NOTES

EXPANDING UNDERGRADUATE RESEARCH IN MATHEMATICS:

MAKING UR MORE INCLUSIVE

Michael Dorff
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Editors

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Expanding Undergraduate Research in Mathematics: Making UR More Inclusive

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About this Book

Why another book on undergraduate research in mathematics? Because there is a gap in the literature. Let us explain. Most of the publications on undergraduate research in mathematics fall into one of three categories.

The first category consists of books and articles that describe established programs or projects that are successfully doing undergraduate research in mathematics. There are many exemplary programs—National Science Foundation (NSF) and National Security Agency (NSA) funded Research Experiences for Undergraduates (REUs), Center for Undergraduate Research in Mathematics (CURM), Preparation for Industrial Careers in Mathematical Sciences (PIC Math), National Research Experience for Undergraduates Program (NREUP), UR programs at national Mathematics Institutes, summer and academic year undergraduate research programs at specific universities. Examples of articles written about some of these programs include

- "Introducing Undergraduates from Underrepresented Minorities to Mathematical Research: the CSU Channel Islands/California Lutheran University REU, 2004–2006" by Cindy Wyels in *Proceedings of the Conference on Promoting Undergraduate Research in Mathematics,* ed. by Joseph A. Gallian, American Mathematical Society, 2007, pp. 27–32.
- "CURM: Promoting Undergraduate Research in Mathematics" by Michael Dorff in *Topics from the 8th Annual UNCG Regional Mathematics and Statistics Conference 1-6,* Springer Proceedings in Mathematics & Statistics, Springer Science-Business Media, New York, 2013.
- "A Student Research Course on Data Analytics Problems from Industry – PIC Math" by Michael Dorff and Suzanne Weekes in *Scholarship and Practice of Undergraduate Research (SPUR),* Vol 2, No. 4, Summer 2019, pp. 37–42.
- "Student-Driven Research at the Mathematical and Theoretical Biology Institute" by Carlos Castillo-Chavez, Christopher Kribs, and Benjamin Morin in *The American Mathematical Monthly,* 124 (2017), no. 9, pp. 876–892.
- "A decade of undergraduate research for all East Tennessee State University mathematics majors" by Ariel Cintrón-Arias and Anant Godbole in *Involve, a Journal of Mathematics,* 7 (2014), no. 3, pp. 281–293.
- "Academic Year Research at Valparaiso University" by Richard Gillman and Zsuzsanna Szaniszlo in *Proceedings of the Conference on Promoting Undergraduate Research in Mathematics,* ed. by Joseph A. Gallian, American Mathematical Society, 2007, pp. 279–283.

In fact, there are edited books that are collections of articles describing such programs. These books include:

- *Proceedings of the Conference on Summer Undergraduate Mathematics Research Program,* ed. by Joseph A. Gallian, American Mathematical Society, 2000.
- *Proceedings of the Conference on Promoting Undergraduate Research in Mathematics,* ed. by Joseph A. Gallian, American Mathematical Society, 2007.
- *Directions for Mathematics Research Experience for Undergraduates,* ed. by Mark A Peterson and Yanir A. Rubinstein, World Scientific Press, 2015.

Publications that describe exemplary programs are useful because they show what types of undergraduate research programs in mathematics are being done nationally. They suggest models that others may replicate or tweak to create new programs.

The second category of types of publications contain books and articles that are collections of actual problems that undergraduate students could research. These often present foundational mathematics content and then give corresponding research problems in that area of mathematics suitable for undergraduate students to work on. These publications focus on a specific mathematical area such as mathematical biology, knot theory, math modeling, etc. Some sample resources include

- *Keeping it R.E.A.L.: Research Experiences for All Learners* by Carla Martin and Anthony Tongen, MAA Classroom Resource Materials, 2011.
- *Explorations in Complex Analysis* by Michael A. Brilleslyper, Michael J. Dorff, Jane M. McDougall, James S. Rolf, Lisbeth E. Schaubroeck, Richard L. Stankewitz, and Kenneth Stephenson, MAA Classroom Resource Materials, 2012.
- *A Primer for Undergraduate Research: From Groups and Tiles to Frames and Vaccines,* edited by Aaron Wooten, Valerie Peterson, and Christopher Lee, Birkhauser, 2017.
- *An Introduction to Undergraduate Research in Computational and Mathematical Biology,* edited by Hannah Callender Highlander, Alex Capaldi, and Carrie Diaz Eaton, Birkhauser, 2020.
- *A Project-Based Guide to Undergraduate Research in Mathematics: Starting and Sustaining Accessible Undergraduate Research,* edited by Pamela Harris, Erok Insko, and Aaron Wooten, Birkhauser, 2020.

The third category consists of publications that provide advice and guidance on how to set up an undergraduate program and better mentor students in mathematics research. The topic of setting up a program involves the challenges of organizing and running an undergraduate research program, and these articles provide a valuable resource for someone who already has an idea of the type of undergraduate research program they want to set up but could use further guidance on setting up the logistics of the program and the pitfalls to avoid in running a program. Examples of such articles include

- ”Adventures in academic year undergraduate research” by Kathryn Leonard in the *Notices of the AMS,* 55 (2008), no. 11, pp. 1422–1426.
- “An Example of Practical Organization for Undergraduate Research Experiences” by Jo Ellis-Monaghan and Greta Pangborn in *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies,* 23 (2013), no. 9, pp. 805–814.
- “Succeeding in Undergraduate Student Research: A Few Helpful Hints for Advisors” by Lora Billings in *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies,* 23 (2013), no. 9, pp. 798–804.
- “Challenges in promoting undergraduate research in the mathematical sciences” by Feryal Alayont, Yuliya Babenko, Craig Jackson and Zsuzsanna Szaniszlo in *Involve, a Journal of Mathematics,* 7 (2014), no. 3, pp. 265–271.
- “Information for faculty new to undergraduate research” by Cayla McBee and Violeta Vasilevska in *Involve, a Journal of Mathematics,* 7 (2014), no. 3, pp. 395–401.

The topic of better mentoring students in mathematics research include such components as choosing appropriate research problems, recruiting students, helping students give talks and write papers, etc. These publications can be very helpful for a novice in learning how to mentor students in research and for an experienced mentor in becoming an even better mentor. For example, some of these publications include

- Trends in Undergraduate Research in the Mathematical Sciences (Chicago, 2012), Special issue of *Involve, a journal of mathematics,* edited by Darren A. Narayan.
- Undergraduate Research in Mathematics. 2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences, 93–98, Math. Assoc. of America, Inc., Washington, DC, 2015.
- *A Mathematician’s Practical Guide to Mentoring Undergraduate Research* by Michael Dorff, Allison Henrich, and Lara Pudwell, MAA Press, 2019.

The focus of this book is different. While it does contain some of the elements from the three previous categories, the purpose is to look into the future. It attempts to predict what will be the emerging areas and new directions in mathematics undergraduate research in the next ten years (i.e., important areas that need more attention and once they receive it, will bring a big step forward to the mathematics undergraduate research community). Most of these areas are already being done by someone so the areas are not totally unknown. But they are areas that are likely to grow and become more important and prominent in the near future. Suppose you were doing undergraduate research in mathematics in the early 1990s. At that time, many undergraduate research programs focused on one-on-one mentoring of advanced students, who were predominantly male. Later in that decade, mentors started focusing on providing undergraduate research opportunities for female students—this could be thought of as an emerging and important area that needed more attention. Soon after that, several exemplary programs for women in mathematics were started such as The Carleton College Summer Mathematics Program for Women in 1995, the EDGE program in 1998, and the Nebraska Conference for Undergraduate Women in Mathematics in 1999. There were similar shifts in focus in doing undergraduate research with regards to moving from one-on-one mentoring to mentoring a group of students, providing opportunities for students from underrepresented racial and ethnic groups, providing opportunities for 1st and 2nd year undergraduate students, etc. Now that we are in the 2020s, what are the emerging areas and new directions that need more attention and that will likely become staples in doing undergraduate research in mathematics in the future? Providing answers to that question is the goal of this volume.

Before we proceed further, let's discuss a few items. First, this book is not a collection of visionary essays that forecast totally new and different things that undergraduate research in mathematics will be doing 50 years in the future and concentrate on only things that no one is doing now. We think such a book would be intriguing but would be mostly speculative. Second, this book does not focus primarily on summer REU programs, although these REU's are still the most commonly offered ones. REU which stands for Research Experience for Undergraduates originally referred to summer programs such as those funded by NSF. For some mathematicians, such programs are what they think of when they think of undergraduate research. However, there is a much larger family that encompasses all of undergraduate research in mathematics. This larger family includes these summer REU programs, academic year undergraduate research, internships, and classes in which undergraduate research is done in fall and spring semester courses. Finally, we—as the editors of this book—prefer to think of undergraduate research as students working on unsolved problems. This includes exploratory analyses of data with a purpose to answer a specific question coming from an industry as done in Chapters 5, 6 and 10; but it does not include just arbitrary exploration of the data set. Why make that distinction? Mainly, that stems from Michael's experience in talking to recruiters and employers in business, industry, and government who hire math students. In talking to over 150 different people, they have said that one thing that makes a math major stick out for hiring is if they have done an internship, which can be thought of as working on an unsolved undergraduate research problem from BIG (business, industry, and government), or have been on a team working on an unsolved undergraduate research problem. Why is the unsolved problem helpful? Because that mimics the work scenario in BIG. In working on unsolved problems, students can't accidentally (or intentionally) find a solution online, they can't find hints on how to proceed in solving the problem, and they experience the uncertainty of not knowing what a solution would look like or even if the problem has a solution. All of that makes the experience more similar to working on a problem when they have a job.

We acknowledge that there are faculty members who do a great job in mentoring undergraduates in solving extensive problems that the students have not seen before and have definitely not seen the solution before. Students working on problems that they have never seen before but that have been solved by someone else earlier is beneficial to students and is an important component of their learning. Therefore, the focus of the book is not to give examples of open and unsolved problems. Neither do all authors talk exclusively about unsolved problems. Rather, we are trying to show what can or should be done so that all interested students would eventually be able to engage in undergraduate research and work on solving unsolved problems.

The book is organized as follows. After a Chapter on the importance of UR and a chapter about the history of REUs, the book is divided into four parts of three to four chapters each.

Part I Integrating UR into the Curriculum
Part II Diversity and Inclusion
Part III BIG and Interdisciplinary UR

Part IV Virtual and Remote UR

From a different angle, readers interested in assessment and summary of long-standing programs will enjoy Chapters 3 and 6. Many chapters include ”How-to” or ”Best-practices” advice and sections, including Chapters 4, 8, 12, and 13.

This volume attempts to predict what will be the emerging areas in mathematics undergraduate research in the next 5–10 years. While most of these new directions are already being done by some people, they are not being done by many people. However, the trend is that these areas will grow and become more important and prominent in the mathematics undergraduate research community in the future. What are these trends?

1. making undergraduate research in mathematics more available to diverse groups of students; this is a recurring topic that is covered through the book from multiple angles, including but not limited to, in Chapter 4 about course-based research, Chapters 11 and 12 about interdisciplinary research, or Chapter 15 about research with high school students,
2. including students at community colleges (Chapter 9), students with disabilities (Chapter 8), and students with limited research opportunities, especially native Spanish speakers and those who would be the first in their families to earn college degrees (Chapter 7),
3. mentoring undergraduate students in mathematics research virtually in an online format (Chapters 13, 14, 15, and 16 in Part IV),
4. having students use mathematics to solve current problems that come from business, industry, and government (BIG) (Chapters 6, 10)
5. doing undergraduate research in math related to social justice issues (Chapter 5).

At this time, there are people working in these emerging areas and these areas are flourishing. They are making undergraduate research more available to a larger and more diverse group of students and thus making mathematics more equitable and inclusive. Yet, there are no books or very few books that address these newer trends. That is the reason we decided to create this book.

1

The Importance of Undergraduate Research in a Mathematics Department

Michael Dorff, *Brigham Young University*

The years from 2016 to 2022 have been a time of change for universities. Some of these changes have been tough and have been brought about by loss of financial support for state universities from the legislatures, declining enrollment at some universities, and attempts to make majors a better match for the current job market. In 2016, the University of Wisconsin System Board of Regents approved new tenure policies [1]. The Board declared the need for flexibility to close programs and to eliminate faculty jobs. Two years later, the University of Wisconsin - Stevens Point announced it would eliminate 13 majors in the humanities and liberal arts and replace them with 13 majors in more technical fields [2]. Later, this move was reversed due to protests. But it demonstrates that uncertain times universities are facing [3].

A similar situation occurred at the University of Akron. In 2018, the University of Akron announced that it was eliminating 80 degree tracks [4] including PhD in applied mathematics, MS in mathematics, and BS in mathematics [5]. The administration claimed that its decision was based on the low enrollment in these majors. Two years later, the university reversed their earlier decision and announced that it would not eliminate these degree tracks, but instead it would cut 97 full-time faculty positions (FTEs) which is 17% of the total number of FTEs [6].

Such changes are not just happening to state universities. Goucher College is a private liberal arts college in Maryland. In 2018, Jose Bowen, the president of Goucher College, announced that the college would be eliminating certain majors including math and physics for low enrollments [7]. Also, Hiram College, a small private liberal arts college in Ohio, reported in 2019 that it would cut 6 faculty positions (9% of the total FTEs) and investigate eliminating certain majors including mathematics [8].

This all happened before COVID-19 which has made such changes more common. In 2020 and 2021,

- the University of Alaska system announced that it would eliminate 39 academic departments [9].
- Elmira College in New York revealed that it was cutting several academic programs [10].
- Ohio Wesleyan University reported that it will eliminate 18 majors in the humanities and certain STEM fields [11].
- University of Vermont declared that it was getting rid of 12 majors, 11 minors and four graduate programs [12].
- Ithaca College in New York announced that it will be cutting some programs, majors, and 116 faculty positions (21% of the total FTE) [13].

It is likely that not all these cuts will occur, but these cases demonstrate a revision in administration thinking about majors and faculty positions in light of hard financial times and decreasing enrollments for some universities, and some of these administrators have discussed eliminating mathematics as a major.

Now I am not a doomsayer prophesying that the university system will collapse as some have predicted. Instead, my point is that changes are and will be occurring in higher education, and we will be at a disadvantage if we act as if the future will be the same as the past. Importantly, we will prosper in actively addressing these changes.

The mathematics major is strong and healthy at many universities and colleges despite it being eliminated by some institutions. However, the mathematics community would benefit from making a solid case for why studying mathematics is useful. Some math faculty attempt to make this case by affirming that studying math trains students to solve problems and think logically. I agree with that statement. However, the difficulty with it is that faculty in almost all disciplines make the same claim. And so when an administrator hears such a statement as the main or sole reason not to cut a major, it will not be effective in changing the administrator's opinion. Often, an administrator will claim the reason to cut a major is low enrollments in the major (it should be noted that this is different than the number of students taking classes in a certain discipline; typically, there are many students taking mathematics classes, but that by itself is not justification to keep a mathematics major). If the number of math majors is low, then it does seem justified from an administrative standpoint to eliminate that major. However, from my perspective, this should not even be a discussion point, because there should not be a low number of mathematics majors. Studying mathematics is an excellent way for students to learn (a) problem-solving skills, (b) the capacity to abstract, (c) paying attention to details, (d) to think of problems in a different way, (e) the competency to learn new concepts and topics on one's own, and (f) the ability to break complicated problems into small pieces, solve each of the small pieces, and then assemble them to get a complete solution. In other words, mathematics is one of the best majors for preparing for a career, especially a career in business, industry, and government.

Where did I get these points? Over the past 15 years, I have talked with over 150 recruiters and employees at companies that hire mathematics majors and have asked them why they want math majors. They have responded by listing the attributes above. Majoring in mathematics can be a great way for students to learn the skills necessary to be successful in careers outside of academia. Of course, this is in addition to the more philosophical reason that studying mathematics trains students to solve problems and think logically. Also, many of the top jobs are in mathematics related fields. Every year, `CareerCast.com`, a job search website, evaluates careers based on the following factors: work environment (both physical factors and emotional factors), income level, outlook (employment growth, income growth potential, and unemployment), and stress level [14]. Then they list rankings for jobs. In 2019, there were 224 jobs listed in the rankings. Six out of the top eleven jobs are common math-related careers. These include data scientist, statistician, mathematician, operations research analyst, actuary, and software developer [15]. 2019 was not a fluke. Similar results occurred in 2018 [16], 2017 [17], 2016 [18], 2015 [19]; I expect this pattern continues back even further, but I decided to just get the results from the most recent five years.

As mathematics faculty, we have not done a good job in discussing math-related careers with students, math majors, and administrators. This brings us back to the president at Goucher College who eliminated math as a major. During his time at Goucher, the president emphasized that "he wanted the school's curriculum to move away from traditional book-based learning to focus more on teaching students how to learn and preparing them for real-world careers" [20]. As mentioned above, the top jobs are those related to mathematics and there is an abundance of careers for mathematics majors and students who are good in mathematics. Somewhere this got lost. As another example, in 2015, I submitted a new study abroad proposal to take students overseas for three weeks to visit companies that hire students with good math skills. In getting university approval for the trip, I encountered an administrator who was concerned about the proposal because he thought there were few jobs available for math majors. I cleared up that misconception quickly. For those who want to know more about mathematics careers outside of academia, a useful resource is the book BIG Jobs Guide by Rachel Levy, Rick Laugesen, and Fadil Santosa [21]. Some additional resources are [22], [23], [24].

Let me reiterate. The status of the mathematics major is strong and its benefits are substantial. But the incidents above demonstrate that there can be a disconnect between what administrators think about career options for math majors and what is happening outside academia. If we consistently and effectively discussed math careers with students, math majors, and administrators. and worked to include some courses that discussed how mathematics is used in non-academic careers, I am confident that math departments would increase the number of math majors and improve the view of mathematics in the administrator's eyes.

What does all of this have to do with undergraduate research? Undergraduate research can increase the number of students who major in mathematics, it can help students have a better experience as a math major, and it can prepare students for careers outside of academia. The combination of these things makes a strong case for the importance of keeping a math major at a university. This is one of the many, but often overlooked, reasons for the importance of undergraduate research in a mathematics department. What are some other reasons?

Undergraduate research is a high-impact practice and consists of activities that have been shown to increase student retention and student engagement [25]. High-impact practices are important aspects of students' learning and are transformative for undergraduate students [26, 27, 28]. Other high-impact practices include First-Year Experiences, Learning Communities, Diversity/Global Learning, Service Learning and Community-Based Learning, Internships, and Capstone Courses and Projects. Many college and university administrators strongly support high-impact practices. This means that if your mathematics department is not a leader of undergraduate research at your institution, it could likely be that the administration does not think of the mathematics department as highly as they could. Also, if the faculty in your department is doing some exciting undergraduate research with students, share this with the administration. Mathematicians are not always good about letting others know the great activities we are doing, and the administration is not going to magically know such things unless someone shares it with them. Let's talk about the benefits of undergraduate research to students. Jeff Osborn and Kerry Karukstis [29] did an extensive search of studies indicating the benefits and summarized them as:

1. Gains in knowledge and skills
   (a) Greater gains in mastering both content and contextual knowledge
   (b) Enhanced ability to put classroom knowledge into practice
   (c) Increased creativity and critical thinking
   (d) Enhanced problem-solving skills
   (e) Enhanced communication skills, both oral and written
   (f) Enhanced technical skills within the discipline
   (g) Greater understanding of the intersections of disciplines
2. Academic achievement and educational attainment
   (a) Higher retention rates
   (b) Greater increases in course grades
   (c) Greater persistence in the major
   (d) Higher graduation rates
   (e) Higher rates of acceptance into and enrollment in post-baccalaureate education (graduate/professional schools)
3. Fostering Professional Growth and Advancement
   (a) Enhanced ability to work collaboratively with others in teams
   (b) Stronger relationships with mentors and other professionals
   (c) Deeper integration into the culture and profession of the discipline
   (d) Enhanced ability to identify and make informed decisions about appropriate career interests
   (e) Enhanced professional credentials
   (f) Higher rates of acceptance into and enrollment in post-baccalaureate education (graduate/professional schools) and/or directly securing employment in the workforce
4. Promoting Personal Growth
   (a) Stimulation of curiosity
   (b) Enhanced ability to learn independently
   (c) Enhanced development of personal initiative
   (d) Increased confidence
   (e) Enhanced ability to understand the philosophy of lifelong learning
   (f) Greater recognition by peers
   (g) Enhanced opportunity to serve as an academic role model

From the list above, the benefits for undergraduate students who do research are numerous and significant. For the purposes of this article, let us emphasize the two areas of increasing the number of math majors and preparing them for careers outside of academia. Undergraduate research can help increase the number of math majors, because it can be an effective tool for recruiting more students to study mathematics and for building an inclusive community in a mathematics department. Then, as these students participate in undergraduate research, they learn, understand, and practice mathematics better (e.g., 1a, 1b, 1c, 1d, 1f above), and they perform better in their classes (2b). Also, such students improve their level of curiosity and confidence (4a, 4d). These factors can help improve the retention and graduation rates of these math majors (2a, 2c, 2d). Similarly, undergraduate research can help prepare students for careers (3e, 3f) by improving the skills they need in business, industry, and government jobs. These include communication skills (1e), teamwork skills (3a), technical skills (1f), and problem-solving skills (1d).

Undergraduate research has been shown to improve the success rates of women undergraduate students and students from underrepresented ethnic groups ([30], [31], [32], [33]). Also, it improves student satisfaction with mathematics as their major and alumni satisfaction with their experience in the department. With undergraduate research helping to increase the number of math majors, improve alumni satisfaction with their experience in the mathematics department, and prepare mathematics students for careers, the administration's perspective of the mathematics department will improve.

Finally, undergraduate research can help improve the faculty's experience in a mathematics department, because many faculty members find mentoring students in research an enjoyable activity. Undergraduate research can strengthen cases for faculty receiving teaching awards at their own institutions, in their MAA section, in the CUR mathematics & computer science division, and in the MAA nationally. Also, faculty having some experience mentoring students in research can facilitate them receiving national grants related to their undergraduate research work such as an NSF REU grant.

All of the above provide strong reasons for doing undergraduate research and its impact on math majors, faculty, and the department. In addition, this can impact the administration's view of the mathematics department and makes possibly one of the strongest cases for the need to keep a mathematics major and a mathematics department at the institution.

Now that we have discussed some of the reasons for the importance of doing undergraduate research in mathematics departments, we invite you to explore the remaining chapters in this book many of which presents ways in which undergraduate research in mathematics can be expanded to make it more inclusive.

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2

History of the REU Program

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The Research Experiences for Undergraduates (REU) is an NSF-wide program. As such, although it is one program, it is managed differently by the disciplines within the purview of NSF. In some NSF directorates, the program is administered at the directorate level but in the Directorate for Mathematical and Physical Sciences (MPS), each of the MPS' divisions administer the program for its discipline. So, MPS' five divisions: Astronomical Sciences, Chemistry, Materials Research, Mathematical Sciences, and Physics, review the proposals for its respective disciplines.

The formal REU program started with proposals being submitted in fiscal year 1987. One of the requirements for participating in an REU program is that you must be a citizen, national or permanent resident of the U.S. From the beginning, the Mathematical Sciences had to overcome the thinking of many in the discipline that undergraduate research is an oxymoron. The sense was that the body of knowledge required to do research in the mathematical sciences was not gained until at least after three years of graduate study. When I hear things like "change is slow" or "real change never happens," one of the things I point to as a counterexample is undergraduate research. These days, we hear things like "we don't accept students into our graduate program if they haven't had an undergraduate research experience," there are conferences where only research done by undergraduates is presented and there are many undergraduates at traditional research conferences. In fact, I attended a research conference once where a complaint was made that there were too many students. (What??)

So, how did we get to such a drastic change? There are a group of mathematicians that I will refer to as the pioneers. They were the people who believed that research in the mathematical sciences by undergraduates was viable even though the conventional "wisdom" said otherwise. During the first year of the REU program, nine awards for REU sites in the mathematical sciences were made. The principle investigators for those nine sites were Robert Brooks (University of Utah), P. Claypool (Oklahoma State University), James Curry (University of Colorado-Boulder), Joseph Gallian (University of Minnesota Duluth), John Greever (Harvey Mudd College), Lawrence Husch (University of Tennessee- Knoxville), B. Frank Jones (Rice University), Robert Robson (Oregon State University) and B. Alan Taylor (University of Michigan- Ann Arbor). In 2020 there were a total of 38 sites supported by the Division of Mathematical Sciences, operating in 19 states. The REU program has expanded to include an option for Research Experiences for Teachers. In addition to the REU sites, the REU program solicitation allows for supplements to either existing or proposed NSF research projects for the purpose of undergraduate research. A Principal Investigator can request funds for one or two undergraduates to participate in the research. In DMS, REU supplements have always been handled by the program officers for that particular field of the mathematical sciences.

One person that must be included in the list of pioneers is John V. (Jack) Ryff. Jack was the program officer for the Classical Analysis program in DMS back when there was such a program in 1987. He was the coordinator for REU sites in DMS from the inception of the program until he left NSF in 1994. Jack was such a believer in the program that he funded the DMS sites from his own Classical Analysis program, regardless of what field of the mathematical sciences the site focused on. He was deeply passionate about the REU program and one could just get that sense from him whenever he talked about it. I remember when I first came to DMS, Jack said to me "Lloyd, have you ever been to an REU site?" When I replied that I had not, he responded, "You have to go to one" and he went on to tell me how great they were. I am sure that my interaction with Jack went a long way towards my interest in undergraduate research. When Jack left NSF, Keith Crank coordinated the DMS REU program from 1994-1996, I did it from 1996 until I left NSF in 2008, then Dean Evasius did it from until he left in 2010. After that, the program has been managed by teams of program officers with Noel Brady, Jennifer Pearl and Tara Smith being the first team in 2011.

Even though program officers are the line managers of the program, these activities do not get funded without the support of the division directors. Judy Sunley, who was the last permanent division director in DMS was there when the program started. Fred Wan (who hired me) was also a supporter of the program. I had the pleasure of working with division directors Philippe Tondeur, Bill Rundell and Peter March, who were instrumental in expanding the number of undergraduates who were supported by the program.

One thing that has been a boon for undergraduates looking for REU site experiences is a project headed by Karolyn Eisenstein when she was at NSF. Karolyn, who is a chemist by training, was a program officer in the Education and Human Resources Directorate at NSF at the time that she coordinated a project with the NSF REU site coordinators that produced a search capability on the NSF web site that allows one to look for existing REU sites that are funded by a particular NSF organization. So, for example, if one is looking for REU sites that are funded partially or totally by DMS, one can easily generate such a list that has hypertext links to the web sites for those programs.

There have been a few conferences for researchers interested in engaging undergraduates in research in the mathematical sciences over the years. The first of these conferences, held in 1999, was convened by John Ewing, when he was Executive Director of the American Mathematical Society (AMS). Funded by the National Security Agency (NSA), this conference brought together researchers who discussed the challenges and opportunities in engaging undergraduates in mathematical sciences research. As a result of this conference, one got a sense of the differences and similarities that researchers experienced in engaging undergraduates in research in the mathematical sciences. A second such conference was held some years later and was also funded by NSA through the auspices of the AMS. A third conference was held in 2012 and was organized by Michael Dorff and funded by the Mathematical Association of America.

Where we are today is that undergraduate research is alive and well in the mathematical sciences. While I don't believe that every undergraduate has to have a research experience, I think the goal should be to provide such an experience to everyone who wants one and I think we're still a long way from that. It's become very competitive and one of the problems is that interested participants self-select out. When I would talk to undergraduates about applying for REU positions, I would often get the reaction of "yes, we've heard of the program but we don't apply because we know the kind of students that get accepted". I have to remind them what the probability of getting into something that you don't apply to is but I do understand their point. Rejection is disappointing but they aren't even giving themselves a chance and we need to expand the opportunities so that they don't feel that way and I mean expand the opportunities in a significant way that doesn't always have to include NSF's REU program. Another issue is that even now there are still mathematicians who believe that undergraduate research is an oxymoron. This is despite the number of undergraduates doing research. When I point that out to those people, I sometimes get the reaction of "Well, that's true in some field of mathematics but not in MY field". So, the battle continues.

3

Assessing Undergraduate Research in Mathematics

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Students who engage in undergraduate research report gains in a variety of intellectual skills, a high level of satisfaction with, and are likely to persist in, their undergraduate education. Because of this, over a decade ago, undergraduate research experiences (URE) were named one of nine high impact practices that positively impact undergraduate students [32]. As a result of this designation and the grassroot efforts of STEM faculty at a wide range of institutions, we have seen a significant increase in the number of mathematics students engaged in URE. STEM faculty members have developed many variations of course-based undergraduate research experiences (CURE) to help engage more students in this important activity. Determining the impact of a CURE/URE, though, is not nearly as linear as the growth in the numbers of experiences being offered. Students bring diverse motivations, experiences and skill levels to their CURE/URE as do their research mentors. Some URE are focused on theoretical questions while others are applied in nature; some UR projects involved one student while others involve students working as a team. For some programs the goal of the URE is to produce new knowledge while for other URE programs the goal it is to help students think like mathematicians. UREs vary in length, content and scope. Thus, determining the impact of a URE can quickly become a complex issue. In this chapter you will learn what is known about the impact a URE has on two major stakeholders- the students and their research mentors. You will learn that the body of literature related to assessing URE is robust when it comes to documenting student outcomes but is more limited with respect to examining faculty, program and institutional outcomes [10]. The literature cited within this chapter will provide you with pertinent, high-quality examples of CURE/URE. You will learn where gaps in our knowledge exist and be directed to some resources that could help you and your colleagues develop a robust plan for assessing your URE/CURE.

3.1 What we know about the impact of URE on students

Early work examining the impact of URE on students focused on retention, GPA, graduation rates and enrollment in graduate study [14, 19, 46, 52]. More contemporary studies focused on the students' personal and professional learning from the URE including new skill development, understandings of disciplinary inquiry, growth in confidence, problem solving skills and working as a team [17, 36, 48, 23, 47]. Recent research studies focus on more nuanced questions about URE including targeted questions about which type of mentoring is the most effective [1, 5, 21, 60] and how students from under-represented groups may be impacted [26, 31, 55, 58]. Student researchers who received culturally relevant mentoring that addressed student's emotional and social issues were positively impacted in terms of research skill development [20]. Students think highly of their URE when their mentors establish regular check-in sessions, set clear goals and benchmarks, are available, are understanding, make them feel part of the community and who give students ownership of some aspect of the research question. Students from underrepresented groups respond well when the research question they are involved with has direct relevance to their lives and communities, benefit from an URE environment that promotes collaboration and empowerment, and from faculty mentors that help their students learn about the earning potential of individuals in STEM research careers. Studies on course-based undergraduate research experiences (CURE) found students gain in most of the skills and competencies that students gain from stand-alone URE. Both URE and CURE improve student's problem-solving, written and oral communication skills, and proficiency with operating equipment used in the field and laboratory. They also help students become competent at finding and understanding peer-reviewed scientific literature, following protocols and their ability to both work independently and as part of a team [4, 15, 17, 29, 30, 39, 42, 54].

Many of the aforementioned studies did not specifically focus on students majoring in mathematical fields but rather grouped math majors with students majoring in other STEM majors. The findings of math-centric studies examining the impact of URE on students mirror those studies on the broader STEM community: increased interest in studying mathematics and in post-graduate studies in mathematics [8, 35, 33]. Fourteen years have passed since the American Mathematical Society's survey of graduates of REUs in mathematics [8]. This study showed that of the 262 REU alumni who responded to the survey, 82% had entered graduate school to earn an advanced degree in a mathematical field. More importantly, many of the respondents indicated that involvement in the URE was key to that decision. The student learning gains of math students engaged in URE are also similar to those experienced by STEM students: problem-solving, communication, and working as a team [6, 56, 13]. Involving undergraduate students in URE is not novel as many mathematical faculty and departments have done so for nearly as long as they have existed [41] and NSF has been funding REU-like experiences since the late 1960's [51]. The University of Chicago, MIT, East Tennessee State University and The College of Wooster have long standing programs [7]. The mathematics community has further demonstrated its interest in URE by having three stand-alone conferences (and proceedings from those conferences) in 1999, 2006 and 2012 [45].

Despite the overwhelming benefits of URE for students, not all students have the opportunity to participate in one. URE are dependent on the availability of research mentors and a survey of COPLAC institutions shows that less than 50% of faculty consistently serve as a mentor of URE [16]. The lack of time, inadequate funding and the level of student preparation are often cited by faculty as barriers to involving undergraduate students in URE [22, 44]. Limited studies have shown increased scholarly productivity when faculty engage in URE [34, 21]; however, Alayont et al. [2] make the argument that the type of research undergraduate students can do with math faculty is significantly different than that of their science colleagues.

Three junior mathematics faculty reported varied institutional support for mentoring undergraduate researchers [24, 38, 53]. There is less support and encouragement for faculty at research institutions to involve undergraduate students in their research programs. However, [40] writes about how the mathematical REU at the University of Chicago catalyzed social interactions between undergraduate and graduate cohorts and changed the culture of the department. For faculty at institutions based on the teacher-scholar model there are existing structures in place in support of faculty-mentored mathematical URE whether it be in the summer, academic year or as part of a CURE [18, 21]. Many faculty, though, report that mentoring undergraduate researchers is only informally considered in annual evaluations [16]. However, there are models of formal support. Some institutions rotate the course credit for teaching a capstone research course while the College of New Jersey allows faculty to 'bank' the time for overseeing independent research across semesters to give a faculty member a reduced teaching load in a subsequent semester [49].

Course-based undergraduate research experiences (CURES) solve many of the issues associated with REU. Because the research experience is embedded in a course, 1) the research experience is factored into a faculty member's workload; 2) it helps to improve equity and inclusivity in that any/all students enrolled in the course gain research experience; 3) it removes the issue of students who are self-confident and who already see themselves as scientists applying for URE at a higher rate than others and 4) helps to scale up the number of students who engage in research. Additionally, CUREs can mobilize a large number of novel researchers to explore a problem which can lead to new insights and directions for research, generate preliminary data, and can help identify students who show promise for independent research.

Most CUREs involve students working in groups on the research project which may introduce a new concern- how to effectively assess an individual's work vs. the group work on a research project. Given the longevity of CUREs it is surprising to find few studies focused on how group dynamics, tasks and processes impact a student's impression of a CURE. A recent study of sophomore computer science students engaged in an academic-year long group structured project found that the group experiences were overwhelmingly positive [28]. Research on sociology students engaged in group research projects showed that groups whose members had a more negative experience earned lower grades on their research project [43].

Assessing an individual's contribution to a group project is a common concern that spans a number of active learning pedagogies, it is not unique to CURES. You are encouraged to reach out to your Teaching and Learning Center director as they can best direct you to resources to assess individual contributions to group work. Common issues that arise include free-riding, interpersonal conflicts, differential commitment to the project and logistical difficulties. Research on group research projects have found that group activities that include turn-taking can help reduce inequitable groups [59] and that making students aware of the learning outcomes prior to the onset of the project helps to alleviate negative group effects [61]. Effective strategies to ameliorate negative behaviors include encouraging informal meetings and to have groups develop strategies to deal with opportunistic behavior such as periodic monitoring and peer assessment at multiple times during the project [27].

3.2 Gaps in our knowledge about the impacts of URE

Although the assessment of CURE/URE has increased in the past decade, there are many questions that have yet to be systematically examined. For example we still do not understand about how and when students feel like they have become a mathematician and/or have become part of a research community [12]. We do not know whether or how students who engage in a year-long URE might benefit differently than those students who engage in a shortened but focused summer URE. We also do not know how involvement in multiple URE influences student learning outcomes-do students gain in the same skills in the second URE as they did in the first? Is it easier for them to feel part of the community? How do they establish and navigate a relationship with their second research mentor?

It is commonly thought that mentoring a student in a URE is a time intensive activity for faculty and that the work an undergraduate student undertakes usually does not produce results that are publishable. The trade-off is that projects undertaken by undergraduates can lead to new avenues of research. Undertaking a systematic study to examine these claims could help develop strategies to recruit more research mentors so that a larger percentage of students can engage in this high impact practice. Evidence-based results might also be used to better understand how involving students in URE impacts progress towards promotion and tenure so that our colleagues can make educated decisions on whether and when to work with undergraduate students. For example, if results show that mentoring undergraduate research has a positive or neutral impact on research productivity a faculty member might work with undergraduate researchers every year. If mentoring undergraduate students diminishes peer-reviewed output, a pre-tenured faculty member might elect to mentor undergraduate students every other year.

Understanding the process(es) departments engage in to adopt policies to encourage growth in URE opportunities is also a worthy endeavor. Was external funding important? An external review? A self-study? Did a greater institutional effort around a Quality Enhancement Plan make the difference? Did it require a critical mass of faculty with a similar commitment? Work done by members of the Council of Undergraduate Research examining institutional cultural shirts in support of URE found these strategies to be important: establishment of awards, funds for students and faculty, building UR into the curriculum and integrating UR into existing campus activities (learning community, community-based research) [38].

3.3 Planning for an innovative assessment of your URE

As indicated in the previous section, there are a number of areas that your work could contribute to our understanding CURE/ URE as long as what you plan to study is well-designed so that it can elucidate relationships among various components of CURE/URE. To ensure your work is impactful you should collaborate with an educational researcher. Educational researchers can help you craft carefully structured research questions and align methods of data collection and analysis. They use rigorous and systematic processes to gather information, usually collecting both qualitative and quantitative data. Educational researchers also have expertise in designing, testing and validating surveys which may be important if you are exploring new areas. There are others on your campus who may have expertise and interest in helping you assess CURE/URE including:

Directors of Offices of Undergraduate Research. Many campuses have an office of undergraduate research and/or staff members tasked with this endeavor. They will have a vested interest in helping you assess and evaluate your CURE/URE and will bring their expertise to a partnership. If your campus does not have a dedicated office to this endeavor the Council of Undergraduate Research has a division called Undergraduate Research Program Directors. Councilors within this division have a history of being collaborative and interest in assessing CURE/URE.

Institutional Research. Every campus is interested in evaluating the student experience. Individuals charged with this task are often found in an office of institutional research. Sometimes the office of institutional research is well-resourced with multiple staff members, sometimes just one staff member might be "the office." In other cases, someone might be tasked with assessing student experiences as part of their assigned duties. They can help you document GPA, persistence by various demographic factors, course enrollment, time to graduation and many other measures. Many campuses have their institutional research offices administer the National Survey of Student Engagement (NSSE) and/or the Student Satisfaction Inventory (SSI) and both of these surveys have questions that specifically address URE providing a good benchmark for your campus. Lastly, the staff within an institutional research office have access to the National Student Clearinghouse for tracking student enrollment in post-baccalaureate degree programs.

Teaching and Learning Centers. Many campuses have staff dedicated to helping faculty create the best learning experience for students in the classroom. These individuals can help you develop and assess a CURE. Remember that assessment of STEM CURE suggest that students enrolled in these courses experience the same gains as students who participate in stand-alone URE [9].

3.4 Surveys, rubrics and other instruments

There are some excellent resources available to help individuals assess your CURE/URE. For example, URSSA [25] is an excellent tool to use. It has been adopted by many institutions and programs such as NSF Biology REU sites. Using URSSA, Drexel University showed that students in their STAR program reported significant growth in understanding what everyday research work is like, increasing their ability to work independently, and explaining their project work to people outside their field [57]. Over the course of the past 30 years, educational researchers have developed, tested, and honed this instrument. The URSSA instrument contain a number of items that are grouped into major categories: thinking and working like a scientist, personal gains related to research work, gains in skills, changes in attitudes and/or behaviors. There are also a number of items that ask students about: what motivated them to do research, aspects about the research experience, and their educational and career plans. Additionally the survey has questions about how the URE is run such as the application process and how well information is communicated.

Evaluate UR (`serc.carleton.edu/evaluateur/index.html`) provides regular scheduled opportunities for students and their mentors to meet to discuss their perceptions of how the students are progressing. There is an extensive pre-research survey that will help the faculty member learn what concerns the student has prior to the work starting, likely creating a better URE. CUREnet is a website with resources for those interested in developing and assessing CURE (`serc.carleton.edu/curenet/index.html`). The VALUE Rubrics of AAC&U [50, 3] assess a variety of student learning outcomes. There are 16 comprehensive rubrics for a given topic (i.e., critical thinking, writing). Rather than use an entire rubric you could select only those sections of a given rubric that directly apply to your URE. Florida Southern College adopted this approach for assessing their QEP focused on creativity activity, undergraduate research, and scholarship [11]. Lastly, there are valid and reliable instruments that measure the imposter syndrome [37] and Belonging and Connections (SBC) [62] that one might use to help determine student connectiveness to your research community.

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Part I

Integrating UR into the Curriculum

Introduction
Integrating UR into the Curriculum

When UR was first started in mathematics, it was typically done in a one-on-one model with a faculty member mentoring a single undergraduate student. With the introduction of more and different NSF-funded summer research experiences for undergraduate (REU) programs, a group mentoring model developed in which a faculty mentors a small group of undergraduate students working on the same problem. This has been beneficial since it allows more undergraduate students to participate in a mentoring experience. However, the faculty's demand to host an REU site as well as the student's demand to participate in an REU quite naturally greatly exceeds the supply. Also, NSF REU grants are quite competitive and the same is true for students' participation. The REU program alone cannot effectively provide a research experience to all undergraduate students. It is therefore important that UR in math is not restricted to summer REUs. What is the next step? Is there a model that permits more undergraduate students a research experience? Borrowing from the laboratory sciences, one way to achieve this is to integrate UR into the curriculum. For example, could mini-research problems be incorporated into mathematics courses such as calculus, linear algebra, differential equations, and others? This section looks at those questions and some issues related to them.

In order to provide every interested student with an opportunity to participate in UR, we argue that UR has to be integrated into the curriculum. Students should be offered a course that provides them with proper research experience. In Chapter 4, Rychtář and Taylor show one possible way to run a semester-long UR course. Their course is based on advice and training they received from CURM (`urmath.org/curm/`) and PIC Math (`maa.org/pic-math`) programs. In Chapter 5, Cohen et al. provide several excellent examples for student projects that could be integrated into a capstone course offered by most math departments. In Chapter 6, Betz et al. demonstrate that UR is no longer reserved for seniors or juniors. They describe a statistics learning community designed for sophomores engaged in year-long research projects. Numerous reflections of past student participants are a testament of success of this growing and scalable model.

4

Course based UR

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4.1 Introduction

Incorporating UR into a classroom is one way to bring the benefits of UR to as many stakeholders as possible.

Undergraduate research benefits students, faculty, institutions of higher educations as well as the graduate programs and employers. Numerous benefits of UR have already been pointed out in earlier chapters of this book and elsewhere; see, for example, [9, Chapter 1]. Students are challenged in thinking in new ways, they connect their learning to settings outside of the university, develop innate potential and many essential skills. Mentors develop and retain skills, generate meaningful scholarship, and transfer "academic DNA." Within the institutions, UR develops and enhances internal and external partnerships, strengthens recruitment and retention of students, faculty, and staff, and overall strengthens the quality of education. The graduate programs and employers that hire former UR students are more likely to have higher retention rates, increased creative/innovative outputs, reduced training time, and higher adaptability.

Course-based UR comes with its own unique benefits and challenges. On one hand, it makes UR accessible and available to a diverse body of students. At the same time, the faculty may not have much control over who is enrolled in the class. Course-based UR creates a definite time that students can and should dedicate to research projects. However, since the semester lasts a finite number of weeks that go by very quickly, students must learn to balance their other classes and duties while also doing research. Faculty also have some additional balancing to manage. The time a faculty member invests in conducting research with an entire class can often take far more time than teaching a typical course. Even determining a method for evaluating students' work and giving a grade at the end of the class requires quite a lot of thought.

In what follows, we give general guidelines and ideas on how UR can be incorporated into a course. We will focus on how the course can be run and how can one turn the challenges into training opportunities and benefits for the students and the mentor. We then provide details of how two such courses were run in Fall 2019 and Spring 2020.

4.2 Research projects

A lot has already been written about what constitutes a good undergraduate research project, see for example [8]. We will therefore not go into those details but rather provide a few pointers that are relevant to course-based UR.

We believe that the research experience should be authentic. This means, that the project should be chosen in such a way that it can be successfully completed during the course and disseminated shortly after the course is over, i.e., submitted for a publication in a peer reviewed journal and also presented at a conference.

The project should require only a small amount of background information so that the students can start working on it relatively quickly. It can still be in a highly advanced and specialized area of pure mathematics. But the necessary amount of information needed for students to work on the project should be trimmed, streamlined and provided to the students.

The project should be solvable. More specifically, the project should be solvable by undergraduate students in one semester. If students work hard for half of the semester and still do not see a way to bring the project to a conclusion, they will get very nervous very quickly. From the faculty perspective, it is also very difficult to fairly grade a research project that is way too hard and thus not complete. As a rule of thumb, a project is solvable if a faculty can see themselves solving it in one working day, i.e., about eight hours. This may be an indication that students can solve it in about eight weeks. Once a particular project is chosen, the faculty mentor should actually solve the project before giving it to the students. This does not mean that one has to write out a complete solution with all the details, but it does mean that the mentor needs to have a reasonably detailed road map towards the solution. Computationally intensive projects may be possible exceptions. The mentor should have a reasonable idea what needs to be coded and how, but the code does not have to be written (as long as the mentor is reasonably sure that the outcomes of the projects can be disseminated regardless of the output of the computations). Making sure a project is solvable is really important as the students should be gaining confidence through this experience. If a student leaves the class thinking there aren't "cut out" for research, that is fine, we just don't want them to feel that way due to a failed project on our end.

Some mentors do not like to solve the projects ahead of the students. They fear that if they know how to solve everything that they will influence the students too much. A simple remedy for that (used by J.R.) is to solve the projects a couple months ahead of the course and then go and work on a different set of problems. This will guarantee that the project is truly solvable, yet by the time students start working, the details will be too vague in a mentor's memory. Also, the students should not ever have to see the mentor's solution. In fact, providing students with too much assistance and details may deprive them of their own creativity and the joy of coming up with their own solutions.

One fairly substantial difference of course-based UR from UR conducted during a summer program is this: a course generally requires much more than just one research project. Topics that naturally split into two or more similar, yet stand alone and different, projects (for example a theorem that can be generalized by going in one of the several directions or a technique of mathematical modeling that can be applied to different diseases) are ideal for the course based UR. Such projects could be worked on simultaneously by different students and will not stretch the faculty too thin into too many unrelated topics.

4.3 Team work

Team work is a necessary component of any UR class. Dividing students into teams of three to four members reduces the number of research projects to a manageable amount. Very few mentors have 20-24 research projects ready and available to be offered to students. However, with a little bit of planning, many mentors can come up with five to six projects. Furthermore, if you enlist a help of one or two more faculty members or collaborators, each one of you need to come up with only two to three projects.

When students work in teams rather than alone, it allows more time for the mentor to provide supervision. As with any group of people coming together, the team members will each have a unique set of skills and strengths. Some are likely already familiar with various components of the projects (be it specific math knowledge, or an experience with LaTeX, coding, etc.) and can thus act as mentors for the others.

By working in the teams, students learn many important skills that are valued by the employers that they simply wouldn't have the opportunity to learn in a typical class. Teamwork is an essential part of workplace success [7]. The students have to learn how to work cooperatively, how to communicate with each other respectfully, how to work on an open-ended problems (ones they can't Google), how to divide the tasks and be responsible in completing their tasks on time, and the list goes on.

4.3.1 Logistics—forming the teams

There are several ways to split students into teams. They essentially fall into one of the two categories—the faculty makes the assignment or the students choose their own teams.

The advantage of the first approach is that one can aspire to create "balanced" teams. For example, we may require interdisciplinary teams where at least one student is a math major, at least one is a biology major and at least one student knows some coding. This approach also mimics how an actual workplace may look and therefore many faculty recommend this method of forming teams [13]. A challenge that arises in attempts to use this approach, however, is that faculty do not normally have control over who enrolls in the course. Consequently, splitting the students into teams that satisfy any requirement created before the beginning of the semester may be an impossible task.

One alternative to the above is the method used by the authors: The faculty mentor specifies all the projects with details on the required background knowledge, coding knowledge, etc. that the student will need to learn. The students then pick the project (or rank them) and essentially form the teams themselves with a little bit of help from the faculty. With this method, students get a larger sense of ownership and responsibility and don't end up on a project that they have no interest in or feel they aren't prepared for. This too helps the faculty mentor to know that each project contains members with strengths (and interests!) necessary to complete that project.

Finally, the teams should be formed as soon as possible. Students can drop or enroll in the class usually during the first week, so the timeline needs to accommodate for that. The team formation activities should start during the first class, but the formal team creation can be concluded only after the end of the transient period.

4.3.2 Teamwork training

While teamwork can be a significant time-saver for the faculty conducting the UR course, it is necessary to provide students with proper training. The faculty needs to spend some time going over fundamentals of teamwork and expectations. This includes the faculty's expectations of them as a group, but also expectations they should have of each other. We highly recommend the prospective mentors to read [13] and adopt many of the recommendations provided there.

Two twenty-minute blocks of the class time spent on teamwork right in the beginning of the semester will save a lot of time and problems later on. In one block, go over the forming–storming–norming–performing model [14]. It is especially important to go over the storming stage. More often than not, the teams will go through this stage and the key is to address and resolve the issues rather than avoid or pretend that they do not exist.

In addition, let the students read *Coping with hitchhikers and couch potatoes on teams* in [13] and submit a half-page essay on how they feel this paper applies to their past experiences. Devote the second block of team building time the discussion the article. Only students that already had a significant team experience in the past seem to appreciate this article but all students still need to read this as early in the course as possible. It is important to repeat this activity once more mid-semester. You may also consider letting the students do one or more of the activities from the Teamwork chapter of [7].

4.3.3 Team agreement

The first thing a team should do is to create a team agreement. Many students are tempted to just start working on the project. However, they need to invest the time to put their implicit beliefs about the team into an explicitly written document. Different students have generally different visions and different ways of dealing with problems and obstacles and often not enough skills necessary to minimize the potential for conflict.

An effective team agreement should address most of the following, see for example [11]: (1) Intent and vision, (2) roles, (3) promises, (4) time commitments, (5) measurements of satisfaction, (6) concerns, risks and fears, (7) renegotiation, (8) consequences, (9) conflict resolution, (10) agreement. The team agreement helps each team member, and the mentor, to address potential issues that may arise.

4.3.4 Issues and problems

The best way to address issues and problems is to prevent them. However, even with all the prevention, most teams will still experience some troubles. The key is to catch the troubles in the beginning and do not allow them to grow into something serious.

Open and frequent communication is necessary. Students should be able to communicate their concerns freely and openly to the team members and the faculty mentor. A great way to start is by asking the students to fill out a little questionnaire in the beginning that contains questions like "How do you prefer to receive constructive feedback from your peers?" and "How do you prefer to receive constructive feedback from your professor?" Making everyone aware of these kinds of things in the beginning can help reinforce that being respectful when being critical is important. Collect feedback from the students regularly (weekly if possible). Make sure the students know you are always open and available to hear them. Provide positive feedback back to the students frequently.

There are times when it seems that everything fails with a team regardless of what training one provides. In these instances it helps to have clear ground rules and procedures in place and on the syllabus. As noted in [13], the most common problems involve team members who do not do their share of the work; domineering team members who want others doing everything their way; resistant team members who refuse to participate or in other ways try to sabotage the team effort; and team members with widely divergent goals—some wanting an A no matter what it takes, others wanting to do just enough to get a C.

The recommended four step procedure addressing any of the above is as follows. (1) The student(s) should notify the instructor as soon as the issue starts; this is most likely to happen during the regular weekly feedback. If all team members provide a similar story, the instructor just needs to note it, provide encouragement and possible suggestions to the team and let the team work it out on their own. Otherwise, it is best to gently inquire about what is going on. A student may have a rough week and their lack of performance could be a one-time thing. However, it is important to prevent any student from getting completely lost. (2) If the problem persists, the team should formally meet with the instructor. The purpose of the meeting is to discuss the problems and concerns openly, to identify what the underlying issue is, and to identify a road map for remediation. (3) If there is still no improvement, the team may consider splitting and regrouping (for example, if one student is still not working, the other three may want to form a team on their own; or if only one student feels like doing all the work, that student may want to work individually). (4) If after another week and thinking everything through the students want to proceed with splitting, they should meet with the instructor once more and have the team split into two or more teams. Depending on how far into the course this happens, the instructor can either have the new teams still working on the same project (just independently), or a new project can be given to one of the teams.

It is important to go over the above with the students early on. Point out that while the teams are not necessarily permanent, it takes at least four weeks for them to dissolve. Also, the teams can be dissolved only if there are some persistent and well-documented issues. The aim is to give students enough time, opportunities and guidance to work out any problems on their own.

4.3.5 Co-mentoring

Course-based UR can be a tremendous strain on the faculty. Getting help from another faculty member or collaborator is strongly recommended. This does not have to be in the form of formal co-teacher; it can be an informal pairing with another professor (possibly even at a different institution). The secondary mentor can be completely in the background and does not have to interact with the students during the course at all. Their role would be to keep you on track when you get buried in the project details and all class related activities. Having another set of eyes who can focus only on the bigger picture of the project is a tremendous help. Finally, while you take the lead during the actual course, the secondary mentor can take the lead once the course is over and make sure the projects get finished into the actual published articles. The opportunities are endless, and it is completely up to you to decide how to handle the work.

4.4 Implementation details

Here we describe a specific course—MATH 380: Introduction to Mathematical Biology—as it was conducted in Fall 2019 and Spring 2020 with J.R. being the teacher and D.T. a secondary mentor in the background.

4.4.1 Basic course information

We start by reprinting the university description of the course taken from the Course Catalog.

> Math 380 - Introduction to Mathematical Biology: One semester course; 3 lecture and 2 laboratory hours. 4 credits. Prerequisites: MATH 200 (Calculus I) and BIOL 151 (Introduction to Biological Sciences, designed for biology majors), both with a minimum grade of C, or permission of instructor. An introduction to mathematical biology. Various mathematical modeling tools will be covered and implemented in a range of biological areas. Additionally, the collaborative research process will be presented and discussed. Crosslisted as: BNFO 380.

This course meets twice a week in a computer lab for 100 minutes over the course of a 15-week semester. The course attracts around 25 students per semester. The enrolled students range from sophmores to seniors. They major in biology (mostly pre-med), engineering, mathematics and bioinformatics. Biology majors constituted the majority of the class in both semesters and their prior math knowledge, on average for the class, did not go much higher than calculus I. Despite this, both classes were extremely adaptive, hard working, and learned new concepts quickly. It was truly amazing.

In the Fall of 2019, the students did not know about the research nature of the course when they enrolled. Each professor is allowed to conduct the course as they see fit within the scope of the course description. On the first day of class, when the students were told they would be working on unsolved research problems, they were all quite excited. Only one student dropped after the first class and another student dropped in the middle of the semester. In Spring 2020, the students already knew what to expect, and the class was full with a populated waiting list. No one dropped during the first week or semester.

4.4.2 Project selection

At the beginning of the semester, the students were given a list of the five broad topics below:

1. Vaccination game theory (ODEs, probability)
2. Evolution in finite populations (probability, stochastic processes, linear algebra, and possibly graph theory)
3. Plant-pollinator interactions (computer simulation, ODEs and PDEs)
4. Owner-Intruder games (Game theory, probability)
5. Stealing behavior (ODEs, probability).

For each topic we had two to three specific projects for a total of about 13 projects from which the students could choose. The first two classes were devoted to team work as described above and mostly to the quick run-through of the broad topics and projects. Students were encouraged to pick a broad topic rather than a project and the teams formed based on their selection. The very first time, everybody picked vaccination game theory (likely due to the instructor's excitement about those projects and the fact that many students were pre-med). We thus had the students make a second round of choices and ended up with three projects in vaccination game theory, one in evolution in finite populations, one in plant-pollinator interactions and one in Owner-Intruder games. For Spring 2020, the list of projects was "replenished" and we ran two projects on vaccination game theory (which now seems largely exhausted), two on Owner-Intruder games, one on plant-pollinator interactions, and one on stealing behavior.

A typical project on vaccination game theory progressed as follows. (1) Pick your favorite preventable disease (for example, Hepatitis B that is preventable by vaccination). (2) Search a literature to find an ODE compartmental model describing the disease transmission and prevention method. If a good model is found, adopt it; if no good model exist (typically because the prevention method is not explicitly considered), extend a similar model. (3) Analyze (or check the published analysis) of the model. This includes finding the disease-free equilibrium, the effective reproduction number, and the endemic equilibrium. (4) Add the game theory component, i.e., assume that individuals behave in such a way that maximizes their own benefits. For example, this means that nobody would vaccinate if the vaccination was so common that the population already achieved herd immunity. (5) Solve the vaccination game, i.e., determine what is the optimal vaccination level from the individual perspective. (6) Determine the disease prevalence in the individual optimum to see how far is it from disease elimination. (7) While doing the previous math, learn as much as possible about the disease, its epidemiology, transmission and elimination efforts.

Such projects are quite well defined and progress in a predictable manner and speed. There are also plenty of examples of similar projects. The fact that students can pick their "own" disease increases their sense of ownership of the project. The biggest issue is that while there are many diseases to choose from, there are also many faculty working on these projects and sometimes a chosen project has already been "taken" by an independent team as discussed later in this chapter.

4.4.3 Team formation

In each course offering, some students knew each other from other classes and had a preference to stick together in a team. Despite advice in [13] to the contrary, we did not object to it and about 50% of the teams thus consisted of students who knew each other. The other 50% of the teams consisted solely of students that did not know each other prior the class. We did not see any significant difference in the performance between the two kinds of groups.

Every project had a computational component to enable students to quickly get started with the project. The engineering students were quite proficient in Matlab which was adopted as the programming language for the class. The bioinformatics students knew Python well and were able to adopt Matlab without problems. One or two teams per semester had no member familiar with any kind of programming. Those teams were given extra attention and help and at the end did not experience any issue.

4.4.4 Templates

Students were given templates which consisted of LaTeX write ups of major steps in the solutions to a 'baby problem'—a related and often significantly simplified version of their research problem. They were also given a well-commented MATLAB file for playing with the baby problem.

The LaTeX and MATLAB templates served as a guide for the students to progress. They allowed them to focus their attention on the actual math rather than on the technicalities associated with writing/coding it. The students started first by modifying small parts of the code and write up to fit their projects. They eventually ventured far away and created completely new elements. We devoted some time (about twice for 20 minutes each time) in the second week of the classes to go over the templates. It was mostly about logistical issues such as downloading and registering MATLAB on their personal computer or accessing and running the VCU's MATLAB app in the cloud. Essentially no student had any prior knowledge of LaTeX. However, they were very fast learners and it did not take them any time to learn all that was needed from the LaTeX template posted for them through Overleaf (`overleaf.com`).

Aside from the template, the following exercise is another quick way to get students up to speed with LaTeX. This activity is to be done individually rather than as a group. Give students a pdf file, about one page, with lots of different mathematical problems and statements (think of writing a math exam for a senior that contains questions from calculus to linear algebra) including bolded text and varying mathematical fonts and ask them to reproduce it exactly, right down to the spacing. Provide for them the preamble of the LaTeX document with the standard packages needed already there. Show them how to put something in math mode and the difference between single and double dollar signs. Explain how easy it is to use Google, Overleaf and Wikibooks for help with LaTeX issues and commands since this is what they will need to do when working on their own and then let them work. Most students who have never used LaTeX at all will finish this activity within a 75-minute class. The exercise is surprisingly fun and effective and leaves students confident that they can use LaTeX.

In general, the students exhibited a high degree of creativity and innovation through out each course. For example, several teams were using overleaf for a swarm-like coding of the MATLAB program before one of the members uploaded the code in actual MATLAB for debugging.

As an example, the template for the teams working on the vaccination game theory project was based on a classical paper in the field [3] that deals with a simple standard three compartment SIR model. The template consisted of all elements of the scientific paper, including guides on the introduction and discussion, with all necessary LaTeX commands. It also contained math worked out in enough details for the simple SIR model (the math calculations were omitted in the published paper [3] because they were considered standard for the regular audience). The MATLAB file was a well-commented code that dealt with many needed aspects of the project. With the templates at their hands, the students picked a preventable disease and their task was to apply the same game theory framework to the disease of their choice. They worked on modifying the templates and they ended up with models consisting of six [5], eight [4], and nine [2] compartments. All of these are now published in high-impact peer reviewed journals.

4.4.5 Oral presentations

Once a week, starting during the third week of classes, three out of six teams had to give an oral presentation. All teams presented every other week and gave a total of six presentations. The first four presentations were relatively short, 10-15 minutes summarizing the team's progress, and showing the class what the team did since the last presentation. They did not have to show every math detail, but they were required to cover at least one mathematical element properly with all necessary background information and reasonably detailed calculations. The presentation was given by two team members. The rest of the class was required to ask questions. The fifth and the final presentations were comprehensive, about 30 minutes long, and covered the project from the beginning to the end. These last two presentations were given by the whole team.

At the beginning of the semester, the instructor delivered lectures to the whole class. As the class progressed, the classroom time was generally spent by the instructor cycling through the teams and discussing the relevant material only with a team that immediately needed it. In general, even on similar projects, teams progressed at different speeds and it was not always the same team that needed help. If the topic was of interest for the whole class (such as determining Nash equilibrium or calculation of the basic reproduction number), the instructor asked the team to present the topic to the whole class during their next presentation.

4.4.6 Written reports

The three teams that did not present in a given week were instead submitting written reports to the instructor as pdf files. The instructor had editing access to the overleaf document, and with the use of the LaTeX package `todonotes.sty` [12], was able to provide detailed feedback right in the documents. The built-in features of the package such as color coding and commands like `\listoftodos`, the package proved to be a quite effective way to handle the work flow. With the help of the custom defined command `\TeamToDo` the instructor was able to give instructions in the exact places where things needed to be done. When the team completed the task or made the necessary correction, they changed `\TeamToDo` to `\TeamDid` which allowed the instructor a quick check that the team indeed completed the task. The written reports were short at the beginning but grew to a full-size research paper at the end of the semester. With the help of `mcode.sty` [10], the students were including their MATLAB codes in their reports as well.

Each set of feedback for the written reports contained a list of tasks that had to be addressed immediately. We were also continuously adjusting the rough road map/timeline of tasks to complete by the end of the semester. Students were not timed and there were no hard set deadlines for particular research tasks to be completed. Yet they knew that by the end of the semester the project is to be finished, and that if they wanted a good grade, the write-up had to undergo several revisions.

Students also realized that the history feature of overleaf documents allows the instructor to have a good overview of who is doing what, and as a result, they were quite good about sharing the workload.

4.4.7 Posters

At the end of the semester, the students were expected to have a poster ready in either LaTeX or Powerpoint format. They were given templates, brief pointers, and a grading rubric. The original plan was to provide them with more detailed help closer to the poster submission date. However, seeing how well they worked on other aspects of the projects, we did not need to spend any time on this.

4.4.8 Grading

In general, setting up the grading was the hardest part of the course. There were two major hurdles to overcome. First, we needed a grading scheme that focused on a team product as a whole but also allowed for variation of the grades within the team. Second, the aim was to have it in such a way that hard working students can get a good grade even if they at the end do not solve the research problem.

The first hurdle was overcome as follows. The teams were graded based on the oral presentation, poster or other submitted work products. However, every team member also submitted regular feedback, rating their teammates as well as themselves. The rating was based on a rubric from [13]. The intention of those ratings was to reflect each individual's level of participation, effort and sense of responsibility, not their academic ability. The team's score was then multiplied by their individual score to receive an individual grade. In particular, no individual received a better grade than the team as a whole.

No grading scheme is perfect and every scheme has a weakness. Yet, there seemed to be a general agreement between the score assigned by others and the "self-score," quickly mitigating early student worries about possible lack of anonymity (the teams were not big enough for the grading to be truly anonymous) and potential retaliation. A rare few cases with a significant discrepancy were addressed by the instructor as soon as they appeared and were resolved quickly.

To resolve the second hurdle, the team grades were based on oral presentations (30%), written final report (30%), poster (10%), and scientific content (30%). The grading rubric of the first three focused mostly on the form. It was emphasized to the students that they can get close to 70% even if they do not solve the problem at the end. Obviously, nobody can present or write well enough without understanding what they are presenting or writing. Consequently, when the students try to present well, they gain understanding of the material. So, when the projects are solvable, chances are the students will actually solve them.

The oral presentations were graded by the instructor and the peers through a google form questionnaire filled immediately after each presentation. The form was based on the rubric from [1]. Students assigned numerical scores for non-verbal skills, oral skills, visual components, teamwork and actual content. Every student was also required to give verbal feedback. The feedback was made anonymous by the instructor and shared with the team.

To help students overcome speaking anxiety, the following two measures were implemented: (1) the lowest score was dropped, (2) the presentations were weighted by their order (the first presentation had weight 1, the sixth had weight 6).

The poster grading rubric was adapted from [6]. The poster was graded by the instructor. The plan was to do first a mock grading of the draft and then a real grading of the final version. However, no draft required any significant changes and so the draft grade became the official grade. In the next iteration, the plan is to have it graded by the instructor and the peers according to this rubric, and in a similar way as the oral presentations.

For all peer-graded scores, the instructor could adjust the score in case of major discrepancies. However, there was no need for any such adjustment. The students were relatively generous but quite fair; about on the level the instructor would grade himself (i.e., forgiving an occasional small slip but not letting major things just go by).

Formally, as far as the written reports were concerned, the instructor graded only the final report and did not assign any score for the interim reports. The grading rubric was relatively simple and required the students to adhere to a basic scientific style appropriate for their project. Students knew that they can be wrong/make mistakes early on and that everything will still be fine as long as they fix their work based on the feedback they receive. Shortly after the submission of their fifth report, the students received feedback with instructions of what needs to be done to achieve at least 90% score on the report.

The scientific content was graded by the instructor and consisted of the following four loosely-defined broad categories

1. Teams ability to do research as directed (or better)
2. Quality and depth of the Introduction section (i.e., background information, literature search) of the final report
3. Quality of the plan and execution of the methodology (math analysis, computer code, etc.)
4. Quality of the conclusions and discussion section of the final report.

From the very beginning, the students knew that the intention is to have their projects published after the class is over. However, they also knew that the grade for the course will be assigned before the submission to a journal begins and that all grading is independent of the submission and any outcome of the submission process.

4.4.9 Time commitment

The class required about two months of preparation before the semester started. That time was devoted to polishing up potential projects and creating the templates for students to use. Since the projects were split into five broad topics, we created one template per topic. Once created, the templates can be reused for similar projects.

During the actual semester, the class demanded about two full working days per week (on top of the class time and the office hours). At the beginning, the time was spent mostly on bringing students up to speed. As the written reports grew in length, more and more time had to be spent on providing feedback and instructions for revision. About one to two hours a week was spent on looking through the feedback and checking the progress of the team and individual students. Another hour or so a week was spent discussing the projects with the co-mentor.

After the semester ended, each of the reports required about two weeks of work to be put into a form that could be submitted to peer-review journal. Once submitted and the editorial decisions came back, it was about another two weeks of work to address the required revisions and eventually get the papers accepted. Students were involved in these aspects as well, and some teams took a very active role in this, however most of this work was done by the instructor and co-mentor.

4.4.10 Bumps along the way

The ride is never completely smooth, and while many issues can be prevented, many issues will still pop up. The instructor and the students have to be flexible and open to adjustments. Here are some of the challenges we had to face in the past year.

In the middle of fall 2019, we learned that a faculty at another university had just submitted a manuscript that was exactly along the lines of the vaccination game theory project one of the teams was working on. We had an “emergency meeting” with the affected team. They were given three options for what to do next. They could keep working on their current project and get the grade as if the other manuscript did not exist (they did not see the submitted manuscript). However, they would not be able to submit the work to any journal. As another option, we could try to add a slightly different spin on the project. It was not clear though whether the novelty would be enough to warrant an acceptance of such a paper. Finally, the last option was to find another disease to work on and restart the project for the new disease. The project would be similar enough that the students would not have to learn any new mathematical concepts, but still different and requiring the students to do a new literature search, solve a new set of equations and rewrite most of the MATLAB code. They picked the third option and worked overtime to catch up with the rest of the class. In the end, their new project was published in a peer-reviewed journal.

Another challenge was when one team thought that they were already done with the math part of their project when they discovered some discrepancies. After endless checking and re-checking of their own work, it turned out, the issue was with formulas and results adopted from another paper that was already published in a well-respected journal. Since the adopted results looked reasonable and convincing, it took quite some time for us to realize that the issue was indeed there and then fix it.

Almost every team experienced one or more down moments. For example, in the vaccination game theory project, such moments typically came when the team had to solve a large system of nonlinear algebraic equations to find an endemic equilibrium. They knew the solution exists and they knew roughly how to get it, but the process of getting the actual formulas was tedious, challenging and prone to many errors. It was excellent practice in discipline for the team to try, fail, and repeat. The hardest moments came when the students thought they finally succeeded only to discover that their formulas do not match numerical solutions. During those moments, students needed a lot of encouragement and support. It helped them to see that even the faculty mentor finds the problems challenging and that there is no simple way to succeed but to keep trying. Another useful tactic was to divert their attention to another aspect of the project temporarily to regain momentum. For example, there always seems to be some work to be done on an introduction and a literature search. By looking at what others have written and done, students may discover what to do in their particular situation.

When a team encountered a “down” moment, they were encouraged to talk about it during their next presentation. It helped other students to see that such moments are natural parts of the research process and that discipline and perseverance often pays off. At the end, nearly all students commented that they appreciated that they were able to work on real projects rather than more simple "change a number here and be amazed at what happens" assignments. They also seemed to learn more from the bumps than if it was just a smooth ride.

4.4.11 Is the class going well?

It is important for the instructors to know whether the course is going well (or not). The regular oral and written presentation serve as a good indicator of how well the students progress on the project. When a team stagnates, a mentor should talk with the group and offer help in getting going again. For example, a team may indicate one week that they are solving a system of equations for the endemic equilibrium. If the system is still not solved the following week, the team may need some help even if they explicitly do not ask for it. The instructor does not need to solve the system for them, but should give them enough opportunity to discuss the obstacles, direct them to the literature where similar systems are solved, or offer alternative approaches.

The regular weekly feedback discussed in the grading section also allows the instructor to gauge how the students are doing. For example, if a student (or their teammates) indicates that they did not spend enough time on the project, it is helpful to talk to the team and find out what exactly is going on. It may be an occasional time crunch caused by obligations of other classes, or it may be a sign that a student is falling behind. In the latter case, a proper assistance should be offered.

Honest and open communication is always the key. Students have to know that they can freely say that they have not been able to devote enough time or that they do not understand something and they will be met with understanding and help as needed.

End of the semester teaching evaluations also offer valuable comments and feedback. For example, after the Spring 2020 semester, a student indicated that they would prefer a more detailed road map for their project. One needs to strike a delicate balance between guiding the students and giving them the solution. However, we are now developing better templates that offer a long term view of the project rather than just one or two next immediate steps.

4.4.12 Students' perspectives

Several students were interviewed for the VCU's news article about the class and here is what they said.

One student in the class who has since graduated and became a student in the VCU School of Dentistry, said her experience in the class gave her professionalism, problem solving and team collaboration skills that she has been using every day. "I took Math Biology simply because I figured it would be an interesting addition to my math minor, but I ended up learning a lot," she said. "The research my team and I did about vaccination strategies and the spread of an infectious disease (polio) was especially relevant, seeing how only a few months after our work was published, the COVID-19 pandemic began."

Another student in the class, biology and psychology major, said he never thought of himself as someone who would become deeply invested in math. "What I really appreciate about the class is that it encouraged critical thinking and problem solving to push through our setbacks. The instructor did not give us the answers, but instead provided us the tools to further develop our project," he said. "I gained a greater understanding of mathematics, biology, coding, public speaking, leadership and teamwork because of this class." He added that it was a thrill to see his name listed as a co-author of a published research paper for the first time. And the experience helped prepare him for his career path.

4.4.13 Teaching the class remotely

About midway through the Spring 2020 semester, during the spring break, the COVID-19 pandemic forced all VCU classes to go online; first for an unspecified amount of time and eventually for the rest of the semester. The students had to move out of the dorms and their lives were disrupted. We had to conduct most activities remotely in an asynchronous fashion rather than a synchronous one.

Given the unprecedented and sudden situation, two major adjustments were made - oral presentations were cancelled and the poster assignment was removed. This was done in the interest of time - we lost one week due to VCU extending spring break. This extension was to allow students and faculty the time needed to regroup and adjust. However, should we teach the class remotely again, there should not be any problem keeping both assignments in the online setting.

For the oral presentations, we would guide the students to create recordings of their presentations (for example in Zoom) which can be done even if the students collaborate only remotely. The recordings could be then shared with the whole class so that they can grade and provide feedback (via a Google form) in a reasonable length of time.

The posters can be handled easily and efficiently through the templates. Whether the class is face-to-face or online does not matter that much for this task.

Overall the class set up seems quite robust even for the online setting. Regular written reports and prompt detailed feedback are the likely reasons for this robustness. This class served as an unplanned pilot for us to run a remote REU during summer 2020. We are now confident that one can incorporate research into the classroom and that one can do it effectively and efficiently even under hard and unprecedented conditions during the COVID-19 pandemic. In particular, we strongly believe that with the proper set up, research projects can be incorporated even into an online curriculum.

4.5 Bibliography

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5

Integrating Social Justice and Undergraduate Mathematical Research

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5.1 Introduction

Social justice is commonly understood to be about the principles 'fairness' and 'equality' for all people and respect for their basic human rights [28, p. xix] The study of social justice has become a common part of the mission of higher education and provides a context for how students may apply their knowledge in their career. For example, according to Brandeis University's definition,

> [S]ocial justice is both a process and a goal. The goal of social justice is full and equal participation of all groups in a society that is mutually shaped to meet their needs. Social justice includes a vision of society in which the distribution of resources is equitable and all members are psychologically and physically safe and secure. [3]

Universities invite students to pursue their ideals while learning more about the potential challenges. As observed by educator philosopher, Daisaku Ikeda, "Those who are privileged to attend a university should spend their lives working for the sake of those who cannot enjoy the privilege" [17]. Our experience with students is that most want to be a contributors to making the world a better place and welcome the opportunity to work on research projects that incorporate social justice issues.

In this chapter, we discuss further what we mean by social justice and give some examples of research projects we have done with our students. We also list some useful data sources and other ideas for social justice research topics. We consider research to be any systematic inquiry into questions whose answers are unknown, including the public dissemination of those findings. We consider social justice research problems to include those where issues of social justice are relevant to the problem and its solution. While other chapters address the process of supervising undergraduate research, we point out a few items that need special consideration when the research is directed at social justice topics. It is our hope that the readers will walk away with both ideas and a road-map to implement them.

[1] When the chapter was written, she was at St. Mary's College of Maryland.

5.1.1 Defining social justice

The term "social justice" has a long history, and is often defined by a set of criteria (see [27] and [29]). Reish [29] describes how a profession may define social justice within its own lens, and culminates in how social workers see social justice. Agencies dedicated to social justice also offer their own definitions. For example, the San Diego Foundation [32] defines social justice in terms of equal rights, equal opportunity, and equal treatment.

One way to think about what we mean by "social justice" would be to examine how mathematicians have defined the term. For example, Karaali and Khadjavi [19, p.5] state:

> Following the spirit of the International Council of Scientific Unions, we use, here and elsewhere, the term "teaching mathematics for social justice" to encompass all mathematics instruction which aims for improved human well-being. Human well-being in turn is defined as a "context-and situation-dependent state, comprising basic material for a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience.

We can also look to authoritative governing organizations such as the United Nations:

> It is important to reflect more deeply on the nature and use of power within both the human and institutional contexts. Individuals who hold power must be willing to submit to certain laws and regulations that limit their freedom to use their authority as they see fit. Those who are privileged to hold political and administrative power must understand that their legitimacy derives entirely from their capacity to serve the community. Social justice is impossible unless it is fully understood that power comes with the obligation of service. In reflecting on the nature, legitimacy and use of power, consideration must be given to self-interest, enlightened self-interest, general interest and the common good. [13, p. 9]

A full examination of these definitions are beyond the scope of this chapter. We note that the examples below fall within the scope of these definitions:

1. **Environmental justice** matters concern who bears the cost of environmental damage. This becomes a social justice issue when polluters abuse their own power and shift costs and harms to those without the political and economic power to respond.
2. **Health Equity** concerns providing access to healthcare for all people.
3. **Citizen science** initiatives aim to empower the population in general to participate in the scientific process. Informed and broad participation in the scientific process can impact overall human well-being when it comes to responding to scientific misinformation on issues such as climate change or vaccination against diseases.
4. **Racial justice** concerns power inequities based around the social construct of race. Kendi [20] defines racism as a power-construct (pg. 238). Economic or political inequalities based on race establish what most would consider to be unfair power distributions. More broadly, McGhee [22] points out that racial injustice costs everyone and harms overall human well-being.

5.2 Examples

We offer some examples of undergraduate research, both one-on-one and course-based projects, that we conducted involving social justice. These examples are intended to contextualize the rest of the chapter, and we also hope that they will inspire ideas for the reader.

5.2.1 Environmental justice

The North Carolina hog industry manages an estimated 9-10 million animals per year. The lagoons where the hog waste is collected house many nutrients, such as nitrates, and phosphate, along with antibiotics and other pollutants. Recently, during three hurricanes in North Carolina, these manure lagoons flooded, spilling raw sewage into the waterways. The excess nutrients contained in the sewage caused a rapid increase in the algae population, called *algal bloom*. As the algae die, their breakdown to nutrient components feed the bacteria that decompose the algae. With more food available, the bacteria thrive and increase in number, which in turn depletes the dissolved oxygen in the water further. The algae cover the water surface, allowing for very little transfer of oxygen from the air into the water through diffusion, and even then, the oxygen goes back into the atmosphere before dissolving in the water. Lack of dissolved oxygen makes the water uninhabitable by the marine life, causing fish and other organisms to die, in millions, wreaking havoc in the environment and causing local economies based on fishing to suffer tremendously [26, 24]. These large CAFOs (Concentrated Animal Feeding Operations) are in largely lower-income areas. The “manure lagoons” used by these facilities contain tremendous volume of animal waste produced which are susceptible to breaches and leaks.

Unfortunately, animal waste is not treated as effectively as human waste in the United States, despite the fact that such waste generally contains more contaminants—including Salmonella and Escherichia coli (or E. Coli) as well as antibiotic residues and heavy metals—than human waste [25]. Allowing big agribusinesses to pollute the air and water for low-income communities, in addition to destroying small fisheries’ main source of income, make this topic a social justice issue.

Köse supervised a research student who designed two mathematical models to predict the size and impact of an algal bloom. These models have shown that the environmental impacts of polluted air and water disproportionately affect low-income neighborhoods [16].

5.2.2 PIC Math and citizen science

In the PIC Math program (“Preparing students for Industrial Careers in Mathematics) [21], faculty identify a professional liaison from Business, Industry, or Government (BIG) to participate in the class by providing a problem for students from their organization that could benefit from mathematical analysis. The liaison would meet with the students to present the problem, get updates on their progress, and finally receive the deliverable at the end of the course with the student’s analysis and recommendations.

When Cohen taught the PIC mathematics course, he found that these projects naturally connected to student’s interest in working on problems with social justice themes. In the first semester, some students worked on a problem from a researcher at Los Alamos National Research Lab who wanted students to design a system of care centers to treat people with Ebola in Western Africa. The design would give the locations and number of beds for each center that would make efficient use of resources. Students used an SEIR model to predict the spread of cases and considered GIS data for accessibility. They also learned a lot about how people lived. A variety of things like the condition of the roads, health care, and burial practices caused them to modify their network. They learned that the data on cases and deaths was filled with gaps and inconsistencies All of these things raised their awareness of the people they were designing this for and figured into their solution.

Another problem which has been ongoing is with the Field Museum of Chicago. The museum houses an archive of specimens collected by research scientists including over three million specimens in their herbarium [6]. Research scientists at the museum considered the question of using crowd-sourcing to measure the leaf structures (lobules) of a variety of bryophytes, microscopic plants collected from endangered ecosystems. Their project aims to increase community interest in the scientific process and raise awareness of the effects of climate change on species diversity. Users of the community science portal Zooniverse viewed randomly selected scans of the plants and marked the endpoints of the major and minor axes of the oval shaped lobules of the plants. Later, a kiosk was launched at the museum that allowed visitors to do something similar. [37]

Students in Cohen’s PIC mathematics class were tasked with examining the data and attempting to answer several questions addressing reliability of the data, sufficiency of user measurements of a single sample, how users’ measurements compare with those of experts, how to modify the instructions to make the system work better, etc. Students became interested in learning more about citizen science and how to get more children interested. Some of the users had answered a brief survey about their age and affiliation, their enjoyment of doing the measurements, and their interest in learning more about the project. Students in the PIC math class analyzed the responses correlated with the accuracy of the measurements to inform their recommendations to modify the instructions. Their goal was to improve the design of science crowd-sourcing projects to make them more accessible to children to support the growth of citizen science.

5.2.3 Racial justice and *The New Jim Crow*

Michelle Alexander's *The New Jim Crow* [1] argues that the collection of policies colloquially referred to as the war on drugs was based on racial rhetoric and is selectively enforced against African Americans. The consequence of this selective enforcement results in ex-prisoners being subject to discrimination in housing, employment, voting, and other fundamental rights, mimicking Jim Crow restrictions.

In Piercey's Contemporary Mathematics class, students read the book and discussed it one chapter at a time. He organized the discussions around the following questions: (1) How did the chapter fit into the overall argument of the book? (2) What were the data that supported the argument in the chapter, and (3) What questions do you have about the data? Immediately following the discussion of the chapter, he delivered short lectures and facilitated brief activities highlighting a quantitative concept raised in the chapter.

For example, one concept concerned how to determine when absolute figures are appropriate and when relative figures are appropriate in making an argument with data. To this end, he asked students to consider whether or not a percentage would improve the following statements from [1]:

- "Between October 1988 and October 1989, the Washington Post alone ran 1,565 stories about the 'drug scourge.' " (pg. 53)
- "In 1972, fewer than 350,000 people were being held in prisons and jails nationwide, compared with more than 2 million people today." (pg. 8)

This discussion requires follow-up on ethical matters, such as how to address a case in which absolute figures and relative figures tell different stories.

Prior to reading the book, the students visited the Jim Crow Museum on the campus of Ferris State University [10]. The purpose of the museum visit was to humanize Alexander's arguments and the data they would examine. The museum is also used to facilitate a follow-up conversation on the difference between quantitative and categorical variables.

After the class finished the book, students came up with questions about contemporary race relations that could be answered with data, and they conducted systematic research online to find answers. Most of the time, students worked with raw data. For example, one student examined frequency counts of tweets displaying certain characteristics right after the movie Black Panther was released, At the conclusion, students presented their findings in a public poster session. See [9] for an early iteration.

5.2.4 Additional topics in social justice for undergraduate research

We described some of our own work above. We also offer a few more potential topics to consider organized around primary data sources:

1. Data Source: Survey of Consumer Finances [35]
   (a) Measure and compare economic inequality over time
   (b) Design compensation plans such as reparations for stolen labor or land
2. Data Sources: Migration Policy Institute [23] and Department of Homeland Security [34]
   (a) Measure the extent of human trafficking
   (b) Analyze immigration and refugee data
3. Data Source: United States Census Bureau [33]
   (a) Design a fair district map of a state, because Gerrymandering aims to create "safe" seats and most often creates segregated districts [30].
   (b) Analyze and compare geometric properties such as measures of compactness of district regions.
4. Data Sources: United States Geological Survey Geometric Information Systems [36]
   (a) Make comparative models of short-term care clinics in various cities.

   (b) Design a network of homeless shelters and temporary housing

5. Data Source: National Center for Education Statistics [12]
   (a) Access to higher education and student loans. Student debt impacts the population unequally, for example, people of color are much more likely to have education debt.[4]

6. Date Source: Johns Hopkins COVID-19 Dashboard[18] and Centers for Disease Control Covid Data Tracker [11]
   (a) Inequality in COVID cases among the minority groups and border states
   (b) High COVID incidence rate among front-line workers and those who are from racial and ethnic minorities
   (c) Unemployment and evictions due to COVID-19
   (d) Managing flexible work schedules and work from home to make it possible for people with family obligations to work full-time. During the pandemic, over 2.2 million women left the workforce between February and October of 2020. [8]

Finally, we wish to note that examining social justice issues within the mathematical community itself could form the basis for undergraduate research projects, which we encourage and hope to inspire.

5.3 How integrating social justice impacts the undergraduate research process

While much of the undergraduate research in mathematics processes transfers well to the integration of social justice, there are a few matters that require some care. The term “social justice” and associated topics may raise political issues that you should address with your students. The work may involve a local community, discussions of sensitive issues, and may require attention to ethical considerations.

5.3.1 Preparing your students

Whether the research is done in a class or as a one-on-one project, students will have preconceived political positions and notions of what social justice is. Some discussion of how social justice is covered in news and social media may be helpful ahead of the work. It is also useful to spend some time developing a common understanding of social justice (possibly using some of the sources cited above), and carefully defining the goals of the research.

On the other hand, try to avoid making assumptions about what your students will bring to the table. When Piercey first used the New Jim Crow project described above, it was with a class evenly split between Criminal Justice students and Social Work students. He expected that the Criminal Justice students would be more pro-police than the Social Work students, and actually hoped to use the different perspectives to generate good discussion. The stereotypes proved incorrect, and all of the students were open to the argument made by the author, eventually finding the confidence to make specific data-based critiques. The point here is that your students’ perspectives may surprise you and, while some caution is called for when introducing a social-justice oriented project, concerns about student reception should not prevent you from running such a project in the first place.

5.3.2 The community as a source for topics

While finding undergraduate mathematics research topics is addressed elsewhere in this book, for social justice issues you may find the community a valuable resource. Cohen’s work through PIC mathematics described above engaged his class with the Los Alamos National Research Lab as well as the Chicago Field Museum. If you take the time to visit with community organizers, community leaders, and local non-profits, you are likely to find potential community problems that your students can work on. The key is determine what data is available and needed to address the problem. For example, a community organization may have problems related to uncertainty when conducting a needs assessment, and probability theory may be very useful in helping them. The value you add to the project is connecting the mathematics to the community need.

Your campus may have resources to help you. There may even be existing community-based university projects that you can get your students involved in. Contact your institution's office for community engagement or service learning to find out what your campus is already doing.

5.3.3 Being responsive to contexts

Work with social justice topics often involves sensitive issues, such as race relations, poverty, health, or issues related to sex and marriage choices. Sensitivity can come from different sources. One source is experience. For example, a student may be triggered while studying structural racism in policing if they had a police officer in their family who was killed in the line of duty or knew someone who was killed by a police officer. Another source can be the student's political views connected to the research topic. A student who believes that a federal medicare-for-all program will solve our health care problems may have trouble accepting their own work if their model indicates that the additional taxes necessary to support medicare-for-all are larger than the premiums currently paid for health insurance.

It is helpful to acknowledge these reactions before proceeding. They are feelings, and we have found that feelings need to be validated before they can be addressed cognitively.

Once acknowledged, pivot the student toward accepting that the goal of the research is to follow the evidence wherever it may lead. Our own experience is limited, and our prior positions can grow and evolve. This is an opportunity to promote the benefit of research, to amalgamate multiple experiences and perspectives, and to draw conclusions based on evidence, whether asking about the existence of a problem or a potential solution.

We should prepare for sensitive conversations. The best way to do this is to lay out expectations and ground rules in advance. If conducting one-on-one research, have a conversation about the underlying social justice issue to prepare students for the road ahead. With class-based projects, have teams establish norms or have the entire class establish norms. These norms can include agreements to "support statements with evidence" and to "listen to understand, rather than respond." Find ways to reinforce these norms, such as having a student assigned each session to enforce the norms (as a rotating assignment) or having class members reflect at the end of each class period on how well they adhered to the norms.

5.3.4 Ethical considerations

As with all research, issues of good scholarship and plagiarism apply. Social justice projects deal with complex social issues, and as such it is important to consider ethical frameworks for responsible research.

Part of the ethical framework involves treatment of human subjects. Confidentiality and privacy needs to be handled appropriately, and data storage has to be carefully addressed. To handle these matters, refer to your institution's institutional review board.

The other part of the ethical framework consists of considering the social consequences of the research, especially unintended consequences. For example, if designing an algorithm to identify a location for a new Amnesty International office in order to maximize its ability to benefit refugees from a crisis, could the algorithm be abused by an organization with a different agenda? If so, how could you adjust either the work product, ownership of the results, or the dissemination of the results in order to reduce this risk?

Ethical challenges, particularly concerning social responsibility, are teachable moments for students. Along with the research, consider having students read about ethics, mathematics, and social responsibility and have regular conversations about those readings. If this is new to you, this is a great opportunity to model lifelong learning. Arguably, ethics and social justice should be part of all mathematical work, as the Cambridge Ethics in Mathematics Society points out [7].

5.3.5 Student feedback

Students in our courses have generally responded positively to the experiences we have provided. They appreciated applying mathematics to solve problems in their communities. Here are a few comments that are typical of the feedback we have received:

- “Our group loved that we had the opportunity to help the museum with their project to have ordinary citizens help with their research.”

- “One thing that I discovered within the project was the impact citizen scientists can have on discovery. It was amazing to see the accuracy of the public in different classifying tasks.”

- ”I have learned how to help my neighbors when they are in trouble.”

- ”I learned that percentages are astonishing proof to change someone’s opinion. I learned that math can be included in social issues like racism, when I thought math was meant for people like engineers. I learned to find important data and exclude data that did not substantially provide good evidence. I learned how to verbally interpret numerical data and discover the true meaning behind it.”

- ”[c]ritical thinking is an under appreciated art. There is a fine line between propaganda and data, only descerning the validity of its message serves as the barrier. Proper math classes can help refine a student’s capability of analyzing data that is constantly surging around them.”

5.4 Conclusion

There is growing awareness of a significant intersection between social justice and mathematics. One side of this intersection concerns integrating social justice into the mathematics curriculum. Karaali and Khadjavi [19] offer multiple modules applying mathematics to issues of social justice, where the mathematics used ranges from calculus to operations research to mathematical interest theory. Special issues/collections of journals such as *Problems, Resources, and Issues in Mathematics Undergraduate Studies* (PRIMUS) [5] and *Numeracy* [15] have been dedicated to social-justice oriented curriculum.

The other side of this intersection is whether the mathematics profession conducts itself in a way to support social justice within the community. In effect, we are asking whether we act as a community according to the values we impart when we teach social justice issues. Lack of diversity within the mathematical community is still glaring, with women occupying 31% of all full-time faculty and only 17% of tenured doctoral faculty in 2017 [2]. The number of full-time faculty positions held by Black, Hispanic, and Indigenous people mathematicians is even lower. Even though we have a long road to achieving true equity in mathematics community, Talithia Williams invites us to “challenge the unacceptable” [38] and proposes faculty actively work on changing their perspective and become intentional about cultivating a growth mindset in students [39].

Whether directing undergraduate mathematics research toward social justice within the mathematics profession, within the local community in which the campus sits, or in broader society, this work can be very powerful for students. Students begin to view themselves as both mathematicians and agents of change, realizing the transformative potential in our teaching. For students from backgrounds that are marginalized within the mathematical community, the experience of mathematical research can itself be seen as social justice. Using that research to address social justice makes the work relevant and important, and fits well within a culturally-relevant pedagogy framework (see [31]). In addition, teaching mathematics from a social justice perspective helps change students’ orientation toward mathematics, and thus contributes to increasing diversity within the mathematical science [14].

Beyond the traditional challenges of facilitating undergraduate research, social justice involves difficult topics and requires an open-mind. The problems may not be solvable. Data may be difficult or impossible to obtain. Many of these topics are subject to partisan disagreement.

These challenges create deeper awareness of the problems and small results can be celebrated. The biggest factor is that students can be inspired by the idea of using math for something meaningful to their life and how they see themselves as a contributor. A significant outcome of liberal arts education is the ability to critically assess complicated contemporary problems using evidence and systematic inquiry. Questions of social justice are complex. If students learn to think about them and discuss them using quantitative evidence and analysis, and even shift their position when appropriate, we will have contributed to improving our national discourse.

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6

Research Experiences in the Statistics Living Learning Community

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6.1 The Statistics Living Learning Community

6.1.1 Overview

Mark Daniel Ward is the inaugural Director of The Data Mine, a unique 1000+ student living learning community at Purdue University. The focus is on enabling students to learn tangible data science skills and knowledge across all domain areas. During 2014–19, Ward was Director of the Statistics Living Learning Community (STAT-LLC), a $1.5-million National Science Foundation initiative that enabled 100 sophomore undergraduate students to pursue 12 months of research with a faculty member. In this book chapter, nine of the STAT-LLC alumni join Ward in reflecting on the impact of the STAT-LLC on their future careers. This collection of reflections is emblematic of the student excitement for data science coursework at the undergraduate level. The reflections also speak to the need for data science skills that support research experiences as well as a wide range of STEM-oriented careers after college.

Undergraduate sophomores from any major in Purdue University were able to participate in the STAT-LLC. No background was required, except for the calculus prerequisite that was necessary for the probability theory coursework. The admissions criteria was holistic. Students were selected because of the potential of the program to make a positive impact on their studies, and because of their desire to live and learn in a community of their peers. In all but one year, at least 50% of the students were women. During the last year of the STAT-LLC, 70% of the students were women, 20% were Black women, and one student was Deaf. The communal aspect of this learning community, and the support that the peers provided for each other, was a hallmark of the experience. To better understand the women student experience in the sciences, we heartily recommend [16, 22].

The STAT-LLC was created to address the "sophomore slump" and similar barriers to achievement in the murky middle of the undergraduate experience [5, 17, 19, 21, 23, 24, 28]. First year undergraduate students (generally) can participate in a broad range of programs, designed to enable a successful transition from high school to college. Research with faculty in the mathematical sciences does not usually occur, however, until the junior or senior year. As a result, sophomore undergraduate students often fall through the cracks. There are not many programs specifically designed for sophomores. On the other hand, sophomore undergraduate students are typically making the transition from introductory, general studies courses into the main part of their major programs of study. At this formative point in their studies, students can benefit from additional layers of mentoring, from faculty, staff, and peer mentors.

For context, the STAT-LLC evolved into a new program called The Data Mine in 2018, growing from twenty students per year to 100 students per year. The program then expanded to 600 students per year in 2019. Since that time, we have enabled graduate students to join the program as well, and we have more than 1000 students enrolled at the start of the fall 2022 semester. Similarly, our Corporate Partners program has grown from 1 company in 2018, to approximately 50 companies in fall 2022. These companies provide research experiences arising in industry, guided directly by mentors from the companies. Ward now reports to the Vice Provost for Teaching and Learning in his role as Director of The Data Mine. He is also the Interim Director for Purdue University's Integrated Data Science Initiative (and reports directly to the Provost in this role). The Lilly Endowment initiative "Charting the Future for Indiana's Colleges and Universities" allocates $10 million to The Data Mine for a 5-year grant (2021–2026), which will enable a statewide expansion of the program to universities of all sizes throughout Indiana. (Jay Akridge, the Provost at Purdue University, is the PI on this statewide grant initiative.) The Data Mine has also just begun a new NSF-funded initiative in the Harnessing the Data Revolution (HDR): Data Science Corps (DSC) program. This grant will provide $1.5 million of funding, for 300 students to receive research stipends. All of the students in The Data Mine's NSF HDR DSC grant will be at Historically Black Colleges and Universities, Hispanic Serving Institutions, Minority Serving Institutions, Tribal Colleges and Universities, as well as universities serving Deaf students. More than 120 universities contributed letters of collaboration for this grant, which is coordinated by the American Statistical Association. (Ward is the PI.)

6.1.2 Curriculum

To support their preparation for research, the STAT-LLC students took several courses as a cohort. The students took a probability course and a new data science course during the fall semester, both taught by Ward, using an active learning methodology. In the spring, they took a mathematical statistics course. The probability and statistics pair of courses constitute the heart of the statistics major and minor programs. The data science course was created as an all-new elective but has now evolved into a permanent course in Purdue's Data Science major.

Throughout the 9-month academic year, the students also had a professional development seminar in which they met faculty at all levels and from all disciplines, research scientists, the Provost, diversity and inclusion staff, graduate students, actuaries, computational scientists, and a wide range of visitors, alumni, etc. The students lived in the same floor of Hillenbrand residence hall.

In the probability theory course, there were forty students, including the twenty from the STAT-LLC, and twenty other students. The students were put into ten randomly generated groups of four students each. The assignments of students to groups was refreshed every day. In this way, the students were able to readily get to know all of their peers during the semester. The students did not have assigned homework. Instead, they had access to 237 videos on probability (created by Ward), a complete set of notes, and hundreds of problems, with solutions, organized by topic. During class, they would receive four problems (usually with several subparts) to solve during the 50 minute class period. Each team was responsible for working toward a written set of solutions for the set of problems. Most of the students were not mathematics majors, so we did not use the word "proof" in the course. Nonetheless, students naturally learned to justify their solutions verbally and in writing, at a level that made sense to their peers. (Ultimately, this is the nature of a proofs-based course anyway.) Ward would walk around the room throughout the class session, talking to students throughout the class time. No lectures were given.

In the data science course, the twenty students in the STAT-LLC and one other invited peer (usually somehow affiliated with the STAT-LLC) would split into seven groups of three students on each project. There were a total of ten projects throughout the semester, each lasting for approximately one and a half weeks. The students were responsible for producing individual solutions to the projects, rather than having one group set of solutions. (This was possible because of the week-long nature of these projects, as compared with the daily nature of the probability course. This style ensured that each student had a good grasp of the data science content before moving on to the next project and the next group/team dynamic.) By meeting two new students in each team, each student worked with her/his 20 peers on exactly one project during the semester. The plan for the overall structure of the 10 team assignments is an interesting problem. Ward relied on Kirkman triple systems to accomplish the plan for the semester placements of students in teams.

The topics covered in the data science course include R, SQL, UNIX, bash, XML scraping and parsing, data visualization, etc. As the STAT-LLC ramped down and The Data Mine ramped up, we added Python to this list of topics. All credit for this concept of blending these technologies goes go Deborah Nolan and Duncan Temple Lang. Ward attended their pioneering workshops in 2008 and 2009 at UC-Berkeley, and started offering his own version of data science courses in 2009. A fundamental idea for the learning methodology is that students are much more likely to understand variables, vectors, vectorized functions, scripts, visualizations, etc., if they apply these concepts directly in data analysis projects. For many students, this is their first immersion into data science. Many have never programmed in any language before. For the ones who do have programming experience, it is often in a high school Java course, which is somewhat orthogonal to the skills learned in our courses. For almost all students, this is their first experience working remotely on a High Performance Computing UNIX cluster, rather than performing the analysis directly on their own computers. It is also usually their first experience working with data that is too large to see with their eyeballs and too large to open in Excel.

The students also learn about many aspects of the data analysis cycle, including how to scrape data from the internet, pre-processing data using scripts, learning libraries in Python and R (which are open source and community developed), conducting exploratory data analysis, performing data transformations, working with databases, building visualizations, creating statistical models, etc. Data munging and data wrangling are recognized as key parts of data analysis, which are frequently not (yet) part of the data science curriculum. Anecdotally speaking, data scientists frequently learn these skills during their first few years on the job in industry or in scientific computing. Students who can learn these skills while still in college are highly sought after, in the job market.

One student made a memorable observation during the 2016–17 academic year, emphasizing that, among the skills learned in the STAT-LLC, *the most important thing was to learn how to learn*. The necessary data analysis skills, tools, languages, etc., will continue to evolve. In contrast, computational thinking (which Jeannette Wing so insightfully described [29]) remains a fundamentally important, underlying framework for approaching all data science problems. The need to solve problems and use computational tools to work on massive data analysis tasks will never go away. Similarly, the benefits of having students participate in living, learning communities are well-known and timeless.

6.1.3 Research culture

In REU programs, students are usually working with a small team of faculty members who work in a dedicated manner as mentors for a whole group of students. In contrast, in the STAT-LLC, each of the twenty students per year was able to pick their faculty mentor from anywhere in the university. As a result, the students were able to see the highs and lows, and pros and cons, of the research environments in which their peers were working. Inevitably, some of the students were more productive than other students, and some of the faculty fostered a more welcoming research culture than others. Throughout the five years of the program, however, creating a welcoming environment for the students in their first undergraduate research experience was the primary goal. The students supported each other and Ward was a constant supporter for their professional development and growth. Even after their graduation, Ward stays in touch with the students and endeavors to continue to support them.

Concerning the research culture that was cultivated in the STAT-LLC: Our overall belief was that, if students successfully felt welcomed and supported throughout their sophomore year, then they were more likely to self-identify as scientists, engineers, etc. Students who felt that they belonged in STEM careers with significant research components would be more likely to consider graduate studies and/or careers in which they would utilize their data science and research skills, first learned during the STAT-LLC.

In our supportive peer environment, the students worked on research with a faculty mentor, from August at the start of their sophomore year, through August at the start of their junior year. Starting in 2017, we took each group of students to the Rose-Hulman Undergraduate Mathematics Conference each April. We encouraged the students to present the preliminary results of their research at this point in their twelve month research experience (between the end of the academic year and the start of the summer), in an environment where most of the attendees were the students' peers.

As with a typical REU, the summer served as an immersion into the research phase of the grant activities. (The summer portion of the 12 month experience comes at the end.) The STAT-LLC students had the advantage of already being friends with each other before the summer started. They also already had nine months of academic year research behind them, before starting their intensive summers of research. This bolstered their comfort levels and their preparation for the rigors of a full summer of research.

The STAT-LLC students authored at least 177 journal papers and conference talks with their mentors. For a full list, please see: `datamine.purdue.edu/research/publications.html` The various students performed research projects in many domains. For a sample of the list of such publications produced from the research, please see: [1, 3, 4, 6, 7, 11, 14, 15, 20, 26].

6.2 Reflections by past students

6.2.1 Reflections by Maggie Betz

I chose Purdue because of the strong actuarial science program and the potential of participating in the STAT-LLC. I entered Purdue set on being an actuary. It seemed like the most straightforward path to using statistics in a career: majoring in actuarial science, taking (and passing) the exams, and getting a job as an actuary. Beyond actuarial science, I didn't know how to apply a degree in statistics to industry. I was accepted into the STAT-LLC during the fall of my freshman year. One afternoon, I stopped by Dr. Ward's office and he had just gotten off a call with the Chief Scientific Officer of CareDx, a biotechnology company in San Francisco, California. They were exploring the possibility of a student conducting research with CareDx in partnership with the STAT LLC. Long story short, I worked remotely with CareDx for two years and in person for one summer.

The experience opened my eyes to possibilities of statistics in industry. During the fall of my sophomore year, I was studying for the probability (P) actuarial exam and working with CareDx. I felt like I couldn't keep up with both and I had to make a decision. I was more interested in a career in statistics and data science, but actuarial science was the "safe" choice. Ultimately, I chose to stop studying for the exam and fully focus on research with CareDx. I presented my work at the Joint Statistical Meetings. I graduated in three years in actuarial science and applied statistics and planned to spend my career in the area of biostatistics. After graduating from Purdue, I interned at Eli Lilly and Co and then started graduate school. The computing skills that I learned helped me stand out when I applied to work at Eli Lilly. Those skills were probably the only reason they offered me a full-time offer so far in advance. I graduated with my Masters in biostatistics, but instead of returning to the healthcare field, I was offered a unique opportunity to return to Purdue for a full-time position with The Data Mine.

Over the five-year course of the STAT-LLC, with 100 students total, 99 worked with researchers at Purdue and I was the only one to work with a company. I was the first Corporate Partners student and now I've come full circle as the Managing Director of Corporate Partnerships, working to offer students the same life changing experience I had as an undergraduate.

6.2.2 Reflections by Peter Boyd

As a sophomore keen on a career in the actuarial field, I had joined the Statistics Living Learning Community with hopes of strengthening my statistical background (as well as my CV), but I had not anticipated the vast, positive developments that would blossom as a result. During my time in the STAT-LLC, I was quickly exposed to the world of statistics and data science. Learning tools such as R and SQL while attending seminars given by statisticians in both industry and academia allowed me to see the intrigue and potential in extensions of statistics beyond the actuarial route. By the end of my time in the STAT-LLC, I found my academic interests and passions shifting and aligning more with my non-academic interests.

Marketing my newly honed statistical skills, I obtained internships with the National Park Service while nurturing my growing interest in environmental and spatial statistics. While the LLC undoubtedly helped my peers that went directly to careers after undergraduate, I found that the LLC tailored my mindset and professional pursuits for research endeavors offered by PhD programs. The dynamic and rigorous foundation of the LLC eased my transition to graduate coursework and research pursuits, having already gone through some of the research growing pains and advanced coursework.

6.2.3 Reflections by Emily Damone

As a current Biostatistics PhD student, I owe a large portion of my career trajectory to the Statistics LLC. To begin my college career, the possibility of becoming a member of the LLC was a deciding factor in attending Purdue University in the first place—an academic space that allowed me to grow as a person and a scientist.

The coursework I took during my sophomore year was advanced and built a strong foundation for my statistical knowledge. I fondly remember sitting in the commons area of our floor completing homework, collaborating on ideas, and memorizing distributions and theorems. This level of interaction with peers closely mirrors how I interact with my PhD cohort and built my comfort and confidence sharing approaches.

The network of support I formed during my year in the STAT-LLC connected me to the project I worked on my final two years at Purdue which solidified my passion for work in clinical trials and ultimately led me to the program I am currently attending at UNC.

6.2.4 Reflections by Christina DeSantiago

My research project with Dr. Lisa Goffman, professor of Speech, Language & Hearing Sciences, studied children with specific language impairment to understand how visual and motor cues impact word learning. The software the lab used for handling and analyzing their data were not the same ones we were learning to use in the STAT-LLC data science course. Despite this, I was still able to apply concepts from the data science course, such as cleaning and preparing data, in my work at the research lab. Also, as a sophomore I had not yet learned about advanced statistical methods such as analysis of variance with more than one factor and repeated measures. However, I was able to connect Dr. Goffman's lab with Purdue's Statistical Consulting Service in order to provide guidance for appropriate analysis. In a joint effort with the Consulting Service, I learned how to write up procedures for statistical analysis and explain statistical findings.

After graduating from Purdue, I attended Southern Methodist University in Dallas, TX for a graduate program: Master of Science in Applied Statistics and Data Analytics (MASDA). The skills I learned through my coursework in the STAT-LLC were very beneficial here. In the MASDA program, I took several courses in mathematical and experimental statistics, which covered concepts I previously learned from the probability theory and statistical theory courses as a sophomore. Since I already had a solid foundation in these topics, this made it easier for me to grasp them at a more in-depth level. The data science course at Purdue in which I learned how to use R also gave me an advantage as a graduate student. My background with the program allowed me to become a teaching assistant starting my first semester, actively guiding undergraduate students on how to use R programming for statistical analysis.

I have been in my current position as a data analyst at GM Financial for one year and frequently rely on the skills first gained from the STAT-LLC.

6.2.5 Reflections by Kent Gauen

The team aspect of taking the same coursework with similar research goals in data science gave me a safe, encouraging space to explore the content related to the research area of my interest; developing algorithms for machine learning models focusing on applications on computer vision. From junior to senior year, I used my experience from sophomore year to become a leader in a large, undergraduate research group at Purdue called CAM2. During my time on this team, I traveled to San Diego, CA; Montreal, Canada; Tokyo, Japan; Singapore; and Seoul, South Korea. My interest in the development of applied statistical methods lead me to become a co-advised PhD Student at Purdue University with my primary adviser, Dr. Stanley Chan, in Electrical and Computer Engineering and my co-adviser, Dr. Vinayak Rao, in the Statistics Department. Currently, I still study algorithms for machine learning models focusing on continuous-time state space models and noisy label problems for computer vision.

6.2.6 Reflections by Katie Lothrop

I have seen a large gap in engineers' understanding of statistics today. Mechanical Engineering students are usually not required to take a statistics class, and as a result many test engineers in the medical device industry only have a rudimentary understanding of statistics. When I was interviewing with DEKA R&D, I learned that they had a big need for engineers that understood how to perform ANOVA analysis. While working at DEKA R&D, I have used the statistics that I learned during the STAT-LLC program to perform ANOVA analysis and create DOEs for multi-factor testing. I have also taught several lead system engineers on the project a better understanding of what a statistical interaction is. If I hadn't been a part of the STAT-LLC and taken statistics classes in this area, I would have struggled to understand ANOVA analysis, main factors and interactions. I led a Python programming course at DEKA R&D where we taught Test Engineers how to parse data files and perform statistical analyses of data files generated during testing. These days, companies want to hire problem-solvers more than any other type of "job," so the more skills and problems engineers have solved, the better off they will be.

6.2.7 Reflections by Mikaela Meyer

I attribute much of my success as an undergraduate and my desire to get a PhD in Statistics to my experiences in the Statistics Living Learning Community (LLC) during my sophomore year. Because of our flipped courses in the learning community as well as my research and social experiences in the LLC, I realized that I could have a successful career in applied statistics research. The flipped class experience gave us ownership of our learning experiences; we could choose to put in as much or as little time as we wished to get whatever we wanted to out of the course. This ownership helped me realize how much I enjoyed learning about these topics and that I was capable of teaching myself new topics with support from Dr. Ward and my peers. Also, I can still remember some of the problems and projects we worked on in these courses, which to me is a sign of how effectively I actually learned the material versus simply memorizing topics.

Outside of the coursework, being exposed to applied statistics research and in particular to research at the intersection of statistics and public policy early in my undergraduate career certainly helped me see how much I enjoyed research. The project I worked on for the majority of that year involved using Bayesian methodologies to understand which factors were impacting Lake Chad's volume the most, with the idea being that we could communicate our findings to policy-makers in the region to inform them of what actions they should or should not take to preserve the Lake. At the time that I started the LLC, I did not know how I'd be able to combine my interests in statistics and public policy into a future career. This project introduced me to this research intersection early in my undergraduate career, and I enjoyed it so much that I sought other research projects where I could use statistics to answer public policy questions. The summation of these experiences, as well as our exposure to graduate students and faculty through the LLC's professional seminar series, encouraged me to look into Statistics PhD programs, which I had not previously considered. The LLC helped me realize that I not only enjoyed research, but that I could be successful as a future academic. I truly believe without this mentorship, I would not have felt that I was qualified enough for graduate school and likely would not have applied. It is also because of my support network that I am interested in mentoring undergraduates, especially women and underrepresented minorities, in statistics because I recognize how critical their encouragement was in developing my confidence in my abilities to be a researcher.

Now, I'm a PhD student in Carnegie Mellon University's joint Statistics and Public Policy program. As a student in this program, I am completing all of the PhD requirements for both the Department of Statistics & Data Science and Heinz College of Information Systems and Public Policy. This interdisciplinary program is a perfect fit for my interests; I am learning more about how to use statistics to inform public policy.

6.2.8 Reflections by Kristen Mori

Two years into my career as a baseball analyst, I've lost track of how many times I've answered the question on every student's mind during lecture: "When will I ever use this?" Moneyball fails to encapsulate just how often probability distributions come up in our line of work. An expertise of R well equipped me for a career working with baseball data, and mine began in a data science course available to me due to the STAT-LLC at Purdue. To this day, whenever a student asks me for advice on getting into baseball, I tell them to "learn to program well" and to pursue some of the types of projects that I was lucky enough to be exposed to via this class.

While I thought I chose a career in industry rather than research, I landed in a Research and Development department for the Houston Astros, and unsurprisingly, many of the components of a seemingly-unrelated undergraduate research project in soundscape ecology still surface in baseball. The truly irreplaceable benefit was the experience I gained in problem-solving. Research, whether ecological or athletic, is as much about how you tackle things you don't know as it is about what you already know. I faced challenges in data transfer, data cleaning, formulating research questions, creating plans to answer said questions, and more, and through peer support, mentor guidance, and my own volition, I persevered and learned then so that those challenges aren't as challenging when they arise in my work today.

6.2.9 Reflections by Ashley Peterson

When I look back on my undergraduate experience, I am grateful for the STAT-LLC for introducing me to the field of data science and analytics. Prior to the STAT-LLC courses and research project, I had limited exposure to applied statistics, analytics, and data science and had yet to determine a career from my mathematics degree. Participating in the STAT-LLC sparked my interest in data science and developed the fundamental skills I use today as a Data Science Consultant. When I reflect on my STAT-LLC research project, I see the parallels with my current consulting engagements. Research taught me how to apply analytics to solve problems, work with an interdisciplinary team, and communicate results which are vital consulting skills. Professor Friedman leveraged a large survey dataset for his work but was limited to using the surveys with complete information, greatly reducing the sample size. My work focused on solving this challenge and gaining information from the incomplete surveys. Throughout the project, I refined my communication skills and understood the importance of effectively explaining the impact and application of my work (rather than explaining the weeds of our code). In addition to my verbal communication, writing a research paper strengthened my ability to clearly communicate and share results. [20] The combination of our statistics courses with our Introduction to Data Science showed the wide range of skills needed to become a data scientist. This led to continuing my education with a Master of Science in Analytics at NC State.

6.3 Closing remarks

Ward has already written at length about other aspects of the STAT-LLC, but this book chapter is the first time that he is writing directly about the impact of the data science training on students' research experiences and future careers. For other discussions about aspects of the STAT-LLC, see these discussions on the role of mentoring [26, 27]; importance of learning communities [25]; curriculum development [9]; case studies about data science curricular developments nationwide [12]; the need for data science curriculum for students from all backgrounds [2, 10]; some aspects on student learning about data wrangling [8]; the assessment of the students in the STAT-LLC experience [18]; and an overview of the first year making the transition to The Data Mine [13].

Ward dedicates this paper to the 100 students in the Statistics Living Learning Community. Working with each of them has been the greatest joy in my academic life.

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Part II

Diversity and Inclusion

Introduction
Diversity and Inclusion

When UR in mathematics was first started, the participating undergraduate students were typically white men, and students who were likely already going to graduate school in mathematics. Later, the demographics of students doing UR in mathematics began to broaden and programs were created that provided opportunities for a diverse group of students. These included women, students who did not have a 4.0 GPA in math, students from underrepresented ethnic and racial groups, first-generation university students, low-income students, and students attending community colleges.

The importance of UR and its positive impact on students is now a well accepted fact. The benefits for students include improvements in problem solving, critical thinking, independent thinking, creativity, and communication skills ([1], [2], [3], [4]). Studies have also shown that for students from underrepresented groups, UR can result in improvements in students' grades, retention rates, and motivation to pursue further education and succeed in graduate school ([5], [6], [7], [8], [9]). It should therefore be a prerogative to offer undergraduate research opportunities to as many students as possible. No group should be excluded and the research opportunities should be available to as diverse a body of students as possible. In this section, we explore UR programs that promote diversity and inclusion.

Leonard and Wyels in Chapter 7 write about the need for a cultural shift in the math department and the math community as a whole to create an inclusive environment for our students. Many organizations supporting mathematicians from marginalized populations already exist and many departments now engage in various efforts for bringing equity and inclusion to their programs. However, there is still a need for a much wider support for these programs so that the "counterculture" becomes the mainstream culture as it should have always been.

Jacob and Obiedat in Chapter 8 write about an important issue of inclusion of students with disabilities; in particular research with deaf and hard of hearing students. While many aspects of the research may seem universal, Jacob and Obiedat do a great job outlining subtle issues that many faculty and students without disabilities take for granted. Their chapter can serve as an excellent resource with specific and concrete suggestions for any faculty who wants to include students with disabilities in their research projects.

Clahane et al. in Chapter 9 make the case that more faculty involvement, administrative support, and student awareness can and should be encouraged in order to build and sustain undergraduate math research activities at community college. This will assure that students at community colleges have equal access to research opportunities as their university counterparts. They also describe a rare REU between Arizona State University and Maricopa County Community College District.

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7

Creating a Counterculture: The CSU Channel Islands Mathematics REU

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7.1 Context

Counterculture: A culture within a larger culture that deliberately challenges or rejects the dominant culture's behaviors, beliefs, lifestyle, norms, and values. [3]

7.1.1 The impact of culture on inclusion in mathematics

For decades, studies have documented ongoing statistical inequities in degree completion and professional attainment in mathematics and STEM generally. Research dispels explanatory myths such as lack of desire among members of marginalized groups to study STEM while no reputable evidence supports myths regarding lack of capability or talent (see [8]). Bill Velez hypothesizes that the culture of college and university mathematics departments is itself a hindrance in attracting and retaining HUGs students [17]. The Mathematical Association of America's (MAA's) Committee on Minority Participation in Mathematics reminds us that "Mathematics instruction and research do not happen in a vacuum" and indeed that "We cannot be effective mathematics teachers if we think that students all enter the classroom with the same sense of value and safety." [6] The Manifesto of the MAA Instructional Practices Guide reinforces "Inequity exists in many facets of our society, including the teaching and learning of mathematics" and challenges mathematicians to "... disseminate mathematical knowledge in ways that increase individuals' access to the opportunities that come with mathematical understanding." [1].

A side note: Organizations and researchers use various acronyms and descriptors: "Under-Represented Minorities" (URM—e.g., National Science Foundation); "Historically Underserved/ Underrepresented Groups" (HUGs); marginalized populations; Black, Indigenous, and People of Color (BIPOC), among others. We recognize that, not only is the language around these issues not static, any label over-generalizes and masks the true diversity of thought, culture and history amongst large groupings of people (e.g., "Latinx" spans continents, languages, race and ethnicities, and myriad indigenous and colonizer cultures, etc.). In this article we will use a variety of terms while asking the reader to maintain mental flexibility when encountering any such term.

We write during the most recent period of widespread protests against racial injustice in the U.S., one in which many commenters note a growing awareness from those who have benefited from systemic racism of its historical and current impacts on individuals, public health, the economy, and more. The mathematical community represents a microcosm of U.S. society. Ignorance or denial of the effects of the dominant culture on mathematical culture(s) continues to affect negatively doing mathematics, who becomes mathematicians, and who thrives as mathematicians. In spite of the evidence, beliefs that mathematics is culture-free, color-blind - both problematic constructs - and effectively a meritocracy persist among many in positions of influence and power within mathematics. Yet organizations supporting mathematicians from marginalized populations speak to the historical and current need for that support. Three examples:

- "Indeed, NAM's founding was a direct result of the marginalization of black people within the professional mathematics community, which then and now serves as a microcosm of the society in which we live... We recognize institutionalized racism as the root cause of the inequities that lead to the underrepresentation and marginalization of black and brown people in the mathematics community and in American society at large." The National Association of Mathematicians (NAM) is a non-profit professional organization in the mathematical sciences with membership open to all persons interested in the mission and purpose of NAM which are a) promoting excellence in the mathematical sciences and b) promoting the mathematical development of all underrepresented groups. The quote selected is from NAM's Statement on the Death of George Floyd. [11]
- "SACNAS encourages the STEM community to reflect on the ways in which we have contributed to the upholding of racial injustice and to be dedicated to removing the systemic barriers that have served to exclude persons of color from full participation in society and the scientific enterprise." According to the SACNAS Mission Statement: SACNAS is an inclusive organization dedicated to fostering the success of Chicanos/Hispanics and Native Americans, from college students to professionals, in attaining advanced degrees, careers, and positions of leadership in STEM. The quote here is from SACNAS Stands in Solidarity with Black Communities & Condemns Racial Injustice. [16]
- "AWM's programs not only support those who participate in them directly, but also *help influence the mathematics culture more generally*, so that young women entering the field today encounter an environment that is more nurturing than that of the 1970's and 1980's." (AWM is the Association of Women in Mathematics. This quote is from the AWM website "About Us" [2], with italics ours.)

Ongoing efforts seek to examine and change or eliminate cultural impediments to inclusivity in mathematics. Individually conceived and maintained initiatives are underway to make visible the contributions of mathematicians from marginalized populations (e.g., Lathisms [10], Mathematically Gifted and Black [14], and Indigenous Mathematicians [5]). Government funded programs promote systemic cultural change in STEM academic settings. As one long-running example, the National Science Foundation has funded the ADVANCE program since 2001, to encourage institutions of higher education as well as other STEM organizations "to address various aspects of STEM academic culture and institutional structure that may differentially affect women faculty and academic administrators" and has more recently included an emphasis on intersectionality.[13] Other government funded efforts include Hispanic-Serving Institution STEM grants, Historically Black Colleges and Universities grants, and Tribal Colleges and Universities grants. Some STEM professional organizations are similarly engaged in such efforts: the STEM Inclusion Study, funded by the National Science Foundation in 2015 and again in 2017, seeks to advance knowledge about potential mechanisms of disadvantage for underrepresented STEM professionals in the U.S.[4]. The faculty who conceived and directed the CSU Channel Islands Mathematics REU designed this program as a counterexample to culturally influenced perspectives on who does mathematics and how it is done. While readers may recognize some aspects of this REU are now practiced by a handful of other REUs – reflecting some movement of general mathematical and societal cultures—this REU was first conceived in 2007, when there was significantly less value attributed to the beliefs and practices discussed in the following sections.

7.1.2 Institutional context

CSU Channel Islands (CSUCI) opened its doors to students in Fall, 2002. As CSUCI matured and grew through the REU years of 2010 – 2017, it embraced more fully its role as a public university committed to providing quality access to higher education for its region. CSUCI became a Hispanic-Serving Institution (HSI) in 2010. Institutional commitment and successful grant writing and activities support an ongoing transformation of institutional capacity to better meet the needs of Hispanic and low-income students. Over the years, the CSUCI student body became majority

Hispanic and first-generation. The mathematics program developed a relationship with *Universidad Autónoma del Estado de Hidalgo* (UAEH), a state university in Mexico, that led to week-long visits of students and faculty between institutions and a longer-term exchange program. CSUCI faculty, led by a group that included the REU directors, worked to create the conditions for robust undergraduate research.

These factors supported the design and facilitation of a countercultural REU in many ways. Campus leaders supported themes of targeting students from underrepresented populations and recognized the transformative effects of undergraduate research. Two of the four pillars of the CSUCI mission revolve around international and multicultural education. The CSU system as a whole and our campus presidents have been vocal in their support of undocumented students. In short: we had moral support and a lack of political headwinds on campus to implementing our REU vision. Of course, as a young, public regional university, CSUCI has always been resource challenged, and this extended to the REU's ability to request help that might defray any of the costs of running the REU.

7.1.3 Faculty directors' values and intentions

The three faculty responsible for envisioning the CSUCI REU (Cindy Wyels, Kathryn Leonard and Geoff Buhl) wanted to offer the transformative effect of engaging in research to students who had mathematical potential but were "on the bubble" in terms of future opportunities. While a few programs were known for engaging students from non-traditional backgrounds, our sense was that most REUs admitted primarily students who attended well-resourced institutions and were well were on their way to graduate school—in other words, students who were already positioned for success within the traditional mathematical culture. The research experience essentially gave them an extra boost on the way. Conversations with faculty engaged in such REUs suggested that the experience was mostly about the mathematics, with publications used as the primary measure of success. We wanted to work with students who—without the REU as an intervention—would likely not be considered good candidates for graduate study in the mathematical sciences and who would not otherwise be ready to make informed choices about such study.

Identifying and recruiting students for whom participation in an REU could make a significant difference in their aspirational and educational outcomes was an initial goal. However, aiding students with non-traditional backgrounds in assimilating into the culture dominant in mathematics was insufficient to change a culture of which we were critical. Since we conceived the REU, we have become aware of work that effectively describes the difference we were seeking. Mathematics educator Dr. Rochelle Gutiérrez cites several researchers to conclude that "…the teaching and learning of mathematics continues to support an agenda of White supremacist capitalist patriarchy" [9]—admittedly, a charge that is difficult for mathematicians to evaluate dispassionately. Gutiérrez distinguishes between teaching students to "play the game"—that is, to succeed in mathematics as it is currently socially constructed in the U.S. and teaching them to "change the game"—to question what mathematics is done, by whom, and to what ends. In hindsight, we see our overall design of the CSUCI REU as an attempt to tweak the game, and to influence the next generation to see the game for what it is and consider changing it further.

7.2 The CSUCI Mathematics REU

7.2.1 Goals

The CSU Channel Islands REU supported 14–15 students in three research groups in 2010, 2011, and 2012; 18–19 students were engaged in four research groups in each of 2015 and 2017. The major goals of both iterations of this REU were:

1. to provide undergraduates with a rich experience of mathematical research;
2. to encourage students to consider graduate school in mathematics by providing information, exposure, and support;
3. to provide students with the skills necessary for success in mathematics, such as proof-writing, technical reading, oral and written communication, and scientific acculturation;
4. to target students with limited research opportunities, especially native Spanish speakers and those who would be the first in their families to earn college degrees; and

5. to build a heterogeneous community of faculty mentors and students committed to excellence and diversity in mathematics.

While goals like 1–3 are common to all REUs, our goals lay out our commitment to creating an REU that specifically sought and supported students from populations underrepresented in mathematics (4) and emphasized counteracting the dominant culture of research mathematics (5). Our proposals also included an informal goal: "We want all students to leave the program feeling excited and prepared to perform graduate-level mathematics in an academic or industrial setting."

7.2.2 Standard features

The CSUCI REU included activities common to most REUs. Participants received expository materials and journal articles related to their groups' topics, along with reading guides, prior to the REU. A major focus of the first week's "boot camp" consisted of students working with their research mentors to construct the mathematical knowledge required to begin their research projects. Mentors simultaneously worked to facilitate students' shift from class-based learning to the types of habits of mind needed for independent research. Beyond the first week, research groups met with their mentors—often for several hours daily—and also worked on their topics individually and in smaller groups. All conducted literature searches and read relevant journal articles. Some used computer experimentation to generate examples and ideas or implement algorithms; some created algorithms to test their models; others generated examples by hand. All groups focused on making and proving conjectures. Weekly presentations of work in progress helped students develop communication skills and solicit feedback from other participants and mentors. Other weekly events included seminars presented by speakers from local academic institutions and industry, weekly professional development workshops, group meals, and social excursions in the area. Towards the end of the 8-week REU, each research group prepared posters and final reports detailing their results. All students presented their final results in at least one national conference; several research groups published papers. Faculty continued to mentor students as students sought advice and letters of recommendation for post-graduation options and additional research or internship programs.

7.2.3 Unusual features

Culture is to humans as water is to fish. (Source unknown)

The CSUCI REU also included several unusual features to help us achieve goals 4 and 5. We intentionally recruited native Spanish speakers, first-generation students and other students from under-represented groups, using networks of academics known to support students from these demographics. Yet the REU was open to all applicants meeting our eligibility criteria, with the application process designed to elicit a sense of whether applicants were open to learning more about and committed to excellence and diversity in mathematics. We built in a partnership with *Universidad Autónoma del Estado de Hidalgo* (UAEH), a state university in Mexico, bringing a faculty mentor and one student from UAEH each year. The Mexican faculty mentor aptly demonstrated that mathematical talent and success could be found across borders and language groups, while also bringing international perspectives on research, mentoring and mathematics. The UAEH students were typically more advanced mathematically than their U.S. counterparts, again providing proof by example that Mexican heritage and mathematical accomplishment are perfectly compatible (contrary to messages common both close to home and throughout the country). The UAEH participants also reinforced the bilingualism sought and celebrated by the CSUCI REU. Indeed, we committed (in our NSF proposals) to having multiple Spanish-English bilingual faculty working with the REU each year (as mentors or directors): by 2017, three of our four mentors were native Spanish speakers. We included an undocumented student from CSUCI in the REU in 2010; in all subsequent years we intentionally welcomed applications from and supported one or two students lacking documentation. This aspect again provided implicit evidence of the wide distribution of mathematical talent and achievement across populations, countering common narratives. Additionally, these students had already developed skills, attitudes and habits to overcome many barriers to pursue their education to the point at which an REU was within reach. They invariably enriched the group experience through their contributions, not least of which were their work ethics and "can do" spirits. Finally, all of the UAEH and undocumented students were native Spanish speakers and first-generation students, in alignment with goal 4.

Our Early Career Faculty Mentoring program (serendipitously initiated in 2012; formally proposed for the 2015–2017 grant) engaged early career faculty in their first intensive research-mentoring experiences, with intentional guidance prior to and throughout the experience. We included four Early Career Mentors (ECMs)—one twice: all were women, three were native Spanish speakers, and two were first-generation college students themselves. This program thus brought additional diversity and bilingualism to our REU, while providing our student participants role models with backgrounds similar to many of the students' and further demonstrating the wide presence of mathematical ability and achievement. The ECMs benefited as the REU provided the structure, logistical arrangements, and resources so the ECMs could focus on developing skills as a research mentor. Meanwhile, we adapted a collaborative professional development structure within which the directors and more experienced faculty intentionally mentored the ECMs not only on how to mentor undergraduates in research projects, but in how and why one might challenge the cultural assumptions in mathematics, and how one could lead students to do the same. In doing so, our focus was to "retrain" the ECMs—and to continue our own retraining—to let go of many of the standard cultural assumptions absorbed in undergraduate and particularly in doctoral programs in mathematics. We found that the diversity in the number of years in the profession and the personal and academic backgrounds of the mentor group, together with the structure loosely adapted from [12], aided all faculty engaged in culturally reflective professional development.

Another unusual feature consisted of having research project options that were designed for early access and activity. In the 1980s and 90s a predominant opinion amongst mathematicians was that undergraduates were not ready to engage in research. Current REU eligibility criteria continue to reflect a preponderance of projects that requires a level of mathematical background that students with less privilege (e.g., no AP Calculus in high school) typically reach just before they graduate, thus leaving REUs out of reach for them. While the practice of incorporating research projects designed for earlier access is becoming more widespread among those wishing to broaden participation, it was less common in 2010 - 2017 (the years the CSUCI Math REU was active).

7.3 Building structure for working with humans

Research on effective teams demonstrates that teams work well when the humans understand each other, give each other equal space to speak, have comfort with experimentation and reflection on processes, and share a collaborative mindset [7]. While we were not aware of this research at the time, many of the structures we built were intended to develop these values and skills. Interestingly, many of these qualities are notably absent in the dominant mathematical culture—one that has historically celebrated lone geniuses rather than collaborative efforts.

In academia, teaching the hidden curriculum provides a foundation for intellectual safety that underpins all these characteristics of a good team. Many of our activities explicitly taught the hidden math curriculum, such as workshops on how to read a good math paper, how to write math, how to craft a good theorem, reviews of proof techniques, and what to do when you are stuck in the research process. Others specifically addressed aspects of effective teaming: conversations about what expectations students needed to feel safe being wrong in public (promoting comfort with experimentation), and activities about roles people play in groups and how those roles affect group outcomes (including the importance of equal speaking time). We implemented other activities designed to foster a collaborative mindset, such as solving fun puzzles requiring team coordination, or sharing kitchen equipment between multiple dorm groups and sharing rides to get places. We also emphasized shared social experiences through weekly meals with speakers, mentors, and directors, and weekend social outings to explore the region surrounding campus.

Aspects of our faculty mentoring program contributed to the aspects of humans understanding one another. The REU directors emphasized an assets-based approach, infusing this mindset among the mentor group at every stage from selection of students for inclusion to offering possible interpretations of student behaviors during and after the REU. Additionally, at least one faculty mentor lived in campus housing with the students every summer. Students interacted with one faculty member's young children, joined a morning running group, rode bikes, engaged in hours-long conversations about living, studying, and working in different countries, and enjoyed other such activities facilitated by proximity.

The faculty-student and student-student relationships built during the REU continued after the REU ended. Each faculty mentor guided the members of their research group through presenting their work at a national conference, responded to questions and requests for guidance, and wrote letters of recommendation on request. Several mentors are still regularly in touch with students who participated in the REU in 2015 and earlier, as they make their way through graduate programs and into various career stages. Students inform us that ties they formed with other students during their REU participation have been critical as they pursue their next steps in the years after the REU. The directors see bonds being maintained over years and across borders through interactions in social media as well.

7.4 Student outcomes

REUs typically use metrics like papers published, software packages created, and students pursuing graduate education as evidence of positive outcomes. Given our values and intentions (7.1.3)—and our belief that emphasizing these metrics only reinforces the exclusionary culture we wish to counteract—we claim **this REU's most significant outcome consists of the creation of a diverse, rigorous research environment with a culture counter to the default academic research culture.** Some evidence for this claim comes from many personal conversations over subsequent years with students reflecting on their experiences in the REU as compared to their experiences while they continued their mathematical studies. Additional support comes from our primary assessment tool: students' self-reported growth in several measures that pertain to a sense of belonging in mathematics, something the REU intentionally supported. Students assessed their comfort with their place in the mathematical community, their confidence in their ability to succeed in the mathematical field, and their ability to become mathematicians. Across all five years, the change in their assessments of their pre-REU selves to their post-REU selves was positive in regards to each of these questions. In 2015, when the REU had really "hit its stride", the average increase in the differences in the mean scores of these questions was 1.67 on a 5-point scale, and was at least 1.0 on the five-point scale for all other years other than the first year. (All p-values were less than 0.0004.) In other words, students moved at least a full level up in their sense of belonging after spending 8 weeks in our REU.

Vis-à-vis the traditional metrics: 10 papers were published in peer-reviewed journals and two new software packages were contributed to Sage Math Cloud for the use of the research community. Every student who participated presented research at a national conference. A preponderance of our students went on to graduate programs in the mathematical sciences—although we intentionally did not select for commitment to this prior to the REU. These achievements again demonstrate the value of looking beyond the traditional sources of REU applicants.

We consider our second most significant outcome to be the diversity of our participants. Among the 44 students who participated in our 2010–2012 REU were 23 women and 21 students from under-represented groups (there is overlap in these two groups). Amongst the 36 participants in the 2015 and 2017 REUs were 11 (of 27) native Spanish speakers, 15 (of 33) women, 13 (of 25) students who represent groups underrepresented in the mathematical sciences, 18 (of 28) first-generation college students, and 12 (of 24) from low-income families. (Students were not required to provide demographic information when they applied; the numbers in parentheses indicate the number of participants who provided data for the corresponding item.)

Final examples of positive outcomes include these:

- Consider the barriers to higher education facing students lacking documentation to reside in the U.S. and recall that our REU began prior to the implementation of DACA. The CSUCI REU benefited from the engagement of six undocumented students contributing their skills and efforts. We are thrilled that one of these six has a PhD, completed a prestigious post-doc and is now in a tenure-track position; another is on track to complete a PhD in 2023; a third is in a PhD program. Another earned an MS and has already spent several years in a coveted position in tech.

- The CSUCI REU garnered attention nationally for its unique emphasis as students presented their work and took their stories back to their home institutions, and associated faculty received multiple invitations to speak and write about the REU. This in turn made it easier to recruit sought-after faculty candidates from historically marginalized populations. The diversity of the groups of mathematics faculty candidates who were invited for phone/Zoom interviews increased dramatically over that of earlier years. Indeed, CSUCI Mathematics was successful in hiring three new faculty from historically marginalized populations (of five tenure-track hires between 2014 and 2022).

7.5 Benefits

Benefits of this REU accrued to the students and faculty who participated, the host institution, and later students of the faculty as well. As one might expect from any REU, students received an intensive research experience and preparation for graduate programs in the mathematical sciences. Students reflected in post-REU surveys that they felt better prepared to decide whether to pursue graduate study as well as better prepared for such study should they choose it. In interviews conducted by an external researcher, students affirmed the "more prepared" theme, and even those who indicated they'd intended to apply to graduate school prior to the REU said they felt more confident they could succeed. Said one 2011 participant: "…it affirmed my knowledge that I could succeed in an intellectually demanding environment." Students felt "empowered" as they analyzed their own growth over one summer. Indeed, 100% of the REU students who applied to graduate programs earned acceptance to at least one program. All of those earned a least an MS, with the majority earning PhDs or still being on track to earn a PhD. In the interviews, students also commented on the "non-math" aspects of the REU. "I thought I walked out of it a more well-rounded person." "It was about math and a good life experience…" "Just the exposure to people from all different parts of the country who enjoy math and getting to do different activities besides math with them and see how they interact was a nice experience too." Several students agreed with one who voiced the opinion that the REU was a lot of hard work but it was also fun; another rephrased that sentiment as challenging yet very rewarding.

The faculty engaged in the CSUCI REU have stayed in touch and supported one another through various endeavors. The Early-Career Mentors particularly felt that participating in this program was beneficial to their careers, and attribute lessons learned and support from senior faculty not only to successful applications for new positions and grants but also to how they work with their students now. One writes "Being part of the REU was a transformative experience for me in so many ways." [15] They understand and act on the importance of building community, being intentional and explicit with students about that. One talks about leveling the playing field—giving first-generation college students the opportunity to "excel and take ownership of mathematics," while another advises faculty in Project NExT, when recruiting students for undergraduate research opportunities from their classes: "Don't always look for the student that is getting 100% on everything in your class… what is most important is excitement about mathematics. Cultivate that excitement!" All of the faculty engaged who discussed the REU in 2020 remain enthusiastic about their participation and the bonds formed.

At the time the CSUCI REU began, the host institution had been open only 7 years. The directors were active in building institutional capacity to carry out student-centered, faculty-mentored undergraduate research. The CSUCI REU became a model for the campus in thinking about summer undergraduate research projects and in writing grant proposals to fund such projects. Lessons learned in planning for and carrying out the REU—particularly in terms of developing faculty capacity to mentor research and to assess student outcomes—led to the development of faculty workshops and were thereby dispersed into other faculty-student research activities. REU faculty learning about issues facing native Spanish speakers as well as first-generation students, students from low-income families, and students who are members of groups underrepresented in higher education carried over beyond student research to significant additional grant writing (e.g., $17 million in three HSI-STEM grants; multiple NSF and foundation grants). Undergraduate research and supporting students from marginalized populations are now two dominant themes amongst talking points the university emphasizes to potential philanthropists.

7.6 Challenges

As much as we might wish otherwise, thoughtful high-touch programs with significant impact rarely come at a low cost, in money or in time. Given the resources available at the host institution and the target per-student funding amount from NSF, the REU directors found the workload unsustainable in the long term.

While the UAEH student and undocumented students were not eligible for NSF funding, the participation of these students added so much value to our countercultural enterprise that we committed to raising funds to enable their participation. (Some sources included MAA SUMMA Tensor grants, local donors, and once, an elite college providing the stipend for its student's participation.) Additional financial costs came from our determination to incorporate faculty mentors from Mexico and other parts of the U.S. (leading to increased housing costs). The low pay for faculty mentors in the REU budget framework requires mentors who can afford to forgo better paying summer opportunities. Yet mentors who are themselves first generation college students or faculty of color—proofs by example for like students—may be the least well positioned to forgo better compensation.

Additional time demands came from those elements that made our REU unique. Faculty mentors met almost daily with their students, for multiple hours. Workshops and other activities demanded additional time and preparation from mentors and directors. Participation in social events and shared meals required time outside regular working hours. Mentoring the faculty research leads, navigating the paperwork for the international mentor and student, seeking funding for non-U.S. participants, and negotiating housing rates with the host institution all required additional time on the part of the directors.

7.7 Charge to the community

One of the most wonderful aspects of the dominant mathematical culture is the spirit of intellectual generosity, where faculty share their intellectual product free of charge to the public. For example, organizations like NAM and AWM run almost entirely on volunteer work. While we strongly support the ideal of intellectual generosity, it is distressing that the enterprises most taxed for funding and support are those designed to address systemic inequity. For example, AMS and MAA both have significant staff support and physical offices. Our REU, designed to address systemic inequity, could not survive with the amount of support it was able to receive unless it was willing to transform into a more traditional REU, where graduate students do the bulk of research mentoring and undergraduate students arrive with fully developed mathematical skills. Mathematicians have tremendous work to do to reformulate our community into the meritocracy we claim that we are. This requires emphasizing community and collaboration, active mentorship, and providing substantial academic and psychological support to individuals from groups we have traditionally excluded. It also requires providing adequate funding and resources for us to engage in that challenging and time-consuming work. We urge you to look around yourselves to see which programs are underfunded and which are well-funded. What can each of you do to generate adequate support for a program addressing systemic inequity? Once we all join together in this work, our culture will be truly shifted.

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8

The Future of Mentoring Undergraduate Research in Mathematics with Deaf and Hard-of-Hearing Students

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When a faculty member thinks about working on undergraduate research in mathematics with deaf and hard-of-hearing (DHH) students, several questions might pop into their mind, depending on how familiar they are with DHH students and the deaf community in general: *How can I recruit DHH students to work on undergraduate research in mathematics? What accommodations should be addressed when working with DHH students? How do I manage a research team that includes both hearing and DHH students? What factors, in addition to those for hearing students, should be considered when selecting research problems for DHH students? What is the status of undergraduate research in mathematics with DHH students?*

The main goal of this chapter is to provide some useful suggestions on mentoring undergraduate research in mathematics with DHH students by discussing the aforementioned questions. The arguments and the suggestions that we provide should in no way be considered as evidence for best practices in mentoring undergraduate research in mathematics with DHH students, as these suggestions are based primarily on our experience working with DHH students, either on class projects, honors projects, capstone projects, or undergraduate research projects. Our suggestions are given in good faith and with the strong desire to draw more attention to the importance of doing undergraduate research in mathematics with DHH students, and to help anyone who would like to work on such research projects.

The students we focus on are DHH college students in the United States, most—though not all—of whom are either at Gallaudet University in Washington D.C. or at the National Technical Institute for the Deaf (NTID) at Rochester Institute of Technology (RIT) in Rochester, New York. Gallaudet is a liberal arts university for DHH students with a department for science, technology, and mathematics, and NTID is a technological college that offers associates degrees as well as bachelors, masters, and PhD degrees as part of RIT. Gallaudet students are bilingual with either English as their first language and American Sign Language (ASL) as their second language or vise versa, while at NTID, students may fall into one of these two categories, or use English exclusively and not ASL. Most of our discussion in this chapter can be applied to research that could be conducted either during the academic year or the summer, paid or unpaid, and to a wide variety of DHH students, including DHH students who use various signed or spoken languages.

8.1 Recruiting and selecting DHH students to work on undergraduate research in mathematics

Recruiting DHH students to work on undergraduate research in mathematics has proven to be a hard task, mainly because the number of DHH students who are interested in mathematics is not large [24]. A research mentor who is looking for DHH students to work on an undergraduate research project in mathematics might first look for students within their institute by consulting mathematics colleagues, for example. It is also good to contact faculty who usually teach a large number of DHH students, or those who are familiar with the deaf community, for example faculty members at Gallaudet University or NTID. Recruiting DHH students into undergraduate research programs that typically serve hearing students should be encouraged, as it gives DHH students the opportunity to work on a research team with hearing peers, and it can positively enhance the research experience of all participants by providing diverse perspectives that can improve understanding [12]. However, it is important not to cross over into the dangerous territory of recruiting students from any group simply to check off "broader impacts" on a researcher's list of tasks. No one feels great knowing they are the token underrepresented student on the team. If this is the only reason a researcher is attempting to recruit a DHH student, it will be clear to the student, and in the long run it will negatively impact everyone involved [13].

To include DHH students in a research team, it is important for research mentors to keep an open mind about what kinds of students they will work with. Having a strict set of criteria—for example, the student must have straight A's in a long list of mathematics courses that aren't necessary for the research—for accepting students can eliminate potential DHH students who could succeed and benefit from the experience. Certainly, some research mentors are in the enviable position of having a line out the door of students who would like to work with them. They need a way to filter these applicants, and to make sure all students involved demonstrate that they have the mathematics skills necessary to successfully work on the research project. Beyond these considerations, however, considering a student's noncognitive skills is another way to evaluate fit. Is the student responsible enough to attend meetings reliably? Will they work well with other members on the team? Are they excited about the topic?

Noncognitive skills are "traits and skills—such as critical thinking skills, problem solving skills, social skills, persistence, creativity, and self-control—that allow [individuals] to contribute meaningfully to society and to succeed in their public lives, workplaces, homes, and other societal contexts" [8]. Measuring noncognitive skills can be difficult. How do you measure responsibility or motivation? While knowing students ahead of time isn't always possible, it can be a huge help when it happens. For example, in the NTID REU, a student's aptitude in the eyes of a mentor who has worked with them is a big predictor of the student's success in the program, at least as strong a predictor as is GPA. This, of course, is not always possible. When reviewing an applicant the research mentor has not met, it's a good idea to take seriously the student's essay and letters of recommendation in combination with traditional measures. If the program requires applicants to write an essay, the essay instructions should specifically include directions to the applicant to address key points of how they fit with the program and how the program fits them. This can help to lead students to discuss personal strengths that align with the program's goals. For example, an REU in which collaboration is highly valued might ask applicants to address why an REU with a great deal of collaboration is the right fit for them.

Most of the challenging situations at the NTID REU have stemmed from bringing aboard a student who was not strong by traditional metrics, and also did not have dependable recommendations nor a strong essay. For example, some faculty who have less experience working with DHH students would chalk up all the weaknesses of a DHH student to the fact that the student is DHH and recommend them for undergraduate research internships that may not be a good fit for the student. While there are always unknowns with any student who is accepted into a research program, particularly an intense program such as an REU, it is recommended that the research program director reach out to colleagues who know the students very well and can provide reliable recommendations. It is also recommended to have follow-up interviews, such as video relay service interviews or Zoom interviews, for example, with appropriate access services. Such interviews could reveal students who lack the motivation to complete the program successfully, and might point out any red flags. The goal of all this is to help make a more informed and fair decision when selecting students, and to lessen stress on the research mentor and students, stress that could result from a selected student not fitting the program well.

For many faculty, a student's writing skills are an important indicator of how successful they will be at research.

However, for English language learners in general, and perhaps especially for DHH students, there may be a large disconnect between the student's aptitude for mathematical work and their skill at writing [3]. While a thorough discussion of DHH people's exposure to language is outside the scope of this chapter, exposure to English is often limited for many DHH people, and lack of early exposure to language at all can be a factor for many DHH students. The main point of this is avoid judging all DHH students by their writing skills, and since many students—regardless of hearing status—lack academic writing skills, having strategies such as those listed in [27] in place to deal with this can be helpful in general.

Many DHH individuals struggle with English and mathematics [1, 15, 20, 22, 28]. Of course, many DHH students are excellent at mathematics, and some are fantastic writers. Many students show excellence in some areas, but unusual gaps in other areas. Some students perform well at mathematics, but struggle with writing. Some students seem to do well at both, but lack confidence in one or both skills. The point of all this is that many of these students fall through the cracks when it comes to undergraduate research opportunities. Noting which skills are truly essential to a research experience may help some faculty recognize students who may have been otherwise overlooked. It's important to remember that broadening participation becomes more likely with a broader pool. Some possible essential qualities include responsibility, curiosity about the project, and a hint of a "mathematical mind." There are many projects that don't assume any specific level of mathematical background, such as graph theory projects that could begin with, "What do you think this quantity is for a path? How about a cycle?" while keeping in mind more advanced projects, such as, "Which graphs will have this parameter equal to 1? Can you make a list? Can you characterize them?" or, "Can you write a program that finds this?"

To summarize, when it comes to recruiting and selecting DHH students to work on undergraduate research in mathematics, the research program director should cast a wide net by stripping off unnecessary expectations, and at the same time take care not to drag a student in if they don't want to be there. Make sure the student fits your necessary criteria, but remember that English is not the first language for many DHH students. Knowing the student beforehand is ideal. If this is not possible, getting a reference from a trusted colleague who knows the student well is the second-best choice, and if this fails, meeting with the student in some form is also a good idea.

8.2 Selecting mathematics problems for undergraduate research with DHH students

As with hearing students, the selection of the research problem depends greatly on the students' level of expertise in mathematics, their availability, and their commitment to doing mathematics research. To understand the additional factors that can impact the selection of the problem when working with DHH students, we will describe three overlapping groups of DHH students. Each of these three groups has specific characteristics that one needs to be aware of when selecting the research problem.

The first group consists of DHH students who consider ASL to be their native language and deaf culture as their way of life and thinking. Some of those students struggle with English language, both in reading and writing comprehension, and they prefer to communicate and think in ASL. As a visual language, ASL uses hands, facial expressions, and body movements, instead of spoken words, to communicate information accurately, sometimes in detail exceeding what is usually conveyed in spoken languages. The language emphasizes visualizing the objects involved in the discourse by including information on their shape, size, movement, and color [5, 25]. This kind of strong relationship between seeing and thinking makes this group of students a unique one in terms of learning and thinking processes, and might add a new perspective on how to look at some mathematics problems and even shed different light on how to solve such problems. So, for students in this group, it might be very fruitful to select a problem that requires visualization and drawings of different cases. Such problems can be found in graph theory, topology, and combinatorics (e.g., problems related to Ferrers and Young diagrams).

The second group consists of students who are taking, or have taken, only precalculus or discrete mathematics. The rationale for working with such a group of students is the fact that the number of DHH students who take mathematics courses beyond precalculus and discrete mathematics is very small, and of those who take such courses, only a few are interested in doing research in mathematics. So, if we want to increase the number of DHH students who do undergraduate research in mathematics, then we need to consider students whose most advanced mathematics course is precalculus or discrete mathematics. While selecting students at such a low level of mathematical maturity to do mathematics research might seem unnecessary or unfruitful when we have the luxury to select from a large pool of students, it is crucial when it comes to selecting from underrepresented groups in mathematics, as is the case with DHH students.

Normally, DHH students take precalculus and discrete mathematics either because these courses are required for their major in STEM, or just because they believe these courses are good for them and that they want to be prepared in case they select a major in STEM. This presents a great opportunity to foster those students' appreciation for mathematics and to increase their confidence about their abilities and fitness for a career in STEM. If we give DHH students the opportunity to work on a mathematics research problem at such an early stage in their education, they will start to think about themselves as intellectual individuals who can contribute to the growth of human knowledge, which will undoubtedly leave a positive and lasting impact on the way they view themselves, and will eventually increase the interest of the deaf community in mathematics [10, 21].

The main goals for doing undergraduate research in mathematics with such a group of students should be enlightenment and recruitment. This might be a paradigm shift of the usual goals of undergraduate research in mathematics, where instead of concentrating on solving the problem itself, we consider research as an opportunity to give students a positive experience of working on mathematics research, an experience that will certainly lead some students to pursue more advanced research in mathematics at a later stage of their lives [7, 9]. So, the research problem should be interesting, not overly hard, and enjoyable. There are many problems in geometry, trigonometry, and combinatorics that satisfy these criteria. For students with discrete mathematics skills, one can choose problems that are similar in nature to those related to Fibonacci sequence or Catalan numbers, and for students with precalculus skills, one can choose problems that include transcendental numbers such as π and e, or problems that are related to the areas and volumes of different geometric figures. For example, problems such as finding the maximum sum of the areas of n circles of equal radii that can be placed inside a circle of radius r where the intersection of the interiors of any two circles is empty. This kind of problem does not require much background in mathematics and it can be easily generalized in many different ways and for many different figures.

Students usually take precalculus and discrete mathematics during their first year of college, together with several other foundation courses that require extra effort to learn. So, when doing undergraduate research in mathematics with such students during the regular semester, one should be careful not to overwhelm students by adding too much work to their plates. As a rule of thumb, the research problem should be selected with the assumption that students will work on it only for two to five hours weekly, and will be able to finish it within one or two semesters. The outcome of this kind of research project can be either full or partial solutions to the research problem. Students should be encouraged to give a talk or a poster about their work, for example during on-campus undergraduate research symposiums or regional mathematics meetings.

The third group of DHH students consists of students whose educational experience is more similar to their hearing counterparts than to DHH students in the first group. Some of those students became deaf or hard-of-hearing at a later stage in their lives, and many of them might have attended mainstream schools. Students in this group are more likely to view English as their first language and might feel very comfortable expressing themselves in written and spoken English. They consider ASL as their second language and view it as a language that provides more access for them to communicate with their DHH friends and families, or with the hearing world by using sign language interpreters. Some of them are more interested in finding ways to participate in the hearing world than to become involved in deaf culture and ASL. For such a group of students, the criteria for selecting the research problem are similar to those for hearing students, namely their mathematics background, availability, and commitment to doing mathematics research.

8.3 Accommodations for DHH students working on undergraduate research in mathematics

A key to success when working with DHH students on undergraduate research in mathematics is to provide appropriate accommodations and make necessary adjustments to the research arrangements so that everyone on the research team feels welcome, respected, comfortable, productive, and beneficial to the team. Prior to the start of the research, the research mentor should put in extra effort to understand the needs and abilities of each student on the team, then make plans to satisfy these needs and to make use of each student's abilities.

Once a research team that includes DHH students is in place, the research mentor can facilitate communication by talking to the students about what works best for them and consulting some basic resources on best practices when working with DHH students in general. See, for example, [6, 14] for a list of resources. Some basic tips for meetings include making sure that the room has adequate lighting and limited background noise, that all group members can easily see each other, and that group members don't talk and write at the same time. Setting up ground rules from the start can be helpful. For example, some groups have a designated object that the speaker must hold when they have the floor so that only one person at a time is talking. While these ideas may seem awkward or even "inorganic," they become more natural with practice, and they help improve group communication for all members.

Strategies like these can be helpful in any group with mixed communication methods, but perhaps most of all when working with a team that has a mix of both hearing and DHH students. Like many hearing faculty, hearing students often have little to no experience working with DHH students. It is therefore especially important in these situations that the faculty mentor encourages and reminds group members to adhere to communication ground rules that allow all students to participate fully.

There are certainly several challenges associated with incorporating a DHH student into a research team that includes hearing students and when ASL is not the language used for communication. Depending on a student's communication preferences, an interpreter, captionist, or other access service may be necessary some or all of the time. Neither writing back and forth nor asking a bilingual group member to interpret is a feasible long-term option, since neither allows all group members to fully participate. We won't dive into the issue of communication in detail since it is heavily dependent on the individual student's needs and outside the scope of this chapter, but ensuring that everyone in the group understands each other will require care, thought, and probably resources such as interpreting [11, 16]. Of course, these challenges have an upside: devoting careful thought to group communication often benefits all group members, but it's important to understand that a slap-dash effort to satisfy diversity expectations simply by recruiting DHH students without giving thought to communication challenges will have its costs.

Related to communication diversity is the fact that a student may have different needs depending on the environment (see page 163 of [17]). For example, some students at the NTID REU have grown up as hard-of-hearing individuals among hearing peers. During a conversation, many people may not realize that the students are actually DHH. However, some students may be comfortable communicating in small groups, but unable to hear well in large gatherings such as a poster session, for example. Many DHH students feel very nervous when they are asked to give a presentation, especially if it is their first time, to a hearing audience. If they decide to use spoken English, they worry people might not understand them well. On the other hand. if they decide to use ASL, they worry that the interpreters might not translate their ASL correctly [2, 19], or that the audience might be distracted by having someone lecture them on mathematics using sign language.

To alleviate students' fears and worries, it is a good idea to first give students the opportunity to give their presentations to their team and, if possible, to talk to the interpreters one or two days before their scheduled presentations, or even to make arrangements so that the student can select one of their regular interpreters. Access services such as captioning or interpreting are critically important, as is picking a good presentation outlet for the student. However, it's important to be aware that different environments can present vastly different challenges to some DHH students, and that some may trigger fears that the student has dealt with for a lifetime. Just because a student appears to "function as a hearing student" doesn't mean that they are truly enjoying the same level of access as hearing students are.

Another important issue is social time. For many programs like REUs, the socialization aspect is an important component of the experience. Even if a DHH student's access and communication needs are met during work time, it is still a challenge to figure out how to meet such needs during social time. While some students are adept at overcoming communication barriers and finding ways to have a satisfying social experience, others prefer to avoid participating in social events, such as team dinners or meeting new mathematicians at mathematics gatherings. The research mentor should be aware of students' communication needs during social events and make efforts to ensure that DHH students have fulfilling networking and social experiences just as their hearing peers on the team do.

Special care should be taken when holding virtual research meetings that include both DHH and hearing students as the interaction mechanism and the technological tools are usually different from those used in face-to-face meetings. Most video meeting programs (e.g., Zoom or Google Meet) provide options for automatic live transcription and real-time captioning. However, unless all DHH participants are comfortable using their voices and reading transcripts, these options should not be used as a replacement for interpreters. It's also important to note that automatic captions are not a substitute for live real-time captioning.

Visual virtual meetings put extra work on the eyes of DHH students, because they need to keep their eyes on the screen, or otherwise they are going to miss some information. Keeping the number of people in the meeting small (up to four people) and the duration short are some of the recommendations to lessen the stress on the eyes of DHH students during virtual meetings. Tips for how to manage virtual meetings that include DHH participants, depending on the number of attendees and the mode of the communication (sign language or audio), are given by Vogler in [26].

Undergraduate research directors who aren't familiar with DHH students may be overwhelmed with the logistics and financial implications when they consider including DHH students, who need access services such as ASL interpretation, on the research teams. One place to get started is the university's disability services or access services office. There also may be opportunities to apply to the relevant funding organization for supplemental funding to cover the cost of access services such as with the NSF, for example [18, E4: Supplemental Support]. Needing access services should never be a deterrent to a qualified student applying for a research opportunity, or for an undergraduate research mentor accepting a qualified student, but the uncertainty can make both undergraduate students and faculty leery enough to create a barrier as something of a self-fulfilling prophecy. This may be one reason that many DHH students, even students who are very strong academically, seem less likely to know about or to seek opportunities for research.

Some conference organizers do not know how to ensure accessible communication for students. ASL interpretation is a skill, which of course comes at a cost, often a cost that seems prohibitive for many local conference organizers. While this can often be worked out on a case-by-case basis, it also means that local conferences, which are often "easy" for hearing students to attend because of their geographical closeness, low formality, and low registration costs, are suddenly prohibitive to students who need interpreters or captioning. While many conference organizers are encouraging and helpful to faculty and students who request access services, others push back, worried about the cost and logistics. Navigating this process may be unpleasant for faculty mentors, but is likely a prohibitive barrier to an undergraduate student.

Finally, it is important to keep in mind that there is not a one-size-fits-all solution for research arrangements and accommodations that fit undergraduate students who are DHH. Some use ASL, and others spoken English. Some students may have other disabilities, such as deaf-blind students, for example. The research mentor should think early about and implement strategies so that group communication is as smooth as possible for everyone in the group. Talk to the students, rather than leaving these issues to pop up. If something is not going well between students, then it's important to solve the issue as soon as possible. Just as with hearing students, each student learns differently. Some students desire and would benefit from an experience among many other DHH students, such as the REU at NTID, while others prefer to be at an REU that serves mainly hearing students, and would not feel they would fit in with ASL users. Students may value being a part of a program that focuses on a different aspect of their identity than the fact that they are DHH, such as a woman student who would like to do research in an environment supportive to women, or a student of color who would like to be involved in a program focused on students of color. It's therefore important to have opportunities for students who benefit from being among other DHH students, but also provide opportunities for DHH students who have different priorities or values. Everyone on the team should feel beneficial to the team. Of course not all students have the same ability: some might be good at writing, others at calculations and programming, others at proofs. It's important to put in enough effort to distributing work fairly among team members and making effective use of everyone's abilities.

8.4 Status of undergraduate research in mathematics with DHH students

There is no formal tracking of how many DHH undergraduate students are doing research in mathematics. There are a few research programs that specifically focus on recruiting students who are DHH into mathematics research. Also, at both Gallaudet and NTID there are faculty members who work with students in informal and formal research experiences. It is likely that there are many DHH students across the country doing research at their home universities. With the wide variety of communication styles of DHH people, and the development of technology, many DHH students may blend in with their peers, to the point that some undergraduate students doing research probably never reveal to their peers or mentor that they are DHH. Since the NSF recognizes that people with disabilities, including DHH people, are underrepresented in STEM disciplines, programs that solicit underrepresented students often encourage students who are DHH to apply. This is not always the case, however.

To the authors' knowledge at the time of writing, there are currently only three programs in the United States that specifically target DHH students to do undergraduate research in mathematics. These programs are at Gallaudet University, RIT/NTID, and the University of Connecticut. The Department of Science, Technology, & Mathematics at Gallaudet University supports undergraduate research internships in mathematics, during regular semesters and in summer, through several federally funded research projects. Students at Gallaudet have worked on a variety of research projects, including cryptography, combinatorics, and data structures. Students usually prepare posters about their work and then present them at the Gallaudet's undergraduate research symposia as well as regional conferences. (See `gallaudet.edu/department-of-science-technology-and-math`). RIT/NTID also hosts multiple research opportunities for DHH students, or mixed groups of DHH and hearing students, each year. One example is the NSF-supported REU Site: Summer Undergraduate Research for Students who are Deaf or Hard-of-Hearing in Applying Mathematical and Statistical Methods to Problems from the Sciences ("REU at NTID"), an eight-week summer research experience in science and mathematics that provides opportunities for students to improve mathematics, statistics, and writing skills in addition to the main research experience (see `rit.edu/ntid/reu`). The REU program at the University of Connecticut (see `mathreu.uconn.edu/`) started targeting DHH students in 2019. In the summer of 2020, the program was able to recruit three DHH students and a deaf PhD student mentor to work on undergraduate research problems within the field of analysis on fractals.

There is no study that provides reliable information on the number of DHH individuals in the United States with graduate degrees in mathematics. To the authors' knowledge at the time of writing this chapter, in the past ten years, only two or three DHH students who became deaf before finishing high school have pursued a PhD in mathematics, and a few others have obtained an MS in mathematics. Thus, the percentage of DHH individuals with graduate degrees in mathematics is very small, compared to the hearing population. There are many reasons why small numbers of DHH students pursue a graduate degree in mathematics, but probably one of the main reasons is the lack of interaction with DHH individuals who have passion for mathematics, be it a friend or a family member. Many DHH students view mathematics as a subject that they struggle with and as something that prevents them from satisfying the school or the major requirements. According to [4, 21, 23], undergraduate research experiences of students underrepresented in STEM increase students' understanding and awareness of graduate school opportunities, confidence in applying for graduate school, and likelihood of acceptance. Hence, undergraduate research in mathematics with DHH students could represent a very important trigger that can increase the interest of the deaf community in mathematics.

At the Joint Mathematics Meeting in 2014, a small group of DHH students from RIT presented their research at an AMS Special Session for undergraduate students. The AMS provided ASL interpreters for their presentation as well as for the students to attend other presentations. During one presentation the students attended, a mathematics professor from another university casually asked the students' mentor, "Are they practicing their presentation?" He saw the ASL interpreters signing the ongoing presentation, and assumed that the interpreter was one of the students. Spotting ASL use at any large mathematics conference is somewhat rare, and he had never seen it before. As DHH undergraduate students participate more and more in mathematics research, we hope that DHH mathematics researchers—undergraduate, graduate, and beyond—become an important part of the future landscape of mathematics research.

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9

Advancing Community College Student Involvement in Mathematics Research

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Enrollments in community colleges make up a major component of the undergraduate student population in the United States. In Fall 2018, 5.7 million students were enrolled in two-year institutions, comprising 35% of the total undergraduate enrollment in the United States [16]. The numbers fluctuate but generally correlate with the unemployment rate [3]. According to data from the National Student Clearinghouse Research Center, about 30% of students transfer from community colleges to baccalaureate programs within six years of their first enrollment [30], and among all students who completed a degree at a four-year college in 2015–16, 49% had enrolled at a two-year college in the previous ten years [12]. The large number of student transfers from two-year to four-year institutions calls for undergraduate research programs tailored to their academic preparations, while ensuring that these students have equitable access to a wide range of opportunities to engage in research. This chapter aims at discussing mathematics research opportunities and mentoring for community college students on community college campuses and at other institutions.

9.1 Differing backgrounds of community college students

Community college students can differ from lower division students at other institutions in several ways, as evidenced by the following estimated statistics from [2].

One aspect of the differences lies in students' commitments in life. A large majority of community college students are part-time; most community college students work while enrolled. For example, 65% of community college students are part-time; 62% of full-time community college students are employed, as are 72% of part-time community college students. Community college students juggle school schedules with longer work schedules than do students at other institutions. Additionally, many community college students are parents. Furthermore, 15% of all community college students are single parents who in turn have higher levels of responsibilities for support and time for their children, compared to students at other institutions.

Community colleges possess a diverse group of students. Community college students are 44% White, 27% Hispanic, 13% Black, 6% Asian/Pacific Islander, 1% Native American, 4% two or more races, 4% are of unknown ethnicity, and 2% are of nonresident alien status. 57% are women and most single parents who are community college students happen to be women. For a large percentage of community college students, English is their non-primary language. As of 2015, approximately 24% of community college students are second-generation Americans, born in the US to immigrant parents. About 29% of community college students are first-generation higher education students. These students often may have little support from their families in pursuit of a college degree. In turn, most (73%) community college students apply for financial aid.

Finally, the knowledge retention in community college students differs from those in other institutions, since community college students have comparatively longer education timelines. The average age of community college students is estimated to be 28 years, while the median age of these students is 24 years. These estimates support the well known fact that community college students are more often returning to college after years of time away from school, compared to their counterparts at other institutions. Involving these students in research activities requires an understanding that these students may have learning loss, while also understanding that any rustiness is only temporary and can be overcome. Because community college students have a higher average age, their appreciation and attitude toward higher education and even research can counter-intuitively be more intense than that of their counterparts at other institutions.

About 20% of community college students use adaptive services that enhance access for students, faculty, staff and guests with disabilities and provide effective, reasonable accommodations and equal access and opportunity. In terms of course preparation, assembly Bill 705 in California has been recently implemented in order to discontinue the practice of directing incoming community college students to enroll in developmental education courses in math, English, or reading. However, nationally, 60% of community college students enroll in one or more of these developmental courses [11].

It is heartbreaking to see community college students with great potential having no choice but to decline to spend time on mathematical research because they must otherwise maintain non-research employment in order to live. For these students, travel to a week-long national conference results in a week of lost pay necessary for their survival. It is important to recognize that our mentored research students are human first and then researchers.

Despite the challenges that our undergraduate research mentees have faced, their participation in mathematical research has resulted in greater opportunities for them down the road. The authors have written many recommendation letters for graduate school and STEM (Science, Technology, Engineering, and Mathematics) careers a few years down the line. We have found that our mentees have experienced a higher level of success in mathematics and mathematics-related fields upon and subsequent to their transfer to a university, than do their peers who have not participated in math research activities during their time on our community college campuses. Many community college students have earned or are currently earning master's and/or doctoral degrees in STEM fields including mathematics after participating with us in math research activities.

9.2 Opportunities and mentoring on community college campuses

Although some community college faculty are engaging their students in math research, most of these campuses have not yet developed ways of ensuring that their students have on-campus opportunities to experience what mathematicians do: research and development of applications in mathematics. Sustained funding of community college student and faculty math research experiences continues to be rare. There is only one NSF-funded community college on-campus REU Site in Mathematics as of the time of writing [24]. In order for undergraduate research to advance equitably for all undergraduate students, these conditions in community colleges must change. We make the case that more faculty involvement, administrative support, and student awareness can and should be encouraged and meaningfully supported in order to build and sustain impactful undergraduate math research activities at community colleges.

9.2.1 The need for change in the mathematical culture

Most community college faculty are sadly not capitalizing on the fact that the most interesting aspects of mathematics to students are its unsolved problems and emerging applications.

Campus culture that is void of mathematical activity beyond the standard curriculum is a primary reason why undergraduate math research has yet to significantly expand to community college campuses at the same rate that it has developed at four-year institutions. Community college students have traditionally not been made aware of what math research actually is, and often, their faculty do not know anything about math research themselves. It is common for students and even faculty on community college campuses to mistakenly view math research work as a superfluous activity that should exclusively be conducted at four-year schools by elite mathematicians and student prodigies. In turn, the community college students are shut out of any potential exposure to the frontiers of mathematics and are misled into thinking that math is merely a set of practical skills and a service subject for other academic areas that are deemed to be more important than math.

We argue that involvement in undergraduate research *is* teaching and should be common on all community college campuses rather than only a few.

Our intent here is not to change community colleges into institutions that primarily emphasize research; instead we intend to expose our students to research experiences that promote success, a love of mathematics, and important problem solving skills as these students transfer, graduate, and enter the workforce. We want these students to maximize their success, as community college students are often quite capable of advanced degrees and success as mathematics researchers and educators.

9.2.2 Challenges in broadening UR to community colleges

Recruiting potential faculty mentors has been the most challenging part of improving a pro-research culture on the authors' campuses. We believe that noting high-quality undergraduate research mentoring as a positive on evaluations is a sound community college practice. At Fullerton College (FC) and Rowan College at Burlington County (RCBC), faculty involvement of students in research activity is noted, though not at all required, as a evidence of good teaching on faculty evaluations. At Rowan College at Burlington County (RCBC), student involvement in research can be used by faculty to fulfill advising hour expectations. It is not unreasonable to incentivize community college faculty involvement with students in research by, for example, support in the form of release time or preferential scheduling.

Currently, a small group of community college faculty are teaming with each other and with four-year college and university faculty to develop support systems for community college math research, and we see an urgent need to grow this group nationally. Although there is only one currently funded National Science Foundation Division of Mathematical Sciences (NSF DMS) grant supporting community college UREM's (Undergraduate Research Experience in Mathematics), we are hopeful that NSF and other funding agencies will begin to see the value of including, rather than excluding community college campuses from meaningful, high quality research endeavors. Conferences such at the Math Research Experiences in Community College Conference (MREC[3]) have been held in order to encourage more faculty to be involved, but such conferences are quite rare at the moment.

A further significant obstacle to promotion of UR is that community college budgets have not historically included meaningful support for these activities. While four-year institutions hold regular math colloquia, community colleges lag in this area quite seriously, almost never having events for such a purpose. In 2009 upon request from the first author, FC began to provide funding for monthly university and industry guest speakers, providing modest refreshments and food for colloquia, providing grant-supported stipends for faculty who organize student research activity, and funding sustenance for Putnam contestants.

The percentage of public two-year colleges in the U.S. that offer as few as merely an annual "special math or stats" colloquium is estimated to have only risen slightly from 16% to 21% from 2010 to 2015. On the other hand, in 2015, the percentage of bachelor's-level institutions offering special colloquia for students was estimated to be 53%, while approximately over $3/4$ of all graduate math degree-granting institutions offered these events for students [8, p. 60]. We argue that since students from marginalized ethnic and socioeconomic backgrounds constitute a disproportionately high percentage of community college populations, then a failure to form and maintain math research-based colloquia and student-faculty math research communities may be a form of systematic discrimination.

On-campus mentors of community college math research students can greatly strengthen ties with math programs at four-year institutions by inviting their faculty to give on-campus colloquium talks. Many four-year college and university faculty are enthusiastic about visiting community college campuses to help broaden participation in mathematical science, so mentors should not be shy about asking for funding for these talks and inviting outside faculty to give them.

Students face many challenges too. Often self-supported, these students may have less time outside of class to invest in research, compared to their peers at four-year institutions. Prerequisite skills in math, software, communication, research design, data collections, and LaTeX may need to be developed as part of the research activity, but our experience shows that students are eager to develop these prerequisites with reasonable guidance and support.

Many community college students have been misinformed that math research is too challenging for them to deserve meaningful support, despite the fact that involvement of these students in research in other fields within STEM is becoming quite common, inconsistently. Community college students and faculty mentors with no previous research experience or exposure may share a lack of awareness of research opportunities that can be engaged in, either on-campus or off-campus. Compared to students at four-year institutions, community college students attend their campuses for significantly less total on-campus time than students on a four-year campus. Therefore, for community college students, lengthy research project goals are not a realistic prospect, while, on the other hand, these students can certainly reach meaningful research goals with guidance. Community colleges have not historically designated funds for students to travel to and attend math research conferences, so external funding may be important if there are no conferences close by. Funding entities have not historically supported community college faculty and student research involvement, so these student mathematicians and their mentors must struggle to balance their lives given that they are engaging in unpaid or otherwise undervalued research activity. Given the diverse math backgrounds of community college students, an additional problem that mentors should think about is how to properly assign or initiate research projects that meet students at an appropriate mathematical level, while not dumbing down the projects or mistakenly restricting the activities only to elite students.

Additionally, community college faculty who wish to establish UREM's on their campus may also experience challenges. There are not many mentor training opportunities, though organizations such as the CURM (Center for Undergraduate Research in Mathematics), the National (Mathematics) Doctoral Alliance and the Pacific Math Alliance are addressing this issue. With high teaching loads, such as fifteen hours of teaching per week and five and one-half hours of office hours with little or no grading assistance, community college faculty have the additional challenge of less time to develop UR projects and mentor students. Research skills-based training for students including forms of communication (verbal, written, presentation), time management, LaTeX, and MathSciNet is nearly non-existent on the nation's community college campuses. It is not yet common for community college faculty to receive teaching credit, release-time, or other forms of compensation for developing UREM's on their campuses. Community college math faculty often are largely adjunct, and consequently are not considered for office space, release time, and compensation for providing students with UREM's.

Most often, community college faculty have no incentive or support base required for fully promoting a student-involved research culture that lifts up mathematical science as a viable option for students as a career path. Although "guided pathways" are being widely formulated and implemented in the nation's community colleges, most community college math faculty are unaware that UREM's are precisely a guided pathway to math careers including higher education, math research, statistics, actuarial science, computational/data science and many other related fields. Community college math faculty are not taking enough advantage of funding that accompanies the guided pathway trend and are instead allowing the funding to be diverted to other fields or to guided pathways that merely support completion of a single college-level math course by under-prepared students. Funds to compensate staff for work on UREM's may be historically non-existent or may require grant writing. Training for community college faculty on conducting math research with students is rare or largely not available. Not having historically seen the value of UREM's on community college campuses, these institutions may state or experience a lack of sufficient computational hardware, specialized equipment, materials, or space for UREM's, or, prospective mentors must navigate the institution's procedures for setting these assets aside for UREM's. Unlike four-year institutions, community colleges may relatively lack industry partners and other potential research partner organizations that can assist in UREM administration.

Although these challenges are real, we believe that it is critical that more community college faculty become involved and supported to overcome them, as a failure to do so will continue to marginalize the national mathematical human resources infrastructure. **At community colleges, significantly more than at other institutions, math as a major is becoming increasingly unpopular, while nearly all of these campuses have not sufficiently prioritized meaningful, consistent focus on the frontiers of mathematics.** While 54% of all declared undergraduate math majors as freshmen in the U.S. end up switching to another major, 78% of all community college students who enter as math majors switch to another major [10, p. 15].

9.2.3 Criteria for successful UREM at community colleges

Successful UREM's maximize the numbers of students who accomplish as many of the following benchmarks as possible:

a. The student, students, or faculty-student team give(s) a talk on some unsolved problem.

b. The student's (team's) talk contains precise definitions of the terminology used in the statement of the problem.

c. For each definition, the student rigorously verifies an example and a non-example, if warranted.

d. The student's (team's) talk contains interesting biographical information about a mathematician or mathematicians who have been involved with the unsolved problem.

e. The student's (team's) talk includes the status of the problem in terms of known partial results as of the date of the talk.

f. The student's (team's) talk describes partial results or results from even crude experimentation, even if these results are rediscoveries.

f. The student converts the talk into a Beamer presentation with the aim of eventually presenting at an external conference.

g. The student (team) converts the talk into a thesis, preferably in LaTeX, with proper formatting and reference citation.

h. The thesis includes new results or original results regarding the problem.

i. The student (team) presents the talk at a regional or national math conference.

j. The student (team) applies for, is accepted to, and completes a paid external UREM.

k. The student (team) publishes the thesis in a reputable undergraduate or other math research journal.

l. The student (team) enrolls in a graduate mathematics program.

m. The student (team) completes a graduate degree in math or some other STEM field.

n. The student (team) publishes a math research paper in a high quality research journal.

Reaching even one of these goals is certainly beneficial for involved students. Accordingly, since community college campuses vary in size and diversity, there are many ways to start and run UREM's on these campuses so that follow-through in the above forms is maximized. We provide some examples below, and mentors should choose a strategy that fits their students and interests in the most inspiring way possible.

9.2.4 Examples of initiated and maintained UREM's at community colleges

Preparation for course based research experience (CURE): In this model, classes are designed to teach students how to effectively participate in UREM's. For example, Howard Community College in California offers a series of one-unit courses that teach students about general UR [15]. The first course in the general series is URSC101 "Introduction to Undergraduate Research" which covers research methods, protocols, data analysis, and research ethics.

Math-specific CURE's have been offered at FC since 2012, in the form of weekly 2-unit math research seminars that train students to work specifically on open problems in math. These students are taught how to give talks, how to create talk and thesis outlines, MathSciNet searches, Overleaf/LaTeX, career pathways, basic logic including quantifiers to maximize the scope of problems that can be discussed, and proper research article formatting and citation.

Independent study courses: A faculty mentor will partner with one or more students to focus on a single problem. Projects are typically one semester long, and course credit or other support can be made available as an incentive for follow-through. The first author tried this approach in 2008 and found that the "independent" nature of such an approach seemed to make it unlikely that students will make significant progress on the above benchmarks in this scenario. He found that more substantive guidance, incentivization, support, and accountability, ensures significantly more follow through by students and their teams on the benchmarks.

Non-independent study CURE: Faculty mentors assign specific projects to be completed by all students in the class. The NSF-funded CURE network[13] has a variety of opportunities and disseminates best practices, including inter-class collaboration, such as between a math and a science class, such that the science class defines the experimental design and data collection while the math class completes the data analysis, for example. This approach has worked well at FC when students are also given their own separate projects to work on simultaneously with the central problem.

Service learning-oriented math research. Students conduct specific research programs for the benefit of the community by, for example, analyzing data. CUREnet [13] supports activities of this kind. One drawback to this approach is the mathematics often may not be interesting enough to persuade students to follow through. On the other hand, data science is an excellent field to get students into.

Partnerships with government agencies and industry: For example, the Oak Ridge Institute for Science and Education (ORISE) offers a variety of opportunities for students to participate in FDA research opportunities [25]. AT RCBC, Lockheed Martin and BSE Systems have funded UR activities campus-wide.

Regular frequent colloquia/seminars: This includes opportunities for community college faculty, external faculty, students and student teams to give talks about open mathematics, with students hopefully reaching as many of the above-listed benchmarks as possible. At FC, such colloquia have been held since 2009, and over 400 student talks have been generated, along with hundreds of student theses on a large variety of unsolved problems.

Research seminar courses. Seeing that many colloquium attenders need incentives to follow through on the above-mentioned benchmarks, the first author created curricula for 2-unit math research seminars that give credit to students for their talk and thesis, these seminars meeting jointly with the FC Math Colloquia. The courses generate much more follow through than simply scheduling guest speakers for the colloquia. This model differs from the CURE model in which the entire class focuses on one problem. At FC, the first author blends this idea with students also working on their own individual projects so that whatever is assigned to them can be molded to fit their specific interests, which may not include the central problem addressed in the colloquia. This blended strategy seems to enhance student persistence in the FC UREM group. Students in this scenario feel more individually supported, and as they see other students giving talks, they learn a great deal more than they would if they were to simply see only polished faculty research talks. Thus the colloquia often are simply mini-conferences containing talks by multiple students and faculty, enhancing the sense of community among the participants.

Joint research seminars with universities or other partnering organizations. Recently, the first author held a Joint Analysis Seminar with California State University, Fullerton. These seminars are very effective in ensuring follow-through by financially supported research students. He still held the central colloquium in addition, because unsupported students were unlikely to have time to follow the details of the lectures in the Joint Analysis Seminar.

A research-focused math club or math association: In this model, faculty and/or students give research presentations in a math club setting. For example, Chandler-Gilbert Community College (CGCC) in Arizona offers the "Research Experiences for Undergraduates Club." Since math clubs tend to be student-led, the productivity of the club in research often heavily depends on who the student leader is during that particular term. The first author started using the math club approach after finding for one semester that independent study was not resulting in meaningful student follow through, but he found that without incentives and support for student research work, math club-oriented research alone does not guarantee much follow through from students. However, math clubs are useful in recruiting prospective research students, because students are the best recruiters of other students! Therefore, at FC, the Math Association (Club) now meets jointly with the Seminars/Colloquia, holding Association business during the first 15 to 20 minutes of each meeting.

Paid research internships: Several FC students who have performed well in the seminar courses have more deeply worked as research interns with the first author in summers and during the regular academic year. When funding has been available, supported students and a paid lab faculty member collaborate together on research projects during specialized informal learning seminars, which have included cancer modeling, Clifford algebras, fractal geometry, and hyperbolic geometry.

UREM Committee: The committee increases faculty and student participation in UREM as diverse opinions can enhance the effectiveness and organization of the program. For example, Mesa Community College in Arizona has established its “Undergraduate Research Initiative Committee” [22]. Mentors should not be discouraged if they are unable to find many faculty who are willing to put time into this effort. All a community college campus needs is one willing mentor in order to generate an exciting UREM for students.

9.2.5 Recommendations for initiating and maintaining UREM

In addition to the activities listed above, we recommend that the interested community college faculty consider the following activities.

Hold meetings with students to make them aware of external, paid UREM’s and graduate school math programs, while also offering to write letters of recommendations for students who follow through by applying. It is recommended that community college faculty mentors follow up with students after their external UREM concludes, to get feedback from their students about the UREM for future reference.

Hold joint research conferences and collaborate on finding financial support for UR students in math through your campus Honors Program.

Work with your campus administration to bargain for work-release in exchange for mentoring students in UR in order to achieve mutually agreed-to benchmarks such as the ones listed above. For example, one unit of release time for each student who is expected to complete a specific set of benchmarks could be assigned, or release time could be awarded on the basis of organizing the UREM to be a guided pathways program. Community college administrations are finding it challenging to find faculty who want to build guided pathways, so prospective mentors should strike now or soon, while that iron is hot!

Ask your math program faculty and campus administration to financially support faculty to work with students on UREM by providing instructors with regularly scheduled lab times requiring no outside preparation, so that UR students and, in particular, paid research interns can be supervised closely enough to ensure that participants stay persistent and diligent on their assigned projects. This approach was tried in 2014 at FC using grant funds and was one of the most productive strategies that the first author has tried. Based on his experience, he believes that regular supervision hours in a lab is most likely to yield research productivity on community college campuses, with expected outside hours for students kept at a reasonable, minimal level that does not distract them from their regular coursework. Paying students to work on a project but not expecting regular work hours with paid, close supervision by the faculty mentor does not seem to be an effective use of funds compared to an expectation that students will work with their mentor at specific, regular times.

Blend research activities such as colloquia with practice for and entry into problem-solving competitions such as the AMATYC Competition and the Putnam Competition. To expose students to research, the third author and his campus colleagues have recruited students from their classrooms to compete in the New Jersey Undergraduate Mathematics Competition, which is held during the Garden State Undergraduate Mathematics Conference (GSUMC). After the competition, there are student talks and a poster session. Students see other students present their research, which naturally motivates students on their own campuses to ask for a research project. Half of the third author’s UREM participants have been recruited with this strategy. The experience also humbles students in a productive way as they view the solutions of problems that are beyond the depth and difficulty of their undergraduate mathematics textbooks, setting up a reasonable expectation that doing research requires a significant time commitment. For the same reasons, since 2009, the first author has combined AMATYC and Putnam Competition problem-solving sessions with the Math Colloquia. Some students who were completely unaware of what math research are nevertheless aware of these competitions. Several of the first author’s students became UREM participants after they attended an AMATYC Student Math League problem-solving session or a Putnam Competition practice session. Reciprocally, nearly all FC students who have positively scored in the Putnam Competition have been Math Colloquium/Seminar regular attenders or enrolled students.

9.3 Partnership models between community colleges and 4-year institutions

In this section, we present a partnership model that vertically integrates community college students into research teams at a four-year institution. Such partnership expands opportunities for community college students to work collaboratively on open-ended topics not traditionally taught in classrooms.

In Arizona, roughly equal numbers of new high school graduates matriculate at two-year and four-year colleges [6]. However, the community colleges in Arizona serve a much larger population of part-time and returning students. For example, the Maricopa County Community College District (MCCCD) enrolled approximately 196,000 students in 2016–2017 on ten campuses, of whom about 40% intended to transfer to a university and 10,000 transferred 12 or more credits to a four-year institution [19], mostly to one of the three public universities in Arizona: Arizona State University (ASU), Northern Arizona University (NAU), and University of Arizona (UA). During the same period, total enrollment at ASU's four metropolitan campuses was about 72,000 [7]. The fractions of women, people of color, low-income, and/or first-generation students enrolled at MCCCD are greater than those at ASU [7, 19]. Furthermore, there are also substantial numbers of place-bound students, such as those with young children, who for family or financial reasons cannot participate in a traditional research experience for undergraduates (REU) program away from home. These local demographics motivated the authors' outreach and mentoring programs in applied mathematics, one goal of which is to facilitate students' transfer plans and increase the participation of historically underrepresented groups in baccalaureate STEM programs in Arizona.

9.3.1 Development of a partnership

The NSF DMS Mentoring Through Critical Transition Points in the Mathematical Sciences (MCTP) program funded an ASU/MCCCD recruitment network from 2013–17 for mathematically talented students who either were currently enrolled at MCCCD or who had recently transferred from a 2-year program to ASU and intended to major in mathematics or statistics. The effort focused on two critical transition points in students' studies of mathematics: (1) the transition from a two-year to a four-year program and (2) the bridge from lower- to upper-division mathematics.

About thirty students from MCCCD each year were selected to participate in a 3-week summer program consisting of three modules lasting four days each, plus one day each week at ASU for various mentoring and research presentations. The course modules included biological data analyses, graphics and animation, population dynamics in ecology, programming and introduction to partial differential equations, and the mathematics of gaming. This pre-REU program served as an enrichment to the regular MCCCD curriculum, and the participants got first-hand information about honors and undergraduate research opportunities at ASU. Overall, about 65% of the MCCCD participants matriculated at ASU, NAU and UA. The MCTP program also recruited students who had already transferred to ASU to participate in an 8-week summer research effort to facilitate the transition between elementary and advanced mathematics courses.

The MCTP program began a fruitful collaboration between mathematics faculty at ASU and MCCCD. The course-module approach also showed that there is a sufficient pool of students at local two-year colleges who have the preparation and initiative to participate in a full REU experience. The present program, described next, evolved from the MCTP pre-REU program.

9.3.2 The (AM)^2 REU

The ASU-MCCCD Applied Mathematics (AM)^2 REU is an undergraduate research program geared towards analytical and numerical studies of problems that arise in engineering and applied sciences. It offers an immersive summer experience for community college students that includes open-ended research questions involving computer programming and data analyses in a small-group setting. One goal is to help students make informed decisions about whether a bachelor's or graduate program in STEM is a good fit for their scientific and career interests. We hope that similar programs can create a bigger pipeline for community college students into STEM programs.

The (AM)^2 REU program contains two integral components. The first component is a 2-week course module developed and hosted by the mathematics department at Scottsdale Community College (SCC) that covers an introduction to programming and a mix of interesting problems. Each week is a separate mini course that runs half-day (8:30–noon) every day. The mini course offers a hands-on introduction and includes visits from research mentors at ASU to describe the upcoming summer research projects in more detail. The second component is an 8-week research program hosted in the School of Mathematical and Statistical Sciences (SoMSS) at ASU, when the community college students join additional REU participants from other four-year institutions.

Each SCC course module emphasizes interdisciplinary connections with mathematics. They are aimed at students who have taken at least two semesters of calculus and preferably calculus III and/or differential equations/linear algebra.

One course module focuses on population dynamics modeling in ecology, epidemiology, and cell growth using a hands-on problem-solving pedagogical approach. Students apply a variety of mathematical tools, primarily from dynamical systems theory, to analyze a problem. The curriculum prepares the students to recognize biological relationships expressed as dynamical models, translate those mathematical models into MATLAB® code, perform straightforward phase-plane analysis of discrete-time planar maps, and derive biological insight from computational and phase-plane analysis of a given biological system.

Another course module is a very brief introduction to partial differential equations and numerical solutions using MATLAB®. Over the course of the week, students learn about finite-difference schemes and program them to explore the transport equation and the heat equation. Applications include various sets of initial and boundary conditions and sources placed at different locations on two-dimensional spatial grids.

Other course modules include statistical analysis of biological data, which focuses on models of physiological processes from invertebrate thermoregulation, metabolism, and water homeostasis. The topics depend in part on the subsequent REU projects and are intended to provide sufficient background to interested students to get up to speed quickly.

At the end of each course module, students report on their progress in a capstone, symposium-style setting that includes their peers and instructors. Teams give 20-minute talks to summarize their analyses and conclusions. For many students, the capstone is their first public-speaking experience or their first oral presentation on a scientific topic. The communication aspects include graphical presentation of data, organization of a scientific talk, poise, and mechanics of managing computer slides with audience engagement, and these are emphasized throughout the course modules and the subsequent summer REU.

The second component of the $(AM)^2$ REU, which is the 8-week ASU program, follows immediately afterward, from late May until mid-July. The projects vary, depending on the participating faculty, but generally include a mix of mathematical modeling, numerical programming, data visualization, error analysis, and uncertainty quantification. The intent is for each project to form an umbrella under which teams of three or four students can find a niche and pursue a sub-problem. The teams are selected based on a survey of students' interests, irrespective of whether they attend a two- or four-year program; generally, we have been able to accommodate everyone's first or second choice.

Each student team is led by an ASU faculty mentor. In most cases, the faculty members' graduate students and postdocs who are pursuing work on related topics are encouraged to participate in the research meetings. Typically, then, each REU participant becomes part of a vertically integrated research team. Also, several faculty members already work collaboratively in their own research, which also facilitates REU student mentoring.

Project topics reflect SoMSS faculty interests, but they typically include mathematical biology (e.g., modeling of brain tumors and prostate cancer), geoscience and sustainability (e.g., the urban heat island), and transport problems (e.g., phytoplankton in ocean gyres and urban ozone pollution and transport). Interested students have the opportunity to run associated models (such as the Weather Research and Forecast model) on the ASU supercomputing cluster.

During the first week of the ASU program, all students meet together with an instructor for tutorials on the use of Unix/Linux operating system, scientific editing with LaTeX, etc. A short course on differential equations, linear algebra, dynamical systems and numerical errors, including computational exercises, extends the SCC program and complements the other participants' prior applied mathematics experience.

The research portion begins with a literature review, led by the respective faculty mentor. Typically, students and faculty mentors meet every day for 3–4 hours for the first two weeks. Faculty and graduate students closely monitor each student's progress. As the students become more independent, the supervision relaxes to 2–3 meetings per week, where faculty mentors check progress, assign tasks, and discuss results, much as occurs with graduate-student research groups.

Irrespective of their particular project, all $(AM)^2$ participants work in a space that is reserved exclusively for them (REU held virtually in 2021 and 2022 due to the pandemic). The community environment is intended to facilitate scientific discussions, personal friendships, and peer support. Students prepare mid-term progress talks by the fourth week of the program in which they present an extended introduction to their research problems and outline their research plan for the remaining time. By the end of the summer, each student rehearses and delivers a self-contained, 20-minute talk that presents at least a small new result. Interested students may continue their research activities following transfer to ASU and to present their results at relevant conferences (which have included the Council on Undergraduate Research REU Symposium, the Southwest Undergraduate Mathematics Research Conference, and others), and, in a few cases, publications in refereed journals.

Additional enrichment activities are offered throughout the 8-week period. Guest speakers include graduate students who describe their own experiences; faculty and staff from other colleges describe STEM research opportunities at ASU, and there is a one-day workshop on career preparation, including résumé writing, LinkedIn profiles, and interviewing skills.

9.3.3 Community college student performance

With the vertical integration of students, it was originally thought that students from 4-year institutions would become the leaders of each group, and tackle the hardest research questions. However, over the course of four cohorts of participants, it is observed that many of the community college students take strong initiative to lead their research team, by organizing group meetings, taking the most challenging problems, discussing the most with graduate student assistants and faculty mentors, presenting as lead speaker at group presentations, and following up after summer to get the research results into journal publications. This is probably due to the fact that these students are more mature and are determined at achieving their goals. So far, after transferring and obtaining their Bachelor's degrees, six community college participants have entered graduate programs. Although not all community college students are as self-motivated, they still actively collaborate with their team members and produce tangible research results. It is also observed that students with only calculus II preparations do spend more effort to fulfill their research duties but nevertheless complete the program.

9.3.4 Things to consider for a similar partnership program

Here, we outline some of the factors that we believe are important to running a successful REU program that includes students from 2-year and 4-year institutions.

Interested faculty should develop a fitting partnership (target audience, appropriate topic and requirements) in your local/regional community as preparation for an integrated REU experience to include community college students.

The program should be appropriate for the level of academic preparation of the targeted student population. Prospective REU programs should understand the available transfer pathways, associated rates of student success, and have built a rapport with the community college faculty who will do much of the student recruitment. We have chosen research topics that provide exposure to problems that we hope will appeal to a broad audience. Our 2-week course-module approach is an attempt to provide students with a smooth transition from a course-based to a project-based mindset. Depending on the pedagogical issues that the target student population faces, a more tutorial approach or a just-in-time learning strategy might be appropriate. Due to the fact that community college students may be enrolled part-time and take less courses per semester, the time interval between a certain prerequisite course taken and the actual time of participation in an REU may be longer than four-year students. As such, mentoring faculty (or their graduate student assistants) should be prepared to spend more time to help the students refresh relevant content; break down conceptual ideas into implementation steps, including numerical codes and their physical interpretations, so students may realize the connection and develop their own understanding; and give students enough independence to come up with their own points of interest to explore in a project. It is also beneficial to integrate the students into vertical research teams that consists of other undergraduate students to form peer support, and possibly have graduate students/postdoctoral associates meet on a frequent basis to help on the projects, so the students will stay on track with technical details. Depending on the participants, peer support can be very important to reduce student anxiety when they may feel that they are working alone, without any help other than the professor/mentor, and improve the success of the REU program.

The projects include open-ended problems suitable for community college students, but can be expanded to PhD dissertations. We find that motivated students are able to understand and articulate the rationale for the particular research project (e.g., why are night-time temperatures getting so hot in the Phoenix urban core during the summer?) Although full details of numerical weather forecast models are too complicated for any undergraduate student, for example, it is feasible to get community college students to understand *what* is being computed, how best to visualize the model output, and how to highlight the most important results to a general audience in a limited amount of time.

Students will need access to the local wireless network, so arrangements must be coordinated with local information technology personnel. Commercial software platforms like MATLAB® are well tested and offer technical support, but they also require licenses. Many universities have site licenses, but two-year colleges generally do not, so one must budget accordingly. Many open-source alternatives, such as Octave and Python, have large user bases, and technical assistance often is available from various online forums; nevertheless, installation problems on students' laptops may not be easy to remedy without specialized expertise. Depending on the institution, one may need to budget for a certain number of hours of information technology support for the REU program.

9.4 Summary

Integration of community college students into STEM research can happen at two fronts. On community college campuses, we strongly urge math faculty to find out how fascinating it can be to work with students on a research project. Any interested community college student can approach open problems in mathematics and make progress. Because community college students have diverse math backgrounds, initial, small wins such as improved mathematical reading and writing skills, presentations, and written reports can boost the confidence of these students and develop intrinsic intellectual curiosity about mathematics frontiers. Since these research, problem-solving and document preparation activities have tremendous positive effects on students, we hope that community college faculty will continue to spread the good word of how rewarding it can be to work with students on math research, regardless of skill level or background.

Making more students aware of what it is like to work on open math problems is an idea that is catching on at community colleges. It is becoming more well known that research experiences for early college students result in significant learning gains [31]. Although in the past they have not been exposed to open mathematics problems, it is becoming more clear that community college students are much more eager to learn about new mathematics, even theoretical aspects, than what was previously thought, and community college faculty can have an important role in supporting these students through the use of research interactions as a crucial teaching tool. We believe that anyone can be guided on a community college campus in efforts to solve an open math problem or to develop a new math application. We are finding that community college students can be taught, in a counterintuitively short time frame, to understand and experiment with a mathematical research problem and typeset any obtained results in LaTeX. These students are happy to put effort into learning how to formulate, test, prove, or disprove conjectures, even if these conjectures involve terminology traditionally not seen in lower division math courses.

Offering research opportunities for community college students at 4-year institutions significantly expands the horizon for these students, as the topics can be chosen across a broad spectrum, not necessarily seen in a traditional math classroom. Students benefit from such opportunities as they see possible mathematics career pathways vastly different from secondary education. However, it is important for the partnership to note that organizing an REU with community college students as the primary audience has similarities and differences from an REU that targets students from traditional baccalaureate programs. The organizing structure, program components, research content and recruitment strategy must be adapted to local needs and academic structures. Research opportunities for these often-underserved and place-bound students may widen the pipeline for a diverse population of STEM majors.

Simply put, we must not miss the opportunity to attract community college students to the math research community.

9.5 Appendix: Opportunities for community college faculty to receive training and support to mentor UR students

The following is an incomplete list of organizations that provide such support, with websites and information specific to community college faculty UREM mentors:

a. *The National Science Foundation Research Experiences for Undergraduates Program* [24].

b. *The Center for Undergraduate Research in Mathematics* [14]: This funding and training program provided the third author's community college with mini-grants to mentor a small group of research students. Three students who participated in this CURM-funded UREM received funding for travel and supplies.

c. *The American Association of Community Colleges* [1].

d. *The Community College Undergraduate Research Initiative* [9].

e. *The Western Alliance to Expand Student Opportunities* [33].

f. *Oak Ridge Institute for Science and Education* [25].

g. *Maricopa County's STEM-CURE Project* [18]

h. *Arizona State University's "ASU's (AM)^2" program* [5]. NSF DMS Grant Number 1757663.

i. *Research Experiences in Community Colleges (REC^2)* [28]: This organization has been supported by National Science Foundation Grant Number 1541911 and provided for the Math Research Experiences in Community Colleges Conference. In addition, community college faculty such as the first author were granted the equivalent of four weeks of full-time summer support to mentor two students who worked on data science projects, specifically, generative adversarial networks, during a ten-week REU program.

j. *The National Doctoral Alliance* [27].

k. *The Pacific Math Alliance* [26]:

l. *The Mathematics Research Experiences in Community Colleges Conferences (MREC^3)* [29]: The first author has been a co-organizer for four years, through 2021 when funding for the Conference ended and during which students and faculty from community colleges have gathered to give math research presentations and discuss strategies for improving community college UREM's. The Conference is one of the activities that fall under REC^2 above, through NSF DMS Grant Number 1541911.

m. *The Research Experience for Undergraduate Faculty (REUF) workshop* [32]: This workshop is held annually by the American Institute of Mathematics and provided the third author with a week-long workshop that prepared him to be part of a math research group for over a year.

o. *Mathematical Association of America (MAA)* [21]: The third author's student who presented at Mathfest in 2018 received an MAA Student Travel Grant.

p. *Lockheed Martin Corporation* [17]: RCBC has received private funding from Lockheed Martin in support of the campus-wide undergraduate research program.

q. *Mu Alpha Theta Mathematics National Honor Society* [23]: This organization provides competition grants that have provided travel funding for students at RCBC, for example, to participate in math contests and to present research results.

r. *American Mathematical Society (AMS) Undergraduate Travel Grants* [4]: The third author's student who presented at the 2020 Joint Mathematics Meetings received support through the JMM Undergraduate Grants support program. in 2018 received an MAA Student Travel Grant.

s. *Youngstown State University Beginning Undergraduates' Mathematical research Preparation (YSU-BUMP)* [34]

t. *Preparation for Industrial Careers in Mathematical Sciences (PIC Math)*[20]

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Part III

BIG and Interdisciplinary UR

Introduction
BIG and Interdisciplinary UR

There has been a tremendous growth in the number of career opportunities for STEM majors including mathematics majors. In 2012 the U.S. President's Council of Advisors on Science and Technology (PCAST) produced its report Engage to Excel [1] describing a need to produce one million more college graduates in the STEM fields. Careers in mathematics and statistics are one of the hottest areas. In 2021, a report from the Bureau of Labor Statistics [2] stated that "the mathematical science occupational group is projected to grow the fastest among all STEM occupational groups." To be successful in a career in business, industry, and government (BIG), students need to develop such skills as solving open-ended problems, learning new materials on their own, taking complicated problems and breaking them into smaller pieces to solve individually and then putting all of the components together to get a final solution, working in teams, and communicating with others. These are the skills students learn by doing undergraduate research.

To help mathematics students be better prepared for BIG careers, it is advantageous if they had experience working on actual problems that come from industry. Such a setting provides students with taking a problem that is phrased in everyday language and thinking about it in terms of mathematics, statistics, and computing. Then the students need to figure out what mathematical tools from their collection can help solve the problem. Finally, after solving it, the students have to rephrase the solution in everyday language that non-mathematicians could understand. The PIC Math program [3] has shown that problems from industry that students can work on are numerous and readily available. Companies often have potential research problems, especially problems related to the analyzing of large amounts of data that the companies have collected but don't have the time or expertise to analyze. The difficulty is that many mathematics faculty are used to working in theoretical areas of mathematics. So to get access to these problems from industry, faculty mentors have to step out of their comfort zone and reach out to companies and businesses.

In this section we see how faculty have achieved this. The four co-authors of Chapter 10 have created and directed successful undergraduate research programs where the research are problems that have come from business, industry, and government. They provide suggestions on how to get started in doing BIG undergraduate research. In Chapter 11 the authors describe a UR program in which undergraduate students work on international applied math problems from Brazil, Colombia, India, and Tanzania. Finally, the interdisciplinary REU at the National Institute for Mathematical and Biological Synthesis (NIMBioS) is detailed in Chapter 12, and the authors discuss what they have learned in improving the training of students for interdisciplinary projects.

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10

Data-driven Undergraduate Research Projects with Business, Industry and Government

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10.1 Introduction

Industries hiring mathematicians want their employees to have experiences that extend beyond the classroom. Some students have the opportunities to attend summer REUs (research experiences for undergraduates) or internships; however, many students cannot take advantage of these summer experiences. Therefore, opportunities to participate in nontraditional activities during the academic year are an attractive option for many students. The authors seek to promote collaboration with business, industry and government into undergraduate research. Whether you are considering the research experience offered within a semester long course or supporting students in participation in data science competitions with problems provided by industry, there are several considerations to keep in mind.

Collaboration with business, industry, or government (BIG) in undergraduate research can provide excellent opportunities for students in terms of novel research and professional development. This chapter will provide suggestions for establishing successful collaborations with BIG on your next undergraduate research project. Before discussing the benefits of bringing industry into undergraduate research, let us define industrial mathematics, research in industrial mathematics, and how it differs from traditional undergraduate research.

10.1.1 What is industrial mathematics?

Industrial mathematics is concerned with transforming technical, organizational and economic problems posed by business, industry or government into mathematical problems; solving these problems by approximate methods of analytical or numerical nature; and reinterpreting the results in terms of the original problems [10]. Projects in industrial mathematics are inherently interdisciplinary. In addition to mathematics, they could include subjects from fields such as business, data science and engineering, and could teach students how to apply mathematical analysis to problems arising in these areas [6].

The modern and interdisciplinary realm of projects in industrial mathematics fits with the original intellectual and creative contribution components of the definition of undergraduate research by the Council on Undergraduate Research [12]. Industrial problems typically have not been previously solved and can be solved using many different approaches.

10.1.2 Benefits of research projects with industry

Undergraduate research projects with industrial partners combine the essence of academic research and hands-on practice outside of the university. The problems have an educational impact that cannot be replicated in the traditional mathematics classroom setting. They can offer a view on the importance and the impact that mathematics can have in providing solutions for problems in industry and business [5]. Undergraduate research in BIG is similar to traditional academic undergraduate research in that it focuses on student growth and letting the students interact with material at a level that goes beyond what they do in the classroom, and introduces students early in their studies to the own discoveries, stimulating further inquiry via open investigations, problem-based learning and project work. However, it also offers the student the chance to work on actual problems from industry.

Industrial mathematics projects develop strong analytical and problem-solving skills built upon a background of computing, mathematics, statistics, and basic science. In addition, industrial mathematics emphasizes written and oral skills along with teamwork, skills which are valued highly in industry, but are not typically emphasized in traditional mathematics programs.

Industrial research experience could help bridge the gap between the problems students learn from textbooks and the problems in industry [4]. Students who engage in research with problems from BIG gain additional experience in:

- Querying clients and finding current baseline solutions to problems under concern;
- Understanding the differences between valuable pragmatic solutions and theoretical solutions;
- Working with existing computer codes and algorithms;
- Communicating effectively to a diverse audience using appropriate data visualization tools.

Keeping all the benefits in mind, consider the possible challenges to make successful collaboration with industry in undergraduate research. We consider three major challenges to beginning research with BIG partners:

- Differing nature of undergraduate research projects in BIG;
- Initializing the successful collaboration with BIG;
- Incorporating undergraduate research project with BIG into curriculum.

10.2 Nature of industrial research projects

The main objectives of research projects with BIG are to

- understand the industrial problems originally posed in a non-mathematical language,
- transform them into appropriate mathematical framework,
- find possible solutions of these problems by suitable methods which may be analytical and numerical in nature,
- test the accuracy, validity and reliability of obtained solutions,
- interpret (or implement) the realizations in terms of original industrial problems [11].

Hence, industrial-based research projects are different than typical academic research projects. These projects

- are typically not in the faculty mentor's area of expertise,
- require a broader background in math/stat knowledge because the problems could be from a wider range of areas,
- are such that students may not have the background to understand the business/industry thinking or approach behind the problem.

Additionally, the faculty mentor's role is different. The faculty mentor serves more of a coach guiding the students than a content expert. Hence, students cannot rely on the faculty mentor to be a content expert providing them content-related answers when they want/need them. Students may feel uncomfortable not knowing how to proceed, but this is a good thing because this is exactly the type of skill students need when they work in a BIG career. In an industrial-based project, there is a clear objective or question the company wants answered. In this type of problem, students must use the concepts they have learned throughout their undergraduate curriculum together with their critical thinking skills they have developed along the way, as well as their ability to learn any additional skills necessary, to answer the specific problem the company wants answered. They must be able to present their solutions with the business goals in mind. Therefore, communication is a key component in these projects. Students need to learn how to present enough of the mathematics behind the solution to give the answer credibility while keeping in mind that the audience is not typically comprised of mathematicians. Therefore, the students need to learn how not to focus on the mathematical details the way they might in an undergraduate research project or in a research presentation, but to instead communicate how the mathematics and statistics lead to actionable solutions of value to the company. In industry, companies might not be as interested in the mathematical or statistical techniques utilized to solve the problem as in how these mathematical insights lead the company to actionable responses for the business. This is not to say that the mathematics and statistical concepts are not important; they are. However, for companies, mathematical techniques are important because they give credibility to the decisions being made by the company.

10.3 Challenges arising in BIG research projects

In addition to the challenges posed by the differences in industrial projects when compared to typical academic research projects, the faculty mentor may experience the following challenges:

- finding suitable research problems from industrial mentors,
- recruiting students into the course,
- effectively managing the student teams and setting appropriate expectations,
- effectively assessing student teams, and
- sustaining the course and maintaining continuity in the program.

For the remainder of the article, we will address how we addressed these challenges in our courses and offer some tips for those planning to incorporate BIG projects into their curriculum.

10.4 Examples of possible projects

There are several key things to keep in mind when seeking appropriate projects. Most importantly, the projects need not be crucial to current business operations. Undergraduate students are still learning, and the industrial, project-based course is another learning opportunity. If your students are working on a BIG research project for just one semester or as part of a one-semester course, then in some cases students will make great strides on a project and produce remarkable results in such a short time period. In other cases, students get bogged down in the details and do not make the progress they had hoped to make. Therefore, in seeking out projects, faculty want to be upfront with their potential business partners and encourage the potential partners to consider projects that might be on the back-burner for the company, i.e., projects in which additional insight would be nice but not crucial to their business. This will help keep expectations reasonable.

Additionally, the projects need to be ones in which mathematical and statistical concepts can be used to effectively provide insight into the problem. For many projects, data are fundamental to the project. In some of the projects discussed below, the data were provided directly by industry; however, in many of the projects, insights were garnered using publicly available data, thus eliminating the need for non-disclosure or confidentiality agreements that can be time consuming to obtain. The project often involves a large amount of data cleaning and preprocessing, which surprises students but is more representative of what they will find when working in BIG upon graduation. Some examples of projects the authors have encountered include:

- A chemical company is using spreadsheets to plan purchases, production and inventories for supply chain planning. Can students make planning more efficient? What recommendations would be made in several different business scenarios where portions of the supply chain might need to be shut down for a portion of time due to maintenance? (Key mathematical content: operations research and linear optimization techniques)
- A supply company transports its supplies to customers in bulk by rail cars that it owns and operates. Each rail car costs $x per year to operate. If there is no rail car available to ship the product once the order arrives, the order might be lost to competitors. However, buying additional rail cars can be costly. In several different scenarios, what rail car fleet size would be recommended given estimated demand while incorporating uncertainty in the demand estimate? (Key mathematical content: optimization techniques and uncertainty analysis)
- Many restaurants and other types of businesses are actively looking for information that they can leverage to influence customer behavior and increase sales, minimize costs and increase profits. Understanding customer experience can help identify the needs and any gaps in existing products or service offerings. Yelp reviews are

a popular source of feedback for restaurants in which reviewers can rate their experience on a 5-star scale and provide additional comments via text. However, many social media sites also have unstructured, underutilized data. Is it possible to predict a quantitative star rating for a Yelp review using the associated text data? If so, it might be possible to further leverage unstructured social media text data by transforming it into quantitative data the company can more easily interpret. (Key mathematical and statistical concepts: text mining and machine learning)

- Risk stratification of populations allow healthcare companies to focus their efforts on the most impactable patients. Is it possible to determine key indicators for predicting rising risk and future costs by reviewing the risk stratification algorithm? (Key mathematical and statistical concepts: statistical correlation and machine learning)
- Using text data from restaurant reviews, is it possible to predict a potential issue from an unclassified review? If so, one can utilize such an algorithm on unstructured social media data to alert a restaurant to specific issues. Can one predict a sentiment of a text review? (Key mathematical and statistical concepts: statistical correlation and machine learning)
- A local company has started using social media to market its services. Given different types of social media posts (problem solutions, birthday wishes, event announcements, etc.) on different platforms (Facebook, Twitter, and Instagram), what types of posts are most significant to the company's website traffic (if any) and what is the best time/day to post to receive the highest number of views? (Key mathematical and statistical concepts: statistical correlation and machine learning)
- To keep up with the need and demand for field crops like maize, genetic engineering has become a popular practice to increase crop yield. Is it possible to identify the geographic growing locations that are most comparable based on their growing season for maize? Within those groups, can one identify the phenotypic and genotypic features that are more significant within similar regions? What are the most predictive variables for grain yield? What type of feature engineering is most beneficial when applying machine-learning algorithms to predict maize yield? Note: this was also a combination of multiple projects for one company. (Key mathematical and statistical concepts: statistical correlation and machine learning)
- The local police department has not realigned its police beats for over 20 years. During that time, the population and crime in the city shifted substantially, leaving officers on some beats with heavy workloads while officers on other beats with lighter workloads. What is the best way to measure workload of officers? How does local geography influence the construction of beats? What recommendations can you make to the department? (Key mathematical and statistical concepts: data analysis and mathematical modeling)
- The local street department has been criticized for inadequate response plowing the city streets after snowfall. Given data on population and traffic counts, as well as the location of schools, hospitals, and other critical facilities, are more efficient routes possible? (Key mathematical concepts: operations research and graph theory)
- Given call data to the local fire department, is it possible to recommend placement of fire stations around the city to meet response time requirements as best possible? (Key mathematical concepts: route optimization)
- Area non-profit agencies have data that may be useful in justifying support from local philanthropic agencies. Can one assess the agencies' neighborhood stabilization efforts by examining crime, property, and demographic data in neighborhoods where these agencies intervened as compared to control neighborhoods? Is it possible to predict housing vacancy based upon publicly-available data such as number of property transfers, age of property owner, neighborhood market type, and tax delinquency? (Key statistical concepts: regression and data analysis)

As shown above, not all projects need to come from profit-bearing companies; there is also a great benefit to working with non-profit agencies. The students feel empowered and enthused to work with a public sector agency as it promotes the feeling that the students are giving back to the local community and providing valuable service. In addition, it increases the students' and agencies' awareness of the power of quantitative analysis and the benefits of hiring individuals with such quantitative and analytic backgrounds. Many of these projects also present ethical dimensions. For example, the project on policing raises issues in social justice that can be discussed among the team and the faculty mentor. In general, these projects provide the faculty member an opportunity to discuss ethical uses of data and modeling and the standards set forth by professional societies that guide such work.

Effective problem selection relies upon recognizing the mathematical or statistical techniques required to solve the problem and how these techniques match with the constraints of the course and students enrolled. It is not always clear that a chosen problem will be successful. Some problems are not well-defined at the start but take shape as the students' discover more about the problem. Others are well-defined but may lead to unforeseen roadblocks or difficulties. The key is to be flexible and willing to take a chance on a project. Many projects may seem like the mathematics or statistics required is not sophisticated, but these projects still provide an important learning opportunity for students.

10.5 Getting ready

All authors started their undergraduate research with industry through participation in NSF-funded MAA PIC Math (Preparation for Industrial Careers in Mathematics) program. This program trains and supports faculty to teach a semester course at their own institution in which students work as a group on a semester-long undergraduate research problem from BIG. PIC Math also builds a network of engaged faculty all over the states, who are helpful and supportive. You can find a list of faculty participants on PICMath website [9].

A majority of the projects involve data science techniques in some form. One way to gain some experience with those is through the Mathematical Association of America (MAA) workshops on data science. These excellent hands-on workshops give a broad overview of the fundamental techniques and software used to solve data science problems [8].

As a means to provide additional experience for students to prepare students for a BIG research project, offer opportunity to students not able to engage/commit to a semester long BIG project, and/or supplement development of skills during or after taking a BIG research project, faculty are encouraged to support student teams engaging in data science and analytics competitions. A quick web search will return a host of annual competitions open to high school, undergraduate and graduate students sponsored by local, regional and national nonprofit organizations such as MinneAnalytics [1] and Women in Data Science [2], by independent colleges or universities, or offered by corporate sponsors such as Adobe Analytics. In addition, many mathematics departments support student participation in the COMAP Mathematical Modeling competitions [3]. In recent years, COMAP added MCM Problem 3: data insights to the competition. Competitions such as MinneMUDAC, and the WiDS Datathon offer opportunity for experiential learning of working with real world data problems connected to opportunity to network with professionals in the field.

10.5.1 Finding a BIG partner

For some faculty, finding a BIG partner is an easy task. Their universities may already be actively collaborating with industry in their graduate programs and want to shift some of those partnerships to an undergraduate level. However, for most faculty, this will feel like a daunting task. If this is your case, focus on contacts in other areas of your life.

- Do you know anyone from your graduate school years who has gone to work in industry? This may lead to an ideal partnership as you and your contact have similar backgrounds and already know each other.
- Is there a speaker series at your university where you can bring in potential speakers from industry for your students? If so, you might approach the speaker after their colloquium about starting a potential partnership.
- Do you have family members or close friends in industry? Do they have any potential projects that might work well for your undergraduate students?
- Do your colleagues have personal contacts that might be beneficial in making that first introduction?

Networking is key to forming these collaborations. Although it is possible to reach out to a company without prior contact, it is easier when there is a personal connection or an introduction to start the conversation.

Public safety agencies are another potential contact for projects and data. Try to call them and talk to the department heads or directors of community outreach. Before making such a call, have a potential project or examples in mind.

10.5.2 Talking to a prospective BIG partner

When talking with a prospective BIG partner, clarify the timeline of the project right away. Companies usually look for the best solution given the time allocated to the project. The industry schedule is in days and weeks and a company may ask for results in two weeks. For mathematicians, the schedule for finding optimal solutions is often in semesters and years!

Be realistic and upfront about potential outcomes of the collaboration. There will be probably no final solution offered by the end of a course. You can promise steps towards the solution and preliminary analysis of those steps. Keep in mind that a company may value an analysis that says “doing such and such does not work”. This could be a challenging issue for students who may think that the result must be something positive. You will have to remind them that showing that a proposed method is not working for a given problem is also an achievement.

Have reasonable expectations about the abilities of your students and be frank about their abilities with the BIG partner. It is important to distinguish the knowledge of theory and ability to try different approaches found on the internet to solve the given problem. Do not underestimate your students. They could be very fast learners. However, they may not be able to grasp a long list of advanced topics beyond the undergrad program.

After the first meeting, follow up via email in about a week for project description. In general, the project description should have, at least, a short introduction of the company and a problem statement along with goals. Try to work out a project with multiple stages and checkpoints. This allows for a mutually positive experience even if only some parts are completed. The details of the description of projects may vary, reflecting the actual environment. Some descriptions could be very detailed, some could include only general information and students should put an effort in communication with industrial partners and request any additional information.

10.6 Incorporation of undergraduate research with industry into curriculum

As with traditional undergraduate research, projects with BIG involved could be a part of the course (project based or Capstone), standalone research project, competition or summer research. Most challenging could be a course-based scenario and we will focus on that.

10.6.1 Administration of the project

Playing the role of a facilitator, the faculty mentor guides students carefully through their experience, keeps teams as independent as possible, foresees possible difficulties and lets them explore possible solutions. In addition, the faculty needs to communicate with a BIG partner, at least initially, as we discussed . Some other responsibilities typically associated with a research project are shifted to industrial partners: confirming strategies and answering all questions about the intellectual part of research during regular meetings or over email. This could be a big relief.

10.6.2 Communication with BIG

It is crucial to make sure the team does not stray away from what the industrial partner wants. Most of the time all communication with BIG partners goes through a faculty mentor. Once you are comfortable with the students’ level of professionalism, you could ask an industry partner if students could communicate directly to the partner via email or project management software.

If not explicitly designated in the syllabus as a requirement, many teams try to avoid the interaction with the BIG partner. Regardless of how intimidating it may seem, the students are responsible for providing updates and asking questions of their industrial liaison. The faculty has to keep an efficient and constant contact with industrial partners parallel to the project to communicate any possible issues and ways to avoid them. Ask the BIG partner for feedback after each meeting with the students. It is one of the key factors of success.

The required meetings with BIG should include: a first introduction meeting, second Q&A meeting, midterm meeting, first draft meeting, and final meeting. This is about 6-8 hours of meetings per 16-week term. Bi-weekly meetings with BIG appear optimal.

10.6.3 Recruiting students

There are various ways to recruit students. A discussion with students regarding undergraduate research in your regular math classes is a very important avenue to promote an interest among students. Reach out to student organizations and clubs. Share information with program coordinators and ask to send an email to all math major/minor students. Invite students who you know and who might be a great fit for this course. Current research students can be the most effective recruiters for potential new students.

Keep in mind that many students already go over the required number of credit hours to complete their degree. Thus, if your department does not already have a capstone course, it helps with student recruitment if the students can substitute your course for another that is already required of them rather than taking it as another elective course. Just keep in mind that your administration may not very enthusiastic about making changes to the curriculum right away. Before your course becomes a regular course, it might be just an independent study course, even on top of your regular teaching load. Over time, the benefits of the course and positive student feedback and demand may lead to a dedicated course.

Do not restrict your recruitment to math majors only. From authors' experiences, students majoring in physics, biology, computer science, engineering and business enrolled in and benefited greatly from the course. Moreover, math education students also found this course intriguing, because they knew they could take the knowledge from this course and share with their future students how they could and have used mathematics in industrial jobs.

10.6.4 Class organization

There are many options on how to organize a research project class. The organization of a course depends on the number of teams and the number of projects. One project with several teams is easier to organize: more projects means more meetings with industry. That could be challenging to organize since they cannot be at the same time.

There are several general tools for success that can be applied for any undergraduate research, but essential for a successful collaboration with industry partners [7]:

- Set expectations early about independence and your role.
- Have clear ongoing deadlines throughout the term.
- Pilot each stage of the project separately.
- Establish frequent checkpoints.

Each team will work closely with a faculty mentor and an industrial partner. Progress tracking and scheduling of the project could be organized through a project management software. It helps to track progress with checkpoints and allows all to effectively share files among all participants. In general, students are required to attend all meetings, group meetings, and industrial partner meetings.

Students are expected to do research outside of class meetings. One of the options is to split the project into stages, two weeks each: background research, initial strategy selection, research on possible implementation, attempt to implement, first results, possible solution building, validation, visualization, and finalization.

Each stage starts with a catch up meeting on status of progress, dedicated for asking questions and raising any concerns. Follow with active research sessions or brainstorm sessions administered by a faculty mentor. Engage every student to participate and lead the discussion. By the end of each week there could be a presentation session, alternating a training presentation for peers and presentation for an industrial partner. The faculty mentor provides feedback and makes suggestions to improve after each peer review session. The general discussion with peers could be initiated afterwards. Presentation to industrial partners is followed by feedback from industry and confirmation on the path of the progress. Getting a confirmation after each stage could be very helpful to stay on a path and not proceed in the wrong direction.

10.6.5 Structure of class

When forming the teams, it is advisable to have the projects determined well in advance of the semester so that the instructor can recruit students with varied backgrounds who offer different viewpoints and angles from which they approach the problem. Although this is difficult in the mathematics classroom, we try to get students with varied majors involved in the projects to give students this experience. Tom and Michele designed their courses based on teamwork aligned with the PIC Math program guideline: Students are divided into teams of 3-5 students during the first week of classes. These are the teams for the entire semester. Teams of three students seem to be ideal as they need to coordinate their schedules (class and work) in order to not only meet during the scheduled class time but also at other times during the week. Moreover, they seem to be able to collaborate easier and have more equal input. Students are allowed to make suggestions about potential teammates, but much of the consideration is given towards the interest in the various projects and making sure each team has a potential leader as well as at least one team member who is more confident in his/her skills as a programmer. All team members are expected to be active in all areas of the project, but they are able to learn from each other when the team is composed of members with diverse skill sets. Moreover, some students forget or want to forget that this is a TEAM project. As such, groups should work as a TEAM! They are graded on how well they actually collaborate and work as a team. This is meant to help build the skills necessary to be a good colleague when they start their career. Therefore, each student is graded individually by how well they are working as part of a team.

10.6.6 Student assessment

- **Weekly or biweekly progress reports and personal evaluations**. This is very important for team-based classes. Each team member is expected to carry their own weight within the group. If they don't carry their weight, they are jeopardizing the project and being disrespectful to the industrial partner. It includes the tasks of the group for the week and how much effort they and each group member put into the project. The evaluation includes percentage evaluations of the effort of each group member for the past week or two, summing totally to 100%. Ideally, if there are four team members, each member is contributing 25% towards the group project. These evaluations are meant to try to prevent the extreme cases where one or two are carrying the full weight of the project.
- **Weekly team presentation.** Each team is required to deliver weekly oral presentations: one week for peers and in the following week to industrial partners. Students gain valuable practice in presenting their work. Every team member is required to deliver a presentation on a rotational basis emphasizing their individual contribution to the project. This activity tries to simulate presenting the project to a business partner who is not intimately involved in the project. The other groups do not know all the details about other projects, so the presentations have to be clear and concise and given in a way that even those outside their area of expertise can understand. The presentations are graded by the students in the other groups on multiple skills and given textual comments by their fellow classmates outside of their team. They can then take these comments (and scores) and know which areas they need to work on to improve their communication skills. During the course of the semester, the ability for each and every student to grow in their communication skills is remarkable. The intermittent presentations also help the students to prepare for their ultimate final presentation for the business liaison at the end of the semester.
- **Final presentation.** Each team is required to deliver a final presentation to industrial partners summarizing their experience. This presentation highlights the project description, defines the objectives, and describes the implementation process and results. We also ask each individual student to prepare a 12-minute video of the final presentation.
- **Final report.** This is the major deliverable for the evaluation. The report has to describe the problem, background literature, approaches taken, results that were obtained, and questions motivated by the research. You should encourage students to begin writing parts of this report as early as the third week of the course, so that you can assist the students in developing a proper style. Final reports are to be submitted to industrial partners. The feedback on the final report could be shared with students afterwards.

The completion of all activities guarantees a passing grade. Teams with publishable results will be encouraged to prepare a conference and/or journal paper. However, not all undergraduate research with BIG leads to a publication. The publication possibility may be limited due to industrial partners involved. Some companies do not want to distribute the results to protect their intellectual property. Some companies might want to sign IP release forms and limit dissemination of the project.

Additionally, you could consider several other assignments, which enable the students to further think about their careers. The homework assignments are short in nature and meant to provide help to them instead of simply being busy work. Ideally, they will make students more competitive in the job market.

- Biweekly Time sheets. It will keep track of the time spent on the project. The actual time spent on each of the following areas are recorded and submitted biweekly: team meetings outside of class, literature search, programming, report writing, presentation preparation.
- Career opportunities summary: students are given multiple different websites which contain a variety of careers. They are asked to look through the careers and write a summary of 2 careers which they find interesting.

- Internship, job or graduate school opportunities summary: If the students are juniors, they are asked to research potential summer internship opportunities. If they are seniors and planning on graduating in the spring semester, they are asked to research either job openings or graduate schools, depending on their interests. They are asked to find three potential opportunities and explain what the opportunity is, why they are interested, what they might need to do to be more competitive for the opportunity, etc.
- Resume: A representative from career services is invited to one of the class meetings to give a talk on resumes after which students create and then revise a resume based on the professor's comments. This is to aid the student in preparing a competitive resume for when they graduate.
- Personal statement: Students are required to write a personal statement. Whether they are applying for graduate school or looking for a job, they can think through why they are passionate about what they plan to do upon graduation. They are asked to show their passion, give personal insights, and give the reader insight into why they would be perfect for the job/graduate program and why this opportunity would be perfect for them. They write and revise the personal statements after feedback.
- Interview questions: Students are given a list of multiple questions that a potential employer might ask. The professor then provides feedback which might help make their responses better in some way or say what they like about their current responses.

10.6.7 Sustainability

Sustaining courses and research experiences of this nature can be challenging. It is recommended to involve more than one faculty member in the course if possible so that other faculty members can be trained and feel comfortable mentoring students in this setting. If the course can be used to satisfy a graduation requirement (such as a capstone), it will also increase faculty, student, and administration buy-in. Successful outcomes in the course can be publicized in local media, which may assist in obtaining industrial sponsors for future course offerings. Additionally, previous industrial sponsors may be willing to continue to participate updating the problem or providing a new problem. This makes finding problems and industrial partners easier, which, at the beginning, is a time-consuming task. Another way to make the program sustainable and make problem identification easier is contacting alumni who took the course or research experience and encouraging them to participate as industrial liaisons. We found that many program alumni are excited to return, participate, and mentor a new group of undergraduate students.

10.7 Conclusion

We hope this overview is helpful when adding industrial collaboration into your undergraduate research activities. It could be really an amazing opportunity to explore new horizons for you and your students. Once again, we should ask ourselves: why do research with undergraduate students and why involve industry in it? Personally, we think the most satisfying aspect of a math professor's job is to see the real growth of students, the passion in their eyes, the simple joy of small achievements, and the pride in their work. In the end, this is what we do: provide students with unique opportunities that they will remember for many years after graduation and prepare them for astonishing industrial careers.

There are many different ways to expose and to engage students in the high-demand field of industrial mathematics and data analytics. If you are unfamiliar or uncomfortable advising students on BIG projects, we encourage you to take a risk, step outside your comfort zone, and give it a try. It is an incredibly beneficial experience for both the students and a faculty member and there is an ever-growing community of faculty who are available to support you in this endeavor.

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11

From Context to Habit: A new CMATH Framework for Undergraduate Research

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11.1 Introduction

Over the last two decades, infectious diseases such as COVID-19 and Zika, hurricanes like Maria and the continuing opioid crisis have all caused havoc in several sectors including education, health, energy, and workforce. These global incidents have not only impacted people's lives but also have reshaped the world from normalcy. While new vaccines and support have helped to alleviate the suffering and losses from these disasters over a long time,some educational good can be reaped from the widespread interest to better understand and collaborate to meet these extraordinary challenges. Research and education can turn ”lemons into lemonade”, as the popular proverb suggests.

One group that has been impacted severely by these incidents, is the undergraduate students.Not only were students adversely affected by university closures and disruption in their education, but also those who need to work struggled financially because of a reduction in hours of operation or even lose of their job. Students with financial need are disproportionately affected by the financial and emotional stress of local and global disasters. Particularly the COVID-19 pandemic has also exacerbated well-documented opportunity gaps that put low-income students in a disadvantage relative to their peers who are better equipped. Many of these students exhibit signals that they are at risk well before dropping out of college. Without a university degree, these students face a life of underemployment and uncertainty in their professional activities.

How do we contribute to retaining and preparing our students for their position as the next generation workforce that require proficiency in the mathematical sciences? How do we as educators engage students so that they are willing to invest in their learning, develop problem solving skills, and be equipped to face the next challenges? Undergraduate students' participation in hands-on research has been an effective way to encourage students to pursue advanced degrees and careers in science, technology, engineering, and mathematics fields [18]. There have been multiple published findings that describe how undergraduate research (UR) in mathematical sciences can help build students' own sense of agency so they realize they have something to contribute to their own education. These students are often more actively engaged in the classroom and are better prepared to meet the demands of future employers in being good communicators, collaborators, critical thinkers and creative problem solvers [6, 20, 21, 22, 8].

In this work, we present concrete examples of UR in mathematical sciences that engaged students effectively, helping them to improve their understanding and appreciation of mathematics, and develop habits of mind to address global challenges. The chapter is laid out as follows. In section 11.2, we introduce a new framework called CMATH to build competencies for solving global challenges from Brazil, Colombia, India, Tanzania and Puerto Rico using mathematical sciences. Following that section 11.3, we provide detailed ideas on how the constructs of CMATH have been used for a set of UR projects. Finally in section 11.4 we conclude by sharing how such UR practices as described in this work can help create change agents at the student, faculty and institutional levels.

11.2 From context to habit

"When will I ever use this?" It is common to hear this phrase from undergraduates who are trying to understand the importance of what they are learning in their mathematics classes and when they will be able to actually use it outside of the university setting. This becomes even more important when mentoring students in UR. To make the UR problem more meaningful to the student it should not necessarily be a problem that fits the current research agenda of the faculty mentor but rather it should also be something that the student is excited to investigate in the first place. The authors of this work have used this philosophy to mentor several UR projects that employ mathematics to solve problems specifically motivated by global challenges. The framework that has been employed includes a holistic approach to UR in mathematical sciences that engages students and faculty in the radical philosophy of "Here is the problem, find the mathematics needed to solve it" rather than "Here is the mathematics, go solve the problem." A related problem-solving philosophy from the business and engineering world is a user-centered approach to problem solving in product development called Design Thinking which is an analytic and creative process that engages a person in opportunities to empathize with users, define problem statements, ideate feasible solutions, create simple prototypes and test the product [4]. While it sounds like a framework for product development in business, there is a lot one can learn from this to conduct research in mathematical sciences to solve global challenges which the authors have used over the years [13, 16]. To explain how this was done over the years, let us introduce a new educational framework for UR in mathematical sciences called **CMATH**.

To engage students in UR involving global challenges, first it is important to introduce them to a real-world **C**ontext. The United Nations Sustainable Development Goals - SDG (See Figure 11.1) which are a collection of 17 interlinked global goals designed to be a "blueprint to achieve a better and more sustainable future for all" provides such a context [19]. One way to engage students is to find out as a mentor which of these goals they are excited to work on.

Once the students select a goal they would like to contribute and make an impact towards, the next step would be to introduce them to the art of developing a **M**odel. This requires the student to understand and define a problem statement that then helps to develop a mathematical model from a physical context. This model could be an algebraic or a differential equation model that describes the dynamics associated with quantities of interest (for eg. growth of a population, size of a spread etc.) associated with the problem. Once a model is developed, then one must **A**nalyze this model to investigate and theorize appropriate behaviors using rigorous mathematical tools. Often such analysis leads to important estimates or bounds that can be used for comparison. As most models representing the real-world may not admit exact solutions, there is a great need to promote the use of numerical methods to next **T**est these complex models developed. This testing step often involves simulation, validation and prediction which help to evaluate the

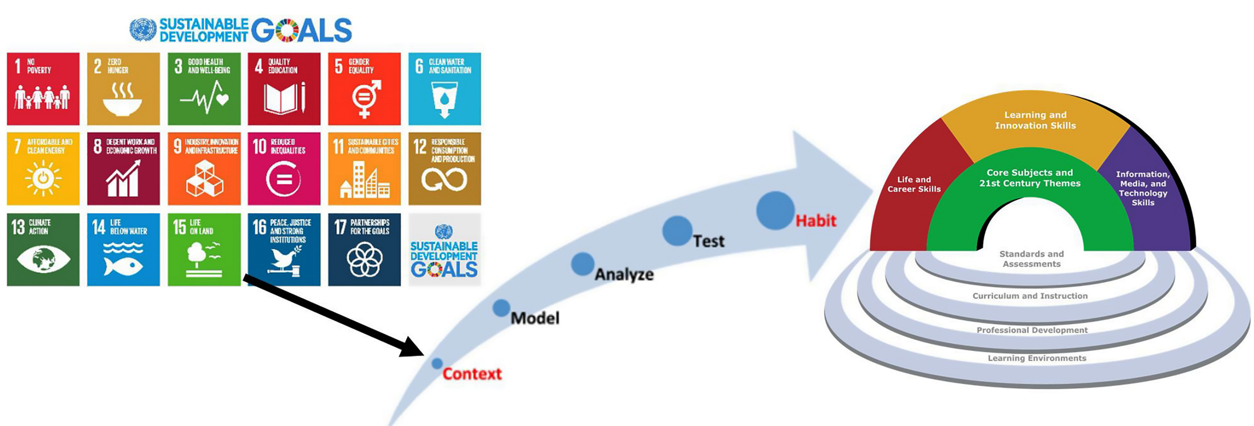


Figure 11.1. From Context to Habit

Mathematical modeling, analysis and simulation of the spread of Zika with influence of sexual transmission and preventive measures [15, 14]
Context: This UR project was motivated by SDG-3 (Good Health and Well-being) and a mathematical model to understand the role of Insecticide Treated Nets (ITN) and Indoor Residual Spraying (IRS) as methods for limiting the impact of Zika transmission in developing countries. **Model**: A new model that builds on classical Susceptible-Exposed-Infected-Recovered (SEIR) epidemiological *single outbreak* model [3, 2] for humans combined with an SEI mosquito model was developed that included both vector-transmission (indirect) as well as sexual-transmission (direct). **Analysis**: The basic and control reproduction numbers and the final epidemic size in the presence of control measures (ITN and IRS) were derived as well as a gross estimate for the rate of sexual transmission, during the initial stages of the outbreak, in terms of prior estimates of the basic reproduction number from related albeit not sexually transmitted arboviral diseases. The model was extended to include multistage dynamics. **Test**: Data from Colombia was used to validate the model along with studying the influence of ITN and IRS and optimal combination levels of the two were identified. **Habits**: Helped to develop threshold combination levels of ITN and IRS to attempt to control the spread of Zika which helped the student to connect the results to inform data driven decisions by the government.
Mathematical modelling, analysis and simulation of the spread of gangs in interacting youth and adult populations [17].
Context: This UR project was motivated by SDG-8 (Decent Work and Economic Growth), SDG-16 (Peace, Justice and Strong Institutions) to understand the spread of gangs in Puerto Rico. **Model**: A modified SIR model [9, 10] with interactions between sub-populations to study the dynamics of illegal gang spread in communities by recruitment of susceptible youth and adults. **Analysis**: A system of six ordinary differential equations were solved using Runge-Kutta methods. Three interactive patterns were defined as preferred, like-with-like and proportional. The reproduction number $\mathcal{R}_0$ was derived along with studies on the stability of the system for particular interacting patterns. **Test**: A parameter estimation algorithm was implemented to obtain the measures of the interactivity of the youth sub-population. Sensitivity Analysis of the model variables to the parameter was obtained with the normalized forward sensitivity index for each parameter. **Habits**: Helped the student to creatively model a complex social problem. Developed observation to find similarities and differences between problems.

Table 11.1. Mathematics for Zika in Brazil with mosquito control in Colombia, and Gang Violence in Puerto Rico

predictive capability of the models. Once students are taken through these steps, we automatically help to develop their essential **Habits** or competencies as an integrated and sustained process. These competencies include collaboration, communication, critical thinking and creativity as outlined in the Partnership for 21st century skills [1].

Table 11.1 showcases some examples of two UR projects in mathematics completed under the guidance of the authors that have had global impact as well as helping to retain the students that conducted these projects with the authors in STEM fields at various institutions. It also illustrates how an infectious disease model can inspire modeling of another problem from a very different context such as gang violence. Both projects enabled participating students to clearly communicate ideas, synthesize and translate them mathematically. They used mathematical tools to analyze stability of solutions and sensitivity to parameters. The students also learned to compute and define the basic reproduction number $\mathcal{R}_0$ as a measure of the potential of a situation to spread or infect in the respective context of the problem. Multiple joint peer-reviewed journal publications were attained [15, 14, 17]. Students from this UR continued to participate in national and international research experiences in the following years along with presenting their work at several conferences. Students also became leaders on campus. The Zika project won the best paper award at the Shenandoah Undergraduate Conference in Mathematical Sciences and the gang-violence project was selected for the CUR Posters on the Hill (POH). The latter gave a chance for the students to meet representatives from Puerto Rico to discuss the impact of their project through mathematics to senators and congressman at Capitol Hill.

Table 11.2 illustrates the mathematics of engineering applications as well as probability models. The student that

Enhancing Groundwater Quality Through Computational Modeling and Simulation to Optimize Transport and Interaction Parameters in Porous Media [23, 24].
Context: This UR project was motivated by SDG-6 (Clean Water and Sanitation) and attempted to create a new mathematical model for cleaning contaminants in aquifers from various sources including industrial processes and uncontrolled sewage in India. **Model**: A study of the movement of the colloids via a Micro and Nanoparticle transport Model in porous media was developed for a non-constant flow velocity and electromagnetic interactions. This included a coupled system of partial differential equations (PDEs) involving multiphysics. **Analysis**: The system of PDEs were solved using an implicit finite-difference discretization along with the iterative Newton's method. The method was extended to incorporate a parareal algorithm implemented using parallel computing. **Test**: Simulated data was used to validate the model and a parameter estimation study was also conducted to quantify parameters of interest. A graphical user interface (GUI) was developed to test with real-data. **Habits**: Produced understanding of the applications of mathematics to engineering and multiphysics modeling and developing strategies for water purification.
Design thinking and computational modeling to stop illegal poaching [13]
Context: This UR project was motivated by SDG-15 (Life On Land) and SDG-16 (Peace, Justice and Strong Institutions) involving the poaching of elephants for ivory in Tanzania Africa, and a proposed solution to develop a monitoring network that include drones for surveillance. **Model**: Using Design Thinking [4, 16] two aspects of this complex problem were modeled: dynamics of drone flight and probability target search. The dynamics of a quadcopter-drone flight used the Newton-Euler equations for rigid body to describe the balance of acceleration of mass and negligible centrifugal force to the gravitational and the four motors thrust. The angular Euler-Lagrange equations described the torque and linear external forces of the thrust. The quadcopter modeling equations obtained from physics were coupled with a Proportional-Integral-Derivative (PID) controller for stabilisation. For the target search problem a probabilistic model was developed using a Bayesian framework to improve the binary decision on the target detection by the evolution of a belief function. **Analysis**: For the flight equations conservation of energy and power relationships were used to derive total thrust in terms of the angular velocities. A heuristic method was used to determine acceleration for control inputs in order to generate a feasible trajectory. For the target search, a binary detection random variable was used to analyze the detection decision on a given cell with a measurement error for missed detection or false alarm. The given sequence of observations and individual cell belief probabilities were computed by iteration. A belief function was defined for a uniform distribution that converges to the prior belief. **Test**: The flight equations with PID control was tested by solving the equations dynamically and running an animation that graphically simulates the quadcopter in flight. For the target detection a simulation of the iterative decision function was programmed and tested in a generated location grid. **Habits**: Helped the participating student to learn teamwork, communicate and solve their model of a global challenge. They learned to build an actual drone for simulation which encourage them to learn by doing.

Table 11.2. Engineering and Mathematics for the groundwater quality in India and elephant poaching in Tanzania

worked in the groundwater UR completed his Bachelors degree from Harvard University in Applied Mathematics and presented the work at multiple meetings. The group of students that developed the second project learned to teach each other, work, write, and present in various forums as a team and had great success as individuals that included scholarships and completion of academic goals. This project also went on to be selected for CUR Posters on the Hill.

11.3 From Olympics to Zika to gangs to opioids

Collaboration is of global value considering each individual of structural importance as a living stone in a growing building of international learners. Through UR we can travel continents via global challenges that give us the opportunity to listen, learn, and share through teaching and research mentorship in our area of expertise. There are endless possibilities of contexts to motivate learning and discovery with the added treasure of an empathic and diverse world-view when considering global challenges. An anecdote that serves to illustrate our limited view of a problem comes from the poaching of African elephants for ivory project. As Americans we see the need to protect these magnificent animals from extinction. On the other hand, according to the world-view of many poor African families, this illegal activity provided for the education of their children, a door to escape from a life of poverty and the assurance of a better future. Therefore, African problems affect America also, an experiential lesson learned from the Covid-19 pandemic. This acknowledgment of the differences in how we view and define a problem is part of the *habits* (competencies) that global challenges and international collaborations brings to UR. In this section we will share a sequence of UR projects that uses mathematics to address a global challenge *context*. We will mention projects by continuously transforming one into the next one that best highlights components of the CMATH framework hoping to preserve, if we may use the mathematical metaphor, some of the underlying topology.

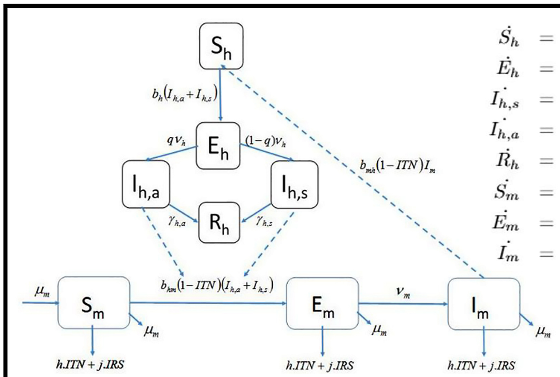
11.3.1 C: Selecting Context

A global context, described in Figure 11.1 referring to the UN SDG, in its local instance motivates the dedication of time and effort of students to a mathematical research task. A UR group of diverse students participating in the 2016 Mathematical and Theoretical Biology Institute (MTBI) at Arizona State University considered important to study the Rio 2016 Olympic Games in Brazil at the time of the Zika virus outbreak in tropical climate countries [12]. They came from diverse institutions and places such as UPR-Mayaguez-PR, ASU-AZ, TSU-TX, Montclair State University-NJ, Foxcroft School-VA, and met at the REU. The World Health Organization (WHO) declared the Zika virus infections a global health threat [7, 11]. Although a rather mild disease, Zika is catastrophic for pregnant women because of the possibility of causing severe birth defects in their infants. This unique vector-borne disease can be also sexually transmitted within humans. An increased threat of spread of sexually transmitted diseases was expected at this mass gathering sporting event. This Zika project motivated from the Olympics in Brazil served as the context of the UR that was modeled as described in the next section.

11.3.2 M: Creating Models

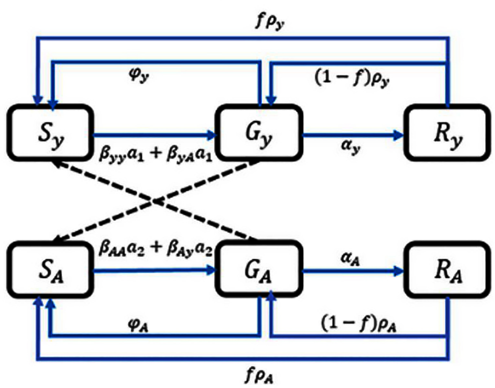
The transmission dynamics of the Zika virus was studied following the classic SIR differential equations model [9, 10]. Three susceptible groups of people in Rio de Janeiro during the Olympics were identified with the potential for sexual interaction: natives, visitors, and local sexual workers. Mixing patterns that model the sexual interaction among these groups were defined as *preferred* and *random mixing*. Building on this Zika project context from Brazil, one of the students from the group went on develop a SEIR-SEI model (See Figure 11.2) that incorporated both sexual transmission between humans as well as vector transmission between humans and mosquitoes motivated by data from Colombia [15, 14].

The interaction between heterogeneous populations in the coupled SIR Zika model from Brazil also inspired the development of another model by students that participated at the 2017 EXTREEMS-QED, an NSF program held



$$
\begin{aligned}
\dot{S_h} &= -b_{mh}\left(1-ITN\right)S_hI_m - b_h\left(I_{h,a}+I_{h,s}\right)S_h\\
\dot{E_h} &= b_{mh}\left(1-ITN\right)S_hI_m + b_h\left(I_{h,a}+I_{h,s}\right)S_h - \nu_hE_h\\
\dot{I_{h,s}} &= (1-q)\nu_hE_h - \gamma_{h,s}I_{h,s}\\
\dot{I_{h,a}} &= q\nu_hE_h - \gamma_{h,a}I_{h,a}\\
\dot{R_h} &= \gamma_{h,s}I_{h,s} + \gamma_{h,a}I_{h,a}\\
\dot{S_m} &= \mu_mN_m - \mu_mS_m - b_{hm}\left(1-ITN\right)S_m\left(I_{h,s}+I_{h,a}\right) - \left(h\cdot ITN + j\cdot IRS\right)S_m\\
\dot{E_m} &= -\nu_mE_m - \mu_mE_m + b_{hm}\left(1-ITN\right)S_m\left(I_{h,s}+I_{h,a}\right) - \left(h\cdot ITN + j\cdot IRS\right)E_m\\
\dot{I_m} &= \nu_mE_m - \mu_mI_m - \left(h\cdot ITN + j\cdot IRS\right)I_m
\end{aligned}
$$

Figure 11.2. SEIR-SEI model for spread of Zika based on human and mosquito interaction



$$\frac{dS_y}{dt} = -\left[\beta_{yy}\, a_1 \left(\frac{S_y\, G_y}{N_y}\right) + \beta_{yA} a_1 \left(\frac{S_y G_A}{N_A}\right)\right] + f_y\, \rho_y\, R_y + \phi_y G_y$$
$$\frac{dG_y}{dt} = \beta_{yy} a_1 \left(\frac{S_y\, G_y}{N_y}\right) + \beta_{yA} a_1 \left(\frac{S_y G_A}{N_A}\right) + \bar{f}_y\, \rho_y\, R_y - \phi_y G_y - \alpha_y G_y$$
$$\frac{dR_y}{dt} = \alpha_y G_y - \rho_y R_y$$
$$\frac{dS_A}{dt} = -\left[\beta_{AA} a_2 \left(\frac{S_A G_A}{N_A}\right) + \beta_{Ay} a_2 \left(\frac{S_A G_y}{N_y}\right)\right] + f_A\, \rho_A\, R_A + \phi_A G_A$$
$$\frac{dG_A}{dt} = \beta_{AA} a_2 \left(\frac{S_A G_A}{N_A}\right) + \beta_{Ay} a_2 \left(\frac{S_A G_y}{N_y}\right) + \bar{f}_A\, \rho_A\, R_A - \phi_A G_A - \alpha_A G_A$$
$$\frac{dR_A}{dt} = \alpha_A G_A - \rho_A R_A$$

Figure 11.3. Modeling spread of gang-violence: Flow Diagram and Equations

at George Mason University that focused on UR in computational and applied mathematics. In this project a new SGR model defined two populations of at-risk youth and adults interacting in the spread of an illegal drug-dealing gang via recruitment using the Zika project experience of one of the students in the UR research team [17]. This is a socioeconomic problem related to human trafficking and poverty contexts of personal interest for another student from Puerto Rico (PR). This complex problem is fueled by the opportunity of luring susceptible minors to commit crimes related to the drug-dealing underground economy in poor communities of the Hispanic U.S. territory of PR. Minors are exploited by criminal gangs because the legal consequences of their criminal behaviour dictated by the juvenile justice system are more lenient for youth than adults. It is also an unfortunate product of unsupervised youth in the streets seeking for the security and care of a gang as a poor substitute for a loving family. Figure 11.3 shows the SGR model for the spread of gang-violence in Puerto Rico. Next, we describe some of the mathematical *analysis* associated with these models.

11.3.3 A: Performing Analysis

A common mathematical construct of interest in the ODE models that describe the spread of infection, be it a disease or gangs, is the *basic reproduction number* $\mathcal{R}_0$. This value describes the number of secondary infections generated by an infected person when the population being considered is composed of primarily susceptible individuals. $\mathcal{R}_0$ determines whether there is an outbreak or not. The next part of the UR is to expose the students to a general approach called the *Next Generation Matrix* approach to find $\mathcal{R}_0$ [5]. The technique can be well understood at the undergraduate level with some Linear Algebra and Calculus concepts. In both Zika and Gang-Violence UR this was obtained by students that benefited from grasping meaningful concepts of the spectral radius and Jacobian calculated as a part of understanding the dynamics of the spread of a disease or the social context of a gang formation. The $\mathcal{R}_0$ formulation can be useful to study the sensitivity of the model to changes in the parameters. Using such techniques and deriving complex expressions reinforces their passion for conducting rigorous mathematics through UR. In this case the basic reproduction number $\mathcal{R}_0$ obtained for the Zika model (Figure 11.2) had the potential of finding dependence and simplifications to the parameters of the model. The reproduction number was shown to be [15],

$$\mathcal{R}_0 = \frac{a_h}{2}\left(\frac{1-q}{\gamma_{h,s}} + \frac{q}{\gamma_{h,a}}\right) + \frac{1}{2}\sqrt{a_h^2\left(\frac{1-q}{\gamma_{h,s}} + \frac{q}{\gamma_{h,a}}\right)^2 + 4\left(R_{0,a}^2 + R_{0,s}^2\right)} \tag{11.1}$$

where,

$$R_{0,a}^2 = \frac{b^2 \beta_{mh}\beta_{hm} N_m \left(1 - ITN\right)^2 \nu_m (1-q)}{N_h \gamma_{h,s}\left(\mu_m + \nu_m + h\cdot ITN + j\cdot IRS\right)\left(\mu_m + h\cdot ITN + j\cdot IRS\right)}$$
$$R_{0,s}^2 = \frac{b^2 \beta_{mh}\beta_{hm} N_m \left(1 - ITN\right)^2 \nu_m q}{N_h \gamma_{h,a}\left(\mu_m + \nu_m + h\cdot ITN + j\cdot IRS\right)\left(\mu_m + h\cdot ITN + j\cdot IRS\right)}$$

Another useful analysis is to engage students in conducting Sensitivity Analysis performed via the normalized forward sensitivity index of the model parameters with respect to the $\mathcal{R}_0$. In the gang-violence context the sensitivity of the

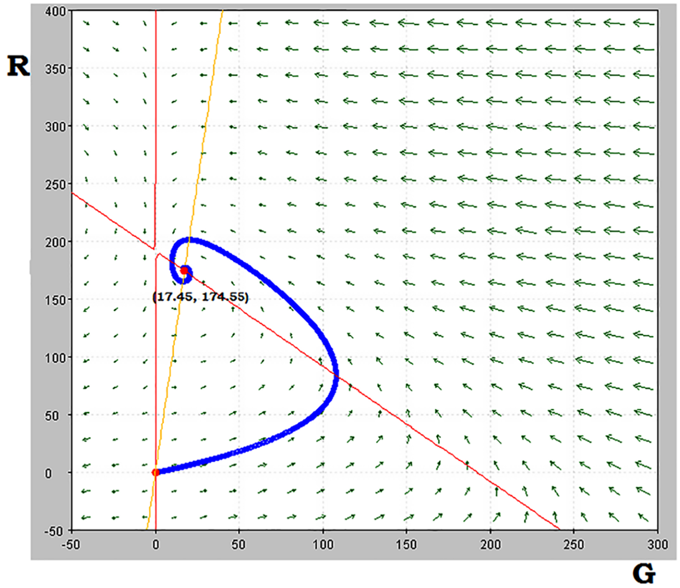


Figure 11.4. Orbit and endemic stable spiral equilibrium node for G=gang members and R=removed in with intersecting nullclines for Gang-violence

Gang membership and Removal variables for youth and adult sub-populations was computed for each parameter. In this case the parameter sensitivity of the variables was preferred over $\mathcal{R}_0$ sensitivity because of the presence of heterogeneous group mixing in diverse proportions of interactions between youth and adults. The research group concluded that the interaction activity parameter is the most relevant to accelerate the gang formation and increase gang membership. Further mathematical analysis is at hand. Stability analysis in two-dimensions is also introduced in the gang-violence work providing a visual-analytical tool of the equilibrium of an ODE system for given parameters and initial conditions in Figure 11.4. The next step in the CMATH framework builds computational skills.

11.3.4 T: Simulating Tests

The students were next taught to verify the two-dimensional stability analysis in the various UR projects using diverse software tools such as MATLAB® or Python. Simulations for numerical methods in a vector-based environment allowed for customized algorithms to solve systems of ODE's developed using a variety efficient algorithms. This step provides the opportunity for the mentor to help UR students learn about the accuracy of using such methods as Euler's Method to Runge-Kutta algorithms as well as graphing solutions and creating user-friendly visualizations and animations that can be used effectively in presentations at conferences and meetings.

Parameter estimation with optimization tools is another important methodology that has an algorithmic implementation that allows students to make predictions that provide for data-analysis. This was particularly significant for the gang-violence model because parameters were not available in literature. For example, the needed data from criminal activity was not readily available to validate the algorithms. For this, the students were shown how data can be simulated from the numerical solutions by introducing noise to simulation results, and recovering the original parameter via minimization of error. This is an opportunity to introduce constrained numerical optimization techniques such as sequential quadratic programming and/or interior point method. These algorithms can then be generalized for any number of parameters. Some parameters obtained in this fashion in the gang-violence problem included the activity parameter for the youth interaction rate.

The UR projects also provide the opportunity to expose students to other approaches to parameter estimation including machine learning (ML) techniques. A ML approach to parameter estimation from data was used to conduct numerical experiments in simulation of a context that is close to home: the opioid crisis. One of the students who was motivated by the gang violence research expanded the model to produce various compartmental models using ODE's to describe drug addiction as a novel approach to model this national problem that affects our socio-economic welfare

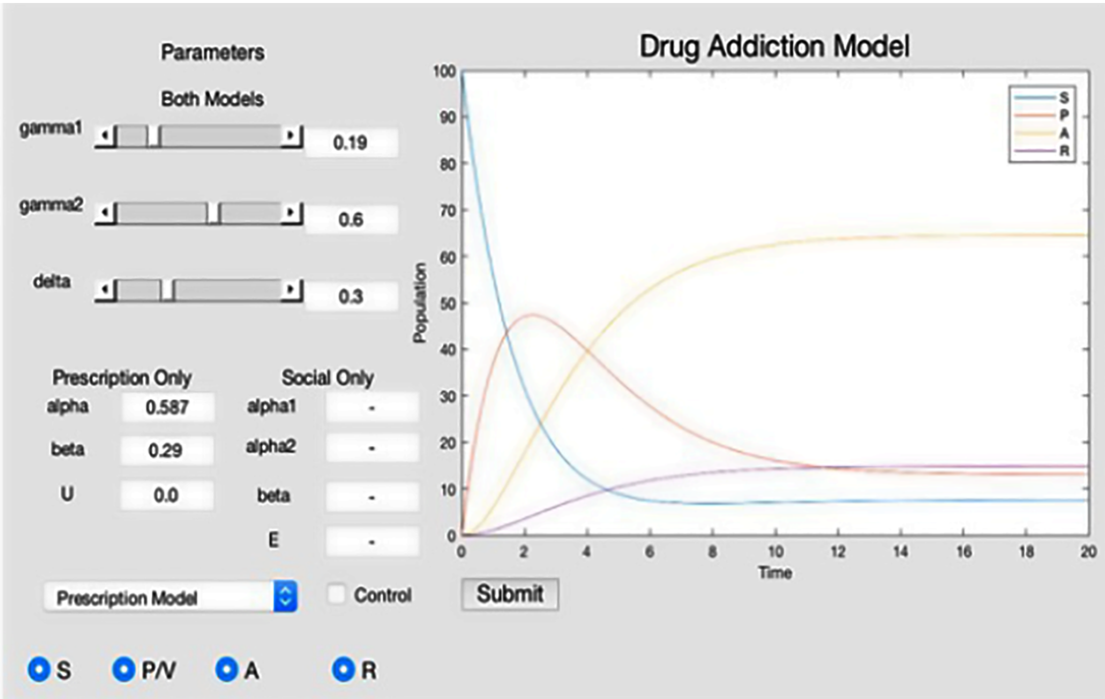


Figure 11.5. GUI for users to interact with the opioid crisis model

and public health. For these experiments parameters for the non-prescribed drug addiction model were estimated using the popular open-source Python® (https://www.python.org/) interface of the machine learning library PyTorch. A neural-network was used with 30 hidden layers, 1000 epochs for training, an SGD optimizer and MSELoss criterion. This work is currently in review and the student is pursuing a STEM degree at the University of California, Berkeley. The opioid-crisis UR also introduced *education* of healthcare professionals as well as the general population as an optimal control similar to vaccination in infectious diseases. This provided the opportunity to introduce optimal control theory using the Pontryagin's Maximum Principle approach by defining the objective functions such as the cost of controlling the opioid crisis using educational campaigns.

Finally, all UR projects have the opportunity to create a product at the end that can be widely used by the general public such as a graphical user interface (GUI). This was done for all the UR projects described in this work as students enjoyed the design and development process provided by the GUI programming platforms. For example in the opioid crisis UR that captured the interest of many, two models were developed: prescription and social models. For this project a comprehensive GUI was developed in Figure 11.5 that allowed the user to enter their parameters and select from the prescription or social model.

11.3.5 H: Becoming a Habit

The end goal of the UR CMATH endeavor has always been to contribute to mathematical competencies and develop *habits* of mind needed in the 21st Century illustrated as a unified and collective vision to strengthen our education in Figure 11.1. Through an engaging Context individuals can appreciate education and define their own value in a global scope. Modeling is as a creative avenue to be continuously built from the simplest to the most sophisticated level with a clear goal of the context ahead. The mathematical Analysis is the power that will guarantee that we will reach the context goal using the model. As we Test the model and do simulations the resulting knowledge will reveal new avenues to build, in a continuous process of learning via research and the consequential growth of new Habits. The sum of skills, knowledge and competencies that we call *habits* will create change agents needed to move toward our highest valued goals. In the next section we propose strategies that produce the needed change that UR generates at different levels including students, faculty and institutions, and share related success stories.

11.4 CMATH for creating change agents

The mathematics community acknowledges the benefits of UR as a big contributor to success in STEM, including underrepresented groups, as evidenced by programs such as the 1998- 2002 Summer Institute in Mathematics for Undergraduates (SIMU) at the University of Puerto Rico—Humacao. In this section we further provide evidence of its impact in women, people of color, and especially women of color that have participated in UR motivated by global challenges. But we also argue that faculty benefit personally and professionally as well from the UR mentoring experience, particularly early career. We propose that UR programs with multiple research mentors should invite junior faculty as co-mentors. This is a strategy for professional development that will help build collaborations, increase research networks, initiate junior faculty in new research areas, produce publications and advance their careers. Next, we share some highlights on how UR in mathematics has impacted stakeholders at multiple levels including students as change agents, faculty professional development and institutional transformation. Specifically, we discuss the impact of UR on the participating students and faculty from Inter American University of Puerto Rico (IAUPR) where the second author is a tenured faculty.

11.4.1 Students as change agents

Students that return to IAUPR after successfully completing their projects have opened doors for others to be selected because of their work. They become leaders that inspire others to apply. Since the 2013 participation of the first IAUPR students at the GMU-UR experiences we have seen a larger group of their classmates applying and being selected to participate in REUs. Many of these students have never travelled outside the island of PR. Some of the institutions that have selected IAUPR students in their UR programs include GMU, Cornell, Virginia Tech, and UPenn. In the past very few students applied despite the efforts of professors to promote UR participation through programs like NSF REU. Students would not apply because they lacked the confidence that they might be selected. Although good students at IAUPR recognized the importance of internships and REUs, the socio-economic challenges that affected their academic background in Mathematics as well as their proficiency in English as a second language also reduced the scale of their dreams. Seeing others like them succeed and share their experience have boosted their confidence and produced better results together with the promotion efforts of the internships and exchange office at IAUPR and REU institutions.

We would like to share success stories of some of the UR students. In 2016 for the first time a Puerto Rican student was selected to participate in the CUR- Posters on the Hill (POH). Mr. A. Baez and the research team of the 2015-EXTREEMS-QED UR at GMU were engaged in the elephant poaching UR project. They published as co-authors [13], presented in various conferences, including both oral and poster formats at the 2016 JMM. Ms. K. McLane received the DoD SMART Fellowship and participated in the NIST Gaithersburg REU Program. Ms. P. Padmanabhan is a very active researcher since high school with both national and international outreach experiences in places such as Suriname, Tanzania, and Myanmar. She has participated in UR including the 2016-MTBI, and has many publications in peer-reviewed journals. She is at the University of Pittsburgh working towards multiple degrees, one of which is in Applied Mathematics. Mr. Castro-Rivera first participated in the 2017 EXTREEMS-QED UR at GMU. He continued his UR at IAUPR during the 2017-2018. He was selected to present at the 2018 JMM and his project was also selected for the CUR Poster on the Hill (POH). He participated in multiple REU's nationally and internationally including: GMU-UR-Gang violence project, Cornell University-mathematical modeling in Solid State physics semiconductors, and in the Universidad Complutense, Madrid, Spain physics laboratory REU. He graduated in June 2020 from the Mechanical Engineering Department of the IAUPR, and was immediately employed at Pepsi-Co, in PR despite the Covid-19 lockdown worldwide economic stress. He is motivated to continue graduate studies in Engineering and Mathematics. Ms. Kumar is another exceptional undergraduate researcher. Hailing from one of the premier high schools in the country (Thomas Jefferson High School for Science and Technology) she founded the organization Shesoft.org motivated to bridge the gender gap in technology, through financially enabling participation of girls in conferences, conducting tech-related community activities and developing computer science curriculum for intermediate school. She conducted research in the opioid-crisis as well as the Covid-19 problem. At present she goes to UC- Berkeley pursuing majors in Computer Science, Electrical Engineering and Business Administration and thinking of picking up a mathematics minor as well.

11.4.2 Facultyprofessional development

The Multidisciplinary REU Program: Research, Education and Training in Computational Mathematics and Nonlinear Dynamics of Biological, Bio-inspired and Engineering Systems (2013) and Extreems-QED UR at GMU (2015-2018) were funded from a proposal written by the first author of this work. It included the participation of a PR undergraduate student and an early career faculty mentor who is a GMU alumni. Through the years a research team of student and junior faculty from PR and GMU faculty worked in various collaborative UR project. This experience has proven life changing for these students underrepresented in mathematics by changing their world-view, developing confidence, learning mathematics and communication skills, impacting their goals and expanding their opportunities. Furthermore, the UR also helped the Puerto Rican faculty member and second author of this work to initiate UR at her home institution of IAUPR. Through the REU she kept in contact with mentors and faculty of the mathematical sciences at GMU. This continued relationship through the summer REU visits provided the fuel to do research despite her teaching appointment. Both authors have collaborated in the design and writing of successful proposals as well as co-mentoring REU students at GMU. This UR experience has resulted in at least three group publications with students as co-authors, and also expanded the UR network in PR. The professional development experience as an UR mentor also produced new collaborations within the island for the Puerto Rican faculty member. Approximately ten other students in PR participated in this line of research through local institutional funding and the PR IDeA Networks of Biomedical Research Excellence (PR-INBRE) organization. This experience also produced the fruitful opportunity for faculty from different departments and campuses to initiate interdisciplinary collaborations through UR after meeting in presentations of the EXTREEMS-QED UR projects at local research conferences.

11.4.3 Institutional transformation

IAUPR, the largest private institution in PR, is a 98% HSI teaching institution with primarily undergraduate programs that has also benefited from the UR experiences. The Bayamon Campus (BC) serves more than 4,500 students of which 4,345 are Pell receive financial aid via the Pell Grant, while 27% are first generation college student. In the past ten years there has been a growth in the research component that includes a higher participation of students in REU's. Through the co-mentorship strategy in REU's of the junior researchers and early-career faculty with senior researchers from GMU, a Tier 1 research institution, proposal collaborations have been successful. In the 2014-2019 period the IAUPR received two awards from the US-DE that aimed to improve minority serving STEM programs develop UR in Data Science, and build new lines of collaboration between the IAUPR-BC and GMU. The co-authors collaborated with the ideas some following successful programs at GMU, and served as PI and key personnel in the awarded projects. New collaborations with other national and international institutions followed. Partnerships with GMU, Harvard University, and the University of Ottawa as well as a private business analytics services company have been achieved developing mentorship relations that impact both faculty and students. Another team that emerged from the UR network will work in the organization of a meeting in PR with the IAUPR-BC as the host, that will bring together both undergraduate and graduate students as well as faculty and speakers from institutions in Puerto Rico, Pacific Northwest National Laboratory (PNNL)-WA, GMU-VA and ASU-AZ.

In 2021, multiple institutions collaborated on a NSF proposal that will use the UR model to connect Universities and Community Colleges. This planning proposal seeks to scale-up a successful collaboration between faculty of UCLA and LA-Community College District to other regions of the nation, including Texas, Virginia and Puerto Rico. If successful this will also help prove the concept that via UR the Community College faculty will have the opportunity to grow professionally co-mentoring their students with faculty from the partner university. The participating Community College students will experience in the UR, a rich and meaningful mathematical sciences world that sparks in them creativity and the desire to achieve more in an environment that provides the needed human interaction through various levels of mentorship.

11.4.4 Potential Challenges and Opportunities

While CMATH may provide the needed problem solving approach in a structured fashion, the application of mathematics to social good comes with certain challenges.

One common challenge in applying this framework is *communication* at multiple levels including a) technical communication where there is a need to translate a societal challenge into an appropriate mathematical model and structure and vice-versa (for example the spread of gang-violence model as a vector-borne infectious disease model) and b) interdisciplinary communication between mathematicians, engineers, sociologists, behavioral and cognitive scientists, government and practitioners (for example the design thinking and computational modeling to stop illegal poaching). These communication challenges also provide researchers potential opportunities such as a) using social media effectively to communicate back to the public about the impact of the model and its benefits through interaction with GUIs or Apps (for example the GUI for the drug addition model); b) using an interdisciplinary multi-tiered mentorship approach across multiple levels including faculty, post-docs, graduate students, undergraduate students and high school students.

Another common challenge that faculty may face is *student motivation* for their mathematical research. This often happens when faculty tend to teach students about what they know rather than what the students want to know. In particular, it is important for faculty to recognize diverse interests of students to solve societal problems and be able to connect their personal mathematical research to the students research interests.

It must be pointed out that when training students in these mathematical research topics that is motivated by societal problems, one must be aware of *ethical dilemmas* and *implicit biases* that often tend to arise. For example, as the research for the poaching was conducted, there was resistance from the local community in Tanzania on foreign ideas proposed that were deemed to affect the local culture, socioeconomic reality and practices. One of the ways this can be mitigated is to create a code of conduct and an open dialogue to empathize with different societal stakeholders prior to creating mathematical models. Being aware of our own biases will also help to coordinate a culturally responsive and equitable research that is inclusive.

11.5 Discussion and conclusion

In this chapter, we present a new framework called CMATH that helps to support UR in mathematical sciences as a way to engage students to not only advance their research portfolios but also to help them connect the mathematical research they are working on to address global challenges. Motivated by a **context**, the students are lead to develop a **model(s)**. These models then undergo an **analysis** and a variety of **tests** before it becomes their **habits** of mind. This chapter helps promote awareness of the need to engage more students in multidisciplinary problem solving that employs mathematics to address global challenges from other fields including infectious disease, gang violence, animal poaching, water purification and much more. These examples along with a structured CMATH provide an overview of the essential foundations needed for UR in mathematics motivated by real-world problems. This UR experience has also helped the students become better communicators, better collaborators, better critical thinkers and most importantly creative problem solvers. Along with the impact on students, CMATH for UR also provides opportunities for faculty development, particularly early-career, and institutions to help build a stronger next generation workforce of change agents.

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12

Interdisciplinary REUs: Lessons from an Institute-based Mathematical Biology Program

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12.1 Introduction

Interdisciplinary research is increasingly important in today's scientific community [6]. This trend is epitomized in the field of mathematical biology, with several professional societies and national organizations calling for multidisciplinary researchers who are able to combine insights from both these fields [4, 9, 17]. Powerful quantitative tools are needed to begin to understand the myriad complexities of biological systems, and the great growth in data from these complex systems increases the need for new tools from the mathematical sciences [12]. Mathematical modeling of these biological systems enhances our ability to better manage natural systems, respond to challenges such as infectious disease outbreaks and to improve medicine. There are challenges in determining effective educational methods to assist students in developing their own skills and insights across multiple disciplines.

The National Research Council has recognized the need for novel training due to the increasingly cross-disciplinary nature of STEM fields [17]. The Common Vision Project [27] emphasized the importance of actively engaging students in learning. There is an array of skills that up and coming students need to develop to compete in the emerging STEM workplace. Programming, data acumen and critical thinking skills are several of the suggested topic areas to allow students to tackle all sorts of problems and prepare them for STEM careers. Students career options will be enhanced if they develop the ability to communicate complex ideas in ways that are understandable to collaborators, clients, employers, and the general public.

Modeling and computation can be used to introduce the scientific method to students to enhance their skills and lay the groundwork for effective communication of quantitative ideas and results. In addition to problem solving skills, experiences that allow students to develop team collaborative skills will be helpful to essentially all career paths [2]. One approach to novel training that addresses these needs is engaging undergraduate students in summer research experiences for undergraduates (REU) programs. Summer REU programs provide students with a unique, focused program for development of skills. Being conducted during the summer, students can focus solely on the research at hand without the distraction of numerous classes and associated coursework and tests. REU programs have increased in number, and many have changed over time from one student working with one advisor to an emphasis on working in teams [8, 13, 11]. In this chapter, our approach to an institute-based REU at the National Institute for Mathematical and Biological Synthesis (NIMBioS) [20] is described.

NIMBioS is a biology synthesis center, funded by the Division of Biological Infrastructure of the National Science Foundation (for its first 12 years) with additional support from the University of Tennessee. NIMBioS activities explore the interface between math and biology and bring together the talents of researchers from around the world to collaborate across disciplinary boundaries and take an integrative approach to address the vast array of challenging questions in this 21st century of biology.

Our Summer Research Experiences for Undergraduates (SRE) program has been an important part of NIMBioS education activities starting in summer 2009 and has hosted 187 undergraduate participants since its inception [19]. Due to the global pandemic, NIMBioS held a virtual SRE program during the summer of 2020. NIMBioS has also supported other undergraduate research efforts by hosting the NIMBioS Undergraduate Research Conference at the Interface of Biology and Mathematics annually from 2009 to 2020.

Although interdisciplinary in nature, students in the NIMBioS SRE program explicitly receive instruction in formulating mathematical models (deterministic as well as stochastic). Since many of the projects are informed by connections to field and laboratory data, the students have been exposed to different techniques for data analysis and parameter estimation. The program also has included activities on collaboration and teamwork, better preparing students to participate in inherently interdisciplinary fields, such as mathematical biology, involving multiple perspectives and diverse expertise.

The depth of research, the variety of types of models, the type of instruction, and analysis techniques have evolved over the years, and we will discuss these changes in some detail below. One of the main changes has been an increased emphasis on preparing students to work in teams, and we describe our corresponding approach.

12.2 Instructional activities

The goal of the NIMBioS SRE program was to immerse undergraduate students into the world of multidisciplinary scientific investigation to learn modeling techniques through their applications to specific projects. NIMBioS SRE students learned fine-scale details of select biological systems, developed mathematical models to investigate questions in those systems, and disseminated their results to the scientific community in written (research report) and verbal (poster presentation and a technical talk) formats. Our specific objectives were to:

1. Provide an opportunity for students to develop collaboration skills and an appreciation for the value of a multidisciplinary approach to research,
2. Provide a multidisciplinary experience that will prepare students to matriculate into graduate or professional programs, and help them to make more informed career choices,
3. Give students background and experience in connecting mathematical analysis, data, and modeling,
4. Conduct research that will result in student work being presented at undergraduate and professional conferences and possibly lead to a journal publication.

To place our instructional activities in proper context, in the open application process that was advertised through the extensive NIMBioS network, students were asked to express preferences for the set of project topics in that year's program. When a student was given an offer to participate, the offer involved the expectation that the student would collaborate on a particular project, which may or may not have been their highest preference topic. Student teams were chosen by the project mentors and NIMBioS leadership to ensure a diverse team for each project. Student teams generally included some pursuing a life science major and some pursuing a quantitative science major, with the mentors also representing a mixture of disciplinary backgrounds. Students were provided a stipend, as well as shared office space at NIMBioS and housing in shared apartments to enhance comradery, collaboration, and peer learning.

The formal sessions offered to students were modified over the years, with one change being an enhanced focus on the topics in data science. Session topics ranged from coding and software management to modeling techniques and tools, with most formal sessions presented during the first two weeks for a few hours a day. The teams began work on their projects upon their arrival, with regular meetings with their mentors to provide early guidance on potential project objectives and methods.

In the first few summers, sessions were provided on MATLAB and R, with additional instructions in areas such as statistics based upon the needs of their particular projects. In more recent years, sessions also were provided on Python coding and the use of Github for version control of codes. Due to increased interest in agent-based models, we added sessions on Netlogo [30], an accessible free software package developed for both educational and research use. Some project topics involved spatial analysis, and thus sessions on appropriate spatial analysis and modeling tools were incorporated, notably software for assembling geospatial data (see details below). The sessions were usually led by postdocs or faculty members and included hands-on activities with sample codes and tasks to complete.

Topics covered in modeling sessions with the full set of students were expanded over the years, going beyond a general introduction to mathematical modeling. Some particular topics covered beyond standard dynamical systems models included:

- Consideration of the building blocks of linear models in discrete time versus systems of linear ordinary differential equations,
- Accounting for the order of events in discrete time models makes a big difference.
- Calculating survival rates in discrete models is done differently than in systems of ordinary differential equations.
- Basic stability analysis was expanded to include the Next Generation Matrix Method as needed for projects.

Particular projects also benefited from including guidance on sensitivity analysis (PRCC, Sobol) of outputs as parameters are varied—see these papers [16, 26], which contains some useful background for your students. Different types of parameter estimation algorithms were presented to certain project teams, depending on their needs.

The above discussion focuses on the quantitative aspects of the program. A tenet of the program was that all students would also gain some biological intuition associated with the underlying area of biology in their particular project. Due to the breadth of life science topics upon which the projects were based (from within-cell, to organism, to populations and communities, and incorporating hosts of modern biology concepts), the life science education components were derived from discussions with the mentors within each project team. Often this involved mentors suggesting readings from texts and recent journal articles, with team meetings including discussions of the key concepts and methods.

Besides projects involving agent-based models, systems of ordinary differential equations, statistical models, and discrete equations, the program recently involved projects utilizing spatial analysis and GIS datasets to investigate the impact of climate change on species distribution. Ecological niche models estimate the abiotic conditions associated with species' presence, under the assumption that climatic conditions are some of the most important determinants of species' geographic ranges [3]. One modeling approach utilizes the maximum entropy algorithm to estimate species' abiotic requirements and geographic distributions from species' occurrence (presence) data and a random sample of background (non-presence) locations from the landscape studied, and the abiotic (climatic) conditions at the presence and background locations [23, 5].

Some projects used the Maxent software application [24] to create ecological niche models with baseline climate data and project the models on future climate conditions. Species occurrence data were obtained from the Global Biodiversity Information Facility [10] and relevant primary literature sources. Climatic data were obtained from ClimateNA [29], which contains 27 bioclimatic variables for North America. ArcGIS [7] and spatial analysis packages in R [25] were used to organize, process, and visualize spatial data. These tools were used in three case studies that modeled an insect-plant disease complex (thousand cankers disease), reductions in specialized pollinator distributions, and the impact of the velocity of climate change on these specialized pollinators. Although students involved in these projects did not develop new mathematical models, they learned certain concepts and skills, such as model validation, coding, variable selection, and the importance of parameters, similar to those involved in mathematical model development.

12.3 Professional development activities

A variety of professional development opportunities were offered to cover the structure and processes of the scientific enterprise and to prepare students for possible career options. To assist in the development of communication skills, student teams gave frequent progress reports to the directors of the program. These reports involved different formats at different times in the program. One format required explaining informally, without slides, the project goals and the current progress. The objective was to assist students in being prepared to give “elevator talks” that quickly provide insight in their overall scientific goals. In these, each student was asked about their work on certain aspects of the projects, since project tasks were typically divided among the team members. Other formats included short talk presentations with slides and practice poster presentations, with some of these given by the full project team. Feedback on the various reports was provided to the students by mentors and the co-directors.

Specific informational sessions provided overall guidelines on writing the final report and giving the final talks. Additionally, the project mentors assisted students in developing the presentations and written reports. The style of the final reports varied according the mentors’ plans for possible publication of the project results. Final talks on each project were presented to the full group of mentors and students, along with invited visitors, on the last day of the program.

The co-directors also led an interactive workshop on developing scientific posters, during which students from each project were given feedback on their draft posters. The students presented their draft poster and associated oral summaries in a short format, with each student contributing some part of the project description and results. The audience of SRE participants, including all mentors, gave suggestions on the format of the poster. The students presented their preliminary results in posters at the campus-wide STEM Poster Symposium (organized by NIMBioS and the University Office of Undergraduate Research). Some students also presented at the Joint Math Meetings Undergraduate Poster Session. Many of the students were funded by NIMBioS to return to campus and give talks or poster presentations at the NIMBioS Undergraduate Research Conference at the Interface of Biology and Mathematics held each fall.

Students were given several other professional development opportunities while in the SRE program. A cross-cultural mentoring seminar and a session on responsible conduct of research were conducted by UT professors or graduate school staff. Dr. Ernest Brothers (Associate Dean of the UTK Graduate School) presented a seminar describing cross-cultural mentoring. The key point in this seminar was that our interactions with a global society continues to expand. The associated dynamics, with demographic, social, and cultural changes, requires reframing our approach to mentoring, and we should include the cultural mores of the individuals involved and cultural aspects within the organization in which they are functioning [1, 14]. Cross-cultural mentoring may be defined as a mentoring process whereby the mentor establishes a relationship with the mentee from a personal, cultural, sociopolitical, and historical context. This approach provides the mentor with context regarding the experiences, cultures and values of the mentee before immersing the mentee into an organization that may be unfamiliar to them.

A co-director led a discussion about the structure of the math and science communities. Topics included the types of the professional math and biology organizations and the structure of the academic positions with lecturers and tenure-track professors (assistant, associate and full professors). Sessions on graduate and professional opportunities in math and science-related fields, including jobs in academia, government and industry, were conducted. A panel discussion with current graduate students was organized. After an opening dinner party, informal out-to-lunch events with mentors gave students a chance to interact with mentors not involved in their projects, and appointments for students were arranged with our faculty based on the career and graduate school interests of each student. Other events included visits to Oak Ridge National Laboratory, the campus planetarium, and the Great Smoky Mountains National Park. Tours of the Knoxville Zoo given by their veterinarian provided students the opportunity to learn about animal behavior and care in an informal setting.

12.4 Teamwork and collaboration

To facilitate interactions between students and mentors, guidelines for mentoring in this program were given to and discussed with all mentors prior to the start of the program. To inform students of expectations and respectful/professional behavior, students were asked to sign a contract detailing these essential expectations of the program. Furthermore, a Team Dynamics activity and corresponding evaluation facilitated effective communication among students within each group and their mentors.

Students participated in the Team Dynamics activity that encouraged them to reflect on their collaboration styles. This activity is based on a method developed by Lancellotti and Boyd [15] and developed for the NIMBioS SRE

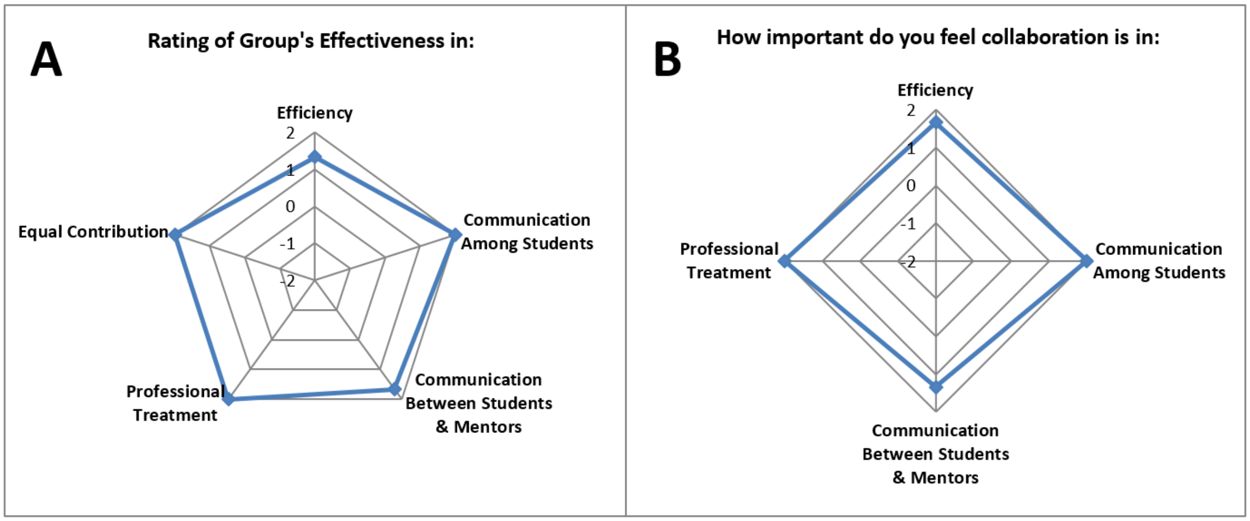

Figure 12.1. Example of results from Team Dynamics group self-evaluation: A) evaluation of within-group collaboration, and B) evaluation of general perceptions of factors important to collaboration. Responses to evaluation questions were assigned number values, and the mean value of student responses was calculated and reported. Number values assigned to answers to evaluation questions are as follows: Excellent (2), Good (1), Neutral (0), Needs Improvement (-1), Needs Big Improvement (-2).

program by Sturner et al. [28]. The first part of this activity uses an oral presentation illustrating various political philosophies to depict how different personalities approach team-based activities. An example of one political philosophy/personality presented is a dictator, with a corresponding statement similar to the following: ”Sometimes we just decide to take over. Do you ever just start talking as if you are in charge, without discussing leadership roles with your teammates?” Students are encouraged to record notes and thoughts on the various personalities and how they relate and/or react to them during the presentation. Following the presentation, students are then asked to consider their own approaches to collaboration and discuss them with their research team members. Because this activity was conducted early in the program, students were made conscious of the importance of meaningful and constructive interactions with one another to enable effective collaboration. After four weeks in the program, students completed a self and peer evaluation of the dynamics within their research team. Responses to these evaluations were compiled and returned to each respective student group (Figure 12.1), so that they could reflect on the findings and continue to develop constructive collaboration skills throughout the program. If results of a group’s team evaluations indicated problems in team dynamics, the directors of the program would meet with that group and propose possible strategies for improving their interactions. The mentors of the groups were also contacted separately to discuss possible needed adjustments in their interactions with the team.

12.5 Program outcomes

A major emphasis of the NIMBioS SRE program was to promote participation of students from diverse backgrounds. In the overall program, 54% of the students were pursuing math majors, 39% were biology majors, and the rest were pursuing majors such as computer science, statistics or health science. Overall 59% of the students were women, 28% were Asian, 10% were Black and 9% were Hispanic.

Student participants in the SRE program were from 101 institutions across the U.S. Of these, 14 are minority-serving institutions (MSIs - i.e., Native American and Alaska Native-serving Institutions, Hispanic-serving Institutions, or Historically Black Colleges and Universities). Participation of students from these minority-serving institutions arose due to a concerted effort by NIMBioS to connect with specific faculty in mathematics and biology at these institutions as part of the overall objective of NIMBioS to facilitate broadening participation in quantitative biology. NIMBioS established a set of MSI partners with established interdisciplinary interests in mathematical biology and supported visits by their faculty and students to NIMBioS activities of all types, as well as encouraging NIMBioS Postdoctoral

Fellows to visit these institutions to share their research. A particularly strong partnership was formed by connecting with faculty at Fisk University, and we assisted with their internal undergraduate research efforts and their development of a mathematical biology curriculum.

From recent surveys of the participants, we found that 91% of respondents who have graduated to date (many respondents are still undergraduates) went to graduate schools. Of these, 70% pursued PhDs, 23% pursued Masters, and 7% pursued medical degrees. These degrees were pursued at 30 U.S. and 3 international institutions. A summary of fields of graduate degrees are as follows: 30% were in biology/ecology, 23% were in computer/data science, 20% were in mathematics, 15% were in statistics, and 12% were in other fields. There were overall 211 participants in the program, including 187 undergraduates, with the remaining being students pursuing a veterinary degree or high school teachers. The veterinary students were included in projects focused on animal diseases with some undergraduates, as an effort to foster appreciation for quantitative methods in veterinary programs. The inclusion of veterinary students was stopped after the first three years, in part due to funding changes and in part due to feedback from evaluations indicating that the differences of maturity and experience level for these generally older students did not mesh well with the expectations of the program from the undergraduate participants. Publications resulting from the SRE program can be found under NIMBioS [18].

NIMBioS had a strong emphasis on evaluation of all NIMBioS activities since the institute was established in 2008. There was a full-time evaluator as part of the NIMBioS staff and in the later years this led to the formation, as an off-shoot of NIMBioS, of the National Institute for STEM Evaluation and Research (NISER) [22], which continues to conduct evaluations of NIMBioS activities and many STEM programs nationally. The SRE program had summative and formative evaluations and exit interviews carried out each year and these were used to inform modifications to the program across the years. These evaluation reports, along with those from many other NIMBioS educational activities are available at the evaluation report listing [21].

12.6 Conclusions

While the leadership of the SRE program had many previous years of experience in directing an REU program, the NIMBioS program presented unique challenges because of its highly-interdisciplinary nature. One conclusion from experience with the applications received over the years is that there is significant demand from undergraduates in the U.S. for novel programs that link disciplines. The SRE program was highly selective and generally only invited about 20% of the applicants to participate. Many of the selected participants were pursuing either double degrees across the quantitative and life sciences or had been pursuing formal course work across these two disciplinary areas. Thus there are many students who realize that there are significant advantages to formally pursuing interdisciplinary programs, and summer programs such as the NIMBioS SRE program are likely to find that there are many more qualified applicants than can be accepted.

A challenge for interdisciplinary programs is effectively building a collaborative culture, so that all students are able to contribute independent of their backgrounds. As we noted above, this involved developing effective quantitative educational experiences in a variety of modes due to the diversity of quantitative background of the students. We wished to avoid the “split” of teams, where the quantitative students were expected to handle all of the modeling and coding and the biology students were to deal with the data and analysis of insights from models. Rather, we encouraged all students to participate as equitably as possible in all aspects of each teams efforts. One of the enlightening aspects of this attitude was the fact that often in the final presentations by the teams, it was just as likely that the quantitative components were being presented by the biology students and the data analysis and biological implications being presented by the math students. We thus encourage having formal discussions of different personality types, how effective teams are successful, and how to enhance effective collaboration be a part of any interdisciplinary REU program.

We also wish to emphasize that success in broadening participation in REU programs arises from explicit and ongoing efforts to foster connections, particularly to institutions with emphasis on the student populations you wish to attract. Diversity in an REU-type program doesn’t just happen and will not occur simply by sending invitations to the institutions that have a potential pool of students. Rather, it requires direct and regular connections with the faculty and leadership at these institutions and preferably would include interactions on research and education beyond those just associated with an REU program. At NIMBioS, this involved quite explicit partnerships with a set of minority-serving institutions that were maintained through regular collaborations with their faculty and an ongoing set of visits back and forth between their institutions and faculty and postdocs affiliated with NIMBioS.

12.7 Acknowledgements

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Part IV

Virtual and Remote UR

Introduction
Virtual and Remote UR

Virtual undergraduate research experiences gained urgency in 2020 when COVID-19 hit and the traditional face-to-face research opportunities could not take place. Many faculty used this opportunity productively and in the spirit of Aradicheva et al.'s "When Life Gives You Lemons, Make Mathematicians", moved their research projects with students online [1].

However, virtual UR has been around long before COVID-19. In 1999, Robby Robson argued that with the help of the internet, faculty can "attract a diversified pool of talented students into research careers in these fields, and to help ensure that they receive the best education possible" [2]. The point is further reiterated in Chapter 13 by Donovan et al. "Removing the physical limitations of meetings can yield a greater flexibility for everyone involved. This flexibility can be particularly advantageous for women, people of color, and especially women of color". They also point out that with today's virtual options and our students having grown up in an always-connected, ever-shrinking world, this generation is poised to take on the technology as well as use it to its fullest extent and in new ways."

The faculty that worked with their students online quickly learned what works and what does not and can now serve as an excellent resource for others who think about the same. After all, having a virtual option for UR is beneficial even if COVID-19 no longer prohibits face-to-face meetings. Donovan et al. in Chapter 13 provide numerous helpful tips for running a virtual research experience for undergraduates, be it during the academic year or during the summer.

There is a considerable community of faculty that believe in the irreplaceability of traditional face-to-face research programs, often held at a single institution. As an alternative to these programs, Jackson and Mateja in Chapter 14 describe an excellent distributed summer research program in physics and astronomy. The multi-institution, multi-site program incorporated in-person and online components, capitalizing on the benefits each modality had to offer in conducting research and developing a community of scholars. Programs like that would be very valuable for the mathematics community as many of its benefits (e.g., inclusion and equity of participating faculty and institutions, building a community of scholars on-campus and in an online environment) are directly transferrable to the mathematics discipline.

At the end of a short essay about virtual REUs from 1999, Robby Robson made the following three recommendations:

1. develop an online clearinghouse for REU problems,
2. establish an online student and mentor network,
3. create an open forum for the electronic exchange of student work.

This is exactly what the program for high-school students (Chapter 15) do. The International Conference on Undergraduate Research (Chapter 16) provides an excellent forum for students to showcase their achievements and many similar conferences have been around for many years.

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13

Mentoring Students Virtually in Undergraduate Research

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13.1 Introduction

Conducting undergraduate research online? Is this even possible or effective? Yes! We often think of meeting face-to-face and working with students as things that go hand in hand. However, with today's technology, and all the lessons we have learned teaching during the pandemic, this pairing is likely to become more scarce as working virtually with students offers greater flexibility and equity than other traditional options. In a field where many collaborate with colleagues at a distance on their own research, offering an undergraduate version of this process has the immense benefit of adequately and accurately preparing our students for their future careers. Interacting online broadens students' soft skills to a virtual level, skills that are in high demand and dominate most career choices today.

Before early 2020, very few research experiences, if any, were run online. Many faculty mentors would passionately argue that there is no substitute for the face-to-face meetings and would not believe that the experience could be replicated in an online setting.

During the early outbreak of COVID-19, most colleges disallowed physical gatherings of any sort on campus, meaning that summer in-person research experiences were put to a halt. While from an educator's viewpoint there is always another summer, the situation is different for students. A missed summer could be devastating to their learning opportunities. Therefore, some faculty mentors decided to run their research experiences in an online setting because "something is better than nothing." The opportunity to participate in an online research experience, work on research projects, and interact remotely with faculty mentors and fellow participants is far more valuable than the alternative of not doing research at all.

This chapter should give you a good jumping off point to offer your first virtual undergraduate research experience. We will begin with an overview of the benefits and limitations of an online environment. We then walk you through how to get started, how to keep things moving, and how to finish up the project. We close with a few key points to keep in mind as you lead your virtual undergraduate research project.

13.2 Why do undergraduate research virtually?

You might be thinking, "Why would I offer a virtual undergraduate research experience? How is this different than a typical face-to-face choice?" With today's virtual options and our students having grown up in an always-connected, ever-shrinking world, this generation is poised to take on the technology as well as use it to its fullest extent and in new ways. This connectivity allows us to recruit a more diverse population of students to work with, an aspect that can drive and increase productivity in the workforce.

Removing the physical limitations of meetings can yield a greater flexibility for everyone involved: time constraints for jobs, child care, and availability to be online can all be worked around with virtual scheduling. This flexibility can be particularly advantageous for women, people of color, and especially women of color – groups who are often offered more opportunities in teaching, advising, committee work, and community service, which takes away from the time they can commit to scholarly activity [9]. Moreover, virtual undergraduate research can typically be done cheaper than a traditional summer on-campus program because housing and food costs are not necessarily a factor, allowing us to offer these career-changing research and learning opportunities to more students with the same amount of resources, especially those who need the flexibility of a virtual environment in order to participate. As a result, this can expose a broader student population to real mathematics research, thereby helping to recruit a larger, more diverse pool of students for graduate school. This is one way that we can help create change and foster a more diverse and inclusive mathematical community.

Another major advantage of virtual undergraduate research is the possibility of collaboration and sharing resources between different teams and institutions. For example, most Research Experiences for Undergraduates (REUs) offer seminars by external speakers, graduate school workshops, boot camps on LaTeX and computational software, and other interesting activities. Indubitably, the online environment is very conducive to multiple REU sites joining forces in running these presentations. This can increase the frequency of these activities and reduce duplication of effort. Participants may also benefit from the interaction with a larger group of students and mentors. In fact, due to such benefits, faculty PIs of future in-person REU sites may consider organizing some of these presentations online.

It is also easy and beneficial for the students to have joint events for the final (or close-to-final) presentations with other REUs. Parents, friends, or instructors from home institutions can show their support of the students by attending these presentations, which is normally impossible to achieve in an in-person REU. Simply sending a web link allows anyone to share these proud moments with the students.

There are several additional benefits of conducting research in an online environment. For instance, all presentations and collaborative work can easily be saved and shared with the group for future reference. After the research experience officially ends, it is easy and almost natural for the research teams to schedule additional meetings, finish their projects, or even begin investigating a new research topic.

13.2.1 Challenges to virtual research

There are, of course, drawbacks to doing undergraduate research online. First and foremost, access to a stable internet connection and computer will be needed. In 2016, 81.9% of people in the US had internet subscriptions, an increase from 77.2% in 2015 [11] and 74.4% in 2013 [5]. Public libraries are continually adding services to their technology offerings. Also, many businesses have set up free WiFi, and cities are now providing free hot spots for users. These might not always be ideal, but a short commute to do research and a good pair of headphones can overcome a few of these obstacles.

Until we become comfortable with meeting virtually, we will struggle with those things that naturally occur when meeting face-to-face: building rapport, visual clues for understanding, audial tone, not talking over one another, and talking in turn, to name a few. However, these items are not insurmountable; they just take extra effort in different areas. For example, building rapport requires open lines of communication. This can happen through some quick, friendly banter or check-in questions at the start of a meeting. Showing the students we are "human" is a good reminder that we are approachable. Additionally, having synchronous video meetings will allow the students to connect with each other (and with us!), alleviating some of those visual issues that arise when in-person meetings are not conducted.

We have found that one of the hardest aspects of virtual meetings is initially getting students to speak out on online platforms and then continuing to move the dialogue along. At the start of any project you might find that students are quiet and less willing to share, especially if they do not know their peers. Ice breakers and easy-to-answer/leading questions can help students open up to share their ideas, whether right or wrong. While shifting to online learning during the pandemic certainly helped ease some of the strained virtual communication, it is a good reminder that some of the same approaches you use to facilitate better discussion while meeting face-to-face also work during online synchronous meetings. For example, repeating student dialogue during difficult conversations and calling on students to solicit feedback are still good strategies, and many online platforms allow hand raising. Moreover, when working asynchronously or synchronously, you often have the ability to message students discretely and individually when there is an issue or you feel a student needs to be nudged to participate. Just remember, being encouraging in your messages can go a long way, so always find something positive to share with the students about their work.

As with any new experience, practice is required to effectively implement the strategies discussed above. In a survey of 10 mentors who ran virtual REUs in the early stages of the COVID-19 pandemic (summer 2020), almost all reported that the social aspects of their programs were below par, in spite of various efforts and strategies employed at different REU sites. One can plan virtual gatherings such as coffee breaks, game nights, or even group dinners. However, such activities tend to be less spontaneous in the online setting. They also compete with the participants' family events or other obligations and are less likely to branch into research discussions. Still, it is important to include these types of get-togethers in an online REU, as they promote team building and conversations that can unintentionally spark research ideas.

Maintaining energy and engagement throughout a research experience can be difficult in a virtual setting. Online meetings week after week can be draining, and many people find that staring at a computer screen for hours at a time is more demanding than a similar in-person event. Again, a detailed daily schedule helps with this challenge. By adding breaks, activities, or parts of the research project that can be done independently, students are able to recharge. Ways to accomplish this include fueling off-topic conversations at times during official meetings and varying the different types of research activities such as literature review, programming, analyzing data, developing models, writing proofs, drafting a manuscript, etc. While this may require a little more effort on the part of the mentors, it really helps preserve the intensity of research and uphold the high expectations of the participants.

Depending on where the research participants reside, time zones can be a factor when scheduling meetings. Nonetheless, it is usually not a major problem as long as mentors and participants can communicate early about their expectations and availability. In fact, such communication should occur before the program starts.

Finally, there will be a time or two that a research team member will have technological issues. Some common hindrances are an unstable internet connection, the inability to connect, a computer crashes (or runs out of battery) when least expected, or a device that decides to perform an hour-long update when it is needed for research! These are all issues that we have faced, and though frustrating, they are rarely the fault of the user. Don't forget to be flexible and understanding – next time, you could be the one facing a technological obstacle.

13.2.2 Online platforms to help facilitate research

Let's talk platforms. In order to connect with students and showcase their thoughts and ideas throughout the process, we recommend you choose among a few of the following:

- A shared storage site, like a folder in the cloud.
- A synchronous conferencing platform that includes video.
- An asynchronous messaging platform.
- A platform for sharing ideas and taking notes.
- A platform for showcasing completed work.

Though we describe these platforms in detail below and then offer suggestions of those you might use in Table 13.1, we do caution that using too many platforms can become overwhelming for both you and students. In order to avoid such pitfalls, take some time to pre-plan your needs for the research project. You probably won't know every detail, but taking time to think about the main technology needs of you and your students can help narrow down the list of choices.

Also, make sure you actually try out a platform before deciding to use it! Nothing is more disastrous than having little or no ability to help students troubleshoot issues and watching them continually be frustrated and distracted from their work. Approach these demos the way a student would: Is the interface easy/intuitive to use? Can I use it on a mobile device? Will it work on both Windows and a Mac? Who knows, you might just find a new platform for other projects you have. And don't forget one other key way to learn about technology: ask your students for feedback. Even though we should *try* to think like students when we check out these products, the way students approach them will still be different. Once you've chosen platforms and your project is underway, see what your students have to say about the programs. This might lead to a discussion about some easy-to-fix frustrations, or great tips for you and the other students on how to do something.

Shared (cloud) storage

This is a necessity whether research is being conducted online or in-person. Students and faculty mentors will need a common place to store and organize background readings, relevant papers, references, and their completed work. If you are planning on submitting your results to a journal, this is also a nice place to store drafts of the papers as well as a copy of reviewer comments and suggestions. Some platforms that allow shared folders are Box, Dropbox, Google Drive, and Microsoft OneDrive. While Overleaf is often thought of as a collaborative LaTeX editor, it also supports shared folders. It might be easiest to ask your students what they are most comfortable using, or even go with the cloud storage that your institution provides.

Synchronous video meetings

In Spring 2020, many faculty gained familiarity with Zoom as a means of teaching classes, holding office hours, attending meetings, and just generally stressing connection with others during the pandemic. There are also other options for real time video interactions: Google Meet, Microsoft Teams, Skype, and WebEx, just to name a few. Moreover, as a response to the COVID-19 pandemic, new platforms and technologies are constantly being created and modified to help with building and maintaining connections. Each has their own distinct options available, so make sure you pick a product that fits your group's needs.

We have found having a tablet or other graphic drawing surface is helpful during these meetings so that we can quickly test conjectures using an online whiteboard (a few examples are given below in the Sharing ideas & taking notes section) and simulate what would be "board work" if we were meeting face-to-face. There are many available options on the market, ranging in price from $25 for a basic USB graphic drawing tablet to above $1000 for a more powerful touchscreen computer. If students and mentors are unable to purchase a tablet or do not already own one, there are ways around this. For example, simply holding up a piece of paper can get a point across, though it can be a bit clunkier.

Communication and messaging

Being able to get ahold of your students and, on the flip side, your students being able to get in touch with you, is key for good collaboration. Most students have a mobile phone these days so text messaging is typically an easy choice for everyone and works for both group and individual messages. Additionally, if your students all have smart phones, they can easily share pictures of what they are working on, thus breaking down that barrier between writing something by hand and formally typing a solution.

If you are uncomfortable sharing your phone number, you can find an app that avoids sharing this information but still allows you to connect: Discord, WhatsApp, and GroupMe are common choices nowadays with the students. Communication apps like these not only come with the same bells and whistles that regular text messaging does, but also have additional features that you might wish to take advantage of: anonymous polling, event reminders, etc.

Sharing ideas & taking notes

Having a way for students to share ideas is crucial to a good collaborative project. Virtual whiteboard applications like Explain Everything, Limnu, Miro, Ziteboard, or Google Jamboards are easy ways for students to quickly communicate their findings, especially pictorially. Also, having one place that systematically (and temporally!) keeps track of ideas will be ideal for organization. Coauthor, a real time, collaborative editor that works with LaTeX, R Markdown, and HTML, was built by Erik Demaine to share information among research collaborators with ease. Slack has become one of the go-to platforms for online collaboration, though easy sharing of mathematics may need to be done more with pictures than with LaTeX in this case, at least until Slack becomes more math-friendly.

Showcasing completed work

Students will need to write early and often in the research process, so having an online collaborative platform for this is imperative. Overleaf is one of the primary choices for virtual collaboration. We have found that the free version of Overleaf meets our needs. With real time editing of LaTeX documents, easy link sharing, and a chat option, this site offers the basic options needed for a collaborative project. With an upgraded plan, one is also able to keep track of changes (as well as see who made them!) and integrate the project into Dropbox or GitHub if one must work offline.

While we find Overleaf to be extremely useful, there are several alternatives. For example, Google Drive has plugins that are able to create output files from a .tex file. However, if you are looking for something broader, consider CoCalc `https://cocalc.com/`, a web-based cloud computing platform for computational mathematics. Not only does it run their Jupyter Notebooks and SageMath Worksheets, it supports a full LaTeX editor, Linux terminal, R Markdown, HTML, and Python. Similarly, an online collaborative platform `https://replit.com` supports many coding languages.

Application suites

Feeling overwhelmed? Both Google and Microsoft have developed a full suite of applications to fulfill the needs of today's businesses. It is highly likely that your institution uses either Google's G Suite or Microsoft's Office 365 Suite. While each of these bundles has its individual pros and cons, they do both provide a host of applications that work seamlessly together. This ease of use might be a big selling point, as both you and your students are likely to be familiar with many of these applications, thus reducing the overall learning curve when it comes to sharing ideas. One word of warning, however: neither platform directly supports LaTeX. As mentioned earlier, there are plugins available to typeset .tex files in Google Drive, and Microsoft's Equation Editor takes many LaTeX commands. However, you will not have the full capability as would be available to you when using a standard LaTeX editor.

As you can see from the above discussion and Table 13.1, there are a LOT of software choices these days. The stand-alone applications all have free versions that can easily fit your needs, while the application suites pull everything together and lessen the choices to be made. Think through what will be critical to your project and mix and match to fit your needs. As long as you are clear with your students the uses for each platform you select, your choices will only make the research experience stronger.

13.3 Building a strong online community and supporting productive work

A majority of students and instructors believe building community online is more difficult than building community in-person [14]. Building community in a virtual experience takes a significant amount of time, more than in an in-person experience. In virtual experiences, it is often easy for students to pop-in for meetings and then check-out, or to miss meetings entirely. Moreover, video-conference meetings tend to be a bit more exhausting than in-person meetings, and thus should be kept shorter [8]. This is why being intentional about building and maintaining community in each phase of your online undergraduate research experience is essential. Stodel et al. [13] studied what learners felt they were missing in online learning. They recommend online instructors "provide examples of strong community building behaviours, remind learners of the important role they have in the discussions, offer constructive feedback, and be present to coach and support learners in their interactions." Similarly, the same can hold for online research. Students need opportunities to build community, be reminded of their important role in meetings and the work, be given ways to improve their work, and be supported.

13.3.1 The planning phase

As in any research project, you must begin by picking an appropriate research problem or problems, advertising the opportunity to students, and then choosing students. There are great tips about how to do this in [4]. Because the online research experience can often be more accessible for students, particularly those with familial and other work

Name	Supportive of LaTeX	Shared Storage	Video Conferencing Platform	Messaging Platform	Idea Sharing & Note Taking	Creating Completed Work
Application Bundles						
Google G Suite	via plugin	Drive	Meet	Chat	Keep & Jamboard	Doc
Microsoft Office 365 Suite	via Equation Editor	OneDrive	Teams	Lync	OneNote	Word
Standalone Applications						
Dropbox		✓			✓	
Skype			✓	✓	✓	
WebEx			✓		✓	
Zoom			✓		✓	
Group Me				✓	✓	
Text Message				✓	✓	
WhatsApp			✓	✓	✓	
Explain Everything					✓	
Limnu					✓	
Miro					✓	
Coauthor	✓			✓	✓	
Discord	via extension	✓	✓	✓	✓	
Slack	via extension	✓	✓	✓	✓	
CoCalc	✓				✓	✓
Overleaf	✓	✓			✓	✓

Table 13.1. Summary of various platforms to aid in virtual undergraduate research. With the exception of some of the applications in the suites, each platform offers a free version that could be used for your research experience.

obligations, we strongly recommend you advertise your opportunity to all students that have met the requirements you are looking for, as opposed to mentioning it to a select few that you think would be interested. It is important to also keep in mind that, because white faculty often lack ways to reach out to underrepresented students, underrepresented students are often uncomfortable and sometimes dissuaded from approaching faculty about research opportunities [3].

Below are our tips for creating a strong research group and building a strong online community within the planning phase:

1. Be clear about the research goals and expectations when advertising the project. Tell the students how much time you expect them to commit per week to the project (both individually and with their collaborators), the agreed upon times of group meetings (including the time zone if applicable), and what you hope will be accomplished throughout the project.

2. Pick students that want to do math! (Again, pick students that want to do math! We can't stress this enough.) The interest and desire to work and succeed is more important than knowledge in most cases, but particularly in an online research project where students naturally tend to work a bit more independently. Find students that are interested in doing research, and find out what motivates them. It is much easier to teach topics than it is to teach interest and motivation.

3. Anticipate that students will need to learn how to do research online. This isn't natural for many, particularly those who take classes in a traditional face-to-face environment. Create spaces and time within your project to invite students to reflect on the obstacles they encounter while conducting online research. As a group, discuss ways to overcome these, working together to build community and understanding of each individual's learning paths.
4. Be prepared to be flexible. When working virtually, you may need to adapt faster than in a typical face-to-face setting. You might have to teach a topic impromptu via Zoom, reschedule a meeting due to internet problems, or have unanticipated one-on-one meetings. Be prepared to adjust to the group's needs, but keep in mind that you not having an answer is also a learning experience for students – even the "experts" don't know everything.
5. Consider partnering with another mentor (or two!). This process is easier with at least one collaborator. Online meetings can be exhausting! It is also a good way to introduce new faculty to undergraduate research or a different research topic, or perhaps even revive a colleague's scholarship. Plus, an additional mentor changes the dynamic, most often for the positive. It is important, however, that all faculty should mentor all students. If the mentors are from different institutions (an added bonus in our opinion), your group will be impacted by the wider and more varied perspectives, making the students' work even stronger.
6. As mentioned in the Synchronous video meetings section, some form of writing tablets are essential for effective communication in an online research experience. However, we should not expect all students to have one or be able to afford one. Hence, program directors may wish to build the cost of writing tablets into their budgets. Depending on how this is incorporated and the funding agency's policies, the tablets might be the property of their home institution, in which case students would need to return them at the conclusion of the program and program directors should budget for the postage.

13.3.2 The first few weeks — starting off right

The first few weeks are essential in setting the tone, building relationships, establishing your role as a mentor, and making clear the expectations. Many find these to be more difficult in virtual research than face-to-face. During this period, you should expect to meet more often than you would in a typical in-person experience. In this section, we will guide you through steps and strategies for your first few weeks that will strengthen your virtual research program.

Before the first meeting

The first virtual research meeting may be a bit awkward. Students will be nervous about the experience, and the virtual nature of the research may be a bit daunting. Moreover, one of the hardest parts of fostering an engaging virtual undergraduate research environment is connecting yourself to your student researchers. To help everyone get to know each other, we encourage you and the other mentors to consider posting an introduction video that tells your background and explains why you want to be a part of the research group. Then, encourage the other students to do the same. If everyone already knows a bit about the other participants prior to the first meeting, they will be a little more comfortable and know who is part of their research community.

The first meeting

After the pre-recorded introduction videos, participants will likely feel ready to kick-off the first meeting. Crystal Wong [15] suggests great ways to help build community and set the tone for the first day of a virtual class, and we have adapted some of her ideas to fit undergraduate research. She recommends that you build community on the first day by doing 15-second introductions, asking students to share something low-stakes (such as favorite food or movie), and then having each student call on the next. At the end of the introductions, asking "What do we all have in common?" will help students see commonalities. Though this activity may seem a bit cliché and is not something one would normally do in face-to-face research, fostering an engaging virtual environment is difficult, and these activities, no matter how cheesy, can help the learners feel less awkward and become more involved with the research experience.

After introductions, you should set the tone for the experience. Start by asking students what their goals are and to set goals for the group. We recommend you type these goals into a document that is kept in your shared folder so that you can refer to them throughout the research project. Highlighting when students hit their milestones, whether they are large or small, will remind students that they are progressing.

Follow up by setting expectations and guidelines for the experience. How often do you expect the group to meet (and at what times)? How much time do you anticipate the students spending each week? How often should they meet together? You can ask the students to use the chat feature of the video conference (or another method, depending on your platform) to identify what group work etiquette will be helpful when engaging in their experience. Then, ask students to read the responses and establish community guidelines together. Finally, establish what methods you will use for communication when not in video meetings (see Section 13.2.2). Again, record the research guidelines, community guidelines, and communication platforms in a shared document for easy reference.

Finally, you want to end the meeting by getting students excited about the experience. What positive feedback have you received from students about research experiences in the past? Or if you have never engaged in undergraduate research before, what are some outcomes from others at your institution or the literature that you can pass on? There are many benefits of undergraduate research listed in [4] and [10]. This is the time to really drive home how interesting the research will be and leave them with a sense of anticipation for the next meeting.

The above will take up a fair amount of time, and because videoconferencing can be much more exhausting than in-person meetings, we recommend you stop the first meeting there. Wait until the second meeting to really dive into the research problem.

Introducing the research questions and getting started

As mentioned previously, you should expect to meet often during the first few weeks. Now, it is time to get to work! The article [1] offers succinct suggestions for beginning your project; however, make sure to adapt accordingly for virtual undergraduate research. First, depending on your students, you may need to provide a significant amount of background. For example, if you are doing research in graph theory, many of your students may not know what a graph is. Prepare worksheets to serve as a place for students' introductory notes. We recommend against spending a significant amount of time lecturing via Zoom. Carefully designed guided worksheets will provide a more interactive approach to help keep student engagement high (and as a bonus may introduce them to one of the platforms you will be using). In these worksheets, you may consider including important definitions, examples, and common terminology. For homework, have students work through basic examples and proofs to gain momentum and familiarity with the topic.

You will want to get the students used to communicating their ideas early, so consider requiring the students to give presentations on the introductory material. This can be done in the next meeting or beforehand through a platform such as FlipGrid or Recap, which are both available as apps on students' personal handheld devices. Use these presentations to stress the importance of attention to detail and using correct terminology.

Once students have an appropriate foundation in the subject, they will need to learn how to find and read research papers. Emphasize that you do not expect students to understand everything they read in a paper. Though we agree with [1] that "Meeting with the students and reading the article together helps develop students' ability to work independently," this may be a tiring task to do in virtual research. Consider having the students initially read through the paper on their own and post, either in a written format or via video, a few key points they noticed as well as any questions they had. Students and advisors can respond to some of the simpler questions in kind, and then all can meet to discuss more complicated questions about the paper.

As students begin delving into the research problem, encourage them to start small and establish conjectures. This will help them build confidence and make them feel like they are making progress (even if small), so that they can carry the momentum and work independently. Also, encourage students to write early and often! One caution is that, in our experience, online student researchers can be a bit more siloed. Learners will need to shift their focus from an individual perspective to one of community. Mentors must prepare to actively guide discussions and monitor student interactions so that the desired level of collaboration is achieved.

In addition, it is imperative to show that not everyone knows everything, including the faculty mentors. Showing that mentors do not necessarily know all the answers (and showing this early on in the process) and talking through how we approach the problems and scenarios will be crucial to the students' learning and helping them move forward in the experience. Remember, your expertise comes in knowing how to find the answer, not in knowing the answer to every question.

Where a faculty mentor may meet with students several times a week for two hours at a time, this may not be desirable in virtual meetings. Consider meeting four to five times a week for no more than one hour during the first few weeks of the project. Also, make sure to set deadlines for the students. For example, "Before tomorrow's meeting, I want you to have read through Student A's conjecture on Coauthor. Create examples and determine whether you believe this should be a theorem or you could find a counterexample. Make sure to also post your work." The clearer and more specific you can be, the better!

Continuing on and maintaining momentum

As students build collaborations and become more familiar with virtual communication, the group meetings can taper to once or twice per week, depending on the length of your project. Due to this research process being online, flexibility can help bring everyone together and provide a greater sense of equity and opportunity for those involved. Just be aware that being too flexible can make things seem disjointed and chaotic. We suggest having some set times to meet, with only a few adjustments should something arise in students' schedules.

To maintain community, you may consider doing some of the following:

- Start the meetings informally. While everyone is logging on and the meeting is just getting started, try to find ways to connect with the students. Ask them what they've been up to, talk about things (besides math!) that you've been doing, generally strike up a conversation with them, or intermittently play some "get to know each other" games. This helps students become more comfortable with their mentors and peers, making the research process feel more friendly and approachable.
- Have students reflect. Give students a short amount of time to share in the chat what they are finding to be difficult about research. Spend a few minutes reflecting on everyone's answers.
- If you have more than one problem that students are working on, make sure the different groups interact. For example, choose different students from each group to write the first draft of an introduction for their section. Have these students work together to gain an understanding of what needs to be written and bounce ideas off one another. Additionally, use students from different groups to peer review another group's work. This "outside perspective" is especially helpful to the process, as those not familiar with the work are more likely to find inconsistencies in terminology and problems with proofs. By engaging with other groups, the students will be more motivated to show what they have accomplished.

When meeting with students, know the central topics or ideas you need to cover, but let the students drive the meeting. Prompt students with questions, have them talk about what they are working on, ask where they have questions, or invite them to give some small examples of what they have discovered. This lets them tell their peers, in their own words, about their progress, but allows the mentors to hear and assess the work being done. If someone has a question, address it, but see if a peer can help explain the idea in a clearer manner. This leads to greater buy-in from the students. Also, don't be afraid if you have to pause to answer a student or admit you don't know the answer!

The meetings will all have a different feel to them, so get ready to adapt as needed. You will need to gauge the feeling of the meeting — are students frustrated, lost, having different production levels, etc.? If everyone is caught up and things are going well, let the students work individually or in small groups for a while. Staying connected at this point allows for questions as they arise. On the other hand, if the students seem confused or discouraged, stop and talk through the issues — maybe they don't understand the vocabulary, or maybe they are stuck on a proof or a certain example. If the "room" seems like a mix of feelings, you can quickly gauge the students with a thumbs up/thumbs down vote on material or, for a more in depth scale, rate their comfort with a topic by holding up fingers for a 1 to 5 scale or even have them use the chat feature of the platform.

Students (even those used to success) will get discouraged, get stuck, and experience failure, and they need reassurance. The article "Crossing Paths: Tips for Undergraduate Research" [6] was recently written by students to help other undergraduates as they engage in their own research experiences. You may consider assigning this article as well as portions of *Living Proof* [7] to students early in the research experience.

While meeting as one large group is good, ultimately you will need other types of meetings, especially if the project can be broken down into distinct chunks. Meeting once a week as a group can keep everyone focused on the same goal, but meeting in smaller groups leads to more focused work. In the smaller group meetings, take some time every couple of weeks to meet with students individually. Use these one-on-one meetings (or many-to-one meetings, if there is more than one faculty member present) to talk to the student about how things are going. Point out all of the things they have learned and the accomplishments they have made. Then suggest ways to improve and move forward from that point, thus driving the project forward. Meeting with one student is also a good way to hear just that student's voice; they may have questions or concerns they do not wish to raise in front of their peers. When conflict does arise, be prepared to handle it swiftly and appropriately. There are several great tips in [2].

Meetings with faculty mentors can be incredibly helpful, but the students also need to find time to meet on their own. Like the other meetings, these should be kept to an hour or less with the focus on talking to one another, discussing problems, asking questions, working through examples, and so on. These student-only meetings form the foundation of a strong collaboration among peers.

Throughout this process, the most important key is to remain positive! As we have said, working virtually can be exhausting. That doubles for your students as they can feel alone and isolated, and these feelings can be exacerbated in an intense learning process like undergraduate research. For most students, every part of this experience will be new and foreign, and they will all cope in their own ways. By following some of the steps above, you should get to know your students, thus helping you know when to push and when to let up.

13.4 Tips for mentoring virtually

Though many of these have already been mentioned throughout the chapter, we thought it best to summarize tips here in a section of their own.

- Take the time to ensure everyone gets to know each other. Oftentimes, students value the relationships they develop with their mentors even more than they value the research itself. Moreover, students who know each other well will become more collaborative and supportive of each other throughout the project.

- Make yourself available outside of meetings. Students will get stuck. If you were on campus, students could just stop by your office and ask a question. However, that is not feasible with virtual research. Make sure the students know the best way to reach you and the group members if a question arises. The last thing you want is for students to stall work for a series of days and lose momentum while they wait for the next virtual meeting.

- Allow students to own the research. As we said in Section 13.3.2, the students should guide the meetings. When students have a sense of ownership of their research project, they will get more out of the the experience and it will in turn become transformative.

- Pick enough platforms to get what you need, but don't pick too many. As mentioned, students can easily become overwhelmed if too many platforms are available. They will wonder what results and information to put in what place. Trying to keep things minimal, but effective, will help streamline your experience.

- Constantly reinforce the community aspect of this collaboration, more so than you would in face-to-face projects. Remind students they are not alone in this process and using you and their peers to help them answer questions is a strength, not a weakness.

- Use the last day for closure. Discuss as a group what worked well and what didn't about the online format and what you could do to overcome these in the future. Ask for feedback.

The article "Ten Salient Practices of Undergraduate Research Mentors: A Review of the Literature" [12] may also be worth the read if you are looking to strengthen your undergraduate research mentoring skills. Many of the evidence-based best practices mentioned can be easily adapted to virtual mentoring.

13.5 Summary

We argue that virtual undergraduate research and online REUs have a place even after the COVID-19 pandemic is over. Rather than trying to replicate all aspects of face-to-face experiences, one should focus on leveraging the uniqueness of the online platform and developing new and non-traditional ways to enable even more students to engage in mathematical research. A great example of this is the Crowdmath project described in detail in Chapter 15.

An online environment has the ability to deliver a research experience to far more students who would not otherwise have the opportunity. Students can easily participate, even if for family or health reasons they may not be able to physically travel to another city or state (or country). Introverted students who may not want to participate in a typical program that is full of social activities can now have an alternative. Finally, the "work-from-home" program provides students with a unique opportunity to work in groups online — an important skill for the future as many companies continue to announce plans for their employees to work remotely.

Merely providing a physical space for a multi-week interaction among students who are passionate and curious about mathematics, and possess a strong desire to learn, will produce a lot of mathematical ideas on its own almost by magic. Providing the space virtually during these new and exciting online programs does have its challenges, but with careful planning and execution, many rewarding experiences will come for the students and faculty.

Yes, you can create a transformative undergraduate research experience virtually and at a distance. However, it will take time and planning, and will look a bit different than the typical face-to-face experience. Just remember, virtual classrooms eliminate many of the natural social characteristics with which students are familiar, and you will need to supplement these interactions with those that better suit the virtual learning world. However, we are constantly looking at ways to engage more diverse groups into the landscape of mathematics, and virtual undergraduate research can be a great way to help achieve this. The only question is: when will you start?

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14

A Distributed Summer Research Program in Physics and Astronomy

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14.1 Abstract

Councilors in the Physics and Astronomy Division at the Council on Undergraduate Research developed a distributed summer research program for students in these disciplines. Faculty research mentors from multiple institutions hosted students on their respective campuses while students interacted online as a virtual cohort until an in-person meeting was held during the final week of the summer program. The program outlined in this manuscript can serve as a template that faculty in mathematics and other fields can follow to engage greater numbers of students and faculty in this important high impact learning paradigm.

14.2 Background and motivation

Engaging undergraduate students in research is a high-impact practice [9] having significant and demonstrable benefit to students [3, 10]. The Council of Undergraduate Research (CUR) is an organization whose mission is to support and promote this particular high-impact practice. The Physics and Astronomy Division of CUR (CURPA) has advocated for the community to provide ALL physics and astronomy undergraduate students with research experiences and for the National Science Foundation's (NSF) Physics, Astronomical Sciences, and Materials Research Divisions to increase support of their Research Experiences for Undergraduates (REU) programs [16].

At CUR's 2011 business meeting, councilors within CURPA decided to move beyond advocating support for undergraduate research experiences to hosting REU students. However, most CURPA councilors were from physics departments that, due to their size, would be unable to individually support a typical 10–15 participant, single-site REU program. Our goal was to develop a program that capitalized on the mentoring skills and research talents of faculty at smaller institutions to provide meaningful research experiences for undergraduate students. Emulating successful multi-institutional, multi-site REU programs (e.g., [15] and [8]), CURPA councilors proposed to pool their resources to establish a multi-institutional, multi-site REU program that would be guided by effective practices identified in mentoring undergraduate students in research [2]. This includes developing projects that provide undergraduate students with:

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[2]Present address: Goldwater Scholarship and Excellence in Education Foundation

- direct faculty mentorship in the areas of physics and astronomy (projects conducted in a small group student:mentor environment),
- intellectual ownership of the project and the development of autonomy, and
- the opportunity to participate within a community of scholars.

Our goal was to identify students who were as early in their STEM education as possible, with particular emphasis on those from marginalized groups enrolled in institutions with little or no on-campus resources for undergraduate research (e.g., community colleges and regional universities). It was our hope that students from these institutions may be more comfortable accepting an REU at an institution that looked and felt similar to their current institution as compared to one having a significant emphasis on graduate students. This manuscript is an expanded version of a prior dissemination of this CURPA distributed REU program [6].

14.3 Applicability to mathematics

Readers may be surprised to see an article that talks about involving students in a physics and astronomy REU in a publication for mathematicians. The answer is simple: there is a need for distributed REUs in mathematics for the same reasons there is a need for them in physics and astronomy! More opportunities for undergraduate research experiences are needed than can be provided by traditional one-host REUs, a “one-size-fits-all” REU model does not necessarily work well for all students, and the community can engage more research active faculty as mentors to the benefit of both the faculty and students.

The benefits from participating in a distributed REU program, irrespective of discipline, are outlined in Table 14.1.

Perhaps the only benefit of the in-person interactions unique to physics and astronomy, as compared with mathematics, was its ability to include experimental research in the list of potential research projects undergraduate students could participate in. A distributed in-person REU model ensured that no research area within physics or astronomy was excluded from participating—another important element of inclusion and equity these programs are striving to achieve.

Finally, as we will outline in this article, a distributed REU appears to have some advantage over the traditional single-institution REU that includes its ability to attract students from traditionally marginalized groups in STEM disciplines. Additionally, we believe the significant geographic distribution of our host institutions enhanced our ability to target community colleges and other schools where we were likely to find larger numbers of students who were early in their post-secondary education and that, for personal reasons, needed to remain in the region. This will be as true in mathematics and other fields as it is in physics and astronomy.

14.4 Key features of the CURPA distributed REU model

Faculty from seven universities from across the United States hosted REU students during this two-year pilot program to test the viability of a distributed REU model. The participating institutions were Central Washington University (CWU), College of Charleston, Embry Riddle Aeronautical University, State University of New York College at Geneseo, University of Wisconsin-La Crosse, Weber State University, and Wright State University.

The CURPA distributed REU had some features that resemble those found in a traditional, single-institution REU and others that are not. The features for these programs are compared in Table 14.2.

14.4.1 The REU cohort of the CURPA distributed REU

NSF’s Division of Physics funded the CURPA distributed REU program as a two-year pilot enabling it to host 6 students in each of the 2014 and 2015 summers (Award No. 1358879). Funding from the Washington Space Grant Consortium, CURPA, and participating institutions enabled the program to support an additional student each summer. Along with standard recruitment processes (e.g., dedicated website, broadly advertised to the community), research mentors promoted the program at their local community colleges and minority serving institutions, often using well established relationships with faculty at these schools to ensure the opportunity was well advertised. We believe the distributed, multi-institutional nature of the program enhanced our effectiveness in reaching a larger number of these

Group	Benefits
Funding Agency and Participating Faculty	The equity and inclusion efforts REU programs are striving to achieve, in regards to engaging different types of post-secondary institutions, is enhanced by a distributed, multi-institution, multi-site model. Such programs provide opportunities for research active faculty at smaller institutions, or institutions where only a handful of individuals conduct research, to participate in a REU program – individuals whom otherwise would not be able to participate solely because of the institution where they are employed. Our distributed REU program provided opportunities for two faculty from traditionally marginalized groups (e.g., African American, women) to serve as a research mentor as well as for two junior faculty to be mentored by a senior faculty member.
Institution and Participating Faculty	Participation in a REU program is an opportunity for the institution and individual research-active faculty to gain outside recognition for their efforts and the quality of research being conducted. Additionally, this provides an opportunity to diversify the students engaged in on-campus research.
Students	Ability to engage mentors who are active researchers at institutions similar to their own. This may be particularly beneficial for students early in their post-secondary education—a key demographic the CURPA Distributed REU program was recruiting.
Program Structure	**Benefits**
In-person interactions	Students had the opportunity to participate in a variety of professional and social interactions with their research mentor, the on-campus research community, and the CURPA Distributed REU cohort—an important benefit identified by the students. We believe these in-person interactions were beneficial in proactively addressing student isolation, particularly given the CURPA Distributed REU program focused on recruiting undergraduate students early in their post-secondary education.
Online interactions	Introduces students to the mechanism by which large-scale multi-institution research groups interact. Even a decade ago, students were already comfortable communicating with one another via social media and other electronic resources. This has only been enhanced due to their experiences during the COVID-19 pandemic along with advances made in, and access to, technology.

Table 14.1. Benefits from participating in a distributed REU program, irrespective of discipline.

institutions than would have otherwise been possible. By the second year, there were over 140 applications to the program.

The program's two-year student demographics included 6 men and 8 women with students self-identifying as Hispanic (2), African-American (1), Asian (2), and Caucasian (9). Two students identified themselves as veterans. Of the 14 program students, eight were from first-generation-college families. Two of the program's undergraduate students were rising seniors, eleven were rising juniors, and one was a rising sophomore. Four students were from community colleges, five students were from comprehensive institutions, and five students were from research institutions.

14.4.2 The cohort experience

The format used by the CURPA distributed REU program each summer was:

Week 1: REU participants traveled to their respective host institution and were introduced to the campus, workplace, and the particular university's summer cohort of research students. Participants completed Responsible Conduct in Research and any relevant safety training along with developing a work plan with their research mentor. This work plan facilitated the discussion of expectations between the student and mentor by creating a daily work schedule and communication protocol for individual and group meetings, developing an outline of the research being undertaken and what was to be accomplished, identifying the type of training needed, establishing a deliverables timeline, and enabling mentors to provide context for how the student's work fit into the bigger scientific picture.

Feature	CURPA REU	Traditional REU(s)
Hosting Institution	Multiple.	Single.
Faculty Mentors	Multiple, experienced researchers, varied mentoring experience.	Multiple, experienced researchers, varied mentoring experience.
Communities	Virtual across multiple institutions and with peers at each host institution. Virtual networking models today's large-scale research group communications.	Peers at host institution.
Exposure	Generally involved students from smaller institutions who were able to work in a similar academic and "community" environment.	Generally, all students, regardless of home school type, work at a university with an expanded research mission that also includes graduate students.
Number and type of research projects	Scalable to any number of host institutions and to any number of project types.	Limited to faculty and projects available at a single institution.
Student feedback	Immediate supervisor, students and faculty participating in the REU project, along with students and faculty participating at host institution.	Immediate supervisor, students and faculty participating in the REU project, along with students and faculty participating at single host institution.
Culminating experience	Brings students with exposure to diverse campus settings and summer experiences together.	Bring students with similar single-campus summer experiences together.
Program cost per week per student (NSF funds only)	approximately $950.	varies; program solicitation indicated participant costs were generally not to exceed $1,200 per week.
Other benefits	Collaboration with Council on Undergraduate Research.	

Table 14.2. Comparison of features between the CURPA REU program and most traditional REU programs.

Weeks 1–9: Students conducted their research during this period. To promote communication among the REU cohort and engage them as a learning community, participants interacted through weekly webinars with additional interactions available through other mechanisms (e.g., discussion board, video conferencing). Several students wrote weekly research blogs using WorldPress' Undergraduate Research Reports.

During weekly webinars, students interacted with practicing physicists and astronomers, discussed their summer research projects, and participated in a number of targeted activities such as the American Institute of Physics' Careers Toolbox [1]. Webinars developed and presented by outside groups, such as the Institute for Broadening Participation, were also provided to participants.

Week 10: Students and PIs traveled to an end-of-summer meeting at the PI's campus (CWU) to finalize the students' deliverables (i.e., presentations, written reports, assessment of the program). This five-day event also provided participants a bonding experience that included activities (e.g., rafting trip) and field trips (e.g., Laser Interferometer Gravitational Wave Observatory [LIGO]).

14.4.3 Research projects and products

Research projects were designed and developed by mentors to (1) advance fundamental knowledge within the discipline and (2) provide undergraduate students with an opportunity to engage in research and make meaningful contributions to the investigation. The research performed by REU participants was either experimental or computational and represented a diverse array of physics and astronomy subfields (e.g., Properties of Cu doped ZnO thin films grown by spray pyrolysis, A search for pulsations in the sdB star EC 20117-4014, Persistence length of microtubules with multi-rigidity segments, Time-resolved tandem Faraday cup, Exploring the limits of planetary habitability using orbital dynamics, Discovery and characterization of terahertz laser emissions). These projects provided students with skill development (e.g., computational, experimental, laboratory safety) and also introduced them to the scientific literature, ethical conduct of research, and dissemination of data and technical information (oral and written) to the professional community. For example, students co-authored several peer-reviewed manuscripts that were published in professional journals [5, 7, 13, 14]. Multiple presentations were made by CURPA REU students at local and regional meetings, including several presentations at the National Conference on Undergraduate Research (NCUR). Two of the students also received local awards for their research.

14.4.4 Addressing reviewer concerns with a distributed REU: Student Isolation

The main concern of those who reviewed our NSF Division of Physics REU proposal was that a "distributed model" would likely lead to student isolation. "The program activities are too dispersed and promote student isolation instead of camaraderie and cohesiveness," was one of the comments. The PIs and host institution faculty sought to implement program elements that would proactively address student isolation. The following measures were implemented.

1. A mechanism was put in place that enabled REU participants to regularly communicate with one another through video conferencing and a discussion board.
2. Ice-breaker and collaborative/competitive activities were conducted throughout the summer both virtually and at the students' host institutions (e.g., t-shirt design competition).
3. A person-to-person, culminating experience was hosted at CWU. The field trips were funded using non-NSF funds (i.e., CURPA and CWU).
4. The PIs communicated at least weekly with CURPA REU students, providing program updates and feedback.

Our host institutions also had robust faculty-mentored undergraduate research programs that were essential in effectively engaging the distributed REU students. When asked what elements of the CURPA REU program had the greatest impact or benefit, the fourteen, Years 1 and 2 REU participants listed:

- "Social interactions with Mentor at the Host Institution" (frequency: 12 times),
- "Social interactions with Students at the Host Institution" (9 times), and
- "End-of-the-summer activity at Central Washington University" (9 times).

14.5 Lessons learned

14.5.1 Evaluation elements

Multiple surveys were either developed or taken from the public domain to assist with assessing the effectiveness of this program. The surveys included pre-experience Preflection Survey [11], post-experience SURE III Survey [12], and a CURPA program specific End-of-Summer Survey, developed by Mateja and Jackson. Mateja also conducted group and individual exit interviews with all participants. In year 2, two independent evaluators (Palmquist and Clay) conducted exit interviews with Jackson, Mateja and students from the year 2 cohort.

Faculty mentors participating in the CURPA REU program were asked to complete two surveys developed by Mateja and Jackson. The first survey was specific to the mentor's work with their summer mentee. The survey asked mentors to describe, among other things, the mentee's research environment, perceived benefits from participating in the program, challenges their student had and how they helped the student overcome those challenges, and participation as part of their respective host institution's summer research cohort. The second survey asked mentors about the specific skills gained by mentees as a result of their research project (i.e., ability to communicate scientific ideas, ability to apply content associated with a typical physics/astronomy curriculum, ability to work and learn independently, ability to apply critical thinking skills, etc.).

The feedback from these surveys, along with the feedback from the independent evaluators and individual conversations with research mentors, were used to identify the strengths of the program along with elements of the program that could be improved.

14.5.2 Activities

With regard to supplemental activities (e.g., webinars, LIGO field trip, end-of-summer group interaction at CWU), of which there were 14 and 15 in Years 1 and 2, respectively, the students rated these activities as having moderate to high impact/benefit 87% of the time. The webinars, which made up 8 and 9 of the activities, respectively, were described by students in the second year of the program as not interactive enough to engender valuable communication among the students. Several students expressed concern that webinars took time away from their research. Instead of live webinars, it was suggested that participants be asked to watch pre-recorded webinars within a given time interval. Requiring the students to post two questions to a discussion board that is monitored by the other REU participants would engender communication among the students, the faculty mentors, and the people who presented the webinar. In this way, participants could watch the webinars during a convenient time, such as when their data was being compiled, while moving their research forward. Today, use of communication tools like Slack, Teams, and Zoom could lead to more robust communication among distributed REU participants.

14.5.3 Overall strengths

The program was successful in introducing participants to the research process. Participants became aware of the depth of knowledge and detail required in the process of research. The specific methods participants employed were highly varied as a diverse set of research projects were performed, covering topics in computational modeling, astronomical observations, and experimental design and measurement. While challenged to learn the skills required to conduct their research, all students indicated they were able to work through and resolve their challenges by the end of the summer. Participants expressed an appreciation for having the opportunity to experience an authentic and challenging research project and only expressed the concern that their summers were too short to accomplish everything they set out to do.

Second, the program either engendered or strengthened the participants' interest in a scientific career. Our evidence of this included direct comments from students, observed gains in student self-confidence as the summer progressed, clearer and more well-defined descriptions of the students' intended career paths, a readiness and eagerness for more demanding research, an increased interest in engaging in learning communities, and clearer indications that they were thinking like scientists. Comments from mentors, gathered through their end of summer surveys, also pointed to an enhanced interest in a scientific career from their mentees'. As with traditional REU experiences, there was an increased interest from the students in pursuing graduate education, specifically a PhD.

Third, participants benefited from exposure to a community of research scientists and overwhelmingly indicated they felt part of a research community at their REU institutions. Participants reported they were surprised by the congeniality and camaraderie within their local group of researchers. A description given by one of the CURPA REU students was "everyone had an opportunity to mentor each other." Students valued the range of experiences of other participants in the program. Participants said they appreciated the diversity of experience gained by their colleagues and the opportunity to share that experience when they convened for the final week of the program. Participants valued hearing about the research results of their CURPA REU colleagues in other subfields and at other universities.

Finally, participants appreciated exposure to career opportunities for physicists, specifically identifying the value they found in webinars focused on career opportunities, stating they "didn't really know what physicists could do outside of academia" until these presentations.

14.5.4 Areas to enhance

While the CURPA REU students felt a sense of community at their host institutions where there were other undergraduate students, a faculty mentor and, in some cases, graduate students, there was less of a sense of community among the REU group until the group convened at the end of the summer. While the webinars were of some help in building community among the CURPA REU supported participants, a majority of the students said it would have been useful to have more interactions with these colleagues earlier in the summer, in agreement with what was found in Ref. [4]. Having said this, the participants did not want any early summer team building experiences if those experiences would reduce the time available for research.

The webinars were rated by participants of moderate to high value/benefit. However, a number of these were delivered rather traditionally and students indicated they were not deeply engaged in most of the webinars, aside from those on career opportunities. The participants valued these webinars for their information and content rather than facilitating interactions among the cohort.

Timing of the webinars was another issue raised by participants. Webinars were held during working hours and often in the middle of the students' research activities, activities they considered to be of higher value than the webinars. Pre-recorded webinars would have been of essentially equal value but would have allowed the students to choose their own viewing time. As the individual host institutions held webinars as part of their summer programs, it is also possible that CURPA REU students had webinar fatigue.

With regard to programming, students participating in the CURPA REU appreciated receiving information on careers in physics and astronomy. We believe this area of programming could be expanded, such as our use of the Careers Toolbox [1] and facilitating discussions with physicists and astronomers working in non-academic settings. With this information, students, like those in our program who were entering their sophomore and junior years, could take courses or attend professional development opportunities that inform their career interests and diversify their skills, subsequently expanding career opportunities.

What the students desired and valued most out of the CURPA REU experience was the time to do research. While they indicated that social interactions were important to them, particularly at their host institution, the students were generally critical of anything that interfered with their research. The suggestion that there be team building activities among all CURPA REU students at the beginning of the experience was desired only if it did not take time away from their research projects.

14.5.5 Impact on career plans

Seven students in the CURPA REU program came into their summer experience interested in obtaining a PhD. By the end of their respective summer programs, 11 students indicated an interest in pursuing a PhD (at least, 8 students have pursued further education, either a second baccalaureate degree in a related discipline, such as engineering, or a graduate degree). Although the decision to pursue a doctoral degree was neither the focus of our program nor was it necessarily an indicator of a program's success, it is indicative of increased student confidence in their ability to perform research. The increased interest of our students in pursuing a PhD after a summer research experience is consistent with survey feedback from nearly 25,000 undergraduate students who participated in more traditional research programs for undergraduates from 2011–2018 [12]. This result also suggests that a distributed REU is as effective in building students' confidence in their ability to do graduate-level work as the more traditional single-campus REU approach.

14.6 Conclusion

The CURPA distributed REU experience appeared to function and impact students in a manner that is similar to a traditional single-institution REU, such as guiding students through a meaningful summer research experience with feedback and support from program participants (e.g., mentors, REU students, other students, staff, and faculty conducting summer research). As we stated earlier in this paper, where we appear to have some advantage over the traditional single-institution REU is in our ability to attract students who come from groups who are underrepresented in physics and astronomy. All fourteen REU participants in our program came from groups that are underrepresented by gender, ethnicity, or school type in physics and astronomy or who were veterans or first-generation college students. The significant geographic distribution of our host institutions, we believe, enhanced our ability to advertise the REU in the "local" area of each host institution, thereby increasing our targeting of schools like community colleges and other schools where we were likely to find larger numbers of students we were seeking to support, including those that, for personal reasons, needed to remain in the region.

While student isolation was a concern of those who reviewed our distributed REU proposal for the NSF, issues of student isolation did not materialize. We attribute this to the fact that all students participated in two student communities, the virtual community associated with the distributed REU and the student research communities that already existed at each of our host institutions. Optimally, in-person team building experiences would be held at the beginning and end of the summer. However, for financial and time reasons, only an end of the summer team-building meeting was held. In the future we would also work to make webinars more interactive and, possibly, pre-record them to minimize the impact they had on the timing of students' research activities particularly since the program hosted students at universities in multiple time zones. There are also more robust technical platforms that could be used today to enhance inter-student communications and activities (e.g., t-shirt design, movie night).

Additional opportunities to engage more undergraduate students in research, including undergraduates in mathematics, are needed. We believe our study has demonstrated that smaller academic institutions with existing undergraduate research programs can work collaboratively to successfully support additional REU sites. Finally, in today's COVID-19 world where the research team density may need to be reduced to ensure appropriate social distancing, this model may have a larger role to play in providing undergraduate students with meaningful research opportunities, either with low-density in-person interactions or in an online environment.

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15

The CrowdMath Project

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15.1 Introduction

CrowdMath is an online program that was created to give math research opportunities to high school and college students around the world who do not have access to other math research projects [1]. Any student from any country can join CrowdMath just by signing up online. The projects are conducted in the CrowdMath section of the Art of Problem Solving website, where participants can post their questions, ideas, and proofs.

CrowdMath removes all barriers to entry for student research besides having Internet access. It is free for students to join CrowdMath. Students do not have to submit any application or request any letters of recommendation. Any student who is interested is welcome to participate.

The idea was inspired by the online Polymath projects for professional mathematicians led by Timothy Gowers and Terence Tao. The Polymath projects started in 2009 on Gowers's blog, where Gowers posted an open problem and invited others to collaborate on solving it in the blog's comment section. The Polymath projects produced multiple papers with substantial progress on major open problems.

In fall 2015, the first and third authors started discussing the possibility of running an online program with the Art of Problem Solving which would be like a Polymath project for high school students. The original Polymath projects were very successful in finding new mathematical results. With CrowdMath, it was initially unclear whether high school and undergraduate students would be able to solve a real research problem collectively in an online forum. In particular, what would motivate them to participate? It was also unclear whether the collaboration dynamics in CrowdMath would differ from those of Polymath.

Since 2010, the first and third authors have been organizers of the Program for Research in Mathematics, Engineering, and Science (PRIMES) at MIT. PRIMES is a selective year-long research program for high school students that requires a problem set, essays, and letters of recommendation for the application. Mentors at PRIMES, usually graduate students or postdocs in the MIT Mathematics Department, work on projects either one-on-one with students, or with a team of two or three students. The students in PRIMES perform very well in science competitions using their PRIMES projects. Since 2011, PRIMES students have won more than 340 awards at national and international science competitions, including 80 top awards in the Siemens competition, Intel/Regeneron Science Talent Search, ISEF, Davidson Fellowship, and S.-T. Yau High School Science Award. Students that participate in PRIMES usually go to the top colleges in the country, and they publish papers in reputable math journals such as *Representation Theory, Transactions of the AMS, Journal of Algebra, Journal of Algebraic Combinatorics, Electronic Journal of Combinatorics, European Journal of Combinatorics, Discrete Mathematics,* and *Journal of Combinatorics.*

Although the students in PRIMES usually have very high mathematical aptitude and a strong work ethic, most of them already go to highly ranked high schools with strong math programs, or their parents pay for them to take online math courses. Before CrowdMath, the first and third authors created two offshoots of PRIMES to provide less privileged students with an opportunity to do math research. Both programs focus on math enrichment rather than proving new results. One of the offshoot programs called PRIMES Circle was created to increase participation in math research by local high school students from underrepresented populations and underserved communities; it runs during the spring term. Another program is MathROOTS, a residential summer program at MIT open to the same category of students all over the United States.

CrowdMath serves a different purpose than MathROOTS and PRIMES Circle, because CrowdMath is focused on proving new results, and it is open to students all over the world, rather than only in the United States. Whereas MathROOTS and PRIMES Circle are in-person programs, CrowdMath is entirely online. All offshoots of PRIMES except CrowdMath require students to submit applications and be accepted into the programs. With CrowdMath, anyone is allowed to participate. The only thing that participants have to do is register on the Art of Problem Solving website.

In early November 2015, the first and third authors invited the second author, then a PRIMES mentor, to be the head mentor for CrowdMath, and the second author enthusiastically accepted. We recruited two other mentors and chose three topics for the first year: pattern avoidance in 0-1 matrices, incidence problems, and optimal codes. At first we restricted participation to only high school students, but later in the year we also opened it to college students since undergraduates were interested in participating. Now we also allow advanced middle school students.

CrowdMath is a good student research model for multiple reasons. For one, CrowdMath removes all barriers to entry for student research besides having Internet access. Even if they live somewhere with no one else who can help them do math research, a student who likes math can join the CrowdMath project and collaborate with the CrowdMath mentors and other participants.

A 2017 survey revealed that CrowdMath contributors often felt that they had been deeply transformed—from learners to researchers. We tried to design CrowdMath so that collaboration would be valued over competition and participants would exhibit empathy and comradery. Participants have commented on the "friendly environment" of CrowdMath and the opportunity for open, informal discussion and creative exploration [9].

In the following sections we discuss CrowdMath in detail. In Section 15.2, we discuss how CrowdMath works including the platform, how the papers are written, and the role of mentors. In Section 15.3, we compare CrowdMath to Polymath in terms of the target audience, the project leaders, the type of problems, and other factors. In Section 15.4, we discuss things that have worked and not worked in CrowdMath, including both successful and unsuccessful research topics, and ways that mentors have increased CrowdMath activity. In Section 15.5, we discuss the advantages of CrowdMath over in-person research groups including the cost, the ability to serve more students, the lack of pressure, and the ability to collaborate from anywhere in the world. In Section 15.6, we describe an example of a CrowdMath research thread from summer 2020 on the topic of metric dimension. Finally in Section 15.7, we discuss the benefits of CrowdMath for students, faculty, schools, and other math research programs.

15.2 How CrowdMath works

CrowdMath projects are conducted entirely on the CrowdMath section of the Art of Problem Solving website. The CrowdMath section includes a message board where mentors and students discuss the research project. The format of the CrowdMath message board is similar to other forums such as Reddit and Twitter, except the Art of Problem Solving website can compile LaTeX code, so it gives CrowdMath participants the ability to display math nicely in their posts. In the CrowdMath section, there is also a page with open problems and a page with references and exercises for each research topic.

Each year, we choose topics that are understandable for students with little to no mathematical background beyond the standard high school courses. In late December or early January, we post several references and exercises for each topic on the CrowdMath section of the Art of Problem Solving website. The exercises usually consist of known results that have short proofs. We also post a list of open problems for each topic.

Throughout the year, CrowdMath participants interact with CrowdMath mentors and each other by posting on the message board. Posts include questions, ideas for possible solutions, full solutions, and new open problems. The research direction goes wherever the participants want to take it. Sometimes a participant asks a new open problem on the message board and other participants start working on it, and this leads to a long research thread. At the end of the year, the mentors compile the results from the year's CrowdMath project and write papers for each topic that has interesting results. The papers appear under the collective authorship of P.A. CrowdMath.

In every year of CrowdMath, we have at least one topic in discrete math, but sometimes there are topics outside of discrete math. Part of this focus on discrete math is because of the second author's background, and another reason is because problems in discrete math usually do not require as much background for students to understand as many other areas of math. Most of the CrowdMath problems that we choose can be described in at most a paragraph. The problem descriptions for CrowdMath are entirely self-contained, so advanced high school students can understand them without having background in the research topic.

People with knowledge of the research topics or ideas for new topics are welcome to participate as mentors. Most of the mentors in CrowdMath are postdocs or graduate students, but recent math PhDs working in industry have also participated, using this as an opportunity to remain active in math research even after leaving academia. The mentoring workload varies, depending on the number of other mentors and student participation, and this provides flexibility for various degrees of mentor participation.

15.3 Comparing CrowdMath and Polymath

Some of the major differences between CrowdMath and Polymath are the platforms, the target audiences, the anonymity of the participants, the type of problems, and the educational support for participants. The target audience is professional mathematicians for Polymath but high school students for CrowdMath. While Polymath can choose open problems that require a sophisticated mathematical background to understand, we choose open problems for CrowdMath that advanced high school students can understand. Moreover, we offer online classes for newcomers on the Art of Problem Solving website to learn more about the CrowdMath topics. Another key difference is in the project leadership. While Polymath projects are led by high-profile mathematicians including Fields medalists, CrowdMath projects have been led by postdocs and graduate students.

The number of participants in Polymath 1 and CrowdMath 1 was similar, but 74% of Polymath 1 participants used their true identity, while only 3% of CrowdMath 1 participants used their true identity. When participants use an alias on the CrowdMath message board, neither the mentors nor the other students know their true identity. Participants in CrowdMath have noted that they like the anonymity of CrowdMath, since it results in less judgment based on demographic differences between participants [9]. CrowdMath 1 had a gender imbalance in participation, but it was not as bad as Polymath 1. While 14% of CrowdMath 1 participants identified as a girl, only 3% of Polymath 1 participants identified as women.

Collective credit was an issue in Polymath projects [10]. With CrowdMath, so far it has never been an issue because CrowdMath was designed to value collaboration over competition. In surveys, one thing that students have liked about CrowdMath is that the papers are written under the name P.A. CrowdMath. One student said that they were "really excited" to publish under the pseudonym, and that working together on the paper was their "best moment." Another student said, "I don't think it would have made much of a difference whether or not I was co-author, because I definitely gained more than having another paper I can list on a CV." The same survey found that making substantial progress toward the solution and publishing a collective paper are motivations for CrowdMath participants, even if the progress was not due to the student's personal contribution [9].

Participants of Polymath and other professional collaborative online forums felt that such forums lacked informal, exploratory aspects and experimentation [11]. On the other hand, most of the math done in CrowdMath has been exploratory. Although CrowdMath participants have solved some known open problems, most of the results in CrowdMath papers are solutions to problems that were suggested by other CrowdMath participants. Experimentation has frequently led to ideas and conjectures in CrowdMath. One survey respondent said that participating in CrowdMath showed them that research math is "about experimenting and playing with things over time" [9]. Some participants in CrowdMath are strong programmers and have been able to find the answers to some of the CrowdMath problems before the solutions had rigorous proofs. This experimentation was useful to other CrowdMath participants, since it

	CrowdMath 1	Polymath 1
Year	2016	2009
Platform for collaboration	message board on the Art of Problem Solving website	blog comment section
Target audience	High school students	Professional mathematicians
Educational support	Online classes, exercises, answers to questions	Answers to questions
Project leaders	MIT students and recent graduates	High-profile mathematicians
Number of participants	35	39
Identity of participants	97% anonymous	26% anonymous
Gender of participants	14% girls	3% women
Type of problems	Exploratory	Major open problems

Table 15.1. A comparison between CrowdMath 1 and Polymath 1

provided conjectures for them to attempt to prove instead of having no idea what the correct answer should be.

Another complaint about Polymath was that it lacked empathy and comradery [11]. By comparison, one respondent in a survey of CrowdMath participants noted that there is no criticism in CrowdMath, and another said that they "really appreciate the friendly environment on CrowdMath" [9]. Another respondent in the same survey noted that contributing to CrowdMath "made the idea of doing research less intimidating," and made them consider "a career in research." Other participants found that CrowdMath taught them that it was okay to post things that were "completely wrong," such as "ideas even if they were just ideas." They also learned that it was okay to ask questions and post if they did not understand something [9]. In comparison, Terence Tao noted that one of the cultural inhibitors to participation in projects like Polymath is that professional mathematicians are "reluctant to say anything on the public record which may end up being wrong, foolish, or naive" since this could damage their reputation [13].

CrowdMath has three rules which are designed to work against some of the toxic elements of the original Polymath: (1) be polite and constructive, (2) make your comments as easy to understand as possible, and (3) it's okay for a mathematical thought to be tentative, incomplete, or even incorrect. The first rule helps maintain CrowdMath's friendly environment, while the second rule helps make research less intimidating for participants. In addition to open problems, CrowdMath also has exercises which are more approachable for participants. These exercises allow students to participate, receive feedback, and learn new math even if they are unable to make progress on the open problems, which helps maintain a friendly environment and makes the idea of doing research less intimidating. The third rule, together with the anonymity of CrowdMath, helps students not worry about asking questions or writing things that are possibly wrong.

15.4 What has worked and what has not

The most successful topics so far have been metric dimension [8], pattern avoidance [3, 4], pursuit-evasion [12], broken sticks [5], and zero forcing [7]. So far, two CrowdMath papers were published in the *Electronic Journal of Combinatorics* [2] and in *Discrete Applied Mathematics* [8], some of the other papers are under review, and all of the CrowdMath papers have been posted on arXiv [6]. All of the topics are easy to describe, and the structures and processes in the problems are relatively easy to visualize. The proofs in these areas are usually pretty elementary, so students can read the reference papers without having to learn extra background. Students often post questions on the message board as they read the references, and the mentors and other students help answer their questions.

Most of the topics outside of discrete math have not been as successful. In the first year, we had some open problems about optimal codes and incidence geometry, but there was no progress on any of the problems for either of these topics. The only progress in the first year was on the problems about pattern avoidance in 0-1 matrices, and we ended up writing three different papers on that topic. Similarly in the third year, we had some open problems about neural codes, but this topic was also unsuccessful, and we ended up focusing more on powerful numbers. All of the unsuccessful topics required more background reading than the successful topics to understand the open problems, and the proofs in the reference papers for the unsuccessful topics were much less elementary, so it was more difficult to understand the references and the problems for the unsuccessful topics.

Another thing that has worked well for CrowdMath is having the mentors post new questions on the message board when activity slows down. In every year of CrowdMath, there are periods of time when no students work on the problems. When it seems like things are slowing down, we usually post some new related problems to try to increase CrowdMath activity. Sometimes when activity slows down, the mentors will also prove results on their own and lead the participants to the solutions by posting outlines on the message board with details to fill in and questions to answer.

15.5 Advantages over in-person and one-on-one or small groups

One of the clearest advantages of CrowdMath over a typical in-person research program with one-on-one or small groups is the ability to serve many more students. A mentor in PRIMES will mentor a research project with between one and three students. In CrowdMath, one mentor interacts with dozens of students. Other research programs usually have meetings between students and mentors, but there are no meetings in CrowdMath, so there is no need to worry about finding a meeting time that works for every participant in the project. Since students participate in CrowdMath over the Internet, there are no costs for transportation or the use of facilities, and the cost of mentors is much less per student.

As mentioned before, there is no barrier to entry for participation in CrowdMath, besides knowing math and having Internet access. Almost every other math research program requires students to apply and be accepted into the program, but CrowdMath will take anyone. There are many talented students who do not know anyone who can write them a reference letter, or who are not eligible for other math research programs, for example, because of their international student status. They can do CrowdMath and still get math research experience. Besides, while most math programs operate only in the summer, CrowdMath is active throughout the year.

Another benefit of CrowdMath is the lack of pressure on the students. With a one-on-one or small group project, there is an expectation that the students will get something done, and the project might be viewed as a failure if the students that were chosen to work on the project do not make any progress on it. With CrowdMath, there are no expectations that any specific student will prove anything, because no one besides the mentors is required to work on the projects. Even if an active participant leaves the forum for a while, someone else will usually fill in for them and continue making progress on the problems.

15.6 The story of a CrowdMath research thread

At any given time, many separate discussions of a variety of research problems are being conducted on the CrowdMath message board. To illustrate the dynamics of collective research in CrowdMath, we will describe one of the most productive research threads from summer 2020, which led to the paper by Geneson, Kaustav, and Labelle [8]. This research thread involved two students and one mentor. That number is typical of research threads in the CrowdMath projects, though many other students participate in CrowdMath by asking questions and working on the exercises.

The main topic for CrowdMath 2020 was metric dimension, a graph parameter that can be used in robot navigation. The robot moves from vertex to vertex in a graph, where some vertices are distinguished as landmarks. We can think of the graph as representing a surface, where the vertices in the graph correspond to a subset of the points on the surface, and the edges in the graph are pairs of vertices for which the robot can step between the corresponding points on the surface. The robot is able to measure its distance to the landmarks, and it uses only those distances to determine its location. The goal is to use as few landmarks as possible, and the *metric dimension* is the fewest number of landmarks that can be used. We also studied a variant of metric dimension called *edge metric dimension*, where the robot moves between edges instead of vertices.

For CrowdMath, we posted several open problems from publications about metric dimension. One problem was to find the maximum degree of a graph of metric dimension at most k. Before CrowdMath, this quantity was bounded between $3^k - k - 1$ and $3^k - 1$. Some similar problems were to find the maximum number of edges, the maximum chromatic number, the maximum degeneracy, and the maximum possible minimum degree of a graph of metric dimension at most k. Another problem was to determine if there exist any families of graphs of metric dimension k where the edge metric dimension is $\omega(2^k)$. A more general problem was to find new triples (a, b, c) of integers for which there exists a graph of metric dimension a, edge metric dimension b, and order c.

In early July, one of the students initiated the progress on the CrowdMath problems by solving the problem about maximum degree, improving the lower bound to $3^k - 1$ with an incredibly creative new construction and proof. Up to that point, there had been solutions to some of the exercises on the CrowdMath message board, but no progress on the open problems. Ten days later, the mentor realized that the same construction would solve one of the other problems: whether there exist any families of graphs of metric dimension k where the edge metric dimension is $\omega(2^k)$. The graphs that the student constructed had metric dimension k and edge metric dimension approximately 3^k, so the mentor posted some hints on how to show that the edge metric dimension was approximately 3^k using the pigeonhole principle, and the same student confirmed the details. CrowdMath mentors post hints when they see how to solve one of the open problems, since it makes the problems more approachable for students, while also allowing the students to generate the final solutions.

The mentor noted that the student's construction also gave new triples (a, b, c), and the same student and a second student found other constructions that gave more new triples (a, b, c). Then the mentor posted a new problem, to find the maximum possible size of a wheel subgraph in a graph of metric dimension at most k. CrowdMath mentors usually post new open problems as the project progresses, even if there are plenty of open problems remaining, since solutions to earlier problems often inspire ideas for new problems. The second student used a modification of the first student's construction to show that the answer for the wheel subgraph problem is 3^k.

Then the first student realized that their construction could be extended to a family of infinite graphs with vertices at lattice points. This family of infinite graphs was very useful for proving results about metric dimension, because all graphs of metric dimension k can be embedded in the k^{th} graph of the family. The result about maximum degree was immediate using this family of infinite graphs, along with several results about metric dimension and pattern avoidance. The family of infinite graphs that the first student discovered became the main tool for proving most of the results in the CrowdMath paper.

The first student used the family of infinite graphs to show that the maximum number of edges in a graph of metric dimension k and order n is approximately $\frac{n(3^k-1)}{2}$ and the maximum chromatic number is 2^k. The first student suggested that the family of infinite graphs could also give bounds for the maximum degeneracy, but wrote that they did not see how to get the lower bound. The mentor saw how to get $\Theta(3^k)$ bounds for the maximum degeneracy, so he first put up hints, and then a proof outline. The first student confirmed the details, and then later they proved a much stronger result. Using a simpler proof, they showed that the maximum possible degeneracy is $\frac{n(3^k-1)}{2}$.

After seeing how useful the family of infinite graphs was for solving problems about metric dimension, the second student asked if a similar family of infinite graphs could work for edge metric dimension. There was some discussion on this, and no similar family was found for edge metric dimension, but it is an interesting question that was included in the paper [8]. Then the mentor suggested that the family of infinite graphs could be used to find the maximum possible minimum degree of a graph of metric dimension at most k. The mentor suggested a construction using a spherical subgraph that seemed to give a lower bound of 3^{k-1}, and the second student used the pigeonhole principle to prove an upper bound of 3^{k-1}. However, the mentor's idea with the spherical construction was flawed, as observed by the first student.

The first student noted that the mentor's spherical construction did not work because it produced a graph of metric dimension greater than k, so the mentor suggested trying some polytopes. First he suggested that using a k-dimensional cross polytope achieved the lower bound of 3^{k-1}. A 2-dimensional cross polytope is a square, and that worked for $k = 2$. A 3-dimensional cross polytope is an octahedron. A few hours later, the mentor observed that using an octahedron was giving a graph with minimum degree too low for $k = 3$, so he changed it to a rhombic dodecahedron, and the students and mentor confirmed that this showed the answer was 3^{k-1} for $k = 3$. The first student suggested a way to generalize the construction to $k > 3$. The students and mentor conjectured that the maximum possible minimum degree is 3^{k-1}, and that it is obtained by a family of graphs formed by using k-dimensional generalizations of rhombic dodecahedrons, but this is still open for $k > 3$.

The entire discussion, from start to finish, took about one month. The students and the mentor took turns making conjectures and offering constructions and proofs. This research thread clearly displayed a collaborative, rather than hierarchical, model of research. The interaction in the research thread was much like a typical math research collaboration, except that the collaborators never met before they started working on the problems, and their work was visible online as they made progress on the problems. The resulting solutions of several open problems provided sufficient material for the research paper by Geneson, Kaustav, and Labelle [8].

15.7 Who benefits

CrowdMath benefits students, faculty, schools, and other math research programs. Students benefit from gaining math research experience and interacting with the mentors and other participants on the CrowdMath message board. Participants have noted that they perceive the CrowdMath message board not only as a place where they can learn "how math research is done" but also how to work with other students. Other respondents noted that CrowdMath transformed them from being a learner to a researcher. One student noted that transitioning from school math to CrowdMath is "like wading in the kiddy pool your whole life and then getting pushed into the deep end" [9]. Students that contribute solutions or other useful ideas to CrowdMath projects can also get letters of recommendation from the head mentor when they apply to other math research programs. For example, several CrowdMath participants have been accepted into PRIMES and other math programs using letters of recommendation from the head mentor.

There are also obvious benefits of CrowdMath for faculty. Experience with mentoring research projects is increasingly important for securing jobs in academia, so mentoring CrowdMath is one way to gain mentoring experience for the faculty mentors. Unlike other math research programs, CrowdMath has a very flexible schedule for the mentor with no meeting times, so faculty mentors can fit CrowdMath around their schedule, even if they would not have time to mentor a project that required regularly scheduled meetings. CrowdMath can also help faculty reflect on how to be a better mentor. For most research programs, it would be impossible for the mentor to remember every single interaction that they had with the student participants. In CrowdMath, every interaction happens on the message board, so it will be on the CrowdMath website as long as the website exists. This allows the mentors to review their posts and see which ones led to more student participation and new results, and the mentor can use this to improve their research supervision skills.

Another less obvious benefit of CrowdMath for mentors and students is the abundance of student participants. One might expect that it would be unmanageable for a mentor to work with many students, and that CrowdMath would become more difficult for the mentor as the number of students increased. However, the opposite has been true. CrowdMath becomes easier for the mentor with more student participants, since the students answer each other's questions and prove many new results. The first year of CrowdMath had the fewest student participants of all the years, and that year was the most work by far for the mentors. Later years had more participants, and those years were much easier for the mentors since the students proved most of the new results and often answered each other's questions. In a survey of CrowdMath participants, students said that other students' posts were a greater source of inspiration than mentors' comments [9].

In the same survey of CrowdMath participants, 79% said that they either definitely or probably want to become a research mathematician [9]. For schools, CrowdMath presents an opportunity to serve students who are interested in learning how to do math research, even if there is no teacher at the school who is willing to mentor a project that interests the student. A high school or college could have a CrowdMath course where students meet and work on the CrowdMath problems together and post on the message board when they have questions, ideas, or proofs. A teacher could oversee them and assign grades at the end of the course, and it would be straightforward for the teacher, since the students would be teaching themselves and interacting with the CrowdMath mentors and other participants. Similarly, schools could have an extracurricular club that works on CrowdMath problems, like a math team or computer team.

Another benefit of CrowdMath is helping other math research programs identify strong students. CrowdMath has benefited PRIMES by providing multiple students that performed well on their PRIMES projects. Some of these students would not have participated in PRIMES if they did not perform so well in CrowdMath. Other students that perform well in CrowdMath can refer to their CrowdMath participation whenever they apply for a math research program. In addition to a letter of recommendation from the CrowdMath mentor, participants can provide links to the

CrowdMath website to show exactly where and how they collaborated on the CrowdMath project. Another possible benefit of CrowdMath for other math research programs is as a template. Whenever other groups try to run online math research projects for college students and high school students, they can look at the CrowdMath website to see what has worked and what has not worked for CrowdMath.

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16

The International Conference of Undergraduate Research (ICUR)

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In the past decade, developments in online conference technologies have opened exciting new possibilities for undergraduates, especially those from lower-income backgrounds, to present their own original research on an international stage. A forerunner in creating this type of opportunity has been the International Conference of Undergraduate Research (ICUR), organized through an alliance of Australia's Monash University and the University of Warwick in the United Kingdom (the Monash Warwick Alliance). Since 2013, this conference has linked over 3000 undergraduate researchers in all fields, including mathematics, to present on panels and engage in question-and-answer sessions with their peers at institutions on five continents. This interdisciplinary conference has been a rewarding experience for students and has helped them develop skills in communicating their research to those outside their field, which has been especially challenging but valuable to students in mathematics.

The International Conference of Undergraduate Research offers a visionary model for how undergraduate research could take place more frequently in a global context in the future. Although this particular conference is open only to students in participating universities, its key innovations and goals might inspire future collaborations across continents for the benefit of students at other institutions as well. ICUR pioneered the use of videoconferencing technology to link undergraduate researchers in a conference that prioritized access and equity for diverse students. The ability to interact with peers globally without the need for expensive travel opened up opportunities for skill and confidence-building and professional development for students who otherwise could not afford such events. As universities explore ways to offer a global education while simultaneously pursuing the increasingly important goal of sustainability, ICUR's use of videoconferencing and social media networking can also be a model for how to hold a conference without the large carbon footprint created by air travel. As the world has turned to the use of Zoom, Teams, and similar platforms during the Covid-19 pandemic, online conferencing has become even easier, and ICUR provides an exciting example for how to use such technology to enrich undergraduate education.

16.1 About the conference

A group of students and staff at Monash and Warwick Universities originally conceived of the idea to link undergraduate researchers across disciplines at universities around the world. The organizers invited other colleagues to participate, and the student-centered event has grown to involve many institutions globally. Baruch College of the City University of New York (CUNY) and Nanyang Technological University (NTU) in Singapore have been yearly participants since 2014. Along with Warwick, Monash's Melbourne and Malaysia campuses, Baruch and NTU, the participating institutions (at the time of writing) have included the University of Leeds in England; Vrije Universiteit Brussel in Belgium; Pompeu Fabra University (UPF) in Barcelona, Spain; CY Cergy Paris University in France; University of Ljubljana in Slovenia; the University of Gothenburg, Sweden; the University of Nebraska–Lincoln and Pennsylvania State University in the United States; the University of British Columbia in Canada; Stellenbosch University in South Africa; and Zayed University in the United Arab Emirates. Previous participants have also included Kyushu University in Japan; Singapore Management University; the University of North Carolina at Greensboro, University of Tennessee, Knoxville, and the University of Washington in the USA; the University of Sussex, UK; the University of Brawijaya, Indonesia; and the University of Western Australia. The range of institutions purposefully encompasses multiple continents.

Based in the Institute for Advanced Teaching and Learning (IATL) at the University of Warwick, the ICUR experience helps students build their skills beginning with their application to the conference. The goal of the conference is to provide students with a great deal of support in developing these skills. Each year in March, April and May, undergraduates are invited to submit a 250-word abstract via the ICUR portal website (icurportal.com). The ICUR website offers detailed guidance on how to write an abstract, and workshops on abstract writing are held at students' home campuses as well as online. Applicants receive detailed feedback on their abstracts, both from local organizers and from the central staff at Warwick. The aim is for students, equipped with this feedback, to revise their abstracts in order to produce a professional abstract that concisely articulates a research question, methodology, findings, context, and relevance and contribution to the field. Since the conference is interdisciplinary, students are urged to explain discipline-specific jargon so that the abstracts can be clear for a non-specialist audience. This skill is especially important for the hard sciences, including mathematics, in which experts use particularly specialized vocabulary. In addition, the students need to view their abstract with an international lens, identifying which information comprises specifically local knowledge that needs to be explained to students in other countries. Writing an abstract for ICUR pushes the students to think about how to make their work legible to a variety of audiences, both globally and disciplinarily.

Once the students' abstracts are accepted, the Warwick-Monash organizers design a schedule for the conference to take place at the end of September. They place students from at least two different campuses onto thematic panels comprising three or four ten-minute oral presentations. The panels are staggered across a 48-hour period, often linking campuses in very different time zones. It's always a bit of a thrill for students presenting in the evening at Baruch College in New York to talk to students just waking up in the morning at Monash in Melbourne. Students also have the option of submitting a poster about their research, as well as a short recorded summary of their project, which is then accessible to conference attendees via the ICUR conference app.

There are a number of logistical challenges inherent in the organization of an event that simultaneously spans a large number of institutions and time zones. Since 2020, a broad outline of the usual conference schedule has been established, but the specifics must still be recreated each year based on the number and type of submissions received from each institution. All panels are designed to bring presentations from different disciplines together under unifying themes, such as "Environment," "Gender and Social Inequality," or "Teaching and Learning." When submitting their abstracts, applicants are encouraged to consider and select from suggested themes that align with their topic so that organizers can place presentations together on thematic panels to foster meaningful dialogue from different disciplinary angles. The focus on inclusivity and developmental experiences for our student presenters creates additional resourcing challenges as organizers strive to maintain a balance between being rigorous and supportive. For example, scaffolding training throughout the abstract submission period or ensuring that each student benefits from the experience of a full review process, while still giving significant opportunities for further feedback and revisions, all take time and result in greater pressure on resources.

Offering this feedback to students and keeping them on task in preparing for the event can be a time-consuming challenge for the conference organizers. A panel, comprised of academics, staff and students from the University of Warwick and Monash University, reviews all the submitted abstracts and sends specific comments to students for improvement. The focus is on the demonstrated ability to communicate research to an international and multidisciplinary audience. Some students can find this particularly challenging, and ICUR organizers purposefully continue to develop support mechanisms in this area. Local organizers also sometimes help students work through their revisions both to the abstract and to the final presentation.

After students receive confirmation of their acceptance, skills building continues with training in giving oral presentations. The website again offers ample advice for giving a successful presentation, and local organizers provide practice sessions for their students. Since 2020, students have been given the chance to practice with others on their panel via Zoom, under the mentorship of a student session chair, in the run-up to the conference.

Another issue for local organizers is ensuring that students who signed up to present follow through on their commitment. The conference helps students with their professionalization, because it demands that they juggle their participation on their panel with their class schedule and other regular duties, such as employment. Local coordinators, in some cases, need to send reminders to students and help them understand the importance of preparation, time management, and prompt communication around deadlines. Helping students either with anxiety about giving a formal presentation or just with staying on top of their assignments comes with the territory. However, organizers should recognize that helping students through this process goes a long way toward instilling confidence in participants that they can handle a wide array of challenges to boost their professional success.

The conference aims to be student-centered, with undergraduates serving as directors as well as presenters, volunteers, and audience members. The directors collaborate with ICUR staff to promote the event and organize logistics on their local campuses. Students also serve as moderators for the panels, introducing the presentation topics and chairing the Q&A segment. They thus learn skills in project management and leadership.

From 2013 to 2019, ICUR used videoconferencing technology to link connected physical spaces on each campus in live spoken presentation sessions. Panelists and local audiences met in these spaces and could see partnered rooms with their respective audiences and panelists on other campuses in real time. Each panelist presented their research, often with accompanying PowerPoint slides, and at the end of the session, panelists answered questions from audience members in the room with them as well as on each connected campus.

During the Covid-19 pandemic in 2020, ICUR organizers pivoted to using available videoconferencing technology, such as Zoom or Microsoft Teams, that made it possible for panelists to present their papers from their own homes where necessary. The ICUR staff developed an app for the conference where panelists and audience members could register for the conference, view the schedule, and communicate with each other. The app also became a virtual conference hub, where audience members from around the world could also connect and launch each presentation session via webinar link, as well as ask questions via the chat function, eliminating the necessity for a physical conference room element. Social media also encouraged student participants to develop connections, which could last beyond the conference.

The ICUR conference was well-positioned to use the push from the demands of the pandemic to innovate in the use of online technology to emerge with a hybrid format that allows participating institutions to combine fully online, in-person, and hybrid formats for their panelists. When the conference went fully online through the app, some students asked for more in-person events to help build community among the participants, but the ability to present work solely through the app has allowed for greater flexibility for organizers and student presenters. Whereas the linking of panels through video-conferencing technology had previously required the use of classrooms or conference rooms equipped with special technology manned by IT technicians, in the post-COVID era anyone can sign on to connect with the conference from a campus location or from home, including family and friends of panelists in far-flung locations.

The ICUR experience has also extended beyond the conference itself, because the Monash-Warwick Alliance also supports the publication of *Reinvention,* an open-access, international, multi-disciplinary peer-reviewed journal of undergraduate research, edited by students at Monash and Warwick. ICUR presenters are encouraged to submit their work to this journal as another step in the process of professionalization for the research world.

16.2 Challenges and rewards of presenting mathematics research

The International Conference of Undergraduate Research has showcased the work of undergraduates in many academic disciplines. Students in both pure and applied mathematics and related fields, such as statistics, have discussed their work on panels with students in very different fields. From 2013 to 2021, around 22 of the presenters have been strictly mathematics students, while others have pursued combined degrees in mathematics and other fields, such as physics (13), economics (2), psychology (1) and statistics (4). The 2022 conference included the largest number of students from mathematics departments up to that point, with 9 students presenting their work in this area and a further 12 students presenting research in fields combined with mathematics. Beyond these numbers, others have used mathematical modeling or methods in their work in other fields. Participating in this interdisciplinary conference has presented mathematics students with the unique challenge of making their work understandable to those unfamiliar with mathematical methods, formulae, and jargon. This task is especially challenging for students pursuing pure math research, which is harder to translate into language that is understandable to non-specialists. However, whether in pure or applied math, interacting with students at ICUR who integrate mathematics into a variety of disciplines, as well as with students from completely outside mathematics or natural sciences, gives the mathematics presenters the opportunity to hone crucial communication skills and to think critically about their work in new ways.

While mathematics undergraduates have a number of different conferences in their field they could attend to talk to others in their discipline, participation in ICUR lends these students a different kind of opportunity to interact with and learn from students in many disciplines, which would help professionalize them for working in a university, for example. The ICUR organizers purposely bring students in different fields together to encourage creative synergies between participants. Mathematics presenters have been put onto panels that push them to think beyond the bounds of their discipline, seeing connections to related fields such as physics or engineering, but also stepping outside their comfort zone to engage in dialogue with more distant fields, such as psychology, business, or medicine. For example, in 2015 Monash mathematics student Musashi Koyama presented "Computing with Topology" on a panel with students giving papers in the fields of computer science and industrial design, and on regulation of antibiotic prescriptions. In 2016, Warwick student Nataliya Dyadyusha presented a paper in the graph theory branch of mathematics, entitled "Stochastic Tournaments and their Application in Ranking." She was on a panel with papers in related fields of theoretical physics ("Extending the Light Bound Skyrme Model," by Leeds student Ben Maybee) and kinetic modeling ("How should we account for errors in kinetic models of surface defects and catalysis?" by Warwick student Harry Tunstall) as well as with a less mathematically-oriented paper bringing together medicine and technology by Leeds student Olivia Coe: "How effective are virtual reality (VR) technologies as a psychological therapy for chronic pain?" In 2017, two NTU students gave mathematics presentations—Zhenyuan Lu with "The Incomplete Inhomogeneous Airy Function" and Quang Trung Ha with "Theoretical Study of a Non-Hermitian Quantum Mechanical Lattice"—on a panel with Sussex student Maddie Atkinson, who gave a paper in psychology "Exploratory Research into the Effects of Covert Facial Familiarity on Face Recognition." Such groupings can be challenging but have yielded interesting conversations. A Warwick mathematics student articulated the challenges and rewards of participating on a panel with students in different fields: "For me, my research was in fact rather unrelated to other presentations in my session. . . .However, I think it was very nice to see presentations from various disciplines and it was definitely an eye opening experience for me!"

In addition to presentations on pure or theoretical mathematics, a number of participants over the years have also been working in applied mathematics or statistics, or have used mathematical modeling as a methodology to research other subjects. Examples have included Philip Luong's 2016 paper from Monash, "Modelling the Syphilis Epidemic in Victoria among Men who have Sex with Men," and NTU student Bobby Ranjan's "Composite Mathematical Modelling of Calcium Signalling behind Neuron Cell Death in Alzheimer's disease." Such papers have been linked on panels with other forms of scientific or social scientific research. Being placed on such interdisciplinary panels forces the presenters to try to explain mathematical concepts in a way that non-specialists can understand. This sort of communication skill can be an important tool in the professionalization of these young scientists and mathematicians. In the future they may need to make their research comprehensible and relevant when applying for grants and explaining their work to varieties of audiences, such as tenure committees or even the general public. Honing these communication skills can help in teaching as well. Since mathematics often uses terminology and methods that are foreign to audience members in the humanities or social sciences, it can be challenging to explain the complexities of mathematical research to a more generalized group. Unlike research in history or sociology, which non-specialists can grasp more easily because the subject matter relates more directly to everyday life, advanced research in mathematics and natural sciences can be difficult for non-specialist audiences to comprehend. As one Warwick mathematics student noted, "Personally, I found that my abstract was easy to be understood by an interdisciplinary audience, but my presentation however, was rather difficult to be understood by audiences without experience in my discipline, though I have tried to only cover the most basic and fundamental parts of my research. I think this is something I am still working on, although ICUR has definitely given me much experience on it!" Audiences may be hard-pressed to find connections to the material or come up with relevant questions. The aforementioned mathematics presenter recounted that “most of the [audience questions] were general ones such as how I got interested in the topic and how the study was affected due to the pandemic.”

However, presenting this kind of research to diverse audiences has its advantages. The interdisciplinary audiences at ICUR can bring new types of questions to the presenters, offering radically different perspectives to their work, which they might not have considered if they hadn’t spoken about their work to those outside their immediate field. As one participant put it, it was “interesting to compare our points of views and different research methods.” In some ways, this interchange can open up new insights into the meaning and relevance of their work and reinforce for undergraduates their level of mastery and competence over their subject.

Students have mostly found this experience valuable; one Baruch participant surveyed said the most useful thing learned at ICUR was “how to make my research understandable and approachable for non-science majors,” and another responded, “being able to communicate technical topics to laypeople.” A Warwick mathematics/physics student similarly reflected, “I really enjoyed the process of writing up my presentation for ICUR because it forced me to put aside technical jargon and focus on the conceptual ideas. I made sure to include examples and relatable scenarios to show the impact of my research and why it’s interesting. As I’ve spent essentially 3 months learning how to be more specific with my communication and ideas, it was of course hard initially to be able to synthesize those months of work into 10 minutes. However, it was a great exercise into seeing the bigger picture and now whenever I describe my research, I use what I’ve done for ICUR.”

Further, attending the interdisciplinary conference can expose mathematics students to broader questions that could help them contextualize the work they are doing in important historical, social, or ethical contexts. For example, some of the papers have examined the gender and racial dynamics of mathematics and scientific professions. At the 2014 conference, Baruch psychology student Renee Cotsis presented “White and Asian Difference in Pursuing Math” and Monash student Natasha Abrahams presented “Women’s Gendered Experience in Undergraduate Physics in Australia.” Other papers have considered the relationship between mathematics and pedagogy or other professions. In 2016, a paper by Warwick student Aminah Khan examined “A Qualitative Case Study of Maths Anxiety in Early Years’ Staff” and Lee Nguyen from Monash presented “Evaluating and Improving the Quantitative Skills of a 2nd Year Monash University Biochemistry Cohort.” Such questions would perhaps be important for future mathematicians to consider as they go forward toward professional careers in academia and in teaching, where they must grapple with questions of the diversity of university departments or deal with student anxieties especially when teaching math to non-majors.

One Warwick mathematics/physics student articulated some key lessons gleaned from the ICUR experience, demonstrating the impact that the conference can have on helping students develop into creative thinkers by considering their own work in relation to other fields represented at the conference: “Be curious and an autonomous learner. In the field of mathematics applied to computer science, there exists a lot of concepts and research that has been done already.... Many times, links can be made from different fields even if they don’t explicitly tackle your issues.”

16.3 A transformational student experience

The majority of students have found their experience participating in ICUR to be a rewarding part of their college experience and a useful step toward graduate study or professional careers. Some students told their campus coordinators how the conference increased their self-confidence and resolve to pursue research further. For example, Baruch presenter in 2018 Veronica Stocker said, “I am beginning to genuinely enjoy presenting and get a lot of fulfillment from these types of events. There was a question and answer period at the end that was quite engaging and stimulated a lot of interesting discussion. It was also fascinating to hear the work being done by other students around the world and it was overall a wonderful experience.... [the] presentation with ICUR has given me valuable insight into the world of research and has inspired me to imagine myself continuing in research as an ultimate career goal.”

A particular advantage of ICUR is the fact that students can engage in a global conference from their home campus at little or no cost. This allows low-income students to have an important chance at professionalization and a broadening of their horizons that they would not have otherwise. This is particularly important at institutions, such as Baruch College, where many of the students are first-generation college students, recent immigrants, and students of color.

ICUR therefore opens up doors for this set of students by providing them with the confidence that they have abilities equal to peers around the world. One student surveyed said the most rewarding thing about ICUR was "being a part of something bigger."

The chance to give a paper at a conference that is professional but also clearly a learning experience is invaluable for boosting students' confidence and helping them to own their status as experts in their original research. Every year some of the students are fearful that they are not good enough, and some want to back out, but after the campus coordinator bolsters their resolve, reminds them that all the participants are students just like them, and gives them a chance to practice, most go on to give their presentations and experience gratification that they were as good as or better than all the other panelists. One Baruch student responded in a survey, "I definitely wouldn't have done something like this if not encouraged. I think that it was a great practice to get over my fear." A Warwick mathematics student reflected on the conference, saying, "I have definitely gained much experience in the process of preparing, practicing and finally delivering the presentation. I have learnt how to narrow my research down and choose the most important parts of my research to be presented, but still make sense of the process of the study. I have also found that it was helpful in boosting my confidence for the presentation through enough practice." To be able to show competence and mastery in dialogue with students from other countries and other institutions can be a transformational experience. Students are sometimes amazed to discover that others around the globe are interested in and impressed with their work, and this can encourage them to pursue this work further in graduate school.

16.4 ICUR and the future of undergraduate research

The past few years have witnessed a troubling trend in which sectors of society have come to question the value and validity of science and even of factual evidence. Given this mistrust of science among some, it is more imperative than ever that students come to value research and that scientists improve communication about their work to non-specialist audiences to restore public trust in data-driven inquiry. An interdisciplinary conference such as ICUR forms a critical part of this necessary endeavor, since it conveys to undergraduate students the importance of research, teaches them the value of methodology, and helps them communicate about their findings to a wide range of listeners, especially those outside their field. Engaging undergraduates in meaningful research will hopefully excite more talented scholars to pursue doctoral studies and to view their future careers within a global context. Providing undergraduates with experiences such as ICUR might play a role in forging a new generation of researchers who are more collaborative and open to learning across disciplines and regions of the globe, who represent more diverse backgrounds, and who can restore a faith in the pursuit of knowledge through scientific inquiry across society.

About the Authors

Emma Barker began her career in book publishing and previously held positions at the Association for Jewish Studies, New York University Press, Cassell & Co, and Continuum Publishing (now Bloomsbury Academic), before joining the Institute for Advanced Teaching and Learning (IATL) at the University of Warwick in 2013. Since then, she has managed a variety of projects focusing on undergraduate research including the International Conference of Undergraduate Research (ICUR) and *Reinvention: an International Journal of Undergraduate Research.*

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Mary Crowe has close to 30 years of experience in higher education and is currently the director of the STEM Accelerator at George Mason University. Dr. Crowe herself carried out undergraduate research projects in animal behavior so understands how transformative an undergraduate research experience can be for a first-generation college student. She has advised and mentored students in her research on the behavior of burrowing animals such as dung beetles and fiddler crabs. Throughout her career she has been an advocate for, and supporter of, faculty as they mentor students in high impact practices such as undergraduate research, study abroad and learning communities. She was a program officer at the National Science Foundation (Division of Undergraduate Education) and had roles at Florida Southern College, Coastal Carolina University (PUIs), Xavier University of Louisiana (HBCU) and the University of North Carolina at Greensboro (R2).

Elizabeth A. Donovan is an associate professor of mathematics at Murray State University. She received her undergraduate education at Worcester Polytechnic Institute, inspiring both her love for graph theory and first exposing her to undergraduate research, before receiving her PhD from Northeastern University. A graph theorist by training, Beth enjoys sharing this passion with her students through a variety of graduate and undergraduate research projects and was a recipient of a Center for Undergraduate Research in Mathematics mini-grant in 2018–2019. She is a strong supporter of active learning techniques in the classroom and works diligently to make the broader mathematical community a more welcoming place for all. In her free time Beth enjoys gardening, woodworking, baking, and playing with her two cats.

Michael Dorff is a professor and former chair in the Department of Mathematics at Brigham Young University, and he earned his PhD in complex analysis from the University of Kentucky. His interests include popularizing mathematics to the general public, promoting math careers in BIG (business, industry, and government), and in advancing undergraduate research. In 2019–2021, he was president of the Mathematical Association of America (MAA). He co-founded and co-directs the MAA PIC Math program that prepares students for careers in BIG, was the founder and past director of CURM (Center for Undergraduate Research in Mathematics), and is currently the executive director for TPSE Math whose mission is to help all students learn and succeed in mathematics. He co-authored the book *A Mathematician's Practical Guide to Mentoring Undergraduate Research.* He is a fellow of the American Mathematical Society and a Fulbright Scholar in Poland. He received the Council on Undergraduate Research national CUR's Fellows Award for his work in mentoring and promoting undergraduate research, the MAA national Haimo Award for Distinguished Teaching of Mathematics, and the BYU Maeser Excellence in Teaching Award. Also, he is a member of the steering committee for the East African Centre for Mathematical Research in Uganda.

Lloyd E. Douglas is an independent consultant. A certified research administrator, he is the former associate director, Office of Contracts and Grants at the University of North Carolina at Greensboro. Before going to UNCG, he was assistant to the vice president for research at the University of Nevada, Reno. He is a former first vice-president of the MAA and has served as its governor-at-large for mathematicians in business, industry and government as well as a member of its Distinguished Lecture Series committee. He is also a former chair of its Committee on Minority Participation in Mathematics and its former interim director of programs. From 1984 to 2008, he worked at the NSF and while there oversaw a large increase in the REU program in the Division of Mathematical Sciences. He also managed the Mathematical Sciences Postdoctoral Fellowships, served on the coordinating committee for NSF's CAREER program and co-chaired the implementation committee for the NSF's ADVANCE program. He received NSF's Meritorious Service Award in 2007. He is a co-principal investigator on MAA's National Research Experience for Undergraduates Program, whose goal is to support the participation of mathematics undergraduates from underrepresented groups.

Pavel Etingof is professor of mathematics at the Massachusetts Institute of Technology (since 2005) and the chief research advisor of the MIT PRIMES Program (Program for Research in Mathematics, Engineering and Science for High School Students) since 2010. Etingof received the MS in applied mathematics from the Moscow Oil & Gas Institute in 1989, and the PhD in mathematics from Yale University in 1994. Igor Frenkel was his thesis advisor. He went to Harvard as a Benjamin Peirce Assistant Professor in 1994, and joined the MIT mathematics faculty as assistant professor in 1998 (professor by 2005). Etingof's research interests are primarily in studies which intersect representation theory and mathematical physics, such as quantum groups. He serves as chief editor of the *Journal of the AMS* and of *Selecta Math.* He has co-authored 8 texts. In 1999 Etingof received a Clay Mathematics Institute Prize fellowship. In 2012 he was selected to be the Robert E. Collins Distinguished Scholar in the Mathematics Department. He was named a fellow of the AMS in the 2013 Inaugural Class. He was selected by the Institute for the Frank E. Perkins Award for excellence in graduate advising in 2015, and again in 2018. In 2016 he was elected fellow of the American Academy of Arts and Sciences. In 2010, together with Slava Gerovitch, Etingof launched the Math Department's MIT PRIMES program, a free outreach program for high school students with a particular focus on increasing the representation of women and under-served minorities in mathematics research.

J.W. Gaberdiel taught math and science to public high school students for nearly a decade, and now teaches math to community college students. He loves to emphasize the importance of "showing work" as the art of clear mathematical communication to an audience. He loves helping Honors students research historical mathematicians, which is one of his own personal passions. J.W. loves to read source materials in ancient mathematics to better understand the differences between how humans solved problems then and now, and the implications for how we might solve problems in the future!

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Slava Gerovitch holds two PhDs, from the Russian Academy of Sciences (1992) and from the Massachusetts Institute of Technology (1999). He has been teaching history of science, including cultural history of mathematics, at MIT since 1999. His research interests include history of mathematics, cybernetics, astronautics, and computing. He is the author of *From Newspeak to Cyberspeak: A History of Soviet Cybernetics* (2002), *Voices of the Soviet Space Program: Cosmonauts, Soldiers, and Engineers Who Took the USSR into Space* (2014) and *Soviet Space Mythologies: Public Images, Private Memories, and the Making of a Cultural Identity* (2015), and of numerous articles on the history of Soviet mathematics. In 2010 he co-founded (with Pavel Etingof) MIT PRIMES (Program for Research in Mathematics, Engineering and Science for High School Students), a free outreach program for high school students with a particular focus on increasing the representation of women and under-served minorities in mathematics research. PRIMES was later expanded to include the Computer Science section (2012), PRIMES-USA (2013), PRIMES Circle in Math (2013), MathROOTS (2015), PRIMES STEP (2015), CrowdMath (2016), PRIMES Circle in Computer Science (2022), and Yulia's Dream for students from Ukraine (2022). Gerovitch is a recipient of the 2020 MIT School of Science Infinite Mile Award and the honorable mention for the 2003 Vucinich Book Prize for an outstanding monograph in Russian studies, the winner of the 2021 Gardner-Lasser Aerospace History Literature Award, and a finalist for the 2016 Historia Nova Prize for the best book on Russian intellectual and cultural history.

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Bonnie Jacob is associate professor of mathematics at the National Technical Institute for the Deaf, a college of Rochester Institute of Technology. Her research interests lie at the intersection of graph theory and combinatorial matrix theory. She also enjoys mentoring undergraduate students in research and writing about more general topics in mathematics. She and her mathematician-husband have two young daughters and a dog. When she's not doing math and can find snow, she enjoys cross-country skiing.

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Michele Joyner received her PhD in mathematics with a concentration in Computational Mathematics from North Carolina State University in 2001 and is currently a professor of applied mathematics at East Tennessee State University. Dr. Joyner started the PIC Math course at ETSU in 2017 and has recently extended this course for graduate students into a yearlong internship course in conjunction with ETSU's new masters in applied data science program. Dr. Joyner especially enjoys mentoring students and helping them explore all the possibilities of what a degree in mathematics and statistics has to offer.

Emek Köse is a computational scientist at Frederick National Laboratory for Cancer Research, prior to which she was an associate professor at St. Mary's College of Maryland. She earned her PhD in mathematics from Drexel University. She has supervised numerous undergraduate research projects, was co-PI on NSF REU grant and co-recipient of a CURM grant. Her research is in mathematical biology with a focus on HIV, currently. In her time at SMCM she received SMCM's Excellence in Teaching Award for Junior Faculty.

Eric Kostelich received his PhD in applied mathematics from the University of Maryland in 1985. He held a post-doctoral position in physics at the University of Texas from 1985–89 before moving to Arizona State University as a faculty member in the School of Mathematical and Statistical Sciences. Dr. Kostelich was the principal investigator on NSF awards for Computational Science Training for Undergraduates in the Mathematical Sciences from 2008–12 and for Mentoring Through Critical Transition Points from 2012–17. He has served as a co-PI with Wenbo Tang on an REU program at ASU since 2018.

Brian G. Kronenthal is a professor of mathematics at Kutztown University of Pennsylvania. He earned his BS in mathematics from Lafayette College (Easton, Pennsylvania), as well as his MS and PhD in mathematics from the University of Delaware. His favorite research problems are combinatorial, often with an algebraic flair. In addition to mentoring research students at Kutztown University, Kronenthal has mentored students at multiple national research programs: he served as a co-principal investigator for a National Research Experience for Undergraduates Program (NREUP), and mentored students as part of Research Experiences for Undergraduates (REU) programs at Lafayette College and Moravian University.

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Kathryn Leonard is associate dean for curricular affairs, and professor and founding chair of Computer Science at Occidental College. Before that, she was a member of the Mathematics Department and director of the Center for Interdisciplinary Studies at CSU Channel Islands, a Hispanic-Serving Institution and the newest member of the Cal State system. She completed a postdoctoral appointment in the Applied and Computational Mathematics department at Caltech after finishing her PhD in mathematics at Brown University. She is the director of the NSF-funded Center for Undergraduate Research in Mathematics, which received the 2015 Programs that Make a Difference award from the AMS. She was president of AWM 2021–2023 and is a founding co-editor-in-chief of AWM's research journal, *La Matematica*. She received a 2013 Henry L. Alder Award for Excellence in Teaching from the MAA, and a 2015 Service Award from AWM. She is a member of the EDI Committee for BIRS and is a board member for STEAM:CODERS, a non-profit bringing STEM education to underserved schools in Los Angeles. Her research, which explores geometric modeling for computer vision, computer graphics and data science applications, has been recognized with an NSF CAREER award and other research grants.

John Mateja, a nuclear physicist, earned BS and PhD degrees at the University of Notre Dame in 1972 and 1976. He joined the physics faculty at Tennessee Technological University in 1978 where he developed one of the first federally funded research programs to involve physics undergraduates in research. In 1988, he took a position at Argonne National Laboratory where he oversaw the lab's college outreach programs. In 1994 he joined the staff at Department of Energy to help establish its new Experimental Program to Stimulate Competitive Research (EPSCoR) program. John became dean of the College of Science at Murray State University in 1998. During his tenure as dean, the College successfully competed for a Howard Hughes Medical Institute award, NSF Collaborative Research at Undergraduate Institutions award, NSF Course, Curriculum and Laboratory Improvement grants, and NSF EPSCoR award. He was the founding director of MSU's Undergraduate Research and Scholarly Activity Office and MSU's McNair Scholars Program. From 2008–2010, John served as a program officer in the Division of Undergraduate Education at NSF. In 2016, John was named the third president of the Goldwater Scholarship Foundation. He oversees the Foundation's scholarship program that awards 450–500 scholarships annually to undergraduates who intend to pursue research careers in the sciences, engineering, and mathematics. John has been the president of the Council on Undergraduate Research and the Chair of the American Physical Society's Committee on Education. He is a fellow of the Council on Undergraduate Research and of the American Physical Society.

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Mohammad Obiedat is a deaf mathematician and computer scientist. He has been teaching mathematics, computer science, and information technology courses at Gallaudet University for more than twenty years. Mohammad became interested in undergraduate research in mathematics with DHH students eight years ago. He worked with several students on their honors research projects in mathematics and information technology, and with problems taken from combinatorics, cryptography, and data structures.

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Victor Piercey received an interdisciplinary BA in humanities from Michigan State University in 1997, a juris doctor from Columbia Law School in 2000, a MS in mathematics from Michigan State University in 2006, and a PhD in mathematics from the University of Arizona in 2012. His dissertation was in algebraic geometry. Since completing his PhD, he has been teaching at Ferris State University in Big Rapids, Michigan. He was granted tenure in 2017 and is a professor in the mathematics department. In 2018, he was appointed director of general education. As a member of the mathematics department, his primary work has involved the interdisciplinary development of the actuarial science program as well as a 2-semester sequence of general education courses entitled Quantitative Reasoning for Professionals. Developing the quantitative reasoning sequence involved collaboration with faculty in business, social work, and health professions, funded by the National Science Foundation. In his spare time, he serves as the treasurer for his local homeless shelter and he reads books on history.

Jan Rychtář is a professor of mathematics at Virginia Commonwealth University. He earned his PhD in mathematics from the University of Alberta, Canada. He has mentored over 150 undergraduate students. He has led REUs, MAA NREUPs and MAA PIC Math projects. He received the 2022 John M. Smith Award for Distinguished College or University Teaching of Mathematics from the MAA MD-DC-VA Section and the 2022 Faculty Mentor Award from the Math and CS division Council of Undergraduate Research.

Padmanabhan Seshaiyer is a professor of mathematical sciences at George Mason University. He works in the broad area of computational mathematics, data science, biomechanics, design and systems thinking, and STEM education. Over the last two decades, he initiated and directed a variety of educational programs including graduate and undergraduate research, K–12 outreach, teacher professional development, and enrichment programs to foster the interest of students and teachers in STEM at all levels.

Wenbo Tang received his PhD from the University of California, San Diego in 2005. Dr. Tang joined the faculty in the School of Mathematical and Statistical Sciences at Arizona State University in 2008 and has been involved in REU style undergraduate research mentoring since then. He was an MAA ProjectNEXT Fellow in 2009–2010 and currently serves as the PI for the $(AM)^2$ REU.

Dewey Taylor earned her PhD in mathematics from NC State University and she is a professor of mathematics at Virginia Commonwealth University. She served as a nationally elected member of the Council of Undergraduate Research, received the 2017 VCU's Outstanding Faculty Mentor Award, and the 2022 VCU's Distinguished Teaching award. She has mentored close to one hundred undergraduate students and served as a PI on REU, NREUP and CURM grants.

Thomas P. Wakefield received his PhD in mathematics from Kent State University in 2008 and is now professor and chair in the Department of Mathematics and Statistics at Youngstown State University. Wakefield finds inspiration in working with students and enjoys accompanying students to regional and national conferences and advising undergraduate research projects.

Jonathan Weisbrod is an assistant professor of mathematics at Rowan College at Burlington County (RCBC), a public community college in New Jersey. He has a master's degree from Rowan University in mathematics and an EdD from New Jersey City University in community college leadership, completing research on course-based undergraduate research (CUREs) at community colleges. He has a 10+ year background mentoring students in undergraduate research. Dr. Weisbrod started at RCBC in 2011. Teaching mathematics at all levels from prealgebra to differential equations is his passion at work. Exploring mathematics with non-STEM majors so they find the math within their interests is particularly gratifying. He lives near campus with his wife and three small children and enjoys gardening, soccer, and home tech toys.

Gregory Wiggins served as education and outreach coordinator for the National Institute for Mathematical and Biological Synthesis from 2017 to 2021. He served as co-mentor on three Summer Research Experiences or undergraduate projects, as well as assisting with organizing and administering the SRE program and various other workshops and training programs. Prior to his time at NIMBioS, he was a research assistant professor in the Department of Entomology and Plant Pathology at the University of Tennessee specializing in biological control of invasive species. He is now the biological control administrator for the North Carolina Department of Agriculture and Consumer Services.

Lesley Wiglesworth is a H.W. Stodghill, Jr. and Adele H. Stodghill associate professor of mathematics at Centre College. She completed her undergraduate degree at Transylvania University in Lexington, Kentucky and her PhD from the University of Louisville. Lesley's current research is focused primarily in discrete mathematics, and more specifically, combinatorics and graph theory. She is very involved in undergraduate research, receiving a Center for Undergraduate Research in Mathematics mini-grant in 2018–2019, and has mentored close to 30 students during her time at Centre. Additionally, Lesley is always developing new ways to broaden students' perspectives, whether with interdisciplinary projects in the classroom or mathematically related study abroad options. She hopes her students learn the value of mistakes and failure in the learning process. When not engaging with students in the classroom, she enjoys spending time with her children and spouse.

Wing Hong Tony Wong completed his PhD in mathematics at the California Institute of Technology in 2013 under the supervision of Richard M. Wilson. Wong subsequently joined the Department of Mathematics at Kutztown University of Pennsylvania, where he is currently a professor of mathematics. His research interests are in combinatorics, elementary number theory, and linear algebra. Wong is experienced in mentoring undergraduate research, such as being a principal investigator for the National Research Experience for Undergraduates Program (NREUP) in 2020, as well as a mentor and senior personnel in the Research Experiences for Undergraduates (REU) at Muhlenberg College and Moravian University since 2018. Wong was recognized as an excellent instructor, winning the Early Career Teaching Award by the Eastern Pennsylvania and Delaware (EPaDel) Section of the Mathematical Association of America (MAA) in 2020, the 40 Under Forty Award by the Reading Eagle in 2018, and the John P. Schellenberg Faculty Award in 2017 at Kutztown University.

Cindy Wyels is a professor of mathematics at CSU Channel Islands, where she co-envisioned the REU described in the chapter with Kathryn Leonard and Geoff Buhl. She co-authored and worked on three HSI-STEM grants ($17m) and an NSF ADVANCE grant and directed the campus' LSAMP program—all efforts focused on creating educational opportunities for students and faculty, particularly those from non-traditional backgrounds. Recognition most meaningful to her includes the SACNAS Distinguished Mentor Award, the MAA's Haimo Award for Distinguished College or University Teaching of Mathematics, and her institution's UndocuAlly of the Year. Her mathematical research interests began in combinatorial mathematics and linear algebra. She now applies data analysis tools to study the effectiveness of educational interventions and to collaborate on environmental issues. She began a 4-year term as the secretary of the Mathematical Association of America in 2022.