NOTES

ADDRESSING CHALLENGES TO THE PRECALCULUS TO CALCULUS II SEQUENCE THROUGH CASE STUDIES:

REPORT BASED ON THE NATIONAL SCIENCE FOUNDATION FUNDED PROJECT PROGRESS THROUGH CALCULUS

Estrella Johnson
Naneh Apkarian
Kristen Vroom
Antonio Martinez
Chris Rasmussen
David Bressoud

Editors

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Contents

1

Introduction

David Bressoud, *Macalester College (retired)*

1.1 Background

No courses restrict access to STEM majors more broadly and effectively than those of single variable calculus. And few courses reveal the inequities of our educational system more starkly. Today, 20% of high school graduates have already studied calculus, around 800,000 per year. But not all students can enroll in this course. In the United States, 48% of high schools do not offer calculus. Although other options often exist, including the opportunity to study calculus at a local community college or a formal arrangement providing for enrollment in online instruction, 17% of students have no access to calculus instruction while in high school. By socio-economic status (SES), 38% of high school students in the highest quartile take calculus. For the lowest quartile, only 7% do (Bressoud, 2020).

Socio-economic inequalities translate into racial inequities. Black students complete calculus in high school at less than half the rate of White students (8% versus 19%). They take the Advanced Placement (AP) Calculus Exam at less than a third the rate (3.7% versus 12.7%). And they earn a respectable grade of 3 or higher at roughly one-sixth the rate (1.5% versus 8.6%) (Bressoud, 2020).

At least three-quarters of the students in college Calculus I are retaking their high school course. For those who were privileged to take this course in high school, this is review, though usually at a more rapid pace and intense set of expectations. Those who are exposed to calculus for the first time find themselves at an enormous disadvantage, especially when instructors set out with an implicit—sometimes explicit—expectation that a certain percentage of the students will fail.

Within many departments of mathematics, there is still a current of belief in calculus as gatekeeper whose role is to deny advancement to those who are not worthy. Yet as Seymour and Hunter (2019), Tough (2019), and others have documented, the deterrence created by calculus instruction has much less to do with actual ability than with the opportunities that were afforded in high school and the nature of the messages conveyed in the course. Students coming out of under-resourced educational opportunities can thrive when the right supports are provided. Those who enter college with a fragile sense of self-identity as a "math person" because of gender, race, ethnicity, or SES are quickly discouraged by instructional and assessment modes that emphasize deficiencies.

It was because of the central role of calculus and the barriers it creates that the Mathematical Association of America (MAA) began a series of studies of college calculus in 2010. The first of these, *Characteristics of Successful Programs in College Calculus* (CSPCC, 2010–2015, NSF #0910240), collected data on the state of calculus instruction in the United States and, via case study visits, identified seven characteristics of successful programs (Bressoud & Rasmussen, 2015; Rasmussen et al., 2014):

1. Regular use of local data to guide curricular and structural modifications.
2. Attention to the effectiveness of placement procedures.

3. Coordination of instruction, including building communities of practice.
4. Construction of challenging and engaging courses.
5. Use of student-centered pedagogies and active-earning strategies.
6. Effective training of graduate teaching assistants.
7. Proactive student support services, including the fostering of student academic and social integration.

The findings from this first study were summarized in *Insights and Recommendations from the MAA National Study of College Calculus* (Bressoud et al., 2015).

1.2 Progress through Calculus

The MAA's second study, *Progress through Calculus* (PtC, 2015–2020, NSF #1430540), has sought to understand the variety of ways that departments have implemented the seven characteristics and to measure the effects of structural, curricular, and pedagogical decisions. This volume summarizes our findings.

In the second study, we made several significant changes. The first was to broaden our scope to include the entire sequences from Precalculus through the last course in Single Variable Calculus. The second was to focus from all post-secondary institutions to those that offer a graduate degree, Master's and/or Doctorate, in Mathematics. This was an easy way of restricting attention to the larger universities and those with a research focus that our initial study had found to have the highest barriers to success in calculus. In addition, we greatly expanded the kinds of data we collected. In addition to surveys, interviews, and focus groups, we obtained transcript data over a two-year period.

For the first stage of our study in 2015, we surveyed all 330 departments of mathematics in the United States that offer a graduate degree in Mathematics. The response rate was a gratifying 75% among PhD-granting institutions and 59% for those whose highest degree was MA or MS. We asked about practices related to the seven characteristics, gathered information on departmental priorities around these courses, and collected enrollment information that spanned the entire year, summer 2014 through spring 2015. The results of this survey are reported in Apkarian and Kirin, 2017.

For the second stage, we selected universities that had been working on some of the seven characteristics and whose progress appeared worthy of further investigation. The data that were collected included case study visits over a two-year period. Protocols were developed for classroom observation and for interviews with deans, chairs, instructors, members of client disciplines, and student focus groups. Special protocols were developed to interview those who could inform us about placement, use of data, course variations, faculty support, GTA training and support, supplemental instruction options, student support services, and any unique programs or initiatives. In addition, we modified the Postsecondary Instructional Practices Survey (Apkarian et al., 2019; Walter et al., 2016) to survey instructors, teaching assistants, and students about their beliefs, perceptions, and experiences in the classes we were monitoring. We also collected transcript data over a two-year period for those students in the classes under investigation.

Dozens of people have worked on aspects of this study. It has given and continues to give rise to many reports and publications that explore and explain our findings in greater detail. Additional information on the study and a continually updated list of publications and reports can be found at `https://maa.org/ptc`.

1.3 Contents of this Volume

This volume draws on just a small portion of the data that have been collected to illustrate the kinds of efforts that are currently underway. While we had restricted our investigation to departments with graduate programs, they are sufficiently diverse in the nature of the challenges they faced that we expect that every department of mathematics will find situations that resonate with their own. The five chapters that constitute the heart of this volume describe existing efforts in multiple universities that address each of the following issues:

2. **Rethinking student support programs.** *What are the ways in which departments can provide structural support to bolster student success and inclusion, especially for students with historically underserved identities?*

3. Alternative course pathways. *What are different ways that departments are offering calculus courses that support students who are most at risk for not progressing through the precalculus-calculus sequence?*

4. Variations in approaches to the content of Calculus. *What are ways in which a department can supplement, modify, or restructure the content in its calculus sequence to increase student interest, improve comprehension, and/or align more effectively with the needs of partner disciplines?*

5. Professional development and course coordination. *How can course coordination and other opportunities that mathematics departments offer contribute to the professional development for their GTAs and faculty?*

6. Program assessment and the use of local data. *How are mathematics departments using local data to make improvements to their calculus programs?*

In addition, three of our case study sites had recently undergone a crisis in which the university administration had threatened to move calculus instruction out of the department of mathematics. Chapter 7 on **Institutional and departmental change** looks at how they have responded to this threat. Chapter 8, **From inspiration to making it happen**, serves as both a summary of the lessons learned from these studies and an introduction to how to think about the changes that may be needed at your institution.

Chapters 2 through 7 share a similar structure. Following the guiding questions and an introduction to the topic, each uses vignettes to describe how this issue has played out at three or four different universities. Each vignette is followed by a few questions asking for reflection on the changes instituted at that university. These are followed by thoughts on how this initiative might be further developed together with additional questions and a final summary of what the research team observed and learned. Each chapter concludes with a discussion of commonalities of these different situations and an overview of the considerations to be kept in mind when attempting similar changes.

To protect individual anonymity, we have chosen pseudonyms for the universities and all individuals mentioned in the study. What follows is a brief introduction to the eleven universities included in this study. Additional information on these universities can be found in the Appendix.

Alpine University (AU). A public university with 22,250 undergraduates and a 57% 6-year graduation rate. The math department has about 30 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 3, **Alternative course pathways**.

Canyon Crest University (CCU). A private university with 14,800 undergraduates and a 71% 6-year graduation rate. The math department has about 25 tenured or tenure eligible faculty and the highest degree it offers is a Masters. Its case study is discussed in chapter 2, **Rethinking student support programs**, and chapter 3, **Alternative course pathways**.

Dandelion State University (DSU). A public university with 11,000 undergraduates and a 44% 6-year graduation rate. The math department has about 22 tenured or tenure eligible faculty and the highest degree it offers is a Masters. Its case study is discussed in chapter 2, **Rethinking student support programs**, and chapter 6, **Program assessment and the use of local data**.

Desert Bloom University (DBU). A public university with 13,500 undergraduates and a 80% 6-year graduation rate. The math department has about 35 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 3, **Alternative course pathways**, and chapter 7, **Institutional and departmental change**.

Dunshire University (DU). A private university with 6600 undergraduates and a 95% 6-year graduation rate. The math department has about 50 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 2, **Rethinking student support programs**, and chapter 4, **Variations in approaches to the content of Calculus**.

Maple State University (MSU). A public university with 40,000 undergraduates and a 63% 6-year graduation rate. The math department has about 55 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 4, **Variations in approaches to the content of Calculus**, and chapter 5, **Professional development and course coordination**.

Pine Grove University (PGU). A public university with 41,000 undergraduates and a 54% 6-year graduation rate. The math department has about 35 tenured or tenure eligible faculty and the highest degree it offers is a Masters. Its case study is discussed in chapter 2, **Rethinking student support programs**.

River Rock University (RRU). A public university with 23,800 undergraduates and a 65% 6-year graduation rate. The math department has about 70 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 4, **Variations in approaches to the content of Calculus**, chapter 6, **Program assessment and the use of local data**, and chapter 7, **Institutional and departmental change**.

Rolling Hill University (RHU). A public university with 21,000 undergraduates and a 61% 6-year graduation rate. The math department has about 40 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 5, **Professional development and course coordination**, and chapter 7, **Institutional and departmental change**.

Sandpiper University (SU). A private university with 3175 undergraduates and a 70% 6-year graduation rate. The math department has about 13 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 3, **Alternative course pathways**, chapter 5, **Professional development and course coordination**, and chapter 6, **Program assessment and the use of local data**.

Tree Line University (TLU). A public university with 14,100 undergraduates and a 76% 6-year graduation rate. The math department has about 20 tenured or tenure eligible faculty and the highest degree it offers is a PhD. Its case study is discussed in chapter 3, **Alternative course pathways**.

References

Apkarian, N., & Kirin, D. (2017). *Progress through Calculus: Census Survey Technical Report*. Available at `https://www.maa.org/sites/default/files/PtC%20Technical%20Report_Final.pdf`

Apkarian, N., Smith, W. M., Vroom, K., Voigt, M., Gehrtz, J., PtC Project Team, & SEMINAL Project Team. (2019). X-PIPS-M Survey Suite. Available at `https://www.maa.org/sites/default/files/XPIPSM%20Summary%20Document.pdf`

Bressoud, D.M. (2020). The strange role of calculus in the United States. *ZDM Mathematics Education*. `https://doi.org/10.1007/s11858-020-01188-0`

Bressoud, D., Mesa, V., & Rasmussen, C. (eds.) (2015) *Insights and Recommendations from the MAA National Study of College Calculus*. Washington, DC: MAA Press. Available at `https://www.maa.org/sites/default/files/pdf/cspcc/InsightsandRecommendations.pdf`

Bressoud D., & Rasmussen, C. (2015). Seven Characteristics of Successful Programs in College Calculus. *Notices of the AMS*, February, 2015, 144-147.

Rasmussen, C., Ellis, J., Zazkis, D., & Bressoud, D. (2014). Features of successful calculus programs at five doctoral degree granting institutions. In S. Oesterle, P. Liljedahl, C. Nicol, & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education Vol. 5* (pp. 33-40). Vancouver, Canada: PME.

Seymour, E., & Hunter, A.-B., Editors. (2019) *Talking about Leaving Revisited: Persistence, Relocation, and Loss in Undergraduate STEM Education*. Cham, Switzerland: Springer Nature.

Tough, P. (2019). The Years that Matter Most: How College Makes or Breaks Us. Boston, MA: Houghton Mifflin Harcourt.

Walter, E. M., Henderson, C. R., Beach, A. L., & Williams, C. T. (2016). Introducing the Postsecondary Instructional Practices Survey (PIPS): A concise, interdisciplinary, and easy-to-score survey. *CBE-Life Sciences Education, 15*(4), ar53. `https://doi.org/10.1187/cbe.15-09-0193`

2

Rethinking Student Support Programs

Attending to Student Identities and Systemic Structures in Introductory Mathematics

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Chapter Question: What are the ways in which departments can provide structural support to bolster student success and inclusion, especially for students with marginalized identities?

2.1 Introduction

Inequities exist within mathematics programs. We take the previous statement as an axiom for this chapter, in which the basis for our reasoning and all other logic follow from this assumption. We do not take this axiom lightly. It is informed by research, data, and the lived experience of individuals in mathematics environments. As such, in order to address and start to rectify inequities, we must attend to issues of diversity and inclusion in all aspects of our introductory mathematics programs (Hagman, 2019). Not centering issues of diversity, equity and inclusion in our mathematics programs will only continue to foster systematic oppression and marginalization. Therefore in this chapter we seek to highlight ways in which students with historically underserved[1] or marginalized[2] identities are supported in introductory mathematics courses through systemic programs or services. So, broadly speaking we were motivated to examine: *What kind of programs exist nationally to support students from historically underserved or marginalized groups in mathematics and how are these enacted at the university?* Throughout the chapter, when we are not referring to a specific identity group (e.g. first-generation, Black, women), we primarily use the term "marginalized" to emphasize both the historical and systemic nature in which certain groups of students are not centrally served by mathematics programs. We recognize the importance of being specific about language and also the limitations in selecting a term which may reinforce deficit views and treat identities as homogenous. Within this chapter we take up this guiding question to critically rethink services and programs for supporting student success. We start first with a national snapshot of such programs and then highlight four programs enacted at different universities across the United States.

[1] We use the term "historically underserved" to highlight the educational debt (Ladson-Billings, 2006) that society owes to minoritized students due to forms of oppression (e.g., racism, sexism, xenophobia) and because it serves to reframe issues of equity from student deficits to structural bias and exclusion. At the same time this term can imply problematic notions that being underserved by mathematics programs is a historic phenomenon that is not present in programs, a premise that we strongly reject.

[2] We use the term "marginalized" to emphasize how systems of oppression act on students to marginalize them and shape their identity.

During the first phase of the Progress through Calculus project, a national census survey was distributed to all institutions offering a graduate program (Master's and Doctoral) in mathematics in the United States to better understand the programs and characteristics around introductory mathematics programs. One component of the survey asked departments to describe any support program for students of "traditionally underrepresented groups" and details related to how they were offered (within the department or university context). We did not define the term "traditionally underrepresented groups" on the survey purposefully since we wanted departments to have flexibility in describing programs that they viewed as responding to their local population and context. We also want to note, that as language has evolved, and as we as researchers have educated ourselves in this journey, we shift away from using the same language of "traditionally underrepresented groups" in reporting results since it potentially further promotes deficit notions and frames the lack of diversity in math programs as an outcome of student motivation and choice. An analysis of the 230 responses revealed that nearly one-third of departments had no knowledge of such programs at their institution, a quarter mentioned a program that was administered by the university, and roughly a quarter of respondents reported a program or service offered within the mathematics department (see Figure 2.1). Consequently, the limited number of departmental programs specifically supporting marginalized groups demonstrates a great opportunity (and need) for mathematics departments to take a more active role in supporting students with a marginalized identity to thrive in STEM (Voigt, et al., 2019).

The ways in which mathematics departments can support students with a marginalized identity to thrive has taken many forms, some which seek to support them within the existing educational system, while others seek to critique and challenge the educational system to remove inequities (Gutiérrez, 2002). For example, mathematics departments can bolster students with a marginalized identity by offering programs or initiatives that aim to support students in being successful within the current system. That is, supports that are designed to increase grades, retention, and representation in STEM courses act to support students to succeed within the system as it is. Mathematics departments can also support students with a marginalized identity by creating and offering support structures that challenge what has been accepted as the university or departmental system. Such support structures work to increase the identity and power of these groups to redefine a system that is informed and molded by diverse perspectives. It is important to note that neither of these types of supports are better than the other, but rather both are critical components to bringing about the systemic and sustainable change needed to address inequities in mathematics programs.

In this chapter we provide vignettes of four programs, with connections to the mathematics department, designed to support students with a marginalized identity. The programs highlighted in the vignettes vary across three dimensions: 1) the degree to which the mathematics department is involved in facilitating the programs, 2) the duration of the program, and 3) whether the program supports students within the existing system or is designed to inform changes

TYPES OF SUPPORTS

Type of Support	Number of Departments	Percentage	
None offered	40	25%	33% no supports
Unaware	8	5%	
Diverse Student Body	5	3%	
University (Specific supports)	13	8%	25% university program
University (generic supports)	23	15%	
Engineering (generic)	3	2%	
AMP (Alliance Minority Participation)	9	6%	13% federal programs
EOP (Educational Opportunity Program)	6	4%	
TRiO	5	3%	
Bridge program	11	7%	23% department program
Department monitoring	2	1%	
Designated course sections	5	3%	
Scholars program	7	4%	
Specific initiatives or programs	12	8%	
Other	8	5%	

Figure 2.1. Breakdown of services or programs to support "traditionally underrepresented groups."

to or challenge the existing STEM culture and educational system. As you read through the vignettes, we ask you to question, How does this program support marginalized students in STEM? How does this program relate to the math department's role - is it complementary? What can math departments do beyond (or within) these programs to support students to thrive? The reader is encouraged to think critically about the affordances and constraints of each program and what the enactment of each might look like at their own institution. To facilitate the reader's own reflection process, we provide a set of targeted discussion questions for each vignette to focus on critical points of emphasis.

The first vignette at Canyon Crest University (CCU) highlights a first-year academic success program focused on providing alternative mathematics courses that operate outside of the mathematics department, but rely on mathematics instructor involvement for the teaching and delivery. The second vignette at Dandelion State University (DSU) highlights a living-learning community that offers specific course sections in mathematics. The third vignette at Dunshire University (DU) highlights a student-focused diversity advisory committee which provides feedback and suggestions for change within the STEM departments and university. Instead of featuring a specific program, the fourth vignette at Pine Grove University (PGU) highlights how the nature of the minority serving institution along with department leadership is providing a learning assistant program for students with marginalized identities to thrive in STEM.

2.2 Canyon Crest University: First-Year Success Program (FYSP)

Vignette

Canyon Crest University is a private not-for-profit high research university. In 2009, Canyon Crest University created a First-year Success Program (FYSP) which acts as a holistic support program designed for students who need to complete prerequisite mathematics or writing courses. FYSP is operated as a university-wide program that provides students the opportunity to take introductory mathematics courses during the summer or winter break that are tuition-free and do not count towards course credit. The program is designed to be responsive to the needs of low-income students by covering the costs of textbooks, licenses for homework systems, and providing transportation passes. In fact, the program grew out of the desire to support students with financial needs. The director of the FYSP program discussed the impetus and design of the First-Year Success Program:

> *We found that students weren't graduating on time because they were running out of money because they were using ... financial aid dollars for courses that wouldn't count toward their degree so the university as a whole is really trying to come together and find out how can we offer these development level courses to ... [local city] students so that they could advance to their degree quicker without cost. So our courses are offered at no cost for tuition and earn no credit but they have the exact same instructors, rigor, syllabi, curriculum, and online tools as a calendar-year academic course.*

As mentioned by the FYSP director, the courses are similar to those offered in the academic year through the math department but are on a condensed timeline of 3 weeks in winter break or 5 weeks during summer term. The program also provides dedicated advisors to students throughout the academic year, access to supplemental instruction, and early course monitoring and intervention (as needed).

The program is open to any students who need to complete prerequisite mathematics courses; however, students are invited to the program If they score below an ACT 19 or have a high school math GPA of 2.75 or less. Additionally, any first-year students who are placed in a developmental math course (based on placement scores) are automatically invited into the program. The program typically serves a more diverse community than the undergraduate population at CCU, with roughly 40% first-generation students, 35% students of color, and 25% transfer students or non-traditional students. The director of the program has been tracking local data to evaluate the success of the program and has shown that students enrolled in the program have a slightly higher persistence rate and perform better in subsequent mathematics courses than those invited to the program who choose not to use the services.

In many ways this program is similar to summer bridge programs that invite students on-campus before the start of the fall semester to provide academic support and socialization into the college culture; however, FYSP is unique from such programs because it is a non-residential program mostly serving commuter students that is free for all students to attend. The choice to implement a non-residential program was met with some tension from student affairs professionals who had concerns that it would not foster socialization into the college culture; however, the administrator leading *the development of the program argued that the cost-savings measure of not paying for room and board allowed the program to serve more than double the number of students with the same budget. As such, the program serves between 200-300 students each summer. The program has been viewed so favorably by administrators, that when the state budget was cut the university board allocated money to continue funding the program, in part because it was cost-effective and because it aligned with the social justice mission of the university to serve and uplift the local community. As one of the Deans stated, "the [FYSP] program is really part of what it means to be [Canyon Crest University]."*

Connections to the Mathematics Department

Although the program is administered at the university level, the mathematics department is an integral part of the success of the First-year Success Program. The mathematics courses offered through FYSP have the same curriculum, syllabi, and instructors as the courses offered through the mathematics department. The major difference is that the FYSP courses are on a condensed timeline and are non-credit bearing. The instructors who teach the courses are paid through the university program, but the chair of the mathematics department nominates instructors who have taught the course before and would be successful in supporting students in the program. The relationship between FYSP and the mathematics department was initially met with some tension, as the chair of the mathematics department had concerns that this would lower the enrollment in courses offered through the mathematics department, and thus impact the financial resources that the department was generating. The Dean of the college assured the chair that this would not negatively impact the mathematics department, and as the program unfolded the Chair's concerns were alleviated. The FYSP program did not negatively impact enrollment in mathematics courses in part because a) the highest-level course offered through FYSP are precalculus and b) success in FYSP meant students could take subsequent courses during the academic year offered through the mathematics department.

Discussion Questions

1. What do you see as the advantages or potential pitfalls for offering developmental math courses as non-credit bearing?
2. CCU operates on the quarter system, thus summer and winter FYSP courses are condensing a 10-week course to a 3-5 week course. How might a university on a semester system of 15 weeks develop a condensed schedule? Would it be possible to offer these over the winter break for a university on the semester system?
3. The initial development of the FYSP program created some tensions with the mathematics department being seen in competition for enrollment numbers. Could such a program be developed through the mathematics department to alleviate such concerns? Or would this create confusion for students?

Further Considerations: Within the System and Changing the System

The FYSP program at CCU focuses on short-term or targeted academic success through condensed course delivery. In some ways the program is operating at the intersection of working within the system and working to change the system. It operates within the original system by offering the same courses, curriculum and instructors as the standard mathematics courses. This serves to perpetuate the same pitfalls that reside within our courses (e.g., curriculum that does not reflect our student's background, taught by predominantly white male instructors). At the same time it challenges the system by offering the courses free to students by operating under a university program. This helps further the question: why should students be paying for developmental math courses, when it represents a failure of our public education system for not providing such educational opportunities? Does requiring students to complete such mathematical prerequisites exploit marginalized students and hinder their educational progress? These are deep questions that likely have no easy solutions, but the FYSP program is one example of a program that is starting to question assumptions about our educational system while also supporting students currently in the system to thrive. The FYSP program speaks to a growing movement in mathematics education to leverage open educational resources (OER) to better serve students with limited financial resources (Hilton, 2016). At the same time, we note that the private not-for-profit nature of the university and its ability to fund the program limits the nature of which other institutions may be able to implement a similar program within their context.

We now turn to a program offered at Dandelion State University that is similar to that of Canyon Crest University as it is a program that operates outside of the mathematics department, but relies on the mathematics instructors involvement. However, the program at Dandelion State is an *on-going multi-semester* living-learning community for first-generation college students, and primarily aims to support students within the existing university structure.

2.3 Dandelion State University: Generation DSU

Vignette
Dandelion State University serves a large number of students who are the first in their family to attend college (˜60% of the undergraduate population are first-generation college students). Local administrators at DSU recognized that the retention rates for first-year first-generation students were 10-15% lower than their peers. In an effort to better support these students and to increase graduation rates, Generation DSU was formed. Generation DSU is a multi-term living-learning community program that offers a variety of supports specifically designed for first-generation college students.

Generation DSU is a multi-semester program designed to provide both academic and social supports for first-generation college students. The program starts with a week-long immersion course in everything college related, including workshops, social events, and service and leadership opportunities. Workshops are mostly focused on academic considerations, like study tips and financial literacy, but they also include opportunities for individual and collective reflection on identity (e.g., what does it mean to be a first-generation college student) and how to navigate balancing family and academic responsibilities or expectations. As part of Generation DSU, students also have the opportunity to live in a residential learning community (i.e., one floor in a residence hall dedicated to Generation DSU students), and the opportunity to participate in several other on-going service, mentorship, internships, and professional development experiences.

Creating and fostering a living-learning community is a central tenet of Generation DSU. When the program first started, students were placed into cohorts where they took three classes together, one each semester for the first three semesters. This means that certain sections of the precalculus and calculus courses at DSU are reserved for Generation DSU students. As the program has grown it has become more challenging to have the students in these cohorts. However, there are still designated Generation DSU sections of courses in a variety of departments, and students still enroll in three of these over their first three semesters.

Generation DSU sections are small sections (less than 25 students) taught by instructors who are hand-selected by the department chairs and the Generation DSU success coordinator. The instructors selected to teach these courses are known as good teachers and are eager to provide instruction and additional support that first generation college students might need. The Generation DSU success coordinator stated:

> *[I]t's really really important to have the right kind of mindset of the professor in that classroom. We're not asking for someone that's going to pass all our students and everybody has A's, and it's all, you know, rainbows and unicorns. But we need good teachers, not necessarily professors. We need people with the mindset of 'How can I say this differently? How can I repeat this differently? Can I give it in a different situation?' Someone that really wants to be a teacher and somebody who really has the heart and patience for those particular students.*

First-generation college students are contacted and encouraged to apply for the Generation DSU program. Students are accepted on a first-come, first-serve basis, with a preference for students who are both first generation and traditionally underrepresented. After their first three semesters, Generation DSU students become mentors to new first-generation students or become mentors to high school students who are interested in attending college. Initially, the program did not have funding, and so the first cohort of Generation DSU students was limited to 20 students. Since its conception in 2014, the program has grown to include 100 first-generation students each year. Currently, there are no plans to expand further as administrators want to be able to intentionally maintain high impact practices and a close connection with these students.

Connection to the Mathematics Department
In order to offer special sections of courses for Generation DSU students, it is critical for there to be a positive and working relationship between the program and the mathematics department. The Generation DSU director regularly meets with the department chair to discuss course offerings for the Generation DSU sections, and the department

ensures that these courses fit into the schedule. Additionally, the program director and department chair discuss instructors that would be particularly well-suited to teach the Generation DSU section of the course.

The instructor of the one Generation DSU mathematics course section regularly corresponds with the program director to discuss student progress. Additionally, the instructor meets with other instructors teaching Generation DSU sections in other departments. They discuss each Generation DSU students' progress in their classes, specifically attending to students who did not do well on the first exam of the semester. This provides a more holistic approach to evaluating students' success and supporting underserved students to thrive.

Discussion Questions

1. Generation DSU serves 100 first-generation students, which is a small fraction of the DSU students who are first-generation college students (6,500). What other supports would you recommend DSU offer in order to reach a greater number of first-generation students, keeping in mind that there are time and budget constraints which make expanding the current Generation DSU program not an option?

2. The Generation DSU program offers on-going support to students throughout multiple semesters, rather than only offering a summer transition class or a bridge program. What are the affordances and limitations of such supports?

3. Note that the initial versions of Generation DSU did not have funding. What supports or structures could you put in place in your department to support marginalized students that require little to no external funding?

4. In what ways could your mathematics department strengthen a connection or commitment to partner with a university-wide organization to support marginalized students? Who in your department would be best suited to lead such an effort?

Further Considerations: Within the System and Changing the System

Generation DSU is an example of a multi-semester program that works to support students within the current educational system by fostering supportive living-learning communities. Namely, this program is designed to help first generation college students understand how to navigate and be successful in college (and beyond) by offering workshops on college-related topics, pairing students with peer mentors, and by providing specific sections of introductory courses for Generation DSU students to enroll. At the same time, such a program places the emphasis on the first-generation students' lack of "cultural capital" and needing to support them in navigating the institutional context without challenging the ways in which the system is designed for continuing-generation students. For example, having designated sections for first-generation students could allow the mathematics department to re-center the curriculum on first-generation experiences either through data exploration or featuring prominent first-generation mathematicians. We encourage departments interested in offering a program similar to that of Generation DSU to consider the ways they can also challenge the normative assumptions that are leading to inequitable experiences.

We now turn to Dunshire University, which developed a different type of on-going program (offered outside of the mathematics department) that worked to support students in the system in addition to providing opportunities to re-invision, challenge, and change the existing educational system.

2.4 Dunshire University: Advisory Committee of Diverse Students in STEM (ACDS-STEM)

Vignette

At Dunshire University, which is a highly-selective private university, most students enter with Advanced Placement (AP) credit in mathematics (thus taking Calculus II or higher in their first semester) and so Precalculus is not offered. Students seeking a STEM degree without AP credit then either enroll in a one-semester Calculus course or a two-semester Calculus course, with disproportionately more students of color enrolling in the two-semester Calculus course. Students generally perceive the mathematics courses at DU to be especially challenging, and will sometimes make decisions about which degree to pursue based on how many mathematics classes are required for the degree *plan. As such, there was a need to hear from, mentor, and support students of color at DU, which resulted in the creation of an Advisory Committee of Diverse Students in STEM (ACDS-STEM).*

ACDS-STEM was created after an academic director for the college, Dr. K, along with the dean of the college, recognized that there was a need to support students of color, in particular, as they pursued their STEM degree paths. Dr. K highlighted that,

> *It was also important for us to kind of hone in on the largest part of attrition for STEM is usually students of color. And if we know that math is one of ... the many gateways into a lot of those majors, these students can provide us a very unique insight, as to not only the student experience of pursuing STEM at [DU], but also students of color. And maybe how math plays into their experiences as well as the other classes.*

Dr. K noted that it was critical to hear from students about what they needed, instead of rushing to create new programming at an institution where students already had access to an overwhelming number of resources. With ACDS-STEM, Dr. K wanted to lean on the students for their expertise because they were the ones who were navigating STEM classes as students of color.

The primary purpose of the ACDS-STEM committee was to create a feedback loop from students of color in order to begin to facilitate change. The director of ACDS-STEM would pass on information about these students' experiences to the dean and department chairs. Dr. K was purposeful in raising the students up as the experts on their own experiences. Students also emphasized that one of their main goals for participating was to "provide as much feedback as possible" and to be part of something that was working towards change so that the experiences of future students of color at DU would be improved. One student described that their work with ACDS-STEM went beyond sharing their experiences, but that sharing solutions and working to improve the system was a critical first step before working to increase diversity in STEM and at DU. She said,

> *I think ... presenting the difficulties of being in STEM and math and everything are good, and a want to increase diversity is good, but ... you don't want to increase diversity to make ... even more of a problem. So if we're here saying the issues that we have and also posing potential solutions, and that's good because increasing diversity - what is that going to do besides hurt more people if there is no solution in place? So this is the preliminary step to increase diversity, because you don't want to come into a broken system. So we're doing the work that should have already been done, I guess, for more diversity to be here.*

Dr. K recruited students to participate in ACDS-STEM by personally selecting and inviting students that she knew, from first-years to seniors, who were pursuing a STEM degree and were students of color. Dr. K highlighted that it was important for her to know the students and to be seen as someone that they could trust. Students involved with ACDS-STEM also emphasized that Dr. K was a "safe space" for them, and that she was someone who was easy to talk to. Students appreciated being part of a program that provided a community and space for them to openly share their experiences with peers and mentors who understood and then could work to improve experiences for others like them. One student said:

> *I feel like it's also like being able to get out your personal frustrations and like know ... other people have gone through what you're going through. But it's also like you realize it's a problem because there shouldn't be a group of people specifically going through these types of problems. So when [Dr. K] points out things that we could possibly do to make a change, and then ... at least there's something that can be done potentially.*

ACDS-STEM met once a month (with approximately 10-20 students attending). A typical meeting would consist of a meal and discussions about what students were feeling and experiencing in their STEM classes. Dr. K often put forth informal prompts to get the conversation started, but as soon as students started sharing their energy would build off one another and there would be a "very back-and-forth, dynamic" conversation. Several students said they initially felt isolated in their classes, but that ACDS-STEM provided opportunities for them to talk with other students and helped them to realize that they were not struggling more than anyone else at school. Further, since ACDS-STEM was composed of students from all years, upperclass students were able to share advice with newer students and serve as role models. Along with ACDS-STEM, Dr. K tried to connect students to other resources around campus and offered *additional programming designed to foster the students' STEM identities and to foster a sense of belonging for students of color in STEM and at DU.*

Connection to the Mathematics Department

A committee like ACDS-STEM affords departments opportunities to hear and engage with student experiences in STEM. The information gained from a student advisory group then can be leveraged to make changes to create more inclusive and equitable learning environments. Although ACDS-STEM was not directly connected to the mathematics department at DU, feedback from ACDS-STEM students played an important role in the development of a different program (a formal mentoring and academic support for STEM students from diverse backgrounds) that mathematics faculty were involved with as mentors. Given that ACDS-STEM was broadly inclusive of all STEM students of color, this likely helped provide a critical mass of interested students to provide feedback to STEM departments, but also removed direct connections and involvements with the mathematics department. As such, mathematics departments seeking to support students to thrive in mathematics may need to take a broader reach in order to hear from the voices of marginalized students.

Discussion Questions

1. ACDS-STEM operates outside the mathematics department, but is designed to empower students of color within STEM to provide feedback to the university via the ACDS-STEM advisor. What are the affordances and limitations of this model?
2. What might a student advisory committee look like in your department or college?
   (a) What resources might need to be secured?
   (b) What would it look like to recruit students to be involved?
   (c) Who would you need to seek buy-in from in order to make changes based on student feedback?
   (d) How willing is your department / administration to make changes based on student feedback?
3. Much of the success of ACDS-STEM relied on one individual, Dr. K. What are some ways that the mathematics department could have supported Dr. K?

Further Considerations: Within the System and Changing the System

Dunshire University offers an ongoing support for students of color that, although not directly connected to the mathematics department, has influenced mathematics departmental programming. Additionally, the ACDS-STEM program works to support students within the current system and works to implement changes to the educational system. Specifically, ACDS-STEM works to build a community of students from all years so that they can support one another and offer advice based on experience as they work to achieve their academic and professional goals. The director of ACDS-STEM connects students with other programming at the university that they could access to support their success. Those involved with the ACDS-STEM program aim to change the system and STEM culture at DU. The director works to do this by elevating student voices to influence change by sharing these students' perspectives and experiences with administrators. The students are also dedicated to this goal and frequently shared that they were a part of ACDS-STEM because they wanted to work towards a better system for future students of color at Dunshire University. ACDS-STEM alo seeks to change the system by giving voice and agency to marginalized students to provide recommendations for support programs that are grounded in their lived experience and not dictated by well-meaning administrators.

Although ACDS-STEM was an impactful program that resulted in changes at Dunshire University, one limitation of this program is that its success and sustainability relied primarily on one individual, the director of the program, Dr. K, who identifies as a woman of color. As such, the responsibility and efforts derived from this program leverage the work of people of color, who are often overburdened with such service tasks, an aspect commonly referred to as the "minority tax" (Carson, 2019). If such a program were within the context of a department, then there would be more opportunity for several departmental members to work together to support these students and establish additional structures to ensure the sustainability of the program. Furthermore, although this program was started at a private highly-selective school with ample financial resources, the cost of the program can be relatively low and thus scaled to different institution types, especially if there is a critical mass of invested faculty in supporting the program. At the same time we note that faculty demands vary by role and institution type and thus the human capital needed to implement such a program could be a barrier at some institutions.

For the final vignette, we highlight a program not specifically designed to support a diverse population of students, but that exists within a Minority Serving Institution (MSI). This is an *ongoing* program that is offered outside the mathematics department but with mathematics instructor involvement, and works within the system.

2.5 Pine Grove University: Learning Assistant Program

Vignette

Pine Grove University is a large, urban institution with a high percentage of students coming from the local community, which has a predominantly Latinx population. As a result, PGU is a Hispanic Serving Institution with nearly two-thirds of the population identifying as Latinx or Hispanic and over ten percent identifying as Black or African American. Similarly, a majority of STEM majors are Latinx and close to ten percent are Black or African American. When we asked instructors and administrators at PGU what the mathematics department or the university did to support this diverse population of students to be successful within STEM, they unanimously responded that they didn't do anything; rather, they are simply serving their local population and this is who their local population happens to be. Instructors, the majority of whom also identify as Latinx, spoke of their dedication to student success, and their strong allegiance to the mission of the institution, which all likely contributed to supporting students to thrive through role models and compassionate instructors. Here, we highlight one such program at PGU that supported underserved students to thrive in STEM - their learning assistant program in STEM.

The Learning Assistant program at PGU was developed based on the program at the University of Colorado at Boulder (Otero et al., 2010), which itself was developed to promote active learning in physics and train preservice teachers to implement active learning in science education. At PGU, this program is funded by the provost's office through a lab fee paid for by students. The LA program serves courses throughout STEM disciplines including mathematics, with about 300 LAs from 9 STEM departments. Learning assistants attend the course they are hired for, and help facilitate active learning. The role of the LA is flexible in design and varies by course instructor, but they typically circulate to assist students with clarifying questions, provide prompts to engage students with the mathematical material, and facilitate dialogue within groups of students.

Data collected by the university shows that courses that utilize LAs typically have a 10-20% higher pass rate. Interviews with first-generation, low-income, students of color at PGU highlighted how LAs were impactful to their success in mathematics courses. They appreciated how the LAs understood the content, helped them with homework while they were in the lab section, shared approaches to solving problems that were different from the instructor, and were able to connect with the learning assistants at times more easily than the professor (more details available in DiGregorio, 2018). In addition to supporting student success, the LA program has had a beneficial impact on the LAs themselves. For instance, LAs for math said that it has changed the way that they view math, and the LA coordinator said that the program was one way to support students by providing on-campus employment.

A major component of the LA program is the professional development course that is required of all LAs. This one-credit course is focused on incorporating active learning and grounded in learning theory and metacognition, and has a prominent social justice component. The director of the LA program shared that this was one of the aspects of the program that she is most proud of:

> *I'm proud of the way that we're able to empower students ... there is a big element of social justice in the program in which we want to think that we empower students to take risks, take responsibility for their own education.*

She explained that social justice is "a thread that we connect through every topic" within the course, including how to create environments where students feel supported to share ideas that may be wrong, and a class period on privilege (including white privilege) and how this shows up in teaching and learning.

Connection to the Mathematics Department

The LA program has supported many faculty within the mathematics department to implement more active learning into their courses, including in the precalculus and calculus courses. In addition to supplying the LAs and preparing them though the one-credit course, the LA program offers workshops for instructors about how to most effectively incorporate LAs into their courses. These workshops have not historically been well attended by the instructors in the mathematics department, which has resulted in inconsistent implementation of the LAs. As such, while PGU is providing greater access to mathematics for underserved populations, the work is largely being done by university programs and through efforts on the part of learning assistants without integration and support from the mathematics department.

Discussion Questions

1. The recruitment of LAs at PGU does not specifically attend to ethnoracial diversity, but results in a racially diverse group of LAs because of the high population of students of color at PGU. What are some steps that mathematics departments can take to recruit a diverse population of LAs that reflect the diversity of students at the institution? How can predominantly white institutions elevate students of color and provide near-peer role models by recruiting a diverse population of students to be LAs?

2. How can a mathematics department at an MSI be purposeful in supporting their diverse student population, rather than relying on the larger number of these students to simply be supported by one another?

3. What are ways the mathematics department can incentivize instructors to take advantage of the LA program, and further to attend the workshops designed to help instructors learn how to incorporate LAs into their teaching?

Further Considerations: Within the System and Changing the System

Because PGU has a racially diverse student population, we initially didn't consider their LA program as a program designed to support a diverse population of students in STEM. This points to not only an initial methodological limitation in our work, but also an underlying and potentially destructive perspective that was present among some instructors and administrators at PGU; that a diverse student population doesn't need targeted support if they are at a university also with a diverse student population. This perspective is problematic for a number of reasons. First, it shifts responsibility for supporting students' success from the department or university to the student population. Second, it assumes that the barriers students from marginalized populations face are removed when these students are in the majority, ignoring the multitude of other barriers these students face in a university environment (such as implicit bias from their instructors, financial constraints of education). Third, this perspective minimizes the work done by people across campus who recognize that a diverse student population will still benefit from supports designed with them in mind, such as the work done by the director of the LA program to integrate social justice throughout the professional development for learning assistants. As such the learning assistant program at PGU serves both to support students within the existing educational system while also challenging normative assumptions of who does mathematics. Students who take mathematics courses and then become learning assistants are positioned as capable near-peers that can serve to boost their own identification with mathematics. Additionally, learning assistants can serve as role models for current students who may not see themselves represented in faculty or national representations of mathematics.

It's important to note that although instructors and administrators at PGU were not explicit in acknowledging how they were supporting structurally disadvantaged students, they were nonetheless doing this work. For example, PGU has a much larger percentage of Hispanic faculty than the national average, and the Math Department Chair was proud of the number of recent instructor hires that are women and Hispanic. PGU has also been acknowledged nationally for inclusive campus-wide student success communication. There are many messages displayed around campus about the importance of student success for all students; including graduating in four years, which was also articulated in math faculty and department head interviews. We provide this information to further the idea that no single program will address the inequity in mathematics, and that we are often not aware of the "waters we swim in" that help support marginalized students to thrive in STEM. Yet at the same time we have a long way to go before we refute the axiom that inequities exist in mathematics programs.

2.6 Discussion and Call to Action

In this chapter, we highlighted four models of support for students from structurally disadvantaged populations that have some connection to the mathematics department. The programs illustrated here included: the FYSP program at CCU which forefronted academic supports through shortened course offerings, the Generation DSU which built a living-learning community with specific course sections in mathematics, the ACDS-STEM at DU which leveraged students of colors experiences to foster inclusive STEM environments, and the Learning Assistant program at PGU which provided near-peer role models to bolster student learning.

The programs detailed above varied based on their duration, from relatively short support programs (e.g., 5-week course at CCU) to programs that offer ongoing support mechanisms (e.g., the student advisory committee). The duration of each of the support programs was inherently linked to the amount of resources and structure that they are intending to provide. Each of the programs mentioned how they were considering issues of scalability with trying to maximize the number of served students while also making sure to provide students with exceptional services. The programs at DSU and DU started with limited to no funding, showcasing that such programs can emerge without institutional support. Furthermore, the programs at CCU and PGU show that once such programs are established and shown to be effective, administrators were willing to support them, especially when they were viewed as supporting the university mission. While departments consider various ways to engage in supporting the populations within their programs that are currently least served, we encourage departments to carefully consider the affordances and limitations of various structures of these programs.

We offer the following questions to encourage further reflection on ways for mathematics departments can actively be involved in supporting a diverse population of students:

1. Is there an organization already existing on campus that the department can plug into, such as Generation DSU, so the department can extend fewer resources with potentially more impact?
2. Are there resources available for the department to directly support this population through ongoing, department specific programming such as the EOP/Precalculus model?
3. What are normative assumptions and structures embedded within your own department that may serve to marginalize student engagement and success?

We also emphasize the need to understand who the population is on your campus that (1) is not being served by the current system, and (2) that can be impacted by such support programs. For instance, the program at CCU was designed for low-income students, Generation DSU was geared towards bolstering first-generation students, and the DU advisory committee served students of color. Each of these programs were designed to meet the needs of particular students and many now reach beyond their intended target. As such, it is imperative that for such programs to be successful that the department understand who the underserved populations are in their local setting. This may be low-income students of all races and ethnicities, or Queer students, or students of color, etc.

We close this chapter by returning to our axiom that, "*Inequities exist within mathematics programs.*" Sadly, this axiom still holds, yet we have highlighted four department efforts aimed at rectifying this issue. Some of these programs have sought to challenge inequities by supporting underserved populations through additional support programs operating within the current educational system (e.g., similar shortened courses or designated course sections). At the same time, some of these programs critique the current system by challenging normative assumptions. Examples of these program elements included: leveraging students' experiences to make direct changes, questioning the utility of developmental math course credit, providing examples of near-peer role models with shared identities, and promoting the importance of academic and social integration (Tinto, 1987).

Yet, in order to fully disprove our axiom, it is clear that mathematics departments need to be taking greater responsibility for supporting students from underserved backgrounds to thrive in STEM. The results from our national survey showed only a quarter of departments had a specific program to support marginalized students in STEM administered through the mathematics department. Our research team sought to investigate such programs, but as we have highlighted here the ways in which such programs have connections to the mathematics department were sometimes limited or administrative in nature. How can we as a community take a greater role in bolstering student success and inclusion, especially for students with marginalized identities? The answer is multifaceted and complex, but starts with acknowledging the inequity and working to take ownership within our own departments, communities, and families.

References

Carson, T. L., Aguilera, A., Brown, S. D., Peña, J., Butler, A., Dulin, A., Jonassaint, C. R., Riley, I., Vanderbom, K., Molina, K. M., & Cené, C. W. (2019). A Seat at the Table: Strategic Engagement in Service Activities for Early-Career Faculty From Underrepresented Groups in the Academy. *Academic Medicine : Journal of the Association of American Medical Colleges, 94*(8), 1089–1093. https://doi.org/10.1097/ACM.0000000000002603

DiGregorio, G. (2018). *Perceptions about math placement and pre-calculus/calculus math courses among college first-generation, low-income, students of color* (Doctoral dissertation, Colorado State University. Libraries).

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning, 4*(2), 145–187. https://doi.org/10.1207/S15327833MTL04023_4

Hagman, J. E. (2019) The Eighth Characteristic for Successful Calculus Programs: Diversity, Equity, & Inclusion Practices, PRIMUS, DOI: 10.1080/10511970.2019.1629555 Hilton, J. Open educational resources and college textbook choices: a review of research on efficacy and perceptions. *Education Tech Research Dev*, 64, 573–590 (2016). https://doi.org/10.1007/s11423-016-9434-9

Ladson-Billings, G. (2006). From the Achievement Gap to the Education Debt: Understanding Achievement in U.S. Schools. *Educational Researcher, 35*(7), 3–12.

Otero, V., Pollock, S., & Finkelstein, N. (2010). A physics department's role in preparing physics teachers: The Colorado learning assistant model. *American Journal of Physics, 78*(11), 1218–1224.

Tinto, V. (1987). *Leaving college: Rethinking the causes and cures of student attrition*. University of Chicago Press.

Voigt, M., Gehrtz, J., & Hagman, J. (2019, January). Programs to support underrepresented students in STEM and the role of mathematics departments. Presentation given at the Joint Mathematics Meetings, Baltimore, MD. Abstract retrieved from: https://jointmathematicsmeetings.org/amsmtgs/2217_abstracts/1145-q1-2635.pdf

3

Alternative Course Pathways

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Chapter Question: What are different ways that departments are offering calculus courses that support students who are most at risk for not progressing through the precalculus-calculus sequence?

3.1 Introduction

In the United States, roughly half of the students who pursue a degree in science, technology, engineering or mathematics (STEM) will ever complete it (National Center for Education Statistics, 2018). As mathematics departments across the country recognize that introductory mathematics classes are serving as a gatekeeper for student progress in STEM (Weston et al., 2019), efforts are being made to increase student success in these courses (Dunnigan & Halcrow, 2020; Wilson et al., 2012). Moreover, universities across the country are eliminating credit-bearing developmental courses and hence there is even greater need to find mechanisms to support students, especially for students with fewer opportunities for mathematical preparation. To be clear, by stating "fewer opportunities for mathematical preparation," we are explicitly calling out the current status of education in the United States and the lack of academic support and opportunities that some students receive compared to their peers.

One approach to ameliorate the gatekeeping status of introductory mathematics courses is to provide course variations that tailor the standard calculus sequence to the needs of particular students (Voigt et al., 2017; 2020). Such course variations re-envision the standard calculus sequence to support students who are less likely to be successful in the standard three semester precalculus, differential and integral calculus courses. We refer to such variations as alternative course pathways. Research on these course variations has documented the ways in which they have successfully mitigated against high failure rates among students who would be the most at-risk for failing in the standard course sequence (Vestal et al., 2015; Voigt et al., 2020; Stone-Johnstone, 2021). This approach to course variations using alternative course pathways stands in contrast to course variations that target particular disciplinary needs (see Chapter 4 for examples).

The design and implementation of alternative course pathways are often informed by the body of research related to student support systems (Johnson & Hanson, 2015). For instance, research has shown that outside support structures such as STEM centers are an effective way to enhance the teaching and learning of mathematics (Carlisle & Weaver, 2018; Biza et al., 2014). Moreover, for Black, Indigenous, and students of color, increased time spent in math learning centers has been associated with increased success and mathematical identity development. For example, Duranczyk et al. (2006) found that 84% of Black students who spent four or more hours in the mathematics learning center passed their mathematics classes and their failure rate dropped from 30% to less than 17%. One takeaway message is that increased contact hours spent working on mathematics is one factor of many (e.g., pedagogy, sense of belonging) that can improve student success.

With this in mind, some mathematics departments have created alternative course pathways that infuse additional contact hours into the course structure itself or have provided off-ramps for students to get back on track sooner. In this chapter, we highlight these alternatives to standard course sequencing, specifically those which are targeted for incoming students who have been failed by the educational system and thus are deemed as mathematically under-resourced. The first vignette describes a co-requisite course that students are automatically enrolled in targeting procedural fluency. This course is designed to support student proficiency of just-in-time material needed to succeed in calculus. The second vignette describes a stretched-out, year-long, calculus course with precalculus supports, which slows down the pacing of the material for students. The third vignette describes a calculus infused with precalculus, but does not extend the duration of the course and instead increases the number of contact hours per week. The fourth vignette describes a structure which is unique from the previous three in that the contact hours are not increased, but the Calculus courses are divided in half terms to provide opportunities for students to restart a particular class midway through the semester.

These vignettes are structured in a way in which the reader might envision the implementation of each in increasing complexity, however, every institution is unique and a later vignette may provide for an easier implementation than earlier ones. With each vignette, the reader is encouraged to think critically about the affordances and constraints of each alternative course pathway and what the enactment of each might look like at their own institution. To facilitate the reader's own reflection process, we provide two sets of discussion questions for each vignette to focus on crucial points of emphasis. Additionally, in Table 3.1 we provide brief demographic information using surveys that were administered in the Fall 2017, Spring 2018 and Fall 2018 terms. We highlight the demographic information to provide a sense of what groups of students these course variations are serving and suggest that the reader reflect on the groups of students that they are serving at their own institution.

3.2 Co-Calculus at Sandpiper University

For the first vignette, we describe a "co-calculus" model at Sandpiper University (SU) which is intended to provide students more in-class time spent on doing mathematics through a low credit co-requisite course model. Typically,

Table 3.1. Gender and Racial Demographic survey responses from course variations and their comparative standard calculus course(s).

Site	Course	Total Surveys	Racial Identity			Gender Identity		
			URM	Non-URM	Not Disclosed	Women+	Men	Not Disclosed
Canyon Crest	Calculus Infused Precalculus	58	36%	59%	5%	67%	28%	5%
	Standard Calculus 1-Calculus 2	224	22%	71%	7%	54%	41%	5%
Alpine University	Stretched-out Calculus 1	1006	9%	84%	7%	39%	57%	4%
	Standard Calculus 1	305	6%	91%	3%	61%	37%	3%
Desert Bloom	Split Calculus 1 - Calculus 2	1812	16%	77%	7%	42%	53%	5%
Sandpiper University	Calculus 1-2 w/ Co-Calculus	641	13%	76%	11%	26%	64%	9%

a co-requisite course is a one or two-credit supplement that runs in tandem with the standard course. At SU there are two co-requisite courses, one for the standard differential calculus and one for the standard integral calculus. The following vignette describes the details of each and how they are designed to support student mastery of mathematical skills necessary for calculus.

Vignette

The majority of SU first-year students have some calculus experience and are by and large prepared for a college calculus course, but there are those who need additional preparation and support to succeed in the calculus sequence. In particular, the co-calculus courses at SU were designed with the goal of shoring up students' procedural competencies in a manner that provided just-in-time review and instruction. Co-Calculus I is a 2-credit co-requisite course for Calculus I (4-credit differential calculus). The unique feature of this course variation is that all students enrolling in Calculus I are automatically enrolled in the co-calculus course as well. The automatic enrollment into the co-calculus course is in addition to the lecture and recitation portion of the course, resulting in students typically having three different instructors (one for the lecture, one for recitation, and one for the co-course).

After enrollment, students are given a gateway exam that assesses procedural fluency related to algebra, trigonometry, exponents, base-10 logarithms, and geometry. The gateway exam is administered in person to students multiple times throughout the semester. If a student earns a score of 90+ (out of 100), they can drop or shift their enrollment in the co-calculus course to pass/fail (with an automatic pass). Students that achieve a score of 90+ also have the option to stay enrolled in the co-calculus course for a letter grade and get a 2-credit A if they continue to attend sessions. Generally, students are encouraged to stay enrolled in case they want to attend sessions later in the semester. For the students whose best passing grade on the gateway exam is between 80 and 90, they receive a B in co-calculus; those who do not get 80+ will fail co-calculus. The delivery of the co-calculus course is provided by senior graduate teaching assistants (GTAs) who meet in small sections with students two times per week.

The course is intentionally structured so that the material covered in Co-Calculus I serves as background knowledge for the upcoming material in the standard Calculus I course. For instance, the main topics covered in Co-Calculus I are algebra, functions, geometry and trigonometry. Overall, there are varied perceptions of the role of co-calculus at SU. There is a mostly favorable outlook of the Co-Calculus I course from faculty as one described it as a course that provides "essential skills that [the students] need to be successful not just in calculus but in physics and other engineering courses." In contrast to Co-Calculus I, which is a credit-bearing (opt-out optional) co-requisite, Co-Calculus II is a non-credit course designed to provide support for students in Calculus II but it is not a co-requisite. Additionally, most students are not automatically enrolled in this course, nor is there currently high enrollment. The Co-Calculus II course was developed in part because students could enroll in Calculus II even if they received a D in Calculus I, so there was a known need for support. Those students with a D in Calculus I are automatically enrolled in Co-Calculus II. As an added measure to ensure that students that need support are receiving it, a Calculus II version of the gateway exam is also used - if students do not pass the exam early in the term, they are asked to enroll in Co-Calculus II regardless of their Calculus I grade.

Vignette Reflection Questions

1. What (if any) do you see as pitfalls of having a test-out option for the co-calculus model?
2. What do you imagine are the reasons for having Co-Calculus II as optional enrollment? What benefits/drawbacks would you anticipate if it was required?
3. How might a co-calculus course be envisioned as an online component?

Strengths of the Variation, Room for Growth, and Implementation Guidance

The co-calculus model at SU is specifically designed to support the mastery of mathematical procedures and help students develop proficiency with essential skills. However, the support courses are not necessarily designed to explicitly connect with the calculus materials throughout the entire semester. This is apparent in that the Co-Calculus I material shifts partway through the semester. Early in the term, while students are still able to pass out, the course functions primarily as concept review for the gateway test preparation. Afterwards, the course pivots to be more about just-in-time review of relevant material. That is, once most of the students have decided to stay enrolled in the course or have opted out of the course, then the structure of the co-calculus course is more explicit about the connection between the co-calculus material and the calculus material. The co-calculus course model is a targeted way to ensure that students are proficient with the procedural skills that most instructors expect them to already know. With that said, however, there are other valuable skills, such as modeling and conceptual understanding, that a course variation could include.

Discussion Questions

1. What are the similarities and differences between a co-calculus course focused on procedural skills and a recitation section that focuses on current calculus topics?
2. What structures are currently in place at your institution to support students who have had less exposure, fewer opportunities to learn, or are less prepared for calculus?
3. How feasible would the implementation of a co-calculus model be at your institution?

Given that students have the option to test out of Co-Calculus I if they pass the gateway exam with a score of 90 or above, most students do decide to opt-out. The reasons students give for opting out of the class vary from already having a full class load to already having a firm grasp on the prerequisite material. However, for the students that are not able to pass the gateway exam on their first few attempts, they experience a smaller and smaller class size. For some, this experience may feel marginalizing, in some way "exposing" them as low-performing students. Readers looking for further ideas around co-requisite calculus models can see Vestal et al. (2015), which describe an alternative co-requisite model and its impact on student course performance.

3.3 Stretched-out Calculus at Alpine University

Rather than the support of a corequisite course, our second vignette details a course variation that stretches out Calculus I into a two-semester long sequence. This doubling of course length is designed to slow the pace of the traditional single-semester Calculus I curriculum, which provides time to incorporate relevant precalculus material throughout the course. Students who enter this stretched-out calculus course complete two sequential semesters of Calculus I material with precalculus woven throughout, providing an alternative to the precalculus-calculus pathway to Calculus II. The following vignette describes the context in which this stretched-out calculus fits, how it is designed to help support students at AU, and perceptions of this alternate pathway from faculty and students at AU.

Vignette

AU has a large contingency of STEM-intending undergraduate students, whose rapid growth influenced the proliferation of Calculus I into two variations: the traditional semester-long Calculus I course and a year-long Calculus I course integrated with precalculus. While both Calculus I courses feed into a single semester Calculus II course, the "stretched-out calculus" course offers a slower pace compared to the standard semester-long course and infuses precalculus concepts and skills throughout. AU designed the stretched-out Calculus because they found that students who placed below calculus rarely completed the calculus sequence but still wanted students to start their university mathematics with a course called calculus.

Incoming STEM-intending students who place into Calculus are required to enroll in the standard semester-long Calculus I course and then continue to Calculus II, and beyond if necessary. Alternatively, students who place below Calculus enroll in the stretched-out calculus, and subsequently take the Calculus II course. Entrance to these pathways is based on performance on the university placement exam (ALEKS), SAT scores, ACT scores, and/or a C- or better in a prerequisite course. Despite the existence of a discipline-based variation of Calculus I designed for engineering students (see Chapter 4), the chair of the math department said in an interview that engineering students comprise about 80% of their stretched-out calculus course, partly due to the placement standards for entrance to the standard Calculus I course. However, they also said that the students who continue into Calculus II from stretched-out calculus do relatively well compared to their peers.

Considering that students are only able to enroll in stretched-out calculus if they place below Calculus, the amount of time spent on Precalculus through Calculus II material in the alternative pathway may only differ by one semester. However, lengthening a course that is commonly prerequisite to many STEM courses outside of the math department *might affect students' opportunity to enroll in courses across disciplines, which may become a hindrance to their overall course-taking trajectory. As such, this might be a disincentive for continuing in a STEM major.*

The student experience in stretched-out calculus, like many math classes, varies depending on the instructor that is teaching the course. Some instructors take advantage of the slower pace to utilize more active learning techniques in class and spend more time on particularly difficult topics. A group of students enrolled in stretched-out calculus stated that they know their class is different from the traditional calculus, but they found their confidence in mathematical ability to be improved, their beliefs around mathematics instruction had changed, and they felt at an advantage compared to their peers in the traditional calculus course. They spoke to a shared belief that students in the stretched-out calculus find the pace to be difficult enough already and that they wonder how anyone could cover the same material in a single semester. While students recognized some benefits of the stretched-out calculus, they also assumed that students in the traditional calculus I are smarter than they are, implying that students that are required to take the stretched-out calculus may feel like they are less smart due to their placement.

Additional support is offered for students in stretched-out calculus including a crew of graduate teaching assistants (TAs) that run the weekly lab sections, organize optional study sessions outside of class, and conduct review sessions for exams. The weekly lab sections are held with the same large class size as the main class session, and the coordinator for stretched-out calculus acknowledged that the TAs could run more efficient and helpful lab sessions if they were broken into smaller sections, but that the budget would not allow it. In response to this, the optional study sessions outside of class were created to provide a small-group feel where just 5-10 students would attend.

Vignette Reflection Questions

1. In what ways could a stretched-out calculus course at your institution provide the kind of support that your students would find useful?
2. Do you have a mathematics placement exam at your institution? How would you decide how to place students into a stretched-out calculus course?

Strengths of the Variation, Room for Growth, and Implementation Guidance

The pathway for students who place into stretched-out calculus provides support for students in multiple ways. First, students are presented material from precalculus in tandem with material from Calculus I, which is designed to promote making connections across essential course concepts. In this stretched-out calculus course, concepts typically found in the traditional Calculus I curriculum are covered throughout multiple stages of tracing back to essential precalculus concepts when they are most needed. This pedagogical approach aligns with Just-In-Time teaching (Novak, 2011), and reaps the benefits of a layered, spiral-style curriculum (Harden, 1999). Students in this pathway also form a cohort for at least one academic year if they continue to complete their intended courses. This cohort of students have the potential to collaborate more frequently, depend on each other in class through group work, and build community while in the extended year-long course, which all have shown considerable benefits for student success (Theil, 2006; Lugosi & Uribe, 2020).

The departmental placement policies of AU, which place a student into stretched-out calculus if they do not meet the required scores to enroll in the traditional Calculus I, also dictate that a student who places into the traditional Calculus I course may not choose to enroll in the slower paced alternative pathway. Students and faculty at AU spoke to the confines in which they felt the placement policies held their students and the lack of agency they provided, which indicates a need for policy change. Allowing students who place into Calculus I to decide for themselves if they would like to enroll in the stretched-out calculus could empower students to choose the pathway that best fits their needs.

AU had consistently low enrollment in Precalculus and poor persistence rates from Precalculus, but they also had a considerable number of students who scored just below the placement cut-off for Calculus I. The creation of the stretched-out calculus course had these particular students in mind, but also offered an alternative pathway for students who have had fewer opportunities or are less prepared for the traditional Calculus I course. Since the time of data collection, AU has actually removed the traditional Precalculus course offering from their catalog relying entirely on stretched-out calculus for precalculus material. While this is not the first time Precalculus has been cancelled at AU, the success of their implementation of stretched-out calculus demonstrates the positive impact a course variation can have on enrollment, placement, and retention issues that affect many math departments. Moreover, unlike the support course at Sandpiper University, students enrolled in stretched-out calculus are provided timely assistance in prerequisite knowledge but without attending a separate classroom and the feelings of exclusion that could potentially be associated with that style of variation.

Discussion Questions

1. How might a stretched-out calculus benefit students at your university? What potential resources would you need to develop a stretched-out calculus course for your department, and who would be involved?
2. Do the benefits of a stretched-out calculus course outweigh a possible lengthening in time to degree completion? What data would you need to collect to show that a course variation like AU's is worth the time and effort?
3. What pedagogical approaches could be well-aligned with a stretched-out calculus course?
4. What instructor characteristics are well-suited for teaching a stretched-out calculus course? Are there dispositions, identities, or beliefs they hold around instruction of mathematics that would align with more contact hours with students, a slower pace, and a spiralling curriculum?

While students at AU are likely to benefit from the stretched-out calculus variation in some way, there exist some potential drawbacks to having this stretched-out calculus option. First, students transition into the same Calculus II course regardless of their pathway leading up to that point. So, students who complete either the stretched-out calculus or the standard Calculus I will be shuffled into the same Calculus II course. This means there is potential for a splintering of that student body between those that recognize and relate to each other from engaging in their respective previous courses and the communities that were formed beforehand. Also, the traditionally fast pace of the semester-long Calculus II course has potential to serve as an unexpected increase in difficulty for students who take the stretched-out calculus who might be accustomed to a slower pace and more contact hours with their instructors. Another drawback of AU's stretched-out calculus variation is that the implementation of this variation excludes students who test into the traditional Calculus I. Thus, this approach neglects the students who test into Calculus I, but fail to meet the expected standards of assessment. These potential drawbacks have implications on student experiences and the retention of STEM-intending students, but might be mitigated through careful consideration of placement and enrollment policies, additional supports built around course variations that create and maintain student cohorts, and increased communication between students, academic advisors, and instructors.

3.4 Calculus infused with Precalculus at Canyon Crest University

The course variations implemented at AU sought to support student success by increasing the amount of contact hours by extending differential calculus into a two course sequence, and thereby increasing the time through the calculus pathway for students that did not place directly into differential calculus (at least 1.5 academic years). In contrast, Canyon Crest University created an alternative pathway that increased the amount of contact hours for each course in the calculus sequence, thereby providing support for requisite materials while maintaining the time to finish the sequence (at least one academic year).

Vignette

Canyon Crest University is a private religiously affiliated university, and the only case study site on a quarter system. CCU developed a course variation, we refer to as Calculus infused with Precalculus, whereby material from precalculus, differential calculus, and integral calculus is covered and presented in a year-long sequence. Normally this material would require students to enroll in a 4-quarter sequence, but instead it is covered in 3 quarters by increasing the amount of contact hours from four to six. The goal of the Calculus infused with precalculus at CCU was to meet the needs of students majoring in STEM who are either out of sequence (e.g., are not able to enroll in differential calculus in the fall) or need additional support learning trigonometry without requiring an additional quarter. The course was originally designed to cover the prerequisite material on an as-need basis, however, as the course has evolved it has primarily led to a sequencing revision where the course includes first the prerequisite material (trigonometric identities) followed by differential and integral calculus.

On the surface, such a course variation seems relatively straight-forward and not drastically different from the standard calculus sequence; however, it was the flexibility of having additional time and a focus on the intended audience that this course variation has successfully supported students' success in the calculus pathway. Specifically, the enactment of well-defined course logistics, cohort modeling, and increased use of evidence-based instructional practices has provided support for students that may not exist at other institutions.

Calculus infused with Precalculus was designed for STEM majors, but having a six credit-hour course for an entire academic year required the mathematics department at CCU to work with other STEM departments to determine course scheduling to avoid conflicts with laboratory sessions. As such, the course is typically scheduled in the early morning the same time each term to avoid any conflicts. Instructors described the logistics of scheduling as a self-selection feature of the course, whereby it typically attracts motivated students who are willing to attend and enroll in the course. Additionally, other STEM departments viewed the course as successful, since it largely meets the needs of the students and "stays out of the way" of their courses.

In addition to being scheduled at the same time each term, calculus infused with precalculus at CCU leverages a cohort model whereby the students and instructor are in the same section of the course for the entire year. That is, whoever the chair assigns to teach the first course in the sequence, was automatically assigned to teach each proceeding section of the course. One of the instructors commented that this helps to keep the curriculum, content, and material consistent across the terms for students; however, one of the implications is that instructors teaching this sequence have a course load assignment equating to five courses (compared to three if they were teaching the standard course). Students we spoke with had overwhelmingly positive things to say about the cohort modeling which fostered and built strong student and instructor relationships throughout the course. Because the course was designed for students who needed additional support and allowed for relationship building, many students communicated that they felt comfortable acknowledging their mathematical anxiety, felt less intimidated, and had built supportive friendships.

The additional contact time in the course allowed the instructors to both expand on topics through lecture and utilize evidence-based instructional practices like group work, interactive quizzing, extended whole-class discussions and active learning. Students were receptive to these strategies and often pointed to them as helpful for learning the mathematical concepts as well as developing a sense of community in the class. In particular, students conveyed that the additional time and slower pace allowed for them to really understand the concepts and ask questions. One student summarized their experience stating that, "I really appreciate the amount of examples of all the different types of problems we will encounter that the instructor provides for us in class. All the handouts are helpful as well. Instead of vigorously taking notes, we are able to pay attention to the process."

Vignette Reflection Questions

1. Calculus infused with precalculus at CCU is based on a trimester system. What constraints do you imagine if this course variation was implemented at a university on a semester system?
2. Calculus infused with precalculus features increased contact time, cohort modeling, and evidence-based teaching practices. What features do you consider the most impactful on student success?
3. Co-calculus at SU features increased contact time through a separate course, whereas Calc infused with Precalculus integrates the time into the existing course. What do you view as the benefits/drawbacks of each model?

Strengths of the Variation, Room for Growth, and Implementation Guidance

Calculus Infused with Precalculus is largely perceived as successful at CCU, and in fact, represented some of the highest ratings of student affect (confidence, interest, enjoyment) and positive climate among all of the course variations in our sample. Furthermore, historical DFW rates since 2001 suggest this course variation is helping support student success even compared to the standard course (18.8% compared to 25.6% DFW). Yet there are also potential limitations with the way the course is currently designed. For example, the course by design is attracting motivated students through early morning scheduling, and thus the positive results may be attributed to student self-selection. As such the course is not widely available to all students who may have personal scheduling conflicts (e.g., childcare, work). Additionally, requiring students to enroll in courses with higher-contact hours can place a larger financial burden on students and may prohibit them from taking a full course load. Lastly, this course variation could go further in re-imagining the sequencing of material and the necessity of particular topics. For instance, although the course was originally designed to feature the calculus material while presenting the prerequisite material in parallel, this was soon scrapped for a more traditional sequencing of topics. One of the long-term instructors said that they had initially tried a more parallel presentation of the precalculus material (trigonometric functions) in the first part of the day and then presenting limits; however, they said this required a lot of cognitive processing and students felt overwhelmed. In Chapter 4, we discuss how a discipline-based variation at MSU re-sequenced the topics in calculus with the aid of mathematics educators. Perhaps a more purposeful re-sequencing of course topics with support from design experts would enable calculus infused with precalculus to be further re-imagined.

Discussion Questions

1. Although the logistics of scheduling seem like a minor feature of the course, they have a large impact on the types of student enrollment and how other departments view its success. How could such concerns be mitigated in scheduling the course while making it more broadly available?
2. Calculus Infused with Precalculus at CCU supports students to understand the prerequisite material such as trigonometric functions, but many are questioning the utility of including such topics for all STEM majors. What course topics might be removed or resequenced to improve such an alternative course pathway?
3. If you wanted to implement a similar course variation at your institution what barriers and what enthusiasm do you anticipate from faculty, registrar, and students?

Although more time on task in and of itself may not always improve student learning outcomes, the features and implementation of Calculus infused with Precalculus at CCU appears to be an effective model. The increased time allows for relationship building and social and academic integration, which is a predictive aspect of students' retention (Tinto, 1975; 1993). Furthermore, the use of cohort groupings throughout the year was not only impactful for CCU but research indicates it supports adult learners (Drago-Severson, 2001) and STEM learners with marginalized identities (Bouniaev, 2014). We encourage readers to consider if there are ways in which their current course designs could incorporate cohort models without needing to adjust the contact hours or curriculum. Designing courses using a cohort model also does not need to be limited to the mathematics sequence and can extend to include other introductory STEM courses so that students are forming connections across discipline settings. Logistically this is not an easy task but could have a large impact on student learning outcomes and well-being.

3.5 Splitting Semester-Long Courses into Half-Semester Courses at Desert Bloom University

The first three vignettes provided insights and various strategies for increasing the time that students have to learn course content. These approaches included the addition of a co-course, a stretched-out calculus over two semesters, and the addition of more contact hours during the semester (and hence more credits) in order to provide precalculus content support. Our last vignette takes a different approach in that it does not alter the amount of time allocated for learning the material, but instead restructures the 15-week term into two 7-week terms. In particular, Desert Bloom University (DBU) adds an option to take a 7-week precalculus course and restructures for all students the semester long differential and integral calculus courses to four, half-semester courses (two for differential calculus and two for integral calculus).

Vignette

As is the case at most universities, the mathematics department at DBU was teaching precalculus and differential and integral calculus in semester long courses. DBU, like many other universities, was experiencing low student success rates and an increasingly unhappy engineering department. This ultimately led to a crisis in which the Provost stepped in and convened a task force to decide what to do, including the possibility that the engineering department take over the teaching of calculus. After much discussion and consideration, the task force decided to keep calculus in the mathematics department, but only on the condition that there be a massive overhaul of the courses. In particular, the decision (made mostly by the Provost) was that the mathematics department hire a non-tenure track Director of

Calculus from outside the university who would be responsible for reimagining their calculus program. This reimagining involved the creation and implementation of half-term calculus courses which were designed with the goal of providing students who do poorly in the first part of course an early opportunity to repeat the first half and hence stay on track with their cohort with less impact on their GPA.

The mathematics department was able to successfully hire someone from the outside to take on the role of Director of Calculus on the condition from the new hire that the university would offer a half semester precalculus course (in addition to the semester long course) and totally restructure the two, one-semester calculus courses, into four, half-semester courses. This is an approach that the new Director had successfully implemented at his previous institution and they convinced the Provost and department of the efficacy of this model. Thus, with the support (and at times pressure) from the Provost, the scheduling, enrollment, and placement offices made this happen.

After a year of putting the policies and practices in place, a half semester version of precalculus, two half semester differential calculus courses, and two half semester integral calculus courses were put in place. Another innovation that was rolled out with the 7-week course pathway was the use of videos for students to watch before coming to class, which offloaded some of the more procedural content so that instructors could incorporate more active learning focused on concepts. All courses are 7 weeks and scheduled in pairs to fit into the regular semester system. For calculus, students enroll in a pair of courses, making sure to sign up for the same section number of each, and thus they will have the same instructor at the same class time for both. However, after 7 weeks a grade is issued and students who do not pass the first-half course will be dropped from the second-half course. They can then retake the course they just failed (there is always a backup scheduled at the same time). In addition, the second 7-week differential calculus course is also offered over the winter break, so that students who either started in precalculus or who had to retake the first 7-week calculus course can stay on track with the engineering courses where calculus is a prerequisite and they can enroll in the integral calculus sequence in the spring. Three potential pathways for students enrolling into integral calculus in the spring are presented in Figure 3.1.

Some of the main strengths of the DBU mathematics program revolve around the half-term structure of the standard calculus sequence, which allows for a more nuanced placement into the sequence because students can place into a wider range of courses, and it allows students multiple opportunities for students who do not pass one of the courses to catch-up and thus not fall behind in other courses that require calculus. In addition, failing a half-semester course is less impactful on a student's GPA than failing a full semester course (as each half semester counts for two credits). While this can also help with students' confidence and belief that they can complete the required calculus sequence, it also may introduce additional high-stakes exam anxiety.

Vignette Reflection Questions

1. In what ways would a half semester system, like DBU's, improve or hinder the current calculus structure at your own institution?
2. The Provost played a large role in supporting a split-term course offering at DBU, how important is it for you to have the support of upper administration? How might you garner that support?

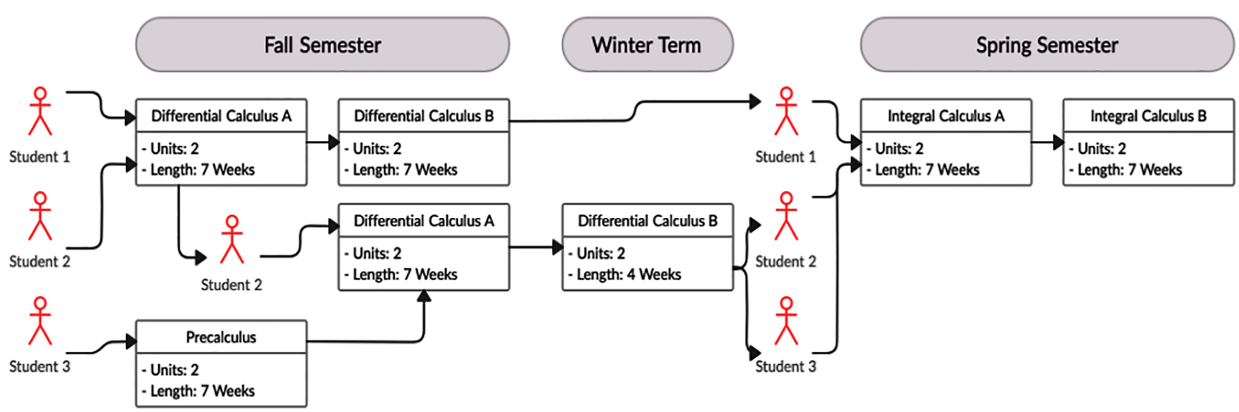

Figure 3.1. Flow diagram of three different trajectories into Integral Calculus in the spring.

3. From a student perspective, in what ways might the two-semester calculus structure be experienced similarly and differently than a semester long version?

Strengths of the Variation, Room for Growth, and Implementation Guidance

The half-semester structure has resulted in quite favorable results in terms of student success through the calculus sequence and improved GPA. The engineering department is also much happier with the changes and resulting improvement on student success. Upper administration is also much happier with the changes and feel that the mathematics department stepped up and addressed the concerns. The hiring of an outside person to be Director of Calculus has been a big success. In addition, the experience for students is also fairly seamless. In our student focus groups we heard many students expressed the view that the half-semester structure did not have a negative impact on their course experience.

Finally, DBU is rethinking the role of the full semester precalculus course. For example, there is debate as to whether this course is intended to be a terminal course or whether it is part of the STEM course pipeline. If the course is intended as an option for STEM majors, an area for continued improvement is to review alignment of course content with the half-semester calculus courses so that students entering calculus from the semester-long precalculus are well-prepared and can pursue STEM majors if they choose. This alignment could include both content alignment as well as pedagogical alignment.

Discussion Questions

1. What student success data would convince you that the half-semester precalculus and calculus structure is worth considering at your institution?
2. What challenges do you anticipate your department would encounter in implementing the half-semester model and how might you mitigate these challenges?
3. What venues and strategies might you use to inform students about the advantages and options that the half-semester affords them?

Certainly the move from a semester structure for calculus to a half-semester structure requires extensive coordination with scheduling and placement offices. For example, DBU needed to put in place rapid mid-semester transitions: exams, grading, and course placement adjustment all required rethinking. The fact that the Provost at DBU was fully behind this change provided useful leverage and top-down pressure to make this happen, and quickly at that. While having such a hands-on Provost does not seem to be an absolute necessity, it does seem likely that garnering upper administrators support for restructuring the calculus sequence would be necessary. Based on the student success data from DBU, which includes lower DFW rates, higher GPAs, and engineering students staying on schedule, the effort to switch to a half-semester course structure is worth considering. The switch to the half-semester calculus structure continues to be well-liked by engineering. They are no longer complaining to upper administration and no longer encouraging their students to take calculus somewhere other than DBU. This is another big marker of success for DBU's restructured calculus sequence.

3.6 Conclusion

For a variety of social, historical, and economic factors, some students enter introductory university mathematics with fewer opportunities for mathematical preparation. In this chapter, we highlighted four alternative course pathways intentionally designed to support these students. The ways in which these support structures are designed are unique, each with coinciding strengths and room for growth. In Figure 3.2 we document the four alternative course pathways to aid the reader in comparing and contrasting each with respect to the academic year (large arrow).

For Sandpiper University, the desired structure is a co-requisite model where students are automatically enrolled into a co-calculus course and must score high enough on a gateway exam in order to opt out of the corequisite class. For the students that do not initially pass the gateway exam or decide to stay enrolled in the co-calculus course, the co-calculus course is structured so that the students receive just-in-time instruction to help them understand the topics

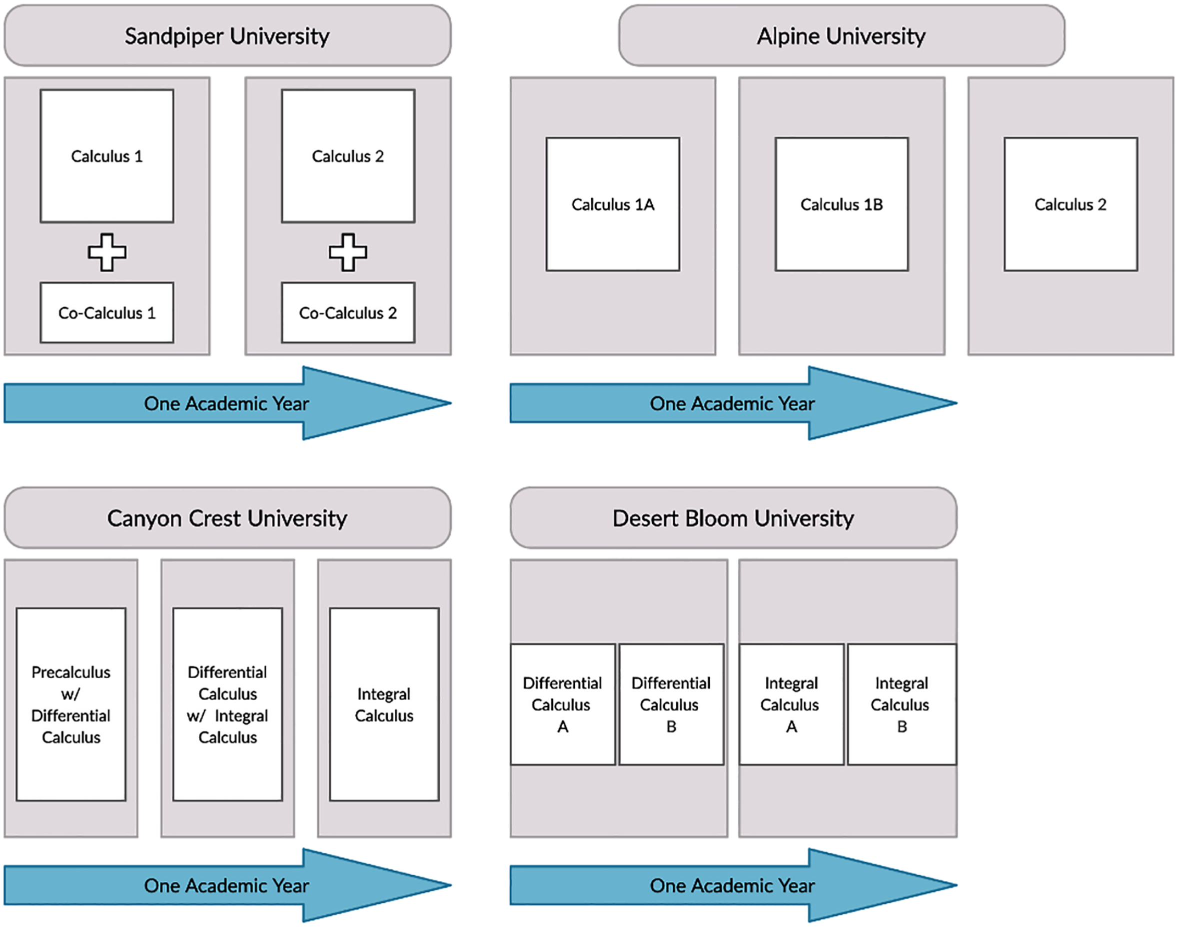

Figure 3.2. Four Course Variations for Students with Fewer Opportunities for Mathematical Preparation.

covered in their calculus class. This structure is intended to ensure that all students have a baseline of procedural mastery with respect to topics traditionally covered in a precalculus course. At Alpine University, for entering students that place below the requirement to enter Calculus I, their first course in the calculus sequence is a calculus infused with precalculus course that is stretched out across two semesters. Having a stretched-out calculus course allows for more time to cover the topics of differential calculus and allows the students to proceed over the course of one academic year with the same group of students. As an alternative measure to ensure more time with the material, Canyon Crest University increased the number of contact hours for each course in the sequence. This method increases the number of contact hours from four to six hours per quarter, but does not extend the length of the course across multiple semesters. In addition, CCU employs a cohort model for instruction where the students are able to finish the entire precalculus-calculus sequence with the same set of students and instructors in one academic year. The fourth model described in this chapter utilized by Desert Bloom University involves splitting each precalculus-calculus course in half to allow students who do not pass an opportunity to catch back up with the cohort they started the academic year with. This structure allows students to get back on track faster while still maintaining enrollment in outside courses that may require calculus as a prerequisite.

3.7 Discussion

In the presentation of the aforementioned course variations, we see that each has a unique set of affordances related to supporting and helping students succeed in calculus. We encourage readers to reflect on the current structures and systems in place at your own institution to better understand how students with fewer opportunities for mathematical preparation are being supported. We would like to highlight that this focus is much different than the traditional course variation that one might naturally think of, such as Bio-Calculus or Engineering-Calculus. These varieties are known as content course variations and will be discussed in the following chapter. To summarize and conclude this chapter, we analyze the four alternative course pathways from the perspective of the following three cross-cutting themes in order to further encourage the reader in thinking about what it means to implement and sustain a course variation: Logistical considerations, Content and instruction, and Student experience.

Logistical Considerations

When it comes to implementing a course variation, there are many factors that one must consider in order to set students up for success. As such, each one of the four variations presented in this chapter come with their own unique set of challenges in terms of staffing, scheduling and finding the appropriate resources to sustain any changes being made. For instance, both SU and DBU must consider issues regarding staffing and scheduling of the appropriate classes. For SU, the right number of corequisite courses must be staffed and the times of the co-calculus courses must not interfere with either the lab portion or with the lecture portion of the course. At DBU, half semester courses must be scheduled in the (likely) event that some students must retake the first half of the calculus course that they have failed. Moreover, clear communication and scheduling with the registrar's office must be in place in order to create a system in which half semesters are possible when the rest of the university is structured on a standard semester system.

As for AU and CCU, the logistics in maintaining a cohort model are not always straightforward. Instructors must be available to teach the duration of the academic year and must be open to teaching more credit hours per week in the case of CCU. In addition, scheduling the proper enrollment procedures for the students must be planned ahead of time to ensure that students stay with the same group of students that they took the class with the previous semester or quarter. The affordances and constraints at your institution may provide for easier implementation of some course variation over another, but just establishing an alternative structure of the course, increased contact hours, or a cohort model does not guarantee increased student success.

Content and Instruction

While an alternative pathway is one way to provide an opportunity for students with historically fewer opportunities for mathematical preparation, it is important to recognize that the structure of a course variation is likely not a cure-all nor a sufficient change. We also need to consider a variety of other features such as the content being covered in class, the goals of the instructors, time on task, the use of evidence based teaching practices, coordinated class structure, professional development of instructors, and other important factors. In the case of SU, some of the GTAs teaching the co-calculus courses had the perception that the course was only in place to prepare the students to test out of the course by passing the gateway exam. At DBU they took advantage of the restructuring to also incorporate the use of videos that students view before coming to class and more use of active learning. A take away here is that the creation of an alternative course pathway is also an opportunity to infuse other features and characteristics of successful calculus courses (e.g., Bressoud, Mesa, & Rasmussen, 2015).

An additional consideration is the messaging of the alternative course pathways for students. In the case of DBU, the half-semester system was designed to support the students, but that point was not always clear for some students. Similarly, some of the students at SU saw the co-calculus course as a hurdle that they needed to get passed instead of a course that was there to support them and help them be successful in calculus. Integrating consistent messaging to help students understand the overall purpose for an alternative pathway in conjunction with evidence-based teaching practices and thoughtful enactment of the course itself is important for providing a high-quality experience in the precalculus-calculus sequence.

Student Experience

Given that the purpose of an alternative course pathway is to support students that might need extra assistance and guidance for succeeding in university mathematics, it is important not to lose sight of the overall student experience. For example, one question the reader might want to ask themselves is this: in what ways do some course variations lead to more exclusionary practices for students that are not scoring as high as some of their peers? For example, for the students at SU that were not able to pass their gateway exam on the first few attempts, they saw a dramatic decrease in enrollment from the co-calculus course. This might lead to feelings of alienation and imposter syndrome (i.e., the feeling of not belonging or having the "right" skillset to fit it). In the case of the alternative pathway at AU, students that placed into precalculus were not given the option of taking the stretched-out calculus class because they did not place low enough. For some students, this might present itself as a problem because they might prefer a slower pace, allowing them to adjust to life as a university student.

In addition, we encourage readers to reflect on how marginalized students are most at risk in the standard precalculus-calculus sequence. What systems are in place to support these students and how might the enactment of an alternative course pathway be helpful in helping these students succeed? One example in particular could be seen by AU and CCU in their enactments of course variations that employ a cohort model to help students develop a strong mathematical identity. As a larger takeaway, we suggest that critical analysis of what it means to change a system for all students versus changing the system for a specific group of students needs to be considered.

References

Axtell, M. (2006). A Two-Semester Precalculus/Calculus I Sequence: A Case Study. *Mathematics & Computer Education, 40*(2).

Bouniaev, M., Edinbarough, I., & Elliott, B. (2014). Lessons Learned in Establishing STEM Student Cohorts at a Border University and the Effect on Student Retention and Success. *121st ASEE Conference*.

Biza, I., Jaworski, B., & Hemmi, K. (2014). Communities in university mathematics. *Research in Mathematics Education, 16*(2), 161–176. https://doi.org/10.1080/14794802.2014.918351

Carlisle, D. L., & Weaver, G. C. (2018). STEM education centers: catalyzing the improvement of undergraduate STEM education. *International journal of STEM education, 5*(1), 47.

Drago-Severson, E., Helsing, D., Kegan, R., Popp, N., Broderick, M., & Portnow, K. (2001). The power of a cohort and of collaborative groups. *Focus on Basics, 5*(2), 15–22.

Dunnigan, G., & Halcrow, C. (2020). If You Don't Build It, They Will Leave: Reforming an Applied Calculus Course by Eliminating Large Lectures and Incorporating Active Learning. *PRIMUS*, 1–24.

Duranczyk, I. M., Goff, E., & Opitz, D. L. (2006). Students' experiences in learning centers: Socioeconomic factors, grades, and perceptions of the Math Center. *Journal of College Reading and Learning, 36*(2), 39-49.

Harden R, Stamper N. (1999). What is a spiral curriculum? *Medical Teacher, 21*(2), 141–143.

Herrick, R. J., Jacob, J. M., & Richardson, J. J. (2003, June). Retention through a coordinated spiral curriculum. *In 2003 Annual Conference*.

Johnson, E., & Hanson, K. (2015). *Academic and social supports*. In D. Bressoud, V. Mesa, & C. Rasmussen (Eds.), Insights and recommendations from the MAA national study of college calculus (pp. 69–82). The Mathematical Association of America.

Lugosi, E., & Uribe, G. (2020). Active learning strategies with positive effects on students' achievements in undergraduate mathematics education. *International Journal of Mathematical Education in Science and Technology*, 1-22.

National Center for Education Statistics (2018). Bachelor's, master's, and doctor's degrees conferred by postsecondary institutions, by sex of student and discipline division: 2016–17. Retrieved from https://nces.ed.gov/programs/digest/d14/tables/dt14_318.30.asp.

Pilgrim, M. E., & Gehrtz, J. (2016). An alternative Calculus I course. *Mathematics and Computer Education, 50*(2), 120.

Seymour, E., & Hewitt, N. M. (1997). *Talking about leaving: Why undergraduates leave the sciences*. Westview.

Thiel, J. (2006). More than just math: A study of collaboration and community building in an undergraduate math classroom (Doctoral dissertation, Capella University).

Tinto, V. (1975). Dropout from higher education: A theoretical synthesis of recent research. *Review of Educational Research, 45*(1), 89–125.

Tinto, V. (1993). *Leaving college: Rethinking the causes and cures of student attrition*, (2nd ed.). Chicago: The University of Chicago Press.

Vestal, S. S., Brandenburger, T., & Furth, A. (2015). Improving student success in Calculus I using a co-requisite Calculus I Lab. *PRIMUS, 25*(4), 381–387.

Voigt, M., Apkarian, N., & Rasmussen, C. (2017). Diverging from the standard fare: Variations in the calculus curriculum. *MAA FOCUS, 37*(1), 32–34.

Voigt, M., Apkarian, N., Rasmussen, C., & Progress through Calculus Team. (2020). Undergraduate course variations in precalculus through Calculus 2. *International Journal of Mathematical Education in Science and Technology, 51*(6), 858–875. https://doi.org/10.1080/0020739X.2019.1636148

Weston, T. J., Seymour, E., Koch, A. K., & Drake, B. M. (2019). *Weed-out classes and their consequences*. In E. Seymour & AB. Hunter (Eds.), Talking about leaving revisited (pp. 197–243). Springer. https://doi.org/10.1007/978-3-030-25304-2

Wilson, Z. S., Holmes, L., Degravelles, K., Sylvain, M. R., Batiste, L., Johnson, M., ... & Warner, I. M. (2012). Hierarchical mentoring: A transformative strategy for improving diversity and retention in undergraduate STEM disciplines. *Journal of Science Education and Technology, 21*(1), 148–156.

4

Variations in Approaches to the Content of the Calculus Sequence

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Chapter Question: What are ways in which a department can supplement, modify, or restructure the content in its calculus sequence to increase student interest, improve comprehension, and/or align more effectively with the needs of partner disciplines?

4.1 Introduction

For decades, mathematics departments have experimented with the ways in which the content of the calculus sequence is presented. This has included the choice of when to start using transcendental functions, the emphasis placed on precise definitions, the time spent studying limits, the choice of theorems to emphasize and which to actually prove. In addition, departments are increasingly sensitive to student complaints that they do not see the relevance of this course to their own academic interests (Seymour & Hewitt, 1997; Seymour & Hunter, 2019). One response has been to restructure the course so that students see from the outset how powerful a tool it can be for modeling a variety of dynamical systems (Callahan et al. 1994, Flath et al. 2013, Bressoud 2018). A more common and less radical response has been to enhance sections of the course with examples directly drawn from one or more of client disciplines such as Biology, Engineering, Chemistry (Voigt et al., 2020). Voigt and colleagues (2020) documented how calculus courses tailored for a particular discipline (e.g. Calculus for Biosciences, Calculus for Engineering) had lower Drop, Fail and Withdraw (DFW) rates compared to standard calculus courses across the United States. In addition, many mathematics departments experience pressure from these other departments to restructure calculus so that the topics and their timing better align with the needs of other courses.

In this chapter we present four vignettes that describe efforts at different universities to implement a course variation that varies in the approach to the content of the calculus sequence to specifically support STEM-intending students. The first vignette offers an account of Dunshire University's course in which a research mathematician highlights their research area during an enhanced laboratory section once a week. This variation was designed to help students build a connection with mathematics research faculty and to gain a broad understanding of the discipline itself. The second vignette describes River Rock University's engineering-based variation, which offers a separate calculus sequence for engineering majors. This sequence was designed to decrease the amount of time spent on mathematics prerequisites before starting engineering coursework and to include more engineering applications. The third vignette describes Tree Line University's physics-based variation of a calculus course that was created in collaboration with the physics department. The goal of this sequence is to coordinate the instruction of both the Calculus and Physics content. The last vignette details Maple State University's restructuring of the course to support mathematics and science majors' meaning-making and conceptual understanding of calculus concepts.

We offer these vignettes to prompt the reader to think critically about re-envisioning the content of the calculus sequence. In what follows, we discuss the four efforts to implement variations of the usual content in the calculus sequence, either supplementing, modifying, or restructuring it for STEM-intending students. For each example, we first describe the variation in the form of a vignette, then engage the reader in a critical examination of the features of the variation, and conclude by connecting the ideas that emerged from the vignette to relevant literature. We close the chapter with a discussion of ideas to consider when aiming to make content-based changes in their calculus sequence, including: (1) promoting diversity, equity, and inclusion, (2) building evidence-based instructional practices into design, and (3) engaging students in authentic discipline-based practices.

4.2 Enrichment Lab Sessions with Research Mathematician at Dunshire University

The first vignette describes a relatively straightforward way to incorporate a richer experience of mathematics into the calculus course. A more comprehensive investigation of the enactment and student experience in the Enhanced Lab Sessions is presented in Tremaine (2021). Here we consider efforts to expose first-year students to the mathematics discipline by offering the students examples of mathematical research in their lab section.

Vignette

Dunshire University is a private university with very high research activity and a large contingent of research mathematicians. The majority of incoming students have some AP credit in mathematics and thus are typically placed into Calculus 2 or higher. Calculus 2 courses were usually in a lecture + lab format in which the students attended a lecture session three times per week and a lab session twice a week. The lab sessions were initially designed to engage students in small group work in which the students collaboratively solved problems and produced reports describing their solutions and strategies. The lecture sections were exclusively taught by professors of practice, lecturers, and experienced graduate students and the lab sections were typically taught by graduate or undergraduate students.

During the 2017–2018 academic year, there was a campus-wide initiative to enrich students' experiences in first-year courses. Part of this initiative grew out of a desire to improve instruction and to increase clarity around grading. The mathematics department responded by piloting a Calculus 2 course variation in which they reduced the groupwork lab component to once a week and introduced a weekly enrichment lab session led by a research mathematician. The research mathematician was a permanent feature of the course as a weekly lecturer, rather than a rotating guest speaker. The primary goals of the enrichment lab were to (a) connect first-year students with research mathematicians, (b) expose students to interesting and cutting-edge applications, and (c) support students in gaining an understanding of what it means to be a mathematician and to conduct research.

In the spring 2018, the mathematics department piloted the enrichment lab in one Calculus 2 section. The research mathematician assigned students to watch several ten-minute videos prior to attending the enrichment lab. In the videos, the research mathematician introduced basic ideas on various topics related to their expertise (e.g., gerrymandering or black holes, see Figure 4.1) that would be further covered in the following enrichment lab session. At the beginning of the lab, the students took a short, open-note quiz on the material covered in the videos or previous lab. Then the lab continued with more in-depth presentation and whole-class discussion of the topics covered in the videos. The research mathematician noted that the quizzes were designed to build the students' confidence in their ability to understand the applications, potentially boost the students' course grades, and motivate the students to learn the material and engage in whole-class discussions during the enrichment lab.

The research mathematician whom we interviewed explained that the enrichment lab was not directly connected to the regular course sessions, except for their contribution to the course grade (the students' enrichment lab quiz grades were part of their overall course grade). The enrichment lab topics were related to calculus concepts but were

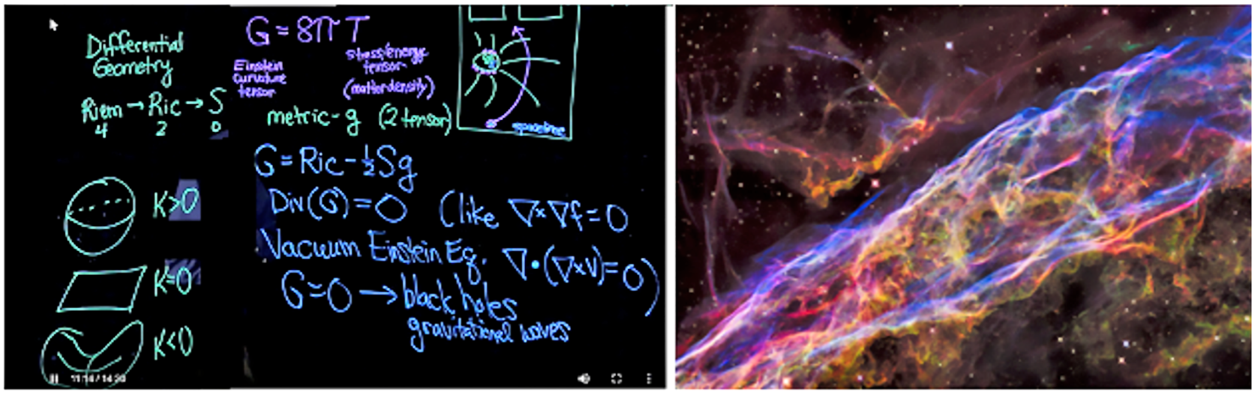

Figure 4.1. Example video of black holes and differential geometry.

not necessarily tied to the concepts that the students were concurrently learning in the lecture sessions. The person we interviewed noted that this design aspect allowed research mathematicians the freedom to share their own areas of expertise. Additionally, this division allowed the lab to be a simple and practical way to enhance the course. The instructor of the primary course (who was a professor of practice) explained that the enrichment lab sessions have impacted the way that they think about the instruction of the course. In particular, the instructor we interviewed expressed the need to address what the students might be missing out on from no longer attending the twice-a-week groupwork lab sessions. For instance, they expressed that they felt the need to fit content that was previously covered in the groupwork lab session into the lecture sessions and incorporate more active learning and problem-solving into the lecture sessions. They explained:

> *I am definitely concerned because we excised the lab material and that's a portion of the class that was about solving a lot of project problems and working in groups and making class time an active time. So I didn't want to lose the active learning component and so I have been thinking about trying to fit that into the regular class schedule.*

The enrichment lab seemed to be perceived as a success by the research mathematician, the primary instructor, and the students. Additionally, at the end of our data collection, the department had plans to scale up the enrichment lab design to the other Calculus 2 sections. According to the instructor, the student feedback was mostly positive and the students reported that the enrichment lab was interesting and not too challenging. The instructor also offered two critiques that students shared: (1) some students who had already chosen a different major than mathematics viewed the enrichment as irrelevant to their interests, and (2) the enrichment lab required additional work that some students did not see as contributing to their overall understanding of Calculus 2 concepts.

Vignette Reflection Questions

1. What challenges do you expect DU to experience when scaling up the enrichment lab and how might these challenges be overcome?
2. The instructor noted that they were concerned with what the students might be losing by not having the twice-a-week lab that focused on problem-solving. How could the enrichment lab design be adjusted to include experiences similar to those in the groupwork labs?
3. Suppose the enrichment lab was designed to avoid the disconnect with the rest of the course. How might a research mathematician make their work accessible and connected to introductory calculus content while still feeling authentic? What would be gained from such an approach?

Strengths of the Variation, Room for Growth, and Implementation Guidance

DU's mathematics department has taken steps toward building connections between first-year students and research mathematics faculty by offering their enrichment lab. The department plans to build on these efforts by scaling up to all of their Calculus 2 sections. We encourage the reader who is considering adding an enrichment lab with an expert from a discipline to think critically about the implications of this design. In particular, at DU the enrichment lab sessions seemed to be at the expense of critical face-to-face time that could be used to engage students in relevant Calculus 2 concepts and active learning. Additionally, bringing an expert from a discipline into a first-year mathematics course sends messages to students regarding who is representative, and potentially, who belongs in the discipline.

Discussion Questions

Suppose that you wanted to bring an expert from a particular discipline into your own university's first-year mathematics course in order to introduce the discipline and its practices to the students.

1. What could be some strategies to support connections between the first-year students and the research faculty *while* actively engaging students in learning about the course content? What additional resources would you need to attend to these goals?
2. Bringing an expert from a particular discipline into first-year mathematics courses situates the expert as representative of the discipline. What should be considered when selecting a faculty member to fulfill this role? And how might this choice influence the ways that students envision themselves as future members of the discipline?

The enrichment lab at DU was designed to support students to connect with research faculty, expose students to cutting-edge research related to calculus, and support students in gaining a broad understanding of mathematics and mathematics research. Even though the enrichment lab is perceived as successful by faculty and students, it is disconnected from the course itself. While there were some benefits of this disconnect (e.g., the implementation was relatively straightforward), there are also potential dangers. For instance, active learning was an important aspect of the old lab-model, but at the time of our visit it was not an explicit goal of the enrichment lab. This was alarming to the primary instructor, causing them to feel pressure to incorporate this goal into the lecture sessions. The literature on team-based teaching suggests that the instructors of the course (and the faculty involved with designing this variation) should establish and clarify shared goals and objectives (Letterman & Dugan, 2004; Thousand et al., 2006). These goals and objectives could involve pedagogy (e.g., engage students in active learning) and/or content (e.g., coordinate topics so that the enrichment lab enhances and reinforces lecture content). We view this as an opportunity for collaboration that can create synergy between the sessions with the primary instructor and the sessions with the discipline expert.

Additionally, we also want to raise awareness of the importance of who is selected to be represented as an expert in the field. Given the overrepresentation of white men in STEM fields, this is an opportunity to select an expert who would challenge normative assumptions of whiteness and masculinity in mathematics or other STEM disciplines. Students' sense of belonging is an important factor in their pursuit of a STEM major (Rainey et al., 2018). Additionally, students', and especially students' with marginalized identities in STEM, benefit from faculty role models with shared identity markers, leading to positive impacts on their persistence in STEM (Gladstone & Cimpian, 2020; Price, 2010; Klopfenstein, 2005; Ko et al., 2020). This type of variation to the calculus content can be implemented not only as a way to support students in gaining a broad understanding of the discipline, but also as a way to challenge normative assumptions about who belongs in STEM disciplines.

4.3 Engineering Calculus at River Rock University

In our first vignette we considered a course variation which involved supplemental content that made connections to cutting-edge research that draws on calculus. Our second vignette expands on the idea of connecting the calculus course to the discipline at a higher cost in terms of resources. Here we consider the creation and maintenance of a calculus sequence that is specifically designed for students majoring in engineering. A more comprehensive investigation of the creation and maintenance of this program is presented in Ellis et al. (2021).

Vignette

River Rock University created an engineering calculus sequence in response to pressure from the college of engineering. The two primary goals of the engineering calculus sequence were to 1) decrease the amount of time students needed to complete their mathematics prerequisites before starting engineering coursework and 2) include more engineering applications to better serve engineering majors.

In order to address the first goal, the three-course calculus sequence was compressed into a two-course sequence. As part of this, topics were reordered (e.g., transcendentals were moved to early in the sequence) with some topics removed from the calculus sequence and placed in later courses (e.g., vector calculus was moved into the partial differential equations course). This resulted in an automatic one-semester decrease in the amount of time engineering students needed to complete their mathematics coursework. As will be discussed in Chapter 6, River Rock is closely monitoring local data (e.g., pass rates, exam performance, and persistence) in order to ensure that this reduction in time is not accompanied by a corresponding increase in attrition.

When the engineering calculus sequence was first created, engineering faculty contributed application problems to be used in the lab components of the courses. However, these were found to be too laden with difficult jargon and involved complicated physics concepts. Currently the course coordinator works with the GTAs to transform "story problems" from the calculus textbook. These problems lack the complexity of authentic applications but do broadly hint at the ways that calculus can be used in engineering and other disciplines. Examples of such problems include calculating the rate of change of the length of a person's shadow as they move toward a light pole, or computing the maximum volume of a box made from folding a piece of paper after cutting squares from the corners. According to the course coordinator, he and the GTAs "sort of break [the story problems] down and deconstruct them a bit and usually flesh them out a bit more so we give [the students] more steps to follow and, in some cases, we go well beyond what the book would do." So, while instructors at RRU do try to take advantage of opportunities to do in-depth applications (e.g., they were developing a lab that investigated the multivariate chain rule in the context of neural networks), most of the applications are best categorized as "story problems on steroids".

The creation of the engineering calculus sequence at RRU was a significant endeavor involving substantial resources. The university approved the hiring of a full-time career line faculty member to create and coordinate the engineering calculus sequence and provided additional funding to allow GTAs to begin their careers at RRU running lab sections (as opposed to teaching their own courses). These resources continue to be deployed to support the maintenance of the engineering calculus sequence. For example, the course coordinator collects extensive data to support the continued improvement of the course sequence and he shares his results in regular meetings with faculty from the college of engineering.

Vignette Reflection Questions

1. What topics in the "standard" calculus sequence do you think could be eliminated in order to create a shorter course sequence for engineers? What would be the best approach to figuring out an appropriate topic outline for such a sequence?
2. How could/should a client discipline be involved in creating a calculus sequence designed to serve their majors?
3. What kinds of resources do you think River Rock University needed to make it possible to create and maintain the new engineering calculus sequence?

Strengths of the Variation, Room for Growth, and Implementation Guidance

The engineering calculus sequence at River Rock University is quite highly regarded by the mathematics department, the college of engineering, and the administration. It is seen as both a success and an example of a successful interdepartmental collaboration. Furthermore, another measure of success as documented in Ellis et al. (2021) is that Engineering Calculus students had similar pass rates compared to the standard Calculus, and reported a friendlier classroom climate and a belief that course activities were more relevant to their life and future work, However, one could reasonably argue that the sequence does not really meet the goal of engaging students in authentic engineering applications. The attempt to use authentic applications was abandoned because such applications tended to involve extensive jargon and complex concepts that students would not have access to prior to completing Physics and starting their engineering coursework.

Discussion Questions

1. Rather than listening to a researcher describe real applications of calculus (as students in the DU course do), students in the RRU engineering calculus course actively solve application inspired problems. What are the relative strengths and weaknesses of these two approaches?

2. Suppose you wanted to use some authentic (or at least more authentic) applications in an engineering calculus sequence at your university. What challenges would you expect to arise (in addition to those already mentioned)?
3. How could the challenges related to using authentic applications be overcome? What kind of resources would be necessary to do so sustainably?
4. What would be some benefits of using authentic engineering applications in an engineering calculus sequence?

River Rock University created and is maintaining an engineering calculus sequence that is viewed very favorably by key stakeholders. Considerable resources were provided to facilitate both the creation and the ongoing maintenance and monitoring of the engineering sequence. These resources seem to have been crucial to the success of the sequence. Indeed, we observed that similar efforts have failed in the absence of such resources (Ellis et al., 2020). It is likely that significant additional effort would be needed to integrate authentic engineering applications into such a course. This could involve more extensive collaboration and coordination with the college of engineering (e.g., the engineering college could offer a first-year course that introduced key ideas and terminology used in the engineering calculus labs). Another approach could be to involve mathematics educators in the design of the labs as their knowledge of curriculum design and student learning may suggest ways to overcome the challenges of implementing authentic applications. Such efforts could have an important payoff both in terms of better leveraging students' interests in engineering to motivate their study of calculus and in terms of their future ability to draw on their understanding of calculus during their engineering careers. The reader is encouraged to see Horwitz and Ebrahimpour (2002) for a description of a collaboration between engineering and mathematics departments that involved enhancing the calculus sequence with applied engineering and science projects.

4.4 Physics Calculus at Tree Line University

In the second vignette, Engineering Calculus at RRU drew on a client discipline for the creation and design of the course, but meaningful application problems were soon abandoned. There were also limited ongoing collaborations with the Engineering college to inform the enactment of the course aside from evaluation purposes. In our third vignette, we feature a course variation with increased and ongoing collaborations with a client discipline department (Physics) and a reconfiguration of the calculus content. A more comprehensive investigation of the enactment and student experience in Physics calculus is presented in Voigt et al. (2021). Here we consider the creation and enactment of a combined Physics Calculus course.

Vignette

Tree Line University created a combined Physics Calculus course where students concurrently enroll in both a Calculus course taught by an instructor from the mathematics department and an introductory physics course taught by an instructor from the physics department. Students enroll in two separate four credit courses and receive individual grades for each course, but they are taught in collaboration with each other. Specifically, the two courses meet five days a week with students spending two days a week on calculus, two days a week on physics, and the remaining day alternating the content based on the needs of the courses. Each class is a mixture of short lectures, group activities, and computer work or experiments.

The course was created in 1998 with collaborations from Mathematics Educators, Mathematicians, and Physicists as part of a grant from the National Science Foundation. The primary goal behind the creation of the course was to enhance connections between the two subjects through a reordering of course topics and choice of applications. As one instructor described it:

> *Physicists can do a better job of teaching physics if they have access to calculus operations earlier in the semester and they don't have to create artificial derivations for a lot of the things that they want to use. So we have reordered the calculus sequence to help facilitate that while still maintaining the conceptual and computational integrity of what you would normally do in Calculus.*

The reordering of topics includes: within the first month introducing rate of change, derivatives, and antiderivatives (mainly in the context of polynomials) and within the second month further exploring these topics but in the context of exponential and logarithmic functions, reserving trigonometric functions for the end of the semester.

In addition to the initial cross-departmental collaborations to design the course, each term the current physics and mathematics instructors discuss the passing and coverage of the course. Based on interviews with recent instructors, collaborations were more frequent and robust when the course was initially developed, but now occur every two weeks to check which topics need to be covered in calculus to prepare students for their use in the physics course. As such, the tight connections between the course topics have diverged somewhat over the years as different mathematics instructors have taken over responsibility for teaching the course and modifying the curriculum. According to one current instructor, "It's not always perfectly handled, but we try to have the mathematical technique show up in some form before it's used in physics." One area of continued overlap in course content and applications is the use of two group projects that are jointly assessed and contribute to the grade outcomes for both courses. One example of a project is an open-ended question around two cars starting in one spot taking different paths, going different distances, and getting to an ending point at different times. Students are tasked to determine whether one was speeding or if one is going faster than another.

In addition to the reordering of course topics and applications, the format of the Physics Calculus course is quite different from the standard calculus sequence at Tree Line University. Driven largely from evidence-based research in mathematics education, Physics Calculus includes a focus on group work, meta-cognition, and conceptual understanding. In order to aid these efforts, the course is taught with smaller class sizes (30-40 compared to 120 in large lectures), meets for longer class periods (2 hours versus 50 minutes), and utilizes teaching assistants in the course to facilitate the activities rather than having recitation sections.

Physics Calculus at Tree Line University typically only offers one or two sections of the course and provides a year-long sequence with the same instructor and students in the course. The course is also targeted for Honors College students which instructors described as "siphoning off the best" students taking calculus, leaving other physics students to take the regular calculus sequence. Instructors pointed to these features as an advantage for students to foster community, "[Students] really get to know one another very well . . . are a tight group and work together on problems, and they also know the instructor better, especially in math because they see us two semesters in a row." In fact, additional analysis from this project revealed students reporting a greater sense of community and support in Physics Calculus compared to students in standard calculus (Voigt et al., 2021).

Vignette Reflection Questions

1. The development of Physics Calculus at TLU was resource intensive with the support of external funding and collaborations across departments. Is this approach feasible without such support?
2. There were several aspects of the course not specifically tied to content collaboration (e.g., group work, contact time, cohort model). In what ways did these contribute to the success of the course variation? Would it have been successful without such structures?
3. TLU partnered with Physics as a client discipline to create the course and re-order the topics. What types of modifications would be beneficial if a course was designed in collaboration with a different client discipline (e.g., chemistry, biology, business)?

Strengths of the Variation, Room for Growth, and Implementation Guidance

Physics Calculus at TLU is largely perceived as successful by instructors, students, and administrators; however, the student population that is recruited for this course is drawn largely from the Honors-College, with highly motivated students, most of whom have had previous exposure to calculus before college. It is important to understand the background and demographic make-up of the student population at TLU and in Physics Calculus in order to interrogate and contextualize this course variation. TLU is a predominantly white institution (PWI) with roughly 83% of students identifying as white, and 56% of students identifying as women. Based on our survey data, of the students enrolled in Physics calculus ($n = 59$), 2% identified with an underrepresented racialized identity (Black or African American, Hispanic or Latinx, or Alaskan Native or Native American) and 47% identified with an underrepresented gendered identity (Woman, Transgender, or Non-binary). This compares to the standard calculus sequence where ($n = 862$), 5% of students identified with an underrepresented racialized identity, and 28% with an underrepresented gender identity. Although we don't make any statistical claims based on this data, the student composition (at the time of the survey) who have access to and enroll in Physics Calculus are more likely to be women and less likely to be underrepresented racialized students. Instructors expressed that this course was not suitable for those who had not taken calculus before because, "the struggle from increasing pacing and the struggle from the activities, kind of compiles on top of one another to make it difficult for them to really get ahead of where they would be otherwise." As such, there are inherent equity issues embedded within this course that privileges and provides richer mathematics and physics access to students who have had prior exposure to calculus. Research has documented that access to calculus in high school is embedded within existing racial and socioeconomic disparities (Battey, 2013). Furthermore, given that women have been historically marginalized in Physics (Ivie & Ray, 2005) and evidence that physics contexts (e.g., rockets, car crashes) often exhibit gendered narratives (Francis et al., 2017), there is the potential for this course to marginalize women.

Discussion Questions
Suppose you wanted to create a joint calculus course with another client discipline.

1. How might you broaden the scope of the course to allow entry for all entering physics students? Or how might you target students who have been historically marginalized or those that have had no prior exposure to calculus?
2. In contrast to Engineering calculus at RRU where client disciplines were involved in the design of the course, Physics calculus at TLU involved ongoing collaboration and synergy. What structures would need to be put in place to assure that continual communication between departments is maintained as different instructors take over responsibility for the course?

Physics Calculus at TLU was designed and created in 1998 and current instructors feel the course materials are somewhat outdated and the activities and projects need to be revamped. The mathematics portion of the course has seen some updates, while the physics portion of the course has remained largely the same. This has caused some disconnect and divergence from the original design of the course. Furthermore, although the course topics have been re-ordered and there are applications within the course, we observed that the innovation at TLU did not involve rethinking the nature of the mathematics that students are learning. We encourage the reader to consider two chapters from the earlier MAA Notes volume *Making the connection: Research and teaching in undergraduate mathematics education* that take seriously the question of what mathematics students should learn before and during their calculus courses (Oerhtman et al. 2008, Thompson & Silverman, 2008).

4.5 Conceptual Calculus at Maple State University

In the above three vignettes, we illustrated how institutions can augment their calculus courses to better serve client disciplines and increase student interest. This has been done by inviting an outside expert speaker (vignette 1), restructuring course content to support students in moving through the math sequence more quickly while also engaging them with engineering applications (vignette 2), or coordinating with a separate client discipline to better align curricula (vignette 3). While these changes involved some re-organization of the mathematical topics, our last vignette illustrates how an institution can take this a step further by focusing on how the mathematics content is structured. To support students in gaining a richer understanding of calculus, Maple State University overhauled its calculus curriculum in order to promote meaning-making and conceptual understanding.

Vignette
Finding the traditional calculus curricula at their institution procedurally focused and conceptually restricting, mathematics education researchers at MSU designed a conceptually focused curriculum that engaged students in making meaning of calculus concepts. Since the traditional and engineering calculus sequences prioritized techniques and procedures, the primary goal for this new calculus course was to provide students with an opportunity to engage deeply with the foundational concepts of calculus and prioritize conceptual understanding. This new curriculum was initially implemented in one or two sections of the traditional calculus, but in a few years it grew to be used in most sections.

This new course variation, referred to here as Conceptual Calculus, restructures the traditional curriculum in a way that creates an intellectual need for each of the topics in the calculus sequence. This restructuring involves a *reversing of the traditional order of topics by starting with integration and then moving on to rates of change (i.e., derivatives). At the core of Conceptual Calculus is the goal to develop a deep understanding of the Fundamental Theorem of Calculus (FTC) by framing integration as the accumulation of a quantity and exploring the rate at which that quantity is being accumulated. More specifically, it focuses on two main points: 1) You know how much of a quantity there is at every moment; you want to know how fast it is changing at every moment. 2) You know how fast a quantity changes at every moment; you want to know how much of it there is at every moment. Understanding these two main points, and the relationship between them is precisely what it means to understand the FTC. A key strategy for getting the students to engage with these ideas is the thoughtful incorporation of technology by way of a graphing program (see Figure 4.2). The technology allows students to solve accumulation problems with integrals in "open form" (i.e., in summation notation, see $A(x)$ in Figure 4.2) before learning integration techniques. Techniques for finding antiderivatives are presented as insights that can facilitate the solution of an accumulation problem but not as essential to its solution. Students use the technology for homework, during class, and even in assessments. As such it becomes a powerful learning tool. As one instructor described:*

> [*the graphing program*] *is really a powerful computational tool and just the students really have to know the math, just the math function notation, to implement—they don't have to code anything but they have to understand what's a difference quotient, [...] rate of change function, [...] or accumulation function, compare it and and visualize everything and calculate*

Using the technology to model and calculate accumulation and rate of change problems requires students to think critically about function notation and pushes students to make meaning of the functions themselves.

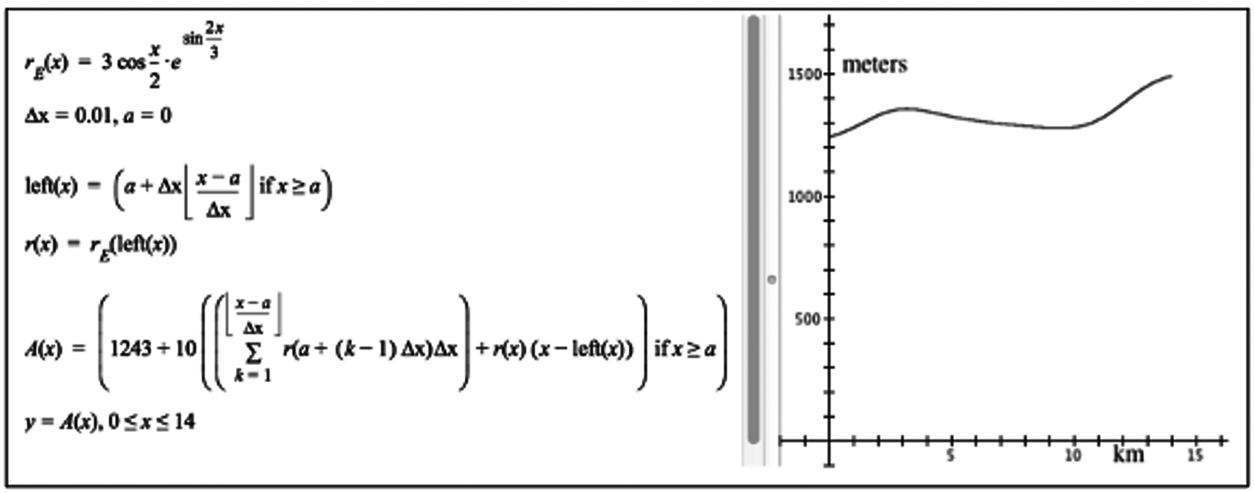

Figure 4.2. Graphing program used in Conceptual Calculus at MSU.

Conceptual understanding is continuously emphasized as a major goal of the course, pushing students to understand the "why" rather than just the "what". Emphasizing the "why" has not only transformed what it means to do mathematics, but with a redesign of assessments, it has also transformed what it means to know mathematics. Unlike the exams from the engineering and traditional calculus, which prioritized procedural and technical fluency, assessments for Conceptual Calculus test meaningful understanding of calculus concepts. Instructors noted that these exams were designed to ensure that students could not pass the course without exhibiting a meaningful understanding of the topics.

This course variation was first implemented in the spring of 2011 in only one or two sections at a time. A few years later, after receiving NSF funding to expand, all but at most one of the sections of traditional calculus were using this conceptually focused curriculum. The mathematics department has been supportive of the efforts to scale up the curriculum by providing access to more calculus sections, allowing for additional training, and involving more instructors and TAs. These courses are taught in large lecture sections (120 students) with smaller recitations (30 students). When registering, students do not have any indication as to which curriculum their course will be using. Instructors have noted that there is a mixed response from the students regarding the curriculum. While some find it *interesting and quickly buy-in to the new approach to calculus, there are many that are perturbed by the conceptual approach. The curriculum designer has noted this through conversation with the students, saying that:*

> *They're really thrown for a loop because they're not- they're unaccustomed to being asked to reason, they're very unaccustomed to being in a situation where they simply don't progress, that they don't make some meaning, and they have strong feelings that a didactical contract has been violated.*

Students who have traditionally succeeded in math courses by memorizing procedures and facts find themselves in a drastically different learning environment. Even with this pushback from students, assessments have shown that students in these courses score significantly better on concept inventories than students in alternative calculus variations (i.e., engineering and traditional).

Vignette Reflection Questions

1. This curriculum requires a complete re-conceptualization of calculus, both in terms of what mathematics is to be taught and what it means for students to understand it. What kind of resources or supports would instructors need to teach this course?
2. One of the main struggles MSU has experienced with Conceptual Calculus is the pushback from the students. How might MSU work with the students to create better buy-in?
3. The rethinking of the calculus curriculum at MSU is quite extensive. What are some more modest approaches that could be taken to creating a variation of the calculus sequence that seriously reconsiders the nature of the mathematics?

Strengths of the Variation, Room for Growth, and Implementation Guidance

Conceptual Calculus at MSU was developed for math and science majors and focuses primarily on a conceptual understanding of calculus concepts. In the redesign of the curriculum, the researchers primarily focused on the course content and materials (e.g., textbook, class activities). In implementing the curriculum, many of the instructors have expressed how the curriculum lends itself to more active learning. This is particularly due to the fact that the curriculum is designed to necessitate student inquiry. While this may be a consequence of the conceptual nature of the curriculum, there has not been explicit attention paid to any specific instructional approaches that should be used. Students who had taken this course described it as primarily lectured based. Based on our classroom observations, most courses exhibited a mix of instructor-oriented discussion with some participation from the students.

Discussion Questions

1. Similar to MSU, the Physics Calculus at TLU involved a reorganization of calculus topics, though their reorganization did not focus so much on the math itself but instead on coordinating with physics topics. How might a reconceptualization of calculus at TLU affect the coordination between calculus and physics?
2. What kinds of things can an instructor do to make a course that reconceptualizes calculus more active? What should they consider to promote equitable student experiences when implementing active learning approaches?
3. The calculus reconceptualization of this curriculum was specifically developed for math and non-engineering science majors. Would there be a different way to reconceptualize the calculus curriculum for other disciplines (e.g., engineering, social sciences)?
4. What kind of support from the department/school would you need to implement such an overhaul of the calculus curriculum at your institution?

The calculus curriculum at MSU was specifically designed to foreground conceptual understanding and meaning-making. Even with the positive results the variation has shown (i.e., better assessment scores), it is no secret that this overhaul of the curriculum required substantial resources and time from researchers at MSU and cooperation from the administration. Additionally, it is clear that more explicit attention needs to be put on the kinds of instructional practices that could better support the conceptual nature of the curriculum. Such a focus on the instructional practices, specifically those that promote active learning in STEM fields, would not only help to increase student outcome (Freeman et al., 2014) but also help close opportunity gaps for students who have been historically marginalized in STEM (Theobald et al, 2020). For further suggestions on tangible equitable teaching practices in STEM education we point the reader to Tanner (2013) who provides teaching strategies for student engagement in the context of undergraduate biology education.

4.6 Conclusion

Mathematics departments across the country are varying their approach to the content of the calculus sequence to better support the needs and interests of their students. In this chapter, we detailed variations offered at four universities. While these were designed with different populations of students in mind, we present them together as a means to promote discussion about approaches to what content is included and how it is presented. In addition, these vignettes raise important issues of implementation and maintenance.

To summarize, Dunshire University (DU) aimed to support first-year mathematics students to gain a broad understanding of mathematics and mathematics research. Their approach to achieve this goal was to restructure the Calculus 2 lab so that instead of attending a session that engaged students in solving problems in Calculus 2, the students met with a research mathematician once a week to learn about applications that were related to calculus and the research mathematician's expertise. River Rock University (RRU) aimed to enable students majoring in engineering to start their engineering coursework sooner and learn more about engineering applications. Their approach was to create an engineering calculus sequence that eliminated content that was not needed or could be postponed and incorporated applications into the lab sessions that broadly connected calculus and engineering. Tree Line University (TLU) aimed to support students majoring in physics to make connections between calculus and physics. Their approach involved creating a Physics Calculus in collaboration with the physics department that reordered course topics and attended to applications. Maple State University (MSU) aimed to support students majoring in mathematics and science to gain a conceptual understanding of calculus that was required for their discipline. Their approach was to reconceptualize the calculus curriculum in a way that promotes meaning-making and conceptual understanding.

4.7 Discussion

In what follows, we take a step back from the individual vignettes and highlight three ideas that emerged as missing from our exploration of the four content-based variations: (1) promoting diversity, equity, and inclusion, (2) building evidence-based instructional practices into design, and (3) engaging students in authentic discipline-based practices. We encourage mathematics departments to think deeply about how to incorporate these ideas into their efforts to re-envision their calculus sequence with the students' needs and interests in mind. In the following sections we discuss these three ideas in detail and then point the reader to additional resources.

Promoting Diversity, Equity, and Inclusion

The potential impact of variations in the approach to content on issues of diversity, equity, and inclusion was especially evident at TLU and DU. In particular, TLU's variation has inherent (in)equitable design features because it primarily serves students who have previously been exposed to calculus before college and features curricular contexts that are embedded with gendered narratives. Additionally, a variation that brings an expert of a discipline into the calculus sequence should take advantage of the opportunity to select experts that challenge assumptions of whiteness and masculinity in STEM disciplines. We encourage the reader to not only think about how an initiative might perpetuate inequities, but also to consider ways to take proactive steps to promote diversity, equity, and inclusion. We view conversations about diversity, equity, and inclusion as fundamental to efforts that aim to make improvements to STEM courses since any effort to change the educational curriculum is systematically linked to issues of diversity, equity, and inclusion (Luke et al., 2013). Without explicit attention, actions can perpetuate issues of inequality and marginalization by default (Johnson et al., 2020).

We point the reader to several equity-focused resources as they consider how to re-envision their calculus sequence with STEM disciplines in mind. First, additional reading about departmental efforts in addressing diversity, equity, and inclusion in the calculus sequence can be found in Chapter 2 of this volume. We also highlight several communities that might support departments' efforts to promote diversity, equity, and inclusion with a discipline-based variation. Conferences on mathematics education often include working groups and/or special sessions that help connect researchers as well as practitioners who are interested in these topics (e.g., Research in Undergraduate Mathematics Education conference has an Equity in Undergraduate Mathematics working group). Additionally, mathematics departments can consult with different organizations at their university that serve to elevate students with marginalized identities, many of which are embedded within STEM disciplines (e.g., Society of Women Engineers, Society of Black Engineers, Society for Advancement of Chicanos/Hispanics and Native Americans in Science, Out in STEM, office for diversity and inclusion). We also encourage the reader to connect with other departments that have similar goals. More informal avenues for networking such as MAA Connect[1] or Facebook's group on Equity and Social Justice in Mathematics[2] can support the reader to connect with others and engage in ongoing conversations.

Building Evidence-based Instructional Practices into Design

While all of the vignettes that we offered in this chapter described efforts to supplement, modify, or restructure the calculus content, not all of them paid explicit attention to the supporting instructional practices. For instance, DU's initiative decreased the students' engagement in active learning to allow time to incorporate a research mathematician's expertise into the course's content. Additionally, MSU's primary effort was geared towards reshaping the curriculum to support students in gaining a conceptual understanding; however, the instruction remained lecture-oriented. Alternatively, TLU was an example that did both; not only did they change the content of the course so that it coordinated calculus and physics concepts, but they also explicitly leveraged group work as a key part of their course. We encourage the reader to consider how changes might be made to their curriculum in such a way that they incorporate evidence-based instructional practices.

The MAA Notes volume *Instructional Practice Guide* can be a helpful resource for evidence-based practices for effective teaching (Abell et al., 2018). There are also open source Precalculus and Calculus textbooks that build active learning into the curriculum (e.g., Active Calculus by Boelkins et al., 2016; Active Learning Materials for Calculus by BOALA, 2019). There are several communities that offer mathematics departments additional support as they incorporate evidence-based instructional practices into course redesigns. For instance, the Mathematics Teacher Education Partnership has an Active Learning Mathematics Research Action Cluster[3], the CoMInDS project[4] offers professional development related to evidence-based practices for graduate student instructors, and MAA Connect has an Active Learning Exchange community[5]. Additionally, many universities have a teaching and learning center which often partners with departments efforts to increase evidence-based practices in classrooms.

Engaging Students in Authentic Discipline-based Practices

At least three of the four universities that we discussed here intended to actively engage students in authentic practices of a discipline (mathematics at DU, engineering at RRU, and physics at TLU). Faculty at RRU expressed that using more authentic engineering tasks was a challenge because of the differences in the language of the two disciplines. Additionally, while we acknowledge that there is room for improvement for the variations offered at TLU and MSU, we also view these variations as relatively successful in engaging their students in more authentic use of calculus. Both initiatives were led by mathematics education researchers and eventually received substantial funding. We view promoting authentic discipline-based practices in the calculus sequence as a commendable goal and also acknowledge challenges that faculty might face when obtaining it. As such, we suggest that mathematics departments connect with the Discipline-Based Education Research (DBER) community[6] (Henderson et al., 2017), mathematics education researchers who have expertise in task design, and their university's client discipline departments. We view collaborations with these communities as critical in designing tasks that promote students to engage in authentic disciple practices. For those with familiarity in DBER practices we point the reader to the theory of Realistic Mathematics

[1]https://connect.maa.org/home

[2]https://www.facebook.com/groups/178344199241717

[3]https://www.aplu.org/projects-and-initiatives/stem-education/mathematics-teacher-education-partnership/mtep-racs/mtep-racs-alm.html

[4]https://www.maa.org/programs-and-communities/professional-development/cominds

[5]https://connect.maa.org/communities/community-home-CommunityKey=1f3d12b6-33c2-49fb-8074-fd5c81e55950

[6]https://serc.carleton.edu/StemEdCenters/Cross-DBER

Education for guidance on creating authentic tasks (see Gravemeijer & Doorman, 1999 for a discussion about context problems with an example from calculus).

In this chapter we discussed four vignettes illustrating different ways that universities have varied their approach to the content of the calculus sequence. We close with a call for mathematics departments to take action toward supporting their students' needs and interests in STEM disciplines. With these efforts, we challenge mathematics departments to (1) promote diversity, equity, and inclusion, (2) build evidence-based instructional practices into the design, and (3) engage students in authentic discipline-based practices. Our hope is that this chapter is a source of inspiration for mathematics departments to better support their students' needs and interests by re-envisioning the content in their calculus sequence.

References

Abell, M., Braddy, L., Ensley, D., Ludwig, L., & Soto-Johnson, H. (2018). MAA Instructional Practices Guide. *Washington, DC: Mathematical Association of America*.

Battey, D. (2013). Access to mathematics: "A possessive investment in whiteness." *Curriculum Inquiry, 43*(3), 332–359. https://doi.org/10.1111/curi.12015

Boelkins, M., Austin, D., & Schlicker, S. (2016). *Active Calculus 1.0*. Grand Valley State University Libraries. Boulder-Omaha Active Learning Alliance. (2019). Active Learning Calculus. Retrieved from: http://math.colorado.edu/activecalc/

Bressoud, D. (2018). Calculus as a Modeling Course at Macalester College. *Launchings*. Available at http://launchings.blogspot.com/2018/08/calculus-as-modeling-course-at.html

Callahan, J., Hoffman, K, Cox, D., O'Shea, D., Pollatsek, H., Senechal. L. (1994) *Calculus in Context: The Five College Calculus Project*. W. H. Freeman. 2008 reprint of original available at http://math.smith.edu/~callahan/cic/book.pdf

Ellis, B., Larsen, S., Voigt, M., & Vroom, K. (2021). Where calculus and engineering converge: An analysis of engineering calculus course design and implementation. *International Journal for Research in Undergraduate Mathematics Education*. https://doi.org/10.1007/s40753-020-00130-9

Flath, D., Halverson, T., Kaplan, D. and Saxe, K. (2013). The first year of calculus and statistics at Macalester College. pp. 39–44 in *Undergraduate Mathematics for the Life Sciences: Models, Processes, and Direction*. Ledder, Carpenter, and Comar, eds. MAA Notes #81. Washington, DC: Mathematical Association of America. www.maa.org/publications/ebooks/undergraduate-mathematics-for-the-life-sciences

Francis, B., Archer, L., Moote, J., DeWitt, J., MacLeod, E., & Yeomans, L. (2017). The construction of physics as a quintessentially masculine subject: Young people's perceptions of gender issues in access to physics. *Sex Roles, 76*(3-4), 156–174.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences of the United States of America, 111*(23), 8410–8415. https://doi.org/10.1073/pnas.1319030111

Gladstone, J., & Cimpian, A. (2020). Role Models Can Help Make the Mathematics Classroom More Inclusive.

Gravemeijer, K., & Doorman, M. (1999). Context problems in Realistic Mathematics Education: A Calculus Course as an example. *Educational Studies in Mathematics, 39*(1-3), 111–129. https://doi.org/10.1023/A:1003749919816

Henderson, C., Connolly, M., Dolan, E. L., Finkelstein, N., Franklin, S., Malcom, S., ... & John, K. S. (2017). Towards the STEM DBER alliance: Why we need a discipline-based STEM education research community. *International Journal of Research in Undergraduate Mathematics Education, 3*(2), 247-254.

Horwitz, A. L. A. N., & Ebrahimpour, A. R. Y. A. (2002). Engineering applications in differential and integral calculus. *International Journal of Engineering Education, 18*(1), 78–88.

Ivie, R., & Ray, K. N. (2005). *Women in physics and astronomy, 2005*. College Park, MD: American Institute of Physics.

Johnson, E., Andrews-Larson, C., Keene, K., Melhuish, K., Keller, R., & Fortune, N. (2020). Inquiry and gender inequity in the undergraduate mathematics classroom. *Journal for Research in Mathematics Education, 51*(4), 504–516.

Klopfenstein, K. (2005). Beyond test scores: The impact of Black teacher role models on rigorous math taking. *Contemporary Economic Policy, 23*(3), 416–428.

Ko, S. J., Marx, D. M., Nickerson, S. D., & Bjorkman, K. (2020). Implementing a peer role model program in college calculus classes to broaden women's participation in STEM. *PRIMUS, 30*(3), 349–366.

Letterman, M. R., & Dugan, K. B. (2004). Team teaching a cross-disciplinary honors course: Preparation and development. *College teaching*, 76–79.

Luke, A., Woods, A., & Weir, K. (2013). Curriculum design, equity and the technical form of the curriculum. *Curriculum, syllabus design and equity: A primer and model*, 6–39.

Oehrtman, M., Carlson, M., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. *Making the connection: Research and teaching in undergraduate mathematics education, 27*, 42.

Price, J. (2010). The effect of instructor race and gender on student persistence in STEM fields. *Economics of Education Review, 29*(6), 901–910.

Rainey, K., Dancy, M., Mickelson, R., Stearns, E., & Moller, S. (2018). Race and gender differences in how sense of belonging influences decisions to major in STEM. International Journal of STEM Education, 5(1), 10.

Seymour, E., & Hewitt, N. M. (1997). *Talking about leaving: Why undergraduates leave the sciences*. Westview.

Seymour, E., & Hunter, A. B. (2019). *Talking about leaving revisited*. New York, NY: Springer.

Tanenbaum, C. (2016). STEM 2026: A vision for innovation in STEM education. *US Department of Education, Washington, DC*.

Tanner, K. D. (2013). Structure matters: twenty-one teaching strategies to promote student engagement and cultivate classroom equity. *CBE-Life Sciences Education, 12*(3), 322–331.

Theobald, E. J., Hill, M. J., Tran, E., Agrawal, S., Nicole Arroyo, E., Behling, S., Chambwe, N., Cintrón, D. L., Cooper, J. D., Dunster, G., Grummer, J. A., Hennessey, K., Hsiao, J., Iranon, N., Jones, L., Jordt, H., Keller, M., Lacey, M. E., Littlefield, C. E., Freeman, S. (2020). Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math. *Proceedings of the National Academy of Sciences of the United States of America, 117*(12), 6476–6483. `https://doi.org/10.1073/pnas.1916903117`.

Thompson, P. W., & Silverman, J. (2008). The concept of accumulation in calculus. *Making the connection: Research and teaching in undergraduate mathematics, 73*, 43–52.

Thousand, J. S., Villa, R. A., & Nevin, A. I. (2006). The many faces of collaborative planning and teaching. *Theory into practice, 45*(3), 239–248.

Tremaine, R. (2021, June 22). Integral feedback: A qualitative analysis of students' experience of an Enhanced Lab Calculus. `https://doi.org/10.31219/osf.io/3c8qg`

Voigt, M., Apkarian, N., Rasmussen, C., & Progress through Calculus Team. (2020). Undergraduate course variations in precalculus through Calculus 2. *International Journal of Mathematical Education in Science and Technology, 51*(6), 858–875. `https://doi.org/10.1080/0020739X.2019.1636148`

Voigt, M., Rasmussen, C., & Martinez, A. (2021). The refiguring of students' mathematical identities: A mixed methods study of three tailored calculus courses. *International Journal of Mathematical Education in Science and Technology*. `https://doi.org/10.1080/0020739X.2021.1940331`

5

Professional Development and Course Coordination
One-time and Ongoing Supports

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Chapter Question: How can course coordination and professional development workshops/courses offered by mathematics departments contribute to the instructional growth of their graduate students and faculty?

5.1 Introduction

In order to improve student learning and experiences in introductory college mathematics courses, it is critical for students to have access to high quality instruction. Although college instructors are regarded as content experts, most receive very little professional development focused on their teaching and on creating equitable and inclusive classes. Consequently, they might not be equipped with the skills and resources necessary to successfully implement evidence-based instructional strategies. In fact, many college instructors only participate in a short professional development program (typically 1–5 days) before their first semester of teaching as a graduate student. There is an increasing number of national professional development workshops and resources available, such as the Academy for Inquiry-Based Learning[1] and Project NExT[2], but in-house professional development remains the most common source of instructor professional development.

Among the mathematics departments that responded to this project's census survey (Rasmussen et al., 2019), most offered a brief professional development experience for new graduate teaching assistants (GTAs) before classes began (Apkarian & Kirin, 2017). Additionally, many reported (∼30%) that their programs did an adequate job of preparing graduate students for their role as instructor, but noted that the program could be improved. One way to improve professional development programs is by making them more robust, which involves extending beyond a single session and incorporating evidence-based activities.[3] There is evidence that robust professional development can positively impact instructors' readiness for teaching.

[1]For more information see `http://www.inquirybasedlearning.org/`.
[2]For more information see `https://www.maa.org/project-next`.
[3]The College Mathematics Instructor Development Source (CoMInDS) is one resource for those who want to create or improve their GTA professional development programs. For more information see `https://www.maa.org/cominds`.

This chapter can serve as a resource for those wishing to create or revise their professional development programs. In particular, we highlight three professional development programs for departments to consider. The programs illustrated in the following vignettes all extend teaching professional development and instructor support beyond a one-time, 1-5 day workshop for new instructors. The first vignette highlights a robust 5-week program specifically designed to develop pedagogical knowledge and skills in graduate students, the second vignette highlights how one calculus course coordinator has worked to embed professional development activities within the coordination structure, and the final vignette highlights a program that employs multiple individuals to support and manage on-going professional development along with course coordination. After the vignettes, we include a discussion about characteristics demonstrated by those providing the professional development in the vignettes – specifically, we discuss coordinator orientations – and point to ways that these orientations might enable coordinators to facilitate instructor development within the context of coordination. Within course coordination, coordinators can nudge instructors to make particular decisions about instruction by intentionally providing certain resources and support (Rasmussen & Ellis, 2015). For example, during meetings, the course coordinator might choose to focus the discussion on teaching strategies (in addition to addressing content and coverage) which can then serve as an opportunity for professional development about pedagogy. We conclude the chapter encouraging the reader to consider how they might leverage course coordination as ongoing, just-in-time professional development for both novice and experienced instructors, and consider how coordinator orientations might support this endeavor.

5.2 Time-Intensive Summer Workshop at Sandpiper University

Sandpiper University (SU) facilitates a single professional development program, specifically for GTAs teaching in the precalculus and calculus courses. These GTAs either teach a recitation section or are the instructor of record for a class. This program is a multi-week summer workshop before the start of the semester that aims to prepare graduate students for the expectations and challenges of teaching university mathematics. The following vignette describes the structure of the program as well as the details of each component of the professional development approach at SU.

Vignette

Shortly before our first site visit to Sandpiper University (SU), SU had merged with a small university that had a strong Masters of Arts in Teaching program. This university offered a six-week summer professional development program to prepare education graduate students for teaching two courses at local secondary schools. Drawing on the strengths of this program, the faculty from the merged education department collaborated with faculty from the mathematics department at SU to design a five-week professional development curriculum for first year GTAs, known as "GTA boot camp." GTAs participating in the boot camp are required to attend Monday through Friday, 8am to noon in the five weeks leading up to the start of the semester.

One distinguishing characteristic of this professional development program is that graduate students from multiple STEM disciplines, not just mathematics, participate and earn course credit for their involvement. The five-week program is divided into three courses: 1) a four-credit hour STEM methods course, 2) a two-credit STEM literacy course, and 3) a one-credit professionalism and teaching course. The STEM methods course covers various aspects related to pedagogy (e.g., questioning techniques) and introduces active learning strategies (e.g., think-pair-share, group-work activities). A main component of the methods course is the weekly teaching labs where graduate students prepare and teach lessons and receive feedback from their peers about the pedagogical strategies used during the lesson. The STEM literacy course includes instruction on the use of proper vocabulary, terminology, and use of the course textbook. Some of the GTAs felt that the STEM literacy course helped them to make the content relatable to students. Specifically, it helped them to recognize that their vocabulary and knowledge might be out of reach for some students, and so they needed to find ways to make the material interesting and accessible. The professionalism and teaching course covers topics such as diversity and inclusion, how to write a professional email, and how to respond to issues with students. In addition, the course serves as an opportunity for the international GTAs to better understand the norms and customs of culture and education in the United States.

Overall, the GTAs liked the boot camp and felt encouraged and comfortable trying new teaching methods before implementing them in their classrooms. Moreover, the GTAs appreciated the focus on STEM literacy as it helped them to make the material of the course easily understood and relatable for undergraduate students. One of the more senior

GTAs, who did not participate in the GTA boot camp, recognized that the boot camp was a beneficial experience for the new GTAs to facilitate a good understanding of what it takes to teach in a university classroom.

Vignette Reflection Questions

1. Of the three components (STEM methods, STEM literacy, Professionalism and teaching) of the professional development boot camp, it seems Sandpiper places most emphasis on the STEM methods course based on the number of credit hours assigned to it. Which component do you see as being most important and why?
2. What are the affordances and challenges of engaging graduate students in a time-intensive professional development experience before they enter the classroom as instructors?
3. What are some of the advantages to partnering with an education department or other STEM departments to offer professional development for new GTAs? What are some of the disadvantages to this approach?

Strengths and Further Considerations
Sandpaper University offers an extensive professional development experience for their GTAs through this five-week summer boot camp. Their approach affords their GTAs opportunities to learn about pedagogy and mathematical content, as well as professional aspects of what it means to be a university mathematics instructor. While the professional development program is generally well-received and the GTAs have reported feeling prepared for teaching their courses, we acknowledge that such an approach requires substantial time and resources. For some mathematics departments, running a summer workshop is logistically not possible. For instance, GTAs may be travelling internationally or funding is not available until the beginning of the semester. We also highlight that this approach offers professional development to GTAs before they have spent time as an instructor in the classroom and have experienced a need to learn more about teaching practices. In the vignettes that follow in the subsequent sections we describe professional development programs that focus on ongoing support provided throughout the semester.

Discussion Questions

1. What are the first steps you could take inside and outside of your department to implement a multi-week professional development program for your GTAs?
2. Who at your institution could lead a multi-week professional development course?
3. What support from upper administration could you leverage to garner the resources necessary to run a multi-week professional development program for GTAs?
4. If a multi-week summer professional development experience is not feasible at your institution, what aspects of each of the components offered through the GTA boot camp at SU would you like GTAs at your institution to experience during their professional development?

5.3 Support Embedded in Course Coordination at Rolling Hills University

At Rolling Hills University (RHU), the mathematics department professional development program has two components. The first is a semester-long professional development course for new GTAs before they start their first teaching assignment. This is in many ways similar to the extended professional development workshop offered at Sandpiper University, but occurs during the academic year rather than during the summer. The second professional development opportunity offered at RHU is embedded in the course coordination system and hence goes beyond a one-time workshop. This second professional development component is intended for all of the instructors of the course, including faculty, postdocs, and GTAs. For this vignette, we focus on the ongoing professional development provided through the Calculus 1 coordination system.

Calculus 1 at RHU meets four hours a week with the instructor of record and does not have a recitation section. Approximately 20 sections of Calculus 1 are offered in fall semesters (and about 15 offered in spring semesters), with an average enrollment of 33 students per section. Faculty, postdocs, and experienced graduate student instructors typically teach the course, with most postdocs and GTAs each teaching two sections of the course.

Vignette

Joshua, the Calculus 1 coordinator at RHU, aims to provide instructional support through course coordination by offering evidenced-based resources and prompting conversations about how to support the development of students' mathematical understanding. Joshua also thinks that it is important to give instructors a great deal of pedagogical autonomy. Consequently, he provides a variety of curriculum resources, and encourages, but does not require, instructors to use them in their classes. Additionally, Joshua assumes as many of the managerial aspects of the course as possible - setting up the online homework system, creating the syllabus, and writing exams. He thinks that the availability of these vetted resources, and not having to attend to the managerial aspects, allows instructors to focus more time and attention on their teaching.

Joshua also regularly engages instructors in discussions that go beyond the managerial aspects of the course, including discussions about content, student thinking, and strategies for teaching Calculus 1. He hopes that this approach will encourage instructors to engage with ways of teaching that are more student-centered and will expand their thinking about students' mathematical strategies. Joshua works to build connections, collaborations, and a community of instructors through these regular coordinator meetings. As a result, instructors regularly discuss their teaching with one another and will ask each other for advice. Joshua has offered to observe classes as an opportunity for instructors to get feedback on their teaching, but very few instructors have taken advantage of this offer.

Joshua writes exams that are very conceptually-oriented in order to influence the instruction of Calculus 1 at RHU. He includes several questions on early versions of exams that are designed to reveal limitations in students' foundational ways of understanding the content. Joshua distributes this version of the exam to the instructors, giving them the opportunity to review and provide feedback several weeks before the scheduled exam date. Consequently, he expects instructors to make changes to their instructional focus (attending more to conceptual concepts instead of procedures) so that their students can be successful on conceptually-oriented exam problems. Joshua then makes some adjustments to the initial version of the exam based on instructor feedback, to create an exam that includes both procedural and conceptual problems. Joshua's approach to exam writing has also prompted instructor reflection on their teaching practices. Specifically, instructors have shared that grading the conceptual exam questions has revealed to them that their students are not understanding the material as thoroughly as they expected. This has resulted in instructors expressing concerns about their own teaching style, and asking to meet regularly to talk about teaching.

Vignette Reflection Questions

1. What are some of the key components of the professional development implemented by this course coordinator that you could see being successful in your department?
2. How, if at all, would you need to alter the course coordination structure so that these components complement what is in place in your department?
3. What are the opportunities afforded by this approach to professional development and to coordination?
4. What are the limitations of this approach to professional development and to coordination? What are some remedies for these limitations?

Strengths and Further Considerations

One strength of this approach to professional development and coordination is that both graduate students and faculty alike have the opportunity to be involved with ongoing professional development through the regular coordination meetings. The professional development activities were well-received by the instructors, including the faculty who particularly valued the evidence-based instructional materials, being able to offer feedback on course materials, and the opportunity for the coordinator to observe their class. Graduate student instructors and postdocs also appreciated the regular discussions teaching the course and about students' thinking about the content.

Institutions that have course coordination systems are in many ways primed to provide activities that align with their goals for course coordination and professional development for instructors. Since course coordination systems typically include coordinated elements (e.g., exams, homework assignments), this provides the coordinator with the opportunity to leverage evidence-based course materials to impact instruction. Further, 22% of Precalculus, Calculus I, and Calculus II courses that operate within a coordinated system already have weekly or bi-weekly meetings (Rasmussen et al., 2019). These meetings can be restructured (if necessary) to promote instructor growth through collaboration and community. It is important to note, however, that not every coordinator may be well-suited to take on this specialized dual role of coordinator and professional development provider as it requires specialized knowledge, time, and energy. Both coordinating a course and providing well-designed professional development activities can require a lot of time and effort in order to fulfill these responsibilities. After the discussion questions, we highlight Maple State University's program where two coordinators have split the coordinator role to share these responsibilities of coordinating logistics and providing on-going professional development.

Discussion Questions

1. What are the benefits and/or challenges of embedding professional development in the context of a specific course through coordination at your institution?
2. What opportunities are there in your department to incorporate ongoing professional development for novice and experienced instructors?
3. What kind of skillset or background would a coordinator need to carry out the work that this Calculus 1 coordinator is doing?

5.4 Curriculum-Centered Support via Course Coordination at Maple State University

In this vignette we focus on the professional development opportunities for instructors in a precalculus course for non-STEM majors because of the extensive professional development opportunities for those teaching this course. The course meets three hours a week with the primary instructor and does not have a recitation. In a typical fall semester there are over 900 students and approximately 16 instructors. Instructors include both graduate students and lecturers. Most sections have class size of 40, with 3-4 sections having enrollment between 80-100. The majority of the sections are taught using the Pathways curriculum (Carlson, Oehrtman, & Moore, 2019). The Pathways curriculum is conceptually-oriented and focused on developing students' mathematical meanings. These sections are highly structured and require dedication and commitment from the instructors teaching these sections.

Vignette

All graduate teaching assistants participate in 4.5 days of professional development facilitated by the mathematics department (and a half day professional development facilitated by the university) before their first semester at Maple State University (MSU). During this week GTAs prepare and give 4-5 lectures that are videotaped and reviewed with a mentor, attend sessions about grading, classroom management, and running recitations (including facilitating group work), in addition to covering logistics about being a GTA and activities to promote a sense of community. The weeklong training helps the course coordinator determine teaching assignments (teaching own class, running calculus recitation, assisting a qualifier class, tutoring and grading, or grading and working in the math learning center). New graduate students also attend a semester-long seminar during their first semester where every other week is spent on pedagogical strategies/discussion or discussions about being a graduate student, choosing an advisor, and research.

Unique to the Pathways course is the existence of two faculty members that oversee and assist instructors. One of these faculty members is the course coordinator, whose primary responsibility is to take care of all course logistics and writing exams. Instructors typically go to the coordinator when they need help navigating university policies or course logistics. The course coordinator does not see their role as providing pedagogical support. All pedagogical support for teaching the Pathways curriculum is overseen by a second faculty member who convenes weekly meetings with the instructors. The Pathways curriculum is focused on concepts and meaning, some of which is actually new to instructors. Hence the weekly meetings often include extensive work on the mathematics itself, research-based analyses of student thinking surrounding the precalculus concepts, as well as instructional strategies to support students' active engagement in course content.

Vignette Reflection Questions

1. What are the advantages of having two faculty members involved in coordinating the course (one focused on logistics and one focused on instructional support)?

2. What are some disadvantages of the approach illustrated in the vignette? What are some possible remedies for these disadvantages?

Strengths and Further Considerations

Many instructors request to teach Pathways because of the strong and deep conceptual focus of the material. Some of the GTAs that teach Pathways are in the mathematics education graduate program and hence have a strong interest in research that examines student thinking and evidence-based instructional practices. GTAs who teach Pathways and are pursuing a graduate degree in mathematics appreciate the exposure to mathematics education research as this strengthens their teaching skills and makes them more marketable for future academic positions.

The mathematics department's structure of two individuals sharing the responsibilities for course coordination and professional development within the Pathways precalculus courses at MSU is one way to provide sustainable on-going professional development for instructors. This approach allows departments to consider the strengths of multiple individuals instead of relying on one faculty member with the time, energy, and skillset to facilitate these activities effectively. Additionally, this approach provides instructors with multiple points of contact for discussing and getting feedback on their teaching or instructional resources. The professional development for instructors teaching Pathways precalculus at MSU was explicitly tailored to the curriculum and the development of instructor (and student) mathematical meanings. Thus the professional development included information that was critical for these instructors' teaching practice.

Discussion Questions

1. What role do innovative curricula, such as Pathways, play in your current professional development efforts?
2. What extra instructional support do you wish you were able to offer your instructors and why?
3. What would it take at your institution to have two people provide different kinds of support for precalculus (or calculus) instructors?

5.5 Summary

In this chapter, we explored three professional development programs for GTAs and instructors that go beyond a one-time, short professional development program prior to the beginning of classes for new GTAs. Sandpiper University's mathematics department's efforts go above and beyond this model by increasing the duration of the pre-semester professional development that occurs the summer before GTAs have their first teaching experience. The SU model allows for a more comprehensive experience to prepare their GTAs to teach their course, focusing on both pedagogy and content. RHU also offers a more time-intensive professional development experience for new GTAs, a semester-long course during the academic year of their first teaching experience. They also leverage their course coordination system to provide ongoing instructional support, as was highlighted in the vignette. Similarly, the mathematics department at MSU offers both a shorter (4.5 day) workshop for their new GTAs prior to teaching the course and utilizes course coordination in precalculus as an ongoing professional development opportunity for those instructors. Figure 5.1 provides a summary illustration of the different approaches to professional development. The lighter boxes represent professional development experiences intended only for GTAs and vary in duration. The darker blue boxes represent professional development for both instructors and GTAs and are ongoing in that the professional development is embedded in course coordination.

5.6 Discussion

The research literature suggests that *robust* professional development workshops (i.e., extended programs that include evidenced-based activities) can positively impact graduate students' preparedness for teaching. For instance, Alvine et al. (2007) reported positive impacts from a GTA professional development workshop that followed a Japanese lesson study, where groups of instructors worked together to plan, execute, analyze, and revise classroom lessons. Looking across the vignettes presented in this chapter, we see departments offering robust professional development in two formats: 1) for new graduate student instructors before their first term teaching, and 2) for all instructors while teaching

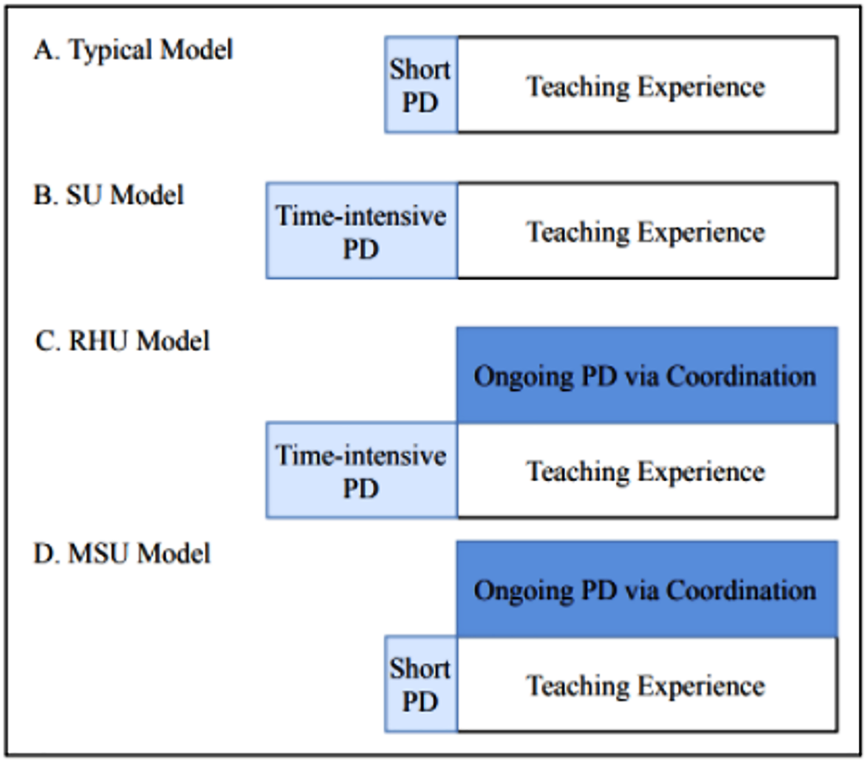

Figure 5.1. Overview of Professional Development Structures

a course through course coordination. We next summarize each of these formats followed by an extensive discussion of how different approaches that coordinators take to their job can support ongoing professional development opportunities.

Robust One-Time Professional Development Before Teaching

One-time professional development experiences provide new graduate teaching assistants with an introduction to teaching at the college level before they are responsible for their own classes. This type of professional development can be very beneficial, providing an overview of various instructional techniques and logistics to teaching at the institution. Additionally, the pre-teaching professional development experience could help new graduate students develop a familiarity with the classroom environment and some confidence in leading a group of students.

Partnering with another department for a one-time professional development also allows for the experience to extend beyond math graduate students to include other STEM graduate students, like the program offered at SU. Further, this creates an opportunity for multiple STEM departments to pull their resources together, which puts less of a strain on any one department. For example, the SU mathematics department was able to collaborate with the education department for the design of the pre-semester professional development program. This one time collaboration before the start of the semester might be logistically easier to achieve than an ongoing relationship throughout the semester.

Robust On-Going Professional Development Via Coordination

While one-time robust professional development workshops offered before graduate students start teaching can be beneficial, it may be an impractical solution to prepare precalculus and calculus instructors. In particular, these types of extended workshops tend to require significant time and funding (like the one we described at SU). Additionally, GTAs might not be able to fully take in and understand pedagogical strategies offered in pre-term professional development workshops before they have some experiences in the classroom. Beisiegel et al. (2019) found that first-year GTAs often struggled to adjust to their role as instructor, stating that GTAs were "struggling to survive." After gaining some experience, however, these GTAs demonstrated an increased ability to reflect on their teaching and began to consider incorporating new strategies into their instruction. Further, in order to develop as instructors who are culturally conscious and who work to promote equity and inclusion, a goal that should be central to college mathematics instructor professional development, a one-time experience before teaching may not be enough time for an instructor to deeply engage with this work. Consequently, just-in-time and on-going professional development can be beneficial to instructors, and specifically for new instructors, as they are more responsive to professional development once they gain experience in the classroom.

We view course coordination as a powerful and practical avenue for providing additional professional development, not only for GTAs but also for other faculty that teach the course. In particular, the coordination structure that centers the professional development in a specific course helps the professional development to be extremely relevant to day-to-day needs and practices of instructors. However, simply having the coordination structure in place does not ensure that it is serving the instructors by providing them with professional development. The coordinator(s) must be intentional about the ways in which they provide instructor support, that go beyond simply providing uniform elements (e.g., syllabus, textbook). A combination of two orientations, which we have identified and called humanistic-growth and a resource-managerial (Martinez et al., 2020) may enable coordinators to provide impactful professional development.

Coordinator Orientations to Support On-going Professional Development

A coordinator with a *humanistic-growth orientation* works to promote the professional growth of instructors by a) attending to the student experience, b) being concerned about others, c) taking action to build community, d) being attentive to instructor differences, and e) providing instructor support (Martinez et al., 2020). This orientation was exemplified by the coordinator at Rolling Hills University. Specifically, the Calculus 1 coordinator provided opportunities for instructor support and professional growth by facilitating regular meetings with instructors focused on collaboration and community, by prompting discussions about instruction and the development of students' mathematical thinking while also allowing for instructor autonomy, and by offering to observe and give feedback on instruction. This coordinator provided numerous vetted evidence-based resources and encouraged instructors to select materials that would work for their instructional approach, demonstrating an attentiveness to instructor differences. He was also willing to make adjustments to these materials (specifically common exams) based on feedback from the instructors. The humanistic-growth orientation was also exhibited by the coordinator at Maple State University who was responsible for facilitating instructor meetings specifically about teaching the Pathways precalculus curriculum. This coordinator demonstrated this orientation in how they focused on providing instructor support by building a community of instructors who were eager to learn with and from each other.

A coordinator with a *resource-managerial orientation* a) draws on their experience and knowledge of the course, department, and institution, b) demonstrates experience teaching and knowledge of teaching the course, c) provides material related to content and curriculum, d) communicates clearly, and e) exhibits strong administrative skills (Martinez et al., 2020). Coordinators demonstrating this orientation also work to provide a coordination system that was informed by course history and departmental/institutional policy. This orientation was exemplified by the course coordinator at Rolling Hills University in the way that he drew on his knowledge and experience of Calculus and teaching to intentionally develop materials in order to provide instructor support and influence instruction. The course materials were rooted in research about student thinking and the conceptually-oriented exams prompted further instructor reflection on their instruction. At Maple State University, the coordinator responsible for course logistics and writing exams demonstrated a resource-managerial orientation to coordination. He was knowledgeable about the institutional and departmental policies and served a critical resource for instructors. This coordinator also demonstrated strong administrative skills, an important component of the resource-managerial orientation to coordination.

Course coordinators are well-positioned to provide instructor on-going support and just-in-time opportunities for professional development that align with both the humanistic-growth and resource-managerial orientations. Specifically because the course coordinator role is often a stable position, with the same individual(s) coordinating for multiple terms. Additionally, many departments that have course coordination systems in place already utilize common course elements, such as exams, homework assignments, class activities, and a suite of available resources. With a coordinator that demonstrates a resource-managerial orientation, these resources could be specifically developed and leveraged to impact instruction. And finally, since instructor meetings are also a relatively common practice, coordinators who exhibit a humanistic-growth orientation could structure these meetings to go beyond the logistics of teaching the course to include professional development activities focused on improving instruction collaboratively. It is important to note that both of these orientations are important and either one (or both) can be leveraged to provide meaningful professional development within the context of coordination, but a combination of these two coordinator orientations is more likely to lead to substantive and sustained opportunities for instructor professional growth.

We close this chapter with four discussion questions that might help mathematics departments to take action by giving their instructors the support that they need to implement high-quality instruction in their courses. Our hope is that this chapter is a source of inspiration for mathematics departments to leverage their coordination systems (or even start coordinating their courses) in order to give the instructors ongoing and just-in-time support as they teach their courses.

Chapter Discussion Questions

1. In what ways is course coordination a feasible way to provide the instructors at your institution professional development at your university?
2. How might your mathematics department support a coordinator to take on an ongoing professional development role?
3. What support might course coordinators need to facilitate professional development that cultivates an equitable and inclusive environment and supports instructors in creating this in their classroom?
4. What might your mathematics department look for when hiring or selecting someone for this position?

References

Alvine, A., Judson, T. W., Schein, M., & Yoshida, T. (2007). What graduate students (and the rest of us) can learn from lesson study. *College Teaching, 55*(3), 109-113.

Apkarian, N., & Kirin, D. (2017). Progress through calculus: Census survey technical report. *Retrieved from Mathematical Association of America website:* `https://bit.ly/PtCCensusReport`.

Beisiegel, M., Gibbons, C., & Rist, A. (2019). Examining obstacles to mathematics graduate students' development as teachers. In (Eds.) Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C., Proceedings of the 41st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. St Louis, MO: University of Missouri

Carlson, M. P., Oehrtman, M., Moore, K., & O'Bryan, A. (2020). *Precalculus: Pathways to calculus: A problem-solving approach.* (8th ed.). Macmillan Learning. `https://www.rationalreasoning.net`

Martinez, A., Gehrtz, J., Rasmussen, C., LaTona-Tequida, T., & Vroom, K. (2020). Promoting Instructor Growth and Providing Resources: Course Coordinator Orientations Toward their Work. In Karunakaran, S. S., Reed, Z., & Higgins, A. (Eds.). *Proceedings of the 23rd Annual Conference on Research on Undergraduate Mathematics Education*, Boston, MA.

Rasmussen, C., Apkarian, N., Hagman, J. E., Johnson, E., Larsen, S., & Bressoud, D. (2019). Brief report: Characteristics of Precalculus through Calculus 2 programs: Insights from a national census survey. *Journal for Research in Mathematics Education, 50*(1), 98-111.

Rasmussen, C., & Ellis, J. (2015). Calculus coordination at PhD-granting universities: More than just using the same syllabus, textbook, and final exam. In D. Bressoud, V. Mesa, & C. Rasmussen (Eds.). it Insights and recommendations from the MAA national study of college calculus (pp. 107-115). Washington, DC: The Mathematical Association of America.

6

Program Assessment & Using Local Data

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Chapter Question: What are some impactful ways to use local data to support improvement of first year STEM mathematics?

6.1 Introduction & National Context

This chapter focuses on the departmental practice of collecting and using local data to assess and improve introductory courses. Mathematics departments often face tremendous pressure from other departments and/or from the administration at their institution to better support the success of students in both STEM and non-STEM disciplines, and to quantify that success. Whether motivated by external factors or their own desire to better serve all of the students that they teach, it is common for mathematics departments to undertake efforts to improve student success by making changes to their programs. These changes can take many forms, from creating new courses, adopting new instructional modes, developing student support programs, or providing professional development for faculty and graduate student instructors (see Chapter 7 for three detailed examples of departmental change initiatives in response to campus pressures and Chapters 2–5 for details about program features at the different sites). In all cases, it is important that these changes be informed by and assessed using locally generated data. Apkarian and Kirin (2017) reported on the results of a 2015 survey of all graduate degree-granting mathematics programs in the US, revealing the following figures (percentages are of responding programs):

- 95% have access to local data for making decisions about their precalculus and calculus courses
- 93% consider local data at least somewhat important for a successful precalculus and calculus program
- 95% consider their collection and review of local data to be adequate for their needs
- 78% report being least somewhat successful at using local data to inform introductory mathematics program decisions

What is meant by local data varies widely across institutions, as does how that data is used. The survey also asked about the kinds of local data that are regularly reviewed to inform decisions about undergraduate programs. Most departments report using measures of student performance in individual courses, such as grades, passing rates, and specific content-focused assessments; most also use student evaluations of teaching to understand their courses. Many departments report using data across courses, such as correlation of grades with incoming placement recommendations, students' success in downstream STEM courses, and program retention patterns.

What data people are looking at is only part of the story; we also consider what people are looking *for* in that data: evidence of successful interventions, indicators for at-risk students, warning signs of program shortcomings, or something else. We observed a variety of approaches to using data at the twelve case study institutions that participated in the Progress through Calculus project. Several institutions monitor their placement process by assessing the relationship between scores and/or method (e.g., SAT scores, transfer credit) and student outcomes to adjust placement criteria or reshuffle content from one course to another in the sequence. Some use student outcomes on common assessment items to identify more and less challenging topics, then use that information to make adjustments to the course and/or support options. In one instance, similar data is gathered from a tutoring center about commonly asked questions and this information is used to structure review sessions prior to high-stakes exams. Most of our case study sites engaged in some kind of sequence-level review including pass rates, persistence, and/or correlation between grades in prerequisite and subsequent courses to identify "problem areas" for further investigation and improvement.

All our twelve institutions shared general goals of supporting students in learning mathematics content, improving pass rates in individual courses, and better preparing students in each course for subsequent courses. Naturally, each program and institution has specific local needs, constraints, populations, and resources. When these features are taken into consideration, it is possible to develop contextually optimal strategies for success which link data, strategies, and goals together (Apkarian et al., 2019; Reinholz & Apkarian, 2018). In the following, we highlight three cases of programs which collected and reviewed local data, for various purposes, and took some action based on that data. Each case illustrates an idea about how data can be used to positively impact student success in first year STEM mathematics. The first case illustrates the idea that data can both motivate and guide an improvement effort. The second case illustrates the role of data in communicating with stakeholders about the impact of an improvement effort. Finally, the third case illustrates the idea of using readiness scores to assess the quality of the program rather than merely as a tool for distributing students among course sequence tracks. In our discussion of these three cases we do not offer our own assessment of student success, but instead focus on the strategies in use by these various sites. Our intention with these discussions, and the associated questions for the reader, is to inspire and support local conversations about ways to collect and use local data to assess and improve student success in introductory mathematics programs.

6.2 Data Guided Change at Dandelion State University

At institutions in which large numbers of prospective STEM students are unable to place into calculus courses, it is important to develop strategies to support these students - who face significant obstacles on their way to pursuing their major area of study. Our first case considers the efforts of a department looking to provide a shorter path to calculus for the large segment of their student population that was being placed into their college algebra course. These change efforts were initially motivated by the analysis of local data and are currently being monitored through a continuing careful study of local data. Our goal in presenting this vignette is to provoke the reader to carefully consider how they might use local data to inform and monitor changes at their institution that could improve the success of their students.

DSU Part 1: Hatching a Plan to Make a Data Supported Change

Dandelion State University[1] (DSU) is a public, comprehensive university which does not have a large STEM program. The majority of incoming DSU students do not place into calculus, and until recently this meant they would need to take a series of three courses (see Figure 6.1) before qualifying to enroll in Calculus 1. The following describes some of the contention at DSU surrounding the preparation for calculus course sequence.

Vignette

Circa 2017, concerns about preparation for calculus courses came from three main groups. Client disciplines (i.e., those whose majors require calculus) were particularly concerned with the length of time required for students to reach calculus. The majority of incoming students had to take the "long track" to calculus, therefore needing two or three semesters to become eligible for calculus, which in turn delayed the start of advanced major-specific work, thus extending time-to-degree and increasing attrition. Also, the DSU administration was raising alarms with the mathematics department about the low success rates (passing rates below 70%) for these same courses. From within the mathematics department, there were concerns that students taking these pre-calculus courses were not being given

[1] All institutions and individuals are referred to using pseudonyms.

enough opportunity to develop "meta-skills," such as "persisting in solving challenging problems." A precalculus redesign committee was formed within the mathematics department, with the guiding principle that "whatever we do, it needs to be something that's informed by data." Eventually, with a push from the (permanent) department chair, they decided on a plan to replace the three three-credit precalculus courses with two four-credit options - despite some concerns that the other problems might be exacerbated by the shortened path to calculus.

Vignette Reflection Questions

1. What kind of data do you think DSU should collect in order to make sure the shorter path to calculus does not harm STEM retention?
2. What strategies could be used to determine whether resulting changes in success rates can be attributed to the changes in the course sequence (rather than unintended differences in grading standards)?
3. What kind of data could be collected to determine the extent to which the redesign achieves the goal of supporting students' development of "meta skills"?

DSU Part 2: What has Happened so far?

Vignette
In Fall 2019, the new precalculus options were implemented. As of yet there are no clear plans to assess students' meta-skill development, but there are now one- and two-semester paths to Calculus 1. As shown in Figure 6.1, there are now two four-credit precalculus courses which can be taken either as semester-long courses or as half-semester courses. This means students can get to Calculus 1 in their second semester either by placing into Precalculus B or taking the two half-semester courses in their first semester. The DSU mathematics department is tracking passing rates and persistence rates (among those who pass) from one course to the next. As shown in Figure 6.1, in the first year of implementation the passing rate was 47% in Precalculus A and 87% in Precalculus B and persistence rates are lower than in the previous semester. Faculty offered two possible explanations for the drop in student success rates. First,

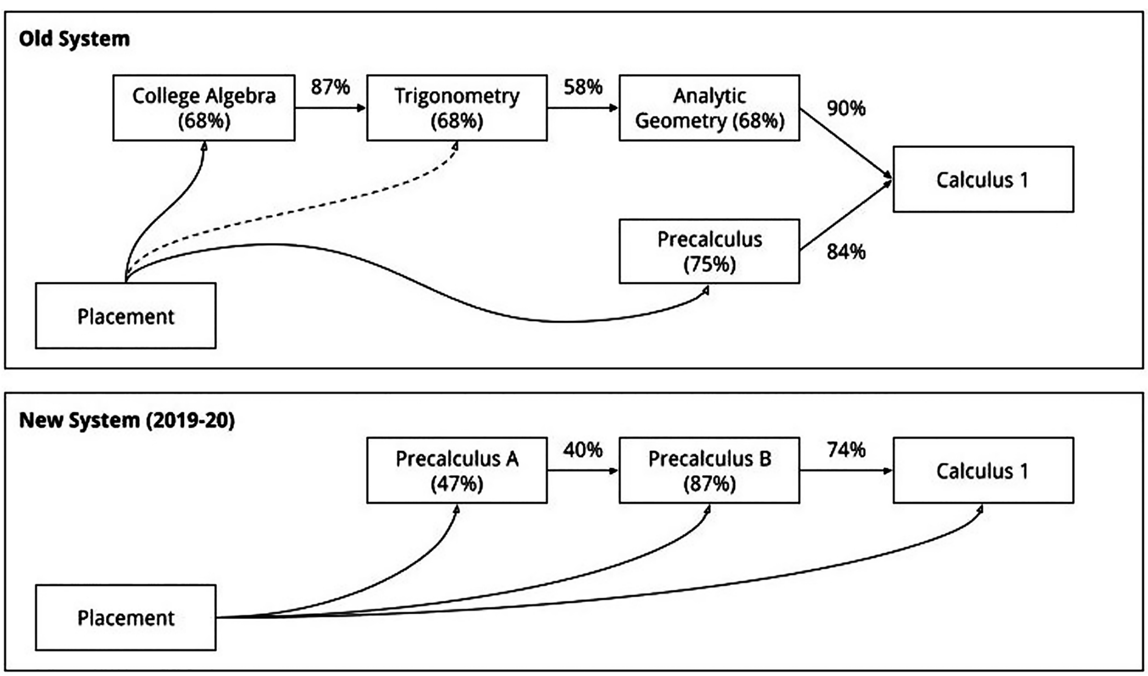

Figure 6.1. Pass rates for each course and persistence into the following course in DSU's old and new system. Passing rates are indicated in parentheticals after each course name. Persistence rates are used to label the arrows from prerequisite to subsequent courses; these indicate the percentage of students passing the prerequisite course who enrolled in the subsequent course in the sequence.

the new courses may be overstuffed and expect students to learn too much, too quickly. Second, some instructors may have increased their grading standards to avoid lowering mathematical standards for rigor. These low passing rates are of serious concern to the department, and they plan to collect and review additional data and revise the courses to achieve their goals of a shorter path to calculus without increasing STEM attrition.

A few caveats must be considered before interpreting these results, which illustrate some of the challenges associated with using data to guide decision-making and change initiatives. First, the data set collected from the older system is more robust because it includes students who enter the track in the Fall and Spring terms, whereas the data for the new system only includes students who started in the Fall. Second, the persistence data for the old model allowed two semesters to enroll in the following course, while it was only possible to check for enrollment in the following semester for the new model. Finally, the second semester under the new model was Spring 2020 when the COVID-19 pandemic interrupted higher education across the United States, which likely impacted the results in important ways. Nevertheless, the data collected so far has already raised important questions for the department to consider as they continue their efforts. Specifically, the data suggests the need to find new ways to measure student learning in order to determine whether the lower passing rates are a function of lower student performance or a result of higher grading standards.

Vignette Reflection Questions

1. What strategies could the department use to figure out why the pass rate was so low in Precalculus A?
2. How should the department deal with the fact that COVID-19 probably had an impact on their Precalculus B numbers?
3. Given that the department does not appear to have any baseline data regarding how the old model impacted students' development of "meta skills," are there ways they could generate data that could help them learn whether their redesign is having a positive impact on the development of such skills?

Summary and Discussion

Dandelion State University faced a situation that is probably familiar to many mathematics departments. Client disciplines were unhappy with the number of mathematics courses students needed before starting calculus, while the administration was unhappy with the success rates in those courses. At the same time, the faculty were concerned that these courses were not supporting students in developing meta-skills such as persistence in solving challenging problems. The results (so far) of their efforts to study the impact of their redesign probably also resonate for many readers. They were able to generate data in the form of rough measures (pass rates and persistence) to allow some comparisons. However, they have been unable so far to develop a good plan for assessing whether they are meeting their goal of supporting the students' development of meta-skills. Further, reality is resisting their efforts to measure the impact of the redesign. They are limited in what they can learn based on this first year of implementation because they do not yet have enough longitudinal data to make good "apples to apples" comparisons and because it is likely not possible to control for the impact of the COVID 19 pandemic on the Spring 2020 data. Nevertheless, they do have some useful data that can potentially guide them as they continue to refine the redesign and their assessment plan.

The DSU vignette provides a context to consider a number of issues related to using data to guide change efforts. The vignette was shared not to provide a model for how this should be done, nor was it shared as a cautionary tale. It is intentionally shared as a work in progress with the hope that thinking about the first steps taken at DSU may help the reader think about how they might imitate a process of using data to guide a change effort at their institutions, and how to build on those first steps to support and document a successful change effort.

Discussion Questions

1. Is there a course or course sequence that your department is (or should be) working to change? What data suggests that change is needed?
2. The DSU data is (or is on the way to becoming) useful for assessing the impact of the redesign ***after*** the initial implementation. Are there some strategies that your department could use to leverage local data to inform and support a change effort before and/or during the process of developing the new approach?

3. DSU generated important information about the success rate in their Precalculus A course, but this information raises significant questions (e.g., are the rates best explained by poorer learning outcomes or by inadvertent changes in grading standards in the new course?). Are there some strategies that could be used to increase the chance that your data collection efforts will produce good information regarding the impact of your change effort?

4. DSU has not yet been able to measure their goal of supporting students' development of "meta skills." What are some goals that you would have for a change at your institution, and how could you go about making sure those goals are addressed by your data collection and assessment plan?

6.3 Communication with Client Disciplines at River Rock University

Many of the students who take first year STEM mathematics courses are (or intend to be) majors in client disciplines such as engineering. These courses can be sources of tension between a mathematics department and the departments that host these majors. Client discipline concerns can include pass rates, impact of prerequisite mathematics coursework on student progress through the major, and how well (or poorly) students appear to understand the mathematics needed in their field. Communication between the client disciples and the mathematics department can help to alleviate these tensions. While it is common for client disciplines or administration to use local data (e.g., low pass rates) when expressing dissatisfaction to the mathematics department, in this section we highlight a different (more proactive and productive) use of local data: The use of local data in communicating successes and ongoing improvement efforts to administration and the client disciplines.

RRU Part 1: Documenting and Communicating Success

As detailed in Chapter 7, River Rock University faced a very serious threat stemming from the College of Engineering's dissatisfaction with first year calculus. The Engineering college felt that students were being inappropriately placed and were receiving poor instruction, resulting in students taking too long to complete mathematics prerequisites. Feeling that their ability to move students through their programs was at risk, the College of Engineering threatened to offer their own calculus courses for their students. The department responded to this threat by creating an engineering calculus sequence that essentially compressed three semesters of calculus into two (shuffling a few topics into advanced courses) and implementing additional application problems. This extensive undertaking was supported by substantial financial resources including the hiring of a career line faculty member, Joshua, to create and maintain the sequence. Chapter 7 discusses many details of the change process, while here we focus on Joshua's use of local data to assess and improve the engineering calculus sequence, and the role these efforts played in improving communication with the college of engineering.

Vignette

One of Joshua's key responsibilities is to facilitate productive and ongoing communication between the mathematics department and the College of Engineering about the engineering calculus sequence. A critical feature of this communication is the sharing of assessment data and ongoing improvement efforts. Each year, Joshua presents a detailed annual report to a dedicated calculus committee within the College of Engineering. There is near universal agreement from stakeholders on all sides that the new program is very successful. The Dean of Sciences also reported that "everybody's very happy" about the engineering calculus sequence. The mathematics chair who initiated the changes reported that "the Dean of the College of Engineering [made] a point to come up to me afterwards and we chatted nicely and he said how happy he was with how this all worked out, so it's all very gratifying." The current department chair explicitly explained the success of the sequence in terms of the assessment efforts saying "assessment is done really vigorously and you can see it in the grades actually. It's very clear." Interestingly, the pass rates in the engineering calculus courses are actually lower than those in the standard option.

Vignette Reflection Questions

1. What kinds of data do you think were collected at RRU in order to assess the success of the engineering calculus sequence? What kinds of results would explain the lack of alarm in response to low pass rates?

2. When we visited RRU we were struck by how universally the engineering calculus sequence was perceived as a success. What kinds of communication strategies do you think were necessary to create this kind of consistent perception, and how much effort do you imagine was involved in executing those strategies?
3. In addition to grades, pass rates, and flow/persistence rates, what kinds of data regarding the engineering calculus sequence do you think the College of Engineering would find compelling?

RRU Part 2: Efforts to Communicate Standards

Vignette

Joshua's primary efforts go toward setting and communicating appropriate standards for the engineering calculus sequence. He noticed tension between engineering faculty who wanted high standards and administrators who were focused on the number of engineering graduates. In response, Joshua engaged in "real rigorous assessment so [they] can make claims with some basis and not just anecdotes." So, rather than presenting pass rates, he collects data about students' understanding of specific concepts (e.g., Taylor series) that are particularly important for the engineering students through 3-6 common final exam items. Student performance on these items (basic and advanced skills) was compared to their final grade using measures of agreement and bi-directional predictive power. Among the results, he discovered that students had been passing the traditional calculus course without understanding polynomial approximations or being able to compute error estimates. He has emphasized how subsequent changes have ensured that students "can't sneak through" without understanding these important topics. Thus, while there are lower pass rates in calculus, the same students are passing the downstream partial differential equations course at extremely high rates.

Vignette Reflection Questions

1. What are the benefits, in terms of communicating with client disciplines, of collecting more fine-grained assessment data in addition to rough measures like pass rates and persistence rates?
2. What are the challenges involved in maintaining and communicating standards, and what role can the collection and analysis of local data play in facing these challenges?
3. What are some of the challenges involved in going beyond pass rates and conducting fine-grained nuanced assessments, and what are some strategies that can be used to overcome these?

Summary and Discussion

While pass rates and grades are important kinds of outcome data to consider when evaluating the success of a program, we argue that it is important to include sources that make it possible to paint both a larger picture and a more detailed picture. At RRU, Joshua (who was also the course coordinator) looked at an entire five course sequence to understand the impact of the engineering calculus sequence. This allowed him to communicate that the low pass rates in the engineering calculus classes were accompanied by a two-semester reduction in time engineering students needed to complete their required mathematics. At the same time, he was assessing specific concepts he saw as important to engineers and working to ensure that course grades reflected the students' understanding of those concepts. The goal was to document whether the engineering calculus sequence was accomplishing its objective which was to provide a shortened path through the mathematics coursework while maintaining high standards.

We shared the story of the RRU mathematics department's efforts to support their communication between the college of engineering with rigorous monitoring of local data for two reasons. First, we wished simply to highlight the value of such communication. The universal perception that the RRU engineering calculus sequence is highly successful can be largely attributed to the aggressive monitoring of local data and consistent communication of the findings. Second, we hoped to challenge the reader to think about how to go beyond grades and pass rates to provide a more comprehensive understanding of a program and facilitate better informed and more nuanced communication about it.

Discussion Questions

1. In your experience at your institution, what are some ways that discussions of grades pass rates can "go off the rails" when used to discuss a course or sequence of courses?

2. Thinking about the clients served by the mathematics courses at your institution, what kinds of data do you think they would find most compelling when attempting to understand whether your courses are serving their purposes?

3. What are some strategies that could be used at your institution in order to understand how well pass rates and grades reflect the students' understanding of the mathematics they need to succeed in their chosen major?

6.4 Placement and Prediction at Sandpiper University

Determining the appropriate mathematics course for students to take, generally referred to as "placement," is considered a critical aspect for successful precalculus-calculus programs (Apkarian & Kirin, 2017; Rasmussen et al., 2019). Most institutions leverage multiple data sources about incoming students to make enrollment recommendations, often using some data to determine whether students are ready for single-variable calculus and where in the calculus sequence they should begin. The most commonly used data source for placement is students' scores on the AP test, used by 71% of all surveyed mathematics departments. Results of the standardized SAT and ACT exams are also common (53%). Internally designed, department-specific assessments are common in PhD-granting departments (60%) but less so in MA/MS-granting programs (28%). Despite being considered an important aspect of a successful program, fewer than half (43%) of all programs regularly review data about adherence to placement recommendations to inform decisions about their program (Apkarian & Kirin, 2017). We use the case of Sandpiper University to illustrate a robust approach not only to assessing student readiness, but to monitoring that assessment process, setting goals, making adjustments, and supporting student success.

SU Part 1: Initial Assessment

Sandpiper is a small, private, research university with a technical focus. All Sandpiper students take a mathematics course in order to graduate, and the majority of students take Calculus 1 in their first term of their first year while simultaneously taking Physics 1 and General Chemistry 1. While a high AP exam score can place a student into Calculus 2 or Calculus 3, the main placement mechanism is a department-designed readiness assessment which all students take at home in the spring before matriculation. The scores are used to recommend summer activities in preparation for enrolling in Calculus 1 in the fall. In conjunction with the mathematics readiness assessment, the Force Concept Inventory (Hestenes et al., 1992) is used to assess students' readiness for Physics. The following vignette describes the situation at Sandpiper circa 2011, when the university implemented a major assessment program.

Vignette

For years, Sandpiper's mathematics department implemented several strategies to support first-term students taking Calculus, and tweaked them regularly. Based on the results of the mathematics readiness assessments, students were told if they were (a) ready to take Calculus 1 in the upcoming fall; (b) should participate in some kind of refresher or brief review before the fall term; or (c) should take a full precalculus course over the summer. Once enrolled, students had the option to enroll in a one-credit co-calculus course, an option which was required of those who struggled to pass gateway exams during the first month of the course. Circa 2011, Sandpiper University contracted with an external educational consulting firm to build a logistic regression model for predicting student success and identifying at-risk students early on. For their purposes success was primarily defined by grades and first-year fall-to-fall retention; variables included measures of prior preparation, demographic information, and student self-efficacy and motivation assessments. In the very first model, SAT Math scores were highly predictive, but the significance of these scores were eclipsed when the mathematics and physics readiness assessments were added to the model. In the first 'complete' model, scores on the mathematics readiness assessment were the single best predictor of first-year success and retention - regardless of summer remediation strategies.

Vignette Reflection Questions

1. Do you think that it is good or bad that the readiness assessment was the best predictor of success regardless of summer remediation strategies? Why?

2. Sandpiper took this information and decided to make some changes. How could they assess whether or not those adjustments improved student success?

SU Part 2: Updates and Evaluation

Vignette

At Sandpiper, the general interpretation of this model was that their first-year program was insufficiently supportive of students. If pre-matriculation mathematical knowledge was more impactful than other factors, then they were not sufficiently supporting students' development of additional knowledge. Several potential contributing factors were identified, including: (1) inadequate or under-utilized summer preparation/remediation for Calculus 1; (2) insufficient support for students taking Calculus 1; and (3) the double-threat of simultaneous Calculus 1 and Physics 1 in the first term of the first year. Steps were taken to address all of three of these factors. For the first factor, the recommended pre-matriculation preparation options were carefully reviewed and amended. Access to summer preparation programs offered by Sandpiper was expanded and the content of those programs was adjusted to better align with Sandpiper's Calculus 1. Further, the benefits of additional preparation were more emphatically communicated to incoming students. For the second factor, adjustments were made to an existing co-calculus course for students so that it became a "test-out" instead of a "fail-in" format, and to better function as "just-in-time" support for students with less proficiency in algebra procedures (see Chapter 3). For the third factor, a "delayed physics" program was implemented (pushing Physics 1 to the spring term), additionally motivated by the fact that nearly 30% of first-year students failed Physics 1 and retook it in the spring term. Students with low scores on both the mathematics and physics readiness assessments are now funneled into that option, and other introductory STEM courses have been revised so that the impact on time-to-degree is minimized. By the time of the first PtC site visits, in the fall of 2017, these programs were fully implemented. The external models have been repeated approximately annually, and "the last two or three times, neither math nor physics readiness appears in the seven most critical factors."

Vignette Reflection Questions

1. How do you think your department would react if your readiness or placement exams were found to *not* be predictive of pass rates or first year retention rates? Would they see this as a good thing?
2. Which of the changes made at SU do you think were the most helpful (and why) in supporting students with lower readiness scores?
3. What should Sandpiper continue to monitor in order to ensure that their calculus program supports student success?

A Sandpiper faculty member explained their interpretation of the changing role of the readiness tests in the student success model and the adjustments which have been made to the program: "[we're] not saying [the readiness tests] don't matter, what [we're] saying is what we've done to counter this has worked and is working, so [we're] saying keep doing it." This overall sentiment was echoed by a member of the institutional research office, who noted that "the readiness exam *was* important, and they're using that to identify students who should delay physics [...] but that is no longer a key indicator in identifying students that aren't going to be retained, so that that would be a key thing to show that, you know, what they're doing is working."

Summary and Discussion

Placement procedures, which often include some kind of entrance exam, are used by most mathematics departments to determine what mathematics course entering students are prepared for (Apkarian & Kirin, 2017). At Sandpiper, it is not so much which course students should take, as there are few options and most take Calculus 1 when they enter the university, but the readiness exams and early term performance determine when, and with what kinds of support, introductory courses are taken. What we draw the reader's attention to is the way in which the placement procedures and support options were assessed using local data.

Across the twelve case study departments discussed in this volume, most analyzed the effectiveness of their placement procedures by looking at the relationship between placement and student success. However, Sandpiper was unique in their approach. They did not make changes to the readiness exam itself, nor to the assessment standards of Calculus 1, but rather to the program, course order, and student supports, in a concerted effort to reduce the predictive power of that exam on student success in their first year at the university. In this way, they have developed a mathematics program which supports each and all of their students and compensates for variation in students' prior preparation. This line of reasoning, that first year student outcomes should not be predictable from pre-matriculation scores, reflects a student-centered program which strives to level the playing field and support success.

Discussion Questions

1. What are some ways the data generated by your placement procedures could guide your efforts to improve your students' experiences in first year STEM mathematics?
2. How predictive are standardized test scores, or your institution's placement test, for student success and first-year retention?
3. What could you do to improve your first year STEM mathematics program so that students' incoming preparation does not determine their level of success?

6.5 Chapter Summary and Concluding Remarks

All three universities discussed in this chapter regularly collect and review local data for assessing and improving their introductory mathematics course sequence. Each embodies a different idea for productively using local data to make decisions about how to improve introductory mathematics courses at university: collecting nuanced data with clear intentions for analysis; using data to communicate with stakeholders across campus; and thoughtfully considering implications of those analyses and aspirational goals. Collectively, these are powerful tools for making locally-informed decisions.

It is important that change efforts be supported by the collection and analysis of local data. However, one can look at many different kinds of data to address many different kinds of questions related to the success of a change effort. While attending to pass rates and persistence is crucial, these rough measures can sometimes defy efforts to draw comparisons and often leave many questions unanswered. Expanding the use of data to include more nuanced information about what is going on currently and the impact of the change is likely to be well worth the effort needed to do so. Departments should commit time to deciding what they really want to know and to thinking through an approach that will give them the best chance to get good answers. And departments should expect their initial efforts will likely not produce clean answers and that additional work will be needed to really understand the impact of their change efforts.

Data should be an important aspect of the communication between a mathematics department and its client disciplines. If the mathematics department takes seriously the task of monitoring their client-serving courses and regularly communicates the results to the clients, it is much more likely that those clients will be satisfied with these courses. Ideally these efforts should capture both the larger picture (allowing the client to understand how the courses are impacting the flow of students through their programs) and the details (allowing the clients to understand how well students are learning the mathematics). These kinds of supportant data can go a long way toward getting beyond opinion-based arguments about what an appropriate pass rate is, and whether those pass rates reflect grading practices or student learning.

Once data has been collected and analyzed, the interpretation of those outcomes is critical when making decisions about the health of a program, what changes to make, and assessments of the impacts of any changes made to the program. When data is considered as part of a broad assessment of student success, inventive solutions can be found which go beyond the traditional placement and course sequence. Taking up the charge to support students, and thus to provide each student with comparable opportunities to succeed, implies that students' prior preparation as measured by readiness exams must not determine their likelihood of continuing. This approach, of course, must be informed by local needs, constraints, and resources.

References

Apkarian, N., & Kirin, D. (2017). *Progress through calculus: Census survey technical report*. Mathematical Association of America. `http://bit.ly/PtC_Reporting`

Apkarian, N., Kirin, D., Vroom, K., & Gehrtz, J. (2019). Connecting the stakeholders: Departments, policy, and research in undergraduate mathematics education. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. it The Physics Teacher, 30(3), 141–158. `https://doi.org/10.1119/1.2343497`

Mengel, F., Sauermann, J., & Źolitz, U. (2019). Gender Bias in Teaching Evaluations. *Journal of the European Economic Association, 17*(2), 535–566. `https://doi.org/10.1093/jeea/jvx057`

Rasmussen, C., Apkarian, N., Hagman, J. E., Johnson, E., Larsen, S., Bressoud, D., & Progress through Calculus Team. (2019). Characteristics of Precalculus through Calculus 2 programs: Insights from a national census survey. *Journal for Research in Mathematics Education, 50*(1), 98–112.

Reinholz, D. L., & Apkarian, N. (2018). Four frames for systemic change in STEM departments. *International Journal of STEM Education, 5*(1), 3. `https://doi.org/10.1186/s40594-018-0103-x`

Uttl, B., White, C. A., & Gonzalez, D. W. (2017). Meta-analysis of faculty's teaching effectiveness: Student evaluation of teaching ratings and student learning are not related. *Studies in Educational Evaluation, 54*, 22–42. `https://doi.org/10.1016/j.stueduc.2016.08.007`

7

Institutional and Departmental Change

Responding to Crisis

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Chapter Question: How can departments respond to credible threats to take calculus away from the math department? Can such threats be avoided?

7.1 Introduction

Among the 12 department case studies, three stories of crisis and recovery stood out. Three departments had, within five years of our study, faced a very real threat of losing calculus teaching to another campus unit; all three came back from the brink, as it were, and now have programs that are better meeting the needs of their students and client disciplines counting on these courses. The idea of removing calculus to another unit on campus is not that new, and in fact received some national attention when the 2012 President's Council of Advisors on Science and Technology (PCAST) report recommended "experiments" of having lower-division mathematics courses taught by other departments (PCAST, 2012). The National Research Council noted this, and generally agreed that the "existing mathematics curriculum would benefit from a significant updating of both content and teaching techniques. There is a real chance that if mathematicians do not do this, others will, and that could exacerbate the erosion in mathematics service teaching" (NRC, 2013, p. 149). At the three sites detailed in this chapter, that chance turned into a high probability. As we describe the challenges at each site, and how they addressed the issue, we raise discussion points for those dealing with the same scenario as well as those aiming to avoid such crises.

This chapter focuses on three similar cases of change in which the weight of concern, particularly from engineering departments, pushed university administrators to consider the removal of calculus from the mathematics department. In these three cases, some resolution was reached through negotiations and the introduction of an actor in a new role. In this chapter, we will share: the instigating problems that lead to the threat, the nature of the threat/demand for change, and the departments' responses. In presenting the departments' responses, we will focus on the position and roles that were created, the changes that were implemented, and the follow-up efforts to monitor and report to stakeholders outside of the department.

7.2 River Rock University

Problems

Circa 2010, RRU changed their placement system for first-year students and began enforcing prerequisites. Many students who would have been placed into Calculus 1 under the earlier system were placed instead into precalculus courses, and all students were prevented from 'skipping' courses, as had been somewhat common practice. As a result, engineering students' time-to-degree increased at a time when the engineering college was facing pressure to increase graduation rates. Frustrations with the delays led to increased complaints from students and increased scrutiny of the courses themselves, which revealed a lack of teacher preparation for the introductory courses. Complaints were escalated up to the Provost's office. The general message coming from the engineering college to the mathematics department was: "you're not placing our students appropriately and you're not teaching calculus in a way that's good for our students, so we're going to teach it ourselves" (RRU, Sp19)[1]*, and the mathematics department promised to "fight them tooth and nail" (RRU, Sp19) to keep control of calculus. This was in no small part because of the financial implications of teaching the high enrollment service courses, and because even though the engineering college was perceived to have the power to take over calculus, mathematics had the practical experience and infrastructure necessary for teaching it.*

Vignette Reflection Questions

1. What caused the threat of losing calculus to emerge in this vignette?
2. What solutions can you envision?

Strategizing

Determined to show that they were serious about keeping their calculus courses, representatives of the mathematics department met regularly with stakeholders in engineering for several months to negotiate a plan. The engineering department clearly outlined their needs during early negotiations: get students through calculus faster, "fix" placement, and include more applications relevant to engineers. A structural solution was envisaged, involving the development of a new mathematics sequence to decrease the time needed for engineering students to cover requisite material. While cooperating with the engineering college, the mathematics department chair worked with the deans of both colleges and the university president to obtain the resources necessary to support their demand - in particular, funding to support a position overseeing the development and management of the new course sequence.

Vignette Reflection Questions

1. From the RRU math department's perspective, what might be gained from implementing the proposed solution? What might be lost?
2. What evidence might the mathematics department chair and other leaders have used to make the case for funding the new position?

Making Changes

In 2012, Joshua was hired as a new member of the mathematics department and tasked with developing and coordinating the new calculus sequence. He met with a committee of stakeholders to create courses that would satisfy engineering's accreditation requirements, maintain mathematical rigor, include applications, and reduce time-to-completion. As a result, the curriculum for engineering was trimmed and rearranged to streamline three semesters of calculus into two semesters (some calculus content was moved to subsequent differential equations courses), lab sections were created where students would work in small groups on applied problems, and more effort was expended on teaching preparation. These went into effect circa 2013. The changes also impacted the teaching experiences of mathematics graduate students, as first-year graduate students went from teaching their own classes to leading lab sections for engineering calculus.

[1] We use this notation to refer to the interview from which the information came. In this case, the information was gathered from an interview conducted in Spring 2019 (Sp19) with a staff member at River Rock University (RRU). We will use "Fa" to refer to an interview conducted in the Fall, for instance "Fa18" would refer to an interview conducted in Fall 2018.

Vignette Reflection Questions

1. For engineering students, what might be some of the benefits of the new calculus sequence? What might be some of the drawbacks?
2. For instructors, including graduate student lab section leaders, what might be some of the benefits and drawbacks?

Success and Monitoring

Overall, the changes are perceived to be successful at RRU. Students' progress has been monitored by the math department for several years, and the students as well as the dean of the engineering college are happy with the results. In the engineering college, one measure of success is seen as particularly informative: in downstream courses, students who complete the engineering calculus sequence are outperforming their peers in non-engineering STEM majors who complete the traditional calculus sequence. At RRU this is attributed in part to the new lab component, which provides additional contact hours and active engagement in engineering applications. Concerns about time-to-completion have also been alleviated, as the shortened and reorganized sequence allows engineering students to complete their programs 'on time.' These data are regularly collected at RRU and are used (a) to ensure continued funding and (b) to facilitate communication and oversight of the sequence. The engineering coordinator (Joshua) serves as a liaison between math and engineering, acting as a broker between communities. He hosts annual meetings with a committee from the engineering college to share data and solicit feedback. These meetings ensure that math standards are kept high as perceived by the math department while still meeting the needs of the engineering department. By keeping multiple stakeholders in communication, potential issues are recognized, discussed, and addressed in a timely manner.

Vignette Reflection Questions

1. Using what you know about this case, what are some examples of potential issues that could be discussed in the annual meeting with the engineering calculus committee?
2. What strategies could engineering and mathematics use to work together to address these issues?

Summary and Application

Faced with the threat of losing calculus, math department actors at RRU were determined to work with their peers in engineering to generate a solution aligned with the complexities of the problem, the institutional context, and competing disciplinary perspectives. Working together, the team quickly recognized that the math department appeared to prioritize mathematical rigor, while engineering seemed most concerned about accreditation requirements, time-to-degree, and applied learning opportunities. With these collective criteria in mind, the group began to develop a solution, and the math department chair and other institutional leaders provided adequate time, funding, and human resources to support this process. Ultimately, the solution included a new sequence for engineering majors, improvements to graduate student teaching preparation, tweaks to the placement process, and hiring a calculus coordinator to develop, implement, and monitor these changes. The math department also now tracks completion rates and downstream performance, and there is an "early warning" system to identify problems and their root causes (e.g., students skipping classes) and make timely changes to prevent future crises. In time and with careful monitoring, the changes produced results that extinguished the threat of losing calculus and satisfied the concerns of students, instructors, the engineering department, and administrators.

Discussion Questions

How might you apply lessons learned from this case to your own department?

1. If you were under pressure to make changes to your calculus sequence, what changes might you be willing to consider? What changes would you be most resistant to considering?
2. If you were thinking of making substantive changes to your calculus sequence, who outside of the mathematics department might need to contribute to these discussions? Why?
3. What sorts of data might you need, and from whom, to make a funding request to support proposed changes to your calculus sequence?

4. What information would you like to have from client disciplines in order to prepare your students better for the client discipline courses?
5. How effective do you think it might be to meet regularly with client departments to uncover, discuss, and respond to potential issues related to the calculus sequence? What do you think would need to happen for these meetings to be productive?

7.3 Rolling Hill University

Problems

At RHU, the mathematics department had a reputation for being inflexible, refusing critique, and resisting change. Having persisted for roughly a decade, this morphed into a campus-wide belief that members of the mathematics department simply did not care about teaching or students. Complaints and concerns, if acknowledged at all, were generally met with statements that the mathematics department was under-resourced and with requests for additional funding. Yet, it was not clear how well the resource concern was understood across campus, nor what strategies had been attempted to resolve the concerns. There were particularly fraught concerns about precalculus and calculus courses, which had low passing rates and functioned as barriers to degree progress for many students—in particular those majoring in engineering. Long-simmering concerns came to a head circa 2010 when a major RHU donor became aware of the situation and threatened to withhold future donations until the situation improved (i.e., removing degree progress barriers and increasing engineering graduation rates). In response, a small committee, which included three mathematicians, reviewed the data and concluded that there were indeed problems with DFW rates, instructional quality, and placement practices. The Provost told the mathematics department that they had two years to resolve these issues, or else the department would be converted to a minimal research unit with no graduate or undergraduate degree programs, and all mathematics teaching responsibilities would be "farmed out" (RHU, Fa17, Sp18) to other units.

Vignette Reflection Questions

1. What brought the "simmering concerns" about calculus to a boiling point at RHU?
2. What are some similarities and differences between the problems at RHU and RRU?

Strategizing

Faced with what appeared to be an insurmountable task—fixing a decade's worth of problems in two years' time and without much in the way of funding—the department turned to their most influential member, Don, for help. At the time, he was away on a research fellowship and unaware of the building crisis. Don had previously been the mathematics department chair, and had good personal relationships with many in the RHU administration. Moreover, his presence in multiple communities allowed for the potential to broker across disjoint communities with dissimilar values. Don immediately leveraged his connections to get in touch with the Provost and some of the most critical donors from afar, convincing them that he could lead the department in solving the problems—but they would need some time and some financial support. Eventually, it was agreed that Don would return as department chair (replacing the current chair before that term was up) and be responsible for developing and carrying out a concrete plan.

Vignette Reflection Questions

1. What about the situation at RHU indicated that bringing Don back on as department chair might be helpful?
2. What could be some drawbacks of bringing back a former chair, like Don?

Making Changes

Don began immediate work to change the "face" of the department. One of his first moves was to appoint a new associate chair, Fred, who was an assistant professor. Don and Fred were both already well-known and respected faculty members in the program, in the department, and across colleges at RHU. Such recognition facilitated the acceptance of the changes that they introduced. Together they set out to: (1) change the impression that the department *is unfriendly and uncaring toward students, and (2) improve DFW rates by supporting student learning in introductory courses. The first goal involved primarily public relations, so Don instituted a new department motto of "don't be a jerk" (RHU, Sp18), asking members to cooperate and work with others on campus. For the second, a detailed three-year plan was negotiated which involved revising the placement system, implementing supplemental instruction, and developing co-requisite supports. These goals were to be supported by hiring mathematics education specialists to help redesign and coordinate courses, increasing the number of teaching assistants, and creating a new learning center, for which they obtained considerable additional funding. These plans were all implemented by the end of Don's first five-year term as chair, after which he was re-elected.*

Led by Don and Fred, the department opened up to cooperating with other stakeholders and changing their departmental practices. Efforts were made to communicate the new messaging and programs to various stakeholder disciplines, administrators, student affairs professionals, and academic advisors, and to take their advice into consideration as courses evolved. Conversations about calculus shifted to focus on mathematical skills and conceptual understanding rather than simply passing rates. RHU stopped using ALEKS as the placement exam, replacing it with in-house readiness tests that assessed students' proficiency with different areas. These new tests allow for more refined placement, including directing students into sections of the introductory courses with additional co-requisite contact hours rather than remedial pre-algebra courses. The additional contact hours emphasize conceptual understanding and provide just-in-time mathematical support.

Vignette Reflection Questions

1. In what ways might the more structural changes have aligned with the department's new motto of "don't be a jerk"?
2. Given what you know about RHU from these vignettes, which changes do you anticipate would be most successful in resolving the situation?

Success and Monitoring

During the first year of Don's return to RHU to make changes, the Provost who issued the initial threat was replaced, but those involved continued to work for improvement. Now the status of the department seems to be stable, with improved DFW rates, specifically in the introductory courses. As part of the improvement plan, more attention is being paid to local data, which has revealed that first-generation college students at RHU have particularly benefited from the changes thus far. These successes were documented and shared outside RHU, and several members of the mathematics department are now heavily involved in statewide efforts to increase student success in postsecondary mathematics. As the (new) Provost noted, "the reputation that they have on campus and off, I mean, they - they've become, you know, state and national leaders in how to implement placement and co-requisite[s] and how to be nice" (RHU, Sp18). Additionally, the new openness to collaboration across departments has resulted in sustained communication about the new mathematics program across stakeholders, leading to courses that are better aligned with the needs of multiple degree programs at RHU. Administrators and donors are pleased with the improved passing rates as well as the increased status of the RHU mathematics programs.

Vignette Reflection Questions

1. How did each of the strategies contribute to successful changes at RHU?
2. Several types of evidence suggest that the changes at RHU have been successful. Which types of evidence seemed to be most compelling to different stakeholders?

Summary and Application

RHU's mathematics department had a decade-long reputation of resistance to change and not prioritizing students or teaching. Precalculus and calculus courses had low passing rates and prevented degree progress, especially for engineering majors. After "simmering" for a decade, these issues boiled over when a major donor threatened to withhold future gifts and a review committee confirmed problematic placement practices, DFW rates, and instructional quality. Under threat of losing their programs, the department quickly instated Don as chair, hoping he could leverage his good relationships across campus to negotiate time and support. Along with associate chair Fred, Don managed to secure funding and oversaw an initiative to change the face of the department. This initiative was supported by the positive relationships Don and Fred each had with important stakeholders and decision-makers across campus. The department adopted a new motto, “don’t be a jerk” and opened up to cooperation and change with those outside the department. New efforts to listen and cooperate with stakeholders across campus informed changes specifically aimed at improving students’ experiences and opportunities to learn calculus, which eventually broadened into state-level education initiatives. The Provost whose actions pushed the department to start this process moved on, but the initiatives continued unabated. With an improved reputation for collaboration and openness to change, courses aligned with multiple degree programs, and donors and administrators (including the new Provost) impressed with the growing status of mathematics programs and improved passing rates, the mathematics department at RHU has experienced sustained success as a result of the changes they made.

Discussion Questions

How might you apply lessons learned from this case to your own department?

1. Generally, to what extent does your department have a reputation for instructional quality, cooperation, and openness to change? If you don’t know, how might you find out?
2. Specifically, to what extent do you believe this case resonates with how your department is perceived by those who depend on your calculus sequence?
3. How, if at all, have you or others in your department tried to address how others (e.g., client disciplines, students) perceive your department?
4. If threatened with losing calculus, how might you identify the members of your department who are best suited to strategize about, implement, and evaluate change efforts?
5. If you made changes at your institution to respond to a threat of losing calculus, which types of evidence of success might be most compelling to different stakeholders at your institution (e.g., client disciplines, upper administration, students)?

7.4 Desert Bloom University

Problems

As with all three of these cases, there were concerns across the university about passing rates in the Precalculus through Calculus 2 sequence. At DBU this was particularly pronounced in the case of the engineering college, with calculus being the largest cause of attrition among first-year majors. The mathematics department generally took the position that the students in the engineering college, not the department’s courses, were the source of the problem—and if the engineering college wanted their students to see applications, they should do it in their courses. The low pass rates (e.g., an 800-person calculus course where only seven A’s were awarded) were well-known enough that academic advisors began recommending that students take calculus elsewhere and transfer the credits. After simmering for years, the situation erupted after a few particularly disastrous terms where Precalculus through Calculus 2 sequence passing rates slipped below 50%, and the Provost’s office decided that something had to be done. An earlier, somewhat similar, challenge on campus involved a first-year writing seminar—that course was removed from the purview of the English department and set up a dedicated center to house that course. Therefore, when the Provost made it known that he was setting up a task force to decide what to do about calculus, it was understood that removing the course from the mathematics department was a true possibility.

Vignette Reflection Question

1. Based on this vignette, what are some potential benefits and drawbacks of separating out calculus from the math department for (a) the math department; (b) engineering students; and (c) engineering faculty members?

Strategizing

At DBU, the Provost’s threat to take calculus courses away from the math department was presented as a last resort, with the hope that mathematics, engineering, and administration could negotiate a solution that kept the calculus *courses under the purview of the mathematics department. This process started with a series of meetings including members of the engineering college, the math department, and the Provost's office, where each party blamed the other: "[There were] three meetings of us yelling at each other before people calmed down and said, 'okay, what are we gonna do?' " (DBU, Fa17). Eventually, it was decided to make an external hire for a new position, one dedicated to overseeing the calculus program. The mathematics department would house the new hire, but while the search committee was formally run by three mathematicians, they were joined by two other members of the task force.*

Vignette Reflection Questions

1. What about the situation at DBU indicated that it may be helpful to hire someone external, rather than appointing someone currently in the mathematics department?

2. What could be some of the drawbacks to this decision?

Making Changes

A staff appointment (renewable five-year contract) for "Director of Calculus" was created through the Provost's office, which had the effects of (1) not replacing a research faculty line; (2) removing the hurdle of tenure for the director; and (3) providing the administration with continued oversight. While the directorship did not take the place of a faculty line, it was heavily implied that the mathematics department would have a hiring freeze until a director was found. Under some pressure to move quickly, the hiring committee unanimously agreed on a candidate and hired Kevork. Although funded and reviewed through the Provost's office, Kevork's position is functionally part of the math department in terms of his teaching and service requirements. Thus, Kevork's position connects him with several units that previously were at odds with each other. This decision, coupled with Kevork's staff appointment, placed him in a peripheral position with respect to multiple communities while he was expected to broker between groups and lead a new system.

The director is primarily responsible for calculus course design and coordination, including common exams. Nearly all calculus courses are taught by graduate student instructors (GSIs), and Kevork is also responsible for most of their teaching professional development activities. Similar to RRU, the new courses presented mentoring opportunities for graduate students teaching calculus, as the new calculus director worked more closely with them. Following discussions and negotiation with many stakeholders, each semester of calculus was divided into two half-semester courses. This structure allows for finer-grained placement into the calculus sequence, more consistent check-in points for students, faster catch-up opportunities for those who fail a course, and a full seven weeks focused on infinite series. (An additional precalculus half-semester was added later.) While Kevork is ultimately responsible for the distribution of content among these courses, decisions were made in close consultation with the mathematics and other stakeholder departments to ensure alignment of objectives and a satisfactory balance of procedural, conceptual, and applied problems. Kevork also pushed a shift from a "weed-out" mentality to one more focused on supporting students, particularly among the instructors of introductory mathematics courses, as well as an improved reputation on campus.

Vignette Reflection Questions

1. Given the situation, what is significant about the reporting structure and staff classification of Kevork's position?

2. What might be some of the benefits and drawbacks of the half-semester model, both for instructors and for students?

Success and Monitoring

Passing rates in calculus and first-year retention rates have soared, and the math department remains satisfied with the rigor of the current courses. A member of the Provost's office described the progress as "incredibly gratifying" (DBU, Fa17), citing a drop in the DFW rate from double to single digits and increased success for underrepresented students. Students' performance in downstream courses has also improved, assuaging fears of a lack of rigor or appropriate preparation. The Provost's office held a party to celebrate the extent of the improvements in the calculus program and, in particular, the increased passing rates. Kevork, and others from the mathematics department and Provost's office, *have presented their results at multiple conferences and continue making efforts to spread the half-semester calculus to other institutions.*

While Kevork is primarily responsible for calculus, along with the associate directors, he is part of two committees that contribute to program monitoring. The first, a curricular content committee, is composed of mathematics faculty who meet regularly (twice a semester) with Kevork to assure themselves and the rest of the mathematics department that their standards of rigor are being met in the new courses. They also sign off on changes to placement procedures. The second is an interdisciplinary oversight committee who meet regularly (currently annually) to review data about student success and discuss any emerging concerns. During the first few years, this committee was key in making additional tweaks to align and reorganize content in both calculus and downstream courses. Major complaints about calculus have essentially evaporated, and when things emerge, this committee ensures that they are discussed and addressed before getting out of hand.

Vignette Reflection Questions

1. How did the various leaders at DBU contribute to the successes of the change initiatives?
2. What are the data sources that provide evidence that the changes at DBU have been successful?
3. What role do the internal (i.e., curricular content) and external (i.e., interdisciplinary) committees play in sustaining success?
4. If Kevork were to leave DBU, what do you think would happen to the calculus courses?

Summary and Application

At DBU, the threat of losing calculus emerged from long standing concerns about passing rates, specifically calculus being the largest cause of attrition among first-year engineering majors, after a particularly disastrous year when passing rates dipped below 50%. The Provost's office led an interdisciplinary task force in deciding to hire externally for a new position dedicated to calculus course coordination, including course (re)design and graduate student professional development. The Director of Calculus was positioned to act as a broker between groups and lead a new system; Kevork took this role on and engaged in discussions and negotiation with many stakeholders. Ultimately, the calculus sequence was divided up into half-semester modules and the curriculum reorganized, a primarily structural change which was intended to address multiple concerns. In time, the new structure facilitated a shift from a "weed-out" mentality to one that promoted students' success, especially among instructors; the department's reputation on campus also improved as calculus passing rates improved substantially, first-year retention increased and students' outcomes improved in downstream courses. The reimagined courses are viewed with satisfaction - if not outright pride - by the mathematics department, client disciplines, and the administration. To sustain the success and work toward continuous improvement, the calculus courses continue to be monitored by two committees: the curricular content committee in mathematics approves changes to placement procedures and ensures the department's standards of rigor are being upheld in the new courses; the interdisciplinary oversight committee reviews student success data and identifies emergent concerns.

Discussion Questions

How might you apply lessons learned from this case to your own department?

1. Imagine that a similar situation emerged in your department, and that it was decided that there should be a new hire to implement strategies for change. Knowing the context of your department and college/university, what would be some benefits and drawbacks to hiring internally vs. hiring someone external to the institution?
2. Similarly, what would be the affordances and constraints of receiving funding for such a position from your academic college, Provost's office, or another academic administration area in which your department is housed?
3. How well do you envision the half-semester calculus model working at your institution? Why?
4. If your institution decided to form an interdisciplinary committee, who would need to be represented, and why? What would need to happen for this committee's meetings to be productive?
5. If you were asked to monitor students' success, what data would you need access to, and how would you use these data?

7.5 Cross-cutting Discussion

These cases offer several considerations for mathematics departments. First, they provide examples of signifiers that a department might be in danger of losing calculus. At RRU, changes to the placement process led to course content issues and time-to-degree concerns for engineering students. At RHU and DBU, concerns had been brewing for some time about DFW rates among engineering majors and these concerns erupted when the problem became so serious that the Provost's office got involved. In all three cases, there were signs of trouble, but the change process only began when administrators clearly communicated to the mathematics department that they were in real danger of losing calculus.

Second, the cases produced what Fullan (2001) calls *restructuring*, or new ways of operating, as well as *reculturing*, or new beliefs, values, and norms. In each case, success involved both restructuring and reculturing, which suggests that attending to both structural and cultural change may be important for departments seeking to eliminate the threat of losing calculus. The cases also point to the importance of developing strategies that align with the nature of the problems and the political, structural, and cultural contexts surrounding them. Accordingly, there were some common strategies across cases, such as considering other departments' needs in the design and development of the new program; having permanent meetings between departments to verify conformity with the program and identify new needs; improving TA training; and creating committees or specific roles that assign responsibility for different aspects that set the change.

For example, at RRU, the nature of the problem involved course content and time-to-degree among engineering majors; the successful strategy was to develop a new engineering calculus series that shifted the culture toward student success. High DFW rates in engineering were the main concern at RHU and DBU, but at RHU, the problem was magnified by the department's poor reputation for collaboration, openness to change, and care for students. At RHU, the changes included several improvements to instructional practices, the curriculum, and support structures for students. However, these were deliberately accompanied by an intentional culture change (i.e., "don't be a jerk") and an associated PR campaign to convey the department's newly adopted cultural norms, all brokered by departmental insiders who were influential and well-liked across campus. On the other hand, at DBU, the Provost's office funded an external hire, who in turn implemented a half-semester structure and collaborated to develop and sustain internal and external committees for monitoring and continuous improvement. In time, members of the department appeared to change their belief about their role in students' learning and success.

In each case, the threat of losing calculus catalyzed a change process. Through multiple stakeholders working together to develop reasonably sound, principled strategies for improvement, the crisis was resolved in every case. However, implementing similar strategies should not require a crisis. Developing and sustaining committees for continuous improvement, for example, would allow mathematics departments to gather multiple perspectives across disciplines (e.g., mathematics, engineering), roles (e.g., graduate teaching assistants, full-time instructors), and organizational entities (e.g., Provost's office, academic departments). Such committees could prevent crises from developing in the first place since they provide regular opportunities to communicate feedback, implement changes, and share evidence of progress. By normalizing external feedback and continuous improvement, mathematics departments can continue to offer calculus while collaborating with others to promote conceptual understanding, high-quality instruction, and student success, ultimately broadening participation in STEM majors and careers.

7.6 Parting Thoughts

From these cases, we offer a bit of advice for all mathematics departments with Precalculus through Calculus 2 sequence (or other foundational introductory mathematics courses) taught to a range of students: establish strong lines of communication and collaboration across units on campus with a stake in those courses - and go into that effort with an open mind. Key units include disciplinary units which require mathematics courses for their majors, administrative units that allocate resources, and student affairs units. These different units will offer diverse perceptions of the state of the Precalculus through Calculus 2 sequence and their impact on student success. One advantage of such communication is as an early-warning system, alerting the mathematics department to concerns that may precede a crisis as seen in these three cases. Another is that cooperative communication may reveal nuance about those brewing concerns and where they come from. For example, concerns about "lack of preparation" might be specifically related to a content area (e.g., students' grasp of exponential growth); time-to-degree concerns might be related to a long sequence of pre-requisites which make it next to impossible for students starting in college algebra to obtain an engineering degree within four years. Further, it gives the mathematics department the opportunity to point out misalignment of the purpose and content of Precalculus through Calculus 2 sequence courses (e.g., content which is not part of the curriculum and therefore should not be expected by subsequent courses).

Understanding the nature of existing or burgeoning concerns, and doing so in a setting where all the concerns can be outlined together, supports mutual engagement in improvement strategies and can sidestep unproductive conflict. Communication and collaboration support the clarification and/or development of shared objectives and mutual understanding, which are prerequisites for sustainable improvement (Kezar, 2011; Reinholz & Apkarian, 2018; Shadle et al., 2017). In such committees, stakeholders and the mathematics department can bring to the table their own objectives, needs, goals, expectation, and resources to plan courses of action which support all parties - and all parties share responsibility for their success. Such groups also have the potential to be more creative in finding solutions to existing problems or preventing their occurrence. For example, shuffling the order of content presentation in the mathematics courses or in a disciplinary course which expects that content; determining that the needs of the engineering and biology majors are different enough to warrant separate courses; adjusting student advising policies to better navigate course sequencing and reflect the reality of pathways to completing a STEM degree. Changes, even small ones, are more effective and more likely to be sustained when they are tightly connected to goals, respect the local context, involve distributed agency, and work *with* existing structures and culture (Reinholz & Apkarian, 2018; Henderson et al., 2011). Developing such initiatives is a more likely outcome when representatives of multiple campus units are sitting at the same table.

References

Fullan, M. (2001). *The new meaning of educational change* (3rd ed.). Teachers College Press.

Henderson, C., Beach, A., & Finkelstein, N. (2011). Facilitating change in undergraduate STEM instructional practices: An analytic review of the literature. *Journal of research in science teaching, 48*(8), 952–984.

Kezar, A. (2011). *Understanding and Facilitating Organizational Change in the 21st Century: Recent Research and Conceptualizations: ASHE-ERIC Higher Education Report, Volume 28, Number 4* (Vol. 155). John Wiley & Sons.

National Research Council. (2013). The mathematical sciences in 2025. National Academies Press.

PCAST. (2012). Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics. President's Council of Advisors on Science and Technology; Office of Science and Technology Policy.

Reinholz, D. L., & Apkarian, N. (2018). Four frames for systemic change in STEM departments. *International Journal of STEM Education, 5*(1), 1–10.

Shadle, S. E., Marker, A., & Earl, B. (2017). Faculty drivers and barriers: laying the groundwork for undergraduate STEM education reform in academic departments. *International Journal of STEM Education, 4*(1), 8.

8

From Inspiration to Making it Happen

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The chapters in this Notes volume offer a number of innovative programmatic structures and practices that can improve students' experience and success in precalculus and calculus. We hope that the details and tactics of these innovations have inspired you and your colleagues to imagine how you might adapt these innovations at your own institution. In this concluding chapter we provide a top level summary of the enacted improvement efforts (or change tactics) followed by recommendations for how to realize your vision for change.

Chapter 1: Introduction. In this brief chapter the author provides the background of the project from which this work arises as well as foreshadows the chapters to come. In addition, the author highlighted findings from a previous study that identified seven characteristics of successful Calculus 1 programs. These characteristics continue to be relevant and thus, similar to the other chapters in this book, can inform or guide new initiatives.

Chapter 2: Rethinking student support programs. In this chapter, the authors highlighted four models of support for students with historically underserved identities. The models included a first-year academic success program focused on providing alternative mathematics courses, a living-learning community with specific course sections in mathematics, a student-focused diversity advisory committee which provides feedback and suggestions for change, and a Learning Assistant program which provides near-peer role models to bolster student learning.

Chapter 3: Alternative course pathways. The authors of this chapter highlighted alternatives to the standard course sequencing, specifically those that are targeted for incoming students who have been failed by the educational system. The alternatives described include a co-requisite course that targets procedural fluency, a stretched out year-long calculus course with precalculus supports, a calculus infused with precalculus, and a half-term calculus course structure.

Chapter 4: Variations in approaches to the content of Calculus. In this chapter, the authors presented four course variations that modify the approach to the content of the calculus sequence. The first details how a research mathematician highlights their research area during a once a week enhanced laboratory section. The second offers a separate calculus sequence for engineering majors while the third is a calculus course that was created in collaboration with the physics department. The fourth variation highlights a course restructuring to support students' meaning-making and conceptual understanding.

Chapter 5: Professional development and course coordination. In this chapter, the authors explored three robust professional development programs for GTAs and instructors. The first approach highlights a 5-week program specifically designed to develop pedagogical knowledge and skills in graduate students. The second approach highlights how a calculus course coordinator embeds professional development activities within the coordination structure, and the third approach highlights a program that employs multiple individuals to support and manage on-going professional development along with course coordination.

Chapter 6: Program assessment and the use of local data. The authors of this chapter described three different approaches for collecting and reviewing data for assessing and improving the introductory mathematics course sequence. One approach focuses on collecting nuanced data with clear intentions for analysis. A second approach uses data to communicate with stakeholders across campus, and the third approach uses readiness scores to assess the quality of the program rather than merely as a tool for distributing students among course sequence tracks.

Chapter 7: Institutional and departmental change. In this chapter the authors highlighted the response strategies of three departments that faced the threat of losing calculus. For each case the authors explain the instigating problems that lead to the threat, the nature of the threat/demand for change, and the departments' responses. In presenting the departments' responses, they focus on the position and roles that were created, the changes that were implemented, and the follow-up efforts to monitor and report to stakeholders outside of the department.

8.1 Realizing Your Vision

Whether your vision for change adapts one or more of the extended stories presented here or seeks to innovate in other ways, the organizational science literature indicates that the likelihood of success will be enhanced if you are guided by a local theory of change. As described by Reinholz and Andrews (2020), a local theory of change is a project specific delineation of the underlying assumptions, starting state, desired end state, indicators that describe the evidence needed to know if the intended goals have been achieved (see Chapter 7 for ideas on useful data sources), and change strategies and tactics.

Assembling a comprehensive local theory of change can feel daunting. However, recent work by Henderson and White (2019) offers a useful tool to assist faculty in creating, monitoring, and evaluating a change effort. The platform is referred to as the Change Dashboard (see `https://ascnhighered.org/ASCN/change_dashboard/index.html`). The Dashboard focuses on the following four key aspects of a change project:

1. The Desired State, which includes big-picture goals, as well as desired characteristics at each important level of the system: individual, department, institution, extra-institution. For example, a big-picture goal might be to increase the number and quality of STEM graduates while a Desired State represents specific changes in conditions (e.g., dedicated advising program, new classroom space for active learning).
2. The Current State, which includes the starting characteristics of each level of the system. Explicating the Current State includes specifying how things look right now as well as aspects of the system that you do not expect to change.
3. Change Strategy, which is the big-picture view of how the project will create the desired changes. As explained below, change strategies specify the focus of change (individuals or environments) and whether the outcomes are prescribed or develop through the change process.
4. Change Tactics, which are the more specific action plans and resources that will be used in the change processes. The chapters in this volume are excellent examples of Change Tactics (e.g., new course variations, student support programs, professional development through course coordination, new roles or positions of leadership).

Figure 8.1 shows the Change Dashboard template, which is intended to be a living document that is created and modified over an extended period of time.

At the start of your improvement efforts the change strategy and change tactics may not be entirely clear. Thus, Henderson and White (2019) suggest an iterative process for detailing these aspects after diagnosing the current and desired states. The Desired State typically includes a mix of concrete things that can be directly measured and the more subtle aspects of the Desired and Current states that tend to be harder to directly measure or observe, what many colloquially refer to as culture (see Reinholz & Apkarian for a more systematic definition of culture). Not attending to aspects of the local culture is often detrimental to a change project, for as the adage says, "culture eats strategy for breakfast" (adage attributed to Peter Drucker).

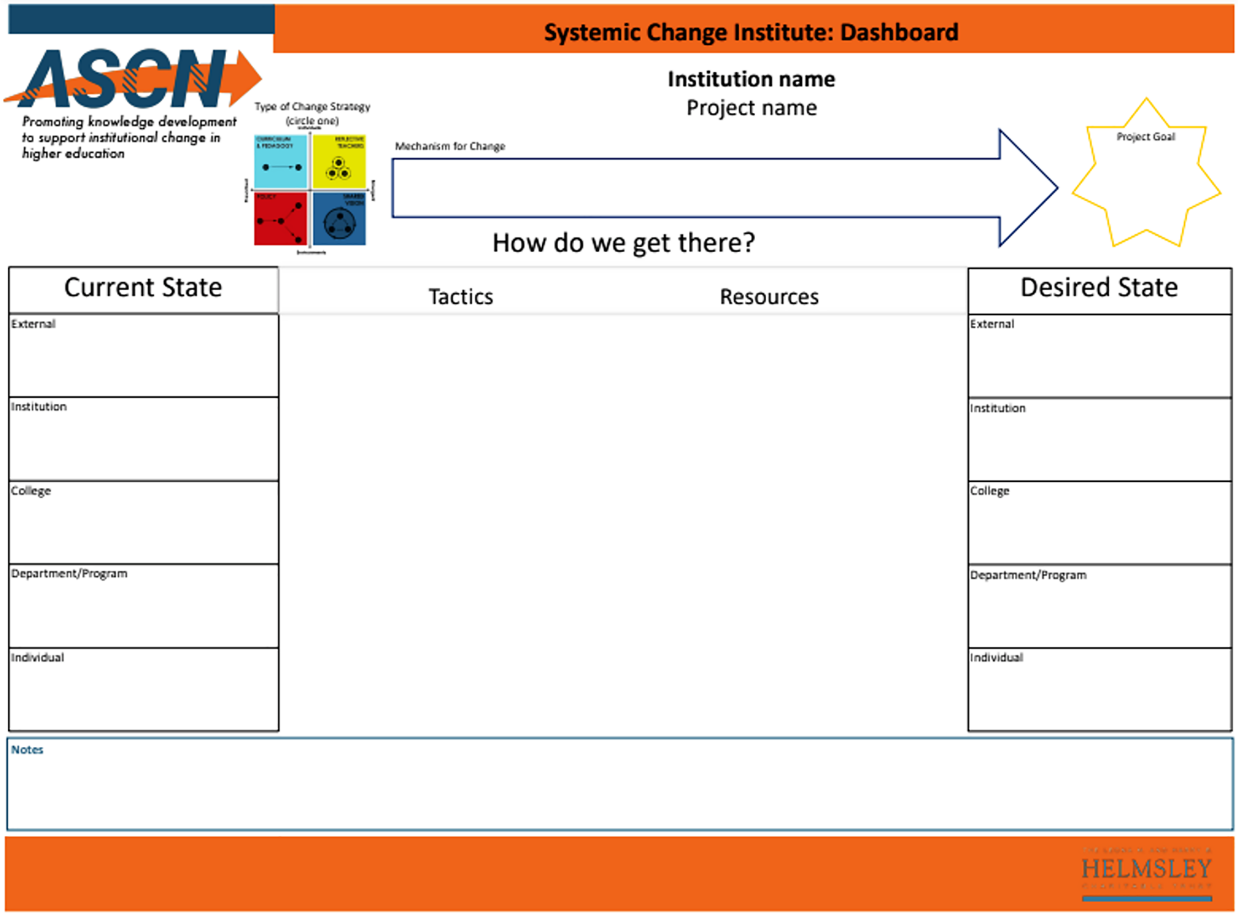

Figure 8.1. Change dashboard template.

It is also useful to distinguish between a Change Strategy and a Change Tactic. A Change Strategy is a global, overarching vision for how the change will take place and the unit of this change. As shown in the near top left of Figure 8.1, there are four basic types of change strategies, each represented by a different quadrant determined by two orthogonal continuums. The horizontal axis indicates whether the intended outcome is prescribed or emergent. The vertical axis indicates whether the change is aimed at individuals or the environment/structures. Change Tactics, on the other hand, are specific activities that are believed to facilitate the movement from the Current State to the Desired State. Experts in organizational science recommend that change tactics be distributed across the four change strategy quadrants (Henderson et al., 2011).

We strongly encourage readers to visit the Change Dashboard website and read the accompanying documents. The research-based suggestions and advice will provide an invaluable source of inspiration that will guide and inform your change efforts. We also recommend the special issue of the journal *PRIMUS* that focuses on ongoing accounts of improvement efforts in precalculus and calculus and corresponding departmental cultural change (Rasmussen et al., 2021). The many stories of change here will provide on the ground advice and suggestions for a successful change effort.

References

Henderson, C., Beach, A., & Finkelstein, N. D. (2011). Facilitating change in undergraduate STEM instructional practices: An analytic review of the literature. *Journal of Research in Science Teaching, 48*(8), 952–984. https://doi.org/10.1002/tea.20439

Henderson, C., & White, K. (2019). The Change Dashboard: A Planning Tool for Successful Change [White paper]. Retrieved from the Accelerating Systemic Change Network, https://ascnhighered.org/ASCN/publications.html.

Rasmussen, C., Smith, W., & Tubbs, R. (2021). Infusing active learning into precalculus and calculus courses: Insights and lessons learned from mathematics departments in the process of change. [Special issue]. *PRIMUS, 31*(3-5).

Reinholz, D. L., & Andrews, T. C. (2020). Change theory and theory of change: what's the difference anyway?. *International Journal of STEM Education, 7*(1), 1–12.

Reinholz, D. L., & Apkarian, N. (2018). Four frames for systemic change in STEM departments. *International Journal of STEM Education, 5*(1), 1–10.

9

Appendix
Site Descriptions

This appendix includes institutional context and program details about each of the eleven case study sites of the Progress through Calculus project mentioned in this volume. Institutional information is drawn from The Carnegie Classification of Institutions of Higher Education (Carnegie Foundation for the Advancement of Teaching, 2011; Indiana University Center for Postsecondary Research, n.d.), the Integrated Postsecondary Education Data System (IES NCES, 2016-20), publicly available data from institution's websites, and local data collected through our research activities. The first table contains basic Carnegie Classification levels, undergraduate enrollment, and indicates which chapters include vignettes about that institution. This is followed by a series of institution profiles with more detail about that site's institution, students, department, and P2C2 courses. While the sections are structured in parallel, due to inconsistencies in how sites collect and report data, the presentation of information varies slightly. Note also that the Carnegie Classification data is from the most recent version of that data, while the other information dates to the time of the case studies.

Regarding "course format" in the following tables: we distinguish between regular class meetings (lecture) and complementary break-out sessions (recitation). Not all meetings referred to as lectures are taught in lecture format, and the recitation section might be locally referred to as a lab, breakout, recitation, discussion section, etc. What is indicated as an "hour" is often in practice 50 minutes, depending on the site.

9.1 Institution Overview

Institution (Pseudonym)	**Carnegie Classification (Basic Level)**	**Approx. Undergraduate Enrollment**	**Chapters**
Alpine University	Doctoral Universities: Very High Research Activity	28,500	Course Var Pathways
Canyon Crest Union	Doctoral Universities: High High Research Activity	23,000	Course Var Pathways DEI
Dandelion State University	Doctoral/Professional Universities	13,000	Data DEI
Desert Bloom University	Doctoral Universities: Very High Research Activity	14,000	Course Var Pathways Change
Dunshire University	Doctoral Universities: Very High Research Activity	6,500	Content Course Var DEI
Maple State University	Doctoral Universities: Very High Research Activity	40,000	Content Course Var PD/Coord
Pine Grove University	Doctoral Universities: Very High Research Activity	41,000	DEI
River Rock University	Doctoral Universities: Very High Research Activity	32,000	Content Course Var Data Change
Rolling Hill University	Doctoral Universities: Very High Research Activity	21,000	Change PD/Coord
Sandpiper University	Doctoral Universities: High High Research Activity	4,500	Course Var Pathways Data PD/Coord
Tree Line University	Doctoral Universities: Very High Research Activity	15,000	Content Course Var

9.2 Individual Site Profiles

9.2.1 Alpine University

Alpine University is a public university located in a small city. They use a semester calendar.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Balanced arts & sciences/professions, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs, with medical/ veterinary school
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, full-time, selective, lower transfer-in
Size and setting	Four-year, large, primarily nonresidential

Admissions information
Of the approximately 21,500 applications, 16,400 students (about 76%) were admitted. The percentage of full-time first-time undergraduate students who received any financial aid is 91%. For the incoming class of Fall 2016, test scores were:

	SAT Reading	SAT Math	ACT Composite
25th Percentile	455	460	21
75th Percentile	560	570	26

Enrollment information
Approximate total enrollment: 28,500

- 22,250 undergraduate students
- 6,150 graduate students
- 4,000 students enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 54% are men and 46% are women
- Of the men:
  - 9% Nonresident (international); 4% Hispanic/Latinx; 0.1% AIAN; 1.5% Asian; 5.5% Black/AA; .06% NHPI; 75% White; 3% two-or-more-races; .2% race/ethnicity unknown
- Of the women:
  - 3% Nonresident (international); 4% Hispanic/Latinx; 0.01% AIAN; 1.5% Asian; 4% Black/AA; .09% NHPI; 83% White; 4.5% two-or-more-races; .1% race/ethnicity unknown

Completion information
First-time, full-time bachelor's seeking retention rate at Alpine University is 79%. They have an overall 6-year graduation rate of 57%. In the July 2015-June 2016 period, 4,550 bachelor's level degrees were awarded. Of these, 22% were in STEM-related fields, of those, 63% were in engineering.

Department Information (AY 2017–18)

The math department has about 60 graduate students and 55 faculty members, about 30 of which are tenured or tenure track. The research faculty includes mathematicians (theoretical and applied), statisticians, and mathematics education researchers. The highest degree offered by the department is a PhD in Mathematical Sciences.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
College Algebra	3 hr lecture + 1 hr recitation (non-STEM) 4 hr lecture (STEM)	200–260 (non-STEM) 40 (STEM)	1431 / 962
Plane Trigonometry	2 hr lecture + 1 hr recitation	200	385 / 930
Precalculus	3 hr lecture + 1 hr recitation	60–70	280 / 249
Calculus 1a with precalculus	4 hr lecture	50–80	477 / 249
Calculus 1b with precalculus	4 hr lecture	50–80	157 / 400
Calculus I	3 hr lecture + 3 hr recitation	25–35	215 / 100
Calculus I Engineering	3 hr lecture + 3 hr recitation	80–90	399 / 240
Calculus II	4 hr lecture	20–40	641 / 605

9.2.2 Canyon Crest University

Canyon Crest University is a private not-for-profit university located in the urban center of a large midwestern city. There are two campus locations, one campus is the primary location for STEM courses and another campus is the primary location for business courses. Canyon Crest University is a large doctoral university, though the highest degree in mathematics is masters level. They operate on a quarter calendar system.

Carnegie Classification

Basic	Doctoral Universities: High Research Activity
Undergraduate instructional program	Balanced arts & sciences, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs
Enrollment profile	Majority undergraduate
Undergrad. profile	Four-year, full-time, more selective, high transfer-in
Size and setting	Four-year, large, primarily nonresidential

Admissions Information
Of approximately 21,000 applications, about 15,500 students (around 75%) were admitted. Canyon Crest University has a test-optional application process, where students applying for freshman admission can choose whether or not to submit ACT or SAT scores as part of the application. They are the largest private institution to implement a test-optional program. The average composite ACT for the entering 2017 class was 25.3

Enrollment Information
Approximate total enrollment: 22,750

- 14,800 undergraduate students
- 7,000 graduate students and 800 Law students
- 450 students enrolled in P2C2 courses in Fall 2017

Student demographics:

- 47% men and 53% women
- 38% are students of color and 7% are international student
- The entering undergraduate cohort race/ethnicity includes 52% caucasian, 21% Hispanic/Latino, 10% Asian, 7% African-American, 4% multiracial, and 2% unreported background.
- 33% of the incoming class are first-generation college students

Completion Information
Canyon Crest University has an overall 6-year graduation rate of 71%. In the 2008-2009 academic year, around 3130 bachelor's level degrees were awarded. Of these, 8% were in STEM-related fields, of those STEM degrees:

- 58% were in Computer and Information Sciences and Support Services
- 26% were in Biological and Biomedical Sciences
- 10% were in Mathematics and Statistics
- 7% were in Physical Sciences

Department Information (AY 2017–18)

The mathematical science department during 2017-2018 had about 25 tenured and tenure-track faculty, 3 full-time faculty and roughly 35 adjunct faculty. Additionally, the department has 5-6 graduate students involved with the teaching of recitation sections in the P2C2 courses. The department offers 5 different Masters degrees, but does not offer a PhD.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / WI 18)
Precalculus	3 hr lecture	30–40	250 / 150
Trigonometry	3 hr lecture + 1 hr recitation	20–30	17 / 17
Calculus I	3 hr lecture + 1 hr recitation	30–40	114 / 37
Calculus II with precalculus	3 hr lecture + 1 hr recitation	30–40	36 / 95
Calculus III	3 hr lecture + 1 hr recitation	20–40	14 / 29
Calculus with integrated precalculus I–III	6.5 hr lecture (year-long course)	30–40	215 / 100

9.2.3 Dandelion State University

Dandelion State University is a public university located in a remote town. They operate on a semester calendar system.

Carnegie Classification

Basic	Doctoral/Professional University
Undergraduate instructional program	Balanced arts & sciences/professions, some graduate coexistence
Graduate instructional program	Research Doctoral: Professional-dominant
Enrollment profile	Very high undergraduate
Undergrad. profile	Four-year, full-time, selective, higher transfer-in
Size and setting	Four-year, large, primarily residential

Admissions information
Of the approximately 11,250 applications, 7,000 students (62%) were admitted. The percentage of full-time first-time undergraduate students who received any financial aid is 89%.

For the incoming class of Fall 2016, test scores were:

	SAT Reading	SAT Math	ACT Composite
25th Percentile	430	440	19
75th Percentile	540	550	24

Enrollment Information
Approximate total enrollment: 12,750

- 11,000 undergraduate students
- 1,680 graduate students
- 765 students enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 36% men and 64% women
- Of the men:
  - 0.7% are Nonresident (international); 17% Hispanic/Latinx; 0.4% AIAN; 1% Asian; 19% Black/AA; 0.6% NHPI; 57% White; 3% two-or-more-races; 0.8% race/ethnicity unknown
- Of the women:
  - 0.6% Nonresident (international); 17.5% Hispanic/Latinx; 0.4% AIAN; 1% Asian; 20% Black/AA; 0.03% NHPI; 57% White; 0.4% two-or-more-races; 0.5% race/ethnicity unknown

Completion Information
First-time, full-time bachelor's seeking retention rate at Dandelion State University is 71%. They have an overall 6-year graduation rate of 44%. In the July 2015–June 2016 period, 2,086 bachelor's level degrees were awarded. Of these, 4% were in STEM-related fields.

Department Information (AY 2017–18)
The mathematics department has about 22 tenured or tenured track faculty, 13 instructors, and 6 graduate students, though no graduate students teach in the P2C2 sequence. The highest degree offered by the department is an MS degree.

P2C2 Course Descriptions

Course Name	**Format**	**Usual Section Size**	**Enrollment (FA 17 / SP 18)**
Plane Trigonometry	3 hr lecture	30–35	124 / 143
College Algebra	3 hr lecture	18–32	338 / 416
Plane Analytic Geometry	3 hr lecture	25–30	70 / 64
Precalculus	3 hr lecture	20	74 / 32
Calculus I	3 hr lecture + 1.25 hr recitation	15–30	101 / 120
Calculus II	3 hr lecture + 1.25 hr recitation	15–30	56 / 64

9.2.4 Desert Bloom University

Desert Bloom University is a public university. It is located in a mid-sized suburban area. They are on a semester calendar. There is a large engineering school at Desert Bloom which accounts for a large portion of those taking P2C2 courses.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Arts & sciences plus professions, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive, no medical/veterinary
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, full-time, more selective, higher transfer-in
Size and setting	Four-year, large, higher residential

Admissions information
Of approximately 32,000 applications, about 13,000 students (40.6%) were admitted. For the incoming class of Fall 2016, test scores were:

	SAT Reading	SAT Math	ACT Composite
25th Percentile	600	630	28
75th Percentile	690	710	31

Enrollment Information
Approximate Total enrollment: 17,250

- 13,500 undergraduate students
- 3,750 graduate students
- 1,300 enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 52% men and 48% women
- Of the men:
  - 8% nonresident (international); 11% Hispanic/Latinx; 0.1% AIAN; 14% Asian; 5% Black/AA; 0.04% NHPI; 59% White; 2% two-or-more races; 2% race/ethnicity unknown
- Of the women:
  - 8% nonresident (international); 11% Hispanic/Latinx; 0.06% AIAN; 15% Asian; 6% Black/AA; 0.08% NHPI; 57% White; 3% two-or-more races; 2% race/ethnicity unknown

Completion Information
Desert Bloom University has an overall 6-year graduation rate around 80%. In the July 2015-June 2016 period, about 3500 bachelor's level degrees were awarded. Of these, 31% were in STEM-related fields.

Department Information (AY 2017–18)
The math department has about 100 graduate students and 55 faculty members, about 35 of which are tenured or tenure track. The research faculty includes some statisticians, and mathematics faculty are primarily pure mathematicians. The highest degree offered by the department is a PhD in Mathematical Sciences.

P2C2 Course Descriptions

Desert Bloom offers most of their P2C2 courses on a half-term schedule. In the enrollment numbers, those followed by (A) refer to the enrollment in the first half of the term, with (B) indicating the second half of the same term.

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
Pre-Calc: Algebra & Trigonometry	5 hr lecture (full term)	20–30	146 / 633
Pre-Calc: Intro to Calculus	4.5 hr lecture (half-term)	20–30	175 / 22
Calculus 1a: Differential Calculus	4.5 hr lecture (half-term)	20–35	842 / 250
Calculus 1b Integral Calculus	4.5 hr lecture (half-term)	20–35	620 / 345
Calculus 2a: Integration Techniques and Applications	4.5 hr lecture (half-term)	20–35	420 / 476
Calculus 2b: Infinite Series	4.5 hr lecture (half-term)	20–35	413 / 394
Honors Calculus	4.5 hr lecture (full term)	20	20 / 0

9.2.5 Dunshire University

Dunshire University is a private, not for profit university. It is located in a mid-sized city. They are on a semester calendar.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Arts & sciences plus professions, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs, with medical/ veterinary school
Enrollment profile	Majority undergraduate
Undergrad. profile	Four-year, full-time, more selective, lower transfer-in
Size and setting	Four-year, large, highly residential

Admissions information
In fall 2016, Dunshire University had an admission rate of about 10%.

	SAT Reading	SAT Math	ACT Composite
25th Percentile	680	700	31
75th Percentile	770	800	34

Enrollment information
Approximate total enrollment: 17,000:

- 6600 undergraduate students
- 10,400 graduate students
- 550 enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 55% men and 45% women
- Dunshire University has a diverse student body, with 41% identifying racially as white, 21% Asian, 9% Black, 7% Hispanic/Latinx, and 10% non-resident (international).

Completion Information
Dunshire University has a graduation rate of about 95%, with a retention rate of 97%. In the July 2015-June 2016 period, 45% of the degrees awarded were in STEM.

Department (AY 2017–2018)
The department consists of about 50 tenured or tenure-track faculty, 3 professors of the practice, 4 lecturers, and 64 graduate students. The mathematics department is highly ranked and very focused on research. Precalculus is not offered, and Calculus I and II are taught exclusively by Professors of the Practice (PoPs), graduate students, and lecturers, with graduate and undergraduate students serving as recitation/lab leaders. The highest mathematics degree offered is a PhD.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
Calculus with Functions I	3 hr lecture + 2 hr recitation	15–25	73 / 23
Calculus with Functions II	3 hr lecture + 2 hr recitation	15	13 / 4
Calculus I	3 hr lecture + 2 hr recitation	20-30	236 / 82
Calculus II	3 hr lecture + 2 hr recitation	20–30	57 / 206
Calculus II w/Applications	3 hr lecture + 2 hr recitation	20–30	174 / 0

9.2.6 Maple State University

Maple State University is a public university. It is located in a mid-sized city. They are on a semester calendar. Maple State University has multiple campuses, for our site visit and the numbers reported here, we are only focusing on the main campus, which houses the College of Liberal Arts and Sciences.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Balanced arts & sciences/professions, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs, no medical/ veterinary school
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, full-time, more selective, higher transfer-in
Size and setting	Four-year, large, primarily residential

Admissions information
In fall 2016, Maple State University had an admission rate of about 85%.

	SAT Reading	SAT Math	ACT Composite
25th Percentile	500	520	22
75th Percentile	630	650	28

Enrollment information
Approximate total enrollment: 52,000:

- 40,000 undergraduate students
- 12,000 graduate students
- 3750 enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 55% men and 45% women
- Maple State University has a diverse student body, with 49% identifying racially as white, 9% Asian, 4% Black, 23% Hispanic/Latinx, and 9% non-resident (international).
- 13% of the student population are non-traditional students (over 25) and 91% receive financial aid.

Completion Information
Maple State University has a graduation rate of about 63%, with a retention rate of 86%. In the July 2015-June 2016 period, 28% of the degrees awarded were in STEM.

Department (AY 2017–2018)
There are about 55 tenured or tenure track faculty and an additional 60 instructors at the main campus. The department houses applied math, theoretical math, mathematics education, and statistics. PhDs in each of these areas are offered.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
Precalculus	3 hr lecture	50–60	926 / 570
Precalculus: STEM	3 hr lecture + 1 hr recitation	50–60	165 / 60
Calculus for Engineers 1	3 hr lecture	60–70	1092 / 580
Calculus w/ Analytic Geometry I	3 hr lecture + 1 hr recitation	100	417 / 184
Calculus for Engineers II	3 hr lecture	50–60	819 / 912
Calculus w/ Analytic Geometry II	3 hr lecture + 1 hr recitation	80–100	284 / 241

9.2.7 Pine Grove University

Pine Grove University is a public university. It is located in a large suburban area. They are on a semester calendar.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Balanced arts & sciences/professions, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs, with medical/ veterinary school
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, medium full-time, selective, higher transfer-in
Size and setting	Four-year, large, primarily nonresidential

Admissions information
In fall 2016, Pine Grove University had an admission rate of about 53%.

	SAT Reading	SAT Math	ACT Composite
25th Percentile	520	510	23
75th Percentile	610	600	27

Enrollment information
Approximate total enrollment: 48,000:

- 41,000 undergraduate students
- 8,000 graduate students
- 3700 enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 43% men and 57% women
- Pine Grove University has a diverse student body, with 67% identifying racially as Hispanic/Latinx, 9% white, 2% Asian, 12% Black, and 7% non-resident (international).
- 22% of the student population are non-traditional students (over 25) and 91% receive financial aid.

Completion Information
Pine Grove University has a graduation rate of about 54%, with a retention rate of 84%. In the July 2015-June 2016 period, 10% of the degrees awarded were in STEM.

Department (AY 2017–2018)
There are about 35 tenured or tenure track faculty and 45 instructors. Almost all faculty teach in the P2C2 sequence. The highest mathematics degree offered is a masters.

P2C2 Course Descriptions

Course Name	**Format**	**Usual Section Size**	**Enrollment (FA 17 / SP 18)**
Trigonometry	3 hr lecture	40–50	333 / 534
Precalculus Algebra	3 hr lecture	55–65	1375 / 1266
Precalculus Algebra and Trig	4 hr lecture	40–50	209 / 252
Calculus I	4 hr lecture	30–40	1097 / 746
Calculus II	4 hr lecture	35–40	701 / 709

9.2.8 River Rock University

River Rock University is a large public university located in a Western urban city. River Rock University is affiliated with their state's system of higher education and is known for high research activity. They operate on a semester calendar system.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Balanced arts & sciences/professions, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs, with medical/ veterinary school
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, medium full-time, selective, higher transfer-in
Size and setting	Four-year, large, primarily nonresidential

Admissions Information
Of approximately 14,300 applications, 11,000 students ($\sim$75%) were admitted. The percentage of full-time first-time undergraduate students who received any financial aid is 87%. For the incoming class of Fall 2016, test scores were:

	SAT Reading	SAT Math	ACT Composite
25th Percentile	520	530	21
75th Percentile	640	660	27

Enrollment Information
Approximate total enrollment: 32,000

- 23,800 undergraduate students
- 8,200 graduate students
- 3,000 students enrolled in P2C2 courses in Fall 2017

Undergraduate demographics

- 54% men and 46% women
- Of the men:
  - 7% Nonresident (international); 9.5% Hispanic/Latinx; 0.3% AIAN; 5.5% Asian; 1% Black/AA; 0.6% NHPI; 70% White; 4.5% two-or-more-races; 0.8% race/ethnicity unknown
- Of the women:
  - 4% Nonresident (international); 14.5% Hispanic/Latinx; 0.6% AIAN; 6% Asian; 1% Black/AA; 0.4% NHPI; 67% White; 5.5% two-or-more-races; 0.5% race/ethnicity unknown
- 25% of all undergraduates enrolled are nontraditional (age 25 and over)

Retention and Completion Information
First-time, full-time bachelor's seeking retention rate at River Rock University is 90%. They have an overall 6-year graduation rate of 65%. In the July 2015-June 2016 period, about 5,200 bachelor's level degrees were awarded. Of these, 19% were in STEM-related fields, of those, 47% were in engineering.

Department (AY 2017–18)

The mathematics department has about 110 graduate students and approximately 70 research faculty (e.g., tenured or tenured track) and 20 career track/instructors. The highest degree offered by the department is a PhD in Mathematics.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
College Algebra	4 hr lecture	45–200	845 / 641
Trigonometry	3 hr lecture	35–165	342 / 370
Precalculus	5 hr lecture	60–85	301 / 158
Calculus I	4 hr lecture + 1 hr recitation	60–143	64 / 785
AP Calculus I	4 hr lecture	20–40	25 / NA
Engineering Calculus I	4 hr lecture + 1 hr recitation	42–141	253 / 244
Accelerated Engineering Calculus I	4 hr lecture + 1 hr recitation	49–50	38 / NA
Calculus II	4 hr lecture	45–185	384 / 596
AP Calculus II	4 hr lecture	20–40	NA / 26
Engineerin Calculus II	4 hr lecture + 1 hr recitation	45–60	109 / 340
Accelerated Engineering Calculus II	4 hr lecture + 1 hr recitation	25–45	29 / 74

9.2.9 Rolling Hill University

Rolling Hill University is a public university. It is located in a town. They are on a semester calendar.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Professions plus arts & sciences, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs, with medical/ veterinary school
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, full-time, more selective, higher transfer-in
Size and setting	Four-year, large, primarily residential

Admissions information
In fall 2016, Rolling Hill University had an admission rate of about 75%.

	SAT Reading	SAT Math	ACT Composite
25th Percentile	480	490	21
75th Percentile	550	610	27

Enrollment information
Approximate total enrollment: 25,000:

- 21,000 undergraduate students
- 4,000 graduate students
- 2,000 enrolled in P2C2 courses in Fall 2017

Undergraduate demographics:

- 50% men and 50% women
- Rolling Hills University is a predominantly while institutions with 68% identifying racially as white, 2% Asian, 4% Black, 8% Hispanic/Latinx, 4% American Indian, and 4% non-resident (international).
- 9% of the student population are non-traditional students (over 25) and 89% receive financial aid.

Completion Information
Rolling Hills University has a graduation rate of about 61%, with a retention rate of 81%. In the July 2015-June 2016 period, 16% of the degrees awarded were in STEM.

Department (AY 2017–2018)
There are about 40 tenured or tenure track faculty and an additional 20 instructors. The department offers undergraduate degrees in Mathematics - General, Secondary Teaching, Applied Mathematics, and Actuarial and Financial Mathematics, but is largely a service department. Addition to PhDs in Applied Mathematics, Pure Mathematics, and Mathematics Education.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
Trigonometry	3 hr lecture	35	708 / 392
Precalculus	3 hr lecture + 2 hr recitation	30–100	131 / 30
Calculus I	4 hr lecture	30–35	638 / 425
Calculus I for Technology Programs I	3 hr lecture	40	78 / 75
Calculus II	3 hr lecture	35	353 / 379
Calculus I for Technology Programs II	3 hr lecture	30	89 / 66

9.2.10 Sandpiper University

Sandpiper University is a private not-for-profit university located in a remote town. While Sandpiper University is officially listed as a liberal arts college, it has a strong technical component and is part of the Association of Independent Technological Universities (AITU). They operate on a semester calendar system.

Carnegie Classification

Basic	Doctoral Universities: High Research Activity
Undergraduate instructional program	Professions plus arts & sciences, some graduate coexistence
Graduate instructional program	Research Doctoral: STEM-dominant
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, full-time, more selective, lower transfer-in
Size and setting	Four-year, medium, highly residential

Admissions information
Of approximately 7000 applications, 68.2% were admitted. For the incoming class of Fall 2016, test scores were:

	SAT Reading	SAT Math	ACT Composite
25th Percentile	520	560	24
75th Percentile	620	663	29

Enrollment Information
Approximate Total enrollment: 4,250

- 3,175 undergraduate students
- 1,075 graduate students
- 600 students enrolled in P2C2 courses in Fall 2017

Student demographics:

- 70% are men and 30% are women
- Of the men:
  - 2% Nonresident (international); 4% Hispanic/Latinx; 0.2% AIAN; 3% Asian; 2% Black/AA; 0 NHPI; 85% White; 2% two-or-more-races; 2% race/ethnicity unknown
- Of the women:
  - 2% Nonresident (international); 6% Hispanic/Latinx; 0.4% AIAN; 4% Asian; 4% Black/AA; 0.1% NHPI; 78% White; 3% two-or-more-races; 1% race/ethnicity unknown

Completion Information
Sandpiper University has an overall 6-year graduation rate of about 70%. In the July 2015-June 2016 period, around 730 bachelor's level degrees were awarded. Of these, 79% were in STEM-related fields, of those 68% were in engineering.

Department (AY 2017–18)
The mathematics department has about 25 graduate students and 20 faculty members including, 13 of which are tenure or tenure track. The research faculty all focus on applied areas of mathematics. The highest degrees offered by the department is a PhD in Mathematics.

P2C2 Course Descriptions

Course Name	Format	Usual Section Size	Enrollment (FA 17 / SP 18)
Intro to STEM Mathematics	3 hr lecture + 1 hr recitation	15	14 / N/A
Calculus I	3 hr lecture + 1 hr recitation	80–100	444 / 64
Calculus II	3 hr lecture + 1 hr recitation	80–100	155 / 355
Co-Calculus (Optional)	2 hr recitation	20	180
Co-Calculus II (Optional)	1 hr recitation	20	30

9.2.11 Tree Line University

Tree Line University is a public university located in a suburban small town. Tree Line University is a large research-intensive doctoral granting institution. They operate on a semester calendar system.

Carnegie Classification

Basic	Doctoral Universities: Very High Research Activity
Undergraduate instructional program	Balanced arts & sciences, high graduate coexistence
Graduate instructional program	Research Doctoral: Comprehensive programs
Enrollment profile	High undergraduate
Undergrad. profile	Four-year, full-time, more selective, lower transfer-in
Size and setting	Four-year, large, highly residential

Admissions information Of approximately 20,000 applications, 15,000 students (75%) were admitted. For the incoming class of Fall 2016, test scores were:

	SAT Reading	SAT Math	ACT Composite
25th Percentile	490	500	22
75th Percentile	560	610	27

Enrollment Information
Approximate total enrollment: 15,000

- 14,100 undergraduate students
- 1,500 graduate students
- 950 students enrolled in P2C2 courses in Fall 2017

Student demographics:

- 45% men and 55% women
- Tree Line University is a predominantly white institution (PWI) with 81% of the undergraduate degree seeking students identifying racially as white. Based on the remainder of the student data, 7% of students are categorized as racial/ethnicity unknown, 4% Hispanic and 2% Asian, and 1% African-American.

Completion Information
Tree Line University has an overall 6-year graduation rate of 76%. In the 2008-2009 academic year, about 2600 bachelor's level degrees were awarded. Of these, 16% were in STEM-related fields.

Department (AY 2017–18)
The mathematics department has about 70 graduate students and 30 faculty members including: 10 lecturers, 5 pre-tenure faculty, and 15 tenured faculty. The faculty represent research areas in core mathematics, applied mathematics, mathematics education and statistics. The highest degree offered by the department is a PhD in Mathematical Sciences.

P2C2 Course Descriptions

Course Name	**Format**	**Usual Section Size**	**Enrollment (FA 17 / SP 18)**
Analysis and Applications of Functions	3 hr lecture + 2 hr recitation (Fall) 5 hr lecture (Spring)	80–120 80–120	227 / 38 227 / 38
Calculus I	3 hr lecture + 2 hr recitation	80–120	331 / 205
Calculus II	3 hr lecture + 2 hr recitation	80–120	225 / 293
Calculus for Social Sciences	3 hr lecture + 2 hr recitation	80–120	132 / N/A
Calculus for Life Sciences	3 hr lecture + 2 hr recitation	80–120	N/A / 358
Honors Calculus I/II	5.5 hr lecture	20–30	26 / N/A

9.3 References

Carnegie Foundation for the Advancement of Teaching (2011). *The Carnegie Classification of Institutions of Higher Education*, 2010 edition, Stanford, CA: Author.

IES NCES. (2018). *Integrated Postsecondary Education Data System*. U.S. Department of Education. Institution of Education Sciences, National Center for Education Statistics.
`https://nces.ed.gov/ipeds/use-the-data`

Indiana University Center for Postsecondary Research (n.d.). *The Carnegie Classification of Institutions of Higher Education*, 2018 edition, Bloomington, IN: Author.