus are introduced and reinforced in the process of applying mathematics to model realistic situations and solve interesting problems that arise naturally from the contexts. We have found that this approach excites the students and encourages them to go further with mathematics by showing them some of the payoff that mathematics provides.

Our materials and the course based on them serve a multiplicity of audiences:

- A one-semester course that lays a different, but very effective, foundation for calculus.
- A one-semester college algebra course that bridges the gap between introductory algebra and precalculus offerings.
- A one-semester math modeling course for business or related students as an alternative to more standard finite math courses.
- A one or two-semester course that stands as a contemporary capstone to the mathematics education of students who do not plan to continue on to calculus. We have seen that the course encourages many of these students to change their minds and to go on to calculus and other quantitatively related courses.
- A course that provides the foundation for further courses in discrete mathematics and related offerings.

Innovations

Our support from the NSF gave us the opportunity to create an entirely new course at the precalculus or college algebra/trigonometry level from scratch. We were not constrained to produce a slight variation of existing texts or courses, but rather to take a fresh look at:

- the mathematical topics that are being used in the reform calculus courses,
- the capabilities of current technology,
- the type of modern mathematics that is being used in other disciplines and on the job,
- interesting and realistic applications that can be used to motivate and drive the mathematical development,
- the different learning environments that have been emerging both in reform calculus and in the high schools.

The challenge we faced was to incorporate all of these principles into a coherent course that would be meaningful, motivating, and accessible to students who traditionally are poorly prepared for college level mathematics. We outline several of our responses to this challenge as follows.

Fitting Functions to Data. Almost anyone using mathematics today begins with a set of data, either a set of experimental measurements from the laboratory or information on some quantity of interest. The problem faced usually is to identify any trend or pattern, find an appropriate mathematical model to describe the process, and use it, if it is a good fit, to make predictions (interpolate or extrapolate) about the process. The simplest case is fitting a line to a set of data in the least squares sense. We focus on this because it is so fundamental an idea, one that is used throughout the physical, biological and social sciences. It also provides the opportunity to study linear functions in detail in realistic settings.

However, we also look at the question of fitting a nonlinear function to a set of data that is clearly not linear. For instance, consider the following data on the growth of the U.S. population (in millions) from 1780 through 1900:

Year	Population	log (Population)
1780	2.8	.447
1790	3.9	.591
1800	5.3	.724
1810	7.2	.857
1820	9.6	.982
1830	12.9	1.111
1840	17.1	1.233
1850	23.2	1.365
1860	31.4	1.497
1870	39.8	1.600
1880	50.2	1.701
1890	62.9	1.799
1900	76.0	1.881

If you examine the ratios of successive population terms, you will observe that they are roughly equal, which indicates that the population values grow approximately exponentially. Figure 1 is a plot of the actual data values which shows the apparently exponential pattern.

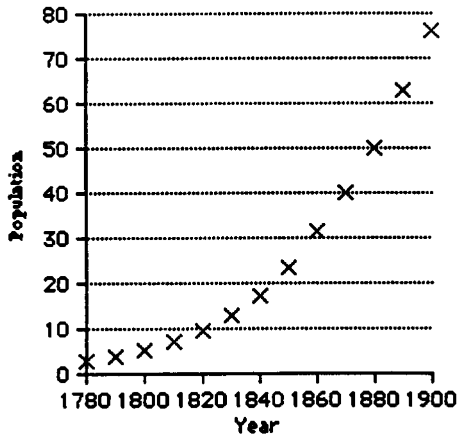


Figure 1

When you suspect that a certain phenomenon follows an exponential pattern of the form $P(t) = Ba^t$, then

$$\log P = \log B + t \log a.$$

That is, log P is a linear function of t and so we should expect that a plot of log P versus t should be linear. Figure 2 shows the associated plot of the transformed data with log (P) as a function of t. It is clear that the transformation has linearized the data.

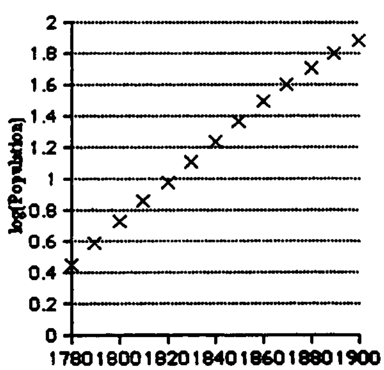


Figure 2

Using the ideas of linear regression analysis previously developed, students now find the line that best fits this transformed data; their calculator or computer program tells them it is generically of the form:

$$Y = .121X + .487,$$

but they must interpret this in terms of the actual variables used as

$$\log P = .121t + .487.$$

They then must undo the original transformation by applying the inverse function and all the pertinent operations to obtain

$$\begin{aligned} P &= 10^{\log P} \\ &= 10^{0.121t + 0.487} \\ &= 10^{0.121t}\, 10^{0.487} \\ &= (10^{0.121})^t\, (10^{0.487}) \\ &= 3.069\, (1.321)^t \end{aligned}$$

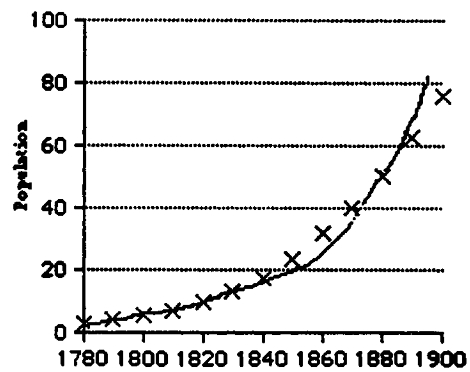


Figure 3

Notice the level of algebra needed to undo this transformation. Interestingly, the students do not complain because they are doing the work in the context of answering questions of interest, not merely to develop some algebraic skills that they see no need to possess. We show how well this exponential function fits the original population data in Figure 3.

Incidentally, notice that the base, or growth factor, for this exponential function is 1.321, so that the corresponding growth rate is 32.1% per decade, or somewhat over 3% per year, as we anticipated. Also, we analyzed the population values only up through 1900 because the rate of growth has diminished considerably during the 20th century (it is currently about 0.7% per year). We come back to study the U.S. population over the entire 1780–1990 period in the context of discussing logistic, or inhibited, growth later in the course.

In a comparable way, if one suspects that a set of data values follows a power function of the form

$$Q = Bx^p,$$

then

$$\log Q = \log B + p \log x,$$

which means that log Q is a linear function of log x. Therefore the data can be linearized by plotting log Q versus log x and the linear regression analysis technique can be used to find the equation of the best fit linear function. The students then need to undo the transformation using all of the usual properties of exponential and logarithmic functions. Again, the level of manipulation is far greater than one would normally expect with simple artificial problems, but the students are willing and able to rise to the occasion.

Modeling Periodic Behavior Periodic phenomena abound in the real world, but they cannot be modeled mathematically by functions as simple as $y = 3 \sin 2x$. For example, the number of hours of daylight on a given day of the year at any particular location is a periodic function of time. If the location is San Diego, say, then the number of hours of daylight can be modeled by

$$H(t) = 12 + 2.4 \sin((2\_/365)(t - 80)),$$

where t is the number of days from January 1 of any given year. What do the different parameters mean? The 365 clearly represents the number of days in a year, or the length of a cycle. The 12 represents the average number of hours of daylight, which occurs on the spring and fall equinox. The 2.4 represents the maximum variation above and below the middle value, so the longest day in San Diego has 14.4 hours of daylight and the shortest day has 9.6 hours. What about the 80? Since it is related to the variable t, it must represent some particular date and, if you count off days, you will find that the 80th day of the year is March 21, the spring equinox, when exactly 12 hours of daylight occur.

In the process of analyzing these parameters, the students achieve a much deeper understanding of what amplitude, period, frequency, vertical shift, and phase shift signify because the terms arise in a meaningful context. Once such a function is available, it is possible to ask a variety of pertinent questions, such as: How many hours of daylight would you expect on a particular date? When will there be 13 hours of daylight in San Diego? Or, we can assign the students a project to determine the comparable formula for the number of hours of daylight in any other city — all they need is to find the data on sunrise and sunset on one day of the year, preferably the longest or shortest.

Modeling with Difference Equations Difference equations can be used to model discretely almost any process that can be studied continuously using differential equations, but at a level requiring no more than standard algebra topics. For example, suppose that 500 pounds of a contaminant are initially dumped into a lake and 10% of it is washed away each year. We denote the level of contaminant in the lake after n years by C_n. If 10% is washed away during that year, then 90% remains at the end of the year, so that the level the following year is modeled by the difference equation

$$C_{n+1} = (0.9)C_n,$$

starting with the initial contaminant level $C_0 = 500$. The corresponding solution is

$$C_n = 500\,(0.9)^n, \text{ for all values of } n \geq 0.$$

This expression can be found either algebraically in an iterative manner or by more formal solution methods; the graph of the solution can be found directly from the difference equation using the difference equation capabilities of many graphing calculators or by graphing the solution function. This solution is an exponentially decaying function that tells us that the level of contaminant will slowly decay down to zero over time.

Next, suppose we change the underlying scenario in a variety of ways:

1. In addition, the factory annually dumps 100 pounds of the contaminant into the lake.
2. The plant increases its production yearly and so increases the amount of the contaminant it dumps by 25 pounds each year starting with the initial level of 100 pounds from scenario (1).
3. The plant increases the amount of the contaminant dumped into the river by 20% per year starting with the initial level of 100 pounds from scenario (1).
4. EPA regulations require that the plant reduce the level of dumping by 25% a year.

The difference equations corresponding to each of these situations are as follows:

1. $C_{n+1} = 0.9C_n + 100, \quad C_0 = 500$
2. $C_{n+1} = 0.9C_n + 100 + 25n, \quad C_0 = 500$
3. $C_{n+1} = 0.9C_n + 100\,(1.20)^n, \quad C_0 = 500$
4. $C_{n+1} = 0.9C_n + 100\,(0.75)^n, \quad C_0 = 500$

Notice how easy it is to introduce the different assumptions. The corresponding closed form solutions, which are all found using the discrete analog of the method of undetermined coefficients (a marvelous way to surreptitiously drill some important algebraic skills, the identical ones they will need for solving differential equations), are:

1. $C_n = 1000 - 500(0.9)^n$
2. $C_n = 2000(0.9)^n - 1500 + 250n.$
3. $C_n = 166.7\,(0.9)^n + 333.3\,(1.2)^n.$
4. $C_n = 1166.7\,(0.9)^n - 666.7\,(0.75)^n.$

How do the various solutions behave? Solution (1) is fairly obvious; the decaying exponential term dies out and so the level of contaminant slowly rises toward a horizontal asymptote of 1000.

However, the others require a bit of analysis. We show their respective graphs in Figures 4-6. In the process of de-

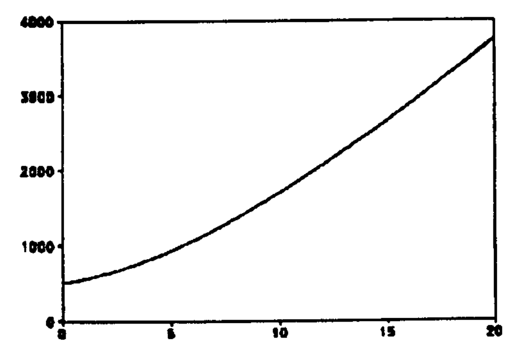


Figure 4

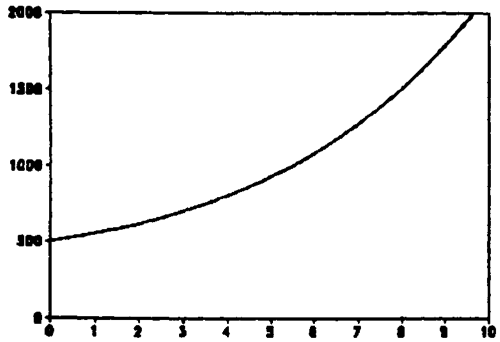


Figure 5

scribing the observed behavior and relating it to the terms in the solutions, students develop a deeper understanding of the behavioral characteristics of each of the component functions.

It is also possible to raise related questions. For instance, in Figure 6, can you estimate the maximum level achieved by the contaminant? What is the effect of changing the initial value C_0? What happens to the solution if values for the other characteristics change, such as the percentage of the contaminant washed out each year or the rate at which the company is required to reduce its contamination?

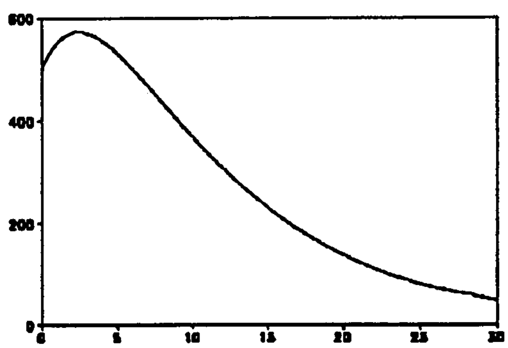


Figure 6

Status of the Project

Preliminary and draft versions of the project materials have been class-tested by about 200 faculty at almost 75 schools around the country These institutions involved cover the full spectrum, including large research universities, four year liberal arts colleges, engineering schools, two year colleges, and high schools. Each subsequent revision of the materials has reflected the suggestions and experiences of the class-testers. The official first edition containing a portion of the project materials was published in November, 1996. Both the preliminary editions and the first edition are accompanied by computer software developed under the project to provide both computational tools and the ability to experiment with most of the mathematical models studied.

PROJECT REPORT

Content

The fundamental idea around which the *Functioning* course is based is the function concept in all of its manifestations with emphasis on its applications to the world around us. The mathematical models developed in the course are based primarily on data analysis and curve fitting, difference equation models, models for periodic phenomena, probability models, and matrix algebra.

Curve fitting techniques are introduced early and used throughout so that students learn how to interpret data values that arise in many different contexts. The data analysis methods simultaneously provide immediate reinforcement regarding the behavior and properties of the different families of functions studied. The focus on difference equations includes applications of first and second order difference equations and systems of first order difference equations. The emphasis is on modeling a variety of situations, interpreting the behavior of the solutions in terms of the situations, and looking at the effects of changes in parameters. Probabilistic ideas are interwoven throughout the course in the context of performing random simulations and studying geometric probability. Matrix algebra is introduced as a unifying tool for investigating a wide array of applications including systems of difference equations, Markov processes, regression analysis, and geometry.

As indicated above, the trigonometric functions are introduced from the point of view of modeling periodic phenomena. For example, students are asked to construct a sinusoidal function to model the temperature in a house where the thermostat is set to turn on the furnace when the temperature drops to 66 and to turn it off when the temperature reaches 70 , and this cycle repeats every 20 minutes. The students are then asked how they would change the model if the house were located in different climate areas. As another example, they are asked to model a person's blood pressure over time given readings of 120 over 80 and a pulse rate of 70.

The *Functioning* course involves a considerable degree of computer or graphing calculator work to explore the implementation of most of the mathematical models. It involves a variety of live classroom experiments to investigate the accuracy of the mathematics in predicting the results of actual processes or to help develop mathematical models based on observed experimental data.

The course also features a series of student investigations to provide a real-life dimension to the mathematics.

The following is an annotated description of the contents of the project materials.

Functions in the Real World introduces students to the function concept from graphical, numerical, symbolic, and verbal points of view (which are used throughout the course) as functions arise in daily life. The emphasis is on the behavior of functions (increasing or decreasing, concave up or concave down, point of inflection, periodicity, etc.). Probabilistic ideas are also introduced via Monte Carlo methods.

Families of Functions includes linear, exponential, power, and log functions, and inverse functions, with emphasis on their applications and their behavior (growth, decay, relative growth rates, concavity, etc.), very much in the spirit of the calculus reform movement.

Fitting Functions to Data includes linear and nonlinear curve fitting to reinforce the properties of the different families of functions, to develop algebraic skills in working with the properties of those functions, and to connect the mathematics to the real world.

Extended Families of Functions includes polynomial functions, fitting polynomials to data, the nature and relative frequency of the roots of polynomial equations, building new functions from old (shifting, stretching, sums, differences, products, quotients, and composition of functions) , finding roots of equations, and finding polynomial patterns (including sums of integers and sums of squares of integers).

Sequences and Difference Equations includes the development and analysis of models for describing population growth, logistic (inhibited) growth, eliminating drugs from the body, radioactive decay, Newton's laws of heating and cooling, geometric sequences and their sums.

Modeling with Difference Equations includes first order non-homogeneous difference equations, fitting quadratic functions to data, Newton's laws of motion, modeling the stock market with a stochastic model, financing and amortization, fitting logistic curves to data, iteration and chaos, etc.

Modeling Periodic Behavior includes using trigonometric functions to model phenomena such as the number of hours of daylight as a function of day of the year, temperatures over the course of a year, and tides; relationships between trigonometric functions; approximating periodic functions with sine and cosine terms; approximating the sine and cosine functions with polynomials; properties of complex numbers; and chaotic phenomena.

Second Order Difference Equations includes the Fibonacci sequence, transmission of information, mechanical systems with simple and damped harmonic motion, non-homogeneous models including a national income model and an inventory analysis model.

Matrix Algebra and its Applications includes a variety of applications of matrices, such as Markov chains in the spirit of a finite math course, not merely the use of matrices for solving systems of linear equations.

Probability Models includes binomial probability and the binomial expansion, geometric probability, estimating areas of plane regions using Monte Carlo simulations, waiting time models, the spread of epidemics, and random patterns in chaos.

Systems of Difference Equation includes the predator-prey model, an arms-race model, a labor-management model, a model for marriage rules among the Natchez Indians, and matrix growth models leading to a treatment of the eigenvalue/eigenvector problem.

Geometric Models includes analytic geometry, the conic sections, parametric curves, the average value of a function, and curves in polar coordinates.

Pedagogy

The *Functioning* course almost forces a change in pedagogy; it is virtually impossible to give the course in a formal lecture format. The non-routine nature of many of the problems make them ideal for having the students attack them in small groups using collaborative learning. Alternatively, the problems can be approached by the entire class (if not too large) as a group with the instructor being the group leader who asks leading questions and who is more of a recorder at the board than the dispenser of all mathematical knowledge.

We also emphasize the use of individual or small group projects related to the mathematical content of the course. For example, students might be required to find a set of data of interest to them and perform a complete analysis of it — finding the best linear, exponential, and power function to fit the data, and asking and answering pertinent questions (i.e., predictions) based on the context. Each student would then be required to write a formal project report. For instance, during the current semester, a sample of the topics my students studied include:

- The number of sexual harassment cases filed as a function of time.
- The likelihood of car crashes as a function of blood alcohol level.
- The growth of the prison population as a function of time.
- The time of high tide at a beach as a function of the day of the month.
- The amount of solid waste generated per person as a function of time.
- The time for water to come to a boil as a function of the volume of water.
- The size of the human cranium over time during the last three million years.
- The results of a serial dilution experiment in biology lab.
- The growth in the Dow-Jones average as a function of time.
- The Gini Index measuring the spread of rich versus poor in the population over time.
- The number of immigrants who entered the US over time.
- The mean annual income as a function of the level of education.

As another example of a project, students are given a set of data of a periodic nature, say historical high temperature readings in a given city every two weeks, and are asked to create a sinusoidal function that models the temperature. This can be quite a challenging problem at first glance, and yet in the process the students really come to understand the meaning of all the parameters in the general equation for a sinusoidal function. A particularly effective way to start this is to have the students begin in groups of three or four, so that someone has to speak up and make some initial suggestion of where to begin. Eventually, I have each student complete the analysis independently and prepare a formal written report.

The fascinating thing about such a project is that there are quite a number of different strategies that can be developed for estimating the various parameters and different students come up with them. One of the most memorable lines by a student that I have ever read appeared in one such report: "The next quantity to be determined is the frequency. This was deceptively simple." How often does a student in a precalculus course describe the frequency of a sine function as deceptively simple, particularly when the value he obtains for the frequency is .0172 or 2_/365?

The use of such projects, however, raises several important questions. First, how do we grade such things? Obviously, there are no right or wrong answers. There should be certain things that we would expect to see, such as correct mathematical formulations and calculations, and it is relatively easy to devise a grading scheme to account for that. However, there is considerable variation from one student to the next in the quality of work they do, and this is much harder to assess. Some students turn in project reports that are truly outstanding, almost professional in nature, and I personally feel that such extra effort should be rewarded in some fashion. Also, special attention should be paid to students for whom English is not the first language.

The second question is, how do we factor this in as part of the overall assessment of a student? In my case, I typically assign three or four such projects during a semester, and count them together as the equivalent of two class tests. In all, I usually give three class tests plus a cumulative final that counts as the equivalent of two other tests, so the projects amount to roughly 2/7 of the final average. Considering the amount of mathematics that the students learn in the process of doing these projects, this seems quite reasonable.

We also encourage the use of "live" experiments in the classroom to collect data to be analyzed. One such activity, using a CBL unit and calculator to study the height of a bouncing ball, is described in the next section. Others might include using a thermometer to measure the temperature of water being brought to a boil as a function of time or the height of liquid in a container having a hole at the bottom (Torricelli's Law) as a function of time.

The Role of Technology

We believe that technology has tremendous implications for the teaching and learning of mathematics. The problem we as educators face is how to use the available technology in the service of the mathematics rather than as an end in itself. To illustrate this dichotomy, consider the question of fitting functions to data. Most graphing calculators have the ability to find the best exponential, power or log function to fit a set of data; they also fit the best quadratic, cubic or quartic polynomial; some can fit the best sinusoidal or logistic function to a set of data. All of this can be accomplished literally at the push of a single button.

However, we firmly believe that pushing that button, at least early on, is a mistake for most precalculus or college algebra students. Rather, we ask that students examine the original set of data, look for a general pattern, appropriately transform the data entries to linearize them, use a calculator or computer to obtain the best linear fit to the transformed data, and then undo the transformation using the algebraic properties of the appropriate inverse function. In this way, the students are learning more mathematics while developing some of the manipulative skills they will need to succeed in calculus and other courses.

Also, the calculator or computer programs will all provide the value for the correlation coefficient or the coefficient of multiple determination R^2 for polynomial fits. But depending solely on this as a measure of how good a fit is can be highly misleading. We also encourage the use of residual plots and emphasize that deciding which fit is best is often a judgment call and attempt to help the students develop that judgment.

The *Functioning* materials were developed from the point of view that each student has a graphing calculator. We do not focus on any particular make or model, however, preferring to keep the descriptions of calculator usage as generic as possible. In addition, the project has developed a package of over 40 computer graphics programs in BASIC for IBM and compatible computers that allow students to experiment with most of the models covered in the course. A multimedia package for both IBM and Macintosh machines is currently under development.

The experiences reported by the class-testers reflect a full spectrum of technology scenarios. Almost all have required a graphing calculator. Some have incorporated a computer laboratory component into the course in which each student has access to and uses a powerful mathematical package such as Derive, Maple, TEMath, or the software package developed under the project. Several have had their students use spreadsheets such as Excel. Several have used the CBL unit to collect and analyze data.

For instance, I typically start the first class by bringing in a basketball, dropping it, and asking the students to sketch the graph of the height of the bouncing ball as a function of time. I have the students compare their graphs with their neighbors in small groups to "break the ice" about talking mathematics to one another.

I then ask for several volunteers and have them run through the same experiment using the CBL unit connected to a calculator with a viewscreen to collect and display the actual data on height as a function of time. I trace the curve on the calculator, indicating the points where interesting behavior occurs — when the ball bounces, when the ball reaches its maximum height, and what those maxima are. I create a table on the blackboard summarizing those results and investigate them for patterns — the times of the ball bounces and the times when it reaches its maximum height follow roughly linear patterns (though the terminology is certainly not available yet) and the values for the maximum heights decay in a decidedly nonlinear pattern. Finally, I isolate one arch of the graph and, using the statistical features of the calculator, fit the best parabola to the curve. All of these activities dramatize to the students that this is not your usual mathematics class. The activities also preview for the students many of the major mathematical themes that will come up during the course.

One of the inherent problems with technology, particularly graphing calculators, is dealing with the growing variety of options available. An instructor can ask for a particular calculator model, but what happens when students come into the course with a different model that they used in high school (a scenario that will become far more prevalent in the next couple of years) or with a used calculator that they bought cheaply from another student who posted a sign in the hallway?

Similarly, an instructor may decide to use a particular piece of software with the course, but what of the students who have learned to use a different piece of software in some other course? If it is just a matter of one spreadsheet versus another, there is no real problem. If it is a spreadsheet instead of a mathematical package such as Derive, say, there may be greater problems, particularly when the other software may do a better job at some chores (such as creating fancy graphs based on data) than the designated one.

Another question to be considered is: how does an instructor teach the technology? Should it be in class, thereby taking a goodly amount of time away from the mathematics? Is it possible for the math department to arrange for a series of technology training sessions, open to all students taking one or a group of related courses, during common hours?

Still another question to be considered is how the technology choices made by an instructor or the entire math faculty compare with the decisions being made by other departments. We suggest that instructors discuss their needs and course requirements with faculty from other fields to try to come up with reasonably uniform goals. Otherwise, one can easily find oneself in the position where the physics department has required an HP calculator, the chemistry department a TI calculator, and the math department a Sharp, and everyone, student and instructor, is frustrated and confused. Actually, we suspect that the choice made in the local high schools will soon trump any decisions made in the colleges.

Day to Day Mechanics

The innovative nature of the *Functioning* course places special burdens on both the instructor and the students. However, our experiences (see the section on Assessment below) indicate that this additional time and effort is well worth investing.

Because the course has so much content that is new to many instructors, it requires considerably more preparation time than a more traditional course where one can often walk in with virtually no preparation. Naturally, by the second and certainly by the third time one offers the course, it does become far less demanding. Similarly, the non-routine nature of the problems requires that considerably more attention be paid to selecting the ones to assign, as well as the number of problems to be assigned. It certainly does not make sense to say "Do problems 1 through 49, odd". Also, we don't recommend walking in "cold" and asking if there are any questions on the homework; many of them require some advance thought.

By their very nature, many of the problems admit to different interpretations. It is a delight to see students coming up with those interpretations. Also, most of the problems can be solved in a variety of ways — analytically, graphically, numerically, and so forth. Consequently, the problems themselves almost automatically change the day-to-day classroom dynamics. Such problems can be used as the basis for cooperative or group learning, if an instructor so desires. Alternatively, the problems can be used in large-scale discussions lead by the instructor as springboards to introduce new ideas and to get the students to think mathematically. Probably the least satisfactory way to approach these problems is to walk in and present polished solutions on the board.

Another challenge faced by many instructors is a personal re-orientation. In traditional courses, the so-called applications tend to be quite artificial. We suspect that most students realize this, which accounts, in large measure, for their not seeing mathematics as a useful subject. In the *Functioning* course, however, one is almost forced to look for truly realistic examples and problems. Certainly, if one is using real data, the situation is inherently realistic. Sources of such examples tend to be non-traditional also, at least for mathematics. We urge both instructors and students, for instance, to look at the Statistical Abstracts of the United States, any information almanac, and even the Old Farmer's Almanac for data on a variety of periodic processes that can be modeled with sinusoidal functions.

The emphasis on realistic applications also tends to reduce the role of the instructor as the all-knowing expert in the room. There will always be several students in any class who are far more knowledgeable about electronics or automotive mechanics or chemical reactions or economic trends. The instructor must be willing to admit to being less informed and drawing such students out (though perhaps turning them off can be a bigger problem) to add to the real dimension of the mathematics. But that is what the course is all about — convincing the students that mathematics applies to all areas of their lives. And when they make that connection, the course really comes alive for them!

Still another challenge is creating new, innovative problems for tests or quizzes. One advantage to having students do projects is that the instructor gains a wonderful pool of examples for subsequent use. (However, the first time through the course, before one has that pool, things are more difficult.)

From the students' perspective, this is also a very different course. We have found it is essential to explain to them, beginning on the first day, why they are being given such a different experience. We admit to many of the problems with more traditional courses and we use many of the same rationales we discussed above for the changing process of doing mathematics in practice. We discuss what technology can do for the students and what they must be able to do that technology cannot do: to think! Furthermore, we repeat many of these messages often throughout the course, so it is not just part of day one background.

Also, we recognize that the students are undergoing a major re-orientation in their view of what mathematics is and how one does mathematics. This is not something that is completed in a day or a week. Students likely buy into the new philosophy the first day or during the first week or so. However, this re-orientation in their thinking typically takes three or four weeks to sink in, so that they become reasonably comfortable with using a variety of mathematical tools (graphing, numerical approaches, algebraic approaches) to solve problems, with using some form of technology, with problems that do not have unique solutions that can be found in only one way, with the use of a variety of different letters (not just x and y) for variables, with thinking about what they are doing, and with discussing mathematics with others.

Often, students who have the strongest traditional math backgrounds (those who have the best algebraic skills) are the ones who are most resistant to this re-orientation. At the other extreme, students whose algebra skills are weak, who previously would be assessed as having little or no mathematical ability, often demonstrate incredible levels of mathematical insight using verbal, graphical, or numerical approaches.

Occasionally, there are some students who simply "do not get it," at least in the classroom. We have found that, with such individuals, it is necessary to drag them to the office for a one-on-one session to reinforce what some of the underlying ideas are. This one session often is all that is needed to bring these students around. Even if it isn't quite sufficient, they are far more likely to come back to the office for help on their own.

Finally, if one is offering a truly different course with different expectations of the students, it is critical to give exams that mirror the new philosophy. Students assess the apparent importance of things by what is tested. If the tests are mostly routine "solve the following" problems with a few template problems tossed in, the instructor will not truly change the culture. On the other hand, it is expecting a lot to give students several "new" problems and expect them to come up with novel solutions under the pressure of an exam. A set of class tests and the final exam for one semester are attached at the end of this article. Our experience is that most students taking this course are capable of far more than we have previously expected. Yet, it is asking a lot of all students. One way we have compensated for this is to grade tests out of about 110 points to give a little leeway. Another is to use projects to provide additional assessment alternatives.

Assessment

Most of the information we have received from the class-testers has been anecdotal in nature, but is often highly indicative of what goes on in their classes. The following paragraphs contain a number of such anecdotes.

Joe Fiedler and Ignacio Alarcon of California State College at Bakersfield report that of 45 students (a majority of whom were minority students) who started a two-quarter sequence, 43 completed the course with grades of C or better. Of these, five changed their majors to become math majors, including the 7-foot tall center on the school's basketball team who dropped basketball because "math was more fun". (They used a computer lab with Derive for all students plus graphing calculators.)

Judy Fethe of Pellessippi State Technical Community College ran a pilot section of college algebra with 24 "high risk" students few, if any, of whom were expected to pass. However, virtually every student did pass, with surprisingly high grades. The people who staffed the department's tutoring facility came to refer to the class as the "honors section" because the tutors themselves were having so much trouble solving the non-routine problems. (Judy used a computer lab with TEMath plus graphing calculators.) She subsequently followed many of these students into a more traditional trig and precalculus course and found them far more open-minded and willing to try new problems than students who had come through traditional college algebra.

Joanne Manville of Bunker Hill Community College taught a pilot section in the spring. She reported a high degree of enthusiasm on the part of the students. She also monitored their performance in a traditional calculus course thereafter. She reported that of the four students who took calculus the following summer, all received grades of either A or B. More telling, however, is an incident that especially impressed the calculus instructor. One of the students questioned an answer in the official solutions manual that accompanies the calculus text. The instructor was prepared to find the student's algebraic error, but the student would have none of that — his problem was that the function, a cubic, could not possibly behave the way that the solutions manual claimed!

Tony Peressini and John Luker of the University of Illinois have given the *Functioning* course for the last three years while coordinating different groups of graduate TA's who have actually taught most of the sections. They report a significantly higher level of attendance (they're not teaching the same old stuff that the students have seen before). All students have graphing calculators and the format is one of collaborative learning. Tony describes their experiences as

> "The use of this book has raised the intellectual level and utility of our basic algebra course significantly. The students find the course to be very challenging but not overwhelming. They seldom question the usefulness of this content as they frequently did (and probably with good reason) that of the traditional algebra course that we offered prior to adopting this text."

John describes their experience as

> "We chose to use these materials because we wanted something very different from the type of algebra our students had seen in high school. Our surveys indicate that our students prefer this course over the traditional course. At least 1-5 students tell me each semester that this is the first time they have enjoyed a math class. I even had one student tell me he looked forward to doing homework. I was glad I was sitting down at the time."

Marc Dancer of Deerfield Academy (a high school) gives the *Functioning* course in a totally group work environment with no lecturing at all. An evaluator who sat in on the class one day and interviewed the students reported:

> "They like this class better than other classes because they're able to see how that stuff they learned in algebra fits into real life. They like the strong focus on applications. They like the text for this reason also. They haven't had any tests yet, so they can't comment on their success (as defined by grades), but most feel like they understand the material better than in other math classes. There were two students in the class who had intended this to be their last math class and who are now thinking of going on to calculus in college because of this class."

Marc's personal comment on the course is: "This is math for the masses, not just mathematicians." (Each student was required to purchase a TI 82 calculator.)

In a more quantitative study, Florence Gordon of New York Institute of Technology (one of the co-authors of the text) had the opportunity to teach two parallel sessions of the same precalculus course, one from the *Functioning* materials and the other from a more traditional text to mollify a conservative department chair. She decided to ask some common questions on the two final exams to compare student performance. However, she felt it would be unfair to ask any conceptual or realistic applied problems of the students in the traditional class. Therefore, as the common questions for both groups, she only posed routine (mechanical) questions or posed the routine part of a realistic application to the traditional group. For example, the following question appeared on the exam for the *Functioning* class:

> At a certain pier, the low water line is 6 feet above sea bottom and the high water line is 14 feet above bottom. If low tide occurs at midnight and high tide at 6 am,
>
> (a) What are the amplitude, frequency and period for this function?
> (b) Sketch the graph, including appropriate scales.
> (c) Write an equation of the water height H as a function of time t.
> (d) How high is the tide at 11 am?
> (e) When is the water 12 feet deep?

The students in the traditional course were simply asked:

> Let $y = f(t) = 10 + 4\sin[(\pi/6)(t - 3)]$.
> (a') Find the amplitude, frequency and period.
> (d') Find $f(11)$.
> (e') $10 + 4\sin[(\pi/6)(t - 3)] = 12$. Solve for t.

For purposes of comparison, the same number of points were allotted to parts (a) and (a'), (d) and (d'), and (e) and (e'). Out of 11 points, the students in the traditional course scored an average of 3.9 while the students in the *Functioning* course scored an average of 9.6. Based on a small sample t-test for the difference of means, this represents a value of $t = 7.014$.

Notice that the students in the *Functioning* course had to understand the context, translate it to a mathematical model, and create the formula that was simply handed to the students in the traditional course. Further, they had to interpret the height of the tide at 11 am as representing $H(11)$. Finally, they had to interpret the meaning of "when is the water 12 feet deep?" as requiring them to set up and solve the equation $10 + 4\sin[\pi(t - 3)/6] = 12$, which was simply handed to the other group.

Incidentally, the students in the *Functioning* course scored better on six of the seven common questions on the two exams, four of them being statistically significant. A full analysis of the results appears in [59].

Working in a realistic context as opposed to an abstract setting appears to make all the difference. The mathematical ideas make sense and the greater mathematical expectations placed on the students becomes acceptable to them. On the other hand, when students in traditional classes are assigned

50 indistinguishable problems every night, all of which look like the same things they think they've seen before, the students are not likely to do many homework problems at all.

I am currently teaching from the project materials in a college algebra course at Suffolk Community College. Among the numerous positive incidents that occurred, one that stands out is the following. Having covered chapters 1-3 on linear, exponential, power, and logarithmic functions, I was about to introduce polynomials for the first time. To do so, I drew a scatterplot of the prices of a stock that followed a roughly cubic pattern and asked the students to describe the behavior of a function that would model such a pattern. The first response was: "Its concavity changes. First it's concave down and then it's concave up."

A second student quickly added: "It has just one point of inflection." A third student then responded: "It changes from increasing to decreasing. First it's increasing, then decreasing, and then increasing again."

A fourth student then chimed in with: "It doesn't have an inverse."

And a fifth student observed: "It has two turning points."

I must admit that, the first time this happened, it left me standing in front of the group with my mouth agape at a complete loss for words! A few years ago, I would have been thrilled to get a comparable set of responses from students after a full semester of calculus. But in college algebra? And this is not the first time this particular exchange has occurred; last year, I got the first four observations, though in a different order.

After leading one such college algebra class to compile a list of behavioral characteristics of cubics several days afterward, one student raised the question: "Is it true that every cubic is centered at its point of inflection?" Wanting to draw her out for the sake of the other students. I asked what she meant by that. With her eyes screwed up as she was trying to visualize her image and with her hands moving in opposing directions, Vicki responded: "Well, if you start at the point of inflection and move in both directions, don't you trace out the same path?" Sure you do! The fact that many professional mathematicians are not aware of this delightful fact only increases the significance of Vicki's insight. What's even more amazing is that Vicki came into the course with a particularly low self-assessment of her mathematical abilities (she had actually been a high school dropout); she viewed herself as an "artsy" type, certainly not a math student.

Another incident in a different section of the same course, again with students just out of an intermediate algebra course, occurred when the trig ratios were first being introduced, starting with the tangent. I asked the students to use their calculators to find tan 0°, tan 10°, tan 20°,..., tan 80°. I then raised the natural question about the behavior pattern of this new function based on the data values. Several students immediately called out, simultaneously, comments about it not being linear, or growing faster than linear, because the successive differences were growing. Then three or four students simultaneously called out that it is growing so fast that it must be exponential, to which another student instantly responded: "No, it's not exponential. I've already checked the successive ratios and it's growing faster than exponential."

Additional student questions and comments displaying their often incredibly insightful understanding of mathematical ideas in reform courses can be found in Gordon [59].

Perhaps most telling, however, are actual written comments made by students on formal course evaluations. For instance,

> "Math is a part of life — everything we experience, whether tides or hours of daylight or population growth or rising medical costs, deals with math. I now have a much better understanding of the patterns of life and how math can be applied to them."

> "My overall reaction to the course was extremely positive. By emphasizing the value of mathematical pursuits through applications first (and theory being derived from the application), the course proved to be constantly interesting. Past math courses seemed tedious. This course never struck me as tedious (challenging? very, but not tedious). Nice to think of math as an intellectual pursuit, a very useful tool, and a way of seeing life — as opposed to 'a course I have to take to complete a chemistry curriculum'. Above all, being able to see how I could use the subject I was learning in the 'real' world."

Problems to be Faced

In implementing a course such as this, or any reform course, there are a number of critical challenges that must be faced, depending on the particular institution. One of the most difficult problems involves the support personnel who typically staff a departmental tutoring center. These are usually students (perhaps graduate students) who have been highly successful in traditional courses and who know nothing about the philosophy of a new course or the reasons it is being implemented, who are unfamiliar with the new content, and who will likely try to answer all questions in a very different way from what the instructor might intend.

For instance, several of my students reported the following exchange. They had gone to our tutoring center for help with some problems on fitting a function to data. The tutor working there, a graduate student at a local university, looked at the problems and exclaimed "You're not supposed to see that until you're in graduate school!" Or, when some of my students in a Harvard calculus course went in for help on graphical differentiation of a function, the response from several other tutors was: "But where's the function? Give me the function, I'll differentiate it for you, and then draw its graph."

Fortunately, this is a problem that solves itself over the course of several semesters as you build up a cadre of advanced students who have been through the new course or courses. During the first semester or two, however, it can be a major hurdle. One partial solution is to offer a number of workshops for the tutors (and pay them for their time, providing the department or the school has the funds for it) to acquaint them with the changes taking place and to teach them some of the "new" mathematics. Another possibility is to identify the best students in a class and have the school or department pay them to peer tutor their weaker classmates.

A related on-going problem arises at schools that use large numbers of graduate students as TA's in lower level courses. These students have been highly successful in traditional courses and likely have not noticed how many of the students around them simply disappeared. Typically, though, most departments already provide fairly extensive training for the TA's prior to the start of any course and weekly meetings throughout the semester. Implementing a reform course such as the *Functioning* course just makes this more important and likely requires more in the way of preparation. The problem here, though, is that at some institutions, the clear emphasis is on completing one's research and teaching is just a way to earn some money along the way. In such an environment, it is very difficult to motivate the TA's to devote the time needed to give justice to such a new course.

A similar problem arises at most other schools that use large numbers of adjunct faculty, who also have been highly successful in traditional math programs. They also need special training before they can teach reform courses. More significantly, such people tend either to hold down full-time jobs elsewhere and the teaching is merely moonlighting or they are teaching as an adjunct at three or four different schools. In either case, there is a natural desire to do things in as simple and fast a way as possible (i.e., the usual) because time is the one commodity they lack. The only advice we can offer is to be highly selective in the choice of which adjuncts are assigned to such a course, picking individuals who have special backgrounds (say who worked as a mathematician in industry) that might make the course special to them, or individuals who have a high energy level who can be turned on by the innovative aspects.

At some schools, perhaps the most severe problem to implementing an innovative course is the resistance of other faculty members. Often, some instructors are philosophically opposed to the innovation because it is not the way that they learned mathematics and how they have been teaching mathematics; some may be intimidated by unfamiliar content; some may fear the loss of topics that they consider important (how many readers remember the Law of Tangents?); some may be uncomfortable with technology; some may be unconvinced about the effectiveness of collaborative learning. Others may simply not want to devote the effort to teaching a new course in a new way.

Some of these objections can be overcome with information. For instance, showing a selection of problems and the level of mathematics, particularly the algebraic tools, needed to solve them, can allay qualms about a course that does not highlight symbolic manipulation. Showing a variety of conceptual problems can convince many people about the mathematical content of an unfamiliar course. Showing some samples of student reports may convince people that there can be a considerable amount of mathematical learning that goes into performing a project and writing the report. Showing information about the changing needs of students and practitioners in other fields may exert some countering pressures regarding the need to change the curriculum. In this regard, for a course such as the *Functioning* course, considerable support can likely be garnered from faculty in other departments because the course content dovetails so well with what they likely would want their students to learn in mathematics. The prevalence of technology throughout our society itself will exert ever greater pressure on faculty to acknowledge that it has a place in the curriculum, though the point should always be made that technology is being used to enhance understanding and learning, not as a substitute or just to provide answers.

Plans for the Future

Although the text for the course has been completed, there remains much to do to affect the mathematics community on a large scale. Most importantly, there will be an on-going need for faculty development and training because there are many innovative features in the *Functioning* course. Much of the mathematical content, such as the notion of fitting functions to data, difference equation models, or even a different slant on traditional topics, may not be familiar to many. For instance, at a recent workshop on collaborative learning at the precalculus level that I ran, I was surprised at the difficulty that experienced faculty were having with the following problem:

> The thermostat in a home is set at 66°. Whenever the temperature drops to 66° (roughly every half-hour), the furnace comes on and stays on until the temperature reaches 70°. Write a trigonometric function that models this situation.

Rather than focusing on the process — the temperature oscillates up and down between 66 and 70 every 30 minutes — they focused instead on the mathematical model $y = A + B \sin(Cx + D)$ and tried to adapt it to the situation. Thus, unlike students in the course who let the process guide them and so have little trouble with such a problem, trained mathematicians had considerably more difficulty.

Furthermore, many faculty members are still not comfortable with the use of technology, and most have never tried to implement the use of projects and writing assignments in mathematics or the possible use of collaborative learning. So there is a need to assist in helping such individuals.

In addition, we intend to continue with a variety of dissemination activities, including talks, workshops, and articles, to acquaint mathematics faculty with the course.

Finally, we see the on-going need to continue working in conjunction with the professional organizations such as MAA, AMATYC, and NCTM to promote changes of this type throughout the mathematics curriculum and to continue maintaining contact with other groups and individuals who are likewise working to enhance and revitalize the curriculum at all levels.

SAMPLE MATERIALS

Text Used: Gordon, Sheldon P., et al. *Functioning in the Real World: A Precalculus Experience*, Addison-Wesley, 1997.

MA 61 (College Algebra) Test #1

1. The accompanying figure shows the relationship between the UV (ultraviolet) index and the number of minutes it takes to get a sunburn.

Ultraviolet Index										
0	1	2	3	4	5	6	7	8	9	10
Low					Moderate			High		
Minutes to sunburn				60		30		20	15	

a) Is the function linear? Explain your answer.
b) Suppose the UV index is 5. Karen estimates from the table that she will get a sunburn in just over 40 minutes. Using your knowledge of mathematics, is her estimate too high, too low, or about right? Explain your answer.

2. One of the following functions is linear, another is exponential and a third is a power function.
(a) Identify which is which. (b) Find the equation of each function.

x	$f(x)$
0	4
1	4.8
2	5.76
3	6.91

x	$f(x)$
0	5
4	40
7	92.6
10	158.11

x	$f(x)$
0	4
4	4.8
7	5.6
10	6.4

3. The 1990 population of California was 29.76 million and growing at an annual rate of 2.3%.
a) Find an expression for the population at any time t.
b) What will the population be in the year 2000?
c) What is the doubling time for the population?
d) If all things remain the same, when will the population reach 50 million?

4. The annual unemployment rate in the United States over a recent period was:

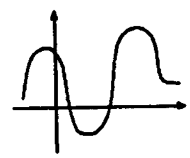
n:	1984	1985	1986	1987	1988	1989	1990	1991
R:	7.5%	7.2	7.0	6.2	5.4	5.3	5.5	6.2

a) Use graph paper to draw the scatterplot for this data and sketch the best-fit line by eye.
b) Estimate the slope of this line and tell what it means
c) What is your best estimate for the equation of this line?
d) Using this line, what is your estimate for the unemployment rate in 1995?

5. Sketch the graph of a function which is increasing and concave down from point *A* to point *B*. Let *M* be the midpoint on this curve. The three points *A, B* and *M* determine three line segments, *AM, MB* and *AB*. List them in the order of increasing slope. Would your answer change if the curve were increasing and concave up?

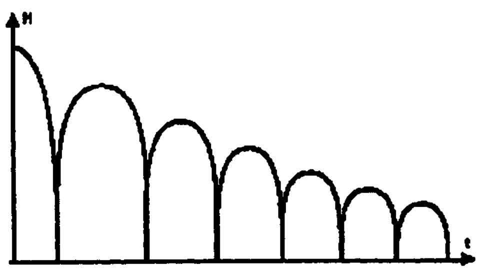
6. Sketch the graph of a single smooth function that has all of the following properties:
   a) $f(0) = 6$ b) $f(x)$ is decreasing for $0 \le x \le 3$ c) $f(x)$ is increasing for $3 < x \le 5$
   d) $f(x)$ is decreasing for $x > 5$ e) $f(x)$ approaches 1 as x approaches ∞.

7. For the function shown, indicate:
   a) all intervals where it is increasing.
   b) all intervals where it is decreasing.
   c) all intervals where it is concave up.
   d) all intervals where it is concave down.
   e) all points of inflection.



MA 61 (College Algebra) Test #2

1. An experiment is conducted in which a ball is dropped from an initial height of 9 feet and its height above floor level as a function of time is recorded and displayed, as in the figure shown. When the curve is traced out, the measurements indicated on the graph are obtained for the time when the ball hits the floor, the times when the ball reaches its maximum heights, and the values of the maximum heights.



n	Time
1	.75
2	2.09
3	3.29
4	4.37
5	5.33
6	6.19
7	6.95

n	Max. ht
1	9
2	7.2
3	5.76
4	4.61
5	3.69
6	2.95
7	2.36

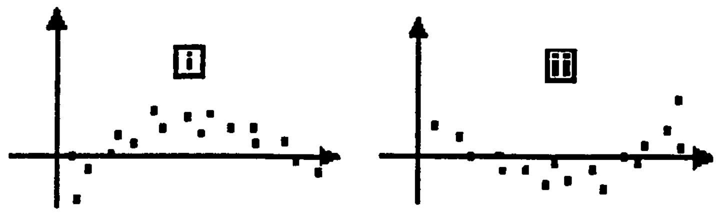
   a) Notice that the times when the ball hits the floor appear to follow a linear pattern. Use your calculator to find the best linear fit to these times as a function of the number of the bounce; that is, bounce number $n = 1$ occurs at time $t = .75$, etc.
   b) Draw the scatterplot for the maximum heights.
   c) Notice that the maximum heights do not follow a linear pattern as a function of *n*. Explain why you would expect this pattern to be exponential rather than a power function.
   d) How would you transform this data to linearize it?
   e) Use your calculator to find the equation of the line that best fits this transformed data.
   f) Undo the transformation to find the best exponential function that fits these data values as a function of n.
   g) What is the practical meaning of the base you obtain?
   h) Use the results you obtained above to predict the next time the ball will hit the ground and the maximum height to which it will rise on the next bounce of the ball.

2. Determine which of the following functions have an inverse. For any that do, estimate $f^{-1}(7)$.
   a) $f(x) = x^2 + 3^x$ b) $f(x) = x^3 + 3^x$

3. A quadratic polynomial has a real root at $x = 2$ and a turning point at $(1,5)$. Find the equation of the quadratic.

4. The best fit line is constructed for each of four sets of nonlinear data. Their patterns are roughly:
   a) increasing and concave up;
   b) increasing and concave down;
   c) decreasing and concave up;
   d) decreasing and concave down.

   Match each with one of the possible residual plots shown below. Explain your answer in each case.



5. Suppose you are told that the real roots of a polynomial are x = 1, 4, -3, 2.5 and -1.5.
   a) What is the minimum degree of the polynomial?
   b) Assuming there are no complex roots, write a possible formula for this polynomial. (Do not expand it.)
   c) Draw two possible graphs for this polynomial.
   d) Indicate roughly where the function has turning points.
   e) Indicate roughly where the function has inflection points.

MA 62 (Precalculus) Test #3

1. Find the complete solution of each of the following difference equations:
   a) $x_{n+1} = 1.2\,x_n$ b) $x_{n+1} = .5\,x_n$ c) $x_{n+1} = -2\,x_n$
   Assuming that the initial value $x_0 = 1$ in each of the above solutions, draw a *very rough* sketch showing the behavior of the solution.

2. The "half-life" of aspirin in the body is 30 minutes. If a person takes 1000 mg (two tablets), what is the effective dosage after 2.5 hours?

3. A can of soda at 70° is put into the freezer kept at a constant temperature of 10° to chill. Suppose that the temperature of the can is 55° after 20 minutes and that its temperature is 45° after 30 minutes. Sketch the graph of the temperature as a function of time and use the concavity of the graph to answer the following: Which of the following temperature readings are possible and which are impossible?
   a) $T(25) = 52°$ b) $T(25) = 48°$ c) $T(35) = 42°$ d) T(35) = 38°
   Give reasons for each of your answers.

4. The population of a certain country was 80 million in 1990 and is growing at an annual rate of 2%.
   a) Write a difference equation for the population P_n in this country after *n* years.
   b) Further, starting in 1990, the country agreed to allow in 1 million immigrants each year. Write a difference equation for the population P_n in this country after n years.

5. The number of new cases of a certain disease each year has been dropping 10% per year since 1950. If there were 8000 new cases in 1960, what is the total number of new cases between 1960 and 1994?

6. A population grows according to the logistic difference equation $P_{n+1} = 1.04_n - .00005\,P_n^2$. (Equivalently, _$P_n$ = $.04P_n - .00005\,P_n^2$) Sketch a graph of the behavior of this population, paying careful attention to concavity, if:
   a) $P_0 = 100$ b) $P_0 = 600$ c) $P_0 = 1000$

7. Consider the difference equation $x_{n+1} = .5x_n + 2n$, $x_0 = 40$. Calculate the next three terms x_1, x_2 and x_3 of the solution sequence from the difference equation and tell whether the solution appears to be increasing or decreasing, concave up or concave down.

8. Suppose you are told that the real roots of a polynomial are $x = 1, 4, -3, 2.5$ and -1.5.
   a) What is the minimum degree of the polynomial?
   b) Assuming there are no complex roots, write a possible formula for this polynomial. (Do not expand it.)
   c) Draw two possible graphs for this polynomial.

9. Perform the first two iterations of the bisection method to estimate the first positive root of the function $f(x) = x^3 - 4x + 1 = 0$. Use your calculator to locate any other real roots correct to three decimal places.

Material for this article was submitted by Sheldon Gordon.
Portions of this article are reprinted with permission from Addison Wesley Longman Publishing.

A Laboratory Course in Mathematics for Liberal Arts Majors

Holly P. Hirst and James R. Smith

Service Course Area

Quantitative Literacy/Special Topics

Institutional Data

Appalachian State University is a public comprehensive university with 12,500 students located in the Blue Ridge mountains of northwestern North Carolina. Its students are predominately undergraduate, residential and of traditional college age. Appalachian is a member of the University of North Carolina system, which includes fifteen other state universities. Appalachian is composed of six schools and colleges, including Arts and Sciences, Business, Music, Fine and Applied Arts, Education, and General Studies. The University offers bachelor's degrees in all of the usual areas except engineering, master's degrees in many disciplines, and one doctoral degree in educational leadership.

The Department of Mathematical Sciences is situated in the College of Arts and Sciences and has 37 full-time and 8 part-time faculty, 35 of whom hold a PhD or EdD. It offers bachelor's degrees in computer science, secondary education mathematics, and statistics as well as in general and applied mathematics. The department also offers a small master's degree program in general and applied mathematics during the academic year (usually 10 students) and a summer master's program in mathematics education. The department graduates about 75 students per year, with computer science and mathematics education being the two largest areas.

Abstract

This course is designed to meet the core mathematics requirement for non-technical majors by introducing them to practical applications of algebra, geometry and trigonometry. The goals of the course encompass enhancing students' skills in:

• Applying previously learned mathematics
• Using technology to solve realistic problems
• Working alone and in groups to solve problems
• Communicating technical results in writing

We have developed class and laboratory materials for four modules. Trigonometry investigates geometric measurement problems. Finance applies the basic formulas to various personal finance situations. Statistics discusses data collection and presentation techniques, simple summary statistics, and basic linear regression. Linear Programming introduces resource allocation and related problems. Our materials are designed to give students an intuitive understanding of mathematical concepts. Traditional lectures are minimized in favor of hands-on activities. Laboratory assignments allow students to use real data, often personally gathered. Students work individually and in groups employing various software packages to investigate the behavior of the models they create. The appropriate incorporation of mathematics, tables, graphs and data into written exposition is emphasized as well.

Feedback from four years and over 2400 students indicates that students leave the course with an increased appreciation of the value and usefulness of their mathematical knowledge. They find their knowledge of computers and statistics immediately applicable in other courses.

THE PROJECT

Motivation for Project

Students go through their lower level undergraduate mathematics courses with one or both of two promises: This is important for the next mathematics course you will take; this will be useful to you in your life. We were teaching college algebra as the terminal course in mathematics for liberal arts majors, and we were failing to deliver on these promises. This was the last course they were to take in mathematics, and algebraic manipulation skills weren't especially useful to non-technical majors. We searched for a course we could teach that would be useful and give our students some mathematical power. We also felt that students needed to use computers throughout the course, not necessarily to learn computing, but to solve problems and accomplish tasks that they could not otherwise. Reading about the reform movement in calculus also inspired us, and forced us to realize that changing the content without changing the approach was impossible. As a result we have integrated writing, computing and cooperative learning into the course.

Project Data

As mentioned above, this course grew out of the need for a high quality terminal mathematics course for non-technical majors. Below is a time line giving milestones in the development in the course and sources of funding we received along the way.

1992

- ILI Grant funded (# 55-5-53364) by NSF and College of Arts and Sciences
- ASU Hubbard Center Summer Grant was received to choose software and develop handouts/lab ideas
- Course officially received C designation—"computer use" course
- The text *For All Practical Purposes* was chosen.
- 22 Macintosh LCII computers installed in lab for Fall semester
- 6 sections of class offered by 6 faculty, Fall semester (131 students)
- Weekly meetings of instructors were instituted to provide support and encouragement
- Software used included MacWrite, MacDraw, Lotus 1-2-3, and ALPAL
- Presentations made at AMS, ICTCM, and Kennesaw State U.

1993

- 22 sections taught by 12 faculty, Spring, Summer and Fall semesters (365 students)
- ASU Hubbard Center Summer Grant was received to write modules incorporating more realistic problems and technology
- Course taught for first time with in-house materials, Fall semester
- Search conducted for better software for linear programming and statistics

1994

- 28 sections taught by 14 faculty, Spring, Summer and Fall semesters (494 students)
- Presentations made at AMS, NCCTM and ICTCM
- Mathematics Education majors began being instructional assistants in lab
- Teacher's Guide first draft tested, Fall semester
- Software used changed to WordPerfect, Lotus 1-2-3, Statistics Workshop and Big ALPAL and TELNET

1995

- 29 sections taught by 16 faculty, Spring, Summer and Fall semesters (633 students)
- Weekly meetings of instructors were replaced by occasional meetings and mentoring of new instructors by experienced ones
- One course released time was granted by department for revision of materials
- Presentations made at AMS, South-Eastern Section MAA
- Course became recognized as *the* mathematics core course for non-technical majors
- Discussions with several publishers culminated in a contract with Kendall/Hunt, Inc.
- Software for statistics changed to Minitab

1996

- 38 sections taught by 17 faculty, Spring, Summer and Fall semesters (796 students)

- Text (11) revised and submitted for publication
- Lab expanded to 25 computers, and software for LP changed to MathProg
- Course officially received W designation – "writing across the curriculum" course
- Presentations made at AMS, South-Eastern Section MAA, ICTCM
- Web use incorporated into class and lab activities
- Teacher's guide materials revised and adapted to be web-deliverable (in progress)

Philosophical Setting

When designing and refining the course, we depended on both our experience with the power of teaching applicable mathematics and on the literature that influenced reform in mathematics education in the 1990's. We were especially influenced by the *NCTM Standards* [122] and by the work on calculus reform. As in the Standards we wanted to "... see classrooms as places where interesting problems are regularly explored using important mathematical ideas." This and other documents ([124, [123], [126], [125], [127]) informed and reinforced our belief that mathematics is best learned actively, in a context related to previous activities, collabor-atively, and using technology. Out of these ideas we developed topics that students can relate to everyday life and that, if treated realistically, require technology. We choose complicated enough problems so that students need to collaborate to solve them, and then will see the necessity of writing solutions in English since the answers are not simply numbers, but courses of action.

We have seen calculus reform make a significant change in content and pedagogy. We feel that liberal arts students deserve the same careful approaches and serious attention that we give to students who will take higher level mathematics. Technology is important to all students. Active learning is good learning no matter the level. We only have one chance, one course, with these students, and we are committed to making the most of it.

Innovations

As we have noted, it was clear from the start that the class we envisioned would not lend itself to the lecture/examine style of teaching. We made the commitment to spend time, energy and resources on improving this non-major class, which for institutions like ours *is* a considerable innovation. As we taught the course, we found several non-traditional teaching and assessment techniques that worked very well. Below is a brief list of what we believe is innovative about our course; please see the sections below on Pedagogy and Technology to read about the details.

Non-traditional coverage of topics: Every topic we cover is mentioned in texts on college algebra. In our course, however, we cover these topics in non-traditional ways.

Collaborative projects: We begin very early, usually the first or second day, with group projects. Then group assignments are interspersed with individual work throughout the rest of the semester.

Alternatives to Lecture: Individual and group activities, class discussions, and mini-lectures replace the traditional lecture approach.

Alternative assessment and writing across the curriculum: We assess students with lab and project work. There are still tests, but even those have open-ended, "what is the best course of action and why" questions. We examine *revised* written and compu-tational work via a student portfolio, emphasizing the *appropriate* incorporation of mathematics, graphs and data into written assignments.

Experimenting in a laboratory setting: We have a two hour lab period each week in which we can allow students to explore a situation, usually but not always using a computer.

PROJECT REPORT

Getting Started

In 1988 Appalachian began an overhaul of its general education requirements. One outgrowth of that effort was a request that the Department of Mathematical Sciences "design and offer a course that covers consumer mathematics, including the interpretation of statistics and graphs and an introduction to computers." The department took this to heart, developed a liberal arts course (MAT 1010) and started teaching it in 1990 without a computer lab. In 1992, Dr. Jimmy Smith and Dr. Gary Kader received funding for a computer lab in the form of an ILI grant from the National Science Foundation, with matching funds from the College of Arts and Sciences. By the Fall of 1992 the computer lab was in place, and the course was expanded to include a two-hour lab component in addition to three one-hour class meetings per week.

We wanted our course to be in consonance with the goals of AMATYC, NCTM, MAA, SIAM and AMS. On pages 40-43, the AMATYC document *Crossroads in Mathematics* [26] details what a college liberal arts mathematics course ought to be. We found this document to be very valuable when refining MAT 1010. Some particularly telling quotes from this report are in the box on the next page.

We started with the text *For All Practical Purposes* [28], but soon found that the text didn't fit our needs very well, especially given our students' access to technology and poor opinion of the beauty of mathematics. Since then a core of five instructors (Ms. Jay Bennett, Dr. Holly Hirst, Ms. Nancy Sexton, Dr. Jimmy Smith, and Mr. Hutch Sprunt) have generated course and lab materials, produced a published text, *How Do You Know? Using Math To Make Decisions* [77], enhanced the computer component to include e-mail and the

"...a few topics studied in depth..."	MAT 1010 concentrates on three or four areas.
"...fresh mathematics rather than a rehash of previously studied topics..."	MAT 1010 topics are not those typically studied in high school, nor is the approach the same.
"Cooperative learning experiences should be devised to use the differing strengths of students."	MAT 1010 starts with a cooperative learning lab and students are subsequently assigned group projects.
"Students should be expected to use technology to solve problems and to write project reports..."	Most of the MAT 1010 labs require computing to solve the mathematical problems, and nearly all of them require a report prepared on the word processor.
"Students should participate in mathematical modeling..."	Each segment of MAT 1010 requires modeling and answering open-ended questions.

web, and implemented the writing component with a portfolio requirement. We had substantial support along the way from the NSF, the department, the college and the ASU Hubbard Center for Faculty Development, as outlined in the Project Data section above.

Content

The main goal of this project is to provide liberal arts majors with a satisfying, useful, technology assisted, terminal mathematics course. To meet this general goal, the ILI proposal that allowed us to start the lab which is so crucial to the course listed the following five goals:

- To develop a style of thinking rather than memorize a collection of techniques
- To explore problems that can be studied, modeled, and solved using mathematics
- To experience mathematical modeling and problem solving through laboratory experiences
- To use appropriate technologies, especially the computer, as tools for solving problems
- To communicate solutions in English

We have designed a course that meets these goals which is built around four independent modules: Trigonometry and Measurement, Personal Finance, Consumer Statistics, and Linear Programming. Instructors have flexibility in the order and depth of coverage. Some choose to cover three modules, treating the non-traditional emphases of group work, writing and skill at computer usage in significantly greater detail or augmenting one of the modules with additional materials. The course meets four times a week, with one two-hour lab and three one-hour classes. The lab usually involves computer work and so is held in the computer lab. In the other class periods, lecture is often minimized in favor of hands-on activities that increase class participation. The textbook is written in a workbook format with large margins for jotting notes and space for working problems. The writing style of the text is informal; students are encouraged to read through the text and to work problems on their own. This allows us to lecture selectively on topics to suit a particular section of students, leaving plenty of class time for hands-on problem solving, experimentation and discovery activities.

On the surface, the four modules covered in MAT 1010 are all traditional topics in applications of algebra and geometry. The mathematical goals for the course — gaining skill at recognizing what to apply and how to apply previously learned mathematics to realistic problems — could be met in a limited way without the use of technology beyond a scientific calculator. Specifically, the students could be exposed to right triangle trigonometry, finance formulas, statistical analysis of small data sets, and solutions to two variable linear programming problems. With the introduction of technology, even in the minimal form of a word processor and spreadsheet, each of these topics can be expanded to include more realism, and the additional goals of computer literacy and improvement of written presentation can be accomplished. Each module was chosen for a specific purpose in meeting these goals.

In the trigonometry module we emphasize realistic problems involving length, area and volume calculations. Trigonometry is introduced from the right triangle relationships, solely to assist students in measuring distances, areas and volumes in situations that might occur in their own lives. For example, students might measure the area of a yard to figure seeding requirements and measure the volume of a patio to figure concrete needs. The experience culminates in a measurement lab in which students actually measure several inaccessible heights and distances using a transit and measure wheels. We encourage students to develop their own approaches to the solutions; traditional texts tend to lay out the description of such multiple step problems by including a clearly labeled picture. We describe only *what* to measure, not *how*. This gives students a notion that the answer is not necessarily obtained by one algorithm. The computer use in this module is somewhat more limited than the others — use of a word processor and drawing program — so it makes an excellent first module.

In finance we emphasize use (not memorization) of the compound interest, annuity and loan payment formulas for solving problems from personal finance. We cover several topics traditionally found in financial planning texts as well, including personal credit, taxes, retirement, debt-to-income ratios, credit card interest and leasing. While many of these topics could be addressed in a limited way with just a calculator, we have expanded upon them considerably by using a spreadsheet. The students build formula driven spreadsheets and are encouraged to experiment and change scenarios to suit their personal situations. Write-ups for this module become more complicated; students learn quickly that the answer to a mathematical question is usually *not* a single number. They also learn that knowing about saving and borrowing money can influence their choices even as students.

In statistics we emphasize the role of statistics in the students' lives; we encourage them to look with a critical eye at the ways data are gathered and presented. We show them how to present data in various forms. We show them simple one and two variable analysis techniques. All of this is done using a spreadsheet and statistics package in order to allow the students to produce professional looking graphics and to analyze large data sets. Unlike most elementary statistics texts, assignments are open ended — 'how would you perform an analysis of the differences in the way men and women estimate time' instead of 'use a box plot to analyze ...' — and the resulting write-ups are required to incorporate numerics and graphics effectively.

In linear programming our goal is not just to teach students two variable graphical solution to problems or simple simplex method manipulations as in standard college algebra texts, but rather to show students how even complicated large scale management science problems can be handled using high school algebra and to convince students that mathematics done by others influences almost every aspect of their lives. Emphasis is placed on building the mathematical model, especially recognizing the objective, the unknowns and the limiting factors. We want the students to understand that there is no mystery to how such problems are related to the mathematics they already know and that common sense plays a large role in mathematical modeling. We use realistic problems whenever possible, so the use of a simplex package for solution of dimension greater than 2 problems is central to this module. The write-ups here are among the most detailed; students are given a scenario and asked to investigate it and several variations. They learn that assumptions are not always explicitly stated, and that the modeling process is often more art than science.

Pedagogy

Teaching a course like this in a lecture style would be putting new wine in old skins. Some of our faculty felt like those old skins when they started teaching it. The NCTM Standards [122] and other recent reports emphasize problem solving, cooperative learning, activity based learning, and using technology. We took them to heart. MAT 1010 is not a lecture course. We talk about good graphs and bad graphs, but students initiate the discussion based on graphs they have brought to class. Experiments abound. Students must take much of the responsibility for their learning in this course. The book is simply written in a conversational style, and most students can understand the ideas presented there. This allows the instructor to use much of the classroom time for more creative work. After introducing a topic, students are encouraged to try problems on their own right in class; we have "you try it" problems with space to work right in the text. We encourage discussion as often as possible; for example, when covering personal finance we ask students to bring questions on things like credit cards, student loans, and investments. In statistics, we have the students collect data about each other; they work as a class to design surveys; students present results of analyses of their own data to the class.

Here are a few other typical examples of classroom activities. After discussing graphing and scale, students are given sales data that is increasing over time, and asked to create one honest graph that emphasizes the increase and another that makes the increase look modest. After discussing finance and statistics, students are given the information below and asked to argue for or against (their choice) whether the schools have been fairly treated by the county.

The Happy Mountain School District gets some of its funds from the county budget. In 1994 the total county budget was $50 million dollars and the school district's allocation was $3 million. In 1995 the county allocated the district $3.3 out of its $60 million budget. Inflation was about 2.7% from 1994 to 1995.

Group work is used in class and in lab to help students develop their interpersonal skills. For example, after discussing compounding in class students can be put into groups and asked to see how long it takes for money to double, with each group looking at several rates and compoundings. Afterwards, the class as a whole creates a table or graph of all the results to see if a pattern emerges. Many of the lab projects involve group work as well. For example, the hands-on measurement assignment requires students to measure several structures on campus; using the transits and linear measure wheels is a three-person-minimum job, so they must plan their approach and work together even to get started on the solution.

All of these problems are more open-ended and more open to alternative solutions than students are used to, and compel students to change the way they think about what mathematics problems are and how they should be approached. By the end of the course the students, when faced with a different type mathematics problem, are much more willing than before to try something, experiment and discuss possible approaches with others in the class than they were in the beginning.

MAT 1010 has had a positive impact on faculty as well as on students. Faculty share teaching ideas more than before because this course is so different. MAT 1010 faculty use the technology and the pedagogy that they discover in MAT 1010 in their other courses. The course has caused even the least technologically inclined to move with confidence into the computer age.

Technology

Using technology is critical to the course and has a major impact on how it is taught. We are currently using Macintosh LCII computers with WordPerfect (word processing), Lotus 1•2•3 (spread sheet), Minitab (statistics) and MathProg (linear programming - simplex method) software packages available from a centrally maintained file server. The particular computer hardware, word processor, spreadsheet, statistics software or linear programming software used is immaterial to the central ideas in the course since the focus is to learn to use technology as a tool for solving realistic problems.

Access to a lab having computers with these capabilities is critical. We would be severely limited in the types of questions we could ask were there no computer lab in the course. Having a linear programming problem solver allows us to state much more realistic problems. When we look at resource allocation, students explore "what if" questions about realistic situations via both experimentation with the parameters in the problem and analysis of sensitivity information provided by the software. In our treatment of finance we can ask "Is buying your dream car better than leasing it?" Preparing a spreadsheet for this is a challenge, but within the grasp of the students, and is certainly richer than answering only questions that can be solved with a calculator. Having a statistics package allows us to study data sets collected from lots of students who have recorded their heights, arm spans and other facts and measurements about themselves. Students easily create box plots, scatter plots, tables or other graphs and do statistical computations they believe appropriate. We don't have to limit ourselves to toy data sets. In trigonometry we use the computer primarily to produce nifty pictures to include in the (word processed) report, but that's a nice change of pace which lets the artistic students use their strengths.

The main ideas and materials can be adapted to allow for reduced access to technology. Many of the individual topics and approaches, including specific assignments, would be appropriate for incorporation as a project into any pre-calculus level mathematics course. Wholesale adoption of the entire set of materials would require some access to computers, minimally ones with a word processor and spreadsheet, but not necessarily in a dedicated laboratory environment. The course could still meet most of the stated goals with only peripheral student access to the technology (e.g., via homework projects to be completed in a public lab), provided some mechanism for classroom demonstration were available. Of course, to implement all of the laboratory assignments most effectively some class time should be devoted to student access to computers, as we have done in the weekly two-hour lab periods.

Day to Day Mechanics

There is not a day-to-day routine in MAT 1010 classes. Students don't know what awaits them as they enter class. They may go outside and measure the height of a tree using a transit and measuring wheel. They may go to the library and sample books to see how many are blue, and then come to class and defend their sampling scheme. They may work in groups on tough linear programming problems. The instructor might even lecture. The students would know about where they are in the text.

The labs are related to the course material and often expand on it. Instructors customize labs to mesh with lecture and student interest. Here are a few examples. We discuss amortization tables in class and do a small one; then in the lab we are able to do a full–fledged one and see how slowly the principal decreases on a long-term loan, experimenting with interest rates. Solving a linear programming problem graphically is a class exercise; translating one with a dozen variables and ten constraints on the computer is different, especially when shadow prices and "what if" questions are part of the assignment. In class we can construct and interpret box plots for small data sets; in lab we can use the computer to construct box plots for a larger version of the data set. We can tie these together by discussing small versus large samples. Many of these class and lab activities then require write-ups in which students must interpret their results.

Appraisal, Comparisons and Conclusions

We have used several measures of effectiveness. We have been especially interested in student reactions to the materials and how their perceptions of mathematics have changed as a consequence of taking our course. The early classes were polled using evaluation forms asking questions about course content, software use, and teaching approaches; the results were used to modify the text materials and the laboratory assignments, and to provide direction for further development. In general, students liked the materials if not the actual mathematics, and gave many helpful comments which we incorporated into the course.

Several sections were surveyed during Spring semester, 1996, to see if attitudes about math changed as a result of MAT 1010. A survey with the questions listed below was given early in the semester and then again at the end of the semester to check for changes in attitudes. Students were asked to circle a response for each question, the possible responses being: Strongly Agree, Agree, Undecided, Disagree, Strongly Disagree. The results are also included below (somewhat represents one change in level and significantly represents more than one change).

The results for some of the questions were very gratifying; there were improvements in how students viewed their general math skills, their comfort in computer use, their level of math anxiety and their ability to handle word problems. We also found much food for thought; there were a few students for whom access to the computer just strengthened their unease. The results of the last question were very disappointing; we had hoped to convince students that math wasn't just memorization, but this wasn't reflected in the results. We have refined the survey and are giving it to this Fall's sections.

To ensure that the course is "teachable," we have encouraged all of the instructors and instructional assistants to submit criticism of and suggest changes to the materials. We have incorporated many suggestions from our colleagues (which is why we claim editor status rather than author status for the text).

We tried many things throughout the evolution of this course and found some great stuff and some not-so-great stuff. One of the most interesting "great" things is the effect teaching the course has had on the faculty and mathematics education majors. Faculty have become much more computer literate and more openly discuss interesting ideas (and frustrating failures)—not just those related to MAT 1010. The course has also had an extremely positive effect on majors serving as instructional assistants. They feel better able to cope with the technology available out in the public schools and have found many ideas for projects that can be incorporated into classes from middle school on up.

Perhaps the single most frustrating thing for us has been finding robust, easy-to-use software for statistics and linear programming. We are currently using Minitab and Mathprog software, but some instructors are unhappy with these programs (even though they are vast improvements over previous choices), and so the search continues. We are also fighting the battle of aging equipment already upgraded to full capacity, which doesn't run the newest and best software well or in some cases at all. Even software upgrades that still run on our computers can be extremely trying; there is always some minor feature that has been "enhanced" in such a way as to change how the students should work through a problem. After four years, we have gotten better at preparing faculty to handle the inevitable problems arising from network software access, computer-inexperienced students, and aging computers. The two golden rules are: Be calm in the face of adversity, arming yourself with an alternate activity just in case, and don't expect 25 students to prepare and print complicated documents (to one or even two printers) during one lab period!

We learned some pedagogical lessons the hard way: Grading writing is tough; even with sections of 25 students, grading time can be significant. It's hard to match assignments with the time allowed for them—making labs too long is common. Good directions on open-ended assignments are essential. Students need considerable guidance at least at first on what mathematical writing should look like. Early on, every new instructor had to learn these lessons individually. Luckily, we have six faculty who have taught the course almost every semester since its inception, so we can mentor new (and not so new) instructors who need support.

Survey

	improved significantly	improved somewhat	worsened somewhat	worsened significantly
I see mathematics as a subject I will rarely use.	5%	5%	0	0
I'm no good in math.	5%	16%	5%	0
I feel comfortable using a computer.	5%	26%	5%	5%
Knowledge of mathematics is important for scientists.	5%	5%	0	0
For some reason, even though I study, math seems unusually hard for me.	10%	10%	0	0
People in business and government use mathematics to solve problems.	0	5%	0	0
Math doesn't scare me at all.	0	21%	5%	0
I feel like I can do "word problems" in mathematics.	10%	32%	0	0
Mathematics is used by other people to make decisions that affect me.	0	5%	0	0
Doing mathematics is mostly a straightforward matter of memorizing and following procedures.	0	10%	26%	0

Future Plans

MAT 1010 is no longer an experiment; 2459 students have taken the course. We foresee more fine tuning than major revision. We have already investigated incorporation of newly emerging technology into the course. Data searches via the world wide web have become possible from all computer labs at Appalachian and many other institutions. We are very interested in investigating the usefulness of the web and other advances in technology from the point of view of: What life skills related to mathematics and the web/other technology would be useful to the students? We are looking with interest at the next generation of graphing calculators as appropriate, cost effective replacements for the computational aspects of the computer use in this course.

We are working on new labs and a few new sections to incorporate into the next edition of the text. The support materials for the text— lab and class activity ideas, suggestions about what works and what doesn't, etc.—are being webified so that anyone can access and customize these activities to fit into their classes. We are planning to submit a proposal to get funding for summer workshops for faculty at other institutions on how to use all or part of the material in courses with differing access to technology.

SAMPLE MATERIALS

Text Used: Hirst, Holly and James Smith. *How Do You Know? Using Math to Make Decisions,* Kendall/Hunt Publishing Co., 1996.

Course Outline

There are four independent modules to choose from. Faculty are encouraged to rearrange topics at will and experiment with incorporating new ideas into their classes. They choose between covering all four modules or covering three of the modules, treating the non-traditional emphases of group work, writing and skill at computer usage in significantly greater detail or augmenting one of the modules with additional materials. We do require everyone to cover the statistics module in order to prepare students for Statistical Methods I, the basic statistics class required by several of our constituent majors (such as psychology and criminal justice).

Below we have included a detailed outline of topics covered including selected examples of lab activities and possible additional topics, where C indicates computer required, W indicates formal writing assignment, and G indicates group activity.

I. Trigonometry and measurement

1. Review of triangle and rectangle geometry including angle properties, areas, volumes
2. Introduction to trigonometric functions through right triangles including sine, cosine, tangent, inverses
3. Applications involving angle measurement

Additional elective topics—Oblique triangle properties, navigation, astronomical measurements

Examples of Labs—learning a drawing program (C), scale drawings (C), measuring a structure on campus (C, G, W)

II. Finance

1. Review of percents and proportions including markups, discounts, simple interest, depreciation
2. Savings and annuities including present value, future value, multiple compounding
3. Loans and amortization including mortgages
4. Personal finance including IRAs, CDs, credit cards, debt-to-income ratios, leasing

Additional elective topics—inflation (in more depth), taxes, other forms of investments

Examples of Labs—learning lotus (C), amortization of a mortgage (C), taxes and mortgages (C, W), buying versus leasing (C, W), retirement savings (C, W)

III. Statistics

1. Collecting data including bias, random sampling, surveys
2. Presenting data including various types of graphs, good and bad graphs, reading and drawing graphs
3. Simple univariate data analysis including distribution shape, measures of center, measures of spread, box plots
4. Regression including scatter plots, least squares regression, coefficient of determination

Additional elective topics—probability, normal curve, nonlinear patterns

Examples of Labs—collecting class data (G), constructing quality graphs (C), survey design (W, G), predicting height (C, W), analysis of particular data sets(C, W)

IV. Linear programming

1. Modeling with mathematics including objectives, constraints
2. Graphical solution including inequalities, feasible regions, corner point principle
3. Solution of large problems including slack variables, shadow prices, sensitivity

Additional elective topics—transportation and assignment problems, integer programming

Examples of Labs—mathematical modeling practice (G), learning MathProg (C), analyzing sensitivity information (C), attacking a real problem (G, W, C)

Syllabus and Instructions to Students

We have included here a typical syllabus and also the textbook's instructions to the students. Together these give a good idea of the flavor of the course.

Syllabus

Textk: *How do You Know? Using Math to Make Decisions*,
Edited by Hirst and Smith, Kendall/Hunt, Inc., 1996

Goal. To introduce non-technical liberal arts students to mathematical problem solving using appropriate computational tools. Four modules will be covered: Trigonometry and Measurement, Linear Programming and Decision Making, Mathematics of Finance, and Statistics. In these modules, you will learn how mathematical concepts studied in high school algebra and geometry are used to describe real life situations and to make decisions. Emphasis will be placed on understanding basic concepts, interpreting results and communicating solutions in writing. You will also learn the basics of several software packages in order to solve problems that are larger and more realistic than can be solved by hand and to present solutions in a neat, professional fashion.

Prerequisites. Sufficient grade on mathematics placement test. By Topic: Basic knowledge of high school geometry and algebra: triangle and angle properties, linear equations and inequalities, percentages, evaluating expressions. Knowledge of basic grammar is also assumed.

Assignments and Exams. The goal in this class is not only to learn to solve problems, but also to learn to write about mathematics. To this end, the assignments will take two forms: traditional homework / labs / quizzes and formal projects. Also, the exams will include discussion questions as well as calculations.

Daily Work. This includes assigned homework problems, quizzes and lab write-ups. Solutions should be neatly presented and may include type written explanations. Grades will be based upon the correctness of your mathematical solution and your ability to use the computer to complete the assignment.

Project Reports. At least twice in each module, you will be given a major laboratory assignment requiring a detailed written explanation of the problem and its solution. Reports will be graded based upon the following criteria:

- Correct mathematical formulation and solution of the given problem.
- Presentation in type written, grammatically correct form.
- Clear and careful explanation of the problem and its solution.
- Inclusion of computer generated graphs, diagrams and charts.

There will be at least eight such projects, on which grading for style and clarity will become progressively harder.

Portfolio. Several assignments in each module will be designated as portfolio assignments. You will choose five to enhance and turn in as a portfolio of your work.

Exams. There will be an exam for each module covered and a comprehensive final.

Instructions to the Student

For most of you this is your first and last college mathematics course. This will be different from your previous mathematics courses in a number of ways:

- You won't memorize many formulas. Neither will you be required to do 47 repetitive problems each night. There won't be nearly as many problems for you to do, but each of them will require considerable work and thought; answers will sometimes be paragraphs rather than numbers.
- Topics will relate to real problems that you may encounter in everyday life.
- You will often work in groups, or at least pairs, rather than by yourself.
- You will have a formal lab and write up detailed laboratory reports.
- You will use computers extensively.

- We will teach no algebra before its time; i.e. algebraic skills won't be taught except as they relate to solving a problem that requires those particular skills.
- We expect you to write in the book a lot. Doing the "you try it" problems is critical to your success in the course.

What then will help you be successful in this course? PARTICIPATE!! Students who passively sit and want to have material fed to them will not appreciate the course, nor will they be appreciated. You must try the problems, mess with the computer, write your answers in paragraphs, speak up in class with questions or answers, and generally be an active learner. This means you'll work hard, which you expected since this is a math course. But your efforts will not be spent in solving equation after equation and in manipulating complex algebra. Instead you will use mathematical principles to evaluate possible courses of action, to reach reasonable conclusions, and then communicate your results.

Standards in this course are high, but reasonable. "A" work will go beyond minimal responses to assignments and also be correct. Routine responses which are basically correct with only an occasional error get a "C." Indolence and absence will receive their appropriate rewards. Careful thinking and hard work is what is needed; there isn't some magic mathematical genetic predisposition that will carry you through if you have it or doom you if you don't.

We expect you to enjoy this course. That may be a bold statement for some of you who have struggled with mathematics in the past, but you can enjoy mathematics if you participate and give it a chance. There will be many opportunities to be creative in your thinking and communication. Taking advantage of these will deeply enrich your experience. We hope the material, the method of presentation, the cooperative effort with the other students, and the support and encouragement of the instructor will all work together with your own efforts to make this a constructive and positive finale to your mathematical career.

Sample Assignments

Below are descriptions for several of the activities that we choose from when assigning labs and projects in MAT 1010. We are in the process of putting these and other activities on-line.

Measuring Using Trigonometry

Your team's problem: The Eggers dorm association wants to drape a long banner on the parking lot side of their dorm for homecoming. They can get fabric that is 15 feet wide and any length they want for $3 a square yard. They want the banner to cover all the way from the roof to the grass. Determine the height of Eggers dorm without leaving the stadium parking lot, and use this height to figure the cost of the banner.

(1) Discuss a measurement strategy with your group members and clear up any questions before leaving to take measurements.

(2) Grab a transit and measure wheel and go take all necessary measurements. Do each one carefully twice to be sure of your numbers.

(3) Come back to lab and work through the calculations to solve the problem. Then increase the largest angle you needed by 2.5° and re-calculate your answer to estimate the amount of error that could result from an inaccurate angle measurement. Calculate % change as described in lab.

(4) Write a report with the following format:

1. Name at top of page 1
2. Restatement of problem
3. Description of how measurements were made, including a picture.
4. Calculations needed to arrive at the solution, including a diagram.
5. Results of calculation when angle is increased by 2.5°, including a percent error.
6. Other possible sources of error in your calculation and what you would have done differently on a second attempt.

Exploring Saving for Retirement (aka Jane/Joan and Fred/Barney)

Investigate the following situations. Write up your findings in the form of two letters.

Problem 1: Jane and Joan were twins. They both went to work at age 22 with identical jobs, identical salaries, and at the end of each year, they received identical bonuses of $2,000. But there was one difference:

JANE: As a young woman, Jane was conservative and was concerned about her future. Each year she invested her $2,000 bonus in a savings program earning 8% interest compounded annually. Jane decided at age 32 to have some fun in life and began spending her $2,000 bonuses on vacations in the Bahamas. This continued until she was 65 years.

JOAN: Joan, on the other hand, believed that as a young woman she should take opportunities to enjoy life and not be too concerned about saving for the future—she had a lot of years later to put money in savings. For the first ten years she spent her $2,000 bonuses on vacations in the Bahamas. At age 32, she decided she should start saving for her future and from that time on she invested her $2,000 bonuses in a savings program paying 8% compounded annually. This continued until she was 65.

Through the years, the sisters became separated. However, they were joyfully reunited at age 65 at a family reunion and exchanged many stories of the events of their lives. Eventually the conversation got around to retirement plans and savings programs. Each sister was proud of her savings activities, terms and accumulations. Which one has more money? Is the answer what you expected?

Problem 2: Fred and Barney were business partners. They bought identical houses and got identical loans for $80,000 at 7% interest payable monthly for 30 years. But there was one difference:

FRED: Fred prefers to make his house payment each month and to put $100/month into his retirement fund (at 6.5% compounded monthly). He will pay off his loan in thirty years, on the day he retires at age 60.

BARNEY: Barney, on the other hand, chooses not to start his retirement plan yet. He puts the extra $100/month onto his house payments. He will end up paying off his loan earlier this way. For the remainder of the thirty years until retirement, he can put both the $100 plus the house payment into retirement.

When Fred and Barney are ready to retire at age 60, who will have accumulated more money? Is this different from what you expected? Does how much the houses are worth in 30 years matter? What if one is worth more than the other?

Taxes and Real Estate

It is January 1, and you want to estimate a friend's taxes for the new year. Your friend earns $40,500 a year, and is a single person with no disabilities or dependents. Your friend has a few minor investments in savings which will net about $800 in interest income for the year. Your friend has an IRA into which he puts $800 each year. Your friend lives in North Carolina and thus pays roughly 7% of income in taxes to the state. Your friend also donates $300 per year to the ASU alumni fund. Your friend is planning to buy a house costing $70,000 and will finance the loan at a bank that requires 20% down for a 30 year loan at 7.25% payable monthly.

Use this information to see what the difference in taxes owed would be between buying and not buying a house. Interest for a home is tax deductible. We need to find the amount of interest paid for one year. (We'll assume that the loan started in January.) Prepare an amortization table for this mortgage. How much interest is paid in the first year?

If the mortgage interest for one year, charitable contributions, and state and local taxes add up to more than the standard deduction of $3,900, you are ahead to itemize your deductions. Calculate the itemized deductions for this situation: One year's interest on the house + charitable contributions + state taxes (assume a 7% state tax rate on entire income).

Fill out a 1040 tax form with the relevant information in two ways: First take the itemized deduction including the mortgage interest, and then take the standard deduction. How much money do you save? Write a report explaining what you did in this lab. (You might find it helpful to think about how you'd explain this to your parents.) Type the report up carefully using a word processor. Be very specific and thorough! Attach this work sheet and the amortization table to the report AND refer to them in the report to explain what you did.

College Tuition

Your task: Prepare a spreadsheet for the following scenario. A suggestion for the format of the spreadsheet is given below. Write a report to the student; think about how you would explain the results to a friend. Print the original spreadsheet and your analysis. In your write-up, which will be in paragraph form, you will include numerical evidence for what you say. Refer to the spreadsheet, which should be in an appendix.

Sarah, a senior in high school, has been accepted at Appalachian for the Fall semester of 1997. She has to pay her own way. She will work on campus during the school year but will only earn enough there to cover her personal expenses. She is planning to work each summer to cover tuition, fees, room and board for the year. Her plan is to work May 15 through August 15. At each payday (June 15, July 15, August 15) she will deposit her check in a savings account that pays 6% annual interest, compounded monthly. On August 15 and January 15 she will pay Appalachian for the coming semester.

tuition = $359 fees = $310.75 room = $685 board = $500

Set up a spreadsheet which will track her bank balance for the first year. Use addresses and formulas when possible in the table so you can enter various incomes and interest rates and see the results.

a. Find the minimum amount she could earn each month of the summer to make the needed payments to Appalachian.
b. What would her hourly wage have to be if she worked 40 hours a week and there are 4 1/3 weeks in each month?
c. How would a higher interest rate affect the needed wage?
d. She has found that she can work at the same job for two weeks during Christmas break and use that money to help pay spring semester tuition and fees. What is her minimum hourly wage then?
e. If she decides not to work at Christmas and also wants an extra $100 at Christmas and Spring Break, how much would she need to make per hour?

Linear Programming

Your team's problem: A federation of three farming communities called the Southern Confederation of Kibbutzim wants to know how to plant their crops for maximum profit. Planning for this organization is done centrally.

Kibbutz 1 has 400 acres of land and 600 acre feet of water available. Kibbutz 2 has 600 acres of land and 900 acre feet of water. Kibbutz 3 has 600 acres of land and 375 acre feet of water.

Only sugar beets, sorghum and cotton are considered for planting next season. The Ministry of Agriculture has set limits on the total acreage that can be devoted to each of these crops. The table below gives this information as well as the water needs per acre and the net return per acre on the three crops.

crop	max acres to plant	acre feet of water needed per acre	net return per acre
sugar beets	600	3	$400
cotton	500	2	$300
sorghum	325	1	$100

The confederation has agreed that each Kibbutz will plant the same proportion of irrigable land. How many acres of each crop should be planted at each Kibbutz? What is the net return for the federation? How much does the "political" proportionality limitation cost the federation? Kibbutz 2 has the opportunity to get 200 more acres of land (but no additional water); how will this affect the original solution?

Your tasks:

1) Translate the problem—carefully write out the mathematical formulation.
2) Solve the problem, answering the questions given about the original formulation.
3) Redesign the problem to answer the two additional questions.
4) Solve the new formulations.

Write a report to the Southern Confederation of Kibbutzim. It could take the form of a long letter. This report should include the following:

1) Problem description—restate and clarify the situation (tables are nice data presentation tools); Remember that the company personnel you are writing the report for aren't math 1010 savvy. Explain carefully how the problem is set up.
2) Recommended solution, including how much profit or cost is involved; (mention that you used linear programming to find the solution - output is in appendices);
3) Information about left over resources and the values of scarce resources (slacks and shadows); Explain carefully any interesting slack and shadow prices.
4) Appendices with the mathematical formulation, solution and sensitivity screens.

Analyzing Data (data taken from Minitab sample data sets)

Wild black bears were anesthetized, and their bodies were measured and weighed. One goal of the study was to find an equation relating weight to some other physical characteristic for forest rangers, so they could estimate the weight of a bear based on another measurement. This would be useful because in the field it is easier to measure a length than it is to weigh a bear with a scale. Also, the rangers were interested in looking at whether the weight of a bear is different for males and females, or differed by age of the bear. You have a data set with the following information:

Column	Name	Description
C1	sex	1 = male 2 = female
C2	age	5 = 0–5 years, 10 = 5–10 years, 15=10–15 years
C3	weight	Weight of the bear, in pounds
C4	length	Body length, in inches
C5	head.l	Length of the head, in inches
C6	head.w	Width of the head, in inches
C7	neck.g	Girth (distance around) the neck, in inches
C8	chest.g	Girth (distance around) the chest, in inches

Your Task: Build a report addressed to the National Forest Service that examines these questions, including appropriate graphs and mathematics to back up your conclusions.

How We Grade and Test

Mat 1010 grades are based on a combination of test grades, the grade on the final, project grades, the portfolio and homework/participation. A typical division is 40% for projects, 30% for tests, 20% for the final exam, and 10% for homework/participation. Some of the project grade is from a portfolio that students turn in at the end of the semester with improvements on their best work during the semester. Even the tests are designed to require a different approach from the usual numeric-answer oriented tests. Here are a few sample questions:

1. Is it better to have $10,000 now or $25,000 in nine years? Discuss how the interest rate affects the answer. What interest rate would make them equal in value?

2. You have the choice of buying a $12,000 car with one of two financing schemes. One gives you a rebate of $1,500 and offers a four year loan at 9% interest. The other offers a low interest loan of 3% for three years. Assume that you use the $1,500 to reduce the amount of the loan.

a. Compute monthly payments in each case.
b. Decide which you would take and make a good case for your decision. The amount of the payment and the amount interest paid on the loan are important considerations.

3. The rainfall totals for Davis, California and for Sacramento, California for the years 1911–1940 are given below.

Davis

9.5	8.7	28.7	20.0	20.9	14.1	9.7	19.4	8.9	17.2
16.6	17.9	9.0	19.2	18.4	19.0	14.8	10.8	13.9	8.7
15.2	9.6	11.0	18.7	17.8	17.9	25.8	7.0	20.4	31.5

Sacramento

9.6	8.0	20.4	17.2	18.3	13.0	10.6	17.2	8.9	16.8
14.2	15.7	8.0	17.7	16.1	17.8	11.6	10.4	13.6	8.4
12.6	8.1	11.6	21.1	20.5	19.8	24.8	9.7	25.1	31.8

The first number in each group is the rainfall in inches for 1911. The numbers are arranged by row. I am interested in which city has more rain. Your task is to compare the rainfalls in these two cities in ways that make it clear to me that what you have said is correct. You will want to use graphs if that is appropriate and other statistical results as you see fit.

4. Mark Twain in *Life on the Mississippi* discussed how the river had gotten shorter over the years as segments of the river straightened themselves out. He gave the following as the length of the Mississippi river:

Year	1700	1720	1875
Length	1215	1180	975

Graph these points. Fit a line through them, then make a prediction about the present length of the Mississippi. Twain predicts that the river will at some point in the future have length 0. When will that be according to your equation? How accurate do you think all these predictions are? Why?

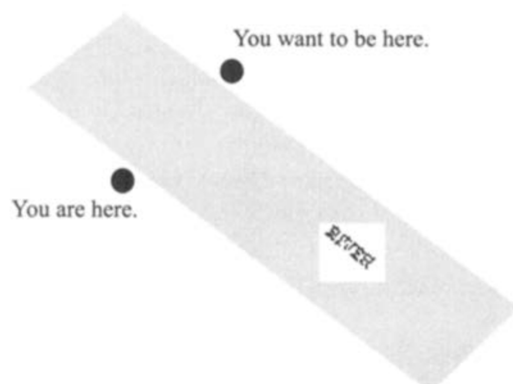
5. Consider the following problem:
A company produces two brands of trail mix, regular and deluxe, by mixing dried fruits, nuts and cereal. The regular must be 20% nuts and 40% cereal and the rest fruit, while the deluxe must be 35% nuts and 30% cereal and the rest fruit. The company has 1200 pounds of dried fruits, 750 pounds of nuts and 1300 pounds of cereal to be used in producing the mixes. The company makes a profit of $.80 on each pound of regular mix and $1.20 on each pound of deluxe mix. How many pounds of each mix should be produced in order to maximize the company's profit?
Answer the following questions. Be as specific as you can.

a. Which, if any, of the ingredients are left over? How much?
b. Tell what the shadow price column numbers mean.
c. If I could buy nuts at $2.00 a pound, should I? Why? If so, how many pounds would I buy if they were available?
d. The company feels that the regular mix is priced too low. How high would they have to raise the price of the regular mix to change the number of each mix that would maximize profit? Would they raise the price that much?

6. You and some friends are hiking in the Amazon jungle and need to cross a river full of man-eating piranhas. You have two choices:

a) Get out your leaky canoe and row across;
b) Hike down river 50 miles to the piranha-free shallows, cross, and come back up the other side.

The main difficulty is that you know you can paddle only 100 feet in the canoe before you sink. How might you decide if the canoe will make it? Explain how you might measure this. You can assume that you just happen to have your trusty trig measurement stuff along!



Material for this article was submitted by
Holly P. Hirst and James R. Smith
Portions of this article are reprinted with permission from Kendall Hunt Publishing.





A New Approach to Calculus for Non-Technical Students

Donald LaTorre, John Kenelly, Iris Fetta,
Cynthia Harris, Laurel Carpenter

Service Course Area

Business Math
Grand Prize Winner

Institutional Data

Located in the foothills of the Blue Ridge Mountains in northwest South Carolina, Clemson University is a publicly assisted, comprehensive land-grant institution. Currently in its one-hundred-and- fourth year, Clemson offers programs leading to baccalaureate degrees in 74 fields of study and 112 graduate programs in 71 areas of study under the colleges of Agriculture, Forestry and Life Sciences; Architecture, Arts and Humanities; Business and Public Affairs; Engineering and Science; and Health, Education, and Human Development. About 15,400 students study on campus, and another 1,100 are enrolled in off-campus programs. Of the approximately 12,700 undergraduates, 73 percent are South Carolinians, and Clemson students represent 50 states and 71 foreign countries. The Department of Mathematical Sciences at Clemson University was a pioneer in integrating areas of the mathematical sciences (algebra/combinatorics, analysis, computational mathematics, operations research and statistics) into balanced educational programs at the undergraduate and graduate levels. Our goal is to prepare students to be problem solvers and to provide them with those skills necessary to compete in a technological society. Degree programs at the undergraduate level and graduate level require breadth and depth of training in the mathematical sciences. Research in the Department ranges from the very theoretical to the very applied. The faculty is active in interdisciplinary and outreach efforts along with educational reform projects, especially in freshman-level general education courses.

Abstract

The project developed materials for an innovative, new approach to calculus for students in such fields as business, economics, liberal arts, management, and the social and life sciences. With the themes of rates of change, accumulation of change, and their interpretations in real-life situations,

the materials are data driven, technology intensive, and feature a unique modeling approach. Along with a focus on fundamental calculus concepts and their relevance and utility in non-scientific careers, the materials stress the development of conceptual understanding without the traditional accent on algebraic skill and technique, emphasize group work on team projects that involve mathematical decision making and the interpretation of results, and use technology as a tool for the learning of mathematics in an interactive classroom environment.

THE PROJECT

Motivation for Project

When the project began, our team was very involved in undergraduate mathematics education at Clemson. Prior to beginning this project, we found the students in the applied calculus courses suffered from mathematical anxiety, had poor retention of algebraic skills, and saw little relevance of the course material to their major fields of study or their lives. There was also ample evidence that the lecture approach very often used in these courses failed to captivate student interest and did not encourage students to become active participants in their own learning. The traditional emphasis on polynomials gave little attention to the importance of growth processes and cyclical patterns in business and life science applications. Realizing that our applied calculus courses suffered from inappropriate content, failed to incorporate realistic applications, were being used more as a filter than a genuine setting for learning, and were for the most part, untouched by technology, we conceived this project in response to a need to restructure the content, spirit, and methodology of Clemson's two-semester applied calculus sequence so as to better prepare non-technical students for their ultimate careers. We were convinced that the development of conceptual understanding, not the mastery of algebraic technique, should be the guiding philosophy of a modern applied calculus course, and we decided to focus on key calculus concepts and their relevance to and interpretation in a world of change. We wanted activities in the course to be just as data driven as would be the future careers of our students.

Project Data

The student population in the courses at Clemson who are impacted directly by the project are undergraduates in majors such as business, economics, health studies, marketing, management, agriculture, animal science, forestry, design, liberal arts, and the social and life sciences; i.e., "non-technical" students in the sense that these students take an applied/business calculus rather than "mainstream" calculus. The majority of those students enrolling in Clemson's first course, Applied Calculus I, are in the Colleges of Agriculture, Forestry and Life Sciences; Architecture, Arts and Humanities; or Business and Public Affairs. The second course, Applied Calculus II, is populated by students primarily in the College of Business and Public Affairs. We therefore decided the emphasis on applications in our materials should be divided equally between business related topics and other types of settings that would be relevant to a variety of major fields.

Having just completed one very successful project that integrated technology into our mathematics sequence for science and engineering majors, we returned to the same source of funds for this new project. We were very fortunate to receive funding for the design, implementation, and evaluation of this project through two substantial grants (1992-95 and 1995–98) from the Fund for the Improvement of Postsecondary Education (FIPSE) in The Comprehensive Program of the US Department of Education. Some of the grant funds provided release time and summer support for up to three members of the project team each year, summer workshops for test site faculty, and travel funds. The grants also supported the establishment of an internet discussion group and a semiannual newsletter. The project team and Clemson sincerely appreciate FIPSE's dedication to innovation in and improvement of postsecondary education.

Clemson students registered in the first semester of the applied calculus course must either have passed college algebra or have been exempted from it with a satisfactory score on the externally administered placement test (College Board Mathematics Level IIC). Students entering the second semester of the applied calculus course must have a passing grade in the first semester, advanced placement credit, or transfer credit for a comparable first-semester course. There are no co-requisites for either course. Clemson students majoring in most business related fields take a calculus based, one-semester probability and statistics course in the Mathematical Sciences Department after completing Applied Calculus II. Other majors allow students to take a one-semester probability and statistics course in the Experimental Statistics Department with no calculus prerequisite while some departments allow their students a choice of the two statistics courses. Other Clemson students complete general education requirements (six hours above either college algebra or precalculus) by taking a finite probability course and Applied Calculus I.

During the 1995–96 and 1996–97 school years, all Applied Calculus I and II courses were taught from preliminary editions of texts developed from the project materials: *Calculus Concepts: An Innovative Approach to the Mathematics of Change, Preliminary Edition and Calculus Concepts: An Innovative Approach to the Mathematics of Change, Preliminary Edition Multivariable Chapters*, both published by D. C. Heath and Company. First-edition manuscripts for a brief text and a complete text were used in the 1997–98 academic year. During that year, Clemson's Applied Calculus I enrollment was about 750 students in 21 sections in the fall semester, and 485 students in 14 sections in the spring session. There were approximately 300 students enrolled in eight sections of Applied Calculus II during that fall semester, and 515 students enrolled in 14 sections of the second course during the spring semester. Approximately two sections of each course were taught during the summer sessions. All classes were taught in the standard class (size 35-43) format. Even though instructors of Clemson's two-semester applied calculus sequence now range from graduate teaching assistants to full professors, the majority of instructors for the two courses are lecturers and graduate teaching assistants.

Clemson's applied calculus courses allow students to use the TI-82, TI-83, TI-85, or HP 48G/GX. Students have calculator-specific instruction for these models using actual examples from the text in the *Graphing Calculator Instruction Guide* that accompanies the text. Possibly due to the Mathematical Science Department's investment in view screen units, nearly all the course instructors use the TI-82. At the present time, students who wish to use models of graphing calculators other than the ones supported in the *Guide* are encouraged to do so only if they are very familiar with their calculators. Most other schools teaching the project materials use graphing calculators, and some have successfully implemented TI-92 and personal computers with Maple, Excel, or other software.

Philosophical Setting

Our objective was to design each of Clemson's two-semester applied calculus courses to improve conceptual understanding and learning by involving students with fresh, new materials in a way that is significantly different from current practices. We have incorporated concepts and topics from calculus that are relevant and useful in non-scientific careers, and which provide experience with mathematical modeling, incorporating real-world data. We focused on concepts and their relationships, and tried to integrate technology so that the course could be offered from graphical and numerical as well as analytical views in an active, constructive environment. Because mathematics today involves much more than calculation and learning rules for symbol manipulation, our competency goals seek to enable students to become confident in their ability to do mathematics and to grasp the implications of the many mathematical concepts that permeate our lives, and we hope to develop in students the ability to communicate and to reason mathematically.

Innovations

In our desire to make applied calculus real for students, we determined that we would avoid the traditional approach of beginning with given equations. Instead, we start many of the examples and activities with a set of real data and ask students to begin by finding a model for that data. When we do give models, they are often referenced with the source of the real data from which they were constructed. We have found this innovative data-driven, technology-intensive, model-building approach very successful in convincing students that mathematics is useful in many aspects of their lives and future careers. To enable students to fit curves to data, we introduce them to models with an emphasis on the underlying behavior of the model. Students learn to analyze data and match the behavior it exhibits with one or more of the six models they have studied. Because many real-life data sets do not fit one particular equation, we introduce piecewise models and discuss in meaningful context certain points at which techniques of calculus may or may not apply. We also take the position that at the core of understanding calculus is the understanding of a function as a rule with associated input and output that in real-life contexts have associated units of measure. We require that students label units on all results and interpret their answers in meaningful, coherent, written sentences. While this is not always popular with students, it is very appealing to curriculum committees in client disciplines.

Departmental/Institutional Support

The goals of the Mathematical Sciences Department and Clemson University foster innovative projects in undergraduate and graduate education. Many faculty in the Department have expressed interest and have been supportive in teaching the project materials in the applied calculus courses. Financially, Clemson has contributed a percentage of the academic year salary and fringe benefits of the principal investigator in addition to an indirect cost reduction. Through an industry grant, the University offers free tutoring to students in all freshman level mathematics courses for two hours each week-night except Friday. Author royalties from all preliminary edition sales are returned to the project for dissemination purposes.

PROJECT REPORT

Getting Started

The project began in the 1992–93 academic year with the development and offering of experimental prototypes of each of the two targeted courses, Applied Calculus I and II. In the summer of 1993, immediately following the prototype courses, the project team modified and redeveloped each of the courses in light of the prototype experiences and preliminary evaluations. They also resolved several significant technology-related issues that surfaced during the prototype courses. The 1993–94 academic year was spent developing and teaching two pilot versions each semester of each of the earlier prototypes of the two courses. The instructional staff for the pilot classes was expanded to include teachers other than project participants in an effort to gather evidence of our approach in a variety of settings with a diverse group of instructors. The summer of 1994 was directed towards refining the pilot courses, preparing organized, written versions of the preliminary materials for both courses, and training Clemson University mathematics instructors.

Due to the nature of the pilot project materials and, at that time, lack of support in the form of an answer key, printed guide for instructors, and supplemental graphing calculator manual, it was obvious from the start that communication of the philosophy of the project and training in using the materials was a necessity for Clemson faculty as well as graduate teaching assistants. This was accomplished by two-day summer workshops and seminars during the fall and spring terms. Weekly "bring your lunch and share ideas" meetings, when faculty other than the project team began teaching using the project materials, were quite successful.

Graphing calculators were loaned to students for the duration of their courses during the first two years of the project because it was considered inappropriate to require them to purchase their own units during periods of experimentation and development. Because of the unusual scope of the project, we were successful in obtaining substantial external support from Sharp Electronics Corporation and Texas Instruments through a loan of 140 Sharp EL-9300 graphics calculators in the fall of 1992 and 150 TI-82 graphics calculators in the fall of 1993. The calculators were loaned to students at no charge under signed loan agreements. Students have been required to purchase their own calculators since the fall of 1994.

Three members of the project team sought an audience in March of 1994 with the 10-member curriculum committee in Clemson's College of Commerce and Industry (since renamed). The presentation was extremely well received; so much so, that the committee requested that we implement our new approach to all their students as soon as possible. It was on the strength of this presentation and its enthusiastic acceptance by Clemson's business faculty that we approached our own colleagues in Mathematical Sciences in April 1994 with the request that Clemson implement the project materials in all class sections of Applied Calculus I and II, starting with the 1994–95 academic year. The request was granted, and the newly developed courses were implemented in all sections of the two courses in the Fall of 1994. Photocopies of the course materials were purchased by students at minimum cost from a local copy shop and replaced a standard textbook in both courses until a published preliminary edition text became available.

From its onset, our project has attracted considerable national attention. We were fortunate to have 13 informal test sites using the project materials during 1994–95, and they provided much valuable feedback on the pilot materials. The spring and summer of 1995 were devoted to final organization of the multivariable chapters for Applied Calculus II, preparing written course supplements, and organizing the preliminary version of a text containing the project materials for publication. Efforts since that time have been involved in preparing the first-edition manuscripts based on preliminary text revisions that were suggested by faculty and students at Clemson and other test sites, writing two additional chapters that were needed by some test sites, studying the transportability of the materials to a variety of settings, and fostering communication with and between preliminary and first-edition users.

Content

The material builds the study of applied calculus through innovative examples and activities that start with real-world data; constructs with technology the mathematical model that best describes the behavior of the data; and then applies mathematical techniques to the model to reach important conclusions about the process that the data describes. We build linear models and through them investigate slopes and rates of change. We build quadratic and cubic models and through them explore tangent lines, derivatives, and optimization. We build logistic models and show their intimate connection with product marketing and constrained growth. We build sinusoidal models and relate the universal nature of seasonal adjustments and business cycles. In all cases the emphasis is on change and the use of calculus to meaningfully explore patterns of behavior. Integration is treated as accumulated change. The area concept is used for support and development, but the emphasis is on dynamics and change. Multivariable analysis is driven by tables of data in a manner paralleling single-variable analysis. Understanding is continually monitored through interpretations of the meaning of results of applications of calculus concepts.

Pedagogy

Contemporary pedagogy is an integral part of the material. It requires student participation in as well as out of class. The students can not be passive learners in the project environment of the activities. *Concept Inventories* at the end of each section of the text provide a list of the key topics under discussion. topics. A *Chapter Summary* relates the major topics under discussion and provides additional insight into reasons for studying those topics. A *Glossary* of important terms is included for student reference. Both short and extended *Projects* at the end of each chapter can be used to further connect the mathematics in the course to real-world applications and emphasize the importance of written and verbal communication as important skills necessary to succeed in today's workplace.

An important aspect of the project materials is the way they explore a concept from multiple representations such as tables of data, models, graphs and scatter plots, and verbal descriptions of the situation and change exhibited. The project team feels that by incorporating these different connections, students become actively involved with the course material. One of the Clemson project's external evaluators states: "Although the *Curriculum and Evaluation Standards for School Mathematics* [122] was written with a focus on grades K-12, its vision of mathematical literacy in today's world justifies the extension of the four overarching standards for mathematics curriculum revision to post-secondary education. These four standards: Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections, provide a clear picture of what every mathematics course should incorporate, regardless of student age or level of mathematics represented in the course. The *Calculus Concepts* courses developed at Clemson University exemplify each of these standards in both their goals and materials."

Technology

One of the remarkable things about the project materials is that they require technology but do not require a particular technology. Because of this, advances in technology will definitely not date the course materials. We require that students have only a reasonably decent tool; a very fancy tool in no way produces an "edge" on the learning or makes our materials obsolete. Technology is used with our materials to produce scatter plots of data, to fit mathematical models, plot and evaluate functions, and perform numerical investigations. Regression models for curve fitting that are not "built into" the technology are handled by supplemental programs. Our materials make use of the fact that today's easy access to technology in the form of graphing calculators and microcomputers breaks down barriers to learning imposed by traditional reliance on algebraic methods. The materials create new opportunities for learning through graphical and numerical representations of real-world information and allow time for verbal and written interpretation of results instead of spending most of that time on burdensome calculations. We exploit all these opportunities by assuming continual and immediate access to technology.

Day to Day Mechanics

Instructional methods with the materials are quite varied. Some instructors preview the material and have the students work in groups the majority of the time. Other instructors effectively involve students in class discussion while presenting material the entire period. Instructors generally use calculator view screens more near the beginning of the course while students are still learning technology basics. As with any new approach to subject material, communication with other faculty teaching the course is often very beneficial. Because of the large number of graduate teaching assistant instructors at Clemson, a course supervisor holds weekly meetings discussing common concerns, new technology techniques, and implementation of the materials. Copies of data collected in student projects as well as a packet of prior tests given in the courses is made available to Clemson instructors for use in designing the content and length of their tests. A printed and computerized text bank is available for instructors using the first edition of the text.

During the class periods our policy is to teach mathematics, not the calculator. Therefore, students often require out-of-class assistance with the technology as well as the course materials. Beginning with the Fall 1994 term, "jump-start" sessions offered at night during the first week of the semester have been effective in helping students begin using technology. We have also found it beneficial to give students a calculator skills quiz during the second week of class to urge them to learn calculator basics. Since the Fall 1994 term, Clemson has provided for student support (approximately four hours a week) an experienced graduate student available for tutoring help with all models of calculators covered in the *Graphing Calculator Instruction Guide* that accompanies the text. In the fall semester of 1996, a Help Center was established by Clemson's Mathematical Sciences Department for students in the two applied calculus courses. The Center is staffed for three hours each week-day afternoon, except Friday, by some of the course instructors who each donate one office hour per week. Beginning with the Fall 1998 semester, additional help with all technologies will be available at a web site and through calculator-specific instructional videotapes.

Appraisal, Comparisons, and Conclusions

The project has been immensely successful in achieving large-scale implementation of its philosophy and products on the Clemson campus. It has evolved from six prototype courses the first year (1992–93) to full implementation in all 61 sections at Clemson in 1998–99 (over 2,100 student enrollments). Unlike students in more technical fields who more clearly understand the need to develop genuine understanding of mathematics concepts, the typical applied calculus student views courses in mathematics as requirements to get out of the way as quickly and as painlessly as possible. We initially knew of this problem and feel that one of our biggest successes is that we now genuinely interest these students by involving real data. We have seen some remarkable changes: non-technical students seem to now be experiencing mathematics in a different way than they have before, with a different emphasis that genuinely interests and involves them. Although we haven't seen a greatly improved withdrawal rate, we have seen a lower failure rate and, more importantly, much improved student attitudes.

In addition to students coming from the first applied calculus course using the project materials, the second course enrollment at Clemson has students that have changed majors to a field requiring a second applied rather than a mainstream calculus course, students who have not taken the first course because of having Advanced Placement credit, and transfer students who have taken the first course at another school using a traditional approach to materials. Studies of these students over the past three years indicate that those students who are dedicated to learning are able to catch up on modeling techniques, calculator use, and terminology from the project's first course. However, we have also learned there are enough students who are not motivated or willing to do this to warrant offering two traditional sections of the second course. Not opening these sections until the transfer students register has solved some of the scheduling problems that result from attempting to satisfy the needs of all students in the second course.

Another problem area arose that we did not anticipate. Students in these courses have traditionally relied on help outside the classroom and the instructor's office. They hire mathematics graduate students as tutors; athletes in the courses rely on tutoring provided by the athletic department; and many students ask for assistance from friends who have taken a traditional version of the course. It soon became obvious that "traditional" tutoring by persons not having knowledge of the calculator or the new approach was only confusing students. We therefore found it necessary to inform the outside tutors of the changes we were making and offer them training.

Because this project represents a totally new paradigm for teaching and learning, it is extremely difficult to conduct comparative studies wherein student performances are measured against traditional methods. In an external evaluation prepared by Dr. James W. Wilson and Mary E. Searcy, Mathematics Education Department, University of Georgia, the following statements are made:

> "A student questionnaire was developed to provide project staff feedback from students at every step of the development process. . .
>
> Data from 1074 Clemson students in 48 sections taught by 20 different instructors. . . show that the students are responding positively to the aims of the course—reducing emphasis on algebraic manipulation, incorporating modeling, using multiple viewpoints, viewing the derivative as rate of change, and using the graphics calculator. About 85% of the students feel they are learning as much or more than students in traditional classes. . .
>
> The profiles of response [for the same data pooled over approximately 200 students in six different college during Spring 1995] are essentially the same as what has developed over the three years at Clemson."
>
> "The project staff recognize that developing procedural skills is a part of developing knowledge. Therefore, a small set of test items appropriate to both traditional classes and project classes were developed for instructors to use. Performance data on the items for the [Applied] Calculus I were obtained from 273 Clemson students and from 247 students at six other colleges during Spring term [1995]. Our judgment is that the performance as a whole is acceptable, and further, the items provide useful formative data for the project staff. A second issue is whether students at other colleges who have studied this material do as well as Clemson students. Clearly, if the performance is satisfactory for Clemson students (and we believe it is) then it is also satisfactory for students at other colleges—since the profiles of the [performance] graphs are basically indistinguishable."

Authentic assessment is a central feature of the material. Project reports from student groups working as consulting firms seeking competitive contracts forces the students to be measured in the way that they use mathematics in oral and written presentations. The external evaluation also states:

> "There are many elements to student performance. In this environment, student projects were a new direction for student performance. ... It is evident that the instructors are learning a great deal about what their students know and don't know from these projects. Although it will be a challenge to construct more project ideas that will provide students with a chance to be creative in exhibiting their knowledge, it seems to be well worth the effort."

During project development, it soon became apparent that we were causing faculty at Clemson and our test sites for the project materials as well as students who had previously been exposed to calculus topics to redefine their concept of "calculus." While we initially thought there might be some resistance to extending our use of technology to these courses, this has not been a cause of concern for most faculty or students. However, group work and projects, less lecturing, writing explanations in English rather than using only mathematical symbols, labeling all numerical answers with units of measure, interpreting answers, modeling, and the lack of extensive algebraic manipulation are new concepts that must be examined and determined to have validity in the courses by those with traditional mathematics backgrounds who teach using the project materials.

Technology is such a natural and integral part of the course that it is difficult for any instructor or student to avoid its use. The project philosophy places more of the responsibility of learning, including effective use of technology, on the student. Observations at Clemson and test sites for the project materials show that students who use the same type of technology as the course instructor are more "comfortable" in the course, but the dedicated student will succeed with occasional outside assistance using any model graphing calculator provided that calculator has the built-in functionality and/or programs necessary for course investigations.

While the student projects are very beneficial in showing applications of the course materials and promoting cooperative learning, many instructors find it difficult to use them the first time they teach the course. Sharing such concerns and interacting with other users and the project directors is an important part of the dissemination of this project. Instructors need to realize that any "reform" course can still be effective without doing everything available the first time through the materials.

Transportability of the project materials in terms of student performance, student opinions, institutional impact, and cost effectiveness is being studied in detail at 10 Beta test sites (1995–96), 10 Adopter sites (1996–97), and 10 additional Adopter sites (1997–98) through the current FIPSE dissemination grant, *Disseminating a New Approach to Calculus for Non-Technical Students.* External evaluation of the test sites gives strong evidence of transportability to other campus environments. From the 13 informal test sites the first year, from high schools to prestigious research universities, there have been more than 75 colleges and universities (as well as some high schools) either piloting sections or going with across-the-board implementation of the project materials.

Comments from Clemson students and faculty throughout two years of using the preliminary edition of the materials in numerous sections of both Applied Calculus I and II as well as discourse with faculty and students at many test sites were used to revise and reshape the preliminary editions of the text containing the project materials into the first editions of the text. Because discussions with test sites showed that many of them cover through integration in their first course, *Calculus Concepts: An Informal Approach to the Mathematics of Change, Brief First Edition* containing differential and integral applied calculus was published in the fall of 1997. We also found that the majority of those schools that offer a second course need more material than was available in the preliminary editions of the text. Thus, two new chapters dealing with cyclical models and trigonometry and differential equations were written. That material, along with the two revised multivariable chapters from the preliminary edition, were added to the brief first edition chapters, and the two-course text, *Calculus Concepts: An Informal Approach to the Mathematics of Change*, was published near the end of 1997. Our discussions with faculty and students at test sites also showed the need for an instructors' test bank and a student solutions manual. These ancillaries as well as both texts and other supplementary materials are published by the Houghton Mifflin Company. Houghton Mifflin's dedication to the success of this project resulted in the April 1998 establishment of a web site for *Calculus Concepts* that can be accessed by clicking on Mathematics at http://www.hmco.com/college.

Future Plans

The project team hopes to continue dissemination activities, such as talks and workshops, at professional meetings as well as to have continued contact with test sites and other users of the project materials. As new technology enters the marketplace, plans call for *Calculus Concepts* supplemental graphing calculator guides adapted to those technologies. Because many business schools are currently using or have plans to require their students to have laptop computers, a manual giving guidelines for using the course materials with spreadsheets on computers is under development. Assessment and study of the transportability of the project materials will continue through a one-year no-cost extension of the FIPSE dissemination grant.

The FIPSE grant and funds awarded from the Summer 1998 Clemson University Advanced Technology Center Instructional Development Awards Program is providing support and a computer for the development of a Clemson web site to assist learning and instruction for *Calculus Concepts.* In addition to having information specifically for students in the Clemson courses, this site will include additional technology assistance, answers to frequently asked questions, practice tests with solutions adapted to each technology supported by the text, interactive quizzes for students to test their knowledge of basic concepts, additional data sets and associated questions, comments on and suggestions for effective group learning, and links to interesting web sites.

Future plans also call for the making of instructional videotapes for each technology used with the text. Students needing additional help with technologies not used by their instructors could go to the Help Center and watch the appropriate video tape.

SAMPLE MATERIALS

Text Used: LaTorre, D., J. Kenelly, I. Fetta, C. Harris, and L. Carpenter. *Calculus Concepts; An Informal Approach to the Mathematics of Change*, Houghton Mifflin Co., 1998.

Bulletin Description

The project materials, *Calculus Concepts: An Informal Approach to the Mathematics of Change*, feature a new classroom dynamics with less lecturing, more student activity, interpretation and mathematical decision making, and group work on team projects. The materials incorporate concepts and topics that are relevant and useful in non-scientific careers, provide substantial experiences with mathematical modeling using real-life data, focus on concepts and relationships, and require the use of technology so that the course can be conducted from graphical and numerical as well as analytical perspectives. The following specifically capture the essence of what is making the project materials exciting for students and teachers alike:

- Rates of change and their interpretation in non-technical setting:
  The overarching theme of our material is rates of change and their interpretation in realistic applications: the derivative as a rate of change and the integral as the accumulation of change."
- Mathematical modeling as a vehicle for learning:
  The project staff has tried to ensure that the subtleties of modeling are not the issue; rather, elementary models are used to obtain functional relationships between variables. We discuss linear models (constant rates of change), exponential models (constant percentage change), quadratic models (constant force for change), cubic models (smooth, transitional change), and logistic models (exponential change with limiting conditions). Using these models presents opportunities to analyze real data that adds validity to the topics under study as well as opportunities to put to rest the classic question, "Where is this ever going to be used?" Interpretation and discussion of results from calculus investigations on the functions given by the models becomes meaningful in the context of real-life situations.
- Interplay between the discrete (the real-world situation) and the continuous (the mathematical model):
  Many everyday, real-life situations involving change are discrete processes. Such situations can often be represented by continuous or certain piecewise continuous mathematical models so that the concepts, methods, and techniques of calculus can be brought to bear.
- A new, expanded role for algebra:
  When everyday, real-life situations that are discrete in nature are represented by continuous or piecewise continuous mathematical models, the role of algebra is to describe and to enable us to reason with the quantities that are undergoing change. This is in contrast to the traditional role of algebra as a collection of organized manipulations and procedures that are applied to obtain a numerical answer to a well-formulated problem.
- Mathematical interpretation and decision making:
  Of equal importance to understanding concepts of calculus in the context of change is the ability to correctly interpret the mathematics in real-life situations. The ability to "make sense" of mathematics is vital to believing in its value and appreciating its usefulness in our lives.

Course Syllabi

The syllabi on the next two pages are based on Clemson University courses and 42, 50-minute class meetings with four tests and two projects the first semester, three tests and one project the second semester, and a comprehensive final exam in each course. In addition to a reading assignment of new material, corresponding sections in the *Graphing Calculator Instruction Guide* that accompanies the text are assigned on the syllabi so students can better prepare to use their calculators in class.

Applied Calculus I (through single variable differentiation):

Day(s)	**Text Material**	
1	1.1	Fundamentals of Modeling
2-3	1.2	Functions and Graphs
4	1.3	Constructed Functions

5-6	1.4	Linear Functions and Models
7	Test 1	
8-9	2.1	Exponential Functions and Models (Assign Project 1)
10	2.2	Exponential Models in Finance
11	2.3	Polynomial Functions and Models
12	2.4	Choosing a Model
13	Test 2	
14	3.1	Average Rates of Change
15	3.2	Instantaneous Rates of Change
16	3.3	Tangent Lines (Project 1 due)
17-18	3.4	Derivatives
19	3.5	Percentage Change and Percentage Rates of Change
20	Test 3	
21-22	4.1	Numerically Finding Slopes (Assign Project 2)
23-24	4.2	Drawing Slope Graphs
25-26	4.3	Slope Formulas
27	4.4	The Sum Rule
28-29	4.5	The Chain Rule
30-31	4.6	The Product Rule
32	Test 4	
33-34	5.1	Optimization
35-36	5.2	Inflection Points
37-38	5.3	Approximating Change
39-40	Project 2 presentations	
41-42	Review	
Final Exam (cumulative)		

Applied Calculus II (single variable integration through multivariable optimization):

Day(s)	Text Material	
1-2		Review of Semester I
3-4	6.1	Results of Change
5	6.2	Trapezoid and Midpoint Rectangle Approximations
6-7	6.3	The Definite Integral as a Limit of Sums (Assign Project)
8-9	6.4	Accumulation Functions
10-11	6.5	The Fundamental Theorem
12-13	6.6	The Definite Integral
14	Test 1	
15-16	7.1	Differences of Accumulated Changes
17	7.2	Perpetual Change
18-19	7.3	Streams in Business and Biology
20-21	7.4	Integrals in Economics
22	7.5	Average Values and Average Rates of Change
23-24	7.6	Probability Distributions and Density Functions
25	Test 2	
26-27	9.1	Cross-sectional Models and Multivariable Functions
28-29	9.2	Contour Graphs
30-31	9.3	Partial Rates of Change
32-33	9.4	Compensating for Change
34-35	10.1	Multivariable Critical Points
36-37	10.2	Multivariable Optimization (Project due)
38	Test 3	
39-40	10.3	Optimization under Constraints
41-42	10.4	Least Squares Optimization
	Final Exam (cumulative)	

Many schools using the project materials cover through Chapter 7 in one course, while some schools go through Chapter 6. Some schools offering two courses include Chapter 8 (cyclical models) and 11 (differential equations). Alternative syllabi for these coverages are available.

Projects

The use of student projects is an important feature of our work that we did not foresee at the onset but began to develop at the end of the first year. It became clear that it is precisely the use of such projects that helps us meet our objectives. At Clemson, we divide each of our first semester classes into teams of three or four, and each team works on a short project consisting of collection of real data of interest to them and also an extensive project that is open-ended in its outcomes (i.e., there is no single, correct "answer" in the traditional sense). On the extensive project, the students do research, collect data, examine various mathematical models of the data, choose a model, determine the model's parameters, and then use the model to analyze the real-life situation at hand. They must make sense of the mathematics and interpret their results in non-technical jargon. We expect an extensive written report that, typically, will include narrative, data, analysis, graphs, models, mathematical results, interpretations, and conclusions. We also require that each team make a 10-to-15 minute oral presentation to a mathematics instructor(s) who is not their course instructor. The oral presentation is expected to be "professional," with proper dress, transparencies, and other visual aids. Some of the student projects included in *Calculus Concepts* are:

- Compulsory School Laws (a data-based analysis of our country's enactment of compulsory school attendance)
- Legislative Turnover (exponential modeling of political legislative turnover in the US. Senate and the former Soviet Union Party Congress)
- Fee-Refund (a critical examination of fee-refund schedules for early withdrawal from college and university courses)
- Super-Highway (design of a model to support safety on a high-speed European highway from Berlin to Lisbon)
- Doubling Time (a modeling-based comparison of financial doubling times for money under various schemes)
- Fund Raising Campaign (a marketing project that incorporates all of the major concepts in differential calculus)
- Estimating Growth (a modeling-based project involving integral calculus)
- Expert Witness (analysis of a multivariable function used to determine skid-mark lengths for testimony at a court trial)

More projects, both short and extended, are given in the text and at the Houghton Mifflin web site. One of the short projects is given here.

Project 5.2
FINDING DATA

Introduction

As mentioned in Project 1.3, newspapers, journals, and government documents are wonderful sources of data. Other good sources include almanacs, statistical abstracts, and world-wide-web sites.

Tasks

1. Find a set of data with more than 12 data points that can be modeled with a non-linear function or a piecewise continuous function that has at least one non-linear part. (If using a piecewise model, make sure that you need no more than three pieces.)
2. Fit a model or piecewise model to the data.
3. Find the derivative formula for your model. If you used a piecewise model, determine the input values at which the derivative does not exist. If the rate of change has a sensible interpretation at these points, discuss alternative methods for estimating the rate of change.
4. Interpret the meaning of the derivative in the context of your data. Illustrate your discussion by evaluating the derivative at certain input values.
5. Use your model to predict a quantiiy one input unit beyond the end of your data range.
6. Use a derivative to approximate the change in quantity one input unit beyond the end of your data range.
7. Compare your answer from Task 6 to that from Task 5. What are the assumptions made for each prediction? Which seems more realistic?

Reporting

1. Write a report in which you describe the meaning of the data, discuss the model you used to fit the data, and answer the questions posed above. In your report you should explicitly state the model, define the variables in the model, and state what input values are valid for this model. You should also include a scatter plot of the data and graph of your model. Properly cite the source of your data using correct bibliographic form. Attach a photocopy of the data on which the title of the article or document appears.
2. (Optional) Prepare a 3-to-5 minute oral presentation of your project. You should incorporate the use of visual aids to enhance your presentation.

Instructions to Students

These materials are written to help you understand how things change and help you build systematic ways to use this understanding in everyday real-life situations that involve change. Calculus is the mathematics of change. The materials are based on three premises: 1) understanding is more important than the mastery of mathematical manipulations; 2) mathematics is present in all sorts of real-life situations—it is not just an abstract subject in textbooks; 3) the graphics technology in today's calculators and computers is a powerful tool that can help you understand important mathematical connections. Like many tools in many fields, technology frees you from tedious, unproductive work, enables you to engage situations more realistically, and lets you focus on what you really do best—think and reason. Use paper, pencil, and your graphing calculator (or computer) when you study. These are your basic tools, and you cannot study effectively without them. Remember that there is no substitute for effective study. You have your most valuable resource with you at all times—your mind. Use it.

Instructions to Teaching Staff

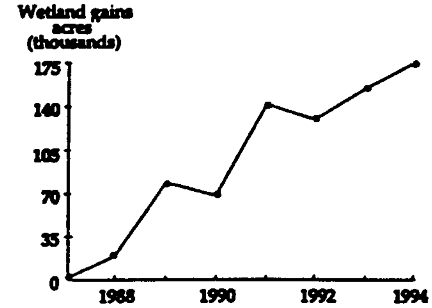
Teaching a course with a new textbook is always challenging. Teaching a course with a new reform textbook and technology is many times more challenging. It is a task that takes an enormous amount of time, energy, and creativity. Teaching *Calculus Concepts* effectively requires a complete change of mindset from the way calculus has been traditionally taught. You will have to change your focus—from algebraic manipulation to conceptual understanding, your emphasis—from the one correct numerical answer to the possibility of several valid answers, and even your grading—from giving credit for the numerical answer to refusing to accept a correct numerical answer that is not expressed in a coherent manner appropriate to the context of the problem. Such fundamental changes do not come easily. In the *Instructor's Guide* that accompanies *Calculus Concepts*, you will find practical suggestions to help you change your teaching, use technology appropriately, and successfully implement projects in your class. The reward of seeing your students enthusiastic about calculus will generously compensate for the investment of your time and energy.

Sample Test Questions

All tests in these courses at Clemson are taken in class with the only aid being the student's calculator. Rather than include one specific test, we give chapter review test questions chosen from Chapters 1-8 of *Calculus Concepts.*

Instructions: Show all work. Label all numeric answers. Give clear explanations of the variables used in all models.

1. In *The True State of the Planet* (Ronald Bailey, Ed. The Free Press, 1995), the graph below showing the yearly gain of wetlands in the United States between 1987 and 1994 is given.
   a. Between 1991 and 1992 was the number of acres of wetlands in the United States increasing or decreasing? Explain.
   b. Estimate the slope of the portion of the graph between 1989 and 1991. Interpret your answer.



2. In *The True State of the Planet,* the statement is made that "the amount of arable and permanent cropland worldwide has been increasing at a slow but relatively steady rate over the past two decades." Selected data between 1970 and 1990 reported in *The True State of the Planet* is shown in the table.

Year	Cropland (millions of square kilometers)
1970	13.77
1975	13.94
1980	14.17
1985	14.31
1990	14.44

a. Find a linear model for the data.
b. Is the model in part *a* discrete, continuous, or continuous with discrete interpretation?
c. What is the rate of change of your model in part *a?* Write a sentence interpreting the rate of change.
d. Do the data and your model support the statement quoted above? Explain.
e. According to your model, what was the amount of arable and permanent cropland in 1995?

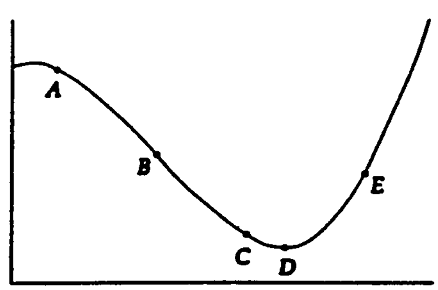
3. The number of subscribers to cellular phone service was 91,600 at the end of 1984 and 30 million at the end of 1995[1]. Assume the number of subscribers grew exponentially between 1984 and 1995.
   a. Find an exponential model for the number of cellular phone subscribers as a function of the number of years since 1984.
   b. What is the percentage change indicated by the model?
   c. According to the model, when will the number of subscribers reach 270 million (the approximate population of the United States)?
   d. Do you believe an exponential model accurately reflects the future growth of cellular phone subscribers? Explain. If not, what do you think would be a more appropriate model?

4. The number of in-hospital midwife-attended births for selected years between 1975 and 1993 is shown in the table[2].

Year	Births (thousands)
1975	19.7
1981	55.5
1987	98.4
1989	122.9
1990	139.2
1993	196.2

   a. Find an exponential model for the data.
   b. Numerically investigate the rate of change of the number of births in 1990. choose at least three increasingly close points. Record the close points, the slopes with four decimal places, and the limiting value with two decimal places.
   c. Interpret the limiting value in part *b.*
   d. Give the formula for the derivative of you model in part *a.* Evaluate the derivative in 1990.

5. Answer the following questions about the graph shown below.

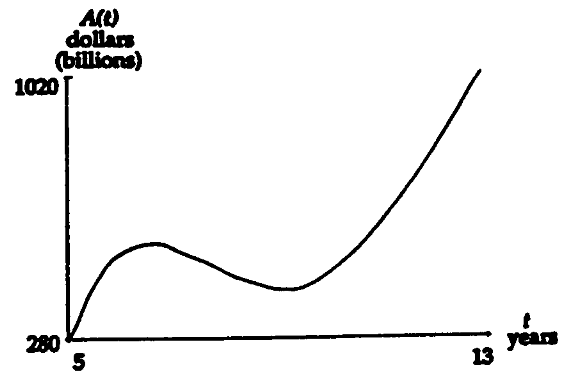


[1]As reported in the *Reno Gazette-Journal,* September 15, 1996, page 1E.
[2]As reported in the *Reno Gazette-Journal,* August 4, 1996, page 1A.

a. List the labeled points at which the slope appears to be
   i) negative   ii) positive   iii) zero
b. If B is the inflection point, what is the relationship between the steepness at B and the steepness at the points A, C, and D on the graph?
c. For each of the labeled points, will a tangent line at the point lie above or below the graph?
d. Sketch tangent lines at points A, B, and E.
e. Suppose the graph represents the speed of a roller coaster (in feet per second)as a function of the number of seconds after the roller coaster reaches the bottom of the first hill:
   i) What are the units on the slopes of tangent lines? What common word is used to describe the quantity measured by the slope in this context?
   ii) When, according to the graph, was the roller coaster slowing down?
   iii) When, according to the graph, was the roller coaster speeding up?
   iv) When was the roller coaster's speed the slowest?
   v) When was the roller coaster slowing down the most rapidly?

6. The total amount of long-term, new mortgages (for 1–4 unit family homes) each year from 1985 through 1993 is shown in the table[3]. Also shown is a model for the data and a graph of the model.

Year	Amount (in billions of dollars)
1985	289.8
1986	499.4
1987	507.2
1988	446.3
1989	452.9
1990	458.4
1991	526.1
1992	893.7
1993	1019.9



$A(t) = -1.74566t^4 + 68.8258t^3 - 966.380t^2 + 5759.1455t - 11863.1488$ billion dollars t years after 1980

a. Use only the data to estimate the rate of change in the yearly new mortgage amounts in 1992.
b. Use the graph to estimate how quickly the new mortgage amount is changing in 1992.
c. Find the derivative of the model in 1992. Interpret your answer.

7. The number of tourists who visited Tahiti each year between 1988 and 1994 can be modeled[4] by $T(x) = -0.4804x^4 + 6.635x^3 - 26.126x^2 + 26.981x + 134.848$ thousand tourists x years after 1988.
   a. Give the rate of change formula for $T(x)$.
   b. Calculate the values T(3) and T(5) and interpret your answers.
   c. Find any relative maxima and minima of $T(x)$ between $x = 0$ and $x = 6$. Explain how you found the point(s).
   d. Find any inflection points of $T(x)$ between $x = 0$ and $x = 6$. Explain how you found the point(s).
   e. Graph $T(x)$, $T'(x)$, and $T''(x)$. Clearly label on each graph the points corresponding to your answers to parts c and d.
   Although $T(x)$ is a continuous function, it must be interpreted discretely since it represents yearly totals. With this in mind, answer the following questions:
   f. Between 1988 and 1994, when was the number of tourists the greatest and when was it the least? What were the corresponding numbers of tourists in those years?
   g. Between 1988 and 1994, when was the number of tourists increasing the most rapidly, and when was it declining most rapidly? Give the rates of change in each of those years.

8. The rate of change of the daily sales[5] of powdered drink mix produced in 1992 by the Campbell Soup Company can be modeled by $r(t) = -0.1439\sin(0.0197t - 3.7526)$ million (reconstituted) pints per day t days after January 1, 1992.

---

[3]Cocheo, Steve. "Give me your delinquents, your former bankrupts, yearning to borrow . . ." *ABA Banking Journal,* Volume 88 (August 1996), pp. 31–63.
[4]Page, Stephen J. "The Pacific Islands," *EIU International Reports*, Volume 1 (1996), page 91.
[5]Based on information in the *Wall Street Journal*, July 7, 1993, page B1.

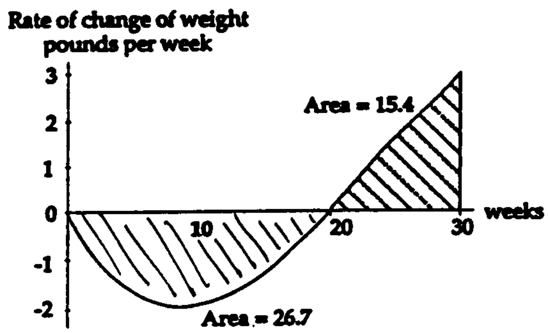
a. Determine when the rate of change of the given sales model was greatest in 1992.
b. Find a model for the sales of powdered drink mix if sales were 14.4 million pints on January 1, 1992.
c. How is the answer to part a related to a graph of the model in part b?

9. A hurricane is 300 miles off the east coast of Florida at 1 a.m. The speed at which the hurricane is moving toward Florida is measured each hour. Speeds between 1 a.m. and 5 a.m. are recorded in the table:

Time	Speed (mph)
1 a.m.	15
2 a.m.	25
3 a.m.	35
4 a.m.	38
5 a.m.	40

a. Find a model for the data.
b. Use a limiting value of sums of areas of midpoint rectangles to estimate how far (to the nearest tenth of a mile) the hurricane traveled between 1 am and 5 am. Begin with five rectangles, doubling the number each time until you are confident you know the limiting value.

10. The graph shown depicts the rate of change in the weight of someone who diets for 20 weeks.



a. What does the area of the shaded region beneath the horizontal axis represent?
b. What does the area of the shaded region above the horizontal axis represent?
c. Is this person's weight at 30 weeks more or less than it was at 0 weeks? How much more or less?
d. If $w(t)$ is the function shown above, sketch a graph of $\int_0^x w(t)\,dt$. Label units on both axes as well as values on the vertical axis.
e. What does the graph in part d represent?

The materials in this article were submitted by Donald La Torre, John Kenelly, Iris Fetta, Cynthia Harris, and Laurel Carpenter. Portions of this article are reprinted with permission from Houghton Mifflin Company.

Successful Use of Computer Algebra Systems in Developmental Mathematics

Nomiki Shaw and Ignacio Alarcón

Service Course Area

Developmental Mathematics

Institutional Data

The California State University Bakersfield, founded in 1970, is one of 22 campuses that form the California State University, the successor of the California State Colleges system. The current enrollment is approximately 5,500. The University serves a socially and ethnically diverse population of 560,000 in an area of 10,000 square miles. The city of Bakersfield is located in the southern San Joaquin Valley, with a population of 213,000. The Department of Mathematics is one of 17 departments in the School of Arts and Sciences. The department consists of 20 full-time faculty. The degrees offered are Bachelor of Science in three tracks: Applied, Teaching, and Theoretical.

Abstract

The Developmental Mathematics Program at the California State University Bakersfield is now entering its fifth year after a profound transformation of the old remedial program. The traditional remedial program used to have very unsatisfactory results, both in the rates of completion of the program and in the subsequent progress of students who finished the program.

The ready access to a Computer Algebra System like DERIVE in the early 1990s made the implementation of the most recent NCTM standards and other initiatives, like the AMATYC Standards for Introductory College Mathematics Before Calculus a real possibility. Our developmental program has a natural adaptation to the second generation graphing calculators with symbolic algebra manipulation capabilities.

The main goal of the Developmental Mathematics Program at the California State University is to usher first time freshmen and reentry students who are in need of developmental mathematics courses, according to the Elementary Level Mathematics placement test, into college level mathematics courses, particularly the General Education Breadth requirement.

The success of the program includes the broadening of students' aspirations and expectations to engage in further mathematical activity. In recent years, the influx of students completing the de-

velopmental program into college level mathematics courses other than the General Education Breadth requirement has been steadily increasing, which is most encouraging for our institution.

THE PROJECT

Prior to 1991, the success rate internal to the Remedial Program was about 50 percent. Additionally, the proportion of students who completed the Remedial Program and went on to satisfy the General Education Breadth requirement, Introductory Statistics, was discouragingly low. A change was sorely needed.

The California State University (CSU) system uses the Elementary Level Mathematics (ELM) test as the instrument for placement in mathematics courses, before enrollment in a course that satisfies the college level mathematics General Education-Breadth requirement. Exemption from the ELM test is given to students with high SAT or ACT scores or who have credit for a baccalaureate-level mathematics course at another institution.

In the Fall of 1991, Niki Shaw, coordinator of the Developmental Program, introduced a pilot course on Intermediate Algebra, the exit course of the program, which heavily incorporated the use of the Computer Algebra System DERIVE. This pilot course was offered also in the Winter of 1992.

Most of the CSUB students satisfy the college level mathematics General Education Breadth requirement by taking the Introductory Statistics course. A statistical analysis was conducted to evaluate performance in this Introductory Statistics course. Three groups were compared: the Intermediate Algebra pilot course graduates from Fall 1991 and Winter 1992; the traditionally taught Intermediate Algebra graduates; and the students who passed or were exempt from taking the ELM test and enrolled in the Introductory Statistics course without any need for remediation. A very brief summary of this analysis is that the Intermediate Algebra pilot course graduates from Fall 1991 and Winter 1992 did not differ significantly from the group of students who passed or were exempt from taking the ELM test, whereas the traditionally taught Intermediate Algebra students performed at a much lower level.

These extraordinarily positive results encouraged the Department of Mathematics at CSUB, with the support of the Dean of the School of Arts and Sciences, to completely transform the remedial program into a program that heavily involves the use of a Computer Algebra system. As of Fall 1994, all three courses in the Developmental Program use this approach.

The Developmental program at CSU Bakersfield consists of three one-quarter courses: Math 70 (Introduction to Algebra), Math 80 (Elementary Algebra and Geometry), and the exit developmental course Math 90 (Intermediate Algebra). Each of these carry five non-baccalaureate quarter units. Math 70 offerings are: three sections during Fall Quarter and one section during Winter and Spring, respectively. Math 80 offerings are: three sections during the Fall Quarter and four sections during each of Winter and Spring Quarters. Math 90 offerings are four sections for each quarter. Each section is typically full at the maximum enrollment allowed of 30 students per section. Our selection of textbooks has changed as newer, more technology adaptive and function-based books have appeared. We have used, for example: *"Modeling, Functions and Graphs: Algebra for College Students"*, Franklin and Drooyan, Wadsworth Publishing Co., 1991., *"Intermediate Algebra: A Graphing Approach"*, Demana, Waits, Clemens and Greene, Addison-Wesley Publishing Co., 1994. Our current selections are: *"Concepts in Algebra: A Technological Approach"*, The University of Maryland and The Pennsylvania State University, Janson Publications, 1995, and *"Intermediate Algebra: A Functional Approach"*, Brant and Zeidman, Harper Collins College Publishers, 1996.

Following is the official catalog description of our Developmental Mathematics Courses.

Catalog Description

Mathematics 70. Introduction to Algebra and Geometry. Relations and rules relating variables, building models of Mathematical relations: tables, graphs. Special attention is paid to functions, with emphasis on linear and quadratic functions. Solving equations and inequalities of first and second degree. Course makes intensive use of graphing utilities and/or Computer Algebra systems. Course does not count toward graduation. Prerequisite: A score of 280-370 on the ELM.

Mathematics 80. Elementary Algebra and Geometry. Algebraic representations of problem situations. Functions and graphs. Special emphasis on linear, quadratic, and higher order polynomials. Solving equations and inequalities involving such functions algebraically and graphically. Systems of linear equations with two variables. Integration of basic geometric concepts. Course makes intensive use of graphing utilities and/or Computer Algebra systems. Course does not count toward graduation. Prerequisite: (1) Satisfactory completion of Math 70 OR (2) score of 380-470 on the ELM.

Mathematics 90. Intermediate Algebra and Introduction to Analytic Geometry. Graphing techniques; solving equations and inequalities involving polynomials, rational, exponential and logarithmic functions, graphically and algebraically. Conic sections. Solving systems of non-linear equations. Arithmetic and geometric sequences and series. Course makes intensive use of graphing utilities and/or Computer Algebra systems. Course does not count toward graduation. Prerequisite: (1) Satisfactory completion of Math 80 OR (2) score of 480 or higher on the ELM.

Students successfully completing the Intermediate Algebra course enroll in any one of the following courses:

Math 120 Business Calculus
Math 140 Introductory Statistics
(College level mathematics, General Education-Breadth requirement
Math 191 Pre-Calculus I
(College Algebra-Elementary functions)
Math 220 Problem Solving
(Preparatory course for Elementary Education prospective teachers)

The traditional efforts at remediation in mathematics were a futile and costly endeavor for both the institution and the students. The students were, once again, subject to a model that had demonstrably failed in their recent or distant past. We were totally committed to implementing an approach that integrates the numerical, algebraic, and graphic contexts as called for in any of the current initiatives that have arisen from the NCTM and AMATYC standards.

The main goal of the developmental program is to "mainstream" developmental students into college level mathematics. We aim to provide students with the background and experiences that will allow them to undertake college level courses with prognosis for success comparable to that of students who are exempt from developmental intervention.

Our program is in alignment with the main goal of the "foundation" mathematics courses as described in the AMATYC Standards for Introductory College Mathematics particularly since we "provide multiple entry points to meet the needs of students who enter college mathematics at different levels of mathematical sophistication" and we "efficiently but thoroughly, prepare students for additional experiences in mathematics."

The themes of connection of classroom learning and real world application are much older than the movements for reform that we are living. The advent and rapid development of accessible technology makes them a good idea whose time has come. It is now possible to explore concepts in nontrivial contexts since mechanical computations are no longer an obstacle and added computational capabilities make the exploration and analysis of real data possible at the developmental stages of mathematical sophistication.

Derive has been the computational platform selected for assisting the transformation in the Developmental Mathematics Program. The main criteria for its selection are its ease of use along with its relatively inexpensive licensing/funding conditions. The hardware requirements for the operation of this Computer Algebra System are not overly taxing. Stand-alone 286 PCs were used for an extended period. We currently have a dedicated Developmental Mathematics computer lab equipped with the University's discarded 386 PCs. Permanent license pricing of Derive is currently at $89 per unit. The software is also available to students at a discounted price.

Our experience with this technology has been very fruitful. It allows the integration of numerical, symbolic, and functional concepts. It also fosters productive group dynamics, making the implementation of cooperative learning techniques accessible and natural.

The courses in our developmental program include sizable writing assignments. The response of the majority of our students shows determination to excel, with the added outcome of accelerating the student's mastery of word processors. We are extremely pleased to observe how our efforts in Developmental Mathematics have become natural allies of the "Writing Across the Curriculum" movement alive in the California State University campuses.

PROJECT REPORT

The format of the courses in the Developmental Mathematics Program is, typically, three meeting of 75 minutes ("the lecture") and one 150-minute meeting ("the lab") per week for a 10-week quarter. During the "lecture" meetings, the instructor facilitates exploration and discussions of new topics as well as reviewing previous assignments. The "lab" meeting is directed by an assignment to be worked in groups of three to five students. The lab material is organized, and each group is required to write up a formal report to be collected within three to five days and is given a group grade. This lab meeting is also an ideal locale for the implementation of cooperative learning techniques.

During the first two years the Developmental Mathematics classes fully integrated the use of technology during only the lab meetings due to limited availability of the computer lab. The "lecture" meetings were held in a traditional classroom where the instructor used a demonstration unit. Since then, all meetings are held in a computer lab equipped with individual stations.

The instructors in the developmental program are full-time lecturers, part-time faculty, and tenured/tenured track faculty who are committed to the program. Training is required of all instructors who teach in the program. Typically, a short period of direct hands-on training is enough for a new instructor to become functional at a basic level of mastery of the computer algebra system Derive used.

We have broadened the assessment of the effectiveness of the program to include all college level mathematics courses that draw enrollment of graduates from our exit developmental course, the Intermediate Algebra course. We have assessed the performance of students in entry college level mathematics courses by breaking down quarter by quarter the course grades received by the whole student body and comparing them to those attained by the students who completed the Intermediate Algebra course.

The success rate of passing a developmental mathematics course has increased from 50 percent pre- technology to 80 percent with technology. The performance of students in our entry-college level mathematics courses has been assessed to include all courses that feed enrollment from our exit developmental course, Intermediate Algebra. We looked at the breakdown, by quarter, of the course grades received by the whole population enrolled in each course as compared to those attained by students who went through the exit course in the Developmental Mathematics Program, Intermediate Algebra. The table below summarizes our findings.

The markedly improved performance of the developmental mathematics students has minimized the number of students who repeat a course within the sequence. The pass rate internal to the sequence has gone up from 50 percent or below to an average of 80 percent. This reflects a savings both in resources to the institution and the student along with enhanced positive student attitudes and confidence in their ability to learn and use mathematics.

Our initial goal for this program was to usher the Developmental Mathematics students into a successful experience in their General Education Breadth mathematics requirement. The program has been successful in this regard with an additional unexpected outcome. Increasing numbers of the Developmental Mathematics students are redirecting their interest to college level mathematics courses. It is illuminating to observe what has been happening in the first Precalculus course. Enrollment of Intermediate Algebra students in Precalculus has increased from 22 percent in Fall 1992 to 48 percent in Winter 1996 while the total enrollments has shown little variation.

	Math 140[a]	Math 120[b]	Math 191[c]	Math 220[d]
Population MRG[e]	2.34	2.12	2.24	2.41
90 Graduate MRG	1.94	1.90	1.78	2.02
Population % Pass[f]	79%	73%	80%	77%
90 Graduated % Pass	73%	76%	82%	88%
Population % Success[g]	67%	61%	65%	74%
90 Graduates % Success	56%	62%	58%	88%

Notes: a Winter '93—Spring '95
b Fall '92—Spring '95
c Fall '92—Winter '96
d Fall '92—Spring '94. As of W'94 lowest passing grade was C–
e MRG: Mean Reported Grade
f Received a grade of D– or better
g Received a grade of C– or better

We are convinced that the students' experience throughout the program aids in the formation of a more universal, less compartmentalized perspective of their learning journey. The drive to excel in the presentation of reports that we observe makes us believe that our students also ultimately value this reporting process.

We need to note that in the implementation of a program like ours, obstacles may arise. At the beginning of the change in the old remedial program to the new developmental program, we experienced only encouragement and support to change an approach that had demonstrably failed. The only goal in mind was to increase the abysmally low number of students who needed intervention who went on to satisfy the General Education Mathematics Breadth requirement. The Developmental Program at CSUB has succeeded in this regard. The additional surprising outcome of the success of this program has been to increase significantly the number of students redirecting their interest into college level mathematics courses other than the Breadth requirement. Some faculty react with trepidation when faced with this unanticipated development. We believe this to be a purely emotional response in reaction to transformation.

We strongly believe in the continuous examination of results when implementing a new program, both internal to the program and in the subsequent impact that they have. This is, we think, the only professional course of action. Unfortunately, paraphrasing Wade Ellis Jr., from West Valley College, in his address *"Making the Waves of Change"* to the 1996 AMATYC Conference in Long Beach, California, not even the most sound statistics will operate to reduce the resistance of those who emotionally oppose change.

Maintenance of the equipment has been onerous in time and resources. The advent of the second- generation graphing calculators with symbolic manipulation, like the TI92 or the HPGX, makes the transition to these devices a natural step to investigate. We will begin pilot courses that implement our program on TI92s available to each student during the Spring Quarter of 1997.

SAMPLE MATERIALS

Texts Used: Brant and Zeidman. *Concepts in Algebra: A Technological Approach*, Janson Publications, 1995.
Brant and Zeidman. *Intermediate Algebra: A Functional Approach*, Harper Collins College Publishers, 1996.

The Math 90 course syllabus is included. In addition, a laboratory activity follows as well. The "Lab" is composed of two parts: the data collection sheet and the report. The data collection phase takes place during the weekly extended meetings of 2 hours and 30 minutes. Each student keeps records of all experiences on the data sheets. One single report for each group is prepared with a minimum set of questions addressed, also provided at the time of the laboratory experience. The grade on the laboratory is based on this report, and, of course, on the satisfactory completion of the data collection phase.

Math 90

Prerequisites

The successful completion of Math 80 or ELM score of 480–540.

Course Description

Fundamental topics: Polynomial, rational expressions and equations, conic sections, exponential and logarithmic functions. Course makes extensive use of scientific/graphing calculators and the Computer Algebra System **Derive.**

Objectives

Student's learning objectives will encompass:
1. The development of good study habits.
2. The reduction of math anxiety.
3. Preparation for the next level of math.

Calculator

Scientific with manual. Bring to class every day! **RECOMMEND:** Graphing Calculator: TI-80, TI-81, TI-82, TI-85, or TI-92.

Supplies

Graph paper, a straight edge, and colored pencils (2 min.).

Learning Experiences

1. The class meets four days a week in the IBM Lab (MB 401).
2. Students will be required to attend all days the class meets. Absenteeism **WILL** affect your grade!
3. Students will be required to do regular homework. This will help develop good study habits. The homework will not be collected but will be discussed at the beginning of each lecture day. Only the students who actually attempted the homework will be allowed to ask questions. Students not keeping up with the homework will find it very difficult to follow the lectures and will do poorly on the weekly quizzes and scheduled exams.
4. Students will be required to attend all labs. All lab assignments will be given during the lab period **ONLY**. All labs will be worked in groups assigned by the instructor. One complete, thoroughly organized, and well written report (preferably typed) will be collected from each group at the beginning of **FRIDAY'S** lecture. The Lab write up will not be accepted any other time.
5. Students will be required to take three exams and weekly quizzes. This will help to reduce math anxiety. No make-ups on exams or quizzes will be given. **NO EXCEPTIONS**! You may take an exam or a quiz early with instructor's permission.
6. Student's passing grade will be determined by overall combined points on lab write ups, group project, quizzes, exams, and final exam. Any student not taking the final exam will not pass the course.

POSSIBLE POINTS

Lab (7 total)	200 pts
Quizzes (10 total)	100 pts
Exams (3 total)	300 pts
Final Exam	200 pts
Total	**800 pts**

7. Student's final grade will be determined as follows:

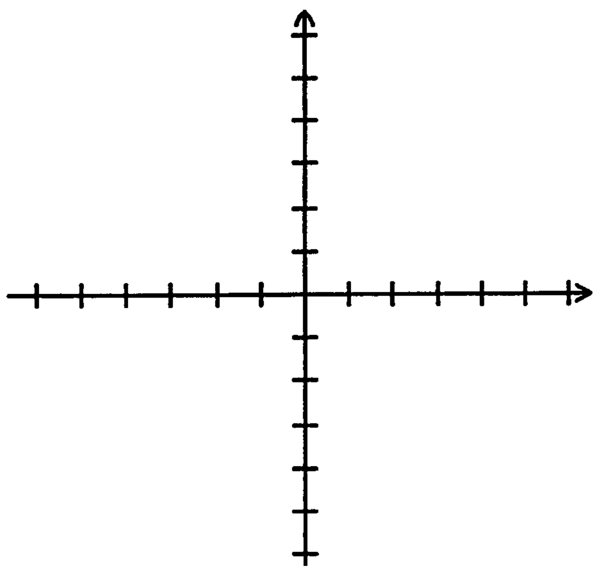
720-800 pts	= A
640-719 pts	= B
520-639 pts	= C
400-519 pts	= D
Below 400 pts	= NC

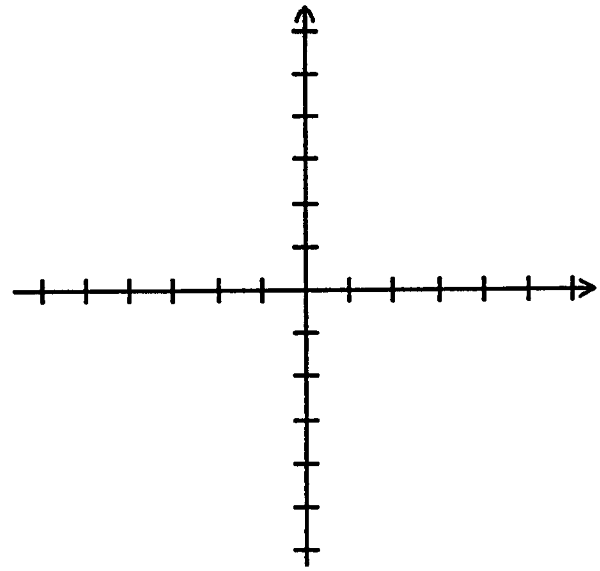
Note: Regardless of the student's total points, in order to get a passing grade in this course one must score **at least** 100 points on the final exam.

PARAMETRIC LINE SEGMENTS

Horizontal and Vertical Lines

1. Plot the three points (–3, –5), (–3, 4), and (4, 4) and record them on the axes on the left below:





SCALE: x: y: SCALE: x: y:

2. The vertical line that connects the points (–3, –5) , (–3, 4) is described by the observation that all of the first or *x*-coordinates are equal to –3. We can use this observation to join these points by a line segment. **A**UTHOR [–3, *y*] and **P**LOT. When you are asked for a parameter range, let y vary from –5 to 4. To join the points (–3, 4) and (4, 4) by a line segment, **A**UTHOR [*x*, 4] and **P**LOT. When you are asked for a parameter range, let *x* vary from –3 to 4. Record the resulting image on the axes above on the right.

   Use the **R**EMOVE command to get rid of all expressions in the Algebra window.

Slanted Segments

Our objective is to create the slanted line segment that connects the points (–3, –5) and (4, 4). In order to do that we will look at simpler problems. A few experiments will help us to understand this situation.

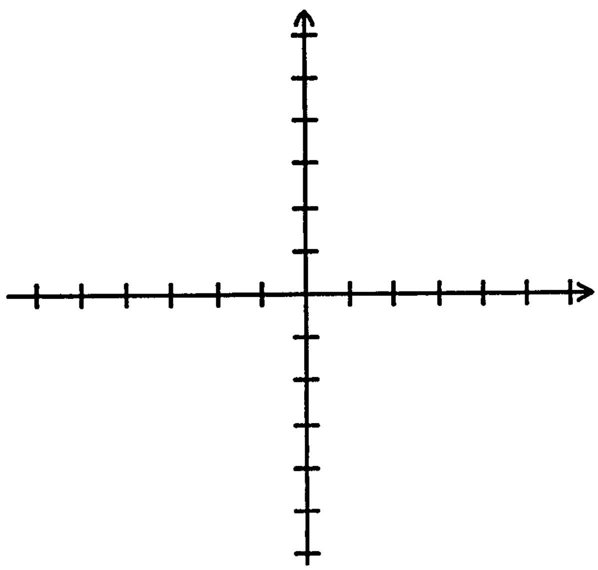
3. **A**UTHOR [*t*, 2*t*] and **P**LOT using the parameter range 0 to 5. Record the resulting graph on the axes below on the right. Label both ends of the line segment with their coordinates.

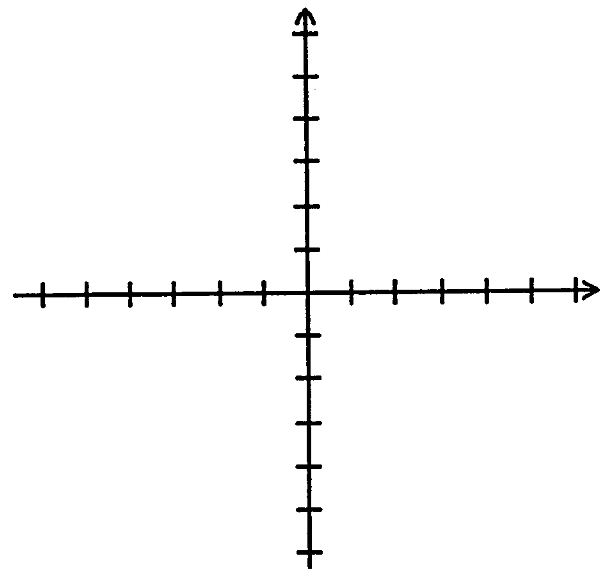
4. Likewise, **A**UTHOR and **P**LOT the parametric expression [2*t*, –5*t*] over the parameter range 0 to 1. Record the resulting graph on the axes below on the left. Label each end of the line segment with its coordinates.

5. Give a parametric expression and parameter range that will connect the points [0, 0] and [-5, 3].

   Parametric expression:

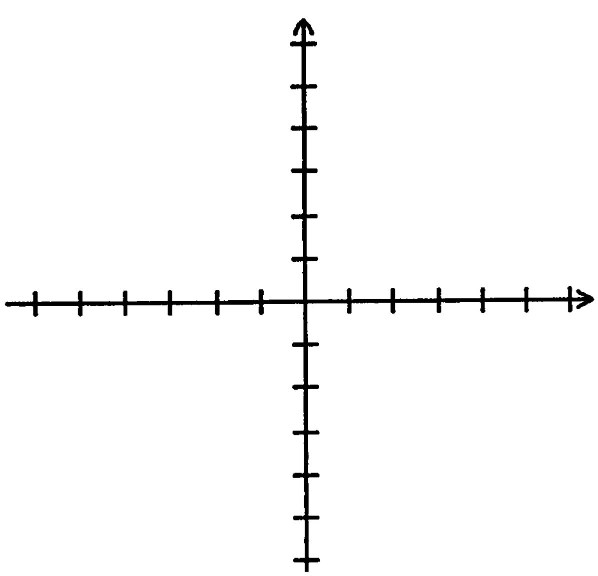
   Parameter range:
   How would you connect the point [0, 0] to any other point [*a*, *b*]?

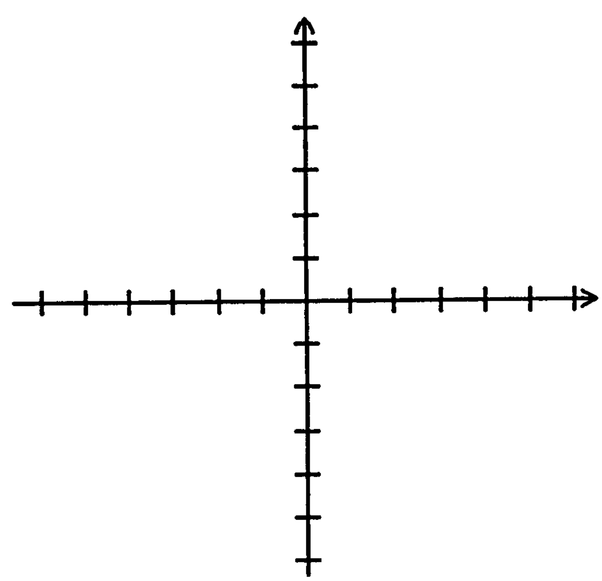




SCALE: x: y: SCALE: x: y:

6. The line segment that starts at [–3, -5] and reaches to [4, 4] travels seven (7) units in the x-direction while traveling nine (9) units in the y-direction. This suggests a strategy of using the expression [$7t$, $9t$] with a parameter range from 0 to 1. But of course this will give us a line segment that starts at the origin and reaches to the point [7, 9]. Nevertheless there is some insight to be gained from the picture. **A**UTHOR and **P**LOT the above expression with the indicated parameter range. Record the resulting line segment on the axes below on the left.





SCALE: x: y: SCALE: x: y:

By hand, add the points (–3, –5) and (4, 4) and the line segment joining them. Examining the resulting graph, we can see that the parametric line segment has the appropriate length and direction and needs only to be moved. We move it with the following expedient:
AUTHOR, **S**IMPLIFY, and **P**LOT the expression: [$7t$, $9t$] + [–3, –5].

7. Construct a parametric expression and parameter range that connects (–1, –4) to (5, –7). Record the plot on the axes above on the right.

Parametric expression:

Parameter range:

8. Construct a parametric expression and parameter range that connects (3, 7) to (1, –4). Record the plot on the axes above on the right.

   Parametric expression:

   Parameter range:
   How would you construct a parametric expression and parameter range to connect any two given points?

Solution of Equations and Inequalities

In what follows, every time that you are going to work on a new plotting screen, press DELETE ALL so that you have a clear screen for every new situation.

9. Determine and PLOT three points that have vertical coordinate equal to 4 (the horizontal coordinate is your choice).

   Results:

   AUTHOR [x, 4] and PLOT. Your settings are going to be such that the three points above are included.

   Parameter range:

   Record both the points and the line segment on the graph on the left below.

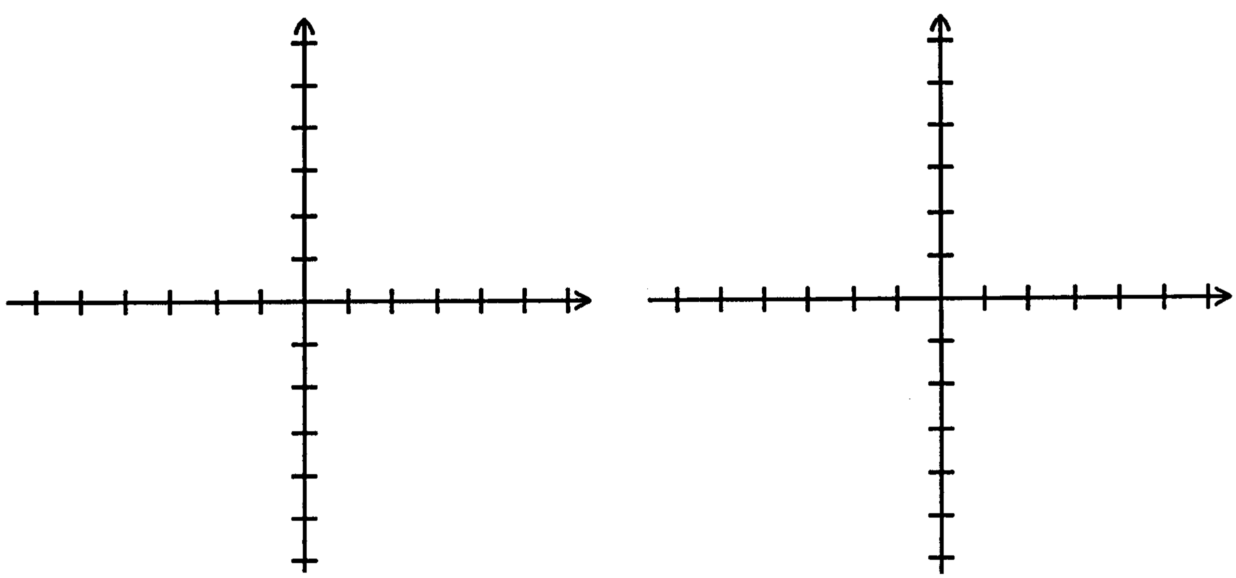
10. Determine and PLOT five points whose coordinates follow the following relation: the vertical coordinate is two units larger than the horizontal coordinate.

    Results:

    AUTHOR [x, x + 2] and PLOT. Your settings are going to be such that the five points above are included.

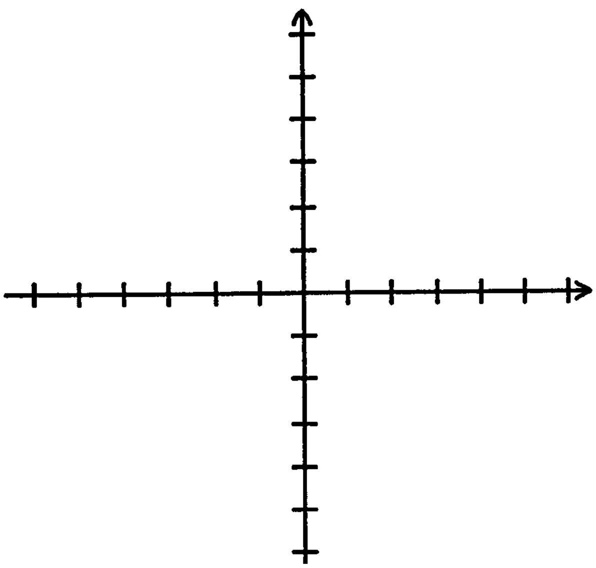
    Parameter range:

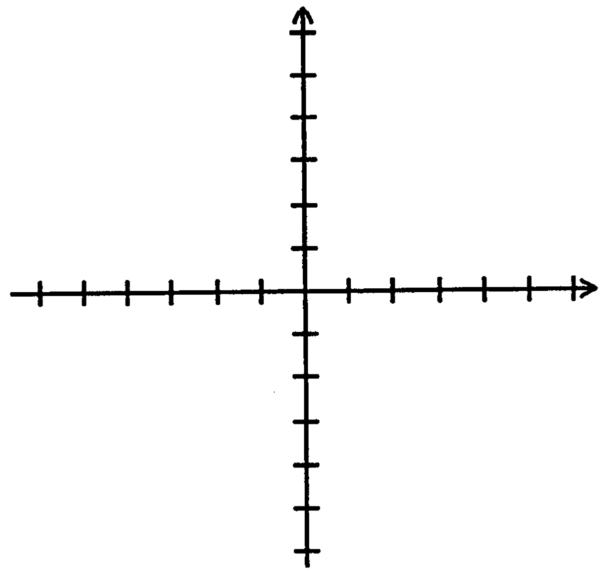
    Record both the points and the line segment on the graph on the right.



SCALE:	x:	y:	SCALE:	x:	y:

11. AUTHOR [a, 3/4 $a + 3$] and PLOT.
    Settings : Min : – 4 Max : 2 Mode : CONTINUOUS
    AUTHOR [a, 0.6] and PLOT.
    Settings: Min : – 4 Max : 2 Mode : CONTINUOUS
    Record both images on the axes on the left below.





SCALE: x: y: SCALE: x: y:

12. SOLVE the equation $3/4\, a + 3 = 0.6$.

Results How does this relate to the image above?

13. AUTHOR and PLOT each of [b, $2 - b/3$] and [b, 8] with a parameter range sufficient to show the two lines intersecting. Record the resulting images, scales, and parameter ranges on the axes on the right above. The settings and the scale are your choice. Include your choices in your report.

    Parameter range:

    Parameter range:

14. Solve the equation $2 - b/3 = 8$.

    Result Explain your result using the graph 13.

15. A tank full to capacity contains 75,000 gallons. The tank is going to be emptied by allowing 1,500 gallons to leak out per day. Fill in the table:

    On the axes below plot the information contained in the table.

Time t in days	Volume left in tank
0	
1	
2	
3	
5	
10	
20	

16. How long does it take for the amount of water contained in the tank to go under 10,000 gallons?

17. How long does it take for the tank to empty out ?

18. When is the volume left in the tank between 30,000 and 10,000 gallons ?

19. Algebraically, the answer to problem 16 is found by solving the inequality: Volume < 10,000. Give an algebraic expression of this inequality.

20. Solve your inequality and compare to your result in problem 16.

21. Algebraically, the answer to 18 is found by solving the inequality: 10,000 < Volume < 30,000. Give an algebraic expression of this inequality.

22. Solve your inequality and compare to your result in problem 18.

Upon completing the data collection, each group discusses and prepares the report. An example of the minimal requirements for this report follows:

PARAMETRIC LINE SEGMENTS

Work through the exercises in the attached DATA SHEET (the following pages), recording your observations as you go. Your personal DATA SHEET must accompany your group's Laboratory Report. The Laboratory Report should be typed and signed by all members of your group. The minimum Laboratory Report must address the following:

- An explanation of what it means when we say that the equation of the line that passes through the points (–3, –5) and (–3, –4) is $x = -3$.
- An explanation of what it means when we say that the equation of the line that passes through the points (–3, –4) and (4, –4) is $y = -4$,
- An explanation of what it means to say that the equation of the line in exercise 3 is $2x = y$.
- An explanation of what it means to say that the equation of the line in exercise 4 is $5x = -2y$.
- A description of how one can use parametric expressions to draw a line segment from (0, 0) to (a, b).
- A description of your method used to obtain the parametric expressions and ranges in problems 7 and 8.
- A general description of how one can construct a parametric expression and range to connect any two points.

- An explanation of the connection between problems 11 and 12.
- An explanation of the connection between problems 13 and 14.
- A description of your attack on problems 16, 17, and 18.
- A comparison of problems 16 and 20.
- A comparison of problems 18 and 22.

The final report is typically collected three to four days after the data collection phase, encouraging out-of-class discussion.

The majority of students resist having to write in a mathematics class on a first approach. We are convinced that their experience throughout the program aids in the formation of a more universal, less compartmentalized perspective of their learning journey. We observe a drive to excel in the presentation of reports that makes us believe that our students also ultimately value this reporting process.

The materials for this project were submitted by Nomiki Shaw and Ignacio Alarcón.

Tier II

This section includes summaries of the middle ten award-winning projects

INNOVATIVE
PROGRAMS
USING
TECHNOLOGY

Projects for Precalculus

Janet Andersen, Todd Swanson, and Robert Keeley

Service Course Area

Precalculus

Institutional Data

Hope College is a four-year liberal arts college located in Holland, Michigan. The college offers, with recognized excellence, academic programs in liberal arts in the setting of a residential, undergraduate, and coeducational college. Hope College has an unusually active science division. It is well known as a leader among liberal arts colleges in the area of student scientific research. For example, in 1996 Hope held five National Science Foundation grants for Research Experiences for Undergraduates (NSF-REU). These were held in the departments of biology, chemistry, computer science, mathematics, and physics.

Abstract

Projects for Precalculus [6] contains projects designed to allow students to use the mathematics they are learning to solve interesting problems. The materials contain 26 projects, drawn from a wide variety of disciplines. Detailed teachers' notes accompany each project. The materials are designed to enhance a precalculus class, and are not meant to replace the text. Using a guided-discovery approach, the projects help students communicate effectively, make connections among the various forms of a function, and see the variety of ways mathematics is used in applications. Students typically work in small groups and use TI-82 graphing calculators.

There are three main goals for this project: The first goal is to help students increase their confidence and ability to successfully comprehend the concepts later taught in calculus. The second goal is to help non-calculus bound students both to appreciate the usefulness of mathematics in solving real-world problems and to understand the broad range of areas in which mathematics is used. The final goal of the project is to provide materials that are easy for instructors to use.

Project Description

Our main goal was to produce and implement projects that can be used in any precalculus classroom at a variety of institutions. This was motivated by the need to better prepare precalculus students for the new approaches being used in calculus throughout the nation. One of the main impediments is the lack of easily accessible problems which encourage the type of interaction now expected in many calculus classes. *Projects for Precalculus* [6] consists of 26 easy-to-use precalculus projects that can be used with most precalculus texts. These projects use a discovery approach to learning the concepts of precalculus, and have been written in a manner that emphasizes the connections among the various forms of a function. Graphing calculators are used to allow students the ability to handle real data and to quickly create graphs and tables to study this data. Since approximately half of all precalculus students do not go on to take a calculus course, the projects have also been designed to help students appreciate mathematics and to see its wide variety of applications.

Most of the projects are applications drawn from a wide variety of disciplines. The projects involve interesting problems that are attractive to students. The materials are designed to transform the precalculus course in much the way that various reform materials are transforming calculus courses. These materials are different from what a student typically encounters in high school; they go into more depth with various mathematical concepts, and show the broad applications where mathematics is utilized. Students are given real problems using actual data that naturally present themselves in a variety of ways. Sometimes a function is presented in numerical form such as a wind chill chart or a daylight chart. Sometimes a symbolic form is presented, such as the one that is found in the instructions for filling out a 1040 income tax form and is used to determine one's income tax. Starting with a graph, students are asked to determine the corresponding symbolic function. Throughout this project the authors have continually striven to show that however a function is presented, in order to understand its nature more deeply, it is important to see its many representations and understand the connections among each of these representations.

The project authors also strove to increase students' ability to communicate effectively both orally and in writing. By working in small groups, students must discuss various mathematical concepts and learn how these concepts apply to a real-life situation. Many of the questions students are asked in the course of completing a project require explanations of mathematical concepts in terms of a certain situation. For example, in a project dealing with a linear function that relates temperature to the number of chirps a cricket makes in one minute, students are asked to explain what the slope and y-intercept mean in terms of temperature and number of cricket chirps. In the project "Looking Out To Sea," that is included in this narrative, students are asked to discuss the asymptotic behavior of one of the functions that relates a person's height above the earth with that of the distance one can see out to the horizon. The student is to explain this behavior through the physical situation, the graph, as well as the symbolic formula.

Because the authors have chosen to use applied problems with actual data, the functions that result are often not nice and clean, but rather are quite cumbersome. This results in a need for technology. A graphing calculator provides most or all the technology needed. Complicated functions can be quickly evaluated and easily graphed, and a table of values can be generated. This allows the instructor to focus on the relationships among the numerical data and the graphical data. Graphing technology is crucial to almost all of the projects. The projects have not been written for a specific graphing calculator, and they are, therefore, highly adaptable to different forms of graphing technology. The projects are also adaptable to other service courses and even to other disciplines. Certain projects have been used in a college intermediate algebra course, a high school algebra II course, secondary mathematics education courses, and high school physics courses.

One of the goals of this project is to have a packet of materials that are easy to use. Instructors of any precalculus course using any text can easily use the projects as a supplement to their course. An instructor can use six to eight projects during a semester or may choose to use a single project as a "capstone" experience for the students.

The project has been evaluated by students, instructors, and outside reviewers. While finding the projects difficult, most students thought that they were a valuable addition to the class and the vast majority thought that the projects helped them relate mathematics to life more than their textbook did.

Instructors who have used the materials have overwhelmingly liked the projects. They particularly like the fact that the projects are mostly applications that use real data. The instructors also like the connections that the projects emphasize among different areas of mathematics.

Sample Materials

Text Used: Christy, Dennis T. *Precalculus, 2nd ed.*, 1993, Wm. C. Brown Communications, Inc.

The following is a list and a brief description of the 26 projects found in *Projects for Precalculus*, followed by the project "Looking out to Sea."

Crickets—Nature's Thermometer
Linear functions that relate the rate of cricket's chirps and temperature are analyzed.

A Taxing Problem
Piecewise functions are studied by analyzing the federal income tax function.

Newton—A Real Swinger
The square root function for the period of a pendulum is examined.

Rolling Along
An interesting square root function is determined that relates the length of a roll of toilet paper to its radius.

The Amazing Golf-O-Meter
A device that measures distance to a flag stick on a golf course is developed using an application of a rational function.

Crossroads
The length of time that a traffic light must remain yellow is determined by using a sum of functions.

More than Meets the Eye
Can the formula for a polynomial function be determined by just using the graph?

Rational Behavior of Rational Functions
The long-term behavior of rational functions is explored as the concept of a limit is introduced.

Setting the Tone
Exponential equations are used to examine the frequencies of the notes of the musical scale.

The Population Problem
Exponential growth is explored using the world's human population.

Risky Business
Some of life's risks are analyzed using a logarithmic scale.

A Stitch in Time
Right triangle trigonometry is used to determine the size of triangular shaped quilting pieces.

Moving Right Along
Right triangle trigonometry is used to analyze a conveyer belt system that sorts pieces of wood.

Looking Out to Sea
Trigonometry is used to determine the distance one can see when looking out to sea from various heights.

As Easy as Pi
Using trigonometry along with inscribed and circumscribed regular polygons about a circle, an approximation for π is determined.

Days of Our Lives
The amplitude, period, and phase shift of a trigonometric function which models the amount of daylight throughout the year are determined by using a table and its graph.

Identity Crisis
The identities for cosine and sine of a sum are proved.

Split the Uprights
Trigonometry is used to compare the difficulty of kicking a field goal in the NFL, the CFL, and the NCAA.

Winging It
Using the Law of Sines (and Cosines), the correct flight plan for an airplane flying with a crosswind is determined.

Life in the Fast Lane
Police officers need to know the law (the law of cosines that is) to stop speeders.

Elliptipool
An interesting property of an elliptical shaped pool table is explored.

Remote Possibilities
The probability of one neighbor's accidentally opening another neighbor's garage door is explored.

Buy Now, Pay Later
One can see that time does equal money after the formula for monthly payments on a loan is derived using a geometric series.

The Point Les Trip
A point named Les takes an interesting journey whose length can be determined using an infinite geometric series involving trigonometry.

Zero to Sixty
Distance traveled and velocity of an automobile are determined as the concept of a limit is introduced.

How Cold is It? or Feeling the Chill in Frostbite Falls
The multivariable wind chill function is analyzed by fixing each variable and examining the resulting behavior.

Looking Out to Sea

The Chicago to Mackinac sailboat race is held every summer on Lake Michigan. The residents along the shore of the lake have an opportunity to view the boats as they make their way from Chicago, on the southern end of the lake, to Mackinac Island, on the northern end. One year the winds died down and stranded many of the boats. If you were standing on the shore you could see about 20 stalled out in the lake. However, if you climbed up one of the tall bluffs near the shore, your vision was greatly enhanced as almost 100 boats came into view. Going just a little bit higher dramatically increases how far you can see. Exactly how much farther? That is the question answered in this project.

1. Study Figure 1. The circle represents the circumference of the earth. The line segment labeled a is the distance one is above the surface of the earth. The line segments labeled b are the radii of the earth, and c is the line of sight distance. Since c is tangent to the circle and b is a radius, the angle where they intersect is a right angle.
   a. The radius of the earth is about 3,950 miles. Using this, find a function whose input is a and whose output is c.
   b. In the function you found in part (a), a and c are in terms of miles. This is a bit cumbersome since we usually don't talk about being so many miles off the surface of the earth. Rewrite your function so that c, your output, is in terms of miles and a, your input, is in terms of feet.

   (Recall that 5,280 feet = 1 mile and $(5{,}280)^2$ ft^2 = 1 mile^2.

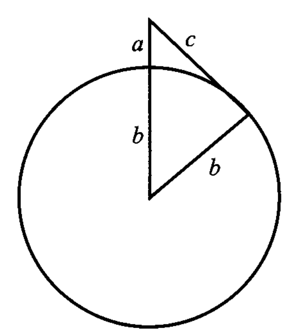


Figure 1.

2. Let's explore the properties of the function from question 1(b).
   a. Fill in the chart below. The first column is how high you are above the earth. The second column is your line of sight distance. The third column is the difference between your current and previous output divided by the difference between your current and previous input, i.e., an approximate "slope" of the function at that point.

height (feet)	distance (miles)	diff in output/diff in input (miles/feet)
5	2.7352	—
10		
15		
20		

Table 1

   b. Using the information in the chart above, could the graph of this function be a line? Why or why not?
   c. Graph the function you found in question 1(b). What is the relationship between the general shape of the graph and the information given by the numbers in the third column of the chart in question 2(a)?
   d. How high above the earth would you have to be in order to see 20 miles out?

3. The distance we have found so far has been the distance from the observer to the farthest point in a straight line, i.e., the line of sight distance. However, often what people refer to when talking about how far they can see is the distance along the horizon of the earth, i.e., the ground distance. This distance is the arc length, s, in Figure 2 below. Find a formula to express this arc length in terms of a where the input is in miles. Also, find another version of this formula where the input is in feet.

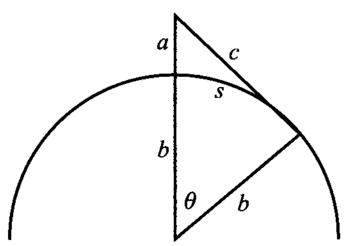


Figure 2.

4. We want to compare the line of sight distance to the ground distance along the horizon.
   a. Compare the ground distance to the line of sight distance when a is 10 feet, 1,000 feet, 10,000 feet, and 200 miles. What do you notice about the difference between the arc length and the line of sight distance as a gets larger? [Note: The formula commonly used for arc length assumes the angle is measured in *radians*, not degrees. Be sure to set your calculator appropriately.]

height (feet)	straight line distance (miles)	arc length (miles)
10 ft		
100 ft.		
10,000 ft.		
200 miles		

Table 2

   b. Using the functions where the input is in miles, graph both the line of sight distance and the ground distance on the same set of axes using a domain of 0 to 500 miles. Then graph using a domain of 0 to 50,000 miles.
   c. We are interested in exploring what happens to both the line of sight distance and the ground distance as a gets larger.

Some functions increase without bound, i.e., keep getting bigger and bigger, and other functions have a limit or upper bound, i.e., a limit as to how large they will get.

i. Think of the physical situation. (Refer back to Figure 2.) The line of sight distance will increase without bound, and the ground distance will have a limit. Explain why. What is the limit for the ground distance?

ii. Look at the second graph for question 4(b). Explain how this graph is compatible with your answer to question 4(c)i.

iii. Look at the behavior of your symbolic formulas as a gets large. Explain how this behavior is compatible with your answer to question 4(c)i.

5. The space shuttle orbits approximately 200 miles above the surface of the earth. Is it possible for someone in the space shuttle to view the entire continental United States? Explain your answer. Use the included map of the United States as a guide.

Portions of this article are reprinted with permission from Saunders College Publishing.

Implementing an Applications-Based, Technology-Supported Linear Math Course

John Engelhardt and Sherry Ettlich

Service Course Area

Business Math

Institutional Data

Southern Oregon State College is one of eight institutions in the Oregon State System of Higher Education. Its primary mission as a small public college is to provide an education in the liberal arts and sciences. The college is designated as a center of excellence in the fine and performing arts and complements its liberal arts emphasis with selected professional and graduate programs. The undergraduate enrollment is approximately 4,200; the graduate enrollment is about 350. The college has four schools: Humanities, Sciences, Social Sciences/Education/ Health & P.E., and Business.

The Department of Mathematics has 11 regular full-time faculty, several part-time faculty and two adjunct faculty. The department graduates eight to 12 math majors annually, with half or more subsequently entering a graduate-level teacher education program. Approximately 40% of the department's courses are service courses for other academic disciplines, and another 40% serve both math majors and other disciplines.

Abstract

This project involves the transformation of an introductory linear math course from a traditional textbook experience to one which is driven by realistic applications that are solved with the aid of technology. The classroom atmosphere is one of cooperation with students often working in small groups to uncover the mathematics. One- and two- week projects are used to focus the material around situations that can be illuminated with elementary mathematics (matrix algebra, curve fitting, etc.). Technological tools such as graphing calculators are used to shift use of classroom time from calculation to exploration. More time is available now for model formulation and solution interpretation. Many of the themes of the mathematics reform movement are incorporated into this course, specifically: use of technology, multiple representations of mathematics, focus on conceptual development, use of collaborative group learning, use of projects, and a focus on students' writing mathematics. Student and instructor feedback indicate that the changes have increased student interest and persistence through the course.

Project Description

This project involved three primary goals. The first was to institute a course which develops mathematics in context, using realistic application settings, and which fosters the appropriate mathematics in project form. The second goal was to use available technology, primarily graphing calculators and computer software, to perform the considerable calculations necessary in such a course. This allowed the instructor to focus students' attention on the underlying concepts and principles involved in the mathematical development. The final goal was to accomplish the above in a spirit of cooperative group learning. Students were expected to work in small groups of three to five on in-class assignments and homework checks, and were encouraged to consult with each other outside of class on their projects.

The mathematics service course transformed by this project is titled *Elementary Linear Mathematics with Applications.* It is an introduction to analytic geometry with an emphasis on linear functions of one or more variables and their graphs. Applications are drawn primarily from the social and management sciences. Topics include lines, planes, systems of linear equations, matrix algebra, and linear programming problems.

This course was developed to include many of the recent reform recommendations for mathematics listed in *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* (AMATYC, 1995) [26]. It is carefully constructed to teach students to "appreciate and use mathematics as a language that expresses, defines, and answers questions about the world" through the use of reasonably realistic applications in which the mathematical content is developed.

Students are introduced to applications which motivate the mathematics discussed. Together, the class discusses each application, looks at various issues that arise, and develops mathematical techniques to address these issues. Students complete mini projects associated with each application that cover a different but related application situation. This results in learning the mathematical content of this course within practical settings that emphasize the importance and/or usefulness of the mathematics. Ideally this makes the mathematics more understandable since it will be developed within fairly realistic situations.

The use of a graphing calculator is an integral part of the course. The TI-82 graphing calculator is the tool of choice for most students; however, any graphing calculator could be used. Besides allowing the students to deal with more complex and realistic problems, the calculator reduces the emphasis on computational skills. However, the students need to have a reasonably good background in two years of high school algebra, as many of the concepts developed throughout the course build on this background.

Students also make use of computer software, typically Quattro Pro's advanced math features, in order to explore optimization of situations involving three or more variables. Additional software for matrix manipulations (NCTM's Matrices package developed by the North Carolina School of Science and Mathematics) is available for use.

Various application packages were developed for this course under three main topic areas: functions, matrices, and modeling. Applications chosen for the function section emphasized linear functions but included at least one non-linear function. Four applications have been developed: modeling world-record times for the mile run, modeling braking distances, using height to predict shoe size, and a study of the federal tax rates. The applications developed for the matrices section included: cashflow of a magazine, modeling of an economy, and population management of a buffalo herd. These applications were chosen to include the development of matrix dimension, notation, operations, inverses, and matrix algebra. Applications in the modeling section require students to create mathematical models, find a solution to their model, and evaluate the solution in terms of the original problem. A linear programming unit has been developed for this section that covers solving two-dimensional linear programming problems graphically and higher dimensional linear programming problems with the aid of the computer. This unit includes maximum, minimum, and mixed constraint situations as well as situations with no optimal solution and those with multiple optimal solutions. Shadow pricing is introduced for decision making. A series of situations has been developed to cover these objectives rather than more extended applications.

A typical application takes a week or two to complete and includes a significant writing component. In the write-up students are expected to explain the mathematical model, including its implications and results, in enough detail to help a classmate who was absent for that project understand what the project was all about. In the course of the time devoted to a topic, students are talking in small groups and are working related problems. They are expected to discuss methods and procedures and turn in either an individual or a group project. As this course is taught by more instructors over the next few years, additional applications should be developed to allow instructors more choice.

The changes made in the redesign of this course did not require external funding or support. These changes are realistic modifications of a typical course existing on many college campuses. This project could serve as a model for course revision in situations where enrollments are limited to 35 or fewer students per section, since substantial faculty time is needed for reading student work. The approach undertaken with this linear math course revision could be extended to other mathematics service courses. It is already being done to some extent in the introductory statistics course where group projects are being introduced.

Since the emphasis of this course has changed, the dropout rate has decreased and a higher percentage who stay enrolled are passing (C– or better). Student evaluations suggest that students are in favor of the applications nature of the course and appreciate the technological tools they have. The instructors feel they are providing a more honest portrayal of mathematics and are getting a higher quality product from students than was previously the case.

Sample Materials

What follows are two projects that were given to students during Spring '95. Following each project description is a student write-up to the last question in the project.

Math 158 Project #1 Modeling Linear Data

The information is taken from the World Almanac. It lists the records for the mile run, given in total seconds. We will explore our ability to predict past and future records based on this data by using a mathematical model. You should show all necessary work and answer all questions on separate paper.

Row	Year	Time	Row	Year	Time
1	1880	263.2	20	1945	241.4
2	1882	261.4	21	1954	239.4
3	1882	259.4	22	1954	238.0
4	1884	258.4	23	1957	237.2
5	1894	258.2	24	1958	234.5
6	1895	257.0	25	1962	234.4
7	1895	255.6	26	1964	234.1
8	1911	255.4	27	1965	233.6
9	1913	254.6	28	1966	231.3
10	1915	252.6	29	1967	231.1
11	1923	250.4	30	1975	231.0
12	1931	249.2	31	1975	229.4
13	1933	247.6	32	1979	229.0
14	1934	246.8	33	1980	228.8
15	1937	246.4	34	1981	228.5
16	1942	246.2	35	1981	228.4
17	1942	244.6	36	1981	227.3
18	1943	242.6	37	1985	226.3
19	1944	241.6			

1. On graph paper, plot every third point (3,6,9, etc.) using Year for the horizontal axis and Time for the vertical axis. Decide which scale to use so that there is enough space to make the points visible (not all jammed together).

2. Does the plot look linear? Compute the median-median line through the data. Show necessary work and draw the line.

3. What does the y intercept mean in this context?

4. What does the slope mean here in this context?

5. Roger Basnister broke the four-minute mile in 1954 by running the mile in 3:59.4. What would the model predict for 1954? Is this a good prediction? Use your *equation* to predict the record time for 1954. Check your prediction against the actual and note the residual. How good is your model?

6. Use your model to predict the time of the mile record for 1993. Convert this to minutes, seconds, tenths of seconds. Find out the mile record for this year and compute the residual.

7. Are there any limitations to the model you came up with? Can we use it to predict the mile run in 100 years?

8. Type (word-process) a one-page essay which discusses what you have done in this assignment. Write it as if you were explaining it to a fellow student. You do not have to go through the computations, but you should describe what computations you performed.

Student Response

Project #1 was a study involving data taken from the World Almanac regarding record times for the one mile run over the past 110 years. Like most records, top times in this event are not broken with any predictable regularity. This is demonstrated by the fact that in one case the record held for 16 years, while in another instance records were broken three times in the same year. The table we were given included 37 record times between 1880 and 1985.

The first step was to plot the data. We did this so as to gain a visual perspective of the data as a whole and, in doing so, to keep a watchful eye for emerging patterns. As there were a substantial number of ordered pairs, we plotted only every third entry. This effectively minimized clutter while still portraying a meaningful outlook of the data.

The plotted data revealed a linear pattern for the most part. However, any line connecting the points would reveal a distinct curve. Because the plot did appear at least mostly linear in nature, it seemed appropriate to generate a line of fit. This was accomplished with the use of a median-median line.

Once the equation for the median-median line was computed and the line itself plotted, the next step was to analyze the equation and determine if the model it furnished seemed realistic to the data. After both the slope and y-intercept were taken into consideration, it became evident that our model would be an effective predictor of record times, with the important exception of extrapolations well outside the range of our data. Further and more specific application of our linear model supported this conclusion.

Math 158 Project # 2 Modeling an Economy

In a mining town there are four industries: Mining, Power, Transportation, and Services. After extensive analysis it was determined how much of each industry contributed to the internal consumption of the others. For every unit of Mining produced, that industry consumes .00 of itself, .23 of power, .16 of trans-portation, and .04 of services. For each unit of Power, .15 of mining, .10 of power, .10 of transportation, and .20 of services is consumed. For each unit of Transportation, .13 of mining, .21 of power, .08 of transportation, and .06 of services is consumed. For each unit of Services, .12 of mining, .20 of power, .12 of transportation, and .10 of services is consumed.

1. Construct the input/output matrix $[A]$ for this town's economy.

2. Consider matrix $E = [1\ \ 1\ \ 1\ \ 1]$. Compute and describe in words what the product EA calculates.

3. Consider matrix $F = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. Compute and describe in words what the product AF calculates.

4. Suppose that in a particular year 580 units of mining, 710 units of power, 435 units of transportation, and 390 units of service were produced. What was the amount of each

industry's output consumed in achieving this level of production? Show how you achieve your results.

5. What is the net production of each industry in the year above? (What is left over for meeting consumer demand?)

6. Suppose the town administrators wanted net pro-duction as follows: Mining 84 units, Power 160 units, Transportation 120 units, and Services 195 units. What would be the gross production needed from each industry to meet this goal? Show work carefully.

7. The labor costs for producing a unit of each sector are as follows: Mining — $.30; Power — $.25; Transportation — $.28; and Services — $.19.

Construct a row matrix L of these labor costs and compute the product of this and the gross production matrix. Describe what the product represents.

8. Type a one-page essay to accompany your work. Write as if to a student in this class who understood all our work prior to Leontief's economic model. Explain the model, its implications, and results, in enough detail to help your classmate understand.

Student Response

Project # 2 was an interesting "real-world" application of the skills we have spent the last couple of weeks refining in the area of matrix mathematics. We were given information concerning the production and internal consumption of a simple four-sector economy. Internal consumption refers to that portion of each sector's production that is required for the production of a single unit within any other sector.

Wassily Leontief, a Nobel prize winning economist, was the first to apply matrix mathematics in this setting. The Leontief Economic Model set the framework for our project.

Our first step was to construct an input/output matrix for the production data we were given (mining, power, transportation, and services). The columns represent the internal consumption necessary to generate a unit of that sector's goods. After two brief exercises aimed at providing a better understanding of the mechanics of multiplying row and column matrices to our input/output matrix, we moved on, in step 4, to employing the model to calculate the internal consumption for a given level of gross production. Step 5 then asked us to calculate the amount that would be left over to meet consumer demand. This was a simple matter of subtracting the internal consumption from the gross production. The resulting net production is then available for consumption.

Moving on to step 6, we had the opportunity to demonstrate the real power of the Leontief Economic Model and, in doing so, apply it to a more realistic economic objective. We were given figures the administration wanted for net production, likely based on projected demand, and we had to use the model to calculate the gross production required to have these amounts left over after taking internal consumption into consideration. We utilized a matrix development based on matrix inversion while also using the identity matrix, much as we had done before in class.

Finally, we applied labor cost per unit figures to determine the total labor costs for the gross production required to meet our given demand.

Note: This example is based on ideas gleaned from a workshop given by Bernadette Perham of Ball State University.

Permission to print material in this article was given by John Engelhardt and Sherry Ettlich.

Graphing Calculators and Cooperative Learning in Elementary Statistics

William E. Fenton

Service Course Area

Statistics

Institutional Data

Bellarmine College is a private Catholic liberal arts college with professional programs in Business and Nursing. While the college offers graduate degrees in Business, Nursing, and Education, the majority of its programs are at the undergraduate level. Most programs offer Bachelor of Arts degrees, with a Bachelor of Science in a few majors. The College has 90 full-time teaching faculty, 78% of whom hold a doctorate degree. The student-to-faculty ratio is 13.66:1.

Abstract

In recent years the author has been involved in the C⁴L Calculus reform project, which has greatly changed his conception of effective teaching. To bring the dynamic flavor of the C⁴L project to Elementary Statistics, he decided to incorporate technology as an exploratory tool and to put a heavy emphasis on cooperative learning. The following goals were chosen: to promote active participation by all students; to make the TI-82 graphing calculator an everyday tool for learning; to bring a problem-solving emphasis to the course; and to use "real-world" data as the basis for exploring and discovering the concepts.

To achieve these goals a course was designed with many components. The TI-82 graphing calculator was used throughout the course. To promote active participation the students formed groups of their own choosing. In these groups the students worked on activities in class and gave suggestions and solutions to these activities; they conducted four statistical projects outside of class and reported on their results; they completed homework assignments which were collected and graded; and they took a group portion of each examination. The TI-82 was used for all of these components. The in-class activities and the projects presented realistic problems and gave the students exposure to real data, much of it collected by the students themselves. The homework came from *Introduction to the Practice of Statistics*, 2nd edition, by Moore & McCabe [119]. This allowed for a strong emphasis on real-world applications. The tests contained, primarily, problems from newspapers and magazines, continuing the emphasis on real-world applications.

Project Description

The primary goal of this course design was to promote the active involvement of the students in their learning. The major features of the course were: cooperative groups, class activities, group projects, graded homework, and graphing calculators. Together these successfully created a classroom in which students were consistently and enthusiastically engaged with the course content. Technology was incorporated as an exploratory tool, and a heavy emphasis was placed on cooperative learning. Lecturing was minimized; instead, a sequence of activities was presented to be done in small groups, usually involving the TI-82 graphing calculator.

For the first three weeks the group work was very informal; the 22 students could work with anyone they wished.

In the second week the class had a detailed discussion of how groups can work and some pitfalls to avoid. In the next week the students made their group commitments for the semester. Although many suggestions have been published on how to set up cooperative groups, the author's experiences suggest that allowing students to select their own group may lead to cooperation sooner than other methods. Regular monitoring of groups by the instructor is important, for an outside view from an instructor often can help a group spot a problem and find a remedy. The author met with each group outside of class three times during the semester to discuss their group.

Many students were reluctant at first to work in a group, and were particularly leery of the group portion of each exam. But after the first exam, group work became very popular. By then the students had overcome the initial awkwardness, and most were comfortable with the technology. It also may be that the exam situation is a "bonding" experience which can improve the interactions between group members.

The class activities were the most critical component of the course. The best were those with a "hands-on" flavor, such as experiments with cubical and tetrahedral dice, the tabulation of M&Ms, and the comparison of thumb measurements. However, even routine activities such as creating boxplots and scatterplots were made more engrossing by the graphing calculator. A student with this technology seems to feel compelled to use it, even if unsure of what to do. The resulting activity, with the guidance and encouragement of a peer group, can be very beneficial.

Many class activities throughout the semester relied on data collected by the students. The first assignment was purely data collection. But there was some difficulty with "dirty" data. For instance, one item was to measure the height of the statue of St. Francis on our campus. The most frequent value was 73", but the second most frequent was 7'3". But even this led to valuable discussion in class. Having the students collect data, either in class or beforehand, gave a "real-world" flavor to the class activities and this was successful in securing the students' attention and interest.

The group projects were highly motivating, and a pleasant diversion from the typical class. Each group conducted four statistical projects outside of class and reported its results. Every student was responsible for one written report and one in-class presentation. Specific guidelines and grading criteria for the project were provided. These were helpful to the students, while still giving them flexibility in choosing a topic and conducting their study.

The chief requirement for implementing this approach is an instructor who is willing to learn about the technology and about cooperative learning, and who has enthusiasm and imagination to apply to the course. The major investment is the instructor's time. The next offering of this course will feature the recently introduced TI-83 calculator. Its extensive capabilities for inferential statistics will substantially change the second half of the course.

Sample Material

Projects

From the syllabus: Part of your grade will be based on your participation in four statistical studies. Your group is expected to choose the topic, collect and analyze the data, and draw a conclusion. A project includes a brief written report on the study and a presentation to the class.

The written report should be typed and be three to five pages long. Include the raw data, any graphs, and an explanation of what you are trying to do in your study. Discuss any factors which may affect the validity of the study. Do not include the calculations, just the results. Turn in *two copies.* Everyone who participated in the project should sign the first page, with the author's name clearly marked.

The in-class presentation should be no longer than five minutes. You are trying to say what the study was about and what you found, not explain the methods or calculations. Be concise!

Project 1: REPRESENTING DATA

Collect at least 30 numerical data values on some topic. Represent your data graphically in two ways. Describe your data numerically in two ways. Discuss the strengths and weaknesses of these four representations and draw a conclusion about your data.

Project 2: COMPARING TWO QUANTITIES

Collect at least 15 pairs of numerical data on some topic. Graph the data. Compute the regression line and graph it on the same set of axes. Compute the correlation coefficient and draw a conclusion about your data.

Project 3: ESTIMATING A MEAN

Select a population and a variable. Design a procedure to get a random sample from this population. Use your procedure to collect at least 20 values. Calculate a 95% confidence interval for the mean of the population.

Project 4: TESTING A HYPOTHESIS

State a hypothesis about a population. Design a procedure to collect a random sample from this population. Use your procedure to collect at least 30 values. Test the hypothesis at the 0.05 level of significance. Draw a conclusion.

Sample topics (selected by the students)

Project 1: Number of blue jeans owned by Bellarmine College students; percentage of pages devoted to full-page advertisements in leading magazines; pulse rate per minute over two days for group members.

Project 2: How much men versus women spend for Valentine's Day; cost versus page count for textbooks; unemployment rate versus crime rate in selected US cities.

Project 3: Mean distance from home for a Bellarmine resident student; mean grade point average of a Bellarmine freshman; mean length of a music compact disk.

Project 4: Do more than 50% of Bellarmine resident students have a car on campus? Does the typical Bellarmine student get 50 hours of sleep each week?

Assignment

Collect the following data:

1. the number of doors in your house;
2. did you wear jeans on Fri., Jan. 12? On Sat., Jan. 13?
3. the height of the statue of St. Francis between Horrigan Hall and Pasteur Hall;
4. the amount of sleep you got on Sunday night, Jan. 14;
5. the number of rooms in your house;
6. multiply your birth month times your birth date. Use this number to locate a page in our text. Count the number of words on that page.
7. the length of the TI-82 calculator;
8. the number of keys on the TI-82 calculator;
9. the name of your favorite Escher print from the ones displayed in the basement of Pasteur Hall;
10. the state in which you were born.

After these data were collected, the groups classified each item as numerical or categorical. Items 1 and 5 were used later in the semester for a correlation example. Item 6 was used to learn about boxplots, and later used as an exam question on confidence intervals. Item 3 was used as an exam question, asking students to compute numerical summaries, draw boxplots and histograms, and comment on the results. Item 10 was used to study proportions. Item 9 was to show where my office is.

Samples of In-class Activities

Jan. 16: Using data from Item 6 (above), create a stemplot, a boxplot, a histogram, and calculate median, quartiles, mean and standard deviation. This also served as an introduction to the calculator.

Jan. 25: Data on egg production in Kentucky was presented. The groups analyzed production levels versus market value with a scatterplot and a regression line.

Jan. 30: The groups analyzed market value of eggs versus time. Predictions were made, and their reliability was discussed.

Feb. 1: Each group sorted a bag of M&Ms by color. The class compared the combined data for plain versus peanut candies in tables and graphs.

Feb. 8: Each group designed an experiment to decide if addition can be done faster with a calculator or by paper and pencil. These designs were reviewed by the class and a single design agreed upon. We then conducted the experiment, using random assignment of subjects. (For the second round I wrote problems requiring a lot of carrying. We then discussed bias.)

Feb. 20: Each group was given a standard cubical die and a tetrahedral die. The groups repeatedly performed the experiment of rolling these dice and recording the larger value. The frequency distribution was created and graphed by the class. Then the groups developed the probability distribution and compared it to the experimental results.

Feb. 29: The groups found the average number of days in a month, and the class compared the different answers.

Mar. 19: Each group measured the lengths of their right thumbs and computed the mean. These sample means were compared to the class mean, and the variation was discussed.

Mar. 21: With the thumb length data, the groups calculated several confidence intervals.

Apr. 11: Using the data from Item 10 on the first assignment, the groups calculated confidence intervals for the proportion of Bellarmine students born in Kentucky. The class then discussed the unreliability of these estimates.

Apr. 16: Using the M&M data from February, the class conducted a chi-square test of candy type versus color.

Questions from the Group Portions of the Exam

1. In the 1990 census, 43% of the population of Magoffin County in Kentucky was classified as living in poverty.
   a. Use the calculator to simulate a random sample of 25 people from Magoffin County and find the proportion of your sample which fall in the poverty category.
   b. The sampling distribution for this situation is approximately N(0.43, 0.111). Find the z-score of your sample.
   c. Does the value for your sample fall in the middle 68% of the distribution? In the middle 95%? Explain how you know.
2. Freshmen at Bellarmine College are required to take a one-credit course known as "Freshman Focus." This is a new requirement at the college. Its intent is to give entering students a smoother transition to college life, in the hope that they will be successful and will graduate from Bellarmine.

   Preliminary indications after one year suggest that the course is achieving its goal. However, the evidence is mostly anecdotal, and there are many confounding variables. So it is difficult to draw reliable conclusions.

   Design an experiment to test the effectiveness of "Freshman Focus." Include a description of how to set up the experiment and what observations will be needed. Also include a diagram of your experiment.
3. Here is a probability experiment: Roll a tetrahedral die (numbered 1,2,3,4) and flip a coin. If the coin comes up

tails, record the number on the die; if the coin comes up heads, record this number plus 2.

a. Find the probability distribution for this experiment and draw a histogram for it.

b. Find the mean of this distribution and mark it on your histogram.

c. What is the probability of getting an odd number result from this experiment?

Permission to print this material was obtained from William E. Fenton.

Basic Geometry Using Computers and Collaborative Learning

Vernon M. Kays

Service Course Area

Developmental Mathematics

Institutional Data

Richland Community College was founded in 1971 as a comprehensive community college offering baccalaureate, technical, continuing education, and community service programs. Its district serves the central Illinois counties of Macon, Moultrie, Piatt, Logan, Dewitt, Christian, Shelby, and Sangamon. Richland moved to its permanent site in the northeast area of Decatur, Illinois, in the spring of 1989. There are 55 full-time faculty and over 125 adjunct faculty. Richland Community College has had a student enrollment between 3,380 and 4,110 over the last five years.

Abstract

This course evolved out of a basic geometry course to meet prerequisite requirements for any student who has not completed a geometry course with some proof writing required. Basic geometry was redesigned using an inductive approach with computer software as the construction tool and with collaborative, group-centered learning. Students learn fundamental concepts, basic interrelationships, and constructions using a computer in a collaborative setting. Since most students have not encountered the computer as a learning tool, computers also become a primary vehicle for forming and maintaining the collaborative groups.

The general goals of the course are:

1. To have students create their own geom-etry via exploration and construction using dynamic computer software and collaborative group processes. Each class builds its own set of geometric definitions and conjectures using the computer software and the collaborative process prior to writing proofs.
2. To enable students' learning of logical thinking skills throughout the process of developing conjectures from their geometric investigations. Deductive reasoning and applications to geometric proof writing are then superimposed over their inductive geometric learning. Students are able to write a simple proof in several different ways by the end of the course.
3. To encourage students to view mathe-matics learning as a group process as well as an individual process.
4. To stimulate students' ability to explore and learn mathematical concepts through the appropriate use of technology.
5. To increase students' ability to apply basic geometric concepts in solving "real world" geometric problems.

Project Description

Many students have completed high school without taking geometry. Many who tried to complete geometry have failed, and geometry was known as the "filter," limiting student access to higher-level high school mathematics courses and college mathematics before calculus. In the late 1980s the State of Illinois mandated an increase in the level of mathematics that all students in college must complete. A new statewide articulation initiative was started in 1991 and implemented in 1995; intermediate algebra was no longer accepted as a college-level course for general education purposes. As a consequence of the initiative, basic geometry became a prerequisite for all mathematics courses, intermediate algebra and above, at the community colleges in Illinois.

Basic geometry, prior to fall 1994, was a typical lecture/homework course with proof writing introduced in the second week of the course. The structure of the course was frustrating because students could not or would not write simple proofs and they lacked a basic understanding of the fundamental geometric relationships developed in the course. Students were frustrated because they could not see how the proofs followed from the material presented and felt that most proof writing was memorization of lists of theorems. They were not developing the necessary connections between the concepts presented via lecture and constructions to the proof writing instructors required. Students did not seem to be able to make the transition from the concrete/visual to the abstract/ logical modes of thinking. The author began to look for different possible modes of student learning. Out of this research, the current course evolved.

The textbook used in this course was *Discovering Geometry: An Inductive Approach* by Michael Serra, Key Curriculum Press, 1989, Berkeley, CA. This course uses *Geometer's Sketchpad*, a dynamic geometry software as the tool for construction. *Geometer's Sketchpad* requires a computer with enough power, disk space, and computer speed to run Windows 3.1 or higher. It is the primary tool used for collaborative group investigations, and it is used to develop most of the conjectures utilized throughout the course.

After a brief introduction to inductive reasoning used in a numerical-mathematical context, students typically spend eight to 10 weeks collaboratively in the computer classroom. The course incorporates discussion, problem solving, reading and writing, student questions, cooperative group work, and lecture. A major portion of the course is student centered and uses investigative techniques. Students are required to work both independently outside of class and in cooperative settings in class. Student participation in the learning process is essential to success in this course.

The objectives of Mathematics 095 (Basic Geometry) include student proficiency and application of the basic skills of an introductory course in geometry. The following topics are included: the foundations of logic, application of proof writing, basic geometric constructions, and application to various geometric problems. An objective that permeates the course is the development of logical critical thinking skills in geometric thought. The course prepares students for further study in mathematics, computer languages, the sciences, and many related fields. This course encourages students to think about mathematics, not merely to do mathematics. This course is fundamentally different from previous geometry courses, not in content but in style. Students create a geometry from their experiences. The focus is more on the student and less on the instructor.

The computer rooms are set up with the focus on the computers. Most are in groups of four computers that students use in their groups and as individuals. Both structures increase the opportunity of collaborative learning by focusing the class on the student and away from the teacher.

Using a student-centered classroom is different. It takes time; the instructor must continually read and attend workshops that focus on those areas of student learning. Most of the students have never used the computer as a learning tool. Thus, all students are starting at the same level. They lack knowledge of how to make constructions using the software. Since all students must learn the rules and procedures of the dynamic software, the computer acts as a focus for collaborative learning. The structure of the computer classroom provides a natural setting in groups of four or less. Because the structure of the class uses an investigative style, students must use the computer first, and then make a conjecture within their group before they ask for instructor input. During the normal class session, the instructor is usually moving around from group to group offering advice only when asked, and often the instructor's response is a question rather than an answer.

The computer is only one format for construction. Students are required to model solutions in a variety of ways, specifically numerically, graphically, symbolically, and verbally. Students present verbal discussions of their conjectures to the class using the technology as a support for their presentation. Presentations are done both individually and by groups. The technology allows for easy and accurate presentation of their investigation. With the technology, students experience much more geometry than they would by simply using compass and straightedge. The software allows students to develop the connections between concepts more accurately and with greater facility than via memorization and poorly formed compass and straightedge constructions. This course is highly adaptable to other institutions. Much of the material is commercially available through Key Curriculum Press. Both the software and the textbook are available and offer some fairly good teacher's guides.

The more difficult area for faculty to include appropriately in the classroom will be developing the collaborative nature of the course. Attempting collaboration and creating effective student learning communities are the most demanding components of this course. Without a supportive text and/or appropriate materials, many students do not like collaborative learning nor do they see how it works. One of the pleasant surprises that has come out of the introduction of technology was that it provided enough of a difference in the classroom structure to allow students to open up to the possibility that collaboration works. Process and modeling are paramount to collaborative learning success. Once the students discover how the process works and observe it modeled by the teacher, they can begin to develop more mathematical independence from the instructor. Lecture must become the mode of last resort. Appropriate use of technology, when used in a collaborative setting, can help to develop more independent active learners.

Prior to changes made in the course and the enforcement of the prerequisites for intermediate algebra and above, enrollment averaged under 15 per year with only one section offered per year, if at all. Enrollment increased with the changes in prerequisites to over 15 per semester for at least two sections. With the introduction of technology, retention has improved to the 75% to 90% range for all classes. This project shows that a reform-based curricular development used in secondary schools can be successfully used in a community college developmental setting.

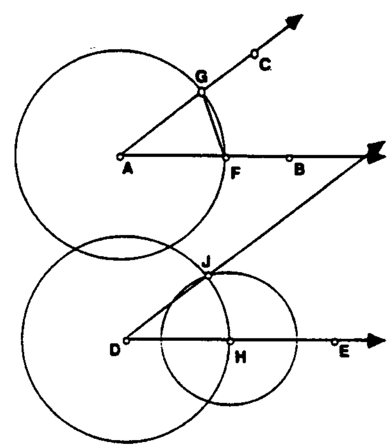
Sample Materials

Text Used: Serra, Michael. *Discovering Geometry: An Inductive Approach*, Key Curriculum Press, 1989.

Construction: Duplicating an Angle*

In this activity, you'll learn how to duplicate a given angle. The method described is equivalent to the method you would use with a compass and straightedge. You might want to follow the first few steps and then try to figure out the rest on your own. This construction is a building block for many other, more complex constructions. You may want to record and save a script for duplicating an angle.

Sketch



Step 1: Construct rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (This is your given angle.)

Step2: Construct $\overrightarrow{DE}$, one side of new angle.

Step 3: Construct circle *AF* and $\overrightarrow{AF}$, with *F* on $\overrightarrow{AB}$.

Step 4: Construct $\overrightarrow{FG}$, where *G* is the point of intersection of the circle and $\overrightarrow{AC}$.

Step 5: Construct a circle with center *D* and radius *AF*.

Step 6: Construct H, the point of intersection of this circle with $\overrightarrow{DE}$.

Step 7: Construct a circle with center *H* and radius *FG*.

Step 8: Construct $\overrightarrow{DJ}$, where *J* is the point of intersection of these two circles.

Step 9: If you wish, hide the circles, segments, and points *H, F*, and *G*.

Investigate

Move points *A, B, C, D*, or *E*. Do the angles remain congruent? Confirm that they're congruent by measuring them. When you try to drag *J*, why doesn't $\angle JDH$ change?

Present Your Findings

Discuss your construction with your partner or group. to present your findings you could create a commented script that duplicates an angle, explaining why it works.

Explore More

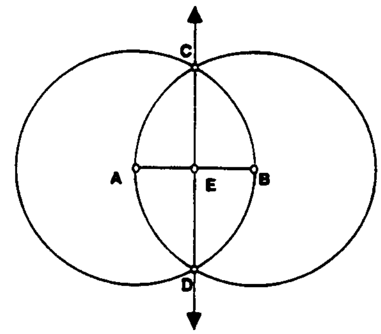
1. Construct two unconnected segments and an angle. Now construct a triangle by duplicating the angle and the two sides on the sides of the angle. Is there more than one way to do it? Are all the triangles you can construct this way congruent?
2. The construction described in this activity duplicates angles only in a counter-clockwise direction. (If you move your original angle past 180°, your duplicate will still have equal measure, but will have a different orientation.) Come up with a construction for duplicating an angle in the clockwise direction.

*This exercise was taken from *Exploring Geometry*, Key Curriculum Press, 1993, p. 57.

Investigation: Perpendicular Bisectors*

In this activity, you'll use Sketchpad's freehand tools to construct and investigate properties of perpendicular bisectors.

Sketch



Step 1: Construct $\overline{AB}$.

Step 2: Construct circles AB and BA.

Step 3: Construct $\overleftrightarrow{CD}$, where C and D are the points of intersection of the circles.

Step 4: Construct E, the point of intersection of $\overline{AB}$ and $\overleftrightarrow{CD}$.

Step 5: Hide the circles.

Investigate

Line CD is the perpendicular bisector of $\overline{AB}$. Move points A and B. What's special about point E? Can you come up with a shortcut for constructing a perpendicular bisector using the construct menu? Construct a point F on $\overleftrightarrow{CD}$. Measure the distances FA and FB. Move point F up and down the line. What can you say about any point on a segment's perpendicular bisector?

Conjecture

Write your conjectures below.

Present Your Findings

Discuss your findings with your partner or group. To present your findings you could:

1. Create a script for constructing a perpendicular bisector with comments explaining why your construction works.
2. Print a captioned sketch with measures illustrating your conjectures.

Explore More

1. Construct the perpendicular bisectors of the three sides of a triangle. Investigate their point(s) of intersection. Can you construct a circle that circumscribes the triangle?
2. Construct $\overline{AC}$ and $\overline{AF}$. Mark your perpendicular bisector as mirror and reflect A, $\overline{AC}$, and $\overline{AF}$ across it. Where is A' (the reflection of A) located? How do the triangles formed by this reflection help explain why C and F are equidistant from A and B?

*This exercise was taken from *Exploring Geometry,* Key Curriculum Press, 1993, p. 65.

A Cognitive Tutor for Authentic Problem Solving in College Algebra

Kenneth Koedinger

Service Course Area

Quantitative Literacy/Special Topics

Institutional Data

The Pittsburgh Advanced Cognitive Tutor (PACT) Center consists of approximately 20 researchers affiliated with the Psychology Department and/or the Human-Computer Interaction Institute (School of Computer Science) at Carnegie Mellon University. It is a collaborative venture between cognitive and educational psychologists, computer scientists, secondary school mathematics teachers, and graduate and undergraduate students and programmers.

The University of Pittsburgh (PITT) and California State University Long Beach (CSULB) are large urban universities offering both undergraduate and graduate programs. The University Challenge for Excellence Program at PITT specializes in remedial curricula in mathematics, science, and learning skills.

Abstract

The goals of this project were to investigate an adaptation of the Practical Algebra Tutor, (PAT), for college-level developmental mathematics, and evaluate PAT at two college sites. PAT is a software environment for learning algebra that presents students with real-world problem situations, modern mathematical representational tools to analyze these situations, and constant background support for a "cognitive tutor"—an intelligent computer tutor based on the Advanced Computer Tutoring project (ACT) theory of cognition. In accordance with previous classroom evaluations at the high school level, "monitored design" experiments at the two colleges showed that PAT students outperformed students in regular classes by more than 50% on a performance-based assessment. This assessment requires students to use mathematical representations to analyze a real-world problem situation—in particular, the relative merits of different cellular phone services. This assessment captured the reform objectives of the PAT approach, which are consistent with new national standards for mathematics, and which have led to efforts to reform the surrounding curriculum in the college test sites.

Project Description

Developmental mathematics courses, defined as college-level courses prior to calculus, are the fastest growing post-secondary mathematics courses in the nation. From 1965 to 1985, total enrollment in college mathematics courses increased 60% while enrollment in college courses in high-school algebra and geometry increased by 250% [108]. Developmental mathematics students tend to have one or more of the following characteristics: (1) high variability in age and prior mathematical background; (2) a prior history of failure and frustration in mathematics; (3) a fragmented, proceduralized understanding of mathematics; (4) a lack of appreciation for the direct importance of mathematics to their lives; and/or (5) poor study skills, particularly, little awareness of how crucial it is to spend time working through problems if one is to succeed in mathematics. The project's goal was to develop an instructional solution that can adapt to the individual background of students, that offers a success experience, that interrelates various mathe-matical skills during problem solving, and that can compensate for poor study skills. Also, this project was designed to prepare students for higher-level mathematics classes and to focus on the application of mathematics to a variety of fields.

A type of artificially intelligent instructional system that addresses these problems has been developed through a collaborative effort between cognitive psychologists, computer scientists, and educators at the PACT center of Carnegie Mellon University. The project investigated the addition of one such cognitive tutor, the Practical Algebra Tutor (PAT), to a traditional intermediate algebra curric-ulum at two college sites.

The current emphasis in mathematics reform efforts is on the use of multiple representations and computational tools to solve difficult real-world problems that involve symbolic expressions. The emphasis in PAT is less on symbol manipulation and more on symbolization as a way to unleash the power of mathematics for problem solving. While PAT covers the traditional content of beginning algebra, these skills are introduced in the context of authentic, realistic problem-solving tasks. PAT provides computerized computation and visualization including a tabling tool, a graphing tool, and an equation-solving tool. Instruction promotes instance-based investigation and a flexible problem-solving strategy. PAT promotes the use of this strategy by asking students first to investigate concrete instances of unknown values and then to formulate the abstract rule.

Cognitive tutors are well suited to the unique challenges of teaching developmental mathematics. Within the tutor is a simulation, developed through psychological research, that can solve mathematics problems in the ways we expect students to solve them, as well as simulate the kinds of mistakes that students make. Like other computer-based instruction, cognitive tutors allow for self-paced use. Unlike other approaches, the cognitive model within the tutor facilitates three additional forms of customized instruction: (1) individualized feedback and advice can be supplied as a student solves a problem; (2) the number and types of problems are individually selected to maximize learning opportunities; (3) it is easy to customize the tutors for particular courses.

At the University of Pittsburgh, the developmental algebra sequence has been restructured with the integration of the Practical Algebra Tutor in a computer lab, a collaborative problem-solving session with specially designed projects to be done with hand-held graphing calculators, and a lecture format made up of an interactive discussion segment, based on questions to be answered from the reference text and exercises, and a metamathematical lecture topic. The PAT system drives the curriculum and serves as a primary resource for student engagement in problem solving. Students are scheduled for one 50-minute lab per week as a class unit. During this session students are expected to complete three problems, working independently, though collaborative discussion is encouraged.

A typical day in the lab finds students attentively engaged in their work. Because the lab is a shared facility, basic algebra students find themselves working alongside calculus students, experiencing an equivalent instructional setting with their colleagues. Students, when help is needed, are encouraged to use the hint feature in the software and will call on the instructor or other students only if they cannot resolve the problem. Questions change significantly during the term from concrete questions about how to use the software to include questions of a more probing nature dealing with the analysis of a solution or the interrelationship between the representations of a problem. Because the students must complete a problem in order to move on to the next, they learn how to persist and sustain engagement even when encountering difficulty. Since the lab-based instruction drives the course material for the week, students must complete the lab by the end of the hour, or at least prior to the next class.

The major difficulty that had to be overcome was the lack of sufficient writeable server space. PAT is capable of storing detailed information on each student's problem-solving process, from the time spent on each problem to the exact sequence of key strokes a student used while constructing a graph. The difficulty was overcome by issuing to each student in the class a diskette on which data was stored. The instructor periodically collected the disks to monitor student progress.

A careful study was done which compared experimental groups using the cognitive tutor with control groups which used a traditional curriculum. The researchers were interested in two major outcomes of student learning: (1) algebraic manipulation skills - the goal of a traditional curriculum, and (2) use of these skills as well as alternative algebraic representations (e.g., tables and graphs) to analyze and solve real-world problem situations - the goal of the PAT curricululm. Performance-based assessment targeted students' problem-solving skills, qualitative reasoning, and ability to communicate effectively about mathe-matics; traditional algebra exams were also administered. A well-defined student survey measured six attitudes: (1) lack of computer anxiety, (2) belief that effort contributes to successful mathematical problem solving, (3) belief that skill in mathematics is useful in life, (4) belief that there are word problems that cannot be solved by a step-by-step procedure, (5) confidence in their own ability to solve difficult mathematics problems, and (6) belief that word problems are an important component of mathematics.

"The Cellular Phone Problem" was contexualized as a requirement given by a "boss" to a "company employee" (the student). The goal was for the employee to decide which cellular phone service would be appropriate for officers of the company, based on the amount of time each officer uses the cellular phone. The basic elements required for the "mathematical analysis" were clearly listed in the problem statement: defining variables, writing equations, making tables, constructing graphs, finding slopes and intercepts, and finding points of intersection. Additional problem-solving tasks were presented as criteria suggested by the boss, e.g., that the analysis should specify the amount of usage each service allows for a total cost of $100, and should state the range of usage for which each service is cheapest. Performance was evaluated by breaking the problem down into eight component tasks that corresponded to the above eight requirements of the problem statement. At both test sites, the experimental classes outperformed the control classes in every category.

The PAT project has met with interest by the media. Outside interest in and demand for materials has been strong. An accompanying textbook to replace the current written documentation is in progress.

Redesigning Intermediate Algebra and Precalculus Using Graphing Calculators

Dani Novak, Martin Sternstein, and Osman Yurekli

Service Course Area

Algebra/Precalculus

Institutional Data

Ithaca College is an independent, comprehensive, predominantly undergraduate institution in Central New York State. The five schools—business, communications, health sciences, humanities and sciences, and music—offer a distinctive balance of professional and liberal arts preparation for careers and for life. A strong emphasis on experiential or "hands on" education pervades the curriculum, including a wide range of internship programs, performance opportunities in music, theater, and media, and in-dependent research in sciences and mathematics. In recent years, the College has risen to the top 6% of private undergraduate colleges as a source of future PhDs. A powerful commitment to outstanding teaching is reflected in internal and external awards to faculty, faculty development funds for instructional improvement, and an active schedule of workshops and e-mail conferencing on enhancement of teaching. In 1995 the College established the Center for Educational Technology, a staffed laboratory of high-end microcomputers and related equipment to train faculty in use and development of multimedia course materials and course use of the Internet.

Abstract

The project goals were: (1) to convince our department to approve a new set of courses that would allow students to take an alternate route to improve their math competency and prepare for business math and calculus courses; (2) to make the newly designed courses interesting, fresh, and motivating for both students and teachers, connecting math and life and helping students learn in a meaningful way by "doing math"; (3) to develop materials which teach traditional skills with new and motivating methods that make use of technology; and (4) to develop materials which prepare students for the new ways of teaching calculus based on the calculus reform movement.

Project Description

This project was motivated by a need to improve student learning in two courses that were basically remedial in nature and were used to prepare students for subsequent math courses. A great deal of thought, debate, research, and testing resulted in two new courses. The first, Power Algebra, is a college algebra course motivated by applications. Students in this course are asked to make conjectures and predictions about real-life applications and to actively participate in constructing simple mathematical models. The focus is on mathematical concepts and thinking rather than on algebraic manipulations. The second, Dynamic Functions, uses a blend of geometric, numerical, and symbolic approaches. A graphical and analytic analysis focuses specifically on exponential, logarithmic, and trigonometric function theory, motivated by practical problems. Students explore and develop precalculus mathematics while focusing on mathematics as a practical tool.

Both courses were motivated by historical perspectives and real-life applications. While most lower-level mathematics courses attempt to introduce applications after each topic is explained, these classes first introduce real-world problems and then derive the mathematics necessary to reach better insight into the problems. Cooperative learning was a valuable tool in these math courses. In small groups, the more mathematically proficient students would help the less proficient students with math techniques, and often the less mathematically proficient students were able to give good insights into underlying themes in new applications. In small groups the students seemed to speak up more as opposed to the regular math classes in which the weaker students often would sit passively in the background. When students explain things to one another, everyone gains. Allowing students to communicate mathematical principles proved worthwhile beyond original expectations. Students actively talked about mathematical concepts and applications.

Graphing calculators were used from the very first day. In a typical class the students may be presented with a set of data from a current newspaper, journal article, or census tabulation. The students enter or plot the data on their graphing calculators. In small groups they discuss what mathematical mode—for example, linear, quadratic, rational, exponential, logarithmic, trigonometric, or piecewise-defined—the data seemed to indicate. Real-life considerations of the data also entered into the group discussions. Each group discovered a mathematical expression for the model they thought best reflected the given data. The students were then asked to make various future predictions based on their model. One person from each group would come forward to explain their models and predictions for a class discussion. Sometimes with more complex models the group work and class presentations were extended into two or three periods. Longer group projects sometimes involved outside class meetings and joint papers.

As a result some math-phobic students began to blossom, talented students became challenged, and almost all students became immersed in a mathematical experience probably different from anything they had seen or done before. Most students quickly overcame initial frustrations at not being told exactly what to do and how to do it. They accepted being pushed to think and discover for themselves, and they were impressed by their own insights and abilities.

The end-of-course evaluations indicated that after becoming familiar with this methodology, students felt that they could "do" mathematics. The students began to appreciate the power, beauty, and accessibility of mathematics. They found that mathematics expanded their understanding of the world around them, and they became comfortable using technology to obtain results and insights into a variety of everyday occurrences, from reading the *New York Times* to choosing among various car loan offers.

The instructors for these courses must be knowledgeable in the use of graphing calculators and should be open to new ways of teaching remedial courses. A willingness to try cooperative learning techniques is helpful. The use of technology in the classroom motivated students to learn. Any apprehension the students had about using graphics calculators disappeared after a few weeks. Math became fun and interesting.

Sample Materials

Exploring Instantaneous Slopes

Enter the following program into your calculator.

```
PROGRAM: INSLOPE (Means Instantaneous slope)
:Input X
:y1→A
:X+.00000001→X
:y1→B
:(B–A)/.00000001
```

To test the program, put x^2 in y1 and run the program INSLOPE. When the ? appears, type 1. The answer should be 2.

Let us check it "by hand": Pick two points, (1, 1) and (1.001, 1.002001). Note: 1.002001 = 1.001^2.
Indeed, (1.002001 – 1)/(1.001 – 1) = 2.001 ♠ 2.

What we want to do now is to use this program to find instantaneous slopes for various functions and see if we can discover a pattern.

y = x ^ 2

x	y	slope
1	1	2
2		
3		
5		
3.5		
–7		
x		
w		
–2		

y = x ^ 3

x	y	slope
1		
2		
3		
5		
3.5		
–7		
x		
w		
–2		

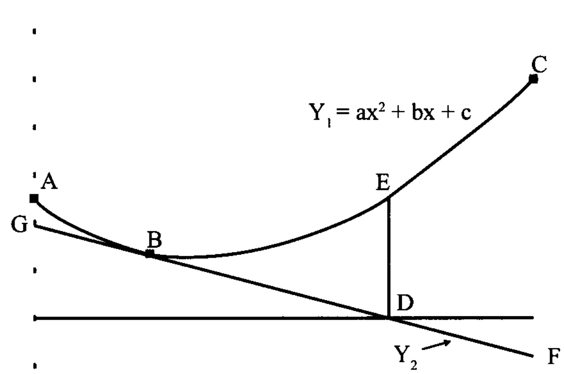
y = x ^ 2 + x

x	y	slope
1		
2		
3		
5		
3.5		
–7		
x		
w		
–2		

General comments for teachers

You can ask your students to compute an approximation to the slope for several cases by direct computation, without the use of the program. This will prevent them from becoming "robots." One can use these exercises to explore the slope of different functions not using algebra, but using "mathematical experiments." After the students become convinced that there is a pattern, they are ready to ask the question "why is there a pattern?" to which algebra (and the concept of limit that will be introduced in a later course) gives the answer.

A "holistic" graphics calculator problem. The problem is visual in nature, allows students to check results on the calculator, and embraces different areas in math.



1. Use the data in the table and the assumption that the equation of the model that passes through these three points is of the form y1=a*x^2+b*x+c to define three equations with three unknowns. Have the students use their knowledge of matrix multiplication and the concept of an inverse of a matrix to solve these three equations. Try not to solve them mechanically on the calculator without having them understand how the calculator technique is related to matrix multiplication.

The data given "by experiment" is

	A	B	C
X =	2.3	3.3	6.8
Y =	2.375	1.275	5.3

where x represents time in years and y represents temperature in degrees.

2. Find the equation of the tangent line through the point B.

3. Find the coordinates of the points F and G.

4. Find xmin, xmax, ymin, and ymax and graph the picture on your calculator. Then trace the graph and report what happened.

5. Find the coordinates of the points D and E using algebra. Then trace the graph and check that the answer is indeed correct.

6. Find the distance between the points D and E.

7. Find the distance between the points C and F.

8. Find the distance between the points C and D.

9. Using the model y1 and your knowledge of algebra, at what times will the temperature be 1.2 degrees? Confirm your answer using the graph.

General Comments

The previous exercise can be called a "holistic graphing problem" because it embraces various areas at once. For example, the problem above asks the student to "reproduce" a picture on a calculator. It uses knowledge of matrices and the idea of the slope. The use of the graphics calculator allows the students to check if their answers are correct and thus encourages independent thinking. The students are not dependent on answers at the end of a book or on a teacher to determine if they are correct or not. They have a tool, the calculator, to help them check whether or not they are correct. The students seem to love these problems. If you are interested in communicating with me, to share your own examples or to receive more examples, contact me at my e-mail address.

Comments and suggestions for teachers about the approach to teaching this material

1. For the holistic graphics calculator problem above, matrices were explained to be an extension of numbers and the students explored the similarities. For example, explain how solving a system of equations is an extension of solving a single equation like 3*x = 5. The graphics calculator allows quick computation. After the students find the equation, they can define the function and test whether they are correct.

2. To find the equation of a tangent line, one needs to find the slope. A simple approach is to take two "close" points. You can also teach your students to program the calculator to find the numeric slope. Also zooming in with the calculator gives a feel for the "linearity" of the function for small time intervals.

Permission to print this material was obtained from Dani Novak, Martin Sternstein, and Osman Yurekli.

Earth Algebra: College Algebra with Applications to Environmental Issues

Christopher Schaufele and Nancy Zumoff

Service Course Area

College Algebra/Precalculus

Institutional Data

Kennesaw State University is a progressive state university in the University System of Georgia, enrolling over 12,000 undergraduate and graduate students. The University offers opportunities for concentrated study in the arts, humanities, the sciences, and the professional fields of business, education, health, and social services.

The Department of Mathematics has 28 full-time faculty, offers a Bachelor of Science degree in mathematics, a joint BS degree in mathematics education, and support graduate courses for a master's degree in K-8 education. The University system has a 10-hour requirement in core mathematics courses for all undergraduate degrees, and therefore a primary role of the department is service.

Abstract

Earth Algebra is an entry-level college algebra course which incorporates the spirit of the NCTM's *Curriculum and Evaluation Standards for School Mathematics* [122] at the college level. The context of the course places mathematics in the center of one of the major current concerns of the world, i.e., environmental issues and global warming. Through mathematical analysis of real data, students gain a new perspective on mathematics as a tool. The course incorporates group work, written reports, simple mathematical models, and the use of technology. All these strategies remove the traditional lecture style as the means of delivery in the course, and make mathematics a "hands-on" subject.

A preliminary edition of a textbook, *Earth Algebra: College Algebra with Applications to Environmental Issues* [149] authored by the project directors, was published by HarperCollins in January, 1993; the first edition was published in 1995. The text can be used for a "Liberal Arts Mathematics" course or can replace traditional college algebra. The course can be taken by all beginning college students with a minimal high school background of Algebra II. Earth Algebra was piloted and is now fully integrated into the curriculum at Kennesaw State, enrolling approximately 2,000 students. It was tested at selected institutions around the country before publication of the text.

Project Description

The impetus for this project was a desire to make college algebra more interesting. The project directors were appointed co-chairs of an ad hoc committee at Kennesaw State with the charge: "Do something about college algebra." The design, implementation, and evaluation of this project were funded by a grant from the NSF and a FIPSE Grant from the US Department of Education. Additional support for the initial development was provided by Georgia Power Company and Kennesaw State .

The project resulted in the formation of Earth Algebra, a college algebra course which can be taken by all beginning college students with a minimal high school background. It is the general entry-level course for non-science/mathematics majors at Kennesaw State, followed by either a one-term business calculus or introductory statistics course or a two-term elementary education sequence, depending on the chosen major. At other institutions with a single college algebra/precalculus course, the Earth Algebra material may be supplemented with additional topics needed for the study of calculus. The intent was to make a course that was more interesting for both the students and the instructors. This goal was achieved by focusing on environmental issues. In our society there is increased emphasis on conservation and problems of the environment, so most beginning college students enter with at least a superficial awareness of these issues.

The course begins with a class discussion of environmental issues, specifically global warming, its causes and effects. Relevant real data is made available to the class and modeled through class participation and discussion led by the instructor. After initially demonstrating modeling procedures, the role of the instructor evolves into that of providing direction, motivation, and guidance for the students. The class is divided into small groups, and much of the work is done in these groups. Communication skills, both written and oral, are required for the group presentations and reports. Some student research to obtain data may be required.

Technology is an integral feature of this project. Graphing calculators or spreadsheet and graphing computer applications are used to reduce the repetition and drudgery of meaningless manipulation. The technology allows the students to analyze real data rather than limiting them to the study of artificial and simplistic functions and equations which are encountered in more traditional textbooks. The calculator removes time-consuming arithmetical computation and allows time for a better conceptual understanding of mathematical topics. If graphing calculators are used, it is unnecessary to have a computer lab. The calculator allows more interaction among students, can be used anywhere, and makes it easy for student groups to work outside the classroom or off-campus. Although computers can also be used, they are unnecessary. The course is designed to be independent of any particular model or brand of graphing calculator and so does not require revision when new models appear.

Throughout the course, the practicality of algebra is emphasized in the context of environmental problem-solving, interpretation of results, and decision-making based on mathematical models. Students of Earth Algebra learn that mathematics can be used as a decision-making tool and understand the relevance of mathematics in modern society.

This course has shown many improvements over the traditional algebra course. The group work and written reports have increased students' abilities to communicate mathematics topics effectively. The use of environmental issues has given students an answer to the age-old question, "What is this stuff for?" Students begin to view mathematics as a tool. This new course has shown a high success rate (82.9% received a C or better). The students did better in subsequent calculus courses than did tra-ditionally-taught students. In addition, Earth Algebra has also improved students' view of mathematics in general.

A second edition of the textbook, *Earth Algebra: College Algebra with Applications to Environmental Issues*, is due for publication in 1998. Although the methodology of Earth Algebra is very different from a traditional lecture course, no special training is required for the instructor. This project has been well received, with adoption at approximately 100 schools. The project directors are currently developing a precalculus course, containing both algebra and trigonometry, in the spirit of Earth Algebra.

Sample Materials

Text Used: Schaufele, C., and N. Zumoff. *Earth Algebra: College Algebra with Applications to Environmental Issues, 2nd ed.*, HarperCollins College Publishers, 1998.

Course Outline

Week 1: Chapters One and Two: Functions; Linear Functions

Week 2: Chapters Three and Four: Modeling Carbon Dioxide Concentration; Composite Functions

Week 3: Chapter Five: On the Beach; Test I

Week 4: Chapters Six and Seven: Quadratic Functions; Systems of Linear; Equations and Matrices

Week 5: Chapter Eight: Modeling Carbon Dioxide Emission from Automobiles

Week 6: Chapter Nine: Modeling Carbon Dioxide Emission from Energy Consumption;

Test II

Week 7: Chapters Ten and Eleven: Exponential and Logarithmic Functions; Modeling Carbon Dioxide Emission from Deforestation

Week 8: Chapter Twelve: Total Carbon Dioxide Emission Functions; Test III

Week 9: Chapters Eighteen and Nineteen: Linear Inequalities; Alternate Energy Sources

Week 10: Chapter Twenty: Creating New Models for the Future Final Projects

(Chapter numbers are based on the first edition.)

Modeling Carbon Dioxide Emission from Autos in the United States

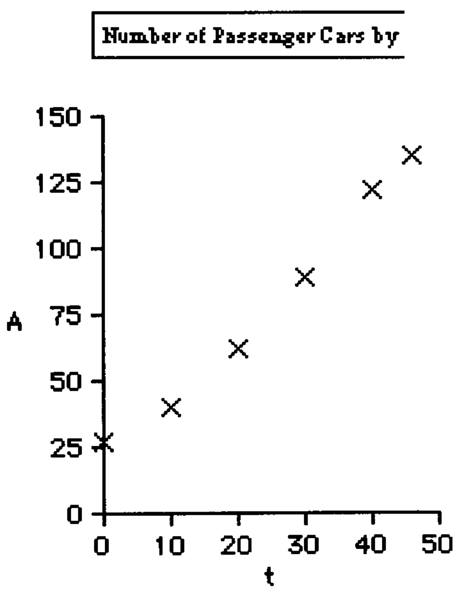
Sample Project 1: Predicting Total Passenger Cars in the Future

The table below shows the number of US passenger cars (in millions) in the indicated year.

Number of Passenger Cars by Year

Years	Cars x 10^6
1940	27.5
1950	40.3
1960	61.7
1970	89.3
1980	121.6
1986	135.4

If we use t for the number of years after 1940, and $A(t)$ for the number of cars (x 10^6) in year 1940 + t, and plot this data on a (t, A)-coordinate system, it looks like these points fall very nearly in a straight line, so we will model this data with a linear function.



You need the best fitting line. Altogether there are 15 possible lines; your instructor will assign certain years to your group, and your group is to find the best line using points corresponding to these years. Then, each group will report its result.

Use your best equation to predict the following information:

1. the number of automobiles in the US in the year 2015;
2. the year in which there will be 200 million automobiles in the US;
3. how many more automobiles there are each year in the US.

After each group has reported its best equation, then you will know what the best overall equation is, and you should save it (recycle all the others).

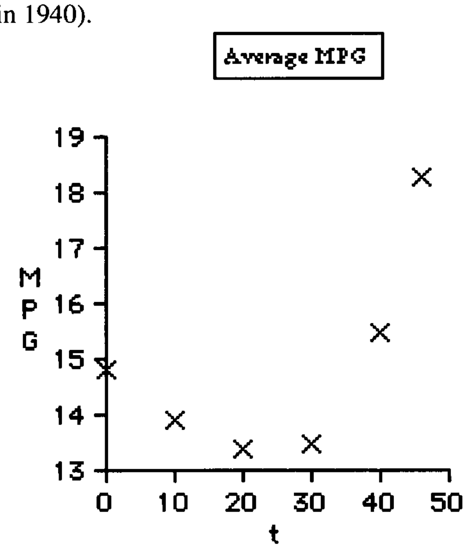
Sample Project 2: Predicting Fuel Efficiency of Passenger Cars

The next factor that we need to complete our model of CO_2 emission from automobiles is the amount of gasoline they use. The data below [2] provide the average gasoline efficiency per car. The table shows the average miles per gallon (MPG) of all US automobiles in the indicated years.

Average MPG By Year

Year	MPG
1940	14.8
1950	13.9
1960	13.4
1970	13.5
1980	15.5
1986	18.3

Plot these points on a (t, MPG) coordinate system (again t = 0 in 1940).



Look at the graph and think about what type of curve will best fit this data. After a moment's reflection, you should all agree that these points fall in a very parabolic pattern. Therefore, we will use a quadratic function to model this data.

It is your task to come up with the best parabola for this data. You should do this in your group after your instructor has assigned years to you. There will be a total of 18 remaining equations. Each group representative should report the group's best parabola to the class. Also, show its graph, do some predictions for future years, and answer this question: "When was gasoline efficiency at its worst?"

Once the reports are done, you will know the best function for gas mileage; call it *MPG*(t); save it for future use.

Discuss what social, political, or physical changes might effect the accuracy of this model.

Sample Project 3: Average Yearly Mileage of Cars

The table below indicates the average number of miles (in thousands) each automobile in the United States is driven in the designated year.

Average miles driven per car by year

Year	Avg. Miles Driven per Car per Year ($x10^3$)
1940	9.1
1950	9.1
1960	9.5
1970	10.0
1980	8.8
1986	9.0
1987	9.6

Take $t = 0$ to be 1940 and plot the corresponding points on a (t, M) coordinate system, where

$M(t)$ = average miles driven (in thousands) per car in year $1940 + t$.

These data are best modeled with two curves, the first going from 1940 to 1970, and the second from 1970 on into the future. These curves should have a common point in the year 1970 to avoid the discontinuity mentioned above. The data for 1940–1970 possibly could be approximated with a line, but a parabola is a better fit. So we choose to model these data with two parabolas which form one function called a *piecewise function*: "piecewise" because its definition changes over different time intervals.

The task is to find the best parabola that models the data from 1940–1970, and then to find the best one that models the data from 1970 on into the future. The equation for any parabola for 1940–1970 must pass through the point (30, 10.0), and the same goes for any parabola for 1970 into the future.

After all this is done, you have two equations, but only one function which models the data in Table 8.3. This is a piecewise function.

Write all possible quadratic equations that model the data from 1940–1970—you must always use as one point (30, 10.0). When you finish writing all such equations (there are only three), find the error using only points corresponding to 1940–1970. The equation with the smallest error is the best one for the time interval 1940–1970.

Next, repeat the same exercise for years from 1970–1987. Again, you must always use (30, 10.0) as one point. Find the one with the smallest error; this is the one to use for years 1970 on.

Use your piecewise function to answer the following questions:

1. Estimate the number of miles each car drove in 1953.
2. Estimate the number of miles each car will drive in 1998.
3. When will the average number of miles driven be 12,000?
4. When was the average number of miles driven 9,700?

Discuss possible limitations of this model.

Portions of this article are reprinted with permission from HarperCollins Publishing Company.

Using Educational Technology in a Precalculus Course at the United States Air Force Academy

Capt. Paul J. Simonich and Capt. Cynthia A. Brown

Service Course Area

Precalculus

Institutional Data

The USAFA offers a wide range of programs in the arts and sciences, all built on a core curriculum. At any one time there are approximately 4,000 cadets enrolled at the USAFA which include about 40 international cadets on exchange programs from other military academies around the world. Females make up about 15% and minorities about 19% of the overall population. All entering cadets must be between the ages of 17 and 22, and be unmarried with no dependents. To be admitted to the USAFA, cadets must meet high leadership, academic, physical and medical standards, be a US citizen (except for international exchange students), and be of high moral character.

Abstract

The US Air Force Academy graduates approximately 1,000 cadets each spring with Bachelor of Science degrees in 28 academic majors representing both technical and non-technical fields of study. All students complete a rigorous "Core Course" sequence which provides a solid foundation in humanities, social sciences, engineering, and basic sciences. As part of their processing into the Academy, freshmen take several math placement exams designed to place them into an appropriate math course to begin their studies. This study focuses on 375 students whose test scores placed them into a remedial, precalculus course during the Fall 1995 semester. Given the stringent technical requirements of the "Core Course" sequence, the purpose of this precalculus course was to prepare at-risk students for the rigorous core requirements.

In the past, students in the precalculus course had minimal exposure to the variety of computer technologies and software they would be required to use in follow-on courses. The Fall 1995 semester offering of the course saw a significant change in this philosophy including: (a) an increased emphasis on integrating technology early into the students' educational experience, and (b) an increased requirement for students to use computer technology. Throughout the semester, the instructors provided not only remedial assistance in precalculus topics, but also provided hands-on instruction including the proper use of the computer hardware and software to perform the tasks required for written assignments, homework, and a capstone project.

The overall objective of this precalculus course was to allow remedial math students a greater opportunity for success in the Academy's rigorous technical core. This objective was to be achieved using four main goals: (a) to increase the overall computer literacy of remedial math students, (b) to provide structured use of specific software packages used later in their academic careers, (c) to improve technical writing skills, and (d) to incorporate technology into the classroom as a way to increase the students' interest level and understanding. Specifically, the students were expected to know how to use Netscape as a research tool, how to prepare reports in Microsoft Word in a professional manner (including integrated graphics), and how to use the software package Mathematica to aid in the analysis and exploration

of precalculus topics. One section of 22 students also were required to use Microsoft Powerpoint to develop classroom presentations of historical figures in mathematics.

Project Description

The precalculus course at the USAFA was designed to assist students who will most likely have difficulty completing the rigorous technical requirements of the core: two semesters of calculus, a semester of statistics, two semesters of physics, and nine engineering courses. Those who choose a technical major are required to take additional math courses. Hence, the precalculus course discussed here is essential to the success of those who come to the Academy with deficient math skills.

The requirement to use computer tools to solve problems was significantly increased from previous semesters. First, students were required to complete a written exercise comparing and contrasting quadratic and cubic equations. They were required to use *Microsoft Word 6.0* and were graded for both content and format/presentation. The students were introduced to the software package *Mathematica* in a one-hour, self-paced (non-graded) tutorial in a computer lab with instructors providing lots of initial help. The students were expected to practice using the *Mathematica* software on the computer in their dorms. Four lessons later, they used *Mathematica* to do a study of transformations of algebraic functions for a graded assignment. Eight lessons later, they were once again given a graded assignment to study transformations of trigonometric functions using *Mathematica.*

The capstone project for the course was available only "electronically" through a link on our Math Department's home page. The students used *Netscape* to go to our home page, do some reading/research, and download pertinent information. They were then required to do analysis on optimizing a "space garbage can" using *Mathematica.* Finally they were required to use *Microsoft Word* to prepare a report of their findings including integrated graphics. The infusion of technical writing into the precalculus students' projects was viewed favorably by the USAF Academy's Department of English.

A copy of the project is included in the Sample Materials and may be accessed on the web. The URL for the project is: http://www.usafa.af.mil/dfms/projects/math130/project.htm

Sixty-five percent of the students preferred the electronic format to the traditional paper/handout format (22%). The most common complaints were network down-time, the slow *Netscape* connection, and problems producing *Mathematica* plots. The complaints the students had were greatly overshadowed by the positive experience. Some of the positive student comments included: "I liked the fact that it combined lots of different things that we've worked on into one project: graphing, equations, *Mathematica.*"; "Having a project like this made the actual work more interesting than if it had just been listed out on a sheet of paper."; "The way it was presented on the net made it seem like a *real* project. This motivated me to learn and try hard to do my best on the project."; "I liked using the net to access information. It was a fresh way to learn."; and "The project was fun to do because it was something different, forcing me to learn how to use certain tools on the computer."

Sample Materials

Group Solve #2
Space Shuttle Experiment Takes on a New Shape

The purpose of this group solve is to use a rational function in an application problem. Besides using Mathematica to do mathematical computations and graphs, we want you also to interpret the numbers and graphs for this particular application problem. You will be examining the use of a new container as a possible housing for the VIPER Space Shuttle experiment. The end goal is to find the dimensions of a square-based container that will minimize material cost while meeting a specific requirement for volume.

Because this is a group solve, we expect you to work very closely with the other members of your group and to share the work as equitably as possible. Other than working with the members of your group, you may seek assistance from your instructor. You may use your textbooks and notes. Turn in only one solution package per group.

The Task

The cadets involved in the VIPER experiment have decided that they should examine other possible shapes and sizes for the container which will house the experiment. Your task is to examine the cost/size relationship for a square-based housing with rectangular sides and a volume of 7,500 cubic centimeters. The material has not yet been ordered, and the engineers have solved the welding problem, so you DO NOT have to create the box by folding. Because the material used for the container is very expensive, you must find the dimensions of the open-topped box which will allow mission requirements to be met while minimizing cost.

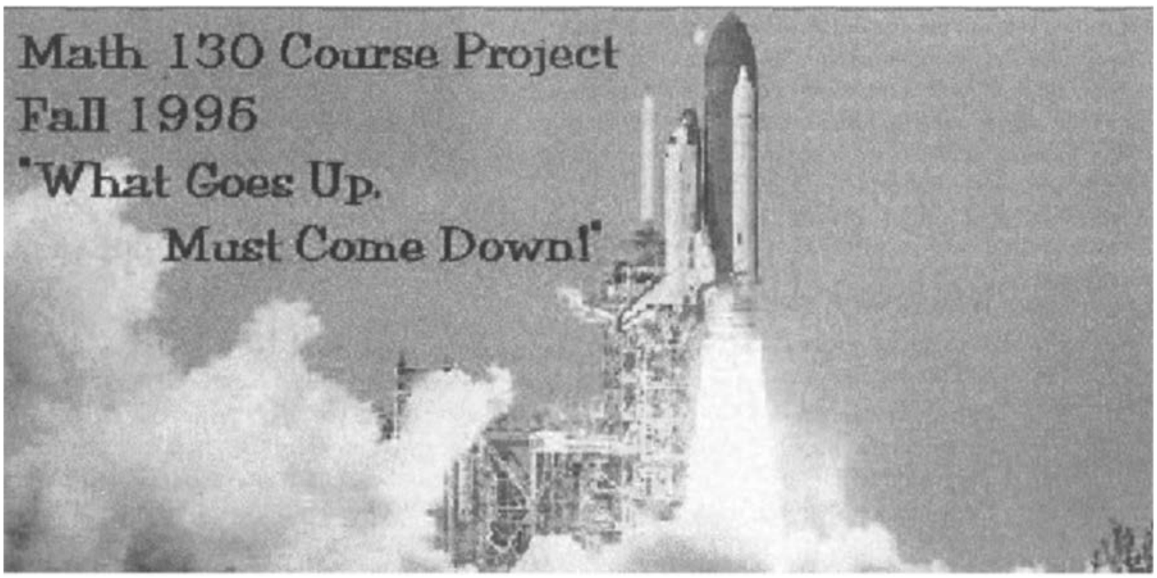
Group Solve #2
Directions & Solution Sheet

1. Draw and label a diagram (hint: square base and four rectangular sides).
2. Use your diagram to write a function in which h represents the amount of material needed to build the container (the surface area of the container). Hint: your function should contain only one variable. Have your instructor check your function before continuing.

3. BEFORE you graph your function, find the following: domain _____ and range _____.
4. Do your domain and range make sense? Explain your choices.
5. What are two Mathematica methods to find the minimum?
6. Where does the minimum occur (accurate to the nearest tenth, include units) of the finished container?
7. What are the dimensions (accurate to the nearest tenth, include units) of the finished container?

   Base _____ by _____
   Sides _____ by _____
8. Calculate the volume of your box based on the answers given above (include units).
9. Does your volume equal the specified volume? Explain.

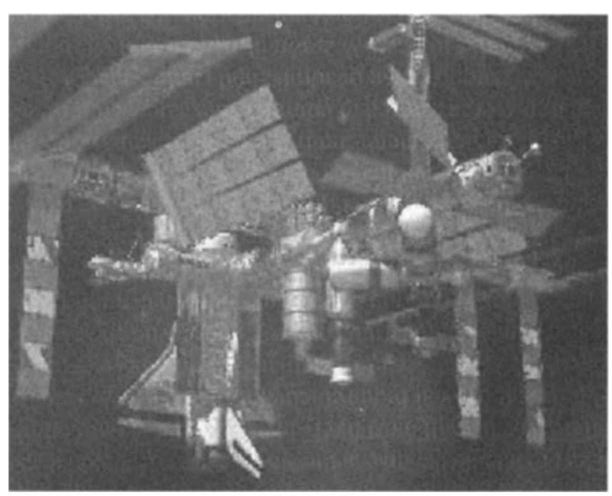
Permission to print this material was obtained from Capt. Paul Simonich and Capt. Cynthia Brown, USAFA



What Goes Up, Must Come Down!

One problem NASA's future space station will have is getting rid of refuse resulting from experiments performed, the manufacture of materials, and the day-to-day waste associated with the workers eating and hygiene habits. This refuse cannot be discharged into space because it would present a hazard to other space vehicles. Also, storage space is at a premium on the station, so it can't be stored in large quantities.

The Space Shuttle will be tasked to not only take workers and supplies to space but also bring back refuse for safe disposal, a very expensive garbage truck at that. While there will be a few large items which will require special handling, most items will be small and need a convienent storage container, a garbage can, that is also relatively small so it can be stored while waiting for the next shuttle to arrive. The proposed containers will be rather expensive since they need to handle a variety of materials and withstand a variety of extreme environments. These containers will need to fit into specified regions of the cargo bay and hence, have a size limitation. NASA would like to make these as large as possible but the cost of each container is strongly tied to its size. Congress is greatly concerned about costs associated with the Space Station and is looking to find the cheapest, yet safe, answer to all issues.



Many of these high-tech garbage cans will be manufactured and used over the years; therefore, *the problem* becomes one of trying to get the most volume at the cheapest cost. These conflicting constraints are very common in many design problems and the designer/engineer's job involves trying to satisfy the needs of both sides of the conflict.

The first female shuttle pilot and one of the potential future shuttle commanders, Lt Col Eileen Collins, also happens to have been an instructor in the Math Department here at the Academy from 1986–1989. Click here to read her biography or click here to read about the shuttle mission for which she was the pilot.

Having been an instructor at the Academy, Lt Col Collins is well aware of how sharp you Math 130 students are and has asked for your support in the design of the containers to be used to haul waste. Details on the specific design requirements can be found by clicking on the Your Specific Assignment link here or on the previous page.

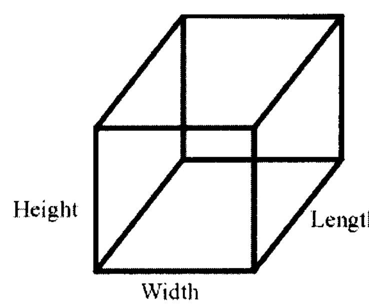
If you have any questions regarding concepts covered in this project please contact your instructor. For any technical questions regarding this project please contact Capt Sandi Beveridge at (719) 333-2891or via e-mail at beveridges.dfms@usafa.af.mil .

Your Specific Assignment

As mentioned in the mission statement, your specific assignment involves trying to make both the cost cutters in Congress and the NASA Space Station Design Team happy. You *need to come up with a recommended design* for this high-tech garbage can that is as cheap as possible while holding as much as possible. You will design your garbage can using the software package *Mathematica* and integrate the results into a letter addressed to both the Congressional Budget Office and the NASA Space Station Design Team. This letter should be formal, include any graphics necessary to support your position, and no more than three typed pages in length. An example of the letter format can be found at x:\df\dfms\math130\letter.doc. You are **strongly encouraged** to bring a draft of your letter to your instructor before it is due. You may also drop by the Air Force Academy Writing Center in 4D43 to schedule time to have your paper reviewed. Please be sure to read the Documentation Statement to ensure you know who you can get help from and, of course, do not forget to use a cover sheet to document such help.

The Garbage Can

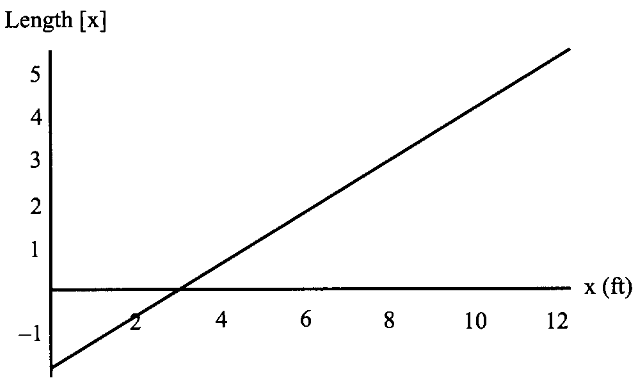
The design of our high-tech garbage can has been greatly simplified by NASA's requirement that our container be rectangular in shape, (its easier to store boxes), as shown in the diagram to the right. Moreover, NASA's research team has come up with some preliminary design data which allows the width, height, and length of the box to be expressed only as a function of the variable "x";. This design data is described in the paragraphs that follow. Since your design will be compared and competed against other designs provided by other organizations, NASA has asked that you express your volume and cost functions only in terms of the variable "x". Also note that the information provided below assumes "x" is measured in *feet.*

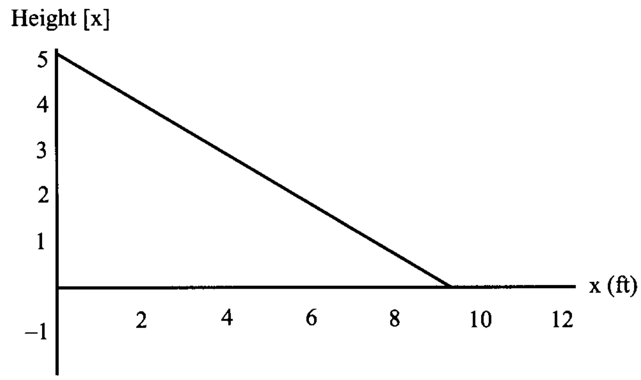


Space Station Trash Receptacle

The Volume Constraints

Due to the design of the shuttle's cargo bay, the maximum value that the width can be is 12 feet, while the only limit on its minimum size is practical considerations. The volume of our box is simply stated by the equation Volume = (width)(length)(height) + C, where the width is given to you as Width = .7(x – 2.86) and the constant C is given as 2.1. The origins of this constant are classified. The height and length functions are strongly tied to such factors as the box's center of





mass, pressure vessel considerations, and other engineering factors. All these factors combined together make height and length linear functions of x as shown below. Using your strong grasp of the Two-Point Form for equations of lines, you should be able to write these equations. Remember that your volume function should be stated as a function of x only. You might want to check these formulas with your instructor before proceeding as they are key to your design.

The Cost Constraints

The cost of each receptacle is a very complicated function that depends on the cost of the materials involved multiplied by cost of machining those materials multiplied by the cost to assemble and test the strength of each receptacle. Fortunately, NASA's cost analysis team has come up with the following information which reduces each of these cost factors down to merely a function the variable, "x", as follows:

- Materials Cost = (.5x +30)
- Machining Cost = (x +18)
- Assembly and Testing Cost = (x^2 - 10.78x + 39.3562)

You'll need to properly combine these factors together to come up with the overall cost formula before proceeding.

Technology Enhanced Connections in Statistics Sequence

Carla J. Thompson

Service Course Area

Statistics and Special Topics

Institutional Data

Tulsa Community College is the largest college in Oklahoma supporting a full multicampus system with over 22,000 students located in Tulsa, Oklahoma. Approximately 82% of the students at TCC reside within Tulsa County. Tulsa Community College is recognized as the leader in Oklahoma in the offering of college-level telecourses via cable television and is known as a leader in technology supporting many technical programs for the community that are based on technology. The two-year college serves students from four campuses who: are preparing to transfer to junior/senior level at a college or university, are preparing for specific occupational careers, are seeking continuing education, are in need of developmental or remedial programs, or are in need of retraining or updating specific career skills.

Abstract

The Technology Enhanced Connections in Statistics Project delivers a triad of innovative course offerings for students generally intimidated or fearful of traditional statistics courses. The statistics sequence includes: Elementary Statistics, Applied Statistical Analysis, and Using Multimedia Environments for Research and Test Analysis in the Social Sciences. All courses in the sequence focus on a hands-on connections and communications environment that incorporates the latest in computing technologies, multimedia, electronic networks, and sophisticated statistical software packages. The Technology Enhanced Connections (TEC) in Statistics Project supports five goals that are based on preparing students in technology and communication, aligning the curriculum in statistics with the reform movement in mathematics, connecting statistics to real-world problems, providing students with community outreach experiences for accessing real data, and positively affecting students to increase their participation and equity in mathematics. The TEC Statistics Project provides a rich, relevant environment for enabling students to access and use the latest technologies as tools for problem-solving, reporting, creating, and presenting information from "real-world" data. The TEC in Statistics Project produces student outcomes that will serve society effectively in the 21st century.

Project Description

The Technology Enhanced Connections Sequence in Statistics involves a triad of courses. These courses offer a wide range of statistical content, community field experiences, alternative assessment strategies, strong use of a myriad of technologies and enhancements, a concentrated effort on connecting statistics to real-world problems, and a focus on oral and written communication skills via technical statistical interpretation and reporting.

The elementary statistics course provides an introduction to basic statistical terminology, organization of data, measures of central tendency and dispersion, binomial and normal distributions, probability, and a variety of other statistical techniques. Emphasis is on using statistics in various experimental designs and applications. Students learn to use statistical software and are introduced to SPSS (Statistical Package for the Social Sciences). One-half of class time is spent in a computer lab with a partner. The students utilize word-processing packages to write up their lab exercises and exams. This course is a prerequisite for the other two courses in the triad.

Applied Statistical Analysis extends the number of concepts that are included in Elementary Statistics by providing advanced statistical types of analyses and methodologies. Emphasis in the course is on applying advanced statistical techniques to specific application areas such as social science, business, and medical fields. Research techniques, field work, and computer analyses are the tools involved in conducting the statistical applications in this course. Students will make substantial use of SPSS for Windows. This course involves many group projects and full lab experiences requiring substantial report writing of results. Students are also given an introduction to hands-on use of multimedia equipment.

Environments for Research and Test Analysis in the Social Sciences focuses on the use of sophisticated statistical software applications in research and test analysis with emphasis in social science fields. In this class SPSS is integrated with other multimedia packages. The entire class time is spent in a computer lab working with a partner. The entire course focuses on hands-on use of multimedia technologies.

The Technology Enhanced Connections in Statistics project supported the following project goals:

1. Technology Goal: To prepare students for the 21st century using technologies and communication skills necessary for effectively functioning in the world of tomorrow.
2. Reform Goal: To align the Tulsa Community College statistics curriculum with the mathematics reform standards posited by the professional organizations (AMATYC, NCTM, MAA, ASA, et al.)
3. Connections Goal: To connect statistics to real-world issues and applications for students to relate statistics to life.
4. Community Outreach Goal: To create partnerships with community agencies for field/on-site statistical experiences.
5. Affect Goal: To provide a positive rich environment for the learning of statistics to dispel math anxiety and to encourage increased participation of all students in mathematics.

The use of technology is an integral component of each of the five goals of the project. Inherent in the Technology Enhanced Connections Statistics model is the need for many types of technologies. The technologies used in the statistics sequence of courses include: Pentium processor microcomputer multimedia stations/labs, laser and color printers, sound cards, CD Rom, speakers, VHS player, laser disc player, camcorder, Quick Take camera, SPSS for Windows, animation, graphics, and presentation packages.

Computer laboratories have changed statistics from a lecture, formula-based, number-crunching topic to a hands-on real world data analysis relevant topic. User-friendly computer environments allow students to feel comfortable and to focus readily on problem solving and reporting/writing and preparing for a technological world. Real data sets of virtually unlimited size and complexity are now solvable for students. Since students are not restricted by data set size or class lecture time constraints, they have more field experience time and on-site problem-solving. Providing a rich relevant environment through the technologies encourages positive attitudes and success in problem-solving for students. The use of multimedia technology as well as electronic networks has recently sparked world interest. Multimedia presentations of statistical findings changes a perceived "dull" topic into one of great interest. Using statistical software for analyzing data provides success for all students. Support for the acquisition of this equipment and software was generated by both faculty and students. Many of the students already owned multimedia machines, and several faculty members needed the SPSS package for conducting evaluation and research projects. The Internet laboratory was requested by all departments and students. Other peripherals were gradually added to the project through yearly requests and small internal grants.

Internal measures of the effectiveness of the Technology Enhanced Connections in Statistics project encompasses an interactive group of individuals affected by the program including:

(a) Students enrolled in statistics: Pre- and post- assessments of affective and cognitive measures have been collected on students enrolled in statistics for the past six years (1991–1996) using the Math Rating Scale [83] for assessing attitudes towards mathematics in the four areas of self-confidence, pressure, motivation, and study-habits. Cognitive assessments in statistics have been accumulated using grades in course, attendance, writing samples, and exam scores. A comparison of cognitive and affective outcomes of students from 1991–1993 with student outcomes from 1993–96 has revealed significant differences in the two time periods, with the latter three-year period demonstrating higher cognitive and affective outcomes, especially on confidence and motivation subscales.

(b) Students not enrolled in statistics: An increasing number of students are choosing the statistics sequence as elective courses and/or selecting statistics in lieu of other math requirements.

(c) Other math faculty and related colleagues: Many of the other math faculty have begun integrating technology into their sequence of math courses.

(d) Administrators: The administration is beginning to take proactive measures to accommodate the needs of students rather than reactive measures.

(e) The instructor: Measures of instructor effectiveness have surged with positive outcomes as compared to the teaching of statistics without technology during the same time period.

Several external measures of evaluation identify the TEC in Statistics effort as an effective program for students as well as the community. Twelve successful student projects have been selected for paper presentations at local and state organizations and conferences. The project director has received many positive letters from staff, personnel, and officials from various community agencies applauding the efforts, findings, and high quality of performance of the students' projects. Students who successfully complete the TEC statistics program at Tulsa Community College and continue their education at other institutions have a high success rate in sequential statistics courses. In addition, a considerable number of faculty members from public and private colleges and universities across Oklahoma have recommended to their students that they take their introductory statistics at Tulsa Community College.

The Technology Enhanced Connections in Statistics Project is ongoing in its progress to develop more and more exciting and creative cognitive and affective experiences for students that transcend present technologies and that transcend present thinking.

SAMPLE PROJECTS DESCRIPTION

I.M.A.G.I.N.E. Enjoying Statistics: Innovative Multimedia Applications Generating Insight into Natural Environments

Imagine learning statistics using:

* a hands-on approach to manipulating real world data
* innovative multimedia computer applications
* powerful statistical analyses software
* high quality computer-generated graphics, graphs, and charts
* sophisticated technological presentation modes
* original data bases generated from local community agencies as field sites
* interfaced video, sound, slides, cartoons, graphics, drawings, and text
* on-line literature searches and reviews
* cooperative partnerships and group mentoring activities
* hypothesis testing and the scientific method for solving real world problems;

and imagine students rather than an instructor conducting each of these activities. These are the components of **IMAGINE**: Innovative Multimedia Applications Generating Insight into Natural Environments for learning and enjoying statistics.

The focus of the **IMAGINE** approach to teaching/learning statistics is to provide a student- centered classroom environment where statistical concepts and applications are discovered and explored through student-created, student-directed investigations using original data from local community agencies. Students select an agency or project of interest for investigation by partners or groups; generate research questions; and submit their proposals to both the agency and the instructor for approval. Students conduct a review of the literature using on-line resources. Students then design their projects, collect data, and direct their studies onsite as determined by the project. Some titles of student-generated projects include: "Assessment of Teenage Pregnancy in Oklahoma": "Examination of Divorce Rates in the US"; "Comparison of Traditional and Reform Methods of Teaching Algebra"; "Analysis of Factors Contributing to Child Abuse in Oklahoma"; "Examination of Income levels by Geographic Region"; "Demographic Examination of Domestic Violence."

After students have acquired their data and completed the field-site portion of their project, they begin the process of coding, data entry, and analysis using highly sophisticated statistical software. Results are examined, conclusions generated, and the student research group begins the process of creating a professional presentation of the research findings. Students access a myriad of multimedia and innovative technology tools for developing their presentations. High quality computer-generated graphics, charts, tables, slides, cartoons, pictures, videoclips, drawings, sound clips, text, and animation are interfaced with statistical output to create a professional quality presentation of findings for the field-site agencies.

Student multimedia presentations serve as portfolios of student accomplishment. Imagine assessing students by their portfolio productions rather than by pencil/paper tests. Imagine allowing students the freedom to explore, create, and acquire knowledge within their natural world. Imagine a classroom atmosphere that promotes critical thinking, cooperative learning, and creative expression while covering traditionally fearful or dry statistical concepts. Imagine "trusting" students to learn under their own direction and impetus. Imagine students enjoying statistics through **IMAGINE!**

Sample Project

The following is the result of a student project. This is the finished product that they would present to a field-site agency.

Divorces in the United States

Student Project

Problem

Is there a difference in the mean of divorces between the Northeast, Midwest, South, and West?

Research Hypothesis

There is a difference in the mean of divorces between the Northeast, Midwest, South, and West.

Null Hypothesis

There is no difference in the mean of divorces between the Northeast, Midwest, South, or West.

Conclusion

In conclusion, our results show that there is no significant difference in divorces between the regions.

Implications of Findings

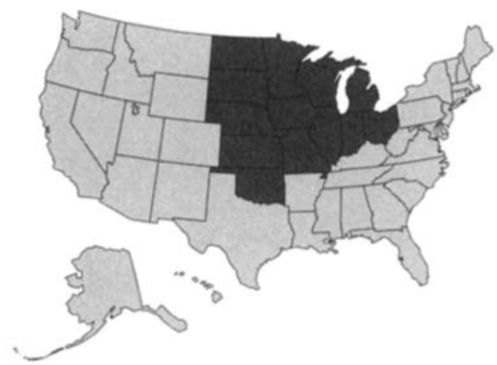
Divorces do not differ in any region of the United States.

Region 1

Region 1 includes all states in the Northeast.

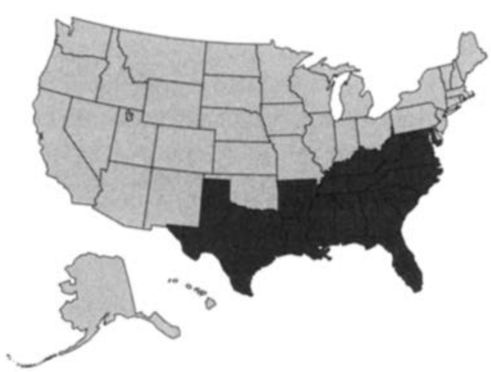
Region 2

Region includes all states in the Midwest.



Region 3

Region 3 includes all states in the South.



Region 4

Region 4 includes all states in the West.

Permission to print this material was obtained from Carla J. Thompson.

Combining Technology with Cooperative Learning in a Foundation Course for Quantitative Literacy

Ed Wright and Don Pierce

Service Course Area

Quantitative Literacy/Special Topics

Institutional Data

Western Oregon State College is Oregon's oldest public institution. Western is a small liberal arts college, offering 34 undergraduate degrees in the liberal arts and advanced degrees in education. The college has an enrollment of approximately 4,000 students. Ninety percent of all freshmen live in residence halls, and a significant number of commuting students transfer in after taking courses at local community colleges. Ninety percent of Western's students are residents of Oregon, with the majority from small city and rural environments. Nearly 50% of all entering students place into a remedial mathematics course.

Western has one of the most comprehensive Local Area Networks for a college of its size. Fourteen Novell servers support all aspects of academic computing, electronic library access, and general faculty support. The network has CD-ROM servers, connections with regional libraries, and an array of scanners and printers. There are approximately 160 PCs available for student use, consisting of both IBM and Apple machines. Students also have dial-in access to the network through the phone system.

Abstract

This course replaces the traditional general education course in both mathematics and computer science with an activity-based interdisciplinary course. Both disciplines benefit from the interdisciplinary approach—mathematics from the incorporation of technology; and computer science from the context given to technology skills. The course has dramatically increased the student success rate for passing the course (from 60% to 95%) while simultaneously cutting instructional costs by over 50%. A comprehensive assessment demonstrated significant student learning gains and dramatically improved student attitudes in the new course.

There is ample evidence that the traditional mode of instruction is not optimal. This is particularly true for students in "required" mathematics courses. Active involvement, cooperation, and personal interaction are much preferred. The instructional delivery system used in this project combines technology with cooperative learning strategies to create a rich experiential learning environment where instructors act as facilitators rather than lecturers, and students learn not only course content, but skills that can improve their performance in subsequent courses and later in life. Working in groups, students explain ideas, discuss strategies and share their knowledge. They help, encourage, and support each other's effort to learn. They are empowered to monitor and take charge of their own learning and, as a result, become responsible for their own progress and less dependent on faculty and staff. In addition, these students develop the qualities industry tells us are most needed in the workplace.

The initial project, *Reforming the Teaching of Entry Level Mathematics in the Electronic Age,* involved the conversion of Western's general education mathematics and computer science courses from separate lecture-based courses into a single interdisciplinary course. The development process

began by defining the role of this course in the lives of future students. The general beliefs of the project directors were that: (1) College graduates should be scientifically literate; (2) The real world is not organized by discipline; (3) Technology can enhance learning; (4) College graduates should be work-ready; (5) Educators must maximize the amount of learning per instructional dollar spent; (6) All students can learn.

Project Description

The design of the course was developed under a grant funded jointly by the Oregon State System of Higher Education and Western Oregon State College. This course was developed because of the belief that every college graduate must have some basic proficiencies in mathematics and technology. At Western Oregon State College, like many colleges today, the study of the mathematics required for a liberal arts degree is limited to a single course. Such courses, because of their role in the curriculum, are populated by students with low or remedial skills—students who have little or no knowledge of the major role mathematics plays in a modern technological society. Environmental and fiscal policy issues facing today's electorate will profoundly affect the future quality of life, and the future habitability of this planet. Every educated person must have the basic mathematics proficiency necessary to understand these issues.

The classroom used for the project is equipped with a networked laserjet printer and an instructor's station connected to a projection plate. The instructor hands out the activity for the day, perhaps gives a brief summary of the activity, and then opens the rest of the class period for group work on the activity. All instruction takes place when students ask questions about the "critical thinking questions" embedded in the activity. The software used is Microsoft Works for Windows. The choice of this very general software makes the project extremely portable, requiring only a word processor and spreadsheet.

Students learn the basics of spreadsheets and word processing early in the course. Daily journals are kept and evaluated. Learning skills, dynamics of working in groups, and measurement of learning are addressed in the class. Students investigate computers and operating systems; a brief history of mathematics; algebra; descriptive statistics; probability and inferential statistics; functions, modeling, and curve fitting; and essentials of the Internet. A tremendous carryover of technology skills to other classes and applications has been observed. With the new paradigm, the availability of personal help in a non-threatening environment has provided a necessary bridge to success for non-traditional students. The teamwork inherent in this project has also benefited students with disabilities. The format enables the student to share information, relate to other students in the class, and journal their thoughts.

A comprehensive third-party assessment of this project was conducted by an independent Teaching Research agency. Results showed that the students in the interdisciplinary mathematics/computer science course outperformed students in the traditional liberal arts math and introductory computer science classes in virtually every regard. An extensive pre/post attitude survey revealed significant attitude changes in the students in the reformed program. There was a significant decrease of anxiety levels in working with mathematics and computers. Students felt that their ability to write a high-quality report, including technical information, increased significantly. Students in the traditional mathematics course did not change their attitudes significantly in any way. Common final examination questions revealed that students in the interdisciplinary course averaged slightly better comprehension on conceptual questions. This is especially significant because over 40% of the weaker students in the traditional class had dropped by the final exam time, compared to only 5% of the students in the new course.

This project also reduced the cost of instruction by reducing the number of math and computer science classes that need to be offered. First, class size is increased to 60 students working in teams of three (with an instructor and two senior student facilitators); and, second, the student completion rate is dramatically increased. All indications are that this is an excellent program with an approach that can be transported to other disciplines.

Sample Materials

Text Used: Wright, E. and D. Pierce, *Skills for Life: A Foundation Course for Quantitative Literacy*, Prentice Hall, 1996.

Course Outline (MTCS 100)

Week 1: Introduction to DOS and Windows
Problem Solving/Team Roles
Week 2: Word Processing
Accessing Information Electronically
Basics of Spreadsheets
Week 3: Descriptive Statistics on Spreadsheets
Week 4: Midterm Exam; Apportionment
Week 5: Variables; Relations and Functions
Linear, Quadratic, Exponential
Logarithmic Functions
Week 6: Power, Inverse Functions
Finance; Curve Fitting
Week 7: Curve Fitting
Week 8: Introduction to Probability
Binomial and Normal Distributions
Week 9: Distribution of Sample Mean
Confidence Intervals;
Hypothesis Testing
Week 10: Hypothesis Testing; Final Exam

Activity 8.8 — Curves of Best Fit II

In this activity, we will use the tools we have developed to model global issues, such as population growth, global warming, and agricultural productivity.

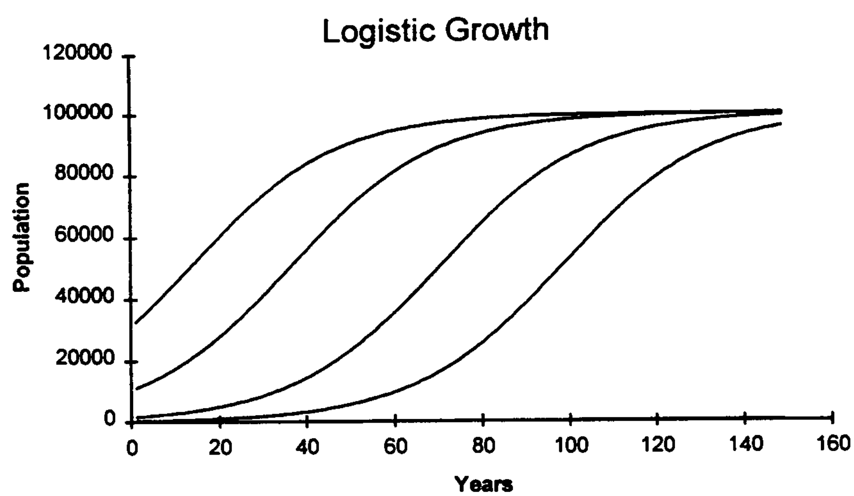
Another model for population growth

In 1838, a Belgian mathematician, P. F. Verhuist introduced a new model for population growth, which he called logistic growth. Today, we refer to the equation he used as the logistic equation. The logistic equation has two parameters: a growth rate *r*, and an environmental capacity of *L*. When the population is small compared to the environmental capacity *L*, the population grows exponentially, but as the population nears the environmental capacity, the rate of growth changes. Any population with finite resources might be expected to experience this type of growth. In particular, this may be the best model for world population.

With $P(t)$, the population at the end of year t, r the maximum rate of growth (expressed in decimal form), and L the environmental carrying capacity, the logistic equation is

$$P(t+1) = r*\left(\frac{1-P(t)}{L}\right)*P(t)+P(t).$$

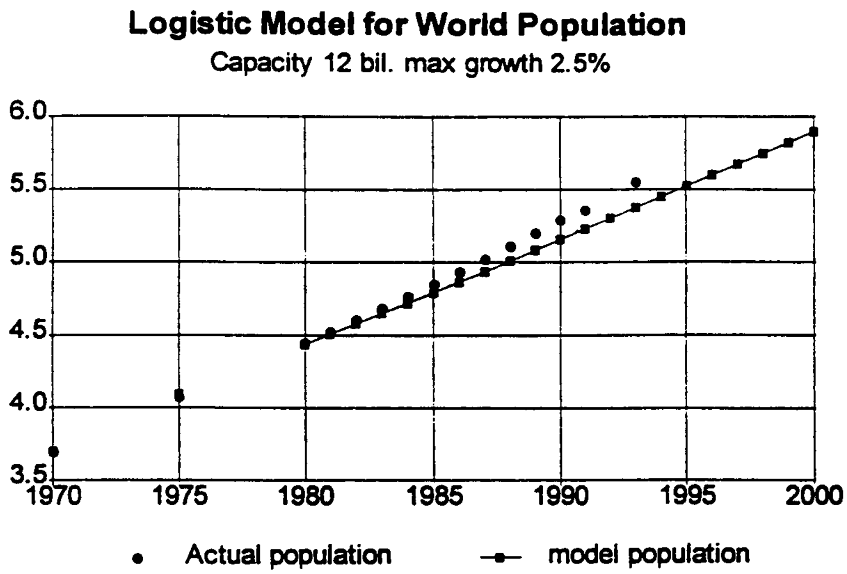
The graph below shows the logistic growth model for several different initial populations, where there is an environmental capacity of 100,000.



To model the world's population, we need to determine the value of both the maximum growth rate *r*, and the carrying capacity. We might start with the assumption that the maximum growth rate is 2.8%, since this corresponds to the doubling time of 25 years, the value observed by Benjamin Franklin early in this country's history. However, since some of this growth was from immigration, we will use 2.5$ as the maximum growth rate. There is widespread disagreement about what the actual carrying capacity of the planet is, but almost all estimates range from between 8 and 15 billion. The source of most of the disagreement is the role technology will play. Many people believe technological breakthroughs will allow the number of humans to become bery large. Others feel that the byproducts of our current technologies are destabilizing the biosphere and will have the long-term effect of lowering the carrying capacity.

1. a) (10 points) Use the embedded spreadsheet to graph a logistic model for the world population. Adjust the carrying capacity until your model achieves the best fit possible with this value of *r*. Report the carrying capacity used in your model in the chart's title. Then set the spreadsheet so the chart showing both the actual data and your model curve shows. Note, the spreadsheet is set up so that you only need to change the value in cell A2 to change the model. Also note, the root mean square error is calculated automatically each time you change the carrying capacity.

b) (10 points) Compare your model to the actual size of the world's population shown by graphing the residuals. Comment on any trend visible in the residuals, size and paste the graph below.

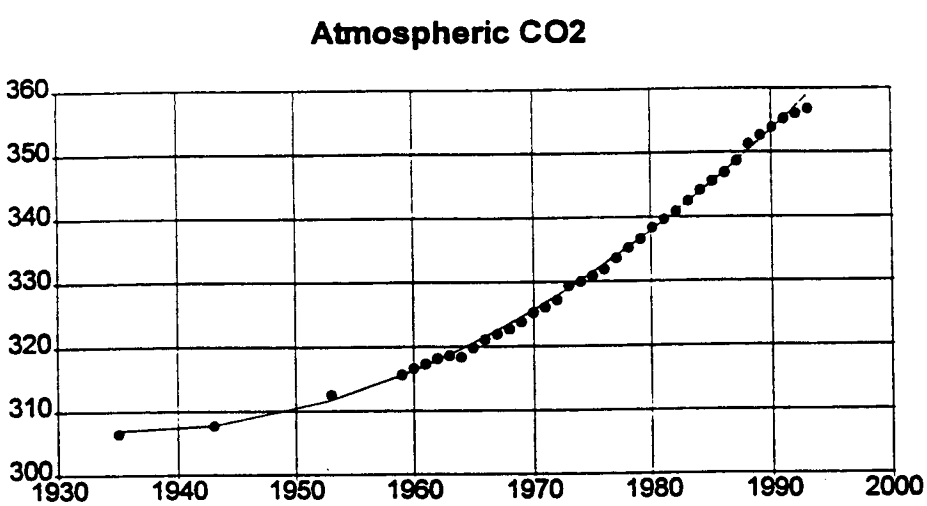


c) (5 points) At the 1994 United Nations population conference in Cairo, a 20-year "program of action," designed to stabilize the world population at about 7.25 billion by 2015, was endorsed. Compare this value to the value predicted by your ligistic model for the year 2015.

d) (5 points) Using the model you have developed, calculate the relative change in the population between 2050 and 2051. Compare this value to the relative change in the population between 1990 and 1991. Comment on the signigicance of these two numbers.

Global Warming

In 1990, the Intergovernmental Panel on Climate Change predicted that each doubling of the concentration of CO_2 in the atmosphere will lead to global warming of 1 to 5° C. Global warming of only a few degrees may significantly raise the sea level, flooding a significant amount of populated land. In addition to the burning of fossil fuels, the destruction of the tropical rainforests further adds to the buildup of carbon dioxide in the atmosphere. The table shows average annual global temperature and the concentration of CO_2 in the atmosphere. There are many short term influences on global temperature, such as sunspot activity or volcanic eruptions, so you would not expect an exact relationship between CO_2 concentration and temperature.



2.a) (10 points) The embedded graph/spreadsheet contains data on the atmospheric concentration of CO_2 from 1935 to 1993, along with the average global temperature for these same years. The chart also shows the quadratice function that best models the data on CO_2 concentration. Use the associated spreadsheet to calculate when the concentration of CO_2 will reach double the 1935 level.

b) (10 points) Use a curve-fitting routine to find the exponential curve that best predicts global temperature as a function of the concentration of atmospheri CO_2. Give the equation of the curve, and comment on how well the model function seems to fit the data.

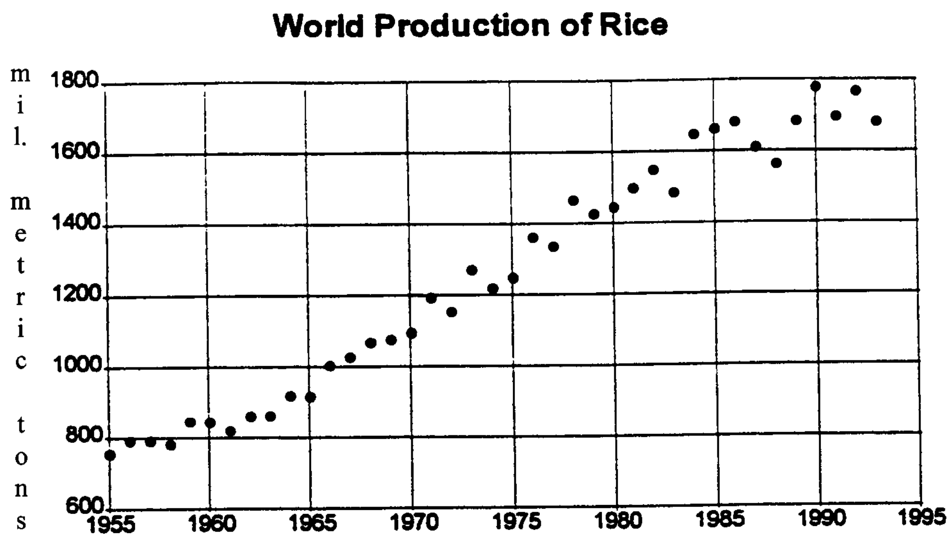
c) (5 points) Use this model to estimate the increase in global temperature that accompanies a doubling in atmospheric CO_2.

d) (5 points) How does your answer in part c) compare to that of the Intergovernmental Panel on Climate Change? In what year will this increase be reached?

Agricultural Productivity

Malthus did not anticipate chemical fertilizers and modern farming technologies that allowed vast increases in agricultural productivity during the industrial revolution. How about more recent times? Rice is a significant portion of the world's diet. The roughly 5.3 billion people in the world in 1990 consumed 1780 million metric tons of rice. That's 3,916,000,000,000 pounds of rice for 5,300,000,000 people or almost 740 pounds of rice for every human on the planet.

3. (10 points) The embedded spreadsheet below shows the world production of rice and world use of fertilizer in millions of metric tons. Use a curve-fitting routine to find rice production as a function of fertilizer use. How closely does the model fit the data?

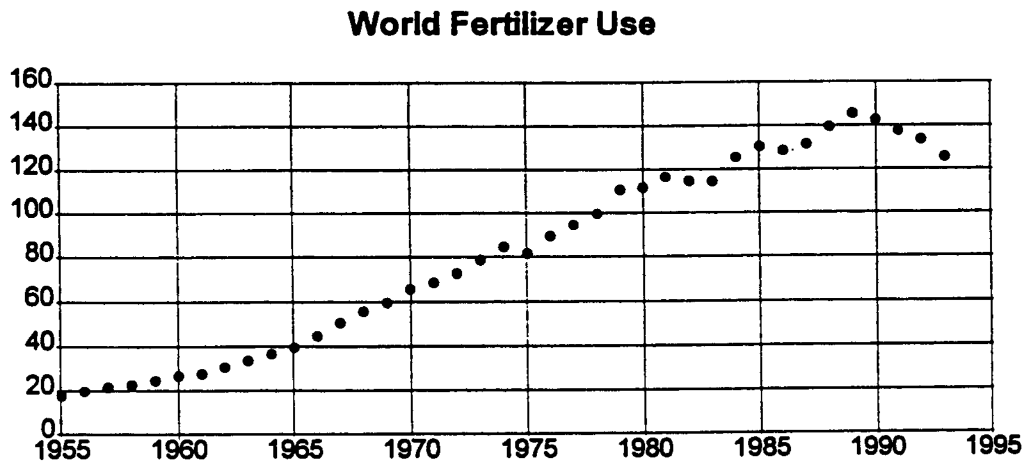


4. The graph below shows the world use of fertilizer. Also in the associated spreadsheet are average grain yields per square kilometer for the same years.

a) (10 points) Use a curve-fitting routine to find world grain yield as a function of fertilizer use. How closely does the model fit the data?

b) (10 points) The graph below shows the average yield of corn per acre in the US from the late 1880s to 1992. Curve fit the data for the first 40 years. Do the yields appear to be increasing linearly? Explain.

c) (10 points) Curve fit the data from 1955 on. Do the yields appear to be increasing linearly or exponentially? Compare your answers for parts b) and c), what do the results mean?



Journal

a) List a few things you learned about the world.

b) How might you benefit in the future from what you learned?

c) What did you learn about the software you were using for this exercise?

d) Where or how might you use this knowledge in the future?

e) What did you learn about or from each of the members of your group while doing this activity?

Tier III

This section includes information about projects that won awards as notable efforts.

INNOVATIVE
PROGRAMS
USING
TECHNOLOGY

Integrating Maple, Spreadsheets, and Writing into Algebra and Introductory Statistics

Linda L. Duchrow

Service Course Area

Algebra, Precalculus, and Statistics

Institutional Data

Regis University is a Jesuit liberal arts and pre-professional university serving students in both undergraduate and graduate programs. Established in Denver over a century ago, Regis is comprised of three academic units: Regis College, The School for Professional Studies, and The School for Health Care Professions. Regis College has long maintained a student body of approximately 1,000 students. Our mission, the education of men and women to be leaders in service to others, is reflected in our primary commitments to teaching, learning, leadership, and service to the community.

Abstract

The 1995 AMATYC standards for introductory college mathematics classes [26] stress the need to: make the student an active learner; go beyond the mathematics classroom; use appropriate technology; and encourage exploration. The goal of this project was to incorporate the AMATYC standards into college algebra and introductory statistics by:

1. Using technology to explore concepts.
2. Using real-world exercises in the mathematics classroom.
3. Having students write, analyze, and communicate about mathematics.

During the 1995–96 academic year, two college algebra classes (56 students) and three introductory statistics classes (109 students) were each given a weekly computer exercise. The exercises were done on Maple for the algebra students and Excel for the statistics students. The students completed an experiment and wrote a short analysis addressing specific topics. The exercises involved using realistic numbers (expected value of a sweepstake), investigating concepts (properties of real numbers), exploring mathematics (families of parabolas), and making decisions (graphing, hypothesis testing). The writing varied from analyzing a result to conveying the information as a newspaper article. The exercises did not have a "right answer" but encouraged students to explore and explain.

Students indicated that the exercises helped their understanding of algebra and statistics. Their mathematical communication skills with each other and with the instructor improved. In addition to being well prepared for their business classes, they wanted to use their new-found skills in other classes by doing projects involving computer problem solving.

Project Description

This project focused on using technology in teaching College Algebra and Introductory Statistics at Regis College. Technology was used as a tool to explore real-world problems and to discover mathematical concepts. Students at Regis College are placed in the highest level math course possible regardless of their major. Placement is made based on high school transcripts and ACT or SAT scores. College algebra is taken primarily by freshmen who have a "C" average in high school. The course objective was to instruct the student in basic algebraic operations. Enhanced by technology, the course focused on the properties and use of the algebraic operations. Introduction to Statistics is taken by freshmen, sophomores, and juniors who have passed at least one college-level math course. The course objective was to present usable statistics to the student for use in business and other sciences. With the use of technology, the objective has focused on drawing inferences using statistics.

The *Regis University Strategic Plan 1996–2001* has a commitment to incorporate more technology into the classroom; thus Regis University already had the hardware and software needed for this project. DEC Pentium PCs (16 meg RAM, 850 meg Hd, color monitor) are available in two classrooms (12 stations each) and one computer lab (24 stations). The lab and classrooms are available to students 24 hours a day. Computers have internet access, Microsoft Office, statistical software, Maple V, and access to the library in addition to various class-specific programs. To use the materials developed for College Algebra, the instructor would need minimal knowledge of Maple V. For Introductory Statistics, instructors would need minimal knowledge of Excel or another spreadsheet.

Using technology allows the students to focus on the mathematics and not get bogged down in the arithmetic. Problems that are more challenging, use larger numbers, and come from real-world situations which can be explored, examined and discussed. Students are able to spend their time and effort on understanding and communicating rather than in trying to get the answer in the back of the book. The speed and ease of using the programs allow students to repeat a problem with minor variations multiple times to see the patterns. They can question and investigate the "Why?" instead of getting lost in the "How?" Students have the time to go beyond the answer and think about the consequences.

The weekly computer laboratory exercise approach with a written analysis can easily be used as a model for any mathematics class. The basic format of *objective, commands, experiment,* and *analysis* that was used in the project would provide a usable model for any mathematics course. As software and hardware become more sophisticated, the exercises can be extended to developments in technology.

Students taking these courses have been enthusiastic about this technological approach. Students indicated that the exercises helped their understanding of algebra and statistics. The exercises have also received endorsements from colleagues at Regis College and other colleges.

Sample Materials

College Algebra
Computer Exercise — Complex Roots

OBJECTIVE: For the quadratic formula, $b^2 - 4ac$ determines if the values of "x" are rational, irrational, or complex.

COMMANDS: Maple uses a capital I for a complex number. For example, to solve $2x^2 + 3x + 4 = 0$,

Input	Maple's Output
2*x^2+3*x+4=0	$2x^2 + 3x + 4 = 0$
solve(");	$-\frac{3}{4}+\frac{1}{4}I\sqrt{23}, -\frac{3}{4}-\frac{1}{4}I\sqrt{23}$

We would write this as $\frac{-3 \pm i\sqrt{23}}{4}$.

EXPERIMENT: Enter and solve each equation.

$x^2 + 3x - 4 = 0$
$x^2 + 3x + 4 = 0$
$5x^2 + 3x - 4 = 0$
$3x^2 + 3x + 4 = 0$
$5x^2 + 4x - 1 = 0$
$5x^2 + 4x + 1 = 0$
$x^2 - 9 = 0$
$x^2 + 9 = 0$
$3x^2 + 17 = 0$

ANALYSIS: Discuss the relation between $b^2 - 4ac$ and the type of roots (complex or real) of the equation.

Statistics
Computer Exercise — Graphing Data

OBJECTIVE: Graphs are an excellent way of summarizing numeric data. The type of graph used may give a different impression of the data.

COMMANDS: The following steps will allow you to create a simple chart.

- Enter the data in tabular form.
- Highlight the data(and titles) to be graphed.
- Click on the "Chart" button.
- Click in the cell below your data.
- Follow the instructions given.

(For more details, review "Charts" in "Examples and Demos".)

To change the style of chart to a pie graph, line graph, 3D graph, or other graph:

- Select your chart by clicking on the middle of it.
- Click on the down arrow next to "Chart Type."
- Click on a chart to select a new chart from those available.

EXPERIMENT: Use the following data to make and print out three different charts. (Add titles and all information necessary for your chart to stand on its own!)

The Amount of Money Spent on Medical Care in the USA

Year	Amount Spent (billions)
1985	$327.5
1986	$357.6
1987	$399.0
1988	$487.7
1989	$536.4
1990	$595.9
1991	$656.0

ANALYSIS:

- Discuss the advantage and disadvantage of each chart.
- Select one of your charts and explain WHY it would be the best representation of the data.

Computer Exercise — Expected Value of Sweepstakes

OBJECTIVE: Advertising for Sweepstakes make it seem like everyone will win A LOT OF MONEY! The expected value gives an estimate of what you would win if you enter similar Sweepstakes over and over and over.

COMMANDS:
All Excel calculations must begin with the equal sign (=).

Excel uses the minus sign for subtraction. For example: **=B2-0.32** subtracts 0.32 from the value of cell B2.

Excel uses the asterisk for multiplication. For example: **=B2*C2** multiples the value of cell B2 by the value of cell C2.

Excel uses the slash for division. For example: **=B2/206000000** divides the value of cell B2 by 206,000,000.

Excel uses the command sum for the sum of a group of numbers. For example: **=sum(B2:B15)** adds all the values from cell B2 to B15.

DATA: The following is a summary of the information from the January 1996 *Reader's Digest* Sweepstakes.

Prize Category	Prize Amt.	No. Given
Grand Prize	$5,000,000	1
First Prize	$175,000	1
Second Prize	$50,000	1
Third Prize	$20,000	2
Fourth Prize	$10,000	4
Fifth Prize	$5,000	10
Sixth Prize	$1,000	45
Seventh Prize	$135	50,750

(In very small print at the bottom. . . .total number of entries is 206,000,000.)

EXPERIMENT: Create a spreadsheet with the following columns:

Column	Contents	Comments
A	Prize Category	Don't forget "LOSE"
B	Prize Amount	Don't forget "0"
C	Number given	Compute the number who "LOSE"
D	Total $ given away	Type in first entry and use **Copy**
E	Probability of Prize	Type in first entry and use **Copy**
F	Net Prize Amount	Subtract 32 stamp from each prize
G	xP(x)	Type in first entry and use **Copy**

- Total columns D, E, and G.
- Compute the total amount of postage used.

ANALYSIS: Discuss

- the cost to *Reader's Digest*,
- the anticipated amount of your winnings, and
- who benefits the most from the Sweepstakes.

Computer Exercise — Hypothesis Testing

OBJECTIVE: Claims are often made by companies about new products. Statistics can help us test these claims to verify their validity.

COMMANDS: Simple computations are used to compute the percent increase. For example: **=(B1-A1)/A** will compute the decimal increase of B1 over A1. By clicking on the "%" button, the value is changed to a percent.

EXPERIMENT: Par, Inc., claims that their new golf ball will increase driving distance by 10%. Forty golfers were given the Par, Inc. new ball and a typical golf ball. Their drive for each ball was measured to the nearest yard. The results are:

Old Ball	New Ball	Old Ball	New Ball
247	284	251	271
244	276	253	279
250	270	270	266
255	273	272	271
241	269	263	287
266	258	255	281
241	269	258	288
249	296	248	283
242	293	243	276
253	271	261	275
246	281	258	269
247	273	264	290
267	269	257	257
246	278	256	260
243	267	246	267
266	288	258	277
238	257	250	270
255	270	262	268
249	285	257	262
245	286	259	270

Compute the % increase for each golfer. Let the null hypothesis be that the drive distance is less than 10%. Test the hypothesis at a 0.05 level.

ANALYSIS: Write a one-paragraph "news article" including your results.

Permission to print this material was obtained from Linda Duchrow.

The Gettysburg Text: An Integrated Precalculus-Calculus Course Using a Computer Algebra System and Spreadsheet Including Business Applications

John M. Kellett

Service Course Area

Precalculus

Institutional Data

Gettysburg College is a liberal arts institution with approximately 2000 students. These students, average over 1,000 on the SAT. There are five computer laboratories on the campus and over 100 additional computers available for student use.

Abstract

The project was designed to create course materials in order to help students achieve more independence, provide opportunities for students to generate conjectures, examples, and generalizations, and to work on non-trivial open-ended problems in a collaborative fashion. The project was also designed to help students find connections between mathematical concepts and applications.

Technology used in the project included spreadsheets and a computer algebra system. The students wrote the text while guided by the instructional materials provided by the instructor. The students involved with this project were completing experiments with the computer, discussing ideas they had generated, or formulating more questions and conjectures. These students learned mathematics by doing mathematics.

Project Description

The course materials are designed to: (1) help students achieve a much higher level of independence in doing mathematics; (2) provide significant opportunities for the students to create examples, search for ideas and insights, make conjectures, test conjectures, make generalizations, refine definitions and generalizations, and evolve theories; (3) greatly increase the opportunities to work on non-trivial and open-ended problems using various methods of attack; (4) help students make connections and see patterns with respect to mathematical concepts and applications; and (5) greatly enhance the opportunities for collaborative experiences.

The students write their own textbook, guided by the materials which offer a directed, paced, discovery approach which consistently asks students to make connections between familiar ideas and new concepts. This is a total immersion program for the students in the use of technology, since to do the work and to take the tests, they must use the computer efficiently, effectively, creatively, and often.

The student-written text consists of chapters as well as summary and review sections. In the chapters, the students explore new ideas and concepts, formulate questions, articulate insights, and make conjectures. A chapter is usually one day's assignment, and unlike a standard text where a single topic constitutes the primary focus of a given day, several different topics are under investigation in each chapter. The understanding of a topic is developed over several chapters to give the students the opportunity to discuss, develop, clarify, reinforce understanding, and to make connections with other topics. By using the computer to investigate several topics on a given day, the students are able to cover more material than they can using the standard approach. Two or three chapters (exploratory sections) are followed by one or two days of summary and review sections so that the students can summarize and consolidate their understandings of the topics under investigation.

This enables the student to gain confidence with the material and to build upon a common base of understanding.

The courseware also requires that the students collaborate, discuss, and describe their results in writing for each assignment. Where the standard text gives an explanation and examples, this courseware gives only questions that provide direction for the students' investigation. The questions build on previous investigations and are designed to allow all students to make real progress at furthering their understanding if they work at it. The open-ended questions challenge the most able and diligent students to make conjectures, raise further questions, and make generalizations.

Classroom discussions of the text are led by students. These discussions are structured to help the students learn to describe their insights and to critique their output, their methods of exploration, and their willingness to take risks at making conjectures.

The focus of the course materials is on the students' intellectual maturation. The strategy emphasizes promoting greater involvement by the students, building student confidence in their ability, and empowering the students to use quantitative methods meaningfully and effectively. The use of a spreadsheet was chosen as an effective tool to teach business applications of precalculus/calculus (also, because the college's Management Department used spreadsheets). This technology made it possible for students to construct their own understanding of the topics being investigated. The process of computing the discrete approximating function, and fitting the models, reinforces the concepts of precalculus while developing an understanding of the concepts of calculus. It became evident that the intimate connection between formulas and numbers—the formula for a cell shows up at the top of the page as a formula and produces a number in the cell—was significant in the student's understanding of functional concepts. By more fully integrating technology into the course, the study of precalculus-calculus becomes a superb experimental science in which students create and do experiments to gain insights. The students acquire the ability to develop good data and use the data to motivate the theories of the subject.

In comparing the use of the Gettysburg text with the Harvard calculus text used in other sections for the precalculus/calculus course, the two-semester success rate (C– or better) for the Gettysburg text was 62% compared to 42% for the Harvard text. In the calculus, the first-semester success rate for the Harvard text was 67% compared to 92% for the Gettysburg text. Students reported spending about twice as much time on their homework for classes that used the Gettysburg text. Comments included: "The material was easy to visualize, learning was hands-on, and working with partners was great!" "It made me think a lot. It made me understand why calculus is used and how to think quantitatively." "Using the computer made math class more interesting. I learned how to use computers and math together. I could relate math to real-world problems." "Having to learn everything on our own helped us to understand better."

Sample Materials

The starting point for a topic is an intuitive description of a functional relationship. Some examples follow. The area function Af(x) is defined for a given function f, as the signed area between the graph of f and the x-axis over the interval (b, x) for a fixed b. The arc length function Lf(x), the volume function Vf(x), and surface area functions are similarly defined. The slope function (the derivative) is defined for a given function f to be that function whose value at x is the slope of the tangent line to the graph of f at the point (x, f(x)). Other functional models that are investigated include the distribution function, the force behind a dam function with height or width or length of the lake as the independent variable, growth and decay functions, and cost, revenue, and profit functions.

Students can construct data for an estimated derivative very simply and can use the data to gain insights as illustrated:

The formula C3 in the table below relates directly to numbers, not hypothetical numbers, and to two points (A2, B2), (A3, B3) on the graph of the f(x) function that they can produce using the spreadsheet. The copy command makes it possible for the students to create the estimated derivative from a specific calculation of the slope between the first two points of the function. Modeling the estimated derivatives reinforces the main concepts of precalculus in an environment where the students have to think and work in a graphical, numerical, symbolic, and conceptual mode. The estimated derivative is in column C.

	A	B	C
1	x-values	f(x)-values	Ef′(x) (Δf/Δx)
2	0	exp(3 * A2)	(B3 –B2)/(A3 – A2)
3	.1	exp(3 * A3)	1 copy
4	.2	1 copy	
5	.3		

Students are asked to answer questions similar to the following:

- Graph f(x) and Ef′(x).
- Describe graphically what is being measured in constructing the function values of Ef′(x).
- Why is this a reasonable estimation of f ′(x)?
- Determine a family of functions that has a member that will provide a good model for Ef′(x).
- Reduce the size of the step and make conjectures about the limiting values of the parameters in the model.

- Why does the reduced step reduce the error in the approximation?
- Consider other functions in the family $g(x) = \exp(cx)$ and make conjectures about limit $Ef'(x)$ as $\Delta x \to 0$.
- Make a conjecture about $g'(x)$.

Permission to print this material was obtained from John Kellett.

Business Calculus Today

Robert L. Richardson and Dona Alejandro

Service Course Area

Business Math

Institutional Data

Appalachian State University is a comprehensive university of over 12,000 students located in the small town of Boone in the mountains of northwestern North Carolina. It is a part of the 16-campus University of North Carolina system. Appalachian offers a broad range of undergraduate programs and select graduate programs. Instruction is its primary mission. Of the 572 full-time faculty, 92% have a terminal degree.

The University maintains a high-speed connection for both video and data to NC-REN (the North Carolina Research and Education Network). Academic Computing Services maintains a cluster of DEC VAX computers with an internet connection and seven microcomputer labs, one of which is the business calculus lab, and a workstation/x-terminal lab. The College of Business maintains several labs, one of which is for coordination between business calculus and information systems.

Abstract

The complete redesign of our business calculus course began in 1988. This new course has been designed to address many of the ills of business calculus as it is currently taught. At present, one can wander through a crowd of 1,000 businessmen and women who have had business calculus, and none of them would ever dream of using any of the concepts or skills presented, if they could even remember them. Math professors are too quick to blame this on the mathematical ineptness of their students. Business professors' opinions of most current business calculus courses vary from suspicion to outright hostility. The problem lies with the course as it is currently taught. With this project the course has been redesigned to take a new and totally different approach. Included topics have survived the axe only by showing themselves to be of direct application in some area of business. The course requires: integrated use of spreadsheets; integrated use of Derive and/or calculators; use of marketplace data; use of outside readings in business papers and journals; and constant application of continuous insights to discrete data.

Project Description

This project was funded by the College of Arts and Sciences and the College of Business at Appalachian State University, and by a grant from the NSF Instrumentation and Laboratory Improvement Program. The course for this project is required for all Business majors at Appalachian State. In an effort to make calculus more meaningful to business students, two new textbooks were developed and used with this project. *Applied Calculus Today with Spreadsheets and Derive* [142], and *Applied Calculus Today with TI-85 Graphics Calculator* [143] present a new approach to teaching business calculus where emphasis is placed on using calculus to understand real-world issues and problems. Within both texts are assignments which require reading and using information from many business-related sources such as *The Wall Street Journal, The Architectural Digest,* the business section of a local newspaper, and others.

Technology is an integral part of this innovative approach to teaching business calculus. Excel, Derive, word processors, and graphing calculators are used extensively to help resolve more complex questions, to help when doing more difficult symbolic manipulation, and, for graphing, to help in understanding and resolving problems. Students are encouraged to use the tools of their trade, namely spreadsheets. Whenever possible, problems are entered into, manipulated, and solved within spreadsheets. When additional insight is to be gained by graphing, carefully designed graphs are required. Instructors require that answers be presented in an attractive fashion with ample explanations within the spreadsheet wherever possible.

Students are first introduced to discrete-type problems and the fundamentals of analyzing them using a computer or calculator. New topics are introduced with an eye to understanding principles. Each new concept is introduced through applications. As the technology frees the student from cumbersome hand calculations, more emphasis is placed on students' interpreting the results of their work.

A major goal of this project was to provide the College of Business at Appalachian State exactly the service course they want and expect for their students, that is, a course in which significant material is presented in such a fashion that the students will be able to apply most of what they have learned in both the College of Business and their first job. As a testament to the success of this project, it was noted that the College of Business was considering dropping this course from their curriculum, but after this technological approach was instituted, they unanimously voted to keep it. It is the wish of the College of Business at Appalachian State that all calculator sections of this course be eliminated in the near future. Plans are to do this as additional computing facilities become available.

Sample Materials

Text Used: Richardson, Robert. *Applied Calculus Today with Spreadsheets and Derive*, Saunders College Publishing, 1996. (The text comes complete with menus so that little prior knowledge of either Derive or Excel is required for the instructors to be successful).

Richardson, Robert, and Dona Alejandro. *Applied Calculus Today with TI-85*, Saunders College Publishing, 1996. (The graphics calculator version, designed for the TI-85,TI-82, and TI-92 comes complete with all necessary menus so little or no training is needed).

The following is a sample of some student work on problems 12 and 13 from our text, *Applied Calculus Today,* published by Saunders College Publishing.

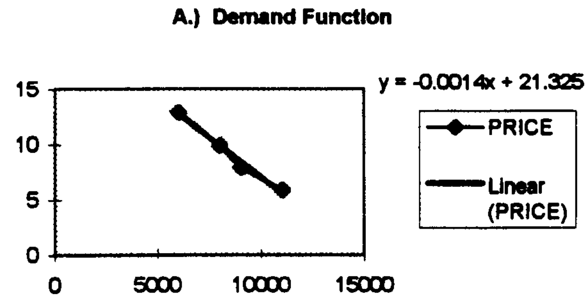
12. You have decided to start a company to manufacture fizzles. In checking out the fizzle market, you discover that the demand curve responds to market conditions as follows:

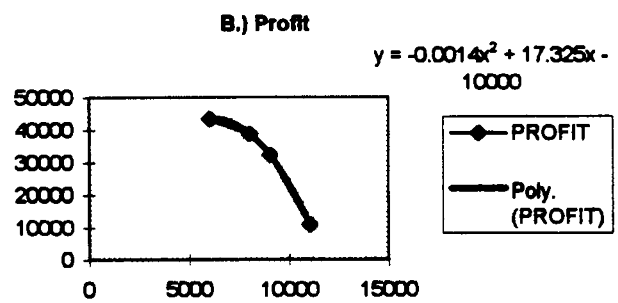
Quantity	11,000	9,000	8,000	6,000
Price	5.98	8	9.98	12.98
Cost	54,000	46,000	42,000	34,000
Revenue	65,175	78,525	81,000	77,550
Profit	11,175	32,525	39,000	43,550

In calculating your costs, you come up with a variable cost per unit of $4 and fixed costs of about $10,000.

A. Graph the given demand points and make a best choice for a demand function.
B. Find and graph the profit function.
C. Find the break-even points.
D. Where would your maximum profit occur, and what is your profit there?
E. Should you actually start up your company? Explain your reasoning.

Student Solution to Problem 12





C.

				BEP	
Quantity	6,100	609	608	**607**	606
Cost	34,400	12,436	12,432	**12,428**	12,424
Revenue	77,989	12,468	12,448	**12,428**	12,409
Profit	43,589	31.69	16.07	**0.446**	−15.18

D.

		Max. Profit	
Quantity	6,100	**6,200**	6,300
Cost	34,400	**34,800**	35,200
Revenue	77,988.5	**78,399**	78,781.5
Profit	43,588.5	**43,599**	43,581.5

E. It would be wise to start this company. In regards to a product, producing 607 items is not an enormous number of items to produce. Therefore, since the break even point is at

607, it would be in the best interest of the future owner to start this business.

13. In exercise 12, you decide to investigate becoming a fizzle middle-man instead of a fizzle manufacturer, using the telephone to receive orders, and then placing lump orders with a supplier in Borneo. The supplier in Borneo will sell fizzles to you at a cost of $3.75 per item plus a flat $500 per order. You predict that you will have orders totaling about 4,000 items during the first year. Storage costs run about $2.50 on the average inventory.

A. Write down the total cost function in terms of order size.
B. Using the revenue function you found in Exercise 12, calculate R(4,000).
C. Using the total cost function, find the cost of placing one order of size 4,000.
D. Using calculus, find the optimal sized order and the cost of placing orders of that size.
E. Using the revenue at the predicted sales level of 4,000, find your best profit and make a comment about whether or not you should become a fizzle middle-man.

Student Solution to Problem 13

A.

ORDER SIZE	# OF ORDERS	# ORDERED	COST	ADD. TO ORDER	ADD. COST	TOTAL COST
200	20	4000	25250	0	0	25250
400	10	4000	20500	0	0	20500
600	6	3600	17250	400	2500	19750
800	5	4000	18500	0	0	18500
1000	4	4000	18250	0	0	18250
1200	3	3600	16500	400	2500	19000
1400	2	2800	13250	1200	6500	19750
1600	2	3200	15000	800	4500	19500
1800	2	3600	16750	400	2500	19250
2000	2	4000	18500	0	0	18500
2200	1	2200	11500	1800	9500	21000
2400	1	2400	12500	1600	8500	21000
2600	1	2600	13500	1400	7500	21000
2800	1	2800	14500	1200	6500	21000

B. 62,900

C. 26,000

D. Size = 1,264.91
Cost = 18,162

E. Best profit = $36,900. Ultimately, the owner is going to make more profit. The maximum profit would be near $46,000 for the owner, while the maximum profit of the middle-man would be $39,600. Besides, more money is better, in my opinion. The owner gets to make all the rules, not just follow rules.

Portions of this article are reprinted with permission from Saunders College Publishing

Powered Aircraft Flight as a Paradigm for Mathematics and Science

George Rublein

Service Course Area

Quantitative Literacy/Special Topics

Institutional Data

William and Mary is a state-supported selective small university. Undergraduate concentrations are available in a wide range of standard disciplinary and inter-disciplinary programs. Doctoral programs are available in American Studies, Applied Science, Computer Science, History, Marine Science, Physics, and Education. The college enrollment is about 5000 undergraduates and about 2500 graduate and professional students. Approximately 30 doctoral degrees are granted each year, together with 180 law degrees and 250 master's degrees. In May of 1996, 1,065 baccalaureate degrees were awarded. Students have dormitory access to college maintained servers, and PC laboratories are available throughout campus. At least one mathematics course is required for an undergraduate diploma. We estimate that in 1997-98, upwards of 200 students (from a cohort of about 1,300) will take the Mathematics of Powered Flight course.

Abstract

The central theme of this mathematical material is that real engineering data will be used by students who will be asked to solve problems that directly bear on an intuitive grasp of a particular physical setting—powered aircraft flight. This data is freely available from multiple sources: daily weather reports, wind histories, navigation maps, runway plates, weights and dimensions of commercial aircraft and current magnetic declination measurements. Computations involve arithmetic, elementary algebra, some geometry, some trigonometry, and a variety of numerical techniques. This course is designed to be accessible to students whose computational skills may be weak. It also provides prospective teachers some tools or outlook on the subject that can be usefully transmitted to others. The student taking this course is led to believe, "Do anything and you will find some mathematics."

Hand calculators are used with many problems. An airport design PC program published by the FAA, together with (locally reduced) wind histories from NOAA, provides students with individualized runway layout problems. In general, problems are selected because they are elementary. Omitted because of mathematical complexity are many problems that would be part of a standard introduction to aeronautical engineering. The Microsoft Flight Simulator is frequently employed to illustrate the results of certain computations.

Project Description

A man who doesn't drink and has no sense of humor goes into a bar. The bartender confronts him:

BT: What are you doing here?

Man: I have no idea. I didn't ask to be in this joke.

Students who need to study mathematics in order to satisfy a collegiate general education requirement frequently play the role of the victim in this parable. By tradition, school experience with mathematics comprises one standard mathematical course after another. That tradition may well be important to the successful training of scientists, engineers, and mathematicians. We have to continually build on students' mathematical training, providing ever more powerful and varied tools that a student *might* find useful in a technical career.

Some students who opt out of technical careers find mathematics interesting. For them, a traditional approach is edifying. Such a student who satisfies a general education requirement by taking calculus or probability or statistics or finite mathematics is quite satisfied with a mathematical agenda as conceived by mathematicians.

What of the others? Many current efforts to get non-technical students interested in mathematics rely on the use of applications that employ real data. For instance, a student might be asked to make an elementary statistical analysis of mad cow disease in Great Britain, using data from the Web. At first glance, the material in "The Mathematics of Powered Flight" at William and Mary bears the appearance of such an approach, merely exchanging mad cows for airplanes. However, we have endeavored to make our material conform to a renegade version of a rule of three.

Our real problems:

1) use real data, reliably gathered by workers in an external discipline;
2) use computations or geometric results that emulate methods used by workers in the external discipline;
3) produce numerical or geometric results that explain the practices of the external discipline.

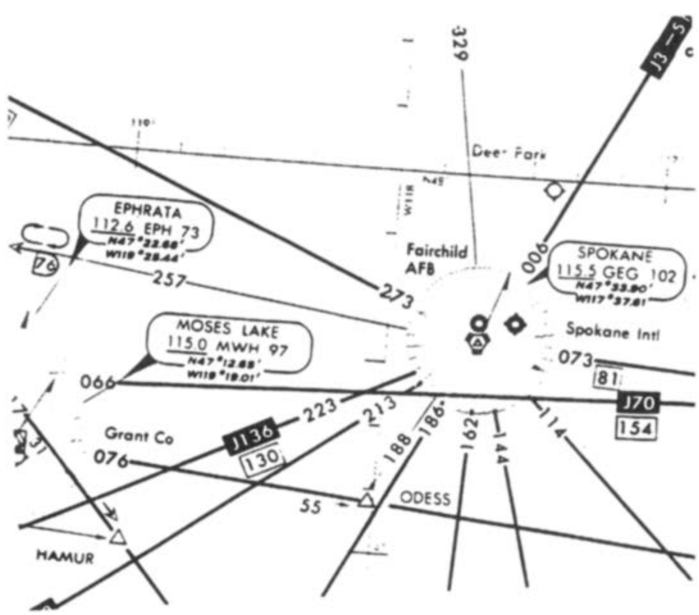
Someone who knows nothing about cows could use, or even explain, the statistical techniques used to study those animals. Once this is done by, or for, general education students, they are apt to ask, "So what?" Far more interesting than the statistical data was the European Community's political response to the outbreak of the disease in Great Britain. There might well be a useful game-theoretic analysis of that response, but that analysis will be much too complex to pursue in an elementary course and will almost certainly pass outside the expertise of the instructor.

Mathematical work that casts an explanatory light on an external discipline might be characterized as "interdisciplinary mathematics," though it would be a good idea to hang on to the quotation marks. As a general matter, our rule of three prohibits us from simply using some calculation as an excuse to elaborate on a particular mathematical development. In other words there is no "new" mathematical agenda for students in this course. This appears to mean that we have to agree to use only the mathematics of a presumed high school experience, say, just geometry and algebra. But we do permit ourselves to develop some ad hoc methods, all of which will be familiar to a mathematician.

The absence of a mathematical agenda flies in the face of certain prescriptions of other authors, as well as of some important organizations, who have studied "liberal arts mathematics." Our students will *not* study a piece of mathematics as a structural system. They will *not* be trained to recognize the generality of the methods they are using. We do *not* make an overt attempt to display mathematics as the beautiful intellectual enterprise that it is. Common to these ambitious missing themes is that they are all the property of the trained mathematician. The respectable argument in favor of adopting them is to say that they are what mathematics is about; to omit them from a course is to abandon its mathematical content. A cynical member of the profession might say, "We have to show them this stuff; it's the only thing we know how to do."

We do omit these themes from our course because our substitute agenda for the students is: "Let's see if we can enhance our grasp of this setting by doing some calculations"— hence, the word "paradigm" in the title of this project. To put this another way, instead of saying to the student, "You might run into this technique some day, and by the way, isn't it pretty", we say, "If you don't do this calculation, you can't understand this particular facet of the (airplane) business."

But the work students do is not a mindless application of formulas. Students must think about the geometric or physical significance of all parts of an analysis. Below is an extract from an air navigation map of the vicinity of Spokane. Question: What is the nominal magnetic variation at Spokane?



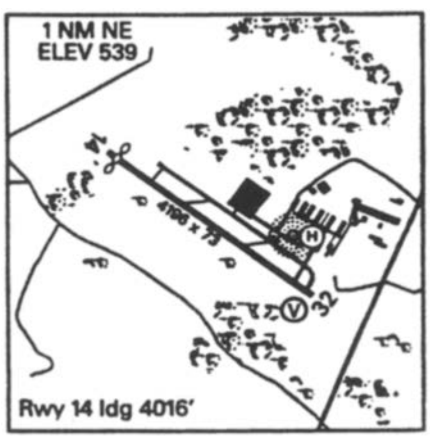
Students must use a protractor and, in essence, rotate a polar coordinate frame at Spokane in order to get the answer to this question. And though the method is quite elementary, the solution to the problem does say something to a user (a pilot, say) about how magnetic compass navigation works.

One of the biggest challenges is to make sure there is enough (elementary) mathematics to do while still adhering to our alternative agenda. Fortunately, aircraft flight is replete with quantitative issues that are accessible to the mathematically naive. Unfortunately, it will clearly be part of the instructor's task to acquire a genuine feel for the practices of this external discipline. By confining our work to a somewhat narrowly defined setting, we can reduce the instructor's external burden.

Sample Materials

The catalogue description of "The Mathematics of Powered Flight" includes the following topics: wind and its effect on airport design and aircraft operation; maps and map projections; static air pressure; lift, drag, buoyancy, and middle ear problems; magnetic declination and its relationship to compass navigation.

On the first day of class, we may display a map of a small airport, for instance, Gaithersburg Airport near Washington, D.C. By calling (301)977-2971, one may hear a report of some current weather information. This information gives rise to a simple trigonometry problem whose solution is important to a pilot who wants to use Gaithersburg. That airport is only one of many country-wide airports where local aircraft-specific weather is available by automated telephone service.



A sample of other types of exercises:

1. An 18-knot wind is blowing from compass direction 220° at Patrick Henry Airport in Newport News, Virginia. Runways 2-20 and 7-25 are installed at Patrick Henry. Which of these four will Air Traffic Control advertise for landing?

2. A plank is supported at each end. A 60-pound weight hangs two-fifths of the way from the left end. How much of that weight is borne by the left-hand support?

3. An FAA inspector is considering whether to turn the nominal magnetic north at the FOOTHILLS VOR (ODF) in South Carolina. This may be found on panel F of H-6. Which way, and by how much, should that inspector turn the VOR? Once this is done, work out new headings for all jet routes out of ODF.

4. A pilot flies along a great circle route whose most northerly point is Reykjavik, Iceland. Determine whether the pilot flies north or south of Cleveland.

5. Two smooth balls are dropped from the Tower of Pisa. One is made of iron and has a radius of 1 inch. The other is made of aluminum and has a radius of 3 inches. Which hits the ground first?

6. Examine the United Nations logo. How can you tell that the map is an azimuthal equidistant projection?

7. Examine a Mercator projection of the world. Which has a larger (true) area, Madagascar or Great Britain?

8. The altimeter in an airplane shows a reading of 15,000 feet, but the altimeter is calibrated at 29.17 inches instead of the correct 29.62 inches. How large, and in what direction, is the altitude error of the altimeter?

As indicated above, three useful technological devices are employed in the course. Hand calculators do most computations. The FAA runway layout program gives students a convenient tool to study the statistical distribution of winds at a site and their influence of (actual) runway layout. The Microsoft Flight Simulator displays a good image of instrument panels on various sized aircraft. During simulated flight, these instruments are the source of a variety of different geometry, algebra, and calculus problems.

A grant from the NSF will fund development of a science (second) half of this project and the purchase of requisite equipment. Pending a determination of the cost of materials and equipment, the transferability of the science course to other institutions is uncertain. As matters stand, the mathematical material is more than ample for a one-semester course for general education students. That course requires only inexpensive data sources, software, and other course materials. The value of this project to the collegiate mathematics community lies in its strength as an exhibition of an innovative approach to curriculum development. Interest at several other institutions has been demonstrated, and the interdisciplinary nature of the project is a positive feature.

Bibliographical Notes

NIMA is the National Imagery and Mapping Agency, an agency of the Department of Defense; NOS is the National Ocean Service, an agency of the Department of Commerce; NCDC is the National Climatic Data Center, an agency of the Department of the Interior; FAA is the Federal Aviation Administration, an agency of the Department of Transportation.

1. Magnetic Variation, Maps 42 (Mercator) and 43 (Polar); NIMA.

2. Climatography of the United States, Number 90. Weather data (including 16-point wind histories) on 163 individual airport sites, paper copy. NCDC.
3. International Station Meterological Climate Summary, CD-ROM. Includes data in #2 above. NCDC.
4. Solar and Meterological Surface Observational Network, 1960-1990. Weather data (including 36-point wind histories) at numerous sites. 3 CD-ROM set. NCDC.
5. Formatted wind data extracted from #4 above. Corrected for magnetic variation. By anonymous ftp from: gopher.cc.wm.edu Files are in the /pub/math directory.
6. Airport Design Program, AD42D. FAA. Located at: http://www.faa.gov.arp/arphome.htm Uses data in #5 above.
7. High level air navigation maps, H-1 through H-6. NOS.
8. US Terminal Procedures (17 volumes), NOS.
9. Low Altitude US Airport Diagrams, Instrument Approach Procedures, (12 volumes), NIMA.
10. US Airport Diagrams, Civil Standard Instrument Departures, (2 volumes), NIMA.
11. Airport/Facility Directory (7 volumes), NOS.
12. IFR Supplement, NIMA.
13. VFR Supplement, NIMA.
14. Map Projections, A Working Manual, John Snyder. USGS Professional Paper #1395.
15. Flight Simulator, Windows 95. Microsoft Corp.

Note: Items 7–13 above may all be purchased from NOS in Bethesda, MD. But items 7–12 are updated every eight weeks, and it may be possible to locate subscribers who will donate obsolete editions. Item 13 is updated every 24 weeks.

Visual Mathematics in a General Education Curriculum: Helping Students Discover the Interconnections Between Math and Other Disciplines

Reza Sarhangi

Service Course Area

Quantitative Literacy/Special Topics

Institutional Data

Southwestern College, a private four-year liberal arts college founded by the Methodist Church, offers a wide range of baccalaureate programs in the arts and sciences, all built on a general education core curriculum. A Masters of Education degree is also offered. The college of 750 students and 50 full-time faculty is organized into a combination of traditional liberal arts and professional programs. The Integrative Studies program is an "across divisions" program, drawing faculty from all other programs.

The Mathematics Department at Southwestern College has established a mathematics laboratory which is open to students six hours each day. The math lab is equipped with computers loaded with software in mathematics and statistics such as MAPLE and MINITAB, TV/VCR units with headphones, slide and overhead projectors, and a big screen computer with a cordless keyboard and mouse. Internet access is also available for class sessions.

Abstract

The objective of Southwestern College's integrative studies mathematics courses, namely "Math and Art" and "Math and Science," is to provide students with a visual approach to mathematics that reaches beyond the traditional concepts of mainstream mathematics. Through a technological approach to these subjects via computer software, videos, and slides, students have access to visual, hands-on learning resources that enhance and excite undergraduate education in mathematics. For instance, see [4], [5], [11], [16] and [38].

The integrative general education curriculum at Southwestern College includes a "mathematics across the curriculum" feature in addition to six other core competencies. High-speed computers, software, videos, and slides available in a laboratory setting enable students to make visual connections between math and other disciplines which they could not "see" as recently as a decade ago ([35], [36], [85], [164]). Two of the courses, in what will ultimately become a four-course sequence, are "Math and Art" and "Math and Science." "Math and Art" concentrates on the study of geometric connections between art and mathematics; "Math and Science" focuses on the study of mathematics connections among different branches of science. The design of both courses addresses national concern about the increasing marginalization of mathematics in general education, declining enrollments in math classes, and the need for a laboratory environment for enhancing student understanding of the fundamental role math plays in nature, the arts, and society ([57], [77], [184], [187]). The Director of the Integrative Studies Program comments, "...we have begun a program which has already generated more excitement and interest in mathematics than we experienced in the previous two decades at the college."

The size of Southwestern College and the budget available to administer the mathematics program is so minimal that virtually any college or university could reproduce the program. The main issue is to bring the idea of such courses to reality.

Project Description

The goal of the project was to develop mathematics courses for the integrative studies curriculum, namely "Math and Art" and "Math and Science," providing primarily sophomore students with a visual approach to mathematics. A Phillips Chair stipend for the development and enhancement of the mathematics department and a Southwestern Bell grant for Innovative Teaching in Mathematics and Science helped to fund this project.

"Math and Art" concentrates on the study of the geometric connections between art and mathematics, including tessellations in different cultures, Fibonacci sequences, the Golden ratio and Golden sections, math and music, and work by artist M.C. Escher ([18], [50], [56], [57], [148], [159]). Lectures, discussions, and demonstrations of a variety of media make up the class structure. Students explore the relationship between mathematical subjects including geometry, number theory, and algebra, and the artistic presentations of these subjects created by scientists and artists throughout the centuries. They are also able to identify the relationship between musical characteristics, including the beat, the tone, the tune, and the song with their mathematical structures.

"Math and Science" uses the same format and includes the investigation of fractals as geometry and fractals in nature, chaos, the mathematics of management, graph theory, Euler circuits, and fuzzy logic and its applications ([13], [112], [133], [146]). Students are expected to identify many of the different aspects of mathematical expressions in science independently in order to enhance their ability to integrate and think critically. The course also enriches students on the story of culture, emphasizing the respect, pride, and understanding generated from the knowledge that all cultures have contributed to mathematics and science [128].

The advent of new technology for the math lab enabled the examination of three-dimensional figures within geometrical contexts, e.g., sculpture and architecture. While two-dimensional representations may be used, technologies, such as videotapes and computer software, which provide a sense of three-dimensionality work much better to develop the intuitive sense of multi-dimensional forms. Prior to technologies which could accurately suggest the three-dimensional nature of artistic production, both teacher and student were all too often trapped in a kind of "flatland" approach to some of the world's most significant works of art. The study of fractals, self-organizing systems, and other chaos theoretical issues, while possible with pictures and charts, would truly be impractical without the technology that provides students with a sense of the dynamic nature of the images.

This project assembled a strong multimedia environment to support the offering of the two courses created. The intent was to increase enthusiasm for mathematics by emphasizing its connections and applications in the fine arts and the sciences. The extent to which the courses depend upon videos and computer software utilized to support the content of the courses may act as a limiting factor in the possibilities for other institutions to adopt this project's curricular developments. Maturation of this project would appear to depend upon more extensive assessment efforts and an exportable set of course materials.

Sample Materials

Math and Art: In the "Math and Art" course students study the five Euclidean Axioms and five undefined terms before talking about the Divine Proportion. They sample deductive reasoning with sketches of proofs for problems such as the irrationality of r(2). They become familiar with terms such as topology and abstract algebra when they study rigid transformations to use in tessellations and in math and music ([56], [57], [159]). Here are some examples of classroom projects.

A series of projects for the classroom may begin with the idea of construction of geometrical shapes using only a compass and a straightedge (a ruler without measurement). This is a challenge the Greeks set for most geometrical constructions. It was used throughout the centuries by artists in the Middle East and North Africa, who created the fascinating designs of tessellations (tilings), and by European artists in the Middle Ages for construction of rose windows. The limitations on construction tools were a means for enforcing mental activities. Though geometry was developed and applied extensively by the Egyptians and Babylonians and transmitted to the Greeks, it was the latter who insisted on the deductive process. They believed such limitations on construction tools were a means for developing the process in the human mind.

In the first few projects, students become familiar with some constructions such as bisection of a given line or a given angle and construction of parallel lines using compasses and straightedges. Then the Golden Proportion (Divine Proportion) is introduced. The division of space is the primary concern of a visual artist. A concern of artists over many centuries was that there must be some way to determine the proportion in different parts of an object beyond intuition alone. One of the most important investigations in this thought was the examining of proportions within the human body. Even though each age and culture redefines its own proportions of an ideal body, our ideas today about this proportion come from classical Greece. The Greek artists were concerned with the idea of balance and symmetry communicated through the nude form of the athlete, and the Golden Proportion was a response to this concern.

The Golden Proportion, which appears frequently in nature and has been used extensively throughout the centuries by artists, is derived from dividing a line segment into two segments with the property that the ratio of the whole segment to the longer part is the same as the ratio of the longer part to the shorter part. The ratio expressed by this proportion is the Golden Ratio: $\Delta = \frac{1+\sqrt{5}}{2}$. This value is the positive root of the Golden Proportion: $\frac{x}{1} = \frac{1+x}{x}$. In this equation, a line segment is assumed to be divided in two parts based on the Golden Cut such that the shorter part is equal to 1 unit and the longer part is x units. The symbol Δ is the first letter in the name of the classical Greek sculptor Phidias, who was one of the artists responsible for the aesthetics of the Parthenon. The following figure represents an example of the Golden Proportion in the human body. For the application of Golden Proportion in art, we refer the interested reader to [18] and [50].

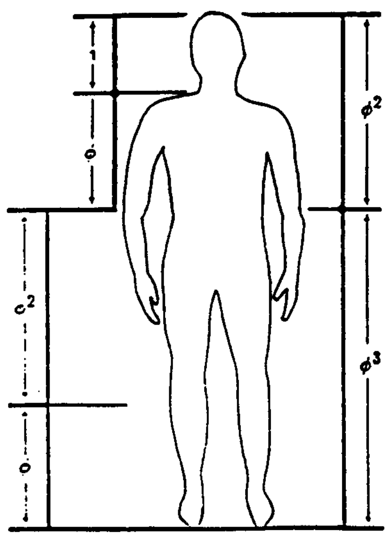


Fig. 1. Golden Proportion and the human body.

An immediate project in this step is the construction of the Golden Cut of a line segment using a compass and straightedge as it has been illustrated in the following figure.

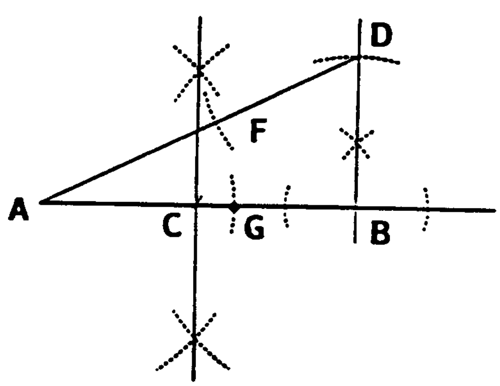


Fig. 2. Construction of the Golden Cut

Sketch of proof

From $AB^2 + BD^2 = AD^2$, $AB = 2FD$, $BD = FD$, and $AG = AF$, we conclude that $AG = (\sqrt{5} - 1)\, FD$ and $GB = (3 - \sqrt{5})\, FD$.

Therefore we have $\frac{AB}{AG} = \frac{AG}{GB}$.

Here we may solve a problem which was left in the second set of projects, the construction of the regular pentagon. For this, we consider circle C with center O and radius OA. We find the Golden Cut of OA. The longer part of this division can divide the circle into exactly 10 parts. Therefore the pentagon, as well as a decagon, can be constructed. A sketch of the proof is discussed with students.

Other interesting related topics are the construction of the Golden Rectangle and the Dynamic Rectangles, the study of the Golden Relationship in a pentagram, the study of the Parthenon and the great pyramid of Gizeh, the study of natural objects and forms, and the study of Fibonacci numbers and spirals [148].

A set of classroom projects may address a special class of geometric patterns called tessellations. The word tessellation comes from the Latin tessella, which was the small square tile used in ancient Roman mosaics. A plane tessellation is a pattern made up of one or more shapes, covering a surface without any gaps or overlaps, which can be extended in the plane infinitely in every direction.

The first set of projects in this subject is to determine which polygons can tessellate themselves. Students work with scalene triangles and discover that any triangle can tessellate itself. They realize that the same property is true for any quadrilateral. Moreover, for such a tessellation they need to arrange triangles (quadrilaterals) in such a way that in each vertex they come up with 360 degrees. They realize it is possible for triangles and quadrilaterals (concave or convex) to tessellate themselves because the sum of measures of any triangle is 180 degrees and for any quadrilateral is 360 degrees. Therefore, arranging 6 triangles (4 quadrilaterals) in each vertex results in a tessellation.

There are only three regular tessellations— equilateral triangles, squares, and hexagons. This is due to the angle of a regular polygon and the number of polygons meeting in each vertex. Let us consider the following values.

p = the number of sides in a regular polygon

q = the number of polygons in each vertex in a regular tessellation

Then we have:

$$\text{angle of a polygon} = \frac{(p-2)\pi}{p}$$

$$q \cdot (\text{angle of a polygon}) = 2\pi.$$

From these two equations we conclude that any regular tessellation should satisfy the following formula:

$$(p-2)(q-2)=4.$$

Another set of tessellations using regular polygons are semi-regular tessellations. A semi-regular tessellation is a tessellation that is formed by a combination of certain regular polygons such that the arrangement of polygons at every vertex is identical.

Due to the size of the angles of regular polygons we are limited to at most six and at least three polygons around each vertex. It can be shown that there are 21 arrangements of regular polygons that fill the space around a point. From these, only eight can tessellate a plane such that the arrangement of polygons at every vertex is identical. The three regular tessellations and eight semi-regular tessellations are called Archimedean tilings. For more information see [159].

A new set of projects in this concept is related to creating tessellations of objects. For this, students need to become familiar with the rigid transformations of translation, rotation, reflection, and glide reflection. Several patterns are studied in this step to help students recognize each of these transformations. They also get acquainted with a topological transformation in general. Several examples help students understand some transformations which are not rigid, and, without touching any serious part of topology, they grasp the idea of this important branch of mathematics. The symmetry group of an equilateral triangle, as well as the symmetry group of a square, allow students to become familiar with abstract algebra's terms such as binary operations, group, and abelian group.

Escher's tessellations in the Euclidean plane arealso studied. Escher's interest in this area was motivated after a repeat trip to Spain, where he visited the Alhambra built by the Moorish artists. In this step students learn how to use rigid transformations in order to modify an equilateral triangle, square, or hexagon to create a motif for their own tessellations. A software program, *Tesselmania* can be of great help for this purpose. Students enjoy working with this software and studying their artwork on the computer screen. Several related videotapes and slides can be shown to students as a part of class lectures.

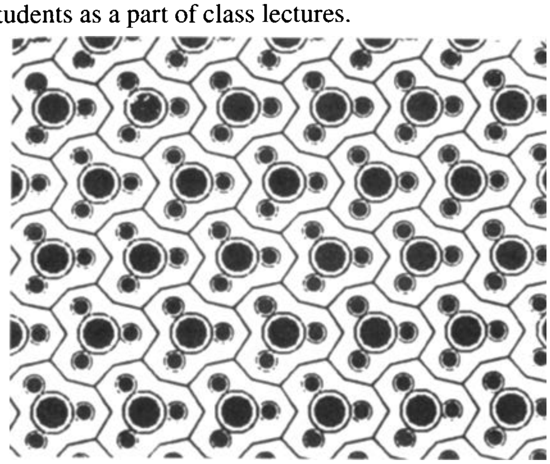


Fig. 3. A tessellation made by Tesselmania

The study of Math and Music may begin with the basis upon which music is built- - rhythm. First, two important components of music are needed to be studied, the measure and the time signature. Students realize that numbers set up the rules of rhythm for a piece of music through the measure and the time signature. Then other related subjects such as polyrhythmic music and counter rhythms are studied. They also study frequency, wave length, wave periods, and the tuning system.

Pythagoras made significant contributions to early Western music history. He and his disciples had many mystical beliefs about rationals and their relationship to the order of the universe. The Pythagoreans believed that all patterns and cycles in nature were musical, and that the study of numbers and their relationships to the harmony of music was the way to purify the soul. Pythagoras traveled to Mesopotamia where he studied and brought back many mathematical ideas. He also studied music through his experiments with the monochord, a simple instrument consisting of a string stretched taut over a movable bridge.

A project that excites students is to repeat what Pythagoras did with the monochord. They notice that the pitch that results from shortening the string to exactly half its original length sounds similar to the original pitch, except that it is one octave higher. The frequency doubles when the string is shortened by half.

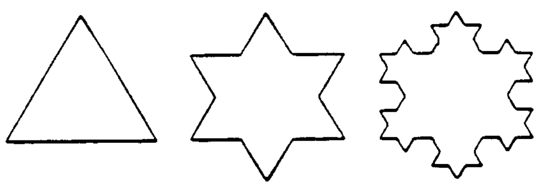
Through his experiments with the monochord, Pythagoras discovered three intervals that he considered to harmonize together—the diapason (f(1,2) of the original length, octave), the diapente (f(2,3) of the original length), and the diatesaron (f(3,4) of the original length). The fractions f(1,2), f(2,3), f(3,4) are made from the first four natural numbers that were very important in the Pythagorean beliefs. The sum of these four numbers, 10, was the Pythagorean holy number.

Based on Pythagoras's experiments, a method of tuning called the Pythagorean diatonic scale was created with intervals expressed as rationals. This tuning is different from today's tuning system which is even-tempered. In even-tempered tuning, an octave is divided into 12 equal parts and the frequency ratio between half steps is equal to $r({}_{12},2)$. So in even-tempered tuning, intervals are all irrationals. Today, some electrical keyboards can be tuned to Pythagorean scales as well. Students who know music may work on several experiments based on the Pythagorean and even-tempered scales. Students in this stage may become familiar with deductive reasoning through the ancient proof of the irrationality of r(2). The proof is by contradiction, and students are encouraged to reason the irrationality of r(3) in a similar manner.

Math and Science: The course begins with an introduction to Euclidean geometry and Newton's Laws of Motion. Throughout the course, students also study rigid transformations and scaling and distortion. They study mathematical logic and its operations before the study of multi-valued logic and hypercubes in fuzzy logic.

A set of projects for classroom use is related to the feedback system and the understanding of chaos theory. For this, a set-up for video feedback with the help of a camera which is looking toward a connected monitor is introduced. The results are compared with the pixel game introduced in [133]. Study of the feedback system and in general the dynamical systems and fractals can be achieved utilizing videotapes and software.

The next set of projects is the study of fractal objects such as the Sierpinski Gasket, Koch Snowflakes, and Fractal Ferns. Students become familiar with the idea of the iterative processing of fractal objects and self-similarities. While students identify self-similarities in nature and in the organisms of living beings, they study the sequences created from fractal objects mathematically. They also have an opportunity to observe the iteration process through networks such as "Fractint".



Perimeter = 1 Perimeter = $\frac{4}{3}$ Perimeter = $\left(\frac{4}{3}\right)^2$

Fig. 4. The Construction of the Koch Snowflake

Bibliographical Notes

1. Alhambra Past and Present, The. Videocassette. Lorraine L. Foster.
2 Beauty of Fractals Lab, The. Software. Springer-Verlag.
3. Chaos Demonstrations. Software. J. Sprott, University of Wisconsin-Madison.
4. Chaos, Fractals and Dynamics. Videocassette. Robert L. Devaney.
5. Chaos, Order and Associate Memory. Videocassette. Spektrum Videothek.
6. Computers. Videocassette. Michelle Emmer.
7 Fantastic World of M. D. Escher, The. Videocassette. Atlast Video, Inc.
8. Fractals for Classroom. Slides. Springer-Verlag.
9. Fractools III, Where Math Meets Art. Software. Bourbaki.
10. Fun with Fractals. Software. Judd Robbins.
11. Mobius Strip. Videocassette. The Great Media Company.
12. Music of Africa. Videocassette. Hollywood Select Video, Inc.
13. Music of India. Videocassette. Hollywood Select Video, Inc.
14. Mystery of the Egyptian Door, The. Slides. Margit Echols.
15. Similarity, Project in Mathematics. Videocassette. Caltech.
16. Spirals. Videocassette. The Great Media Company.
17. Symmetry and Tessellations. Videocassette. Michelle Emmer.
18. TesselMania: A software inspired by the artwork of Escher.
19. Transition to Chaos. Videocassette. Robert L. Devaney.
20. Visualization in Computational Science. Videocassette. MIT Press.

Permission to print this material was obtained from Reza Sarhangi

Conclusion

In this final section the four INPUT project goals are reviewed in the context of the need to hasten the pace of change in introductory college mathematics. The award-winning projects are then discussed in terms of the three sets of standards issued by AMATYC in 1995 and the eight INPUT criteria. Common themes and issues that emerged from that discussion suggest implications for the continuation of the reform effort in mathematics instruction. After an assessment of the extent to which each of the four INPUT project goals has been achieved, the section concludes with a challenge to introductory college mathematics instructors and administrators, professional organizations and accrediting agencies, former students, and their employers.

INPUT Project Goals

Goal 1: To discover important changes in curriculum, pedagogy, and assessment inspired by the use of technology

The previous sections of this handbook described the successes of faculty who have used technology to make important changes and pioneer reform in their introductory college mathematics courses—developmental mathematics, precalculus, business mathematics, introductory statistics, and quantitative literacy. These mathematics faculty responded to a wave of calls for reform that led to the development and preparation of *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* [26]. The Standards were designed as a framework for the development of improved curriculum and pedagogy. The INPUT Competition culminated in the collection of projects that exemplified these standards. The project directors featured in this handbook have all used technology to rethink their mathematical content and teaching. Guskin [70] notes that we are quick to tell faculty that they "need to restructure what they do" and that they "need to introduce information technology into their courses," but we don't give any examples of what is meant, how this might be done, or how it relates to what we know about student learning. This handbook has provided detailed examples of how to use technology in teaching mathematics.

Goal 2: To reward instructors who took advantage of technology to rethink and improve the mathematical content of their courses

To hasten the pace of change, the INPUT project team has directly addressed the issues of reward and support for change. Chosen by a distinguished advisory board in an international competition, the project directors featured in this handbook were rewarded for their innovations. They were presented with cash awards ranging from $500 to $5,000. Award winners were featured on the 1997 annual conference programs of the American Mathematical Society/Mathematical Association of America, the American Statistical Association, and the American Mathematical Association of Two Year Colleges. Award winners were also featured in the videotape, *The INPUT Project: Rethinking College Mathematics for Non-Majors,* and on the INPUT web page (http://www.cmich.edu/~mthaward).

Goal 3: To disseminate detailed examples of reform in mathematics service courses

Exemplary Programs in Introductory College Mathematics was designed to illustrate that programs modeling the AMATYC standards are realistic and can be successful. Designed to do for introductory college mathematics what *Priming the Calculus Pump* [179] did for calculus, this handbook provides concrete, detailed descriptions of what some instructors are doing, how they are doing it, what technologies they are using, and how to contact them. In addition, the handbook includes selected references to materials recommended by the award winners.

Goal 4: To encourage more faculty to incorporate technology into the curriculum and to re-evaluate their course content

By providing detailed examples of realistic and successful introductory college mathematics reform, and by rewarding the faculty engaged in those reform efforts, it is anticipated that increasing numbers of faculty will be motivated to rethink their own courses.

AMATYC Standards

As recommended in *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* [26], many of the award-winning projects focused on the standards for intellectual development, content and pedagogy.

The standards for intellectual development include problem solving, modeling, reasoning, connecting with other disciplines, communicating, using technology, and developing mathematical power. All of the projects featured in this handbook engaged students in substantial mathematical problem solving. Many of the problems were selected for their realistic content. More than half of the 20 winning projects required written and oral communication involving mathematics. Letters of support from English faculty and administrators were extremely positive about the contributions these instructors had made to building students' written and oral communication skills. In the project "Using Educational Technology in a Precalculus Course at the United States Air Force Academy," students learned to communicate their understanding of mathematical modeling by integrating the use of Mathematica, web-browsers, and presentation software. In the project "Visual Mathematics in a General Education Curriculum," featuring integrative studies linking mathematics and art, students explored the connections between number theory, algebra, geometry, and art. In "Projects for Precalculus," the development of students' mathematical power was realized through activities designed to help boost their appreciation, confidence, and comprehension of mathematical situations.

The standards for content include number sense, symbolism and algebra, geometry, function, discrete mathematics, probability and statistics, and deductive proof. These standards are meant to serve as a guide for introductory college mathematics courses. Each project included a number of these content standards, but not all. For example, "Earth Algebra: College Algebra with Applications to Environmental Issues," focused on numerical information and functions using "real-data." In "Basic Geometry Using Computers and Collaborative Learning," students developed conjectures while completing geometric investigations. Both deductive and inductive reasoning were important facets of this project. "Graphing Calculators and Cooperative Learning in Elementary Statistics" used newspaper and magazine articles as the source for many problems. Students in this project completed a variety of "hands-on" activities while learning probability and statistics.

The standards for pedagogy include teaching with technology, interactive and collaborative learning, connecting with other experiences, the use of multiple approaches, and experiencing mathematics. Use of technology was an integral part of all projects. Every winning project used collaborative learning to a varying degree. In Martha Aliaga's Statistics 100 class, students worked in small group settings where "students debate and question each other," despite a large class format. Connecting with other experiences was featured in Carla Thompson's statistics sequence. This project provided opportunities for student-directed investigations with original data from local community agencies. Most of the projects used graphing calculators, spreadsheets, and/or computer algebra systems that allowed multiple approaches to problem solving. The Clemson Business Math team exemplified all the standards for pedagogy. As taken from their abstract, their project emphasized "group work on team projects that involve mathematical decision making and the interpretation of results, and use technology as a tool for the learning of mathematics in an interactive classroom environment." The innovations found in the projects featured in this handbook include significant changes in pedagogy. The sage on the stage has been replaced by a facilitator who invites discussion, utilizes technology, cajoles when necessary, and lectures only when needed.

The INPUT Criteria

All project winners featured in this handbook described various ways that their projects attempted to satisfy the goals of NCTM, AMATYC, or other professional organizations. Although none of the projects were designed to fit the criteria established for the INPUT Project, many provided evidence that a number of these criteria were met.

IN-1 Instructor, Student, and Administrative Feedback

Most of the projects submitted contained evidence of positive feedback from instructors, students, and administrators. Many contained students' comments, anecdotes, and letters of support from other faculty or administrators. The advisory board considered this criteria crucial in ranking the entries and saw a need for a more comprehensive assessment of student learning and attitudes. The vast majority of the winning entries had at least one co-author. Where there was only a single author, it was clear that there was an extensive effort to gain support of colleagues and administrators before proceeding. Entries that did not contain evidence of peer support expressed a clear sense of frustration when they tried to "go it alone."

IN-2 Appropriate Support of Client Departments

More than half of these projects contained descriptions of how they obtained support of the appropriate client departments. However, it was not clear that there was an initial collaboration with the client department in selecting the course content. Some of the client departments included Art, English, Business, Education, Physics, and the Sciences. As evidenced in letters of support, faculty from client departments were impressed with how serious mathematics instructors were in their use of more realistic problems. The INPUT Project team anticipated that more projects would break departmental barriers in a true spirit of collaborative effort. The small number of collaborative projects submitted may be an indication to college administrators that more opportunities for interdepartmental collaboration should be made available and that these efforts should be rewarded.

IN-3 Potential for Adoption at Other Institutions

The evaluation team found that more than half of the projects could be or had been adopted at other institutions. Some of the projects had been tried at only one institution, while others were mature enough to have been adopted at more than 100. Most, if not all, of the project leaders were willing to work with instructors at other institutions or to make presentations at conferences to show others their innovations. Many of the project directors have commercially published materials, and some are sharing their projects at workshops, conferences, and on the world-wide web.

Another major aspect of portability was the consideration that the innovations in content could take place without specific technology. Most innovators made an effort to have materials available in multiple formats. For example, Aliaga had materials for both the graphing calculators and statistical software. Richardson has parallel textbooks for use with a spreadsheet package or graphing calculator. It was clear that most project winners found that graphing calculators were an ideal format. The cost to the student was greatly outweighed by the ability to access the technology both in and out of the classroom. The newer versions, such as the TI-83, have many statistics and table features that previously were found only in more expensive software. Graphing calculators also made collaborative learning possible in large lecture sections in contrast to the smaller classes often dictated by computer technology.

IN-4 Extended Accessibility for Non-Traditional Students: and
IN-5 Improved Efficiency (cost effectiveness)

Unfortunately, there were no projects claiming to extend accessibility to nontraditional groups, and only four projects showed that they were actually cost efficient. Not all of the projects were designed to address these criteria specifically, and many had not collected data to address these issues. Some members of the advisory board were looking for successful use of distance learning in extending the accessibility for non-traditional students. However, none of the projects included distance learning. Most entries addressed cost-effectiveness in a way that was natural for instructors, but not for administrators. Instructors do not generally consider actual costs in achieving desired student learning, as this is an administrative responsibility. If student learning is improved and the technology is available, then we have the "undefined ratio of positive benefit to zero cost." Without a common understanding of cost/benefit ratios, continuing financial support from administrators is unlikely.

These criteria were originally suggested by the project monitor, Steve Ehrmann, in conjunction with his work for AAHE (American Association of Higher Education), *Project Flashlight* [49]. In that project Ehrmann presents the triple challenge to administrators to rethink the use of technology in order to fulfill their missions. The triple challenge is to improve:

1. 21st century learning outcomes — to help graduates lead better lives and enrich their communities;
2. accessibility — to enroll, retain, and graduate more non-traditional students; and
3. cost effectiveness — to attempt to control the costs per graduate.

The latter two challenges are primarily institutional goals, not traditionally within the domain of most instructors. The Project team hoped that it would receive entries that had encouraged the use of technology to accomplish these institutional goals. Because the initial flyers were sent to departments of mathematics and statistics and not to the administrators of the institutions, the project teams consisted wholly of faculty members. Faculty could adequately respond to the first challenge but seemed unaccustomed to thinking in terms of institutional goals. Better communication between faculty and administrators may help both groups better understand the triple challenge.

IN-6 Revitalized Course Content with the Use of Technology

All of the instructors of the winning projects indicated that they had to spend a great deal of time and energy collaborating with other faculty to rethink the entire course content, not just a few isolated problems. This is clearly evidenced by the detailed course syllabi submitted by the top five winners. The majority of the projects changed pedagogy and student assessment as well. Only a few of the non-winning entries concentrated on changing the content to fit the specific needs of the available technology. All members of the advisory board were in agreement that technology was a means to rethinking the mathematical content of the courses and not an end in itself.

IN-7 Coordination of Content with Sequel or Related Courses: and
IN-8 Collaboration with Other Disciplines, Colleges, Business, and Extracurricular Agencies

When sequel courses were within the same department, most often mathematics, there was a strong effort to coordinate content. This was not apparent when sequel courses were outside the department. Although instructors of the winning projects communicated with other departments, businesses, and community agencies in obtaining realistic data, we did not find true collaboration. Because the university environment is not often conducive to interdepartmental collaboration, faculty themselves are often unfamiliar with the teamwork approach. It is not hard to understand why many instructors are reluctant to try collaborative learning approaches in their classroom when they have not experienced it themselves. As W. Edwards Deming [40] has said, the team approach is foreign to the independent American spirit. This is a very important milestone for administration to encourage and reward. The departmental infrastructure promotes independent faculty research but discourages interdisciplinary efforts, particularly because of policies about the allocation of student credit hours.

Summary

In the descriptions of the award-winning projects, a number of common themes begin to take shape, namely:

1. Instructors' use of technology has led to a rethinking of the content in their introductory college mathematics courses. The students have gained access to ideas and problems that would be difficult or impossible to pursue without the technology.
2. Survey results showed that students perceived these courses as requiring more time than the traditional "chalk and talk" courses. Despite this, student attitudes improved and course withdrawal rates significantly declined.
3. Attempts at assessing the "traditional content" showed that the average scores have not declined even with the smaller withdrawal rate. Further, assessment of students' problem-solving skills showed them to be significantly better than those of students in the "traditional courses."
4. Non-traditional assessment tools were required to measure the scope of students' understanding. Extensive written and oral assessment was a common theme of the winning projects.
5. Successful projects were collaborative in fact as well as in content. Not only did they effectively use teamwork as a pedagogical tool, they also relied on extensive departmental teamwork to initiate, sustain, revise, and assess the project. The broader the realm of collaboration and support, the more INPUT criteria the projects met.
6. While not every project had external support, it was clear that additional resources of time and support were needed to initiate and maintain a successful project.
7. Because students were given, or gathered, actual "messy" data, descriptive statistics and least squares curve fitting played a major role in all introductory subject areas.
8. Technology in the winning projects was used to enhance the content and not to "drive it." The use of existing technology shortened the time to display the data and experiment with multiple functions and approaches.
9. Increased student confidence in their ability to analyze and solve real problems was the most commonly reported result.
10. Feedback from other departments indicated that students from the "innovative" courses were more willing to attempt mathematical modeling in subsequent applied courses. They expressed the belief that the revised content courses were truly more service oriented than more traditional topic oriented courses.
11. Winning projects were easily portable to other institutions in the sense that their formats were flexible enough to be adapted to a variety of software packages and/or calculators.

Where do we go from here?

These winners have demonstrated that students, faculty, and their institutions benefit from more meaningful teaching and learning experiences in introductory college mathematics. We have learned that some of our old axioms of student learning and student needs are no longer beneficial for the majority of introductory mathematics and statistics students. As with most scientific discovery, there is a need to examine examples and counterexamples to understand and develop the model further. The INPUT project originally hoped to obtain examples of both "successful" and "unsuccessful" projects. Even though no entries claimed to have unanticipated or unsuccessful results, there is still a great need for instructors to share both their victories and their defeats. A great deal can also be learned by forming a stronger relationship with K–12 teachers who have had more time and experience with the reform effort. For the sake of consistency of goals, all introductory college mathematics instructors should be aware of the revised standards as set forth by the NCTM in 1989.

All of the winners were successful in describing various ways that their projects exemplified the standards for intellectual development, content and pedagogy as set forth in the AMATYC publication, *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* [26]. Because the INPUT goals had not been previously published, there was not a general consideration for some of these goals. In particular, the goals of extended accessibility and improved efficiency are primarily institutional goals.

It became clear that most instructors may not measure cost effectiveness in the same way that administrators do. Administrators who may be looking toward technology as a means of reducing the cost per student may not be pleased with results that demonstrate much more learning and much more positive student attitudes for slightly more money. Service courses, which have traditionally been low cost, may not be viewed as a priority for increased resources. Without a common understanding of accessibility and cost/benefit ratios, continuing support from administrators is jeopardized.

In order to maintain support for mathematics service programs, more communication with administrators, client departments, former students, and their potential employers is imperative. Mathematics/statistics departments can no longer rely on the fact that client demand will always be there. There is a growing perception that many mathematics/statistics courses are not serving the immediate and long-range needs of today's students. Since more than 70 percent of mathematics/statistics credit hours are generated by non-majors, mathematicians may feel the need to "sell their wares." Like it or not, the new academic accounting systems are forcing all departments to be more accountable. In particular, traditional service areas such as English and Mathematics will be obligated to justify their existence in light of declining majors. This justification may be cost effectiveness and/or better client orientation. This will mean that faculty among many departments will need to collaborate on the content and assessment of the introductory courses. A strong tradition of departmental independence poses a difficult challenge for administrators: to open the doors to interdepartmental collaboration. National accreditation agencies, professional organizations, and administrative organizations can play a stronger role in initiating rewards for collaboration between departments.

Collaboration is also necessary within the department because economic constraints may not allow institutions the luxury of offering separate client-centered and traditional topic-driven courses. Unless dialogue occurs within departments, the decision of course approach may be taken out of their hands. Client departments and employers are also encouraging the use of more collaborative activities. But how can faculty espouse collaborative efforts in an environment that has traditionally rewarded individual achievement? The courses exemplified in this text have demonstrated that instructors need to support the student collaboration necessary for the analysis and solution of complex problems. Similarly, the complex problems inherent in decision-making regarding course offerings and content require support for faculty collaboration. As the INPUT project has recognized the need to reward the collaborative efforts of these innovators, a broader commitment to encourage and reward faculty collaboration within departments is also needed.

Challenge

The primary goal of the INPUT Project is to encourage more faculty to reevaluate their course content in light of existing technology. Whether or not we accomplish this goal depends upon: the efforts of faculty and administrators; recommendations from professional organizations and accrediting agencies; and the cooperation of former students and their employers. The winners of this competition have broken important ground. By following their lead, you can also make changes that will enhance teaching and learning. Although you may not use the exact materials or the particular technologies they used, we hope that the examples will help you to rethink your options. By disseminating specific examples of innovative projects in this handbook, the INPUT Project team hopes that others will be inspired to collaborate with peers to design and implement their own innovative courses.

Appendix A

INNOVATIVE PROGRAMS USING TECHNOLOGY

APPLICATION FORM for the First Annual Awards for Innovative Programs Using Technology (INPUT) in Mathematics Service Courses (Sponsored by the Annenberg/CPB Project, Central Michigan University and NSF)

Deadline: June 1, 1996

OVERVIEW

Purpose: The purpose of the Second Awards Competition for Innovative Programs Using Technology (INPUT) in Introductory College Mathematics Courses is to stimulate the rethinking of the mathematical content of these courses using technology. Innovative programs using technology (INPUT) should feature engaging, cost-effective programs that provide broader access to higher education for traditional and non-traditional students. The competition will reward exemplary instructors and departments that have made the effort to rethink introductory college mathematics courses and whose efforts can inspire or console those who follow.

Application forms are available 1) on the World Wide Web Server (http://www.cmich.edu/~mthaward) 2) by e-mailing listserv@cmich.edu with the one line message GET INPUT APPLICAT or 3) by writing Susan Lenker, Project Director, Department of Mathematics, Central Michigan University, Mt. Pleasant, MI 48859. Questions can be asked by sending an e-mail address to INPUT@cmich.edu.

OBJECTIVES:

1) For the purpose of this competition, a mathematics service course is defined as a course (or sequence of courses) involving: developmental mathematics, quantitative literacy/special topic, algebra/pre-calculus, business mathematics, and introductory statistics.

2) The competition will result in 20 cash prizes:

One	$5000 award (?)
Four	$1000 awards (?)
Ten	$500 awards (?), and also
Five	$500 prizes (?) for 'notable efforts' (efforts that did not assess as well as expected)

An attempt will be made to distribute the awards among all areas. The notable efforts prizes are to encourage entries from instructors (departments) who tried, but for some reason did not meet their desired goals. This recognizes the fact that there is often as much to be learned from false starts as from successes.

3) An analysis of the top entry in each of the five categories will be included on a World Wide Web page that also features links to other projects, discussion groups, conferences and workshops that are focusing on mathematics service courses.

4) Award winners will be asked to help with the dissemination of their results which may include mentoring an electronic discussion group, moderating presentations at workshops and national meetings or creating a WEB page.

THE PROBLEM:

Students' lack of interest in mathematics and poor technical skills for entering the job market have become a source of national concern. For years, collegiate mathematics service courses have served a gate-keeping function, shutting the gate on ill-prepared students. Although 80% of most 4-year colleges and as many as 90% of most community college mathematics credit hours are generated by service courses, very little of this revenue has been used to provide technological support for teaching these courses. For years, mathematics service courses have lagged behind the technological revolution, choking in chalk dust. Those instructors who attempt to introduce technological innovations to service courses are rarely rewarded. While the areas being served have frequently progressed to full integration of technology, many math departments are either opposed to, indifferent to, or incapable of integrating the technology into service courses. For those who are willing, there is little support. Furthermore, there are very few rewards for innovations in service courses. All but the most secure are discouraged from straying off the traditional path of theoretical research.

REMEDIES:

A small but growing number of mathematics instructors has begun to focus on using technology rather than rote drill to enhance students' skills in critical thinking, pattern detection and collaborative problem solving. These instructors understand that such skills cannot be "added" or "squeezed in" to a traditional curriculum. The use of technology can free both faculty and students from many traditional tasks, but calls for a change of paradigm regarding the goals and objectives of service courses.

The First INPUT Competition was held in 1996 and culminated with an Award Ceremony held at the joint AMS/MAA conference held in San Diego in Jan., 1997. A handbook entitled *Exemplary Programs in Introductory College Mathematics* was published (1997) featuring the award winners of the competition. A video of the top 5 award winners was also produced by the Annenberg/CPB Project. The WWW page (http://www.cmich.edu/~mthaward/ also displays the top 5 projects along with more information regarding the competition. The second competiton will continue to reward exemplary instructors for their efforts and provide an international forum for sharing their ideas with others. It is the sponsor's (INSERT NAMES) that you will be challenged and rewarded for your effort in rethinking and rejuventing the content in introductory college mathematics courses.

ELIGIBILITY REQUIREMENTS

Eligible Programs:

The purpose of the First Annual Awards for Innovative Programs Using Technology (INPUT) in Introductory College Mathematics Courses is to identify an exemplary course or sequence of courses involving lower division mathematics and statistics courses that fall within the general description of: *developmental mathematics, * quantitative literacy/special topics, * algebra/pre-calculus, * business math, or * introductory statistics.

Entries may describe any type of technology-enhanced course, where the term technology is to be broadly interpreted to include (but not be limited to) the use of calculators, computers, video, multimedia and telecommunications. The project must have been taught to one or more classes during at least one term at an institution and use one or more types of technology in an innovative way. Preference will be given to entries that: 1) include instructor, student, and administrative feedback, as well as the appropriate support of client departments; 2) have the potential for adaptation at other institutions, given the proper level of technology; and have used technology to: 3) extend accessibility for non-traditional students; 4) improve cost efficiency; 5) revitalize and coordinate the content of impacted courses(s) with sequel courses; and/or 6) foster collaboration with other disciplines, colleges, business and extracurricular agencies.

Eligible Institutions:

Entries are invited from educational institutions in the U.S. and abroad including: two-year and four-year colleges, four-year universities and consortia of institutions, nonprofit, non-academic institutions that are directly associated with educational research activities. Entries from a formal consortium should be submitted by the consortium; entries from an informal consortium should be submitted by one of the member schools. Entries must be submitted in English. Applicants are expected to sign a release to have a description of their project appear on the WEB page http://cmich.edu/~mthaward/.

Target Audience:

The First Annual Awards for Innovative Programs Using Technology (INPUT) in Mathematics Service Courses focuses on using technology in innovative ways to teach introductory college mathematics courses. The categories for the project are:

•developmental mathematics
•quantitative literacy /special topics
•algebra/pre-calculus
•business mathematics
•introductory statistics

The project encourages entries from a target audience of institutions that support these types of innovative technological programs focused on the lower division mathematics curriculum. The goal of the project is to disseminate these programs to a wide spectrum of undergraduate institutions for use by large numbers of students.

Preparation and Submission of Proposals

General Information:

This section sets forth basic information needed to initiate planning for application submission. Applicants may consult with the Project Director for additional guidance or may view the World Wide Web page (http://www.cmich.edu/~mthaward/) for frequently asked questions. All applicants should be aware that the purpose of this project is to disseminate information about the projects as described in your application . As you write your application, remember that some or all of the text you submit may be published in the future for use by colleagues around the world. Please write in a way that will be of greatest benefit to them.

Application Preparation

The standard application should contain the following information, assembled in the order indicated. Note the stringent page limits on Form 2 and the project narrative. All forms are available on the World Wide Web Server (http://www.cmich.edu/ ~mthaward) or by e-mailing listserv@cmich.edu with the one line message GET INPUT APPLICAT or by writing Susan Lenker, Project Director, Department of Mathematics, Central Michigan University, Mt. Pleasant, MI 48859. Questions can be asked by sending an e-mail address to INPUT@cmich.edu.

The original application and 5 copies are to be submitted together. A complete application consists of the following:

I. Cover Sheet (Form 1)
II. Institution Information (Form 2)
III. Table of Contents
IV. Project Narrative
V. Appendices

I. Cover Sheet

The first page of the application should be the cover sheet (Form 1). It is important that the cover sheet be completed with the full information requested. Most of the items are self-explanatory. One copy of the cover sheet must carry the original signature(s) of the applicant(s) and the authorized organizational representative (for possible publicity and releases). The Title of the Applicant's Project is one of several items used to direct the application to the appropriate categories and to announce and advertise to the general public and the mathematics community the nature of the projects. The title should include informative key words that indicate the course(s), the student population, and the nature of the innovation, for example, "Using Spreadsheets in an Introductory Business Math Sequence."

II. Institution Information (Form 2)

The second page of the application should be Form 2. It contains information about the institution(s) involved with the project, the general student population and available technology.

III. Table of Contents

IV. Project Narrative

In general, the narrative section must not exceed 20 double-spaced pages (3 lines per inch). The narrative presents most of the information that determines whether or not an applicant will be awarded recognition and financial awards. The application narrative should follow the outline below and should keep in mind the points made in the eligibility requirements.

A. Course Profile

1. Provide a bulletin description and syllabus of the course or sequence of courses involved.
2. Describe the context of this course(s) in the curriculum it serves. Describe all course prerequisites, co-requisites and course sequels that are appropriate for this course(s).
3. Describe the student profile.
4. Describe the instructors and any special training required.

B. Methodology and Soundness

1. Describe the project goals.
2. Describe the degree to which the project is grounded in the literature. Include citations in Appendix C.
3. Describe the degree to which the goals are in alignment with some of the goals of the national organizations (AMATYC, AMS/MAA, NCTM, ASA, or NRC).
4. Describe how the use of the technology supports these goals.
5. Describe the extent to which this project can be used as a model for others to follow.

C. Measures of Effectiveness

1. Compare and contrast the course objectives with and without the new technology.
2. Describe any measures of internal effectiveness (i.e., success rates, test scores, attitude surveys, etc.). Projects that have assessments from instructors, peers and students will be viewed more favorably.
3. Describe any external measures of effectiveness (i.e., letters from other instructors, departments being served, administrators, employers, success rates in sequel courses, demand for supporting materials, publisher interest etc.).
4. Provide some evidence of the cost-benefit ratio of student learning outcomes.

D. Technology

1. Describe the technology and the extent to which the technology supported the change in the course(s).
2. Describe the rationale for the choice of the technology and the extent to which it received peer, interdepartmental and institutional support. Describe any difficulties that had to be overcome to implement the project (i.e., acquisitions, maintenance, etc.)
3. Describe the extent to which the project can be adapted to other institutions with different levels of technology.
4. Describe the extent to which the project could be extended to other service courses and/or other disciplines.
5. Describe the degree to which the innovations will be able to transcend present technology.

E. Dissemination

1. Describe the degree to which you have publicized or would be willing to publicize your project at conferences and workshops.
2. Describe the degree to which you are willing to disseminate your project (WEB page, texts, draft documents, electronic bulletin boards, moderating professional presentations at conferences, mentoring a listserv list, etc).
3. List any recognition for the project; briefly describe the nature and extent of the award.

F. Other Information (Sampe activities, test items etc.)

V. Appendices

A. Biographical Sketch(es). The biographical sketches should be limited to one page per participant and address only the teaching experience and research that directly relates to this project.

B. Letters of Support. As appropriate, include official letters only that verify specific institutional support for the project. These letters should include a description of the institutional goals and how the project helps attain those goals.

C. Bibliography

Submission

The required materials should be postmarked no later than June 1, 1998, and sent to:

Department of Mathematics
ATT: Susan Lenker
214 Pearce Hall
Central Michigan University
Mount Pleasant, MI 48859

You must meet the following requirements:

- Submit five copies of the application delineated in the previous section.
- All materials submitted to the Mathematics Department must be contained in a single package. Secure packaging is mandatory. The Department can not be responsible for the processing of proposals damaged in transit.
- Each copy of the application should be on standard size paper of regular weight. It should be stapled only in the upper left corner. It should not be bound by means of glue, spirals, wire, clasps or any other means. All narrative and appendices pages must be numbered.
- One copy must be signed by the applicant(s) and by an administrative official(s) who is(are) authorized by the institution(s) involved in the project. Typically, this might be the department chair, the dean, or provost.

For further information:

1. e-mail INPUT@cmich.edu or

2. phone Susan Lenker (517) 774-6520.

FORM 1 Project Information (Cover Sheet)

Project Title:

Project Director(s):

Funding source(s) for design, implementation, and evaluation of the project:

Technologies used in the project:

Service Course Area (Circle all that apply):

- Developmental Mathematics • Algebra/Pre-Calculus
- Quantitative Literacy / Competency
- Business Math • Introductory Statistics

Describe the student population in the course(s) for this project.

Course(s) and program(s) impacted directly and indirectly by the project:

Project Abstract (250 words or less) Include a description of the project goals.

FORM 2 Institution Information

Institutions/Departments involved with this project:

Describe the institution(s):

Describe the technologies available at the institution(s):

Describe the student population(s) of the institution(s):

Appendix B

INNOVATIVE
PROGRAMS
USING
TECHNOLOGY

Bibliography

1 Air Force Academy Admissions Office. *United States Air Force Academy Catalog, 1995–1996*. United States Air Force Academy, CO, 1995.

2 Albers, D. J., D. O. Loftsgaarden, D. C. Rung, and A. E. Watkins. *Statistical Abstracts of Undergraduate Programs in the Mathematical Sciences and Computer Science in the United States, 1990–91 CBMS Survey*. MAA Notes, Number 23. Washington, DC: Mathematical Association of America, 1992.

3 Aliaga, M., and B. Gunderson. *Interactive Statistics*. Prentice Hall, Inc. 1998.

4 American Association for the Advancement of Science. *The Liberal Art of Science: Agenda for Action. The Report of the Project on Liberal Education and the Sciences*. Washington, DC: AAAS publication 90 - 13S, 1990.

5 American Association for the Advancement of Science. *Science for All Americans: A Project 2061 Report on Literacy Goals in Science, Mathematics, and Technology*. Washington, DC: Author, 1989.

6 Andersen, J., T. Swanson, and R. Keeley. *Projects for Precalculus*. Philadelphia, PA: Saunders College Publishing, 1997.

7 Anderson, B. J. "Minorities and Mathematics: The New Frontier and Challenge of the Nineties." *Making Mathematics Work for Minorities:Framework for a National Action Plan, 1990–2000: Report of a Convocation*. Mathematics Sciences and Education Board, 1990.

8 Anderson, J. R. *Rules of the Mind*. Hillsdale, NJ: L. Erlbaum Assoc., 1993.

9 Anderson, J. R., C. F. Boyle, A. Corbett, and M. W. Lewis. "Cognitive Modeling and Intelligent Tutoring." *Artificial Intelligence*, 42, 1990, pp. 7–49.

10 Anderson, J. R., A. T. Corbett, K. R. Koedinger, and R. Pelletier. "Cognitive Tutors: Lessons Learned." *Journal of the Learning Sciences*, 4, 1995, pp. 167–207.

11 Association of American Colleges. *Integrity in the College Curriculum: A Report to the Academic Community*. Washington, DC: Association of American Colleges, 1990.

12 Astin, A. W. *What Matters in College?*: Four Critical Years Revisited. San Francisco: Jossey-Bass, 1993.

13 Barnsley, M. *Fractals Everywhere*. Boston: Academic Press, 1988.

14 Becker, J. R., and B. J. Pence. *The Teaching and Learning of College Mathematics: Current Status and Future Directions*. In J. J. Kaput and E. Dubinsky (Eds.), *Research Issues in Undergraduate Mathematics Learning*. Washington, DC: Mathematical Association of America, 1994, pp. 5–14.

15 Beckmann, C. "Effects of Computer Graphics Use on Student Understanding of Calculus Concepts." *Dissertation Abstracts International*, Vol. 50, (5-B), November 1989.

16 Blackwell, D., and L. Henkin. *Mathematics: Report of the Project 2061 Phase I Mathematics Panel*. Washington, DC: American Association for the Advancement of Science, 1989.

17 Blum, W., and M. Niss. "Mathematical Problem Solving, Modeling, Applications, and Links to Other Subjects—State Trends and Issues in Mathematics Instruction." *Educational Studies in Mathematics*, 22, 1991, pp. 37–68.

18 Boles, M., and R. Newman. *Universal Patterns*. Second edition, Bradford, MA: Pythagorean Press, 1987.

19 Brito, D. L., Peterkin Professor of Political Economy at Rice University, and D. Y. Goldberg, Professor of Mathematics at Occidental College. "Calculus for Business and Social Science Students." In L. Steen (Ed.), *Calculus for a New Century*. MAA Notes, Number 8. Washington, DC: Mathematical Association of America, 1987.

20 Brown, A. L. "Design Experiments: Theoretical and Methodological Challenges in Creating Complex Inter

ventions in Classroom Settings." *Journal of the Learning Sciences*, 2, 1992, pp. 141–178.

21 Brown, C. A., and H. Borko. *Becoming a Mathematics Teacher*. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan, 1992, pp. 209-239.

22 Burrill, G., ed. *From Home Runs to Housing Costs, Data for Teaching Statistics*. Palo Alto, CA: Dale Seymour Publications, 1994.

23 Carpenter, L. *Instructor's Guide with Complete Answer Key to Accompany Calculus Concepts: An Informal Approach to the Mathematics of Change*. Houghton Mifflin Company, August 1998.

24 Choike, J. R. "A Report of Activities for 1992-1993: The Equity 2000 National Mathematics Committee." New York, NY: The College Board, 1993.

25 Cobb, G., J. Garfield, W. Meeker, and D. Moore. "Statistics education fin de sielce." *The American Statistician*, 1995.

26 Cohen, D., ed. *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges, 1995.

27 Cohen, V. "How Do You Know It's True? Probable Fact and Probable Junk." *NewsBackgrounder*. Los Angeles, Foundation for American Communications, 1994.

28 COMAP. *For All Practical Purposes: Introduction to Contemporary Mathematics* New York: W. H. Freeman and Co., 1994.

29 *Contemporary Precalculus through Applications*. The North Carolina School of Science and Mathematics. Janson Publications, 1991.

30 Corbett, A. T., and J. R. Anderson. "Student Modeling and Mastery Learning in a Computer-Based Programming Tutor." In *Proceedings of the Second International Conference on Intelligent Tutoring Systems*. Montreal, Canada, 1992.

31 Crocker, D. A. "What Has Happened to Calculus Reform?" *The AMATYC Review*. Volume 12, 1, 1990, pp. 62–66.

32 Crocker, D. A. "Constructivism and Mathematics Education." *The AMATYC Review*. Volume 13, 1, 1991, pp. 66–70.

33 Crocker, D. A. "Cooperative Learning." *The AMATYC Review*. Volume 14, (1), 1992, pp. 78–83.

34 Cunningham, S., and W. S. Zimmermann , eds. *Visualization in Teaching and Learning Mathematics* (MAA Notes number 19). Committee on Computers in Mathematics Education, 1991.

35 Daniel, D. "A Modest Proposal for Education in the Next Century." *Earth Literacy Link*, Volume 2 (4), 1994, pp. 13–16.

36 Daniel, D. "Earth Literacy and the Universe Story." *Earth Literacy Link*, 3, 1995, pp. 11–13.

37 Davidson, N., ed. *Cooperative Learning in Mathematics: A Handbook for Teachers*. Menlo Park, CA: Addison-Wesley Publishing Co, 1990.

38 Davis, P., and R. Hersch. *A Mathematical Experience*. Boston: Houghton-Mifflin, 1981.

39 Demana, F. *College Precalculus Courses: Challenges and Change*. In A. Solow (Ed.), *Preparing for a New Calculus*. (MAA Notes number 36). Washington, DC: Mathematical Association of America, 1994, pp. 14–20.

40 Deming, W. E.. *Quotations of Dr. Deming: the little blue book*. Greyden Press, 1994.

41 "Developing and Maintaining a Course in Computer Literacy." *Proceedings of the 4th Annual Southeastern Small College Computer Conference*, 1990.

42 Dion, G., and I. Fetta. "Calculator Graphs: Magic? or Mathematics?" *Mathematics and Computer Education*, 27, 3, Fall 1993, pp. 180-197. Spanish translation appearing in *Las calculadoras graficadoras en el aula*, Departamento de Mathematica Educativa, CINVESTA, June 1994.

43 Dossey, J., I. Mullis, M. Lindquist, and D. Chambers. *The Mathematics Report Card: Are We Measuring Up?* Princeton, NJ: National Assessment of Educational Progress, ETS, 1988.

44 Douglas, R., ed. *Toward a Lean and Lively Calculus*. (MAA Notes Number 6). Washington, DC: Mathematical Association of America, 1986.

45 Drummond, P. *Earth Algebra Project: A Final Evaluation Report*. Submitted to FIPSE as evaluation of EA Project. Kennesaw State College: Marietta, GA., 1993.

46 Dubinsky, E. et al. *Readings in Cooperative Learning for Undergraduate Mathematics*. (MAA Notes number 44). Washington, DC: Mathematical Association of America, 1997.

47 Dunham, P. H. "Does Using Calculators Work? The Jury is Almost In." *UME Trends*, May 1993, p. 8.

48 Dunham, P. H., and T. P. Dick. "Research on Graphing Calculators." *Mathematics Teacher*, Volume 87, 1994, pp. 440–445.

49 Ehrmann, S. C. "The Flashlight Project: Spotting an Elephant in the Dark." In *Assessment Update* vol9(4), July, 1997, p. 3.

50 Emmer, M. *The Visual Mind: Art and Mathematics*. Cambridge, MS: MIT Press, 1993.

51 Erickson, T. *Get it Together*: math problems for groups, grades 4-12. Berkeley, CA: EQUALS, Lawrence Hall of Science, 1989.

52 Fetta, I. *Graphing Calculator Instruction Guide to Accompany Calculus Concepts; An Informal Approach to the Mathematics of Change*. Houghton Mifflin Company, August 1998.

53 Flores, A. "The Shadows of Mathematics." *Arithmetic Teacher*, 40, 1993, pp. 428–429.

54 Foley, G. D. "Accessing Advanced Concept with Technology." *The AMATYC Review*. Volume 11, 2, 1990, pp. 58–64.

55 Garfield, J.. "How Students Learn Statistics." *International Statistical Review*, 63, 1, 1995, pp. 25-34.

56 Garland, T. H. *Fascinating Fibonaccis: Mystery and Magic in Numbers*. Palo Alto, CA: Dale Seymour Publications, 1987.

57 Garland, T. H., and C. V. Kahn. *Math and Music: Harmonious Connections*. Palo Alto, CA: Dale Seymour Publications, 1995.

58 Goldenberg, P. E. "Mathematics, Metaphors, and Human Factors: Mathematical, Technical, and Pedagogical Challenges in the Educational Use of Graphical Representation of Functions." *Journal of Mathematical Behavior*, vol. 7, 2, August, 1988.

59 Gordon, F. S. *Assessing How Well We Do Precalculus while Functioning in the Real World*. PRIMUS, Vol. V, 1995.

60 Gordon, S. P. "Implementing Change in the Mathematics Curriculum." *The AMATYC Review*, 16, pp. 8–13, 1994.

61 Gordon, S. P. *A Roundtable Discussion with the Client Disciplines*, in *Calculus: The Dynamics for Change*. Wayne Roberts, et al, eds. MAA Notes, Number 39. Washington, DC: MAA, 1996.

62 Gordon, S. P. *Functioning in the Real World: Probability and Data Analysis in Precalculus*, in Proc. Seventh International Conference on Technology in Collegiate Mathematics, 1996.

63 Gordon, S. P. *Functioning in the Real World: A Reform Precalculus Experience*, in Proc. Seventh International Conference on Technology in Collegiate Mathematics, 1996.

64 Gordon, S. P. *Out of the Mouths of Babes: Student Questions and Comments in Reform Courses*. PRIMUS, 1997.

65 Gordon, S. P., and D. H. Hallett. *Lessons from the Calculus Reform Effort for Precalculus Reform*, in *Preparing for a New Calculus*. Anita Solow, ed. MAA Notes, Number 36. Washington, DC: MAA, 1994.

66 Gordon, S. P., et al. *Functioning in the Real World* software package, Instructor's Manual, and Student Solution Manual, to accompany textbook. Addison-Wesley, 1996.

67 Gordon, S. P., et al. *Functioning in the Real World: A Precalculus Experience*. Reading, MA: Addison-Wesley, 1997.

68 Grouws, D. A., ed. *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillan Publishing Co., 1992.

69 Guirr. *Nurturing Science and Engineering Talent*. The Government-University-Industry Research Roundtable, 1987.

70 Guskin, A. E. "Reducing Student Costs and Enhancing Student Learning, Part II: Restructuring the Role of Faculty," *Change*. Volume 26, 5, 1994, pp. 16–25.

71 Harris, C. "Exploiting the Modeling Capabilities of Graphing Calculators in Business Calculus." *Mathematics and Computer Education*, 27, 3, Fall 1993, pp. 226–232.

72 Haver, B., and G. Turbeville. "An Appropriate Culmination Mathematics Course." *The AMATYC Review*, 16, 1995, pp. 45–50.

73 Heermann, B. *Teaching and Learning with Computers: A Guide for College Faculty and Administrators*. San Francisco: Jossey-Bass, 1988.

74 Heid, K. M. "Resequencing Skills and Concepts in Applied Calculus Using the Computer as a Tool." *Journal for Research in Mathematics Education*, Vol. 19, 1, 1988, pp. 3–25.

75 Hillel, J., L. Lee, C. Laborde, L. Linchevski. "Basic Functions through the Lens of Computer Algebra Systems." *Journal of Mathematical Behavior*, 11, 1992, pp. 119–158.

76 Hilliard, A. G. III. *Changing Attitudes. Making Mathematics Work for Minorities*. A project of the Mathematical Sciences Education Board, National Research Council, 1990.

77 Hirst, H., and J. Smith. *How Do You Know? Using Math to Make Decisions* Dubuque, IA: Kendall/Hunt Publishing Co., 1996.

78 Hogg, R. V., and M. C. Hogg. "Continuous Quality Improvement in Higher Education." *International Statistical Review*, 63, 1, 1995, pp. 35–48.

79 Howard, R. M., and S. Jamieson. *The Bedford Guide to Teaching Writing in the Disciplines: An Instructor's Desk Reference*. Boston, MA: Bedford Books of St. Martin's Press, 1995.

80 Illinois Prairie Higher Education Consortium: Distance Education — Class Schedule Interactive Video Network. Spring Semester, 1996.

81 Jacobs, A. *An Application-Driven Curriculum: The Maricopa Mathematics Consortium Addresses the AMATYC Standards*. Scottsdale Community College: Scottsdale, 1995.

82 Jacobs, H. H. "Mathematics Integration: A Common-Sense Approach to Curriculum Development." *Arithmetic Teacher*, 40, 1993.

83 Johnson, D., and T. Mowry. *Mathematics, A Practical Odyssey*. Boston, MA: PWS Publishing Co., 1995.

84 Johnson, D. M. *Mathematics Rating Scale Forms A and B*. Unpublished instrument from University of Tulsa: Tulsa, OK, 1990.

85 Johnston, W. B., and A. E. Packer. *Workforce 2000: Work and Workers for the Twenty-first Century*. Indianapolis: Hudson Institute, 1987.

86 Kaput, J. "Information Technology and Mathematics: Opening New Representational Windows." *The Journal of Mathematical Behavior*, 5, 1986, pp. 187–207.

87 Kaput, J. J. "Technology and Mathematics Education." In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillan Publishing Co., 1992, pp. 515–556.

88 Kaput, J. J., and P. W. Thompson. "Technology in Mathematics Education Research: The First 25 Years in JRME." *Journal for Research in Mathematics Education*, 25, 1994, pp. 676–684.

89 Keith, S. Z. "Writing for Education Objectives in a Calculus Course." In A. Sterrett (Ed.), *Using Writing to Teach Mathematics*. Washington, DC.: Mathematical Association of America, 1990, pp. 6–10.

90 Keith, S. Z., and J. R. C. Leitzel. "General Education Mathematics: A New Challenge for Departments." *UME Trends*, 1994, pp. 6–7.

91 Kenelly, J. "Calculus as a General Education Requirement." In R. Douglas (Ed.), *Toward a Lean and Lively Calculus*. MAA Notes, Number 6. Washington, DC: Mathematical Association of America, 1986, pp. 61–69.

92 Kenelly, J., ed. *The Use of Calculators in the Standardized Testing of Mathematics*. Published jointly with The College Board, 1989.

93 Kenelly, J. "Technology in Mathematics Instruction." In W. Roberts (Ed.,), *Calculus, the Dynamics of Change*. MAA Notes, Number 39. Washington, DC: Mathematical Association of America, 1996, pp. 24–28.

94 Kloosterman, P., and F. K. Stage. "Measuring Beliefs about Mathematical Problem Solving." *School Science and Mathematics*, Volume 92 (3), 1992, pp. 109–115.

95 Kober, N. *EdTalk: What We Know about Mathematics Teaching and Learning*. Washington, DC: Council for Educational Development and Research, 1993.

96 Koedinger, K. R., and J. R. Anderson. "Abstract Planning and Perceptual Chunks: Elements of Expertise in Geometry." *Cognitive Science*, 14, 1990, pp. 511–550.

97 Koedinger, K. R., and J. R. Anderson. *Illustrating Principled Design: The Early Evolution of a Cognitive Tutor for Algebra Symbolization*. Manuscript submitted for publication, 1995.

98 Koedinger, K. R., J. R. Anderson, W. H. Hadley, and M. A. Mark. "Intelligent Tutoring Goes to School in the Big City." In *Proceedings of the World Conference on Artificial Intelligence in Education*. Charlottesville, VA: Association for the Advancement of Computing in Education, 1995, pp. 421–428.

99 Kozma, R. B. "Learning with Media." *Review of Educational Research*, 61, 1991, pp. 179–211

100 LaTorre, D. "Business Calculus: Restructuring Teaching and Learning for Non-technical Students." *Proceedings of the Seventh International Conference on Technology in Collegiate Mathematics*. Addison-Wesley, 1995.

101 LaTorre, D., J. Kenelly, I. Fetta, C. Harris, and L. Carpenter. *Calculus Concepts; An Informal Approach to the Mathematics of Change*. Houghton Mifflin Company, August 1998.

102 Leinbach, L. C., et al., eds. *The Laboratory Approach to Teaching Calculus*. Washington, DC: Mathematical Association of America, 1991.

103 Leinwand, S. J. "Sharing, Supporting, Risk Taking: First Steps to Instructional Reform." *Mathematics Teacher*, NCTM, Vol. 85, No. 6, September 1992, pp. 466–470.

104 Leitzel, J. R. C., ed. Committee on the Mathematical Education of Teachers. *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*. Washington, DC: Mathematical Association of America, 1991.

105 Lincoln, Y. "Trouble in the Land: The Paradigm Revolution in the Academic Disciplines." In J. Smart (Ed.), *Higher Education: Handbook of Theory and Research*. New York: Agathon Press, 1989, pp. 57–116.

106 Long, C. T., and B. H. West. *Mathematics at Southern Oregon State College: A Critique of the Department and its Programs*. Unpublished report, Washington State University and Cornell College.

107 *Lotus 1.2.3*, Release 1.1. Cambridge, MA: Lotus Development Co., 1992.

108 Madison, B. L., and T. A. Hart. *A Challenge of Numbers: People in the Mathematical Sciences*. Committee on Mathematical Sciences in the Year 2000, National Research Council. Washington, DC: National Academy Press, 1990.

109 Massy, W., and R. Zemsky. *Using Information Technology to Enhance Academic Productivity*. Educom reg. 1995. Interuniversity Communications Council, Inc.

110 Mathematical Association of America. *Quantitative Reasoning for College Graduates: A Complement to the Standards*. Washington, DC: MAA, 1995.

111 Mathematical Sciences Education Board. *Reshaping School Mathematics*. Washington, DC: National Academy Press, 1990.

112 Mcneill, D., and P. Freiberger. *Fuzzy Logic*. Simon & Schuster, 1994.

113 Mesterson-Gibbons, M. *A Concrete Approach to Mathematical Modeling*. Reading, MA: Addison-Wesley, 1989.

114 Meyer, W., ed. *Math 101–102: A New Start for College Mathematics*. USA: COMAP, 1992.

115 *Minitab*, Version 8.21 for Macintosh. Reading, MA: Addison Wesley, Inc., 1992.

116 *Minitab Example Data Sets*. Reading, MA: Addison Wesley, Inc., 1994.

117 Mokros, J., and S. J. Russell. "Children's Concepts of Average and Representativeness." *Journal for Research in Mathematics Education*, 26, 1995, pp. 20–39.

118 Monk, S. G. "The Development of Student Understanding of Functions." In *Proceedings of the Fourteenth Annual Meeting of the Canadian Mathematics Education Study Group*. Martyn Quigley, ed. Simon Fraser University, 1990, pp. 133–149.

119 Moore, C. C. *Recognition and Rewards in the Mathematical Sciences*. (Report of the Joint Policy Board for Mathematics Committee on Professional Recognition and Rewards.) Providence, RI: American Mathematical Society, 1994.

120 Moore, D. S. and G. P. McCabe. *Introduction to the Practice of Statistics, 2nd edition*. New York: W. H. Freeman, 1993.

121 Moskowitz, H. and J. B. Henderson, "Calculus for Management: A Case Study." In L. Steen (Ed.), *Calculus for a New Century*. MAA Notes, Number 8. Washington, DC: Mathematical Association of America, 1987.

122 National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM, 1989.

123 National Council of Teachers of Mathematics. *Professional Standards for Teaching Mathematics*. Reston, VA: NCTM, 1991.

124 National Council of Teachers of Mathematics. *Assessment Standards for School Mathematics*, Reston, VA: NCTM, 1995.

125 National Research Council. *Everyone Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, DC: National Academy Press, 1989.

126 National Research Council. *A Challenge of Numbers*. Washington, DC: National Academy Press, 1990.

127 National Research Council. *Moving beyond Myths: Revitalizing Undergraduate Mathematics*. Washington, DC: National Academy Press, 1991.

128 Nelson, J., and J. Williams. *Multicultural Mathematics*. Oxford University Press, 1993.

129 Nickell, G. S., and J. N. Pinto. "The Computer Attitude Scale." *Computers in Human Behavior*, 2, pp. 301–306.

130 Nicklin, J. "Liberal-Arts Educators Unveil a Plan to Improve Undergraduate Programs in Math and Sciences." *The Chronicle of Higher Education*, February 1991, pp. A11–A12.

131 Norwood, K. S. "The Effects of the Use of Problem Solving and Cooperative Learning on the Mathematics Achievement of Under Prepared College Freshmen." *Primus: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, Vol. V, Number 3, September, 1995, pp. 229–252.

132 Orton, A. "Students Understanding Differentiation." *Educational Studies in Mathematics*, 14, 3, 1983, pp. 235–250.

133 Peak, D., and M. Frame. *Chaos Under Control: The Art and Science of Complexity.* NY: W.H. Freeman and Co., 1994.

134 Prevost, F. "Rethinking How We Teach: Learning Mathematical Pedagogy." *The Mathematics Teacher*, NCTM, Vol. 86, Number 1, January 1993, pp. 75–78.

135 Price, J. J. "Learning Mathematics through Writing: Some Guidelines." *College Mathematics Journal*, 1989, pp. 339–401.

136 Prichett, G. D., VP for Academic Affairs and Dean of the Faculty of Babson College. "Calculus in the Undergraduate Business Curriculum." In L. Steen (Ed.), *Calculus for a New Century*, MAA Notes, Number 8. Washington, DC: Mathematical Association of America, 1987.

137 *Quantitative Reasoning for College Graduates: A Complement to the Standards*. Washington, DC: Mathematical Association of America, 1995.

138 Ralston, A., ed. *Discrete Mathematics in the First Two Years*.

139 Rauff, J. *Math Matters*. New York: John Wiley and Sons, Inc., 1996.

140 *Resources for Calculus*. A Calculus reform project funded by the Mathematical Association of America and published as MAA notes Numbers 27–31.

141 Reynolds, B., et al. *A Practical Guide to Cooperative Learning in Collegiate Mathematics*. Washington, DC: Mathematical Association of America, 1995.

142 Richardson, R. L. "Business Calculus Today with Spreadsheets and Derive." Saunders College Publishing, 1996.

143 Richardson, R. L., and D. Alejandro. "Business Calculus Today with TI-85." Saunders College Publishing, 1996.

144 Richardson, R. L., and M. Gilchrist. "Computers: Understanding and Using Them," Gorsuch Scarisbrick, Publishers, 1986.

145 Richardson, R. L. and M. Gilchrist. "Introduction to Computers and Computing: A Hands-on Approach." Gorsuch Scarisbrick, Publishers, 1984, 1990 (2nd ed.).

146 Robbins, J. *Fun with Fractals*. San Francisco: Sybex, 1993.

147 Roberts, D. M. "The Impact of Electronic Calculators on Educational Performance." *Review of Educational Research*, 50, 1, 1980, pp. 71–98.

148 Runion, G E. *The Golden Section*. Palo Alto, CA: Dale Seymour Publications, 1990.

149 Schaufele, C., and N. Zumoff. *Earth Algebra: College Algebra with Applications to Environmental Issues*. HarperCollins College Publishers: New York, 1995.

150 Schoaf-Grubbs, M. M. "The Effect of the Graphing Calculator on Female Students' Spatial Visualization Skills and Level-of-Understanding in Elementary Graphing and Algebra Concept." In E. Dubinsky, A. H. Schoenfeld, and J. Kaput (Eds.), *Research in Collegiate Mathematics Education*, I. Providence, RI: American Mathematical Society, 1994, pp. 169–194.

151 Schoenfeld, A. H. "Mathematics, Technology, and Higher Order Thinking." In R. Nickerson and P. Zodhiates (Eds.), *Technology in Education: Looking toward 2020*. Hillsdale, NJ: Lawrence Erlbaum, 1988, pp. 67–96.

152 Schoenfeld, A. H., ed. *A Source Book for College Mathematics Teaching A report from the MAA Committeee on the Teaching of Undergraduate Mathematics*. Washington, DC: Mathematical Association of America, 1990.

153 Schoenfeld, A. H. "Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics." In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan, 1992, pp. 334–370.

154 Schofield, J. W., D. Evans-Rhodes, and B. R. Huber. "Artificial Intelligence in the Classroom: The Impact of a Computer-based Tutor on Teachers and Students." *Social Science Computer Review*, 8, 1990, pp. 24–41.

155 Searcy, M. Doctoral dissertation. J. Wilson, advisor. Mathematics Education Department, University of Georgia, 1997.

156 Secada, W. G. "Race, Ethnicity, Social Class, Language, and Achievement in Mathematics." In D. A. Grouws (Ed.), *Handbook on Research in Mathematics Education*. New York: Macmillan, 1992, pp. 623–660.

157 Selby, C. C. "Integrated Mathematics, Science and Technology Education." *The Technology Teacher*, 1988, pp. 3–5.

158 Selden, A., and J. Selden. "Constructivism in Mathematics Education: A View of How People Learn." *UME Trends*, Vol. 2, Number 1, 1990, p. 8.

159 Seymour, D. and J. Britton. *Introduction to Tessellations*. Palo Alto, CA: Dale Seymour Publications, 1989.

160 Siegel, M.J. "Discrete Mathematics." In Reshaping College Mathematics (ed. L.A. Steen) MAA Notes Number , Washington DC: Mathematical Association of America, 1989, pp. 623–660.

161 Sigma Xi. *An Exploration of the Nature and Quality of Undergraduate Education in Science, Mathematics and Engineering*. New Haven, CT: Author, The Scientific Research Society, 1989.

162 Simon, M. A. "Reconstructing Mathematics Pedagogy from a Constructivist Perspective." *Journal for Research in Mathematics Education*, 26, 1995, pp. 114–145.

163 Skemp, R. "Relational Understanding and Instrumental Understanding, Mathematics Teaching." *Journal of the Association of Teachers of Mathematics*. Great Britain, December 1976.

164 Smith, D. A, G. J. Porter, L. C. Leinbach, and R. H. Wenger, eds. *Computers and Mathematics: The Use of Computers in Undergraduate Instruction.*: A Project of the Committee on Computers in Mathematics Education. Washington, DC: Mathematical Association of America, MAA Notes (Number 9), 1988.

165 Solow, A., ed. *Preparing for a New Calculus*. (MAA Notes, Number 36). Washington, DC: MAA, 1994.

166 Sons, L. R. *Quantitative Reasoning for College Graduates: A Complement to the Standards*. (Report of the CUPM Subcommittee on Quantitative Literacy.) Washington, DC: Mathematical Association of America, 1995.

167 Steen, L. A., ed. *Calculus for a New Century: Pump, Not a Filter*. (MAA Notes, Number 8). Washington, DC: Mathematical Association of America, 1988.

168 Steen, L. A., ed. Committee on the Undergraduate Program in Mathematics. *Reshaping College Mathematics: A Project of the Committee on the Undergraduate Program in Mathematics*. MAA Notes, Number 13. Washington, DC: MAA, 1989.

169 Steen, L. A., ed. *Reshaping College Mathematics: A Project of the Committee on the Undergraduate Program in Mathematics*. (MAA Notes, Number 13). Washington, DC: MAA, 1989.

170 Steen, L. A., ed. *Heeding the Call for Change: Suggestions for Curricular Action*. Washington, DC: Mathematical Association of America, 1992.

171 Steffe, L. P. and T. Kieren. "Radical Constructivism and Mathematics Education." *Journal for Research in Mathematics Education*, NCTM, Vol. 25, Number 6, December 1994, pp. 711–733.

172 Stehney, A. K. "Mathematicians Write: Mathematics Students Should, Too." In A. Sterrett (Ed.), *Using Writing to Teach Mathematics*. Washington, DC.: Mathematical Association of America, 1990, pp. 3–5.

173 Sterrett, A., ed. *Using Writing to Teach Mathematics*. Washington, DC: Mathematical Association of America, 1990.

174 Stiff, L. V., M. McCollum, J. Johnson. "Using Symbolic Calculators in a Constructivist Approach to Teaching Mathematics of Finance." *Journal of Computers in Mathematics and Science Teaching*, 11, 1992, pp. 75–84.

175 Swetz, F. "Incorporating Mathematical Modeling into the Curriculum." *Mathematics Teacher*, 84, 1991, pp. 358–365.

176 Swetz, F. and J. Hartzler. "Mathematical Modeling in the Secondary School Curriculum." Reston, VA: National Council of Teachers of Mathematics, 1991.

177 Szetela, W. and D. Super. "Calculators and Instruction in Problem Solving in Grade 7." *Journal for Research in Mathematics Education*, 18, 3, 1987, pp. 215–219.

178 Tucker, A., and J. Leitzel, eds. *Assessing Calculus Reform Efforts*. (MAA Notes Number 6). Washington, DC: Mathematical Association of America, 1995.

179 Tucker, T. W., ed. *Priming the Calculus Pump: Innovations and Resources*. Committee on Calculus Reform and the First Two Years, (MAA Notes number 17). Washington, DC: Mathematical Association of America, 1990.

180 U S Department of Education. *National Goals for Education*. Washington, DC: U. S. Government Printing Office.

181 "Using Technology in Business Calculus." *Centroid*, NCCTM, Vol. XX, 1993.

182 von Glasersfeld, E. "Learning as a Constructive Activity." *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. J. C. Bergeron and N. Herscovics, eds. Montreal: University of Montreal, 1983, pp. 41–69.

183 Webb, N. L., and T. A. Romberg. "Mathematics and Community." *Reforming Mathematics Education in America's Cities*. New York: Teachers College Press, 1994, pp. 8–23.

184 Weingartner, R. *Undergraduate Education, Goals, and Means*. American Council on Education Series on Higher Education. Phoenix, Oryx Press, 1993.

185 *What Work Requires of School: A SCANS Report for America 2000*. The Secretary's Commission on Achieving Necessary Skills. Washington, DC: U.S. Department of Labor, 1991.

186 Wheatley, C. L. "Calculator Use and Problem-Solving Performance." *Journal for Research in Mathematics Education*, 11, 5, 1980, pp. 323–334.

187 Wilcox, S., P. Schram, G. Lappan, and P. Lanier. *The Role of a Learning Community in Changing Preservice Teachers' Knowledge and Beliefs about Mathematics Education*. (Report No. NCRTE-RR-91-1) East Lansing, MI: National Center for Research on Teacher Education. ERIC Document Reproduction Service No. ED 330 680, 1991.

List of Winners

Tier I Winners

1. MARTHA ALIAGA
   Department of Statistics
   1459-A Mason Hall
   University of Michigan
   Ann Arbor, MI 48109
   e-mail: martha.aliaga@umich.edu
   Phone: 313-764-5493/ 763-3519

2. DONALD R. LATORRE
   Mathematical Sciences Department
   Clemson University (Retired, June 1996)
   Clemson, SC 29634-1907

   JOHN W. KENELLY
   Mathematical Sciences Department
   Box 341907, Clemson University
   Clemson, SC 29634-1907
   e-mail: kenellj@clemson.edu
   Phone: 864-656-5217

   IRIS B. FETTA
   Mathematical Sciences Department
   Box 341907, Clemson University
   Clemson, SC 29634-1907
   e-mail: ibbrh@clemson.edu
   Phone: 864-6565-5217

   LAUREL L. CARPENTER
   Lansing, MI
   e-mail: dcarpen@voyager.net
   Phone: 517-272-9083

   CYNTHIA R. HARRIS
   Sun Valley, NV
   e-mail: cindy@cs.unr.edu
   Phone: 702-674-2670

3. SHELDON P. GORDON, Project Director
   Suffolk Community College
   Selden, NY, 11784
   e-mail: sgordon@ccmail.sunysb.edu.
   Phone: 516-451-4270

   FLORENCE GORDON
   New York Institute of Technology
   OldWestbury, NY 11568

   BERNARD A. FUSARO
   Mathematics Department
   Florida State University
   Tallahassee, FL 32306

   MARTHA SIEGEL
   Mathematics Department
   Towson State University
   Baltimore, MD 21204

4. HOLLY PETERS HIRST
   Department of Mathematical Sciences
   Appalachian State University
   Boone, North Carolina 28608
   e-mail: hph@math.appstate.edu
   Phone: 704-262-3050

   JAMES R. SMITH
   Department of Mathematical Sciences
   Appalachian State University
   Boone, North Carolina 28608
   e-mail: jrs@math.appstate.edu
   Phone: 704-262-3050

5. NOMIKI (NIKI) SHAW
   California State University, Bakersfield
   Bakersfield, California 93311-1099
   e-mail: nshaw@csubak.edu
   Phone: 805-664-2135

   IGNACIO ALARCON
   California State University, Bakersfield
   Bakersfield, California 93311-1099
   email: ialarcon@csubak.edu
   Phone: 805-664-2139

Tier II Winners

1. Janet Andersen
Department of Mathematics
Hope College
Holland, MI 49422-9000
e-mail: andersen@math.hope.edu
Phone: 616-395-7909

Todd Swanson
Department of Mathematics
Hope College
Holland, MI 49422-9000
e-mail: swanson@math.hope.edu
Phone: 616-395-7679

Robert Keeley
Holland Christian High School
Holland, MI
e-mail: rkeeley@calvin.edu

2. John Engelhardt
Department of Mathematics
Southern Oregon State College
Ashland, Oregon 97520.
e-mail: engelhardt@wpo.sosc.osshe.edu
Phone: 514-552-6579

Sherry Ettlich
Department of Mathematics
Southern Oregon State College
Ashland, Oregon 97520.
e-mail: ettlich@wpo.sosc.osshe.edu
Phone: 514-552-6576

3. William E. Fenton
Department of Mathematics
Bellarmine College
Louisville, KY 40205-0671
e-mail:wfenton@bellarmine.edu
Phone: 502-452-8430

4. Vernon M. Kays
Instructor of Mathematics
Richland Community College
Decatur, IL 62521
e-mail: vkays@richland.cc.il.us

5. Kenneth Koedinger
Computer Science Department
WEH 4212
Carnegie Mellon University
Pittsburgh, PA 15213
e-mail:koedinger@cmu.edu

6. Dani Novak
Department of Mathematics
Ithaca College
Ithaca, NY 14850
e-mail: novak@ithaca.edu
Phone: 607-274-3595

Martin Sternstein
Department of Mathematics
Ithaca College
Ithaca, NY 14850

Osman Yurekli
Department of Mathematics
Ithaca College
Ithaca, NY 14850
yurekli@ithaca.edu

7. Christopher Schaufele
Department of Mathematics
Kennesaw State University
1000 Chastain Rd.
Kennesaw, GA 30144-5591
e-mail: cschaufe@ksumail.kennesaw.edu
Phone: 770-423-6327

Nancy Zumoff
Department of Mathematics
Kennesaw State University
1000 Chastain Rd.
Kennesaw, GA 30144-5591
e-mail: nzumoff@ksumail.kennesaw.edu
Phone: 770-423-6327

8. Captain Paul J. Simonich
Department of Mathematical Sciences
The United States Air Force Academy
USAFA, CO 80840-6252
e-mail: simonichpj.dfms@usafa.af.mil
Phone: 719-333-3099

Captain Cynthia A. Brown
Department of Mathematical Sciences
The United States Air Force Academy
USAFA, CO 80840-6252
Phone: 719-333-3099

9. Carla J. Thompson
Mathematical Sciences and Engineering Division
Tulsa Community College
3727 East Apache
Tulsa, Oklahoma 74115
Phone: 918-595-7505

10. Ed Wright
Academic Computing
Western Oregon State College
Monmouth, OR 97361
e-mail: wrighte@cs.wosc.osshe.edu
Phone: 503-838-8480

Don Pierce
e-mail: donald_pierce@heald.edu
Phone: 415-864-5060

Tier III Winners

1. Linda L. Duchrow
Mathematics Department
Regis University
3333 Regis Boulevard, D-16
Denver, Colorado 80221-1099
e-mail: duchrow@regis.edu
Phone: 303-458-4164

2. John M. Kellett
Gettysburg College
Gettysburg, PA 17325-1486
e-mail: John.M.Kellett@cc.gettysburg.edu
Phone: 717-337-6635

3. Robert L. Richardson
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
e-mail: RichardsonRL@conrad.appstate.edu
Phone: 704-262-3050

Dona Alejandro
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
e-mail: alejandro@appstate.edu
Phone: 704-262-2859

4. George Rublein
Department of Mathematics
College of William and Mary
Williamsburg, VA 23187-8795
e-mail:gtrubl@math.wm.edu
Phone: 804-221-2028

5. Reza Sarhangi
Department of Mathematics
Southwestern College
100 College Street
Winfield, KS 67156-2499
e-mail sarhangi@jinx.sckans.edu
Phone: 316-221-8373

INnovative Programs Using Technology

This handbook will help you improve your teaching by showing you how you might take advantage of current technology to teach introductory mathematics courses. Browse the winning entries in the INnovative Programs Using Technology Project (INPUT) and learn how others have used technology to teach developmental mathematics, precalculus, business mathematics, quantitative literacy and introductory statistics.

The INPUT project, (funded by the Annenberg/CPB Project, the National Science Foundation, and Central Michigan University) sought to stimulate change in the way mathematics service courses were being taught by recognizing and rewarding college instructors who have rethought the mathematical content of their "introductory" mathematics courses. The award winning projects of that competition are featured here. You will find detailed descriptions of what some innovative instructors are doing, how they are doing it, what technologies they are using, and how you can contact them to learn more about their particular project. Also included is a list of reference materials for further exploration.

ISBN 0-88385-159-8

9 780883 851593

Visit the INput website:
http://www.cmch.edu/~mthaward