notes

SHIFTING CONTEXTS, STABLE CORE: ADVANCING QUANTITATIVE LITERACY IN HIGHER EDUCATION

Luke Tunstall
Gizem Karaali
Victor Piercey
Editors

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Shifting Contexts, Stable Core

Advancing Quantitative Literacy in Higher Education

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88. Shifting Contexts, Stable Core: Advancing Quantitative Literacy in Higher Education, *Luke Tunstall, Gizem Karaali, and Victor Piercey,* Editors.
89. MAA Instructional Practices Guide, *Martha L. Abell, Linda Braddy, Doug Ensley, Lewis Ludwig, Hortensia Soto,* Project Leadership Team.

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Dedication

In memory of Caren Diefenderfer, a much loved and admired teacher and scholar. Caren was distinguished not only for her outstanding achievements as a faculty member but also for her gentleness, generosity, modesty, and incredible care and consideration in interactions with the many whose lives she touched. She is an inspiration to all of us—optimistic, encouraging, caring, and guiding.

Foreword

Considering Quantitative Literacy in the Context of Dewey, Data, and the Ever-shifting Landscape of a Democratic Society

Susan L. Ganter
Embry-Riddle Aeronautical University

> As societies become more complex in structure and resources, the need of formal or intentional teaching and learning increases. As formal teaching and training grow in extent, there is a danger of creating an undesirable split between the experience gained in more direct associations and what is acquired in school. This danger was never greater than at the present time, on account of the rapid growth in the last few centuries of knowledge and technical modes of skill. —John Dewey, 1916

The purpose and substance of a foreword is defined as a discussion about the relationship of the author to the story the book tells [11]. My interaction with the story of quantitative literacy (QL) began in 1998, when Lynn Steen and Bob Orrill asked me to participate in a design team they were forming as part of the work of the National Council on Education and the Disciplines (NCED). The purpose of the design team was to discuss and develop a working definition for QL, in light of Steen and Orrill's recent work in *Why Numbers Count* [9].

The design team first met in early 1998 and quickly realized the complexity of the task that had been assigned. Because QL as an academic subject was new, conversations turned to historical contexts that would support the development of a definition for QL. Among many others, the works of John Dewey served as a rich source of information and inspiration for our attempt to define QL in a concise way. Specifically, Dewey urged Americans of the early 20th century to remember that change is always more prominent than the status quo, implying that our interpretation of literacy must constantly evolve, that this "is the need of a human nature and of a society that are themselves in process of constant change" [4, 8]. In fact, Dewey said, the very success of our democratic society is dependent upon individual citizens who can "think for themselves," "judge independently," and generally be able to tell the difference between good and bad information [4, 8].

While the difficulty of the task at hand only increased as the design team moved forward, the profound importance of the work would not allow for anything less than complete commitment. Even 20 years ago, in the infancy of the internet and a worldwide superhighway of information, it already was increasingly clear that the magnitude of information at our fingertips would only continue to increase. "[W]e are drowning in data, and there is unimaginably more on the way" [1, p. 4]. No more will data be clarified and interpreted for us by an informed elite—every individual will have direct access to the information they desire. And with that information comes an increasing responsibility to ensure that every citizen has the tools to understand and interpret the many implications of these data on their decision-making and well-being.

The response to this urgent demand—and the result of many hours of discussion among the design team and others—was *Mathematics and Democracy* [8], a collaborative collection of writings led by Lynn Steen and introduced by the design team's narrative "The Case for Quantitative Literacy" [10]. In lieu of a seemingly-unattainable definition, the design team chose instead to focus on elements, expressions, skills, and challenges of QL. In this way, the opening chapter could set the stage for a new way of thinking about quantitative information through ideas such as cultural appreciation, interpreting data, logical thinking, and decision-making. Mathematics in context was highly-emphasized, with the notion that all citizens "need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning" [p. 2]. Also important was the distinction of QL as "inseparable from its context," unlike mathematics, statistics, and many other traditional school subjects.

The primary challenge for QL set forth by the design team—and to which multiple authors responded in the papers that comprise *Mathematics and Democracy*—is the current state of numeracy as the "ugly duckling" in a family of three concepts that form literacy: reading, writing, and 'rithmetic. It is universally accepted that every citizen must be able to read and write—in fact, the ability to read often is stated interchangeably with the word "literacy," in spite of the fact that it really is only part of the knowledge base needed to function as a literate citizen in modern society. However, 'rithmetic and mathematics—and consequently QL—are still seen as subjects to be mastered only by a few. The necessity and many uses of quantitative skills are widely misunderstood, and a failure to attain these skills often is declared as a badge of honor. To the point, "public apathy in the face of innumeracy may itself be a consequence of innumeracy" [8, p. 18].

And so, the charge set out by the bold statements in *Mathematics and Democracy* was to find mechanisms by which the ideals of QL could be operationalized and incorporated into the K-16 curriculum in a comprehensive way. In the years that followed, I had the privilege of working with Lynn Steen and Bernie Madison, under the guidance of Bob Orrill, on numerous QL publications and we successfully worked with many others to launch the National Numeracy Network (NNN), a non-profit organization whose purpose is to develop and disseminate a wealth of materials and professional development opportunities that focus on QL for all citizens. As the founding Director of NNN (prior to the achievement of 403(c) status in 2004), I worked with wonderful and dedicated colleagues such as Len Vacher, Dorothy Wallace, Caren Diefenderfer, Milo Schield, Jerry Johnson, Kim Rheinlander, Judith Moran, and others to initiate a network of faculty who help students at all levels to understand and work with quantitative information, regardless of their anticipated profession. I have watched NNN develop from a small homegrown group to a robust organization with a thriving electronic journal, *Numeracy*. From these initial efforts have sprung committees and special interest groups in the Mathematical Association of America (MAA) and elsewhere, plus countless academic programs and centers devoted to QL.

Fast-forward to a conversation on the Michigan State University campus in 2016. Mathematicians Luke Tunstall and Victor Piercey were discussing QL, and observed that it had been relatively quiet on the QL front since the flurry of publications and activities during the decade from 1998-2008. The limited publications devoted to QL (aside from the many articles published in *Numeracy*) certainly is not for lack of courses, conversations, and activities—the QL community is more vibrant than ever. As such, it is an appropriate time for a new volume that highlights how the work of QL has evolved since those early days, providing QL updates to a broader audience. Gizem Karaali joined the team as a third editor, and this volume—*Shifting Contexts, Stable Core*—was born.

The premise for the title came from the fact that while the ethos for QL established via *Mathematics and Democracy* and the work of the decade that followed has remained relatively stable, the contexts in which QL is practiced—and the ways in which we talk about the construct—are ever-shifting. Madison highlights these shifts and a new vision in the closing chapter of Part I, **Quantitative Literacy: A Bird's-Eye View**, discussing changes we have seen in QL resources, centers, projects, and organizations, as well as the ever-changing relationship between QL and mathematics. Craig, Guzmán, and Harper discuss the changes in another important part of the QL landscape: QL as a critical skill for informed citizens. As already observed through the works of Dewey, this conversation about an informed citizenry has been a part of society in one form or another for generations. Steen [9] re-emphasizes Dewey's important ideas a century later:

> *An innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time.*
>
> —Lynn Steen, 1997

In the spirit of the original charge set out by NCED to operationalize QL by incorporating it into the K-16 curriculum, the current volume continues with a modern look at the **Curricula for Quantitative Literacy**. Today's QL courses continue to evolve, with connections to such areas as the liberal arts (Fung), media and the news (Boersma, Diefenderfer, Dingman, and Madison; Schwab-McCoy), sustainability (Baird, Nikbakht, Marland, and Palmer), and finance (Dorée and Balbach). Common skills and practices can be observed in these QL courses, and Gaze provides an overarching context for creating and teaching QL offerings.

As already outlined, QL has struggled for decades to find its place—both in the academic curriculum and throughout society. It is fitting that a section of the volume, **QL in an Institutional Context**, which provides a variety of perspectives on how institutions have adapted and adopted QL as part of their culture. And, as QL continues to evolve, the pressures for accountability in higher education place greater demands for QL assessments that are reliable, accurate, and easy to administer. While the importance of developing such valid assessment measures for QL has been frequently discussed (initiated by a seminal 2008 study of QL [2]), current ideas for promoting assessment such as those presented in this section (Kiliç-Bahi and Cahoon; Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt; Zerr) are increasingly critical to the future of QL.

The International Life Skills Survey [5] calls Quantitative Literacy "an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work." In "The Case for Quantitative Literacy," the design team stated that, "Only by encountering the elements and expressions of numeracy in real contexts that are meaningful to them will students develop the habits of mind of a numerate citizen." The design team went on to say that "like literacy, numeracy is everyone's responsibility" [10]. This final statement presented a concern, discussed by Bernie Madison [6] in "Quantitative Literacy: Everybody's Orphan": that is, instead of being embraced as everyone's responsibility, what if QL became the purview of no one? Fisher's opening chapter in the current volume revisits this concern, stressing that QL sits at the intersection of numerous disciplines and communities of practice. However, we have witnessed in the intervening years since Madison stated his concern that while QL may be more complex than a simple definition can convey—and the debates about the definition and place for QL are likely to continue, as Fisher suggests—a much broader appreciation of the importance of quantitative skills for all has been achieved through the hard work of the rapidly expanding QL community. Quantitative Literacy is definitely here to stay—and happily, as Madison now discusses in the pages that follow, QL is "an orphan no longer."

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Contents

Opening Remarks

Luke Tunstall, Gizem Karaali, and Victor Piercey
Michigan State University, Ponoma College, Ferris State University

The thrust of this volume makes clear that in more ways than one, quantitative literacy and reasoning (QL/QR)—to use a metaphor from Bernie Madison—is no longer an orphan within the realms of U.S. higher education. Through seminal pieces for what has become a community of scholars passionate about QL/QR, Paulos [9], Steen [8, 11], and others have firmly established an imperative that our citizenry—and college students in particular—be quantitatively literate for their own sake, for the health of our workforce, as well as for democracy itself. This appealing and intuitive rhetoric has propelled conversations about QL/QR into the new century, spurring the creation of organizations, curricula, publications, as well as institutional networks, each with the purpose of advocating for QL/QR within and across the college curriculum. Furthermore, such work has allowed for conversations about collegial first-year mathematics courses to transition from, for example, ideas for improving college algebra, to the difficulties in scaling up QL/QR classes at larger institutions. Without question, our thriving community is indebted to Lynn Steen, Bernard Madison, and a host of others who have championed QL/QR on the ground over the past four decades, for this evolution in thought. Countless individuals—from members of the MAA or National Numeracy Network, to K-12 teachers creatively adapting their day-to-day curriculum—have been and continue to be involved in the cause, and so any prospective listing would necessarily leave out key figures. Because of this vast group's work, such evolution in thought about QL/QR—like evolution itself—is constantly underway.

For this reason, the theme we take up in this collection is *Shifting Contexts*, *Stable Core*, the idea being that while the construct of QL/QR has remained relatively stable over recent years, the application of this construct remains perpetually transient. For example (and discussed in significant detail in Craig, Mehta, and Howard in this volume), since the 2001 publication of *Mathematics and Democracy*, a number of new phenomena and technologies have flooded the realm of public consciousness, providing new contexts to practice QL/QR—a constant flux that can be at once both exhilarating and overwhelming. As Gillman [4] notes, for some this ephemeral quality suggests that we might only know QL/QR when we see it. Fortunately, while this suggestion might resonate with many readers, the ways in which QL/QR has "grown up"—conceptually, institutionally, and practically—have forced us to make decisions based on some operationalization of the construct. The sheer number of individuals and institutions making such decisions has resulted in a myriad of artifacts that invoke different meaning in terms such as *numeracy*, *quantitative literacy*, and *quantitative reasoning*. Confronting the nuance head-on, Karaali, Hernandez, and Taylor suggested that upon putting complexity aside, the three terms connote a common spirit "of a competence in interacting with myriad mathematical and statistical representations of the real world, in the contexts of daily life, work situations, and the civic life" [7, p. 25]. That is, the term quantitative literacy and its etymological siblings form a solid core from which individuals may advocate for QL/QR. It is these advocates whom we have had the privilege of working with in the past months to develop the present volume.

Thematically, we have organized this collection around four motifs: **A Bird's Eye View**, **Curriculum for Quantitative Literacy**, **Quantitative Literacy in an Institutional Context**, and **Perspectives from the Quantitative Literacy Community**. While the reader may choose to explore the chapters in any order, we have structured the volume so that the sequential reader will experience a "campus visit" to "QL University" (or QLU).

Part I, **A Bird's Eye View**, can be thought of as gathering expectations during the flight to QLU and looking down on the campus as our flight nears the airport. To start our preparation, Fisher directs us to consider just what we mean by quantitative literacy. He offers the following as a definition: Quantitative literacy is the facility to participate in the intersecting quantitative practices of many different communities (each with their own patterns of discourse). This is the foundation of the stable core of QL/QR that we have asked our writers to consider as they prepared their contributions for your tour. We move on to two visions of how quantitative literacy has and could change, the first offered by Craig, Mehta, and Howard focusing on the impact of technology, and the second offered by Cardetti, Wagner, and Byram with an emphasis on the relationship between QL and various partner disciplines. These chapters are broad reflections on how the core of quantitative literacy is impacted by shifting contexts. After our plane lands, Madison drives us from the airport to the campus and shares his perspective on how quantitative literacy has evolved over the decades. This last chapter provides closure to our "pre-visit" reflections.

Once we arrive on the campus of QLU, Part II of our trip involves classroom observations to examine **Curriculum for Quantitative Literacy**. This allows us to see, in a concrete way, how the stable core of QL/QR curriculum has evolved into shifting contexts. We first peek into Gaze's classroom to observe fundamental pillars of our stable core in action. To Gaze, the core of quantitative literacy has three pillars: proportional reasoning, spreadsheets, and probability and statistics. Next, Fung's chapter and Schwab-McCoy's chapter bring us into two first-year seminars emphasizing QL. First-year seminars have been identified by the Association of American Colleges and Universities (AAC&U) as a "high-impact practice" [6] and provide a relatively new context for teaching quantitative literacy. Next, we visit Boersma, Diefenderfer, Dingman, and Madison's class, where they use news from old and new media to teach QL. Part of the shifting contexts is understanding quantitative information in an era in which news is accused of being fake (for an interesting discussion of fake news and its impact on the classroom, see [10]). Next we move into a couple of classes where we see other branches that are growing out of the stable QL core. Baird, Nikbakht, Marland, and Palmer are teaching QL through sustainability, while Dorée and Balbach are focusing on finance. A notable characteristic of their classrooms is that they need not be exclusively centered on QL. Having visited various classes, Edwards, Melfi, and Satyam sit with us to talk about what we observed and consider how rigorous QL/QR really is. It is a common belief that quantitative literacy and reasoning courses are somehow weaker and lack the rigor of courses such as college algebra. Edwards, Melfi, and Satyam take issue with this belief, pointing out a parallel between the cognitive growth for students in QL/QR and the cognitive growth for mathematics majors in Introduction to Proof courses.

Part III of our visit involves talking with faculty, administrators, staff, and external parties to examine **QL in an Institutional Context**. As the contexts for quantitative literacy and reasoning have shifted, it is important to think about an entire institution's commitment to QL/QR. First, we step into Salamone, Smith, and Parsons' office where they provide a hopeful vision of expanded offerings of quantitative literacy among a variety of institutions, defining a "quantitative literacy program" and describing how supportive "QL centers" have grown. Not every story ends happily, however, and Dewar, Larson, and Zachariah tell us about obstacles they experienced in trying to sustain a quantitative literacy innovation at an institution. Getz, Richardson, Hartzler, and Leahy bring us into their office to describe a counterpoint to Dewar, Larson, and Zachariah by describing efforts to expand access to QL/QR by developing courses at scale. With an institutional commitment to QL/QR comes the need for assessment. We step into the assessment lab where Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt describe a model for situated assessment using authentic tasks. Zerr steps over to us to offer his own perspective that quantitative literacy is a skill that is learned throughout a program or experience and should be assessed accordingly. Kilic-Bahi and Cahoon conclude this part of our trip, focusing on the institutional perspective, by discussing key assessment efforts in recent years.

We conclude our visit with Part IV, **Perspectives from the Quantitative Literacy Community**, in which we go to dinner and discuss philosophical questions concerning how each of us arrived at quantitative literacy and reasoning, and where we think the community is heading. As you can imagine, there will be some disagreement. Bolker shares his personal experience evolving as an instructor and how he found his way to quantitative reasoning. Craig, Guzmán, and Harper critically engage in the political nature of quantitative literacy, and discuss how we might re-envision a quantitatively literate citizenry. Philip and Rubel continue the political discussion by proposing that a quantitative literacy class can be used as a model for democratic citizenship, challenging us to create more opportunities to incorporate quantitative literacy into a broader sense of democractic deliberation. Our dinner concludes with an interview with Len Vacher, the founding co-editor of *Numeracy*. Vacher came to quantitative literacy through computational geology. He shares his perspective on how quantitative literacy has evolved over the years, providing a different view on this than Madison, and emphasizing the importance of conversations among multiple disciplines in further growth.

As we wait at the airport or during our flight home, we think about our visit to QLU. We hope that you reflect on the chapters you read, the classrooms you visited, and the people you met. There is a variety of reasons a reader may have picked up this volume and visited QLU. Some readers may be curious about theoretical insights into QL/QR and its history and evolution. Other readers may have a more immediate need for concrete suggestions for teaching QL/QR in their own classrooms. Others may be charged with assessing QL/QR at the institutional level. Regardless of what drove you to pick up this volume, we hope that you found what you were looking for in addition to some unexpected conclusions, as we certainly have.

Though the tone of this volume is civil and communal, it is certainly worth noting that—as with any field of research and practice—there are issues about which thoughtful people disagree. For instance, it would be misleading to claim that there exists agreement concerning the extent to which mathematical skills are necessary for quantitative literacy (e.g., [3, 5], Boersma, Diefenderfer, Dingman, and Madison in this volume). Furthermore, as we see from Craig, Guzmán, and Harper in this volume, the ways in which we evoke or talk about quantitative literacy are not without critique. We encourage readers to engage with these arguments, discuss them with colleagues, and even with the authors themselves. Indeed, given that the roots of QL/QR are couched in discussions of democracy, it is imperative that we encourage democratic deliberation among ourselves as the community grows and shifts.

As we were collecting proposals, drafts, and revisions of each of the chapters, we were struck by the extent to which QL/QR has matured. Not only is QL/QR no longer an orphan, but QL/QR has reached a stage in which practitioners are more confident in the stability of the core while branching into a multitude of shifting contexts. Practitioners can engage in social justice initiatives, guide consumers in their encounters with sophisticated financial products, or participate in dialogues that "deconstruct" the very notion of a classroom space as a laboratory of democracy. The chapters in this volume illuminate how far the quantitative literacy community has come over the past decades, demonstrating that Steen's 2004 *Urgent Challenge* [13] for universities has not gone unanswered. In all of these cases, the core of QL/QR, whether as defined abstractly by Fisher in Part I of this volume or as enumerated concretely by Gaze in Part II, ties these branches together. This is a milestone in the QL/QR journey. We hope the reader is as excited as we are to reflect on this point, and to engage in creating more milestones to come.

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Part I

Quantitative Literacy: A Bird's Eye View

We begin this volume with a "bird's eye view" of quantitative literacy. These chapters are dedicated to describing a vision of what we mean by quantitative literacy, as well as the journey quantitative literacy has taken over the past three decades. Taken together, our aim in these chapters is to provide a framework for thinking about what we mean by quantitative literacy in the parts that follow.

In the opening chapter, "What Do We Mean by Quantitative Literacy?" Forest Fisher lays a foundation both for this part and for the rest of the volume. In this chapter, Fisher grapples with the problem of defining quantitative literacy, asserting that it is no longer sufficient to claim that we "know it when we see it." By tracing the meaning of quantitative literacy and diving into its relationship with literacy more broadly, he offers readers a nuanced operationalization for the term that situates it within conversations that go beyond the mathematics community. In particular, Fisher arrives at the following: Quantitative literacy is the facility to participate in the intersecting quantitative practices of many different communities (each with their own patterns of discourse).

The next two chapters push Fisher's definition in different (although related) directions. Jeffrey Craig, Rohit Mehta, and James Howard's chapter, "Quantitative Literacy to New Quantitative Literacies," builds on Fisher's notion of literacy as a social practice, all the while striving to make our understanding of quantitative literacy more dynamic. Using media literacy as a framework, Craig, Mehta, and Howard argue that quantitative literacy should adapt to sociocultural and technological shifts in our society. In the next chapter, "Intercultural Citizenship as a Framework for Advancing Quantitative Literacy Across Disciplinary Boundaries," Fabiana Cardetti, Manuela Wagner, and Michael Byram focus on the interdisciplinary nature of quantitative literacy and the need to encourage quantitative thinking across the curriculum.

Bernard Madison concludes Part I by describing the progress made in defining and conceptualizing quantitative literacy. Madison was involved in the earliest discussions of quantitative literacy. Initially, Madison argued that quantitative literacy was an "orphan"—a mode of inquiry with no disciplinary home. In his chapter in this volume, "An Orphan No More," Madison contends that quantitative literacy has "grown up" and found its own place in the world. We encourage readers unfamiliar with quantitative literacy to read these chapters before reading the remainder of the volume.

1

What Do We Mean by Quantitative Literacy?

Forest Fisher
Guttman Community College (CUNY)

1.1 Introduction

In his introduction to MAA Notes #70, Gillman describes Quantitative Literacy (QL) as "one of those things about which we say 'I know it when I see it' " [21, p. vii]. He then admits that quantitative literacy is quite difficult to describe precisely, and after listing several potential topics that might be covered under this banner (numeracy, some geometric, algebraic, and algorithmic skills, etc.), he finally settles on the definition found in the bylaws of the MAA's SIGMAA on QL: "Quantitative Literacy (QL) can be described as the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in individuals' private, civic, and work lives" [41].

Gillman is not the first person to struggle with the definition of quantitative literacy. Everyone seems to agree that QL is important, but few can agree on what it really means. Indeed, Madison in this volume recalls how many of us "experience difficulty in conveying the meaning of QL/QR to others." In the UK, QL was first referred to as "numeracy" in the 1959 Crowther Report where authors sought to "coin a word to represent the mirror image of literacy" [13]. The 1982 Cockcroft Report expanded upon this definition by suggesting that the word numerate should entail two attributes: "The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands on his everyday life. The second is an ability to have some appreciation and understanding of information, which is presented in mathematical terms, for instance in graphs, charts, or tables or by reference to percentage increase or decrease" [11, p. 11].

Already this description sounds quite different from the "geometric, algebraic, and algorithmic skills" identified by Gillman, but the bigger problem is that this definition is not helpful to instructors who may be entrusted with teaching a QL course. It is not clear what content or approaches an instructor should pursue in the classroom to engender "at-homeness" with numbers. The implicit comparison between quantitative literacy and the traditional notion of written literacy also seems like a stretch; should we use the term literacy whenever we want students to feel at home with a particular set of skills or ideas?

The Association of American Colleges and Universities (AAC&U) [1] offers another equivocal definition. QL is a " 'habit of mind', competency, and comfort in working with numerical data." There are many habits of mind that we might hope to arouse in our students. Should all of them be thought of as a type of literacy? And specifically, what does it mean to be comfortable working with numbers? What content and practices should instructors employ to develop this habit of mind? The AAC&U at least offers some guidance in observing that "individuals with strong QL skills possess the ability to reason and solve problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc. as appropriate)" [1].

Much like the AAC&U, Steen defined QL as a "practical, robust habit of mind anchored in data, nourished by computers, and employed in every aspect of an alert, informed life" [44, p. 4]. Notice that his definition yields a special status to computers, whereas the other definitions do not even mention computers. Even Steen himself admits that "beyond 'the basics,' there is little agreement about specific goals appropriate for tomorrow's world," and so authors "express contrasting views about the nature and importance of quantitative literacy" [42, p. xvi-xvii].

It took mathematicians over 100 years to agree upon a formal definition of the limit, but that consensus created a deluge of new results and understandings [6]. Quantitative literacy is at a similar point. "I know it when I see it" will simply no longer cut it. We need to think critically about what we mean by quantitative "literacy." In particular, can we justify using the word "literacy"? Does QL have more than a superficial resemblance to traditional notions of literacy?

This chapter takes a social linguistics[1] approach to quantitative literacy. I will look at research on reading-and-writing literacy, and apply it to the study of QL to show that quantitative literacy does in fact resemble reading-and-writing literacy in many ways. I will argue that all forms of literacy involve a representational medium that is shared by different social groups, each with its own unique practices surrounding that medium. As such, literacy is an inherently social phenomenon, and we cannot divorce the study of QL from the social contexts in which it is realized. In the final section, I will propose the following definition:

> Quantitative literacy is the facility to participate in the intersecting quantitative practices of many different communities (each with its own patterns of discourse).

For example, many different communities use numerals, algebraic expressions, graphs, charts, and/or computers to represent, interpret, manipulate, and communicate about quantities. However, not all all communities are situated in the same way with respect to these practices, so performing a calculation or reading a chart may look very different from one social context to another. I will close the chapter with a discussion about the pedagogical implications of this definition.

1.2 Technologies of Comprehension and Social Literacies

Written language is a sort of technology of communication. It is true that the human brain is uniquely wired for the production and comprehension of speech [38], but written language is different from speech. Only a handful of civilizations in human history have independently invented written language [10]. Like other technologies, writing requires a physical medium (paper, pencil, typewriter, computer, etc.,) and an invented means of representing spoken language using this medium.

Written language is also very much a technology of *comprehension*. There is an enormous body of research demonstrating that literacy changes the way humans think about the world. Writing enables a person to store thoughts on paper; it often serves as a sort of external memory storage device. The classicist Eric Havelock [25] studied ancient Greek poets like Homer and argued that in an illiterate society—one in which most people could not read and write—these poets were the keepers of knowledge. They used repetition and character-driven stories to help their audiences remember and pass on information. People in the oral culture of ancient Greece thought about the world differently than we do today.

Ong [35] contrasted thought and expression in oral versus written cultures as

1. additive (combined through expressions like "and") rather than subordinative (combined through complex grammatical embeddings),
2. aggregative (pieces of thought come clustered together as in "valiant knight" rather than "knight"),
3. redundant and formulaic (as in the repetition of Homeric poems),
4. involving a conservative structure that inhibits experimentation,

[1]For a good introduction to social linguistics (also called sociolinguistics), see Gee [20]. Readers may also want to take a look at Craig, Mehta, and Howard in this volume, who also explore quantitative literacy through a social linguistics lens and Karaali, Villafane-Hernandez, and Taylor [29], who anticipated many of the ideas in this chapter.

5. empathetic rather than impersonal,
6. situational rather than abstract.

It seems that writing enables human beings to put their thoughts down on paper, and in doing so, they can rearrange and reorganize them in complex ways.

Numbers are also very much a "technology of comprehension." Much as the human brain has built-in capabilities for spoken language, newborn babies and even animals are capable of a sort of crude counting called "subitizing" [14]. Subitizing is the quick assessment of small quantities; it is more about estimation than cardinality, and is not really a reliable way to count numbers bigger than about three or four. Pica, Lemer, Izard, and Dehaene studied the Mundurukú Indians in the Amazon whose language does not have words for numbers bigger than five [37]. Through a series of different experiments, they showed that the Mundurukú count in ways that resemble subitizing. In other words, they have not really invented numbers.

Likewise, the linguist James Hurford demonstrated that the first three numbers have a special status in language [27]. To count beyond these numbers, early human beings relied on their fingers and toes [27]. As pre-numerate people ran out of body parts to count, they had to invent "disembodied" numbers. The earliest known example of this appears in bones found from the Aurignacian period (35,000 to 20,000 BC) [30].

Early written systems of numbers had several limitations. Just imagine using long division with Roman numerals to divide CDLXVIII by XXXIX; it is practically impossible! Math historians are mostly in agreement that the Romans relied on abaci and fingers to perform calculations and used Roman numerals exclusively to inscribe the results [9, p. 116–118]. The long division algorithm explicitly takes advantage of the way numbers are represented in decimal notation, making it a product of the notational system itself. Different technologies of communication permit different ways of comprehending quantities.

Indeed, we take numbers so much for granted that we do not recognize the monumental technological accomplishments involved in "simple" things like Arabic numerals, the decimal system, and the number 0. These things had to be invented. They were not readily apparent to ancient cultures.

Porter [39] observes that as recently as two centuries ago, nearly every human being was quantitatively illiterate. Even scientists rarely used numbers, and the largest practitioners of counting were merchants. The widespread use of numbers did not really come about until French scientists invented the metric system in 1790. This allowed 19th century bureaucracies to overcome the idiosyncrasies of different local measurement systems. As Porter reminds us, the word "statistics" originally referred to the collection of data by the state [39]. In Germany in the 17th century, a particularly vocal group of social scientists advocated the use of statistical tables and tabular inference under the banner of *Die Tabellen-Statistik*. From 1687 onward, this movement was known throughout Britain as the "Political Arithmetic" [4].

Across the Atlantic, [12] Harvard University did not make basic arithmetic a requirement for admission until 1802. Again, merchants had more exposure to arithmetic than academics, although their methods of calculation were so closely linked to specific social practices (calculating board feet to build a home, determining discounts and interest rates, etc.) that they often could not transfer techniques from one context to another.[2]

These observations demonstrate how technologies of comprehension, inventions like numbers and written language, depend upon particular social institutions to sustain them. It is no coincidence that the state was the first major proponent of numbers, and that their widespread adoption was accelerated by a socially agreed upon standard like the metric system. There are very specific social practices associated with reading and writing. Knowing how to read and write has as much to do with these practices as it does with being able to decode a word on a page.

One more example may help to illuminate key facets of this point. Consider the technology of maps, which are the oldest form of quantitative graphic known to humankind, dating all the way back to clay tablets in 3800 BC [4]. Maps do not fit neatly into either category of literacy discussed so far (reading-and-writing or quantitative literacy), but they nicely illustrate several key points about technologies of comprehension.

For example, the anthropologist Edwin Hutchins studied navigational practices aboard a U.S. helicopter carrier ship in the 1980s [28]. In his observations, he pointed out that maps encode information collected by thousands of human

[2] This brief history is clearly centered around Europe and the U.S., although similar things could be said about the development of quantitative practices in other parts of the world. For example, record-keeping with numbers goes back to Babylonia, Mesopotamia, Egypt, China, and so on. In each instance, these practices were sustained by social institutions much like the role of the state in the European examples.

beings over hundreds of years. They also contain information that no single human being may have ever calculated on their own. It is quite possible that no one person has ever measured the distance from Fairbanks, Alaska to Auckland, New Zealand, and yet this information is readily available in a map. Since this calculation was made possible by multiple people's work but not known by any of these people individually, Hutchins concluded that cognition extends beyond the individual mind to include groups of people.

If cognition is distributed among multiple people then culture is a powerful mediator in this process. Exploring this proposition, Hutchins compared the practices of Micronesian navigators in the 1980s with their counterparts in the U.S. Navy. Navigators in the Caroline Islands use a sophisticated "star compass" to chart canoe trips between islands that are often separated by over one hundred miles of ocean. This "compass" indicates quite a bit more information than cardinal directions and could be more accurately described as a "star map." The basic elements of the technology are "linear constellations," sets of six to ten stars that move in a path across the sky from the eastern horizon to the same point in the western horizon. By comparing the position of their boat in relation to these star paths, navigators can accurately locate themselves without consistent landmarks or birds-eye view maps. These star maps are not written down, so it would be easy to think of them as passive observations of the physical world. Yet, as Hutchin's observes [28, p. 68],

> Seeing the night sky in terms of linear constellations is a simple representational artifice that converts the moving field of stars into a fixed frame of reference. This seeing is not a passive perceptual process. Rather, it is the projection of external structure (the arrangement of stars in the heavens) and internal structure (the ability to identify the linear constellations) onto a single spatial image. In this superimposition of internal and external, elements of the external structure are given culturally meaningful relationships to one another.

It would also be a mistake to view written, birds-eye view maps as passive representations of reality. If one were to plot a line of constant course across the surface of the earth (a so-called rhumb line) then this line would trace a path through the poles and around the globe. The Mercator projection, found in many birds-eye view maps, distorts the features of the earth so that rhumb lines correspond precisely with the straight horizontal and vertical lines on a map. This distortion has many advantages for navigation, but it greatly exaggerates the size of Greenland and Antarctica. Both birds-eye view and star maps are constructed with the practices of specific social groups in mind. One needs to be aware of these practices in order to read the map [23].

Much like maps, numbers are also not passive representations of reality. The navigators in the Caroline Islands projected external structure (the arrangement of stars) and internal structure (the ability to identify certain constellations) onto a single spatial image. In the same way, counting on one's fingers and toes involves the projection of external structure (body parts) and internal structure (cardinality) onto a single spatial image. This practice gives fingers and toes culturally meaningful relationships to one another such that even a casual phrase like "high five" is loaded with mathematical meaning. In both examples (counting and star maps), the technology is a lens, a certain way of imposing structure on the how we see the world. Since these technologies do not involve a physical artifact like a written map, they are often invisible to their users. That is, we forget that counting on one's fingers and toes is a culturally mediated act.

Similar observations can be made about written numbers. The inscriptions "9" and "IX" are more than impartial records of quantity. Different symbolic systems permit different ways of comprehending numbers. For example, certain practices like long division are not even possible in an inscription system such as Roman numerals. The technologies of mathematics are constructed with the practices of specific social groups in mind, and again, one needs to be aware of these practices to use numbers.

Tools like written language, numbers, and maps enable us to accomplish things we could not otherwise do. DiSessa [15] refers to this phenomenon as *material intelligence* since these material objects enable us to be intelligent in new ways. People often lament the effects of new material intelligences, observing that students are reliant on calculators to perform simple arithmetic or dependent on their phones to get around town. Fewer people seem to lament the fact that they are reliant upon written language and numbers, although these are technologies of comprehension just like cell phones and calculators.

1.3 A List of Skills

Most of the thinking about literacy in the previous section relates in one way or another to tools like written language and numbers. Perhaps then, we could define literacy by listing the skills needed to use these tools? This section examines the idea of defining quantitative literacy as a list of skills.

Literacy is a broad field of research that encompasses many different perspectives. See [2] for a concise historical account. Traditional literacy research focused on the skill of decoding words on a page (phonetics) and tended to approach this practice through a cognitive or behaviorist lens. In the 1980s, some researchers began to critique this approach, noting that a person needed to draw on a wealth of social knowledge in order to comprehend the meaning of a sentence. Some of these scholars also began to recognize the plurality of literacy, noting that there were several different socially defined practices connected to print. The emergence of the internet meant that a person might encounter text along with images and other media while reading a web page. Reading was no longer simply about decoding words on a page; it involved "multiliteracies" [19, p. 53–63]. Today, researchers work from a variety of perspectives: behaviorist, semiotic, multiliteracies, cognitive, sociocultural, critical, feminist, and so on. See Baker [3] for a good cross-section of these perspectives.

In this section, I draw on the perspective of Andrea DiSessa who coined the term "computational literacy" to describe an aspiration that reading and writing computer programs might become as common as reading and writing words. His ideas closely parallel work in the more established field of social linguistics [20], but DiSessa provides a particularly good framework for thinking about quantitative literacy. Steen [44] emphasized quantitative literacy as a "habit of mind anchored in data, nourished by computers" and so DiSessa's focus on computers is quite useful. This technology-driven perspective led him to consider how literacies and their requisite skills change over time. In particular, DiSessa initially proposed the following definition [15, p. 19].

> Literacy is a socially widespread patterned deployment of skills and capabilities in a context of material support (that is, an exercise of material intelligence) to achieve valued intellectual ends.

By this definition, traditional literacy involves the deployment of reading and writing skills through the material support of written language, pencil, paper, the printing press, etc. By the same token, quantitative literacy ought to involve the deployment of skills used in the comprehension and manipulation of numbers with the material support of numerals, the decimal system, arithmetic, graphs, calculators, computers, pencil, paper, etc.

It is tempting to define quantitative literacy in this way, simply listing mathematical skills like learning objectives in a textbook. Many of the definitions mentioned in the introduction took precisely this approach. Before long, the inevitable question arises: "which skills are necessary for the comprehension and manipulation of numbers and which ones are not?"

We can all agree that counting, arithmetic, and number sense are important. Probably, most people would agree that proportional reasoning is necessary (see Gaze in this volume). What about algebra? Some feel that quantitative literacy might include algebra [18, 21]; others position QL in opposition to algebra [24]. Should we include "constructing and interpreting tables and graphs?" Determining the margin of error? Using hypothesis tests to make inferences about the world? Applying the law of sines to determine an angle? Finding marginal cost using a derivative? Balancing a checkbook? Writing a computer program? Calculating interest on a loan? Counting cards in poker? The list of skills could go on forever. How do we decide which skills are the right ones?

In their original Case Statement for QL, Steen et al. [43, p. 15–17] listed arithmetic, data, computers, modeling, statistics, chance, and reasoning with several sub-skills under each of these headings. Karaali, Villafane-Hernandez, and Taylor [29] conducted a frequency analysis of the fifteen essays in *Mathematics and Democracy* [43], comparing which skills were most frequently associated with terms like "quantitative literacy," "numeracy," and "quantitative reasoning." They concluded that "quantitative literacy in these two edited volumes encompasses *confidence with number*, *appreciation for mathematics*, *ability to interpret data*, *to think logically*, *to make decisions logically*, and *to use mathematics in context*."

There is no way to list every skill, and as DiSessa further observes [15, p. 21],

> Given skills may change their affect on and relevance to valued accomplishments with the development of other skills. For example, arithmetic may be a valued skill, but it changes its entire context—its community association, if not its essential meaning—when quantitative sciences give arithmetical computation new

> reach. Not just accountants but also engineers and scientists use arithmetic, and for each, it is relevant in a different way.

Indeed, DiSessa has identified a trap in which many QL advocates find themselves. In 1982, the MAA's Committee on the Undergraduate Program in Mathematics (CUPM) released a special report on QL in the *American Mathematical Monthly*. The report attempted to answer the question, "what mathematics should every graduate of an American college or university know?" After conducting numerous surveys, the panel "found such great diversity that it could not describe an everywhere attainable goal" [21, p. 5]. The list of relevant skills are constantly changing and their meaning changes from one social context to another. We cannot create a meaningful definition of quantitative literacy simply by listing skills. As Craig, Mehta, and Howard in this volume argue, "Literacy is not a fixed entity because what and how people read and write changes over time and space."

With this in mind, DiSessa shifted his focus to ask the seemingly innocuous question, "What do the following have in common: newspapers, magazines (from *People* to *Soldier of Fortune* to *National Geographic*), scientific papers, pulp fiction, poetry, advertisements, tax forms, instructional manuals, and financial prospectuses?" [15, p. 23]. The answer is clearly that (1) they all involve the medium of reading and writing, and (2) each of these genres of writing serves the needs of a particular social group. Pulp fiction is not sustained on the skills of reading and writing alone; it requires a community of avid fans who share an interest in reading pulp fiction. It has its own social conventions—its own patterns of discourse—that set it apart from tax forms and scientific papers. Hence, DiSessa proposed the following much-improved definition [15, p. 24].

> A literacy is the convergence of a large number of genres and social niches on a common, underlying representational form.

DiSessa's definition also means that literacies are implicitly wrapped up in identity. Being an avid reader of pulp fiction means claiming a certain identity just like tax accountants and poets claim other identities.

Scollon and Scollon demonstrated this idea beautifully while studying literacy among the Athabaskans, an indigenous group in Alaska and northern Canada [40]. They argued that discourse patterns reflect a certain worldview, and so adopting new patterns of discourse meant adopting a new identity. The Athabaskans engage in patterns of discourse very different from the dominant patterns of discourse in the U.S. and Canada. They are protective of their individuality and thus avoid conversations unless all perspectives are known. Likewise, an Athabaskan in a subordinate role is expected not to demonstrate their learning, but rather to sit quietly and observe. In contrast, Americans are expected to exert their individuality by expressing their opinions, and are likewise expected to outwardly demonstrate their understanding of new ideas to show they are following along.

When it comes to writing, the Scollons observed that essays are a disembodied form of discourse. The writer must imagine a fictional audience since the person to whom they are speaking is unknown. Writing down ideas in the explicit language of an essay is thus problematic in a culture where reading between the lines is so highly valued. To write an essay, the Athabaskan would need to know their audience and their audience's perspective, but the audience is not present. School literacy presents a worldview that the Scollons call "modern consciousness" because it is about abstract rational minds communicating with one another. We will see this idea resurface in the next section where I discuss the patterns of quantitative discourse encountered in traditional school mathematics.

1.4 School Mathematics and Other Quantitative Discourses

As the above discussion demonstrates, literacy implicates different social niches, each with their own patterns of discourse. To find a workable definition of quantitative literacy, we need to consider how discourse in school mathematics differs from other quantitative discourses. In this section, I will investigate the discourse practices of school mathematics as they relate to quantitative literacy.

If the "modern consciousness" worldview is about abstract rational minds communicating with one another, then mathematical discourse takes this worldview to its logical extreme. For example, mathematical discourse flattens time [31]. A statement like

"You had two marbles, and then I gave you three more. Now you have five."

becomes simply

$$2 + 3 = 5.$$

O'Halloran [33, 34] describes mathematical discourse as "multisemiotic," meaning it combines mathematical symbolism, visual display, and both written and spoken language. She observed that mathematical discourse tends to nominalize processes so that a verb like "added" becomes a noun like "the sum." These nominalizations are more complicated than those in everyday English; they can sometimes occur across semiotic mediums. A textbook problem might talk about a baseball that was thrown into the air. When a student graphs the trajectory of this ball, suddenly the verb ("throw") in written language has become an object (the graph) in a different semiotic medium!

Through nominalization, mathematical expressions can embed multiple clauses within one another. The symbols 7 and x are combined to form the clause $7x$. This is combined through the process of addition with 3 to form the clause $3 + 7x$. This expression is then squared to form the clause $(3 + 7x)^2$, and so on. This process perfectly mirrors Ong's suggestions that orality is additive whereas literacy is subordinative. These embeddings often create confusion since spoken language does not permit the same types of complex subordinations. For example, the statement "three divided by x plus 4" could be interpreted as either of the algebraic expressions

$$\frac{3}{x} + 4, \qquad \text{or} \qquad \frac{3}{x+4}.$$

O'Halloran's perspective is deeply rooted in the history of mathematics and the different social practices that created mathematical language [33]. Citing Boyer [5, p. 180], she acknowledges three stages in the development of algebraic expression.

1. A *rhetorical stage* in which algebraic expressions are written out in words as in "three divided by an unknown quantity plus four."
2. A *syncopated stage* in which symbols were adopted for some words as in "3 divided by x plus 4."
3. A *symbolic stage* in which full algebraic symbolism was realized: "$3/x + 4$."

The standardized symbols that mathematicians use today developed over hundreds of years through the contributions of many people, and as Cajori [8, p. 337–338] notes, "[o]ften the choice of a particular symbol was due to a special configuration of circumstances (large group of pupils, friendships, popularity of a certain book, translation of a text), other than those of intrinsic merit of the symbol." In other words, the evolution of algebraic symbols was social, incorporating the work of many people spread across space and time. Much like navigating with a map comprises cultural norms and several generations of minds, a student who solves an equation for x has not solved this problem on their own.

Clearly, there are specific patterns of discourse in school mathematics, and these patterns privilege a certain worldview. The discourse of mathematics is abstract rather than situational, impersonal rather than empathetic [36], and subordinative rather than additive. It is multisemiotic, and it seems to push an extreme "modern consciousness" worldview.

This abstract, impersonal discourse is certainly far afield from the descriptions of quantitative literacy encountered in the introduction. As suggested by DiSessa, literacy involves the convergence of a wide array of genres and social niches. Each niche has its own patterns of discourse and its own ways of making meaning with the representational form. Pulp fiction writers and tax accountants both use written language, but they use it in very different ways.

Conversely, if there were only one social niche invested in a representational form then it would be difficult to label it a literacy. There is a thriving community of cross-stitchers who share their designs online. They have both a social niche and a representational form, but it sounds absurd to talk about cross-stitch literacy precisely because there is one and only one social niche invested in this representational form. The term literacy applies when a representational form has multiple social groups invested in it, not just a single group.

Whether we want to admit it or not, school mathematics is a discipline with its own very specific practices. Unlike cross-stitchers, mathematicians are not the only people who use numbers. Accountants, merchants, computer programmers, scientists, engineers, bureaucrats, doctors, insurance companies, navigators, members of professional sports teams, poker players, and many other social groups all use numbers, and each of these communities has their own practices, their own patterns of discourse surrounding numbers.

For example, a mathematician might write the number $\sqrt{2}$ and state that this representation is "more accurate" than a decimal approximation. Embedded within this representation is procedural information about how to calculate the number $\sqrt{2}$; it is a nominalized expression containing the instructions to "take the square root." On the other hand, the accountant may not perceive this nominalization as being more precise, and may instead write this number as 1.41 since she or he deals primarily with monetary values. A scientist might choose to round this number to a different decimal place based on significant digits. There are many different ways to represent numbers. The thing that sets quantitative literacy apart from traditional mathematics is that it does not privilege one discipline's patterns of discourse over another's. While school mathematics tends to favor the quantitative practices of a single social group (mathematicians), quantitative literacy admits a more level playing field, recognizing the quantative practices of numerous discplines.

1.5 Conclusions

By now, it should be clear that literacies are socially constituted. They require a technology like an alphabet or numerals, and they require social institutions to sponsor these technologies. I demonstrated several similarities between written language and numbers. They can each be thought of as cultural extensions of a biological process. Writing is very much an extension of speech, and numbers are an extension of subitizing. I showed how these technologies are historical, rooted in specific cultures with specific social practices. To become literate means to change one's identity, to join a new social group, or adopt a new culture. With this in mind, I suggest the following definition.

> Quantitative literacy is the facility to participate in the intersecting quantitative practices of many different communities (each with their own patterns of discourse).

For example, many different communities use numerals, algebraic expressions, graphs, charts, and/or computers to represent, interpret, manipulate, and communicate about quantities. However, not all all communities are situated in the same way with respect to these practices so performing a calculation or reading a chart may look very different from one social context to another.

One does not have to accept this definition, but it is difficult to dispute its logic, and the pedagogical implications are very important.

First, given the social nature of literacy, a course that addresses quantitative literacy must provide ample opportunities for students to engage in discussions about numbers. There are many ways to make this happen. Students could engage in small-group discussions, classroom discussions, writings on a blog, discussions in an online forum, etc. Whatever the method, there simply cannot be a successful QL course where the instructor is the only one talking.

In restructuring the communication in our classrooms, we must also be conscious of how certain forms of communication favor one community's discourse over another's. Forman, McCormick, and Donato studied discourse in a reform-oriented classroom, and showed that the instructor unintentionally "privileged one particular form of explanation over others that are mathematically equivalent" [16]. They suggested that "reform requires the prior establishment of a community with a shared set of norms, mutual respect, and a common means of communication that is different from those of a traditional classroom." Likewise, Tunstall [45] used critical discourse analysis to examine how the language in mathematical textbooks and the language spoken by instructors in class may foster or inhibit quantitative literacy. He argued that textbook authors and instructors need to be conscious of whether or not they position the reader with agency. The way we communicate in our classrooms matters, and if we are not careful, quantitative literacy initiatives could have inequitable effects on our students.

Much along these lines, the history of reading and writing is littered with literacy initiatives that failed to deliver on promises of social mobility because their efforts did not provide agency to all participants. For example, Graff [22] studied and critiqued literacy efforts in 19th century Sweden that sought to teach the poor how to read the Bible, but did little else to socialize them in practices of reading and writing. Swedes achieved near-universal (reading) literacy before the 18th century, long before their peers in Europe, and yet they were unable to differentiate themselves in terms of social equality, economic development, and cognitive growth. If we fail to socialize students' quantitative practices in equitable ways, we may also find that QL is not an empowering subject.

Second, in preparing faculty to teach QL courses, we should steer away from lists of skills, and instead focus on the social contexts in which numbers arise. Rather than asking faculty to "cover ratios" the first week, and "charts" the second week, we might ask them to identify different social groups that use ratios or charts, and think about how each of these groups uses them differently. Authentic learning opportunities are more likely to grow out of this exercise than from perusing a list of decontextualized skills.

Third, if quantitative literacy involves the convergence of many different social niches then it needs to be an interdisciplinary endeavor. We need non-mathematicians to share the burden of QL so that it is embedded in a wide variety of courses across the curriculum. This idea has been suggested before [26, 7, 17], but the social linguistic perspective offers a particularly pointed argument in favor of this approach.

Fourth, central to any social group's practices are the tools and technologies used by this group. Students cannot develop quantitative literacy if they do not learn how to use appropriate technologies of comprehension. It would be strange to calculate a 20% tip in a restaurant by setting up a proportion and solving for x. It would be especially strange to use long division to solve this problem by hand. These are not the practices of restaurant patrons; most people would use a calculator in this context or heuristics like "multiply the tax by two." Students of QL should be given the same technologies to solve problems as would be used in the corresponding social context. For example, if accountants use Excel then students who are learning about the quantitative practices of accountants should be encouraged to use Excel.

Academic mathematics is basically the only quantitative discipline that (sometimes) discourages the use of calculators and computers. Even then, this discouragement is often inconsistently applied with remedial students being asked to calculate with pen and paper while advanced students use software like MatLab and SPSS. The representational mediums of mathematics have changed and will continue to change over time. We would not socialize students to count with Roman numerals or to use the bones of the Aurignacian period. Those are antiquated technologies, and we are living in the 21st century. We should encourage our students to use 21st century technologies of comprehension.

This discussion does not mean that every classroom task needs to align precisely with the technologies and practices of a specific, narrowly defined community. Many authentic tasks are in fact quite rote, and at times, the classroom needs to be its own space for students to reflect on and negotiate the manner in which the different practices of these communities intersect. The point is more that some technologies (numerals, graphs, and algebraic notation) are taken for granted or perhaps not even recognized as technologies while others (calculators and computers) are treated like invasive species that have tainted the mathematics ecosystem. If QL differentiates itself from traditional mathematics by not privileging the practices of a single community, then it must also make space for the technologies used by these other communities.

Finally, we need to temper our expectations for quantitative literacy classes. If literacy requires the commitment of many different social groups across many different dimensions of society then one college course is not going to suddenly create a literate population. Bob Moses acknowledged this fact beautifully while describing his own "mathematical literacy" work with the Algebra Project [32, p. 18]:

> This is a cultural struggle, the creation of a culture of mathematical literacy that's going to operate within the black community as church culture does. And that means that math won't be just school-based, but available as reading and writing are. Kids now routinely assume that someone will be able to explain some word to them, or teach them how to read a sentence if they don't understand it. They also take it as a matter of course that no one can help them with their "higher" math studies. Projecting several generations down the road we can see a youngster who has grown up in a black neighborhood being able to get his or her questions about mathematics as easily answered in the neighborhood.

As this discussion makes clear, quantitative literacy is a "literacy" in all the same ways as reading and writing. Just as traditional literacy makes use of written language, QL makes use of technologies such as numerals, graphs, and charts to communicate and comprehend quantitities. These technologies extend the biological process of subitizing, but more importantly, they arise in—and exclusively make sense in—specific social contexts. Quantitative literacy is thus as much about the communities that practice quantitative skills as it is about the skills themselves. As educators, we need to focus more on these communities of practice and less on the decontextualized skills that crowd so many definitions of quantitative literacy.

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2

Quantitative Literacy to New Quantitative Literacies

Jeffrey Craig, Rohit Mehta, and James P. Howard II
The University of Arizona, California State University, Fresno, University of Maryland University College

2.1 Introduction

Steen and colleagues [43] made their *Case for Quantitative Literacy* based on the premise that the 21st century, primarily due to technology changes, is a significantly more quantitative environment than any previous time in history. The design team who wrote the case made rhetorical observations about the increasing prevalence of numbers in society in the United States; phrases like “[a] world awash in numbers” [43] precede nearly every piece of literature regarding quantitative literacy (or numeracy). These authors used this rhetoric to position people relative to the demands of social systems in the world. Specifically, numeracy has emerged as a partner to literacy because the social world is being integrated with numbers due to recent technological advances. Steen argued that “as the printing press gave the power of letters to the masses, so the computer gives the power of number to ordinary citizens” [41, p. xv]. Through the internet, this power of numbers is extended to instantaneous and unmediated communications. Philip and Rubel have provided a detailed accounting of the potential for new quantitative literacies to be used for positive change by students within classrooms. Accordingly, “ordinary citizens” should be empowered to use and understand numbers in meaningful and liberating ways. With comparisons to power and literacy, as when Steen claimed “an innumerate citizen is as vulnerable as the illiterate peasant of Gutenberg’s time” [41, p. xv], it becomes important to consider the ways literacy research has interrogated illiteracy. An important development in literacy scholarship is the emergence of social theories of literacy [2, 16] as counterparts to the psychological theories.

As the discursive move to tie numeracy to literacy echoes throughout the scholarship, an ongoing, concerted effort to learn directly and productively from this longstanding field should continue. It is to that effort that we are trying to contribute this chapter, where we are beginning with a media literacy framework that offers four dimensions for thinking about the transformations of literacy. We use that framework to think about quantitative literacy in a similar way. In part, these efforts are challenged due to the rapidity of quantitative changes to our societies when, as Bailey [1, p. 15] forecasted twenty years ago, “today we are drowning in data, and there is unimaginably more on the way.” Happily, literacies scholarship has worked through the same technological and sociocultural shifts that scholars argue created the vacuum for quantitative literacy. For example, research on multiliteracies (e.g., [10]) and on new literacies (e.g., [30]) investigate the impacts of new social practices and new technologies on literacy, teasing apart modes of representation that make social spaces to understand literacies in those spaces. We draw parallels with this research in multiliteracies and new literacies and use its adaptability with emerging technologies as a framework to better understand quantitative literacies in the new media age—which began when electronic displays challenged what it means to read and write in ways that are not predominantly alphabet-centric [26].

Specifically, the main purpose of this chapter is to reconceptualize quantitative literacy as a social practice (also see Chapter 1 by Fisher). At times this involves thinking about how the skills of quantitative literacy [43] are changing, and what might be added to their list. More centrally, however, we want to move to thinking about quantitative literacy practices that are social and goal-driven and not reducible to skills, instead being dynamic cultural interactions. In our exploration, we adapt an existing framework used to analyze media literacy (e.g., [31]) to provide a structured approach to quantitative literacy in the new media age [26]. This framework analyzes the effects of multimodal media on quantitative literacies by considering four interrelated aspects of media: access, analysis, evaluation, and content creation.

In embracing the media literacy framework, first, we explore the potential of taking a sociocultural approach to literacies, wherein practices are considered as culturally and contextually embedded. Second, we tackle the *quantitative literacy as a skill* perspective, because it focuses on what people *have* or *should do* rather than what they *do*—a more sociocultural approach. It calls for a more descriptive and less judgmental approach. Therefore, we argue for a perspective that emerges from transformations of mathematical and statistical skills enabled by technological changes, but fundamentally created by technology users. This perspective evolves into the practices engaged by people interacting with quantitative information in new and sociocultural ways. We call this perspective *new quantitative literacies*. One benefit of this approach is that it poses a direct challenge to quantitative literacy discourse as it is constituted normatively; instead, it suggests clearer educational and social science research and attention to what students (and people generally) can do, attempting to mitigate the common deficit perspective taken toward quantitative literacies.

2.2 Literacies as Social Practices

Gee [16] argued that the changing literacy practices emerge from new social practices, including those evolved upon advent of new technologies. He found that many scholars from different fields had begun to focus on literacy practices as something people do *in the world*, rather than solely in their heads. Hence, whereas “traditional psychology saw readers and writers as engaged in mental processes...NLS [New Literacy Studies] saw readers and writers engaged in social or cultural *practices*” [16, emphasis in the original]. As social and cultural practices, literacies are not static: Leu, Kinzer, Coiro, Castek, and Henry [29] argued that literacy is deictic, or constantly changing. Its dynamic nature derives from conceptualizing literacy as more than the ability to read and write, where it is “always literacy for something—for professional competence in a technological world, for civic responsibility and the preservation of heritage, for personal growth and self-fulfilment, for social and political change” [25, p. 76]. Literacy is not a fixed entity because what, how, where, and why people read and write changes through time and context. For example, the emergence of social media (e.g., Twitter) and use of smartphones has changed the activities of reading and writing, where use of multimodal texts, such as GIFs, memes, and emojis, has become a common practice. That is, technology, especially digital technologies powered by internet, provide ever-changing contexts and multimodal spaces for reading and writing, which makes defining literacy somewhat irrelevant, but rather shifts focus to understanding its plurality.

Leu, Kinzer, Coiro, Castek, and Henry [29] identified several central principles of literacies that hinge on the transformative power that technology generally, and the internet specifically, have had on literacy. The authors claimed that the internet drives the emergence of new social practices of literacy by creating new literacies, systematically transforming existing literacy practices and supporting people engaging in new social practices of literacy. To understand digital literacies on the internet goes beyond finding the new places that the same literacy practices exist. New social practices include text messaging, email, and viral information sharing, and transformed existing social practices include reading the newspaper [37], reading comprehension [30], and academic scholarship [19], among others. The multimodality of literacy also decentralizes language from literacy because to be literate involves not only words, but also images, speech, sounds, gestures, and other modes expression and representation [22]. Textbooks, for instance, have embraced the internet as e-books and can be organized interactively around images and hypertexts [26, 27].

A similar multimodality is involved in quantitative literacy, where the integration of quantities not only within contexts, but also with words, images, and videos. Steen et al. [43] pointed to technological changes as the fundamental driver for calls for quantitative literacy, just as Kellner and Share [23] argued for re-conceptualized media literacy because "the twenty-first century is a media saturated, technologically dependent, and globally connected world." They organize their theorization of critical media literacy around the concept of public pedagogy; that is, the idea that media constitutes another source of public education that intersects with but is distinct from public schooling. The great challenge of media literacy in this age, they argue, is that "it is insufficient to teach students only to read and write with letters and numbers" [23], because the multimodal format of media involves the interaction of script (lexical and numerical) with other visual images such as graphs, sounds, and multiple mediums simultaneously.

Forest Fisher opens this volume by suggesting an alternative conceptualization of QL based in social practices and situated knowledge. His suggestion follows a tradition of understanding QL and literacy from a social perspective [9, 35]. Brian Street, for example, has contributed significantly to social practice theories of literacy and of numeracy [45]. He and colleagues [46] produced a volume on the potential of social practice theories for numeracy. They constructed their theory both from their knowledge of social practice theories and from ethnographic accounts of students' school and home numeracy practices.

2.3 New Quantitative Literacies

As noted by Steen et al. [43], the practices required to be quantitatively literate can be static, such as arithmetic, or dynamic, such as data and information literacy. The latter, however, change with emerging technologies and social spaces. Instead of new technologies simply increasing the importance of quantitative literacy, for example, it is possible that new technologies might involve new social practices of quantitative literacies that make the technologies important to our scholarship. The interactions between media and quantitative literacy are numerous, and offer rich opportunities for classrooms to develop an array of important skills and dispositions (e.g., Boersma, Diefenderfer, Dingman, and Madison, this volume; Schwab-McCoy, this volume).

In addition to the rich pedagogical interactions, emergent media involves new technologies and social practices. Digital and internet-based platforms afford spaces that allow for quantitative information to be used in new activities and practices. For example, interactive websites and multimedia are simply traded via Twitter and Facebook. Users have more control over not just what they wish to consider, but also how they wish to consider it. Just as new literacies scholars argued that literacies practices do not simply translate from print to screen—a rigid movement—we also argue that quantitative literacy skills do not simply translate; instead, we argue they transform constantly with co-emerging social practices and technologies.

Here, we call these *new quantitative literacies*—the social and cultural practices and interactions that co-emerge with use and creation of quantitative information. The media framework we use [31] discusses the effects of multimodal media on literacy by considering four interrelated dimensions of media: access, analysis, evaluation, and content creation. We reimagine quantitative literacies within each dimension: more *access* to quantitative information and arguments, proclivity to *analyze* quantitative information faster and more readily, and changes to those analytical strategies used, new, or transformed, concern about *evaluating* the credibility of quantitative arguments, and agency to use the internet as *content creators.*

2.3.1 Access

A first dimension of new quantitative literacies is in terms of absolute access to information, which includes the quantity, diversity, and openness of large data sets, and competing interpretations of those data. At the end of 2014, an estimated 2.8 billion people had access to the internet, including 276 million people in the United States [8]. This growth in access to the internet has effects on many expressions of quantitative literacies. For instance, in 2005 students in the United States reported using more time reading online than offline [38], and notably, 20% of the 2015 advertising market was internet-based [32].

Increased access to the internet is related to information *virality*, which describes the "process that gives any information item the maximum exposure, relative to the potential audience, over a short duration, distributed by many nodes" [3]. The internet creates a new potential for quantitative information virality because of: (1) the number of nodes, (2) the speed of sharing, and (3) the low cost of sharing. At a societal level, [24] found that internet virality is four to five times more influential than mass media. Although mass media often play an intermediary role in the information that goes most viral, the mainstream media are less likely to initially construct the story than they were before the internet emerged. However, virality and access alone do not equate critical comprehension [28]. Still, these changes to access affect quantitative analytical skills, evaluation of quantitative information, and quantitative content creation.

Another way that access to quantitative information has changed quantitative literacies is through open-sourced databases. Beyond being available to the public, more refined data sets and representations are also accessible. Further, these representations are fundamentally more complex and different from the skills of quantitative literacies [43] listed, and these complexities transform how quantitative information can be analyzed and evaluated. We consider the changes in access to quantitative information as resulting from the ways technologies have made quantification cheaper and faster. For instance, Google Trends uses so-called "found" data accumulated through search entries, quantifies those trending searches, and opens that quantification to the public. Another example of changing access to quantitative information is socially constructed rankings. Yelp, for instance, involves user reviews and recommendations that couple qualitative stories with what are essentially public comment cards. The same premise of social ranking and quantification is transformed by access to being able to rank—for example compare who can rate restaurants through Yelp, Facebook, or Foursquare to those who rate for the Michelin Guide. These changes to access offer space for educators to consider meaningful learning experiences related to measurement, including distinguishing between facts and opinion through analysis.

2.3.2 Analysis

The new literacies theory argues that new analytic skills and strategies are necessary to become literate in new practices [28]. Similarly, new quantitative literacies practices emerge from people's use of the internet and related technologies, and these new practices involve new skills and strategies. A second transformation of new quantitative literacies undergirded by the emergence of the internet, therefore, involves the qualitative differences between quantitative information on the internet and computers and quantitative information without those technologies. Not only is the amount of and access to quantitative information, representations, and arguments affected by the internet, but also the nature of the quantitative representations has changed.

Data visualization and infographics, for example, involve new skills of quantitative literacies that rely on different interpretive skills than traditional imagery accompanying news items. Many researchers in statistics education have acknowledged the prevalence and pervasiveness of statistics, data, and representations in the digital information age [14, 4, 18, 11]. The nature of those statistics, data, and representations, however, is not static over time. The most recent transformation supports new and innovative forms of communication that blend graphical and lexical representations in creative and distinct ways [20]. This transformation, called *Web 2.0*, is defined by the interactive and responsive websites that began appearing after 2004.

Web 2.0 has distinct participation structures and informational media. Infographics are one such structure that uses data visualization to persuade and inform their audience; the blended nature of infographics reflects the flexibility of representation and communication Web 2.0 offers, and the analogous multimodality of quantitative literacies. Although infographics existed prior to widespread internet use [36], the internet has transformed their nature by increasing their prevalence and facilitating their creation and distribution. Additionally, prior to computers, quantitative representations were usually static and how people could manipulate those representations was limited. For example, using a scatterplot to investigate or demonstrate the relationship between two variables was limited. To compare change in the relationships over time, several graphs would have to be made, placed next to one another, and analyzed. To investigate relationships with other variables would require creating a new plot, and could require collecting new data.

The website Gapminder provides an example of how technology alters skills related to quantitative literacy—in this case quantitative representations. Its interfaces enable users to manipulate scatterplots constructed automatically after selecting variables backed with data sources. A user has the ability to transform the data from linear to logarithmic, as well as to change the variables being compared fluidly, without needing to create the scatterplot; the creation of quantitative representations and argumentation thereafter transitions from relying on the ability to construct accurate scatterplots by hand to the ability to analyze and compare scatterplots to one another—a significant transition. In addition, the nature of the representation shifts completely from a two-variable comparison to a many-variable comparison. A scatterplot drawn by pencil represents the relationship between the two variables on the axes, and little else. To understand the source of each point on the plot, a person would need to refer to the original data. In contrast, standard

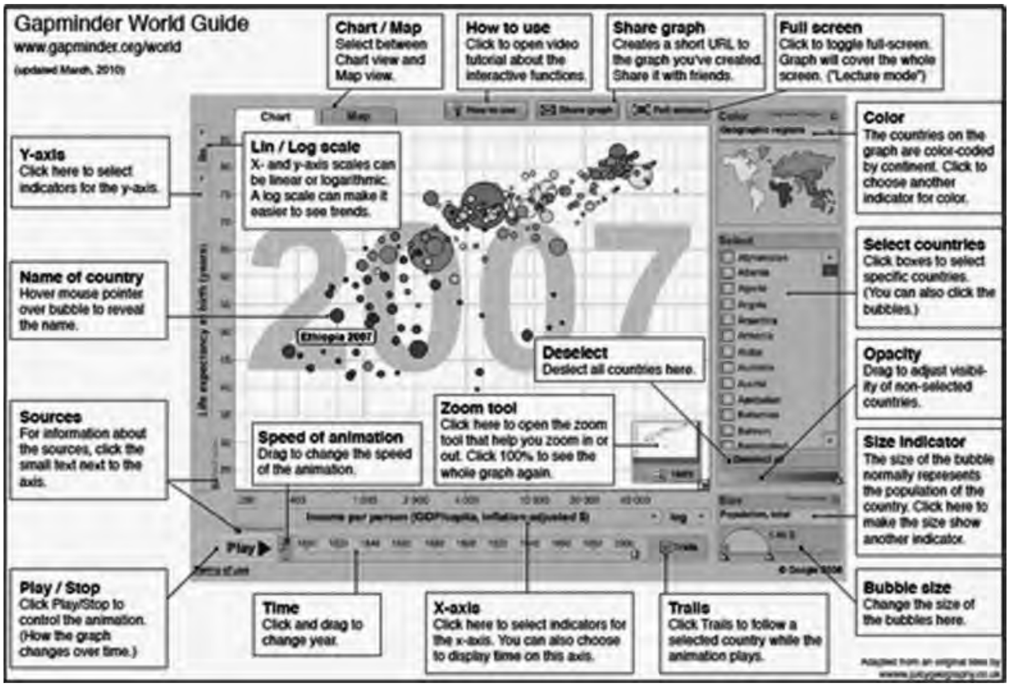

Figure 2.1. Gapminder World Guide, showing some of the analytical skills of quantitative literacy. Adapted from "Gapminder World Guide" by `www.gapminder.org`. Originally adapted from `www.juicygeography.co.uk`.

Gapminder scatterplots include at least six variables:

1. The horizontal axis variable
2. The vertical axis variable
3. The country (accessible by hovering the mouse over each point)
4. The continent of each country (identifiable by the color of the country bubble)
5. The population of each country (represented by the size of each countrys bubble)
6. The year of the data (shown in the background).

The analytical skills of quantitative literacy related to considering Gapminder graphs is outlined by the guide they created, shown in Figure 2.1.

Introducing additional variables and allowing for dynamic movement place new demands on people analyzing the graphs [39]. Additionally, Gapminder supports looking at trends over time by providing the user with a play button that shows the transformation of the same variables over time by country. The ways in which Gapminder extends and transforms traditional scatterplots is one example of how the internet generates new affordances for quantitative literacies.

And such affordances make Gapminder well-suited as a tool for use in the quantitative literacy classroom. The software affords a distinct focus on quantitative analyses—such as considering correlation or hypothesizing causal links—in rich, meaningful real-world contexts. For example, in a course in quantitative literacy, students could consider the United Nations Millennium and Sustainable Development Goals using the data sources and representational flexibility offered by Gapminder. Because of that flexibility, however, there are changes to what students are doing with analyzing scatterplots: students make choices among different variables and analyze trends through time with only a few button-clicks. As a pedagogical tool, Gapminder shines, and as an example of how analytic practices involving quantitative literacy change, it demonstrates the affordances of skill-building within complex problem spaces.

Ideas like representational fluency, data and statistic interpretation, and logical reasoning, are all arguably central to quantitative literacy [43]. Such skills and elements are not static when the kinds of representations, the amount of data, the contexts of statistics, and the purposes of reasoning, are all changing. Instead, those quantitative literacies change as people make new representations, interact with new data (e.g., Google Trends), find and generate statistics in domains not traditionally quantified, and debate what quantitative information offers. As what is analyzed is changed, how those analyses get used has also changed. People evaluate those analyses consistently, from thorough investigation of source trustworthiness to the casual recirculation of an analysis without critique.

2.3.3 Evaluation

Steen [43] conceptualized reasoning about and evaluating quantitative evidence as part of the new quantitative literacies. Gal [14] suggested that people should evaluate the source and purpose of the data being represented, the appropriateness of the representation, and additional information that might be omitted. The internet and related technologies have changed the form of quantitative evidence because of more multimodal representations and decentralized content creation. Researchers developing ways to unpack infographics and data visualizations [15, 6] have argued that these representations of evidence require different skills of quantitative literacy to evaluate.

Pasternack [36] claimed that readers of print newspaper infographics use them primarily to summarize information. He argued that for newspaper infographic creators, how well people could analyze their creations was primarily concerned with aesthetics or shock value. Siricharoen [40] argued that the transformative effect of the internet on news media reverberated into the creation and purpose of infographics. Instead of primarily being references or summaries of the text, infographics and other audio-visual representations have become “increasingly prominent as carriers of meaning” [7], often being a complete news story without traditional exposition. Varis [48] found that social media posts, such as infographics, often get shared without reading associated links; that is, distributors of information through the internet might only see a quantitative representation and a headline before they share, and validate, the associated arguments and becoming the complete story described by [7]. These phenomena mediate the new ways in which quantitative literacies are used to evaluate the credibility of information.

Infographics also change as they become more dynamic. Media is sometimes supplemented with interactive data visualizations as it expands to include actively circulated blogs and websites like *FlowingData*,[1] for example. Along with Gapminder, people interact with these types of data compilations and analyses with varying levels of criticism and investigation. One major transformation in social practice of evaluating quantitative information is the ability of people to actively share analyses. We consider the effects of this practice of validating and then sharing quantitative analyses to be one way that people are newly practicing quantitative content creation.

2.3.4 Content Creation

Bezemer [7] used social semiotics to investigate the evolution of creating multimodal representations of information with potential for learning. They found that moving from one media to another yields affordances and constraints. Through the process of *arranging* the information as an infographic, words with particular mathematical or statistical meanings (e.g., a percent, relative sizes, etc.) are often exclusively translated into and shared in graphical form, further integrating quantitative literacy and multimodal writing and composition. The move of multimodal representations from static textbooks to dynamic websites recontextualizes [5] the social meaning of the representation. Specifically, the social structure in which knowledge is generated and shared affects and is affected by the mode of representation. Infographics online, therefore, are qualitatively different than their static counterparts because of the social structure of the internet: infographics can be shared with comment, wherein consumers of information become distributors who reconstitute and validate the information as knowledge—a process referred to as *social repositioning* [5]. Social repositioning acts as people share information over the internet because they act as a validator or critic of the information being shared. Specifically, different criteria and skills of quantitative literacy are used to decide whether to share something and how to create an infographic to increase virality. Part of the changes in content creation skills involve creating representations that leverage the context on which the content focuses.

Certainly, questions can be asked about how people can be more critical or thorough in their validation. But, these questions run perpendicular to the social practice of being a validator, because when people rapidly—and perhaps nonchalantly—redistribute analyses, a variety of factors, including time, are involved that quantitative literacy scholars need to consider. The criteria people often use when choosing to share an analysis intersect with quantitative literacies, including a person’s history with the media and data sources, their agreement with the argument or conclusion, as well as their relationship with who introduced them to the piece of media.

[1] See `flowingdata.com/`.

Recognizing that sharing can be considered a form of content creation has important implications for new quantitative literacies practices. These acts of content creation intersect with the evaluation practices discussed earlier, and extend whatever interpretation and evaluation made by a consumer of quantitative representations as an act of production. Unsurprisingly, the acts of content creation that are often focused upon are those arguably misleading or misinterpreted things that go viral. Besides those instances being potential opportunities to understand and explore ideas such as quantitative persuasion, overattention to those situations obscures the massive amount of sharing that is ostensibly about alternative interpretations, and the creation and recreation of perspectives, analyses, and evaluations. The potential for sharing as a quantitative literacies practice to function in educative ways should be further explored; often, complex infographics or other multimodal representations of quantitative information are coupled with comments and responses that demonstrate the complexities and intentionality involved in what can be simplistically discussed in terms of simple mathematical or statistical principles. In fact, with any significant amount of digging, likely deeply analytical and philosophical quantitative conversations are easily found.

Beyond sharing as an act of (re)creation, there are also new quantitative literacies emerging as websites such as piktochart.com that facilitate people acting creatively to represent and interpret quantitative information in new ways. That people can create and share quantitative representations should be welcomed and embraced by the quantitative literacy community in its claims that quantitative literacy supports democracy. First, these acts involve engagement in, and recognition of, the possibilities for numbers to describe and shape the world. Second, to engage in the creation or sharing of quantitative information necessarily means disposition to *engage*, which may also involve qualitatively new and different practices of engagement. Finally, although having more content creators with less quality assurance on one hand creates new challenges to analysis and evaluation, it but on the other hand suggests a better distribution of power to create and shape messages.

2.4 New Quantitative Literacies, New Challenges

In the Preface to *Mathematics and Democracy*, Orrill [34, p. xvi] cautioned “this unprecedented access to numerical information promises to place more power in the hands of individuals and serve as a stimulus to democratic discourse and civic decision making.” Simultaneously, he cautioned that discomfort with numerical information is problematic as it pervades most of our society. Similarly, new quantitative literacies can represent both promise and caution, and it would surprise us if our scholarly community stopped focusing on the cautions. But, arguably the focus on cautions sometimes overshadows the promises of quantitative literacies to effect change and ignores new possibilities. The debate as to whether this scholarship acknowledges something wonderful or threatening for democracy can be shifted by a perspective change, one that recognizes the tenuous nature of ascriptions of numeracy, or innumeracy, to social practices that are interconnected, complex, and subject to different normative views. Nevertheless, any just conceptualization of quantitative literacy involves understanding how quantitative literacies practices intersect with moral and ethical concerns, in particular when the data has been interpreted correctly, but is used to push agendas antithetical to open democratic institutions—agendas that present quantitative information in deceiving and dehumanizing ways that disadvantage a group of people.

As social media users interact with potentially sharing data or an analysis, they necessarily evaluate the credibility and accuracy of each share, regardless of how thoroughly they investigate it. In each case, quantitative literacy plays a role in both the spread and response of each artifact. For example, people supporting anti-vaccination movement create and share memes that often rely on false equivalences and the assumption that correlation and causation are linked. Specifically, one meme presents three data points, giving the years 1983, 2008, and 2013. As the years increase, so do the number of vaccines recommended by the Centers for Disease Control. In addition, on the same graph, the rate of autism diagnoses is also shown to increase. Although the numbers are accurate, the infographic does not facilitate readers in analyzing and evaluating them, but rather misleads them. The meme creators likely intend for the reader to draw the conclusion that the autism diagnosis rate is tied directly to the number of vaccines a child receives, despite researchers thoroughly rejecting the link [47]. Attentiveness to the ethical concerns related to quantitative literacies practices is also important for content creators, echoing what Victor Piercey calls *quantitative ethics*; for example, after the website *FiveThirtyEight* produced an image showing that only the men voting would lead to an overwhelming Republican electoral victory in the 2016 election, it went viral with the hashtag #Repealthe19th — a reference to the nineteenth amendment to the United States Constitution guaranteeing women’s suffrage. In terms of static, abstracted ideas of numeracy skills, this hashtag represents an accurate reading of data combined with an effective legal change, yet is an offensive, oppressive interpretation. We argue that no interpretation of quantitative literacy is complete without ethical considerations; therefore, understanding quantitative literacies practices involves holistic understandings of intentionality, rhetoric, and purpose. The virality of the infographic was likely supported by the reputation and content creation abilities of the designers. The *Washington Post* found, however, that many comments featuring the hashtag were outraged responses to the original idea [17, 33], and not continuing the initial cause of the hashtag. This example in social media shows some of the complexities involved in considering quantitative literacies as social practices.

2.5 Conclusion

In this chapter, we used a media literacy analytical framework to understand and unpack conceptualizations of quantitative literacy. The four dimensions of the media literacy framework—access, analysis, evaluation, and content creation—are all affected by the same technological changes driving much of the scholarship in media literacy. If indeed, "As the printing press gave the power of letters to the masses, so the computer gives the power of number to ordinary citizens" [41, xv], we need also to recognize that literacy practices did not emerge during the 1400s and remain static. Many of those changes in literacy practices occurred before technologies such as television, computers, and tablets had been created; instead, literacy practices changed when people created new genres, new styles, and new purposes for text—changes that have continued or accelerated with technology changes. Likewise, people are constantly creating new genres, styles, and purposes for quantitative information. We should simultaneously question and embrace these new practices, recognizing that age-old discussions of free speech and propaganda have not yielded to claims that people need particular skills, beliefs, and dispositions; neither have they relented to shifts from reading and writing skills to reading comprehension. Similarly, we hope that quantitative literacy scholarship is not confined to considering what people do not do, and instead learns from our literacy colleagues that new perspectives and theories can help us imagine different possibilities. For mathematics teachers, embracing the idea of multiple numeracies has implications for practice. The technological changes facilitating the evolution of new numeracies poses a challenge to how we formulate curricula for classes designed to learn about QL/QR, and any applied mathematics class. We must continue to ask what role mathematics education has in guiding numeracies, and how that can be accomplished.

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3

Intercultural Citizenship as a Framework for Advancing Quantitative Literacy and Reasoning across Disciplinary Boundaries

Fabiana Cardetti, Manuela Wagner, Michael Byram
University of Connecticut, University of Connecticut, Durham University

3.1 Introduction

Calling for systemic change in mathematics education, Kasi Allen [2, p. 1] stated: “Adults and children alike joke about being terrible at math, seemingly unconcerned about how this innumeracy hinders full participation in our democracy or the realization of their individual goals, hopes, and dreams.”

In this opening quotation, Allen pinpoints not only the existence of innumeracy and the problems it causes, but also the insouciant attitudes towards innumeracy in the young and the old. There is therefore a responsibility of all educators, and mathematics educators in particular to address the issues directly. In this chapter, we offer a critical reflection on one of the central challenges that has concerned mathematics educators over the years: the disconnect between the mathematics we want our students to learn and their understanding of and attitudes toward the need and use of mathematical knowledge and skills outside the classroom. At the heart of the struggle are deep-rooted beliefs about what mathematics content students should learn, how it should be taught, the nature of mathematics as a discipline, and its relevance in today’s society. These critical points have been preoccupying educators and researchers in mathematics and in many other disciplines, especially those concerned with building students’ quantitative literacy and quantitative reasoning skills to support deep understanding across the disciplines on the one hand and, on the other hand, to equip students with the knowledge and skills they will need to understand and play an active role in our democratic society. Indeed, when referring to quantitative literacy as numeracy, Steen asserted “numeracy nonetheless nourishes the entire school curriculum, including not only the natural, social, and applied sciences, but also language, history, and fine arts” [28, p. 8].

At the college level, mathematicians tend to believe that these connections, though implicit in most programs, are embedded in our students’ course of study. Having experienced college education successfully, many of us believe that as students move through their studies, their ability to integrate conceptual knowledge within and across disciplines will grow organically. However, when we look closely at our students’ capacity for integrating knowledge and using it to solve routine or novel problems [17], we quickly find out that our students are not where we believe they should be.

One of the distinctive characteristics of quantitative literacy is the engagement with problems that are embedded in specific contexts, which in turn provide rich opportunities for integrating knowledge (e.g., [3, 11]). The reality for many of our students is that they do not make the content associations one expects unless they are given the opportunity to think about those links or asked to solve problems that call for those associations. In addition, they do not develop skills to communicate mathematically unless they have the opportunity to engage in oral and/or written communication in their mathematics courses. Furthermore, they often do not reach the critical thinking level that allows them to support or create judgments, conjectures, or arguments on the basis of mathematical knowledge, resources, and reasoning. It is important to recognize this reality because the absence or inadequacy of these types of learning experiences not only negatively impacts students' understanding of mathematics, but, more importantly, prevents them from developing the quantitative literacy that they need to deal with the challenges and demands of the new century.

Cognitive scientists suggest that learning key ideas and frameworks is more powerful than learning individual or fragmented ideas, and helps learners construct stronger knowledge schemata [6, 13]. Educational experiences focused on an understanding of the complex connections between key ideas within a subject and how they interact with key ideas in other subjects help students apply what they have learned in meaningful novel situations. We agree with Steen [28] that this in turn has an impact on their preparedness to interact successfully with the complexity of modern society in which they will be faced with problems that cannot be solved within one discipline. Weaving together key ideas and important skills from different disciplines allows students to reinforce the content knowledge specific to each subject. It encourages integration across the content in the different disciplines that serve as tools for problem solving and, more generally, for critical analysis of societal issues, whether local or global.

Using the perspectives outlined by the National Numeracy Network [22], we consider quantitative reasoning as different; namely, quantitative literacy (QL) describes the comfort, competency, and habit of mind in working with numerical data, while quantitative reasoning (QR) emphasizes the higher-order reasoning and critical thinking skills needed to understand and to create sophisticated arguments supported by quantitative data. We contend that in order for students to become informed and active participants in our democratic society, they must be exposed to authentic learning experiences that develop their quantitative literacy early on in their education (K-12). Furthermore, colleges and universities should infuse quantitative literacy and quantitative reasoning across the curriculum so that it becomes a meaningful and integral part of a student's learning experience.

As Darling-Hammond [10] stressed, ensuring the presence of democratic processes in educational institutions provides students the opportunity to learn as members of a community in which they have a voice and can participate in making decisions, leading students to an authentic understanding of multiple perspectives. Noddings, who writes about the role of dialogue for social awareness within the mathematics classroom, acknowledges that a challenge to education becomes the furthering of the development of students as competent and contributing citizens [25]. Integrating "intercultural citizenship" [5] into the development of quantitative literacy is, we suggest, a meaningful approach to Nodding's challenge. In this chapter we focus on Byram's model of Intercultural Communicative Competence [4] in combination with his concept of Intercultural Citizenship [5] to offer an approach to advancing students' quantitative literacy and quantitative reasoning that cuts across disciplinary boundaries.

We argue that, building from the theories of intercultural communicative competence and citizenship, we can move our students from mere consumers of mathematical knowledge to critical thinkers who can (a) use their skills to discover new (even if only to them) meanings, (b) communicate and collaborate with others thoughtfully and effectively, and (c) "*read and write the world* with mathematics"([14, 16]) so that they can find solutions to issues in our society.

In what follows, we present the intercultural competence model, its origins and underlying theories, and describe the extension of the model to intercultural citizenship, which involves practical applications through active citizenship. We then discuss how we see the role of intercultural citizenship in the development of quantitative literacy and reasoning as both a vision for curriculum change and as a resource for the development of curricular materials. Finally, we provide a few illustrative examples and key considerations for anyone looking to incorporate our proposed approach into their own approaches to advance students' learning of quantitative literacy and reasoning.

3.2 Furthering the Development of Students as Competent and Contributing Citizens

Our previous work (e.g., [7, 32]) has involved the integration of mathematics and world language with learning process founded in Byram's models of Intercultural Communicative Competence [4] and Intercultural Citizenship [5].

Although the framework of Intercultural Communicative Competence was originally developed for world languages education, there are a number of important connections between the skills of Intercultural Communicative Competence and skills required in mathematics.

In world languages education, "communicative competence" is focused primarily on grammar/linguistic competence and appropriateness /sociolinguistic competence. The purpose of teaching "*intercultural* communicative competence" is to enable students first to interpret and understand the cultural contexts of their interlocutors, second to be able to interact with them accordingly, and third to act as mediators between two groups with mutually incomprehensible languages and cultures, or "languacultures" [27]. One might note here that the intercultural skills and attitudes involved in intercultural communicative competence in world language learning are also required when speaking a shared language with someone from a different cultural background or group, e.g., someone from Canada speaking English with someone from India, or someone from the east coast of the U.S. speaking with someone from the west coast or a lawyer speaking to a scientist about their respective professions. It is here that we can make the connection to other disciplines, such as mathematics, since people who interpret and reason about mathematics in different ways are acting as members of different cultural groups. Students who can successfully negotiate meaning in mathematics for themselves will also need to understand how others negotiate their own meaning, and be able to interact with them. This in turn can lead to their ability to become "mediators" in mathematics. For example when students are exposed to two distinct solutions for finding the integral of a function (e.g., by parts or by substitution), they need to analyze the different approaches to decide whether either is correct, what the advantages of each one might be, and which one, if either, would be the better strategy to use for the particular type of functions represented in the problem. Using their mediator skills, students will then be able to help those who have used one solution to understand and evaluate the other solution and vice versa, thus creating a dialogue analogous to a dialogue among people of different national, regional, or professional cultures.

These skills not only help students investigate mathematical connections, but also positively impact their ability to communicate and collaborate with others in general. The need to be able to communicate with diverse people in a variety of contexts and to address challenges in local, national, and international contexts has never been more urgent. Teaching for intercultural competence, mediation, and citizenship provides a useful and meaningful framework which helps students make interdisciplinary connections, as well as connections between what they learn in school and the challenges they face in the world.

More specifically, the components of intercultural *communicative* competence include linguistic/grammatical and sociolinguistic/appropiacy skills of *communicative* competence with skills, knowledge, and attitudes of *intercultural* competence:

- knowledge (of cultural groups and how people of different groups interact)
- attitudes (of openness and curiosity)
- skills of interpreting and relating (e.g., comparing ideas and behaviors of different cultural groups)
- skills of discovery and interaction (e.g., observing and questioning of people in another cultural group)
- critical cultural awareness (e.g., making evaluations on explicit criteria).

The attitudes, skills, and knowledge help students learn to work with others, to problem-solve together in hands-on projects, and to think critically and creatively about problems by tapping into individual talents, knowledge, and creativity. Critical cultural awareness—closely linked with social justice education (e.g., [26])—is a particularly important element in this construct, entailing the ability to critique and evaluate an event or behavior on the basis of a conscious awareness of the criteria one is using, and to do so with respect to one's own group's behaviors and assumptions as well as those of other cultural groups. Thus intercultural competence helps students become more meaningfully involved in their learning experiences in ways that are not possible using traditional teaching (i.e., formula-focused and decontextualized) approaches. For it is clear that the skills and practices listed above are crucial not only in the foreign language classroom, but also in students' success in all disciplines and, ultimately, in the 21st century workforce.

Moreover, it is especially useful to address complex real world problems requiring creative solutions and input from a variety of sources. In this way, students' intercultural competence is combined with "action in the world," which is included in intercultural citizenship, which in turn links to quantitative literacy "as an expression of just action" (as discussed by Craig, Guzmán, and Harper in this volume). In the model of intercultural citizenship, Byram [5] took the model of intercultural (communicative) competence a stage further to address the demands and opportunities of students as "intercultural citizens." Intercultural citizens can put knowledge, attitudes, and skills to use at any time, applying what they learn in different disciplines, and collaborating with people of other (cultural) groups to offer solutions to social issues.

3.3 How Can Intercultural Citizenship Help to Rethink QL?

While we strongly believe that all subjects and areas should collaborate to teach knowledge, skills, and attitudes that foster quantitative literacy and reasoning, we also know that teachers need to approach the teaching of complex issues such as this within their subject area. Ostensible goals for our mathematics courses include that they should provide opportunities for students to learn how to make their mathematical reasoning visible, unpack mathematical procedures, and discuss connections, strategies, and solutions. These experiences will better prepare them for interdisciplinary problem situations, where knowledge from other disciplines is necessary to support inferences and conclusions. In this section, we explore the links between the development of quantitative literacy and reasoning and intercultural citizenship. Specifically, we explore the question: How can the model of intercultural citizenship help instructors rethink the way in which quantitative literacy and reasoning are promoted in the mathematics classroom?

The approach to mathematics learning that sees QL as a matter of interest far beyond mathematics teaching and learning has already generated some change in the ways we think about how mathematics should be taught at the K-12 level with the wide adoption of the Common Core State Standards for Mathematics (CCSSM) [17], which were developed with guidance from several national and international documents [23, 21, 13]. These new standards are based on what has been learned about mathematics education since earlier curriculum documents, and incorporate recent scientific advances that have helped us understand how people learn mathematics as evidenced, for example, in the more coherent and rigorous distribution of the mathematics content across the grades.

Another significant advancement in CCSSM is the inclusion of Standards for Mathematical Practice that describe the mathematical proficiencies and habits of mind to be fostered and developed at all grade levels, assuming the goal is for students to successfully learn the proposed content, as well as be better prepared for postsecondary education and 21st century lives and careers. Although different scholars use slightly different frames for conceptualizing these practices, the intent is similar: to empower students with an understanding of mathematics, thus giving them access to opportunities in a society of the future—one, of course, we cannot imagine at the present time. Thus, at the school level, the CCSSM are paving the way for a common vision of mathematics that has the potential to support quantitative literacy and reasoning. At the same time, we recognize that more work needs to be done especially for advocating the use of authentic applications and promoting interdisciplinary work [18].

An additional encouraging development at the school level that might foster quantitative literacy and reasoning is the advent of the new Next Generation Science Standards (NGSS) [1]. These standards are based on the Framework for Science Education [24], which promotes a shift towards engaging students with content in ways that simulate the practices of scientists and engineers, with a focus on key scientific concepts that cut across the scientific disciplines. The new science standards disrupt the status quo of science teaching, requiring teachers to choose contexts and examples that will be meaningful to their students so they can develop deep understanding. Teachers need to identify interdisciplinary contexts that "relate to the interests and life experiences of students or be connected to societal or personal concerns that require scientific or technological knowledge" [24, pp. 2–6]. The NGSS promotes a science classroom that relates to students' lives in which they design experiments and collect and analyze data, which will in most instances require them to use their quantitative literacy and reasoning skills.

Thus, taken together, these new initiatives convey a more consistent message across the country for school teaching of mathematics and sciences in ways that will better prepare students for future study and for their lives. The shift towards interdisciplinarity is apparent in the NGSS. Science teachers consider, for example, issues related to human health or global sustainability to provide authentic contexts. These same topics can be used to integrate science with mathematics, as well as other subjects such as language arts, the social sciences, and world languages. Such promising changes at the K-12 level point toward an enhanced mathematical preparation of students that might help them reach higher levels of quantitative literacy and quantitative reasoning than ever before, generating momentum to think about how to strengthen QL and QR in the undergraduate curriculum.

It must, however, be recognized that there are concerns as to how this can be implemented while maintaining a well-founded base to guide decision-making. Common concerns range from the possible creation of less rigorous courses, sometimes referred to as *watered-down mathematics* [29], or the development of truly authentic tasks (*camouflaging traditional tasks*, [30]), to ensuring that QL and QR are addressed following the guidance of research on learning and evidence-based practice [19]. To address these concerns, we suggest that the concept of intercultural citizenship provides an approach that is tried and tested, albeit mostly in world language education. The emphasis in intercultural citizenship research and teaching to date is on finding out about, understanding, and interacting with people of other languacultures in their countries, and their solutions to problems in the world. Specifically, our focus here is on understanding oneself and others and collaborating to find solutions to problems in the world where mathematics is a significant instrument for analysis and action. We see the model of intercultural citizenship as a tool that can help guide our efforts, because it identifies and defines specific skills and knowledge that educators should consider in their planning of opportunities for QL and QR that ensure the competences are explicitly taught and learnt and not left to chance.

In the terms of the intercultural competence model described in the previous section, we already have in mathematics a focus on mathematical knowledge. However, the model of intercultural competence draws attention to skills, attitudes, and critical cultural awareness as being of equal importance. This suggests that in mathematics education we lack intentional focus and understanding for how to develop crucial knowledge of self and others, as well as the attitudes and skills that truly get at the heart of developing QL for what it is, and for its potential impact on students' lives beyond the classroom walls. One of the habits of mind that students need to form is to express their mathematical reasoning so that others can follow their thinking—orally and in writing [6]. That is, students need to be able to move away from "I know the answer, I just don't know how to explain how I got there," a very common response students give to each other (or to us) when pressed for providing support to their "right" answers. The ability to explain their reasoning is crucial not only to recognize the mathematics that is being invoked to get to the solution, but also for the student and others to pinpoint misunderstandings or develop creative paths to a solution. As Grawe points out, "even students with strong math skills may come up short in QR from inability to translate that strength into an argument." [6, p. 42]. Thus, it is imperative that students learn to communicate how they reason quantitatively, as otherwise the inability becomes an impediment to becoming active participants of the world they live in, as we shall explain in detail below. Here is where *intercultural competence* is relevant. Students need a good understanding of themselves and others as members of similar or different groups to be able to interact with them, ask questions, present their hesitations, and communicate their mathematical arguments and ideas in ways that make sense to others. They need to find their own identity, as members of a group of learners that thinks and reasons about data and contextualized situations in a certain way that may be different from that of others. This is especially important if we expect our students to be able to work collaboratively with others on authentic problems (e.g., [8, 12]). For this is very likely the scenario they will face in the workplace in a world in which online or face to face collaboration across the globe is not only likely to happen, but is an expected and crucial part of more and more job positions.

Intercultural competence in the sense described above can only be achieved if we help our students develop attitudes of openness and curiosity that are strongly tied to productive disposition, one of the challenges highlighted by Madison and Dingman [19] for teaching and learning QR. Development of these attitudes allows students to: be willing to engage with others to find out how they are using mathematics in a given situation, seek out other perspectives on approaching a problem that may be similar to or different than their own, be open to think carefully about what others have to offer, and ask questions that help them clarify someone else's rationale. Our students need to feel comfortable with the idea that perhaps they may not understand another's reasoning right away and know that there are ways to find out how others are approaching a problem, as this will enrich their own and others' ways of thinking and of expressing their ideas with each other. These attitudes will unquestionably serve them well not only in QL-focused mathematics courses, but also in other subjects and outside education as well.

It is clear that the attitudes that foster intercultural competence are also closely linked to and reinforced by the skills of interpreting and relating in the model of intercultural competence. In order for students to develop "the habit of mind to consider both the power and limitations of quantitative evidence in the evaluation, construction, and communication of arguments in public, professional, and personal life" [6, p. 41], they need both to be open to others' perspectives, and have the ability to interpret and relate them to their own. Communication of arguments, motivated by the pursuit of the relationship between seemingly different ideas that address the same problem (e.g., issue, idea, calculation) to find out how they compare and relate to one another, is likely to result in deeper understanding of the possibilities and constraints that the situation affords. Combining those skills and attitudes with the skills of discovery and interaction, students can take these new understandings to stretch their thinking forward by exploring, observing, questioning, and interacting with newfound possibilities and negotiating the content with other students in ways that help them discover new applications or solutions that may not have been apparent at first glance. When students then address real world problems as members of a national (or indeed international) society in the here and now or in their future lives, they use their intercultural competence as intercultural citizens.

If students acquire the knowledge and apply the attitudes and skills from the model of intercultural competence, this should lead to their ability to judge an event, problem, task, or text critically, based on specific criteria that they are able to make explicit, this refers to "critical cultural awareness" in the model. In other words, they then are able to judge an event, problem, task, or text, critically, based on specific criteria. They are able to support this judgment with arguments based on cogent evidence that uses mathematical insights (as needed) and is backed by their interpretations and interactions with the relevant issues surrounding the problem. It is self-evident that our students need to become skillful in this quantitative reasoning in their daily lives as it becomes more crucial to evaluate the readily available information we are faced with every day. Among other things, using critical cultural awareness to consider fundamental questions that impact society (e.g., questions of social justice and human rights) is part of what differentiates mathematical learning as we know it, and what we today mean by quantitative literacy and reasoning.

3.4 Enhancing our Students' QL and QR Development with Intercultural Competence: Examples and Practical Considerations

Most of us would agree that creating learning experiences that help our students develop quantitative literacy and reasoning skills sounds exciting and daunting at the same time. Indeed, other chapters concerning curriculum development in this volume certainly bring this tension to life. In this chapter, we have put forward an approach for embedding QL and QR in the curriculum that is based on the model of intercultural competence and citizenship. This approach emphasizes depth of mathematical understanding, communication, and reasoning with a focus on exploration and (interdisciplinary) collaboration to solve authentic, real world problems. The approach thus offers a framework for building learning opportunities in a systematic way that ensures attention to each of the crucial aspects that are necessary for advancement of quantitative literacy and reasoning, using contexts that are meaningful to students in terms of their daily life and culture, as well as relevant in today's society. It is important to highlight that this can be used for small curricular changes (at the activity level in one course), as well as to implement larger curricular changes that cut across disciplines.

As an example, we provide an outline here of an interdisciplinary unit we created for intermediate Spanish and mathematics courses at the high-school level. The unit was implemented in a Spanish classroom at a high school during fall 2016. We designed the unit to support students' understanding of experiences of immigrants from Latin American countries and to support their QL development. Specifically, as part of the unit, students explore applications and modeling of different functions, as well as collaborate in groups to develop a financial plan for immigrant families upon arrival in our state of Connecticut. Students examine the connections between the costs and the needs, conditions, and habits of families in specific scenarios; they consider families' cultural backgrounds, educational and job opportunities, as well as housing, childcare, and transportation options. They gather data, reflect on, and interpret their findings continuously to help refine their plans. Using different algebraic expressions and representations and interpreting these mathematically they draw conclusions about their assigned scenario. They discuss and compare findings and approaches from different groups to gain a better understanding of the advantages and shortcomings of the different mathematical models. Most importantly, they use this knowledge to critically analyze the realities of immigrant families' day-to-day lives, their own cultural and community realities, as well as what can be done to make a positive change in the world around them. They then create action plans based on these critical analyses that can be acted upon (e.g., students create websites) or be brought up to the school or larger community in the form of presentations, among other options. We hope that the reader recognizes the many opportunities educators have to make meaningful connections to intercultural citizenship knowledge and skills, as well as the connections to quantitative reasoning, as a detailed unit plan would go beyond the scope of this chapter. By offering an example at the school level, we hope to convey the message that meaningful learning experiences do not require higher levels of mathematical knowledge, nor are they exclusive to advanced mathematical courses in order to achieve desired levels of quantitative literacy and reasoning using the framework of intercultural citizenship. As pointed out by Madison and Steen,

> Everyday contextual situations are heavily utilized in early school mathematics (and non-mathematics) studies but become much less evident in middle school, high school, and college mathematics. The data analysis, statistics, and probability strand in school mathematics does maintain some everyday contextual connections, but in college statistics courses are usually separate from mathematics courses. Many college statistics courses are methodological or theoretical and have minimal everyday connections [20, p. 5].

This same unit we presented above can be used in our college algebra classes with appropriate changes according to the course's goals, allowing for a wider range of data possibilities, higher levels of sophistication to analyze and interpret results, and potential engagement with other disciplines.

Yet another example in college-level calculus is a unit to explore renewable energy via water splitting. This topic allows for further development of QL and QR to examine basic principles of this chemical reaction that is part of the entry-level general chemistry curriculum. Students start by working in groups to investigate energy shortage in the context of different fossil fuels (e.g., oil, natural gas, coal). Each group is assigned a specific fossil fuel to identify its peak production year and generate a prediction of the terminal year on the basis of actual data over the past century, analyses of graphs and rates of change in consumption and reserves levels. Groups also explore sustainability options (e.g., energy conservation, efficient consumption) and the plausibility of implementing these in their community. Students are then guided to look into renewable energy alternatives focusing in particular on understanding artificial photosynthesis, or water splitting. This is a very appealing concept because it involves a chemical reaction that generates hydrogen (H_2) from water to use as clean fuel. However, there are some important challenges for the reaction to produce a high yield of hydrogen. Working in groups, students can discover these challenges by analyzing the rate of the chemical reaction using the Arrhenius equation ($k = Ae^{\frac{-Ea}{RT}}$) with different values of activation energy (Ea) or different temperatures (T) and interpret their findings with respect to the reaction's energy profile (energy graph). Using posters, slides, or reports all groups share the results of their investigations for their assigned fossil fuel, including the rationale behind the predictions for fuel depletion. They also discuss sustainable opportunities within their communities, which is an element of their commitment and activity as citizens in their society. Finally, they share their findings for combating energy shortage via water splitting and the challenges related to this renewable energy source, which is again a basis for potential citizenship activity.

Other examples of how QL and QR can be addressed at the college level are presented by colleagues throughout this volume. We highlight here two examples from this volume where the theories we have introduced can be easily incorporated to address intercultural competence in a systematic and rigorous way. In both examples real-world connections are already present and a citizenship education perspective would lead faculty to stimulate students to take action or at least to imagine what actions are feasible in their own communities. Specifically, we would like to point to the chapter in this volume by Edwards and colleagues exploring the connections between a Quantitative Literacy course and a standard Introduction to Proofs course, honing in on their similarities and suggesting ways to adapt both research approaches and pedagogical moves. Another example is the work presented by Fung in this volume. She describes a series of projects and activities to embed quantitative literacy in the curriculum and more broadly how quantitative literacy impacts our ability to answer big questions about our world such as the impact of population growth, climate change, or human trafficking to name a few. She presents the details of these explorations in the context of first year seminars at Worcester State University but does so in a way that can be effortlessly adapted to fit curriculum settings in other institutions.

Finally, we offer the following important considerations for the reader wishing to use our proposed approach. We encourage you to think about these before getting started with curriculum development and then again during the implementation phases:

- Start small: developing a single activity or a project is preferable to attempting to develop a full course.
- Identify a core mathematical concept or idea that you wish to address; plan backwards. In our example, this was the ability to collect relevant data, to analyze and interpret it, as well as to represent the relationships between the variables involved so as to highlight the features of their results specific to the problem.

- Know your audience: What are their day-to-day realities, their cultural and historical backgrounds, and how might you leverage these? If you do not know this, how can you find out?
- What sparks your students' interest? Identify topics that are meaningful and relevant to their lives, and related human and social issues that can be considered.
- Identify the learning objectives for quantitative literacy and reasoning and for the human and social issues you wish to tackle and create assessments that address your learning objectives (see for example the works by Craig, Guzmán, and Harper, and by Philip and Rubel in this volume). That is, what do you want your students to be able to do at the end of the unit or activity?
- Design your activity with the quantitative literacy and reasoning skills, as well as the social issues at the forefront. Only by connecting these two will the students benefit from understanding the connections themselves and thereby learn to apply their QL and QR in real-life situations.
- Take some time to connect with colleagues from the humanities and social sciences, among other areas, to understand the key issues and challenges related to the topics you selected or to learn about new ones. This will help you develop materials that address these non-mathematical aspects with authority.
- Come back to this chapter and look for how your activity encourages or cultivates QL/QR as well as each of the intercultural competence dimensions we have elaborated on here.
- Branch out; development of QL and QR through the intercultural competence model does not have to be limited to the mathematics classroom. Connect with colleagues from other disciplines that relate to what you are planning to address, or that might already be addressing similar issues. Explore possible collaborations and interdisciplinary learning opportunities.
- Try it, reflect on the results, revise, modify, and try again!

3.5 Concluding Remarks

In conclusion, we wish to share that although we were convinced of important connections between intercultural citizenship and our respective fields, we continue to be surprised by emerging and exciting findings. The more we work in exploring the theoretical and practical connections between world languages and mathematics teaching, the more relevant they become and the more enriching they are to the learning of each discipline, across the disciplines, and to education for intercultural citizenship. In developing and implementing units in various contexts, we experienced first-hand how much richer our teaching has become through the systematic effort to focus on current social issues through the model of intercultural citizenship. Students can see the connections between schoolwork and the real world, and enjoy the learning process apparently unaware of the specific content objectives they are achieving. Knowing these objectives in advance enables students to be more conscious and autonomous in their learning. We also learned that it is helpful to be intentional and clear about our goals. Our students know that we are after making connections between content areas, applying what we learn to real contexts, and developing their intercultural citizenship. This in turn enables them to see how education can be an important part of their own development of becoming informed citizens in an increasingly diverse world, which offers numerous opportunities as well as challenges that require the integration of the knowledge and skills from the different disciplines. We believe that the significance of quantitative literacy and intercultural citizenship cannot be overstated, and "being terrible at math" can no longer be considered a joke.

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4

Quantitative Literacy: An Orphan No Longer

Bernard L. Madison
University of Arkansas

4.1 Introduction

In August 2016, I was interviewed by one of the producers of National Public Radio's Science Friday about Common Core and quantitative literacy. The interviewer kept pressing me to be more explicit about the strong or weak points of the Common Core State Standards in Mathematics (CCSSM) as preparation for quantitative literacy (QL). So, what was I to say? CCSSM is too "mathey"? CCSSM does not include good contemporary applications? CCSSM skips over the foundational issue of quantification by defining a quantity as a number with a unit? One who had contributed to the writing and interpretation of CCSSM and who has studied QL for more than a dozen years should have taken this question and romped with it. But, I did not. Revisiting what I wrote for *Numeracy* [30] about CCSSM supporting QL revealed that I copped out there, saying that it depends on the implementation and assessments of CCSSM. Reflection on why I did so led me to remember, as we wrote in describing a rubric for assessing QL work [11], that the core competencies for QL are not tightly connected to mathematics or statistics content, so identifying QL as some kind of mathematics (or statistics) or some knowledge of mathematics (or statistics) can be misleading. Yet, that is the easiest way to proceed because many accept that they understand what mathematics and statistics are. Thus the Science Friday angle on QL. Distinguishing QL within or from mathematics and statistics has dominated QL discussions over the past couple of decades. Many of us in the QL community, an expanded version of what Lynn Steen referred to as a "band of rebels" [48, p. 12], have moved beyond trying to define QL or quantitative reasoning (QR) or numeracy and have accepted that QL/QR is what it is and worthy of support, further study, and development. Of course, as do the many who accept mathematics and statistics, we experience difficulty in conveying the meaning of QL/QR to others, as I did to the producer of Science Friday.

In 2000 when I wrote about QL in *MAA FOCUS* [27], "Quantitative Literacy: Everybody's Orphan," and in the AMS *Notices* [28], "Quantitative Literacy: A Challenging Responsibility," it seemed that if QL was to survive and thrive that mathematicians needed to take the lead and adopt QL as part of their responsibilities. Others have pointed this out more forcefully. Hugh Burkhardt [13] expressed astonishment with the view in *Mathematics and Democracy* that QL should not be taught by mathematicians. He believed that this was wrong, and he listed five reasons for disagreeing. Several at the 2001 national forum on QL and at Wingspread expressed the view that unless mathematicians taught QL it would not be taught. Because of the structure of K–12 education, QL seems destined to be a part of the mathematics curriculum. In post-secondary education, however, QL is moving toward cross curricular responsibility as predicted in *Mathematics and Democracy*. In the sixteen years since I characterized QL as an orphan, the situation has changed rather significantly. Although QL is no longer an orphan, the identity of its supporting family structure remains hazy and unconventional. Herein I attempt to shine some light on this family structure and how it has developed so far.

In the "Evolution of Numeracy and the National Numeracy Network (NNN)," Lynn Steen and I (2008) described the historical roots and the 20th century phases of QL, known outside the U.S. as numeracy. My focus here picks up in the late 20th century and describes some circumstances within the past couple of decades that have brought QL to its current status in the U.S. One of the major contributions to QL development over the past fifteen years has been strong, visionary leadership, most notably Lynn Steen and Robert (Bob) Orrill. I recall some of my interaction with Lynn and Bob in a sidebar to this chapter. Obviously, there have been other strong leaders of the QL movement, but attempting to name all of them invites failure. The following describes how various courses, publications, projects, and organizations have contributed support to QL. As with the leaders, all such entities defy inclusion, and omissions are not judgments of merit but rather reflect limitations of the author's memory, knowledge, and space.

4.2 Mathematics and QL: A Foster Relationship

Historically [36], the mathematics of everyday life has been quite distinct from the mathematics of mathematicians that strongly influences school mathematics. From the beginning of quantification of Western society in the thirteenth century [16] and the germs of probability and statistics [5] in the fifteenth century, the mathematics of Euclid, Euler, and Archimedes has had minimal connections to the mathematics of commerce and everyday life. This separation continued through the American colonial period [14] and the evolution of the U.S. educational system to today's separation of school (and college) mathematics from the mathematical thinking required to understand contemporary U.S. society. It is into this breach that QL/QR lays. Looking back, the issue is not two mathematics, rather two ways of reckoning and reasoning for different purposes. Mathematics as practiced by U.S. mathematicians is quite different from QL/QR. Paul Halmos [24] in his 1968 essay in the *American Scientist*, in explaining what mathematicians do, noted, "To begin with, mathematicians have very little to do with numbers" [p. 376]. Because mathematicians, as characterized by Halmos, hold tremendous sway in U.S. mathematics, there has been reluctance to adopt QL in the form that it developed. Many mathematicians, for example, view QL as inherently elementary and thus the responsibility of K–12 schools rather than colleges. Discussions at a 2004 international workshop, Numeracy and Beyond, held in Banff, Canada, were hampered greatly because of these conflicting views of the nature and complexity of numeracy or QL. The views of QL that are emerging are much more integrative and far-reaching than the initial lists of mathematical skills in the mathematics phase of numeracy as outlined by Maguire and O'Donoghue [38]. As examples in this volume, Fisher gives a linguistically based definition of QL, and Craig, Guzmán, and Harper offer perspectives based on collectivism and activism. This is quite a long way from the earlier discussions when QL was being fostered largely by mathematics.

Whether or not QL is part of mathematics or statistics was one of two questions posed often in the early years of the recent movement. (The other question was: What is QL?) It was not until about 2007 that the discussion turned from these questions to a more substantive exploration of how to educate for QL/QR. Maura Mast [39], in the abstract of her review of the proceedings of the Wingspread workshop, *Calculation vs Context,* noted that "The book . . . marks a turning point in the quantitative literacy movement as 'QL explorers,' as Lynn Steen calls them, move beyond issues of definitions and content to a discussion of how to bring quantitative literacy into a broader setting."

Most current courses in QL/QR are mathematical and statistical, but they are quite different from traditional mathematics and statistics courses. Being a part of mathematics or statistics is not the issue—using mathematics and statistics is. There are examples of scholarly areas that may help to understand the relationship between QL/QR and mathematics and statistics. One is critical thinking. As described by the Foundation for Critical Thinking [21], "critical thinking is that mode of thinking about any subject, content, or problem in which the thinker improves the quality of his or her thinking by skillfully analyzing, assessing, and reconstructing it." As this implies, like QL/QR, critical thinking cannot be confined as a part of any discipline. Another scholarly area is ethnomathematics [17], which is the study of the relationship between mathematics and culture. Ethnomathematics provides many examples of the practices of various communities that Fisher (in this volume) indicates converge into the meaning of QL/QR. Ethnomathematical issues are manifestations of culture that generate practices leading to methods and theories (e.g., the methods of carpenters, athletes, and street vendors). There are more obvious examples in the interaction of mathematics with U.S. society. For example, the Dow Jones Industrial Average is a fixture in U.S. society, but the "average" is not a traditional mathematical average, and it is not an index by any reasonable definition of an index. Another example is the so-called body-mass index, also not an index. What this indicates is that QL/QR, like critical thinking and ethnomathematics, is not part of mathematics but heavily dependent on the logic and methods therein.

The Wingspread workshop was held just as the NNN journal *Numeracy* was preparing its first issue. The interdisciplinary discussions at Wingspread and the subsequent interdisciplinary nature of the articles in *Numeracy* did signal the movement of QL/QR into a new phase, one where discussions focused not on what QL was or where it belonged but how it could be achieved and flourish. Its cross-disciplinary status was evident and thinking on QL/QR was broadened and connected to many other scholarly movements. Recent papers in *Numeracy* document the shift and increased diversity of thinking (e.g., [40], [26], and [51]). Several papers in this volume also (e.g., that from Craig, Guzmán, and Harper) plainly point to expanded and scholarly-networked discussions of QL/QR.

As noted above, if QL is to be a part of U.S. K–12 education, then, at least for the foreseeable future, it will be a part of the mathematics strand. In college, while the situation is far from clear, prospects appear encouraging. Over the past decade, a variety of structures in colleges and universities have supported QL. These include courses in mathematical departments and science departments, courses and student support in quantitative centers, extensions of professional societies into institutions, and externally funded multi-year projects, several supported by the National Science Foundation (NSF) and the Keck Foundation. Some illustrative examples of these structures are below. Most courses in mathematical departments are general education courses while many in science departments are QL-in-discipline courses. A longstanding example of the latter is Computational Geology at the University of South Florida [43].

4.3 QL Course Resources

Currently, the number of collegiate enrollments in QL/QR courses is not systematically gathered. Every five years, the Conference Board for the Mathematical Sciences (CBMS) conducts a survey to record the enrollments in the mathematical sciences, consisting mostly of mathematics but some statistics and computer science. The 2010 CBMS report [9] reveals that QL/QR courses have not been among the reporting options. However, the 2015 survey, not yet compiled, does contain references to QL in two-year colleges but falls short of asking for numbers of enrollees. One of the likely reasons for these circumstances is the lack of a common course name for QL or QR courses. For example, some such courses carry names such as "Liberal Arts Mathematics." Further, as indicated by Bolker in this volume, QL/QR courses by the name QL or QR have existed for about a decade, covering only two CBMS surveys.

One indication of the increase in QL courses and enrollments is the increase in the number of textbooks. An online search for quantitative literacy textbooks returns over 20 references, but several are not textbooks and there are duplications. This list and my own knowledge about such books points to at least a dozen commercial QL textbooks, almost all published within the past five years. In addition, there are several books that are very useful as supplementary materials for QL courses. For example, the U.S. version (*The Numbers Game*) of *The Tiger That Isn't* by Michael Blastland and Andrew Dilnot [10] contains a dozen essays on understanding and misunderstanding data and quantities. Social scientist Joel Best's books, *Damned Lies and Statistics* [6], *More Damned Lies and Statistics* [7], and *Stat-Spotting* [8] often provide humorous explanations of how statistics are produced and reported. Jeffrey Bennett's *Math for Life* [4] gives numerous examples in clear and forthright terms of quantitative knowledge that is important in life.

One of the interesting and perhaps predictive instances is the appearance of books for QL/QR in high school. One ambitious effort in this direction is The Advanced Quantitative Reasoning Project [2] directed by Greg Foley at Ohio University. Over the past seven years this project has produced three versions of a high school textbook [20] that is now being used by nearly 14,000 students in over 200 high schools.

Courses in QL/QR have been slower in development than would have been the case if there were canonical models for such offerings. While historically there has been an absence of canonical models for QL/QR courses, some are beginning to emerge. In this volume, Eric Gaze outlines a course with a "a firm foundation in proportional reasoning and modeling with spreadsheets," and Boersma, Diefenderfer, Dingman, and Madison outline possible courses using media articles. The lack of such models prompted me [29] to write "How Does One Design or Evaluate a Course in Quantitative Reasoning?" What I offered there were design principles based on collections of content standards and research findings on learning. In addition to the possibility of a blueprint for a course, there have been several descriptions of QL/QR courses appearing in the literature, most of them in *Numeracy*. Shannon Dingman and I [18] described various aspects of our course at the University of Arkansas in two papers in *Numeracy*. Later, we joined Stuart Boersma and Caren Diefenderfer [11] to further describe our courses and the assessment of student work. More recently, several authors [50] describe designing two courses at Michigan State University and Todd and Wagaman [49] describe designing a course for a two-year college in North Carolina under an articulation agreement with the University of North Carolina System. As one can surmise from reading about QL/QR courses, there is no single canonical model, and there is not likely to be one. The nature of QL/QR is such that it can be studied and practiced in a variety of courses. There are dozens of venues for reasoning and many of these have quantitative content.

4.4 Quantitative Reasoning Centers

As noted below, the NNN was originally organized in 2000 as a confederation of QL/QR centers. Lynn Steen and I [32] described several of these centers in the first issue of *Numeracy*. Since then many other centers have been created. Karaali, Choi, Sood, and Grosfils [25] describe a multidisciplinary center. One former NNN President (Corrine Taylor) was director of the QR center at Wellesley College, and the current NNN President (Eric Gaze) directs a center at Bowdoin College. Such centers are common in four-year liberal arts colleges in the northeast U.S. One likely reason for this is the long tenure of the Northeast Consortium on Quantitative Literacy (NECQL) that has been active and holding annual meetings for more than a decade. Some centers offer QL/QR courses, some manage assessments, and most provide QL/QR learning support.

4.5 Four Multi-year Projects

The following four examples of projects indicate several things. First, the four are, on the surface, very different and are housed in different collegiate environments. The first is spread across much of a private four-year liberal arts college; the second is a statistical literacy project housed in a college of business; the third is led from a department of geology at a research university; and the fourth has three foci in mathematics departments at a research university, a regional university, and a four-year liberal arts college. Yet, the four have had many mutually supportive interactions over the past decade. Leaders of all four are active in the NNN and have promoted critical reading, writing, and argumentation as well as assessment of QL/QR.

QuIRK at Carleton College—The Carleton Quantitative Inquiry, Reasoning, and Knowledge (QuIRK) initiative is a project designed to create curriculum and practice around the teaching of quantitative reasoning. QuIRK began in 2005 and has been supported by grants from the NSF, the U.S. Department of Education's Fund for the Improvement of Post-Secondary Education, and the W. M. Keck Foundation. The focus of the project is how QR is used in the development, evaluation, and presentation of principled argument. Assessment of writing containing the principled arguments is at the core of QuIRK. [1]

Keck Statistical Literacy Project at Augsburg—In 1998, Milo Schield proposed a Quantitative / Statistical Literacy course at Augsburg College where statistical literacy is defined as critical thinking about statistics in arguments, somewhat akin to the goals of QuIRK at Carleton. Augsburg approved the course in general education and it has been taught every year since. In 2001, the W. M. Keck Foundation awarded Augsburg College a grant to develop statistical literacy as an interdisciplinary curriculum in the liberal arts with Schield as the project manager. The Statlit web site contains a myriad of information about QL/QR that has been collected by Schield over the past fifteen years. [2]

Spreadsheets Across the Curriculum—This sequence of NSF-funded projects produced approximately 100 learning modules on various topics aimed at developing QL in various disciplines. The lead institution was the University of South Florida (USF) where Len Vacher coordinated most of the effort. Riccezza and Vacher [43] describe this project and relate it to other in-discipline efforts at teaching QL/QR in geology at USF. The modules are available on the SERC website [3]. There are two other across-the-curriculum projects that are intertwined with this spreadsheets example. They are Mathematics Across the Curriculum at Dartmouth College [4] and Mathematics Across the Community College Curriculum [5]. These across-the-curriculum projects point plainly to the interdisciplinary nature of QL/QR.

QRCW at Arkansas, Hollins, and Central Washington—Quantitative Reasoning in the Contemporary World (QRCW) originated at the University of Arkansas in 2004 when I began teaching a QL course aimed initially at journalism majors. Subsequently, the project expanded to Central Washington University and Hollins University as Stuart Boersma

[1] For more information, see `apps.carleton.edu/quirk/about/` or the several papers on QuIRK, (e.g., [23]).
[2] For more information, see `web.augsburg.edu/schield/`.
[3] See `serc.carleton.edu/sp/ssac`.
[4] See `math.dartmouth.edu/~matc/`.
[5] See `www.amatyc.org/mpage/mac3micro`.

and Caren Diefenderfer joined the project that had obtained NSF support. The core idea of QRCW was the use of news stories to teach QL/QR. For more information see *Numeracy* and *Peer Review* articles about QRCW [18], [31], [11] and [19]. Further results of QRCW are summarized by Boersma, Diefenderfer, Dingman, and Madison in this volume.

4.6 Organizations

Below are brief descriptions of how five U.S. professional organizations have supported and are supporting QL/QR educational efforts. The first is a professional association of colleges and universities that strongly promotes liberal education. The second and third are major mathematics and statistics professional individual and institutional membership organizations. Each of these two has sections or special interest groups that focus on quantitative literacy education. The last two were created specifically to support QL and have their roots in the seminal work of Robert Orrill.

AAC&U—The Association of American Colleges and Universities (AAC&U), consistent with its promotion of liberal education, has been very supportive of both QL education and the fledgling organization NNN. One of the AAC&U's journals, *Peer Review*, has had two issues specifically focused on QL, the first in 2004 and another ten years later. In 2009, as a part of a larger initiative entitled Valid Assessment of Learning in Undergraduate Education (VALUE), quantitative literacy was one of nine intellectual and practical skills for which rubrics were developed to assess learning from undergraduate programs.

ASA—One of the early uses of the phrase quantitative literacy was as a title of an NSF-funded joint project of the American Statistical Association (ASA) and the National Council of Teachers of Mathematics in the mid-1980s. This effort produced materials and workshops to introduce middle and high school teachers of mathematics to basic concepts of data analysis and probability, eventually evolving into a strand of K-12 mathematics standards. The connections between QL and statistics are numerous, not only historically, but also in pedagogical practice. As Richard Scheaffer [44] points out, "there are strong ties between statistical thinking, data analysis, and quantitative literacy in terms of historical developments, current emphases, and prospects for the future." ASA has a Statistical Education Section that sponsors sessions at the annual Joint Statistical Meetings. One of the frequent topics of these sessions is the promotion of statistical literacy referenced above as the focus of the Keck Foundation project at Augsburg College.

MAA—The Mathematical Association of America (MAA) first formally recognized QL through the work of a subcommittee of the MAA Committee on the Undergraduate Program (CUPM). In 1989, the MAA appointed a Subcommittee on Quantitative Literacy Requirements (QL Subcommittee) of CUPM. This subcommittee began by considering the question: What quantitative literacy requirements should be established for all students who receive a bachelor's degree? In 1994, the QL Subcommittee issued a report [45] that highlighted four conclusions emphasizing the collegiate responsibility for QL education, but the report did not have much immediate effect on collegiate mathematics. Although the QL Subcommittee continued to work after 1994, the need for a more substantial presence of QL in MAA activities was evident from the work produced by the National Council on Education and the Disciplines (NCED) initiative and the substantial attention to general education issues in the 2004 CUPM Curriculum Guide [15]. This need led to the creation of the SIGMAA QL, a special interest group of MAA members. SIGMAA QL was formed by action of the MAA Board of Governors in January 2004 aiming to provide a structure within the mathematics community to identify the prerequisite mathematical skills for QL and find innovative ways of developing and implementing QL curricula. The MAA published *Current Practices in Quantitative Literacy*, a volume of descriptions of QL programs edited by Rick Gillman [22], who was a driving force in organizing SIGMAA QL. In addition to publishing the Gillman volume, the MAA published *Achieving Quantitative Literacy* [47] and *Calculation vs. Context* [35]. The MAA has published two classroom textbooks on QL, *Understanding our Quantitative World* [3] and *Common Sense Mathematics* [12].

NCED—In 1997, the College Board, under the guidance of Robert Orrill and Lynn Steen [46], published *Why Numbers Count*, the first of what was to be four volumes in the most recent initiative concerning QL. Subsequent to the publication of *Why Numbers Count*, with support from the Pew Charitable Trusts, Orrill founded and directed the NCED whose goal was to promote discussion about core literacies at the transition from secondary to postsecondary education. Its first initiative, *Mathematics and Democracy* [41], offered an apologia for QL together with responses from professionals in a variety of careers. Subsequently, in cooperation with the National Research Council and the MAA, the NCED hosted a national forum on quantitative literacy at the National Academy of Sciences in Washington, DC [33]. In a brief fourth volume, *Achieving Quantitative Literacy* [47], Steen offered a synopsis of major issues raised at the Washington forum in order to create a national debate about the place of quantitative literacy in higher education. As of now, the NCED is inactive but its role in getting the current initiative on QL underway deserves notice. One of the vestiges of the NCED's work is the principal QL/QR supporting organization, the NNN.

NNN—The National Numeracy Network[6] (NNN) was part of Bob Orrill's vision to promote QL at the transition from secondary to postsecondary education. Modeled somewhat on the National Writing Project,[7] the NNN was initially a confederation of quantitative literacy centers. In 2004, the NNN was reconstituted as an interdisciplinary membership organization. Since then the NNN has promoted education and scholarship in QL, mostly at the postsecondary level. The most substantial evidence of the NNN's work is its journal, *Numeracy*, which began in 2008 and has blossomed into a significant international publication due largely to the able stewardship of co-editors Len Vacher and Dorothy Wallace. Numeracy's growth has both tracked and spawned an international community of scholars looking at QL from numerous directions, and Numeracy's authors reflect the interdisciplinary nature of this QL community of practice [52].

4.7 The Future

As apparent from the discussion above, QL/QR still does not have an organizational disciplinary presence in higher education, being scattered among mathematics, statistics, sciences, and interdisciplinary centers. This makes it more difficult to explain QL/QR to outsiders, but it evidently does not create insurmountable barriers to further scholarly and academic development, and, in fact, may promote growth. A large community of practice has been built and is sustained by an unofficial alliance of various organizations, projects, centers, and, most importantly, common goals. No longer an orphan, but with a non-traditional and somewhat hazy supporting family, QL/QR continues to flourish. The reason? The rationale for stronger QL/QR is overwhelming, and the strong interdisciplinary community of practice is forging underpinning scholarship that will eventually garner respect within academe. The foregoing description provides significant evidence pointing to continued robust development.

As a final comment, I have spent much of my career as a research mathematician and teacher of canonical undergraduate and graduate courses in mathematics. That experience leads me to conclude that, in general, many mathematicians require a significant shift in disposition as instructors of QL/QR. Faculty from other more integrative disciplines may find their dispositional adjustment easier. All this may portend the utilization of QL/QR as a bridge among disciplines, and releasing QL/QR from counterproductive disciplinary restrictions, may furnish a fertile foundation for growth.

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4.9 Personal Addendum on Working with Lynn Steen and Robert Orrill

In 1999, I had just finished two decades in university administration: ten years as Department Chair and ten years as Dean of my College. In addition, I had finished a term (1995–1999) as Chief Reader of AP Calculus. Bob Orrill and Lynn Steen contacted me and invited me to join them in the QL initiative that they were leading. I agreed to help and began playing catch-up by both learning what they were doing and what exactly this QL thing was. Playing catch-up with Lynn and Bob was no easy task. They had been thinking about QL for several years, had compiled and edited *Why Numbers Count: Quantitative Literacy for Tomorrow's America* in 1997, and were moving toward a second publication, *Mathematics and Democracy: The Case for Quantitative Literacy.* More daunting was the intellectual power that Bob and Lynn wielded. Both had incredible, comprehensive senses of history (Bob's discipline), the humanities, and education. Over the next dozen years, I learned an enormous amount through Bob's vision about literacy initiatives and Lynn's insightful interpretations of how this vision could play out within QL. With support from the Pew Charitable Trusts, Bob founded and directed the National Council on Education and the Disciplines (NCED) whose goal was to promote a "national reexamination of the core literacies—quantitative, scientific, historical, and communicative—that are essential to the coherent, forward-looking education all students deserve." Bob and Lynn had laid out a plan for four volumes, referred to in our discussions as QLI, QLII, QLIII, and QLIV. The first was *Why Numbers Count* [46], the second *Mathematics and Democracy* [41], the third the proceedings of a national forum [33], and the fourth a summary of what had been learned about achieving QL [47]. One of the reasons Bob and Lynn invited me to join their efforts was my previous experience with directing a forum at the National Academies, namely, directing the 1987 Calculus for a New Century. Thus the national forum, planned for 2001, became my responsibility to coordinate. Fortunately, I was able to spend the 2001 calendar year at the MAA in Washington, DC as a visiting mathematician and bring the resources of the MAA to bear on planning the forum that would be held across town on Constitution Avenue behind the intriguing statue of a reclining Albert Einstein. It was during the preparation for this forum that I began collecting news articles that eventually led to three editions of *Case Studies for Quantitative Reasoning* [37] and two QR courses at my university.

Much of my interaction with Lynn and Bob was by way of email. Sometimes a chain of emails would be triggered by a naive question I would ask. Sometimes Bob would venture an opinion or cite some historical analogy to something we were facing. Occasionally, Bob would ask if something was a QL issue. One such query was whether or not Pascal's Wager was a QL issue. We decided that it was, and I filed that away as something to be used in an honors course on QL that I had in mind, but never managed to realize. Some of our discussions concerned meanings of phrases and terms associated with literacy. One such discussion compared the term numeracy that was used in most of the world rather than the term quantitative literacy being used in the U.S. In an email, somewhat out of the blue, Bob described QL as a cultural field where language and quantitative constructs merge and are no longer one or the other. From this perspective, quantitative literacy is a more inclusive term than the narrower word numeracy. To get a sense of how Bob's creative mind works, one has only to read Bob's discussion of humanism and quantitative literacy rooted in a happenstance 1890s conversation between philosopher George Santayana and Harvard President Charles Eliot [42].

Lynn Steen and I co-authored and co-edited several publications over the period 2000–2015. Writing with Lynn was inspiring, humbling, and educational. I have never known or read a better writer, and I firmly believe I will never know a better editor. Two examples of Lynn's cataloging and summarizing often remind me of his myriad talents. One was his "interview" of me [34] as the opening session of the 2008 NNN meeting at Carleton College. I knew nothing of what he was to ask, but he pulled statements I had made over the past several years and tossed them out to me. As Lynn described what was to take place, "He [Lynn] would simply pretend to be Tim Russert to interview him [Bernie] and try to pin him down on some of the things he actually has written about QL to see if he still believes them or if he's learned anything about how to get around some of the problems that have emerged." It was fun, reflective, educational, and I trust informative to the audience. But it was Lynn's judgment, interpretation and commentary that made it work. The second example is recorded on pages 75–90 of *Achieving Quantitative Literacy* [47]. Lynn had been preserving the email exchanges among him, Bob, and me that had occurred over the years 2001–2003. Many of those were discussions on questions that Bob would ask Lynn and me as mathematicians. Lynn recorded these as conversations between Outsider (Bob) and Insider (Lynn or me). Of course, Lynn edited the conversation and no doubt made it more articulate and informative, but the original essence is there.

Those years working with Bob and Lynn are the best of my academic memories. I so miss having Lynn to ask for guidance by email, which he always answered promptly. Occasionally, I exchange emails with Bob, who, in his own words, is continuing his education in retirement in Bucks County, Pennsylvania.

Part II

Curriculum for Quantitative Literacy

Having looked at quantitative literacy from a 10,000-foot perspective, we are ready to start examining some of the new ways this is put into practice in the classroom.

Eric Gaze starts us off with his chapter, “Thinking Quantitatively: Creating and Teaching a Quantitative Reasoning Course.” While Fisher (in Part I) gave us a theoretical definition of quantitative literacy, Gaze shows us what this might look like in practice, arguing that quantitative reasoning rests on three pillars: proportional reasoning, spreadsheets, and probability and statistics.

To Gaze, these proposed pillars are part of the stable core of quantitative reasoning. The two chapters that follow Gaze’s illustrate what this core looks like in a new context: first-year seminars. First-year seminars are one of the higher-impact practices recommended by the Association of American Colleges and Universities. Maria Fung, in her chapter “Quantitative Literacy in a First-year Seminar for Liberal Arts Students,” describes first-year seminars in general and how she has woven quantitative literacy into projects and activities in this environment. Aimee Schwab-McCoy contrasts quantitative literacy with what she calls quantitative fluency. In her chapter, “‘Life in the Data Deluge’: Developing Quantitative Fluency in a First-Year Seminar Course,” she shares how she uses new media in the first-year seminar to support the development of quantitative fluency. While the focus of Fung’s chapter is on the interdisciplinary nature of her quantitative literacy projects, Schwab-McCoy’s focus is on communicating using data from media sources. The next chapter, “Teaching Quantitative Reasoning (QR) with the News,” by Stuart Boersma, Caren Diefenderfer, Bernard Madison, and Shannon Dingman, leaves the first-year seminar behind and continues Schwab-McCoy’s discussion of media and news into their own QL/QR classrooms.

The next two chapters are each focused on a new, interdisciplinary context for quantitative literacy. In “Revealing the Mathematics of Sustainability,” Alana Baird, Sherareh Nikbakht, Eric Marland, and Katrina Palmer illustrate one way in which quantitative literacy serves the world around us. Suzanne Dorée and Eleonore Balbach, in “It’s a Question of Money: Class Projects that Build Students’ Quantitative Literacy,” illustrate the role quantitative literacy plays in empowering us to control our own lives (specifically through financial mathematics). It is worth noting that the examples and projects described in these two chapters are versatile. They can be adapted to courses beyond those that were developed to satisfy quantitative literacy requirements. This is important in that it illustrates that quantitative literacy need not—indeed should not—be siloed into designated courses, but rather spread throughout the curriculum.

Part II concludes with “Yes, But Is it Rigorous? Seeing the Mathematical Complexities in a QL Course,” by Abe Edwards, Vince Melfi, and Rani Satyam. Much of the mathematics we see in this part of the book appears to be elementary insofar as the mathematical content itself is taught in elementary- and middle-school classrooms in the U.S. For example, Gaze’s focus on proportional reasoning puts ratios and proportions at the center of quantitative reasoning—elementary topics whose inclusion in college-level courses we are right to be skeptical of. Nonetheless, the power of quantitative literacy is not in the sophistication of the mathematics, but the sophistication of the reasoning in the contexts in which the mathematics is applied. The higher-order thinking skills required are demonstrated throughout the chapters in this book. Edwards, Melfi, and Satyam address this observation by examining the cognitive transitions required for a successful quantitative literacy student. In their inquiry, they find that the cognitive demands of quantitative literacy have much in common with those of mathematics majors in transition courses. While this chapter can be read on its own, the sequential reader is encouraged to think about the examples that were described in the other chapters in this part and challenge themselves to use them to test the ideas put forth in this final chapter of Part II.

5

Thinking Quantitatively: Creating and Teaching a Quantitative Reasoning Course

Eric Gaze
Bowdoin College

5.1 Introduction

The text *Mathematics and Democracy* [13] carefully lays out the case for quantitative literacy (QL) in terms of requisite elements, expressions, and skills. The authors of the design team provide the rationale for a robust quantitative reasoning (QR) curriculum:

> Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them the tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are the skills required to thrive in the modern world [p. 2].

This chapter will explore elements to consider for a QR course that addresses these challenges by providing a firm foundation in proportional reasoning and modeling with spreadsheets. I believe proportional reasoning underlies the skill set required to effectively communicate with numbers, while spreadsheets provide a platform for engaging with complex problems that are "anchored in data derived from and attached to the empirical world" [13, p. 5]. QR curriculum is distinguished from traditional mathematics in that the context drives the content in QR, and in teaching QR, "content is inseparable from pedagogy and context is inseparable from content" [13, p. 18]. To develop in our students the habits of mind of a numerate citizen we must engage them with course material and pedagogical approaches embedded in meaningful, real contexts. To this end, I will discuss the use of articles and reading assignments alongside worksheets and problem based learning. Assignments that scaffold QR skill development while deepening students' reasoning capabilities will be discussed. To truly empower our students to actively participate in today's data driven society, we must actively engage them with well thought-out QR curriculum and active learning strategies. There seems to be a growing consensus on the critical components of a meaningful QR course [8, 12, 16, 19, 3]:

1. Proportional Reasoning
2. Modeling with Spreadsheets
3. Probability and Statistics

The rationale for each of these components and how they are implemented in a specific QR course will be explored [5]. There exist other popular texts of course, some addressed in other chapters of this volume, and they vary in content and emphases. The intent of this chapter is to motivate the above three components and connect them to the vision laid out in *Mathematics and Democracy*.

5.2 Proportional Reasoning

Proportional reasoning is based on the fundamental concept of ratio. Simply comparing the relative size of two quantities is the most basic form of quantifying relationships. Proportional reasoning specifically involves maintaining a sense of multiplicative scale in a relationship between quantities. If two triangles are in proportion (or similar), then the ratio of paired sides will all be the same. Thus multiple comparisons are inherent to proportional reasoning. All of this is predicated on the multiplicity of representations of ratios using equivalent fractions and decimals and the related idea of proportionality.

Co-variation is also inherently linked to proportional reasoning. If we travel at a constant rate, then our distance and time are both changing (or co-varying) in a precise way. Doubling the distance will double the time, the quantities vary in proportion to one another. This example alone leads to the subtleties of proportionality. If the miles you drive is directly proportional to the gallons of gas you use, then these two quantities *co-vary,* but in a precise way captured by the constant of proportionality—miles per gallon. Co-variation is thus intimately connected to *rates* (miles per gallon) which are ratios with the second quantity set to a meaningful standard. The fact that a rate of change is *constant* gives you some sense of our students' conceptual difficulties in understanding how change and constancy are two sides of the same coin.

Results from the National Assessment of Adult Literacy [12] conducted in 1992 and 2003 justify the need for proportional reasoning. In both years, 87% of U.S. adults scored below "proficient" with regards to quantitative literacy, meaning only 13% could calculate annual costs from monthly costs or determine flooring units per square foot. In other words, proportional reasoning was the criterion used to define being "numerate." The first challenge in creating a QR course is thus to systematically develop the requisite proportional reasoning skill set in a rigorous but accessible manner. There are many who would argue that such "middle school content" has no place in a college credit-bearing curriculum. The simple answer to this challenge is that QR will present students with problems that require sophisticated reasoning using elementary mathematics, in contrast to much of traditional mathematics which involves sophisticated mathematics but elementary reasoning [13, p. 6]. The Program for International Assessment of Adult Competencies (PIAAC) [16] in 2012 confirmed that U.S. millennials, those born between 1980 and 2004, are to a great extent quantitatively illiterate, with the U.S. coming in last among developed nations on the numeracy test that again tested proportional reasoning skills.

The minimum benchmark for numeracy on the PIAAC scale was not reached by 64% of U.S. millennials compared with the OECD average of 47% not achieving minimum numeracy proficiency. Breaking this statistic out by race is even more troubling in the U.S., as 54% of white millennials, 83% of hispanic millennials, and 88% of black millennials did not achieve the minimum numeracy benchmark [7]. This naturally speaks to unequal access to quality education. In general, the innumeracy of young adults in the U.S. is getting worse as seen when comparing the 2012 PIAAC assessment with the Adult Literacy and Lifeskills Survey scores conducted between 2003 and 2008 [10]. Figure 5.1 provides further evidence of this worsening problem as young adults age 16-24 scored even lower than the overall millennial cohort. Questions such as mileage reimbursement for a trip, converting energy units to determine how many wind stations would be needed to replace a nuclear reactor, or simply interpreting a stacked bar chart with years on the x-axis and percentages on the vertical axis all underscore proportional reasoning skills in context. In our data driven society, economic success is strongly linked to numeracy [1]. If we truly want to provide access to a better life for all of our students, it is imperative that we address the failings and inequities of our current educational system, and the mathematics curriculum in particular. Proportional reasoning needs to be systematically developed.

Mathematics educators are aware of the vital place proportional reasoning and the closely associated concept of co-variation have in our mathematics curriculum. Robert Mayes cites Piaget on the importance of proportional reasoning for understanding in mathematics: "Proportional reasoning encompasses complex cognitive abilities which include both mathematical and psychological dimensions. It requires a significant conceptual shift from concrete operational to formal operational levels of thought. It has been proposed as a major barrier to a student's development of math-

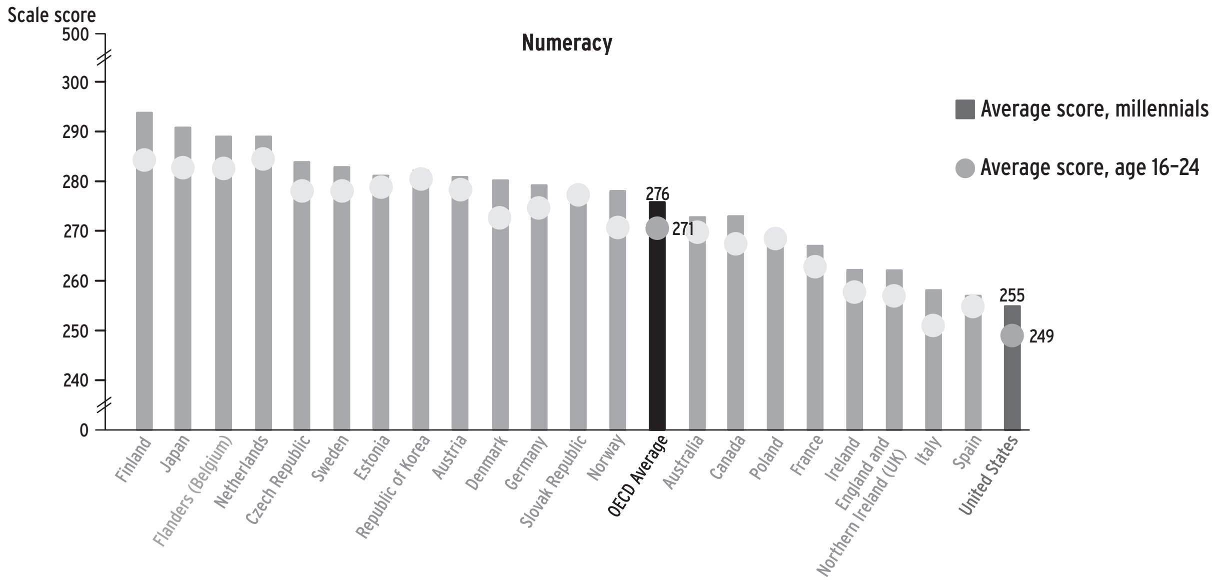


Figure 5.1. Average Scores on the PIAAC numeracy scales for adults age 16–34 (millennials) and adults age 16–24, by participating country region in 2012 [10].

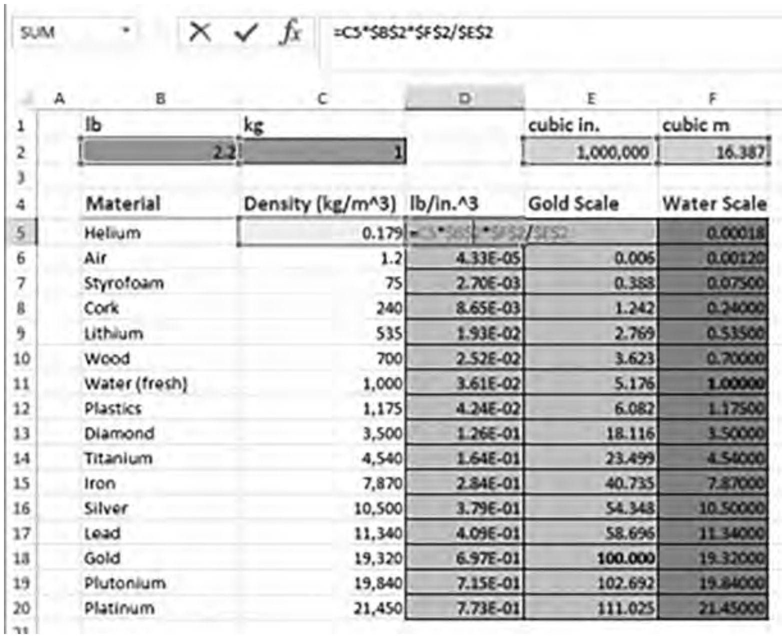
ematical understanding" [11, p. 17]. Mayes also describes proportional reasoning as being critical to QR in science: "Proportional reasoning is often a major conceptual barrier to students, inhibiting their ability to reason quantitatively in science. Here we include pre-proportional reasoning skills such as an understanding of fraction, ratio, percent, and rates all leading up to proportions" [11, p. 17].

Fractions in particular are consistently identified as a major stumbling block for children that prevent them from successfully moving on to higher mathematics. The National Mathematics Advisory Panel was convened by presidential order in 2006 to assess the state of mathematics education in the United States. This panel of experts conducted an exhaustive review of over 16,000 research publications and policy reports in math education, and came to the conclusion that our current system of mathematics education is "broken." Their published findings call for an emphasis on fractions: "A major goal for K8 mathematics education should be proficiency with fractions (including decimals, percentages, and negative fractions), for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped" [14, p. xvii]. In the paper by Alan Tucker, "Fractions and Units in Everyday Life," he makes the case that "understanding fractions well by relating them to units is both important and intellectually rich. It is definitely worthy of a college level course in quantitative literacy" [22, p. 85]. Hung-Hsi Wu echoes this sentiment in his article, "What's Sophisticated About Elementary Mathematics?" [23].

Lynn Steen offers some insight into where the problem lies with teaching such mathematics: "Current schooling is strikingly deficient in achieving a primary goal of middle school mathematics, namely to convey the interrelated meanings of fractions, percentages, proportions, decimals, and rates. Kepner noted that one reason for the decline in comprehension of these topics by high school graduates in the last half century is that teachers taught them according to different algorithms required for calculation rather than as *different perspectives on a common topic* (emphasis added)" [21, p. 21].

The concept of ratio can provide a common theme to convey the interrelated meanings of fractions, percentages, proportions, decimals, and rates. I use ratios to tie together all of these disparate topics in a QR course. For example, conversion factors between units (e.g., $1\text{ EUR} = 1.12\text{ USD}$) are simply ratios used to set up proportions for converting. Scales are ratios indicating a change in magnitude from a model of the real world to the real world. Spreadsheets provide the perfect tool for analyzing data sets and developing students' understanding of conversions and scales.

Figure 5.2 illustrates an example converting densities from kilograms per cubic meter to pounds per cubic inch. There is a lot going on in this problem. First, the conversion of compound units is notoriously challenging (as you will experience when teaching this). Asking a student to convert the density of gold at 19,320 $\frac{kg}{m^3}$ to $\frac{lb}{in^3}$ is a difficult arithmetic problem. Asking this student to determine the correct formula to type into cell **D5** that can be filled down (to convert each density in the data set) turns this into a challenging algebra problem. Spreadsheets develop students'



SUM =C5*$B$2*$F$2/$E$2

	A	B	C	D	E	F
1		lb	kg		cubic in.	cubic m
2		2.2	1		1,000,000	16.387
3						
4		Material	Density (kg/m^3)	lb/in.^3	Gold Scale	Water Scale
5		Helium	0.179	=C5*$B$2*$F$2/$E$2		0.00018
6		Air	1.2	4.33E-05	0.006	0.00120
7		Styrofoam	75	2.70E-03	0.388	0.07500
8		Cork	240	8.65E-03	1.242	0.24000
9		Lithium	535	1.93E-02	2.769	0.53500
10		Wood	700	2.52E-02	3.623	0.70000
11		Water (fresh)	1,000	3.61E-02	5.176	1.00000
12		Plastics	1,175	4.24E-02	6.082	1.17500
13		Diamond	3,500	1.26E-01	18.116	3.50000
14		Titanium	4,540	1.64E-01	23.499	4.54000
15		Iron	7,870	2.84E-01	40.735	7.87000
16		Silver	10,500	3.79E-01	54.348	10.50000
17		Lead	11,340	4.09E-01	58.696	11.34000
18		Gold	19,320	6.97E-01	100.000	19.32000
19		Plutonium	19,840	7.15E-01	102.692	19.84000
20		Platinum	21,450	7.73E-01	111.025	21.45000

Figure 5.2. Density homework using Microsoft Excel to convert units and create scales.

arithmetic and algebra reasoning skills simultaneously! This is an example pointing out how students apply reasoning skills to interpret data. Asking students to create two new scales emphasizes the arbitrariness of our units:

1. Scale gold to 100 and the other densities proportionally.

2. Scale water to 1 and the other densities proportionally.

There is no real significance to the absolute number 19,320 in relation to gold. Numbers only have meaning in context when we compare them to other relevant quantities. Is 19,320 $\frac{kg}{m^3}$ a lot? The gold scale tells us gold is roughly twice as dense as silver, and the water scale tells us silver is roughly 10 times as dense as water. This is true no matter what the scale. What matters is the relationship between the numbers, their ratio. We can see this beautifully by creating two column charts of density, as in Figure 5.3.

A robust understanding of scale is also critically important for statistical thinking and leads to the notion of z-scores. An SAT score of 650 is meaningless until we know that the mean is 500 and the standard deviation is 100. Converting this score to $z = 1.5$ allows us to compare this SAT value to other standardized tests like ACT scores.

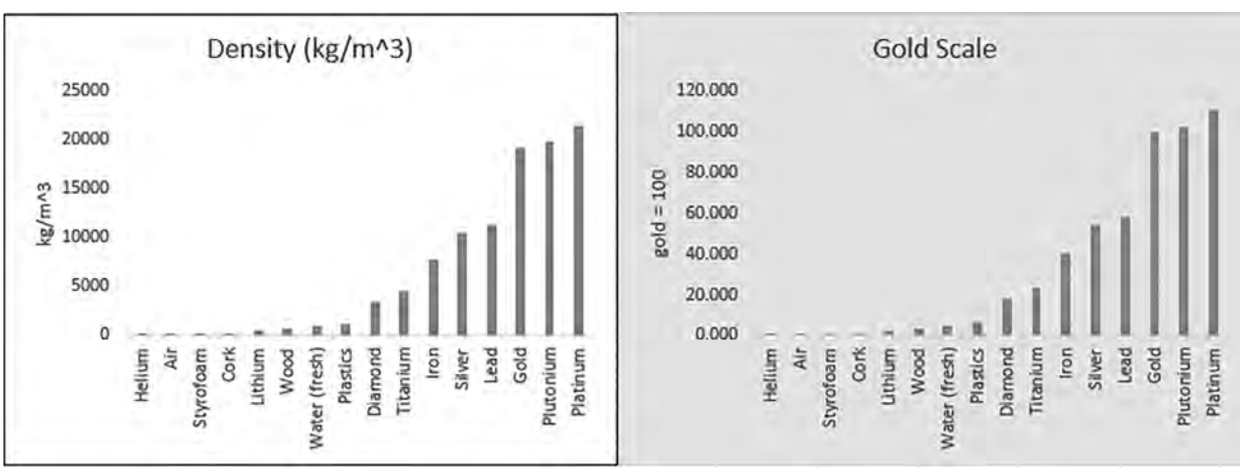


Figure 5.3. Two scales showing densities in proportion.

F4 =C4/$C$1*100000

	A	B	C	D	E	F
1		2010 Population (US Census 2010):	308,745,538			
2	Rank	Cause of Death	Number	Proportion of Total Deaths	Percent of Total Deaths	Crude Death Rate (2010)
3		All causes	2,468,435	Decimal	100.0	799.5
4	1	Diseases of heart	597,689	0.2421	24.2	193.6
5	2	Malignant neoplasms	574,743	0.2328	23.3	186.2
6	3	Chronic lower respiratory diseases	138,080	0.0559	5.6	44.7
7	4	Cerebrovascular diseases	129,476	0.0525	5.2	41.9
8	5	Accidents (unintentional injuries)	120,859	0.0490	4.9	39.1
9	6	Alzheimer's disease	83,494	0.0338	3.4	27.0
10	7	Diabetes mellitus	69,071	0.0280	2.8	22.4
11	8	Nephritis, nephrotic syndrome and nephrosis	50,476	0.0204	2.0	16.3
12	9	Influenza and pneumonia	50,097	0.0203	2.0	16.2
13	10	Intentionalself-harm (suicide)	38,364	0.0155	1.6	12.4

Figure 5.4. Death data for U.S. in 2010.

Rates

Rates are the most common way we see ratios presented as statistics. From birth rates per 1,000 people (12.6 in 2010 in U.S.), to fertility rates per 1,000 women of child bearing age (62.3 in 2013 in U.S.), to death rates per 100,000 people (799.5 in 2010 in U.S.), to homicide rates per 100,000 people (18 in Chicago in 2015), to miles per gallon (28 for a 2016 Subaru Forester) and students per teacher (9 at Bowdoin College in 2016), we are inundated with ratios being presented as single numbers. The second quantity in the comparison has been scaled to a "nice" number. The next example shows how a given statistic can be represented in multiple ways, making for a perfect application of spreadsheets with real world data.

In 2010 in the U.S., 597,689 people died from a disease of the heart, or 193.6 deaths for every 100,000 people, which, at 24.2% of all deaths, makes heart disease the leading cause of death. The male heart disease death rate is 1.6 times the female rate making men 60% more likely than women to die from heart disease, while blacks are 30% more likely than whites to die from heart disease. Students are baffled by this complexity of different ways to represent data. They tend to think of statistics as simple absolute numbers, like 30,691 men committed suicide in 2010. This number is just as meaningless as the 19,320 associated with gold, it needs context and comparison to make meaning of it. Turning this statistic on male suicide into a male to female ratio of death rates for suicide equal to 4 takes a lot of mental processing to figure out what is meant! Never mind the important aspect of the social construction of statistics in terms of questions like: "How are we counting suicides?" "What is the definition of suicide?" "Who is doing the counting?" All of these decisions influence the actual statistic [2].

Percentages

Percentages are certainly the most common form in which we encounter rates, and yet our students struggle with sentences like the following from an article on income in the U.S.: "While the bottom fifth of households increased their share of the nation's income, by the Census' definition, to 3.4 percent from 3.3 percent, the richest 5 percent kept 21.8 percent of the pie" [17]. And these percentages are all relatively straightforward by themselves, just parts of a whole; but the whole keeps switching. Part to part ratios further confuse matters, especially when we scale the second quantity to 100 as governments do when computing their dependency ratios. The old age dependency ratio is the ratio of the number of people 65 and over to the number of people 18-64, with the second quantity scaled to 100, which for the U.S. in 2015 was 23.9. Ask your students to use this number in a sentence, and you will be surprised by the varied responses. Johanna Drucker in *Graphesis*: *Visual Forms of Knowledge Production* refers to "humanistic fields where interpretation, ambiguity, inference, and qualitative judgement take priority over quantitative statements and presentations of facts" [4]. Such a statement is a common misconception—that there is no interpretation

	A	B	C	D	E	F	G	H	I	J	K
1	Table 1.3—SUMMARY OF RECEIPTS, OUTLAYS, AND SURPLUSES OR DEFICITS (–) IN CURRENT DOLLARS, CONSTANT (FY 2005) DOLLARS, AND AS PERCENTAGES OF GDP: 1940–2018										
2	(dollar amounts in billions)										
3		In Current Dollars			In Constant (FY 2005) Dollars				As Percentages of GDP		
4	Fiscal Year	Receipts	Outlays	Surplus or Deficit (–)	Receipts	Outlays	Surplus or Deficit (–)	Addendum: Composite Deflator	Receipts	Outlays	Surplus or Deficit (–)
70	2005	2,153.6	2,472.0	-318.3	2,153.6	2,472.0	-318.3	1.0000	17.3	19.9	-2.6
71	2006	2,406.9	2,655.0	-248.2	2,324.6	2,564.3	-239.7	1.0354	18.2	20.1	-1.9
72	2007	2,568.0	2,728.7	-160.7	2,413.1	2,564.1	-151.0	1.0642	18.5	19.7	-1.2
73	2008	2,524.0	2,982.5	-458.6	2,288.1	2,703.8	-415.7	1.1031	17.6	20.8	-3.2
74	2009	2,105.0	3,517.7	-1,412.7	1,901.0	3,176.8	-1,275.8	1.1073	15.1	25.2	-10.1
75	2010	2,162.7	3,457.1	-1,294.4	1,929.1	3,083.6	-1,154.6	1.1211	15.1	24.1	-9.0
76	2011	2,303.5	3,603.1	-1,299.6	2,013.7	3,149.8	-1,136.1	1.1439	15.4	24.1	-8.7
77	2012	2,450.2	3,537.1	-1,087.0	2,093.4	3,022.2	-928.7	1.1704	15.8	22.8	-7.0
78	2013 estimate	2,712.0	3,684.9	-972.9	2,271.4	3,086.2	-814.8	1.1940	16.7	22.7	-6.0
79	2014 estimate	3,033.6	3,777.8	-744.2	2,487.4	3,097.6	-610.2	1.2196	17.8	22.2	-4.4
80	2015 estimate	3,331.7	3,908.2	-576.5	2,674.3	3,137.1	-462.7	1.2458	18.6	21.8	-3.2
81	2016 estimate	3,561.5	4,089.8	-528.4	2,799.0	3,214.3	-415.3	1.2724	18.8	21.6	-2.8
82	2017 estimate	3,760.5	4,247.4	-486.9	2,894.3	3,269.0	-374.7	1.2993	18.8	21.3	-2.4
83	2018 estimate	3,974.0	4,449.2	-475.3	2,995.4	3,353.6	-358.2	1.3267	18.9	21.2	-2.3

Figure 5.5. 2013 Federal Budget Data from the Office of Management and Budget. Source: www.whitehouse.gov/omb/budget/.

or ambiguity inherent in statistics—and actually privileges quantitative analysis over humanistic modes of inquiry. A single statistical "fact" can trump a well-reasoned argument because of such a misconception. Exploring the levels of interpretation and ambiguity in statistics make for great projects. The dependency ratio project [6] has students focus on writing with numbers, comparing and contrasting these ratios for various countries, and writing an article on their findings. Articles can be assigned every week on current events, and take home quizzes assigned connecting the contextual data from the article to the mathematics content of the QR course.[1]

Given the prevalence of percentages found in articles with quantitative information, "Percentage of what?" is probably the most important question we can teach our students to ask (and answer) when it comes to their quantitative literacy. Interpreting the dependency ratio, 23.9 : 100, as a percentage is challenging because of its part to part nature.

Percent change presents a whole new arena for confusion: "The data, which measured how Americans were doing six years into the economic recovery, shows that incomes in the middle, measured in 2015 dollars, were still 1.6 percent below the previous peak of $57,423 a household, which was attained in 2007, just before the economy sank into what has come to be known as the Great Recession" [17]. This is a very dense sentence: just trying to figure out what year we are talking about is mind-boggling, before we even get to the "1.6 percent below" phrase. In addition we have the innocuous comment about 2015 dollars, introducing the time value of money that is needed to truly understand financial statements like data on our federal budget over time.

Remember I started this chapter with a quote from *Mathematics and Democracy*, the title of which emphasizes that we are trying to teach our students the skills they need for informed citizenship. Certainly that would mean giving them the skills to comprehend the data given in Figure 5.5 on the federal budget, but the sheer amount of information being presented in this table is overwhelming. The terminology alone, from "Current Dollars" to "Constant FY 2005 Dollars" to "Receipts and Outlays" to "GDP," takes a lot of unpacking before we can even begin to process what the quantitative information is telling us. These are the sorts of datasets (and spreadsheet activities built around them), that our students need to grapple with to build their confidence in "asking intelligent questions of experts" [13, p. 2].

The activities shown here and found in the course must be a mix of procedural skill building exercise and contextual problem solving, as Fisher in the opening chapter calls for with his discourse specific definition of QL. Homework assignments should ask students to compute percent change and convert units as skills drills, in addition to immersing students in inquiry-based learning [24]. Without such a skills foundation, the danger is that students will flounder on the activities. We want them to struggle, but it should be productive struggle.

[1] A resource for such articles and quizzes can be found here: https://thinkingquantitatively.wordpress.com/.

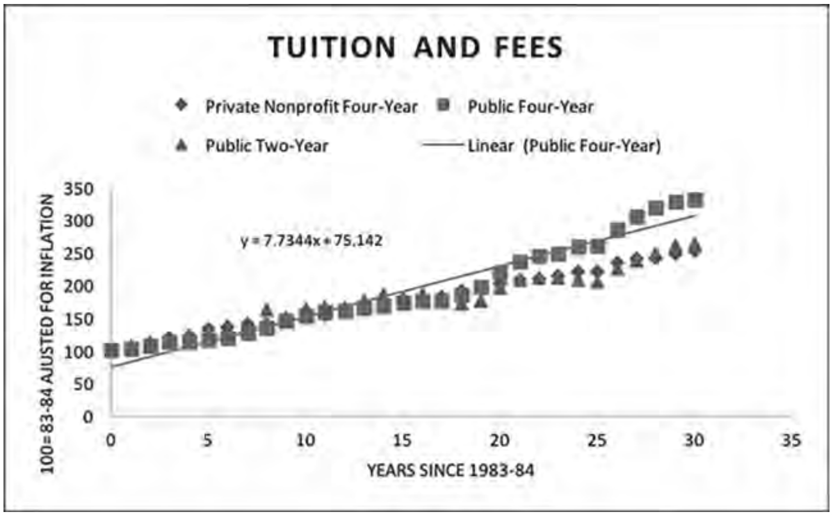


Figure 5.6. U.S. Average College Tuition and Fees in 2013. Source: The College Board Annual Survey of Colleges, available at `professionals.collegeboard.org/higher-ed/recruitment/annual-survey`.

5.3 Modeling with Spreadsheets

The section above on proportional reasoning demonstrates the power of spreadsheets for engaging our students with real world data. The variety of contexts seen in the examples naturally lead students to an appreciation for the value of developing their proportional reasoning skills. Once familiarity with the spreadsheet interface has been established, we can move to more sophisticated modeling applications. It is important to reiterate that you do not have to teach in a computer classroom in order to use Excel (or spreadsheets in general). Worksheets with screenshots of Excel greatly aid the learning process, as students can take notes on how to use Excel, and then implement on their own for homework. The students absolutely know how valuable spreadsheet skills are, so they readily learn Excel. This may require some selling on the faculty member's part, but it is not hard to convey how valuable spreadsheet skills are in today's data driven economy. It is incumbent upon us as educators to give students the skills they need (spreadsheets, not graphing calculators) to succeed in the workplace. Modeling requires a solid foundation in total change and percent change which are necessary for covering linear and exponential functions. A quantity with a constant total change every year grows (decays) linearly while a quantity with a constant annual percent change will grow (decay) exponentially. Once again, the use of spreadsheets greatly facilitates the relevance of these topics. Coverage of linear functions is common in QR courses, but coverage of exponential functions tends to be optional depending on your emphasis and student population. The conclusion at the end of the chapter discusses these options in more detail. Here I am giving multiple examples to illustrate why spreadsheets can facilitate the learning of mathematical content and engage students with the material.

Excel computes the equations of the trendlines for us, allowing us to focus on the meaning of the models and what the numbers are telling us. We can see in Figure 5.6 that private non-profit four-year tuition and fees have a larger annual increase than the other types of institutions, and the slopes allow us to quantify these increases.

Given the large gap between tuition and fees at private non-profit four-year and the other types of institutions we can scale all the quantities to 100 in 1983–84 for a more accurate visual comparison of change over time. In particular, we can ask which institution type has been increasing tuition and fees the fastest. This is a somewhat ambiguous question, as one could argue the largest slope gives the answer, but private non-profit four-year schools charge more so we should expect their increase to be greater. Comparing the slopes to the original value in 1983–84 allows us to answer the question in terms of "relative" increase.

From the scatterplot in Figure 5.7 we can see that public four-year colleges have the largest relative annual increase in annual tuition and fees. I will leave it to the reader to interpret the slope and y-intercept of the given equation. Modeling exponential functions is of course just as easy with a spreadsheet.

Wind energy production in particular has been growing exponentially and Excel represents the equation in Figure 5.8 using the continuous form: $y = 1.3069e^{0.2021x}$. This is somewhat of a blessing and a curse. It forces us to grapple

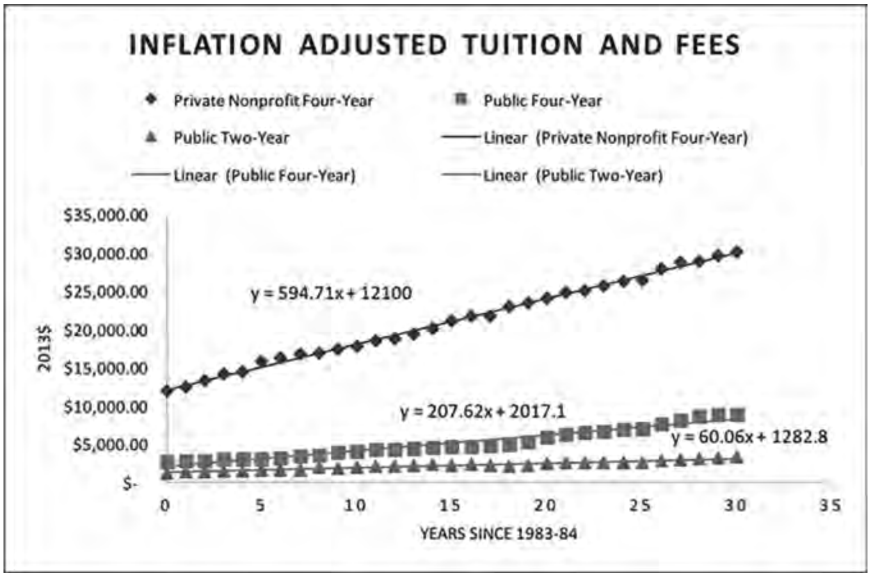


Figure 5.7. U.S. average college tuition and fees adjusted for inflation. Source: The College Board Annual Survey of Colleges, available at `professionals.collegeboard.org/higher-ed/recruitment/annual-survey`.

with Euler's number but using this form of the equation is beneficial for those students interested in STEM fields. In any case, it is easy enough to rewrite as $P = 1.3069(1.224)^t$ or rewriting it further to emphasize the growth rate as $P = 1.3069(1 + 22.4\%)^t$. We can now say that wind energy production has been increasing by 20.21% continuously per year since 1990 or by 22.4% annually since 1990. The subtle difference between these statements is a perfect segue into compounding over different periods. Financial literacy is an important component of a QR course and this discussion gets at the heart of the distinction between APR and APY. Discussing doubling times and half-lives lead us to the development of logarithms.

At this point I have laid a solid foundation for students who wish to pursue STEM fields. Some believe QR courses are terminal math courses for students wishing not to pursue STEM fields, but that is not a universally held belief. Modeling with linear and exponential functions, dealing with unit conversions, and applying logarithms to doubling times (half-lives), and semi-log plots are all crucial concepts that students can build on as they move into more sophisticated math and science coursework. With this background, we can dive more deeply into linear applications such as correlation and causation, or exponential applications related to financial literacy, or modeling applications using

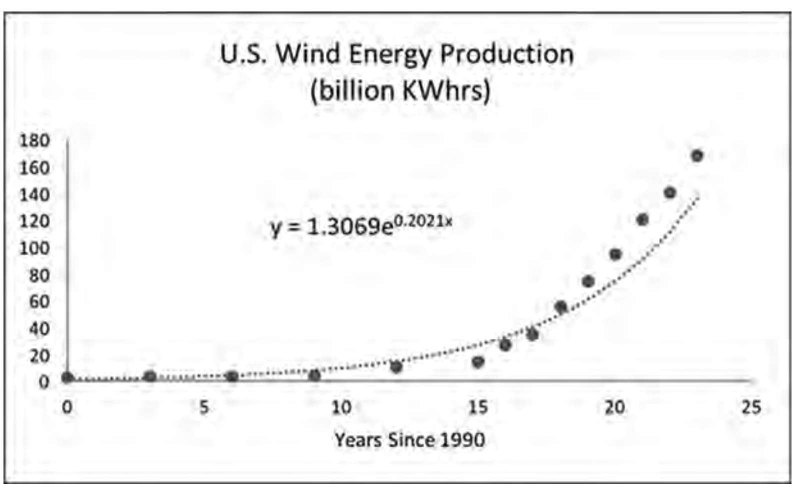


Figure 5.8. U.S. wind energy production. Source: U.S. Energy Information Administration, available at `www.eia.gov/renewable/data.php`.

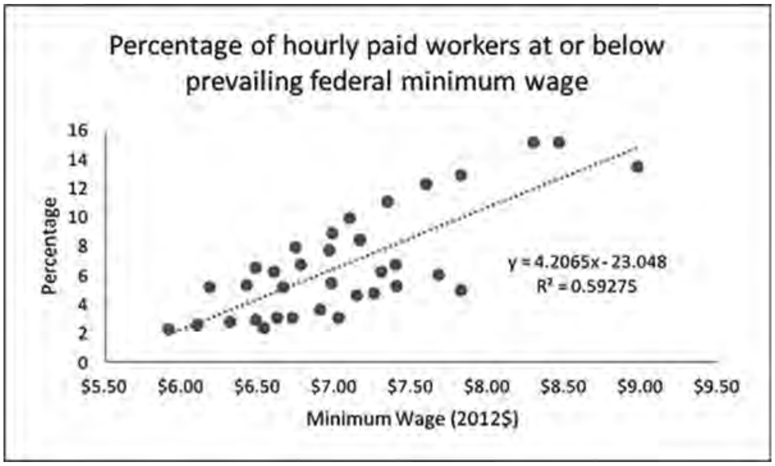


Figure 5.9. U.S. federal minimum wage. Source: U.S. Department of Labor, available at `www.dol.gov/whd/state/stateMinWageHis.htm`.

Excel's built-in logic functions. One of the most abused and misunderstood statistics is the correlation coefficient, and its offshoot the coefficient of linear determination. We need our students to appreciate the concept of confounding factors and spurious correlations. Important public policy issues can be reduced to simple bivariate correlations like the scatterplot in Figure 5.9 showing the percentage of hourly paid workers at or below the prevailing federal minimum wage against the actual minimum wage.

There are those who would argue that raising the minimum wage will reduce the number of employees businesses will hire. This graphic seems to provide counter-evidence to that claim. Remember from the Introduction that we want to empower our students to actively participate in our democratic society and "ask intelligent questions of experts" [13, p. 2].

The confidence to do this stems from a solid foundation in quantitative reasoning. Understanding that the R-squared value in Figure 5.9 is telling us that 59% of the variability in the percentage of hourly paid workers at or below the prevailing federal minimum wage can be attributed to variability in the minimum wage is an important take-away from this model. The discipline of statistics allows us to quantify variability, helping us to be certain of our uncertainty!

The line of best fit provides a perfect opportunity to recap the statistical concepts covered in the course. We know that a fish's weight is strongly correlated to its length, and as shown in Figure 5.10, Excel readily reports the equation of the least squares regression line and the R-squared value close to one.

A beautiful way to derive the equation is to recognize that a fish with above average length should have above average weight. But we have different units on the axes so we cannot just say a fish that is 50 mm above the mean

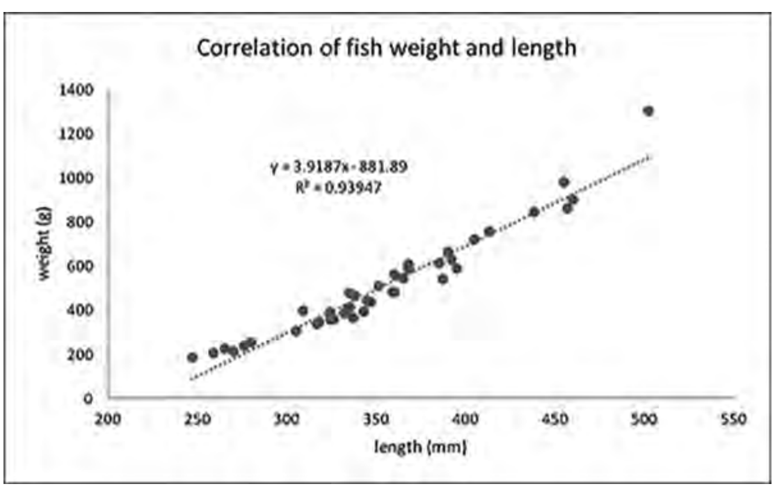


Figure 5.10. Fishy data. Source: WA Department of Ecology Report.

length should be 50 g above the mean weight. We of course use z-scores to compare deviations from different scales. So a fish that is one standard deviation above the mean length should be "roughly" one standard deviation above the mean weight. How do we make precise the fuzzy word "roughly"? If the data is perfectly correlated we could expect a precise match between the two different standard deviations, but if the data is say only 94% correlated, then we can "weight" the y-value's z-score by the correlation coefficient:

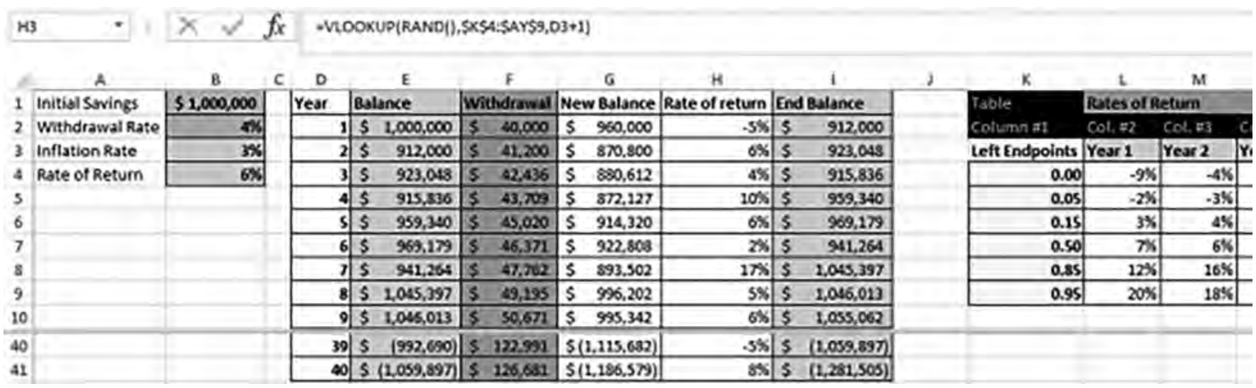
$$\text{Predicted weight} = (\text{mean weight}) + (\text{predicted distance from mean})$$

$$\hat{y} = \bar{y} + z_Y \cdot SD_Y$$

$$\hat{y} = \bar{y} + R \cdot z_X \cdot SD_Y$$

These equations exemplify the critical distinction between a traditional math course and a QR course. We do encounter such mathematical statements in QR, but only to provide some justification to intuitive understanding. Excel gives us the equation and R-squared value, and what we emphasize is how to interpret this information. The two equations also gives us a nice way to interpret the correlation coefficient as the ratio of the z-scores for the associated x- and y-values.

In addition to developing statistical literacy throughout a QR course, I also incorporate financial literacy systematically. Starting with functions for interest and payments, ratios such as price-to-earnings and the consumer price index, percent change leading to compound interest and the difference between APR and APY, I consistently ground the math in financial contexts. All of this culminates with a chapter devoted to financial literacy and the built-in Excel financial functions. Finally I introduce Excel's logic functions, and use IF and VLOOKUP to create financial models involving random simulations with the RAND function. These are shown in Figure 5.11.


H3 =VLOOKUP(RAND(),$K$4:$AY$9,D3+1)

	A	B
1	Initial Savings	$ 1,000,000
2	Withdrawal Rate	4%
3	Inflation Rate	3%
4	Rate of Return	6%

	D	E	F	G	H	I
1	Year	Balance	Withdrawal	New Balance	Rate of return	End Balance
2	1	$ 1,000,000	$ 40,000	$ 960,000	-5%	$ 912,000
3	2	$ 912,000	$ 41,200	$ 870,800	6%	$ 923,048
4	3	$ 923,048	$ 42,436	$ 880,612	4%	$ 915,836
5	4	$ 915,836	$ 43,709	$ 872,127	10%	$ 959,340
6	5	$ 959,340	$ 45,020	$ 914,320	6%	$ 969,179
7	6	$ 969,179	$ 46,371	$ 922,808	2%	$ 941,264
8	7	$ 941,264	$ 47,762	$ 893,502	17%	$ 1,045,397
9	8	$ 1,045,397	$ 49,195	$ 996,202	5%	$ 1,046,013
10	9	$ 1,046,013	$ 50,671	$ 995,342	6%	$ 1,055,062
40	39	$ (992,690)	$ 122,991	$ (1,115,682)	-5%	$ (1,059,897)
41	40	$ (1,059,897)	$ 126,681	$ (1,186,579)	8%	$ (1,281,505)

	K	L	M
1	Table	Rates of Return	
2	Column #1	Col. #2	Col. #3
3	Left Endpoints	Year 1	Year 2
4	0.00	-9%	-4%
5	0.05	-2%	-3%
6	0.15	3%	4%
7	0.50	7%	6%
8	0.85	12%	16%
9	0.95	20%	18%

Figure 5.11. Retirement investment random simulation.

By incorporating variable rates of return each year we can model actual real world risk in investment decisions. The insights generated from such models can lead to informed decision making. It is important to reiterate that this does not require having to teach in a computer classroom in order to use Excel. Worksheets with screenshots of Excel can be used in class, students take notes on how to use Excel, and then implement on their own for homework.

5.4 Probability and Statistics

Statistical literacy is an integral component of QR and the third of three key components mentioned at the beginning of this chapter. A QR course should provide students with the quantitative skills needed for informed decision making in their personal, professional, and public lives. Often we are faced with situations that involve chance, or have incomplete information resulting in likelihoods and risk assessment. To deal with uncertainty, we need a firm grasp of probability.

To make an informed decision as a citizen regarding public policy issues, one must understand that life is filled with uncertainty. Smoking does not always lead to lung cancer, and scientists cannot predict exactly how much temperatures will rise given the best available data on greenhouse gas emissions, as shown in Figure 5.12. We also need to appreciate that chance is not absolute but relative to other factors. The chance of a 55 year old person dying from a heart attack is highly dependent on whether they smoke. Figure 5.13 gives death rates per 1,000 of each cohort over the next 10 years.

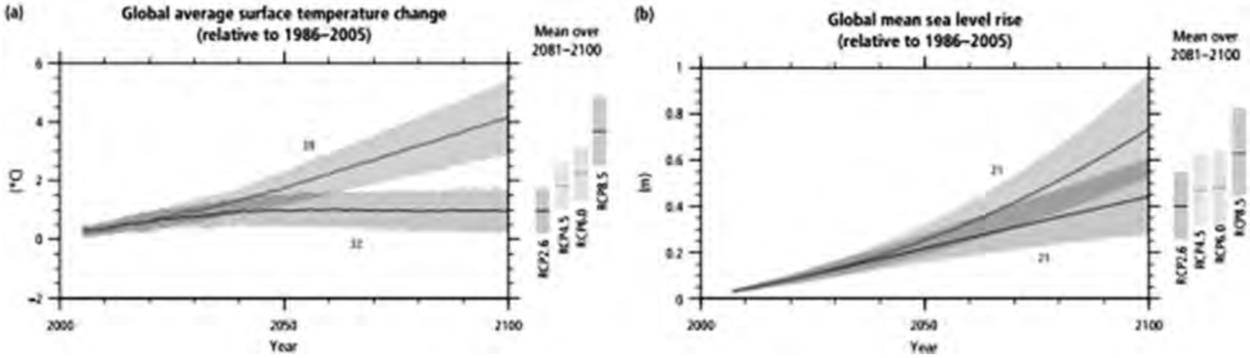


Figure 5.12. Projected simulations for four emission scenarios. Source: `www.ipcc.ch/pdf/assessment-report/ar5/syr/SYR_AR5_FINAL_full.pdf`.

			Vascular Disease		Cancer					Lung Disease		
Age	Sex	Smoking Status	Heart Attack	Stroke	Lung	Breast	Colon	Prostate	Ovarian	COPD	Accidents	All Causes
55	M	Never Smoked	19	3	1		3	2		1	5	74
55	**M**	**Smoker**	**41**	**7**	**34**		**3**	**1**		**7**	**4**	**178**
55	F	Never Smoked	8	2	2	6	2		2	1	2	55
55	**F**	**Smoker**	**20**	**6**	**26**	**5**	**2**		**2**	**9**	**2**	**110**

Figure 5.13. Death rates, with numbers gives deaths per 1,000 of each cohort over the next 10 years. Source: Woloshin, Steve, Lisa Schwartz, and Gilbert Welch. 2008. "The Risk of Death by Age, Sex, and Smoking Status in the U.S.: Putting Health Risks in Context." *Journal of the National Cancer Institute* 100: 845–853.

The chance of dying from a gunshot is 30 times higher in the U.S. than in England. Table 5.1 compares the risks of dying from a gunshot in other countries with the U.S., and shows living in the United States makes you much more likely to be a gun homicide victim.

Living in Chicago in 2015 raises the rate of gun deaths to 178 per million, while being poor, black, male and living in Chicago raises the rate even more. In 2016 gun control and race relations were topics that dominated the headlines and polarized the debates of our Presidential hopefuls. Should we vote for stricter gun control measures? Just how dangerous is our society? Does racial profiling impact police tactics such as stop and frisk? Answering such questions requires understanding risk and assessing various interventions. We can reduce our risk of dying from a car crash to zero by walking everywhere and similarly reduce our risk of a heart attack, stroke, or diabetes by adopting a wholly food-plant based diet. We need to provide our students with the quantitative skills to help make such decisions. Certainly, they are going to need to understand the basics of probability with a good sense of conditional probabilities. This can lead to applications of Bayes' theorem, where we update prior probabilities when confronted with new evidence [20].

Table 5.1. Comparison of death rates (2007–2012 averages). Source: `nyti.ms/2jTYUat`.

Being killed with a gun in:	Is as likely as dying from a ______ in the U.S.	Deaths per million
El Salvador	Heart attack	446.3
United States	Car accident	31.2
China	Plane crash	1.6
Japan	Lightning strike	0.1

Table 5.2. Bayes table given the data that someone had a heart attack.

Hypotheses (H_i)	Priors $P(H_i)$	Likelihoods $P(D \mid H_i)$	Products $P(H_i) \cdot P(D \mid H_i)$	Posteriors $P(H_i \mid D)$
Smoking	20.0%	4.1%	82	$\frac{82}{234} \cdot 100\% = 35.0\%$
Not Smoking	80.0%	1.9%	152	$\frac{152}{234} \cdot 100\% = 65.0\%$
Totals	100%		234	100%

Let us assume you believe that there is a 20% chance that a 55 year old male is smoking, so there is an 80% chance of not smoking (these are our hypotheses). If they experience a heart attack how does that change your belief? We can use the likelihood of a heart attack (our data) from Figure 5.13 for death rates (this is a proxy). Plugging all of this information into Table 5.2 and applying Bayes' theorem gives us an updated posterior probability of 35% that this male is smoking.

This is a real world application of probability worthy of a QR course! Statistics can be infused throughout a QR course. Functions are a natural place to introduce descriptive statistics, and then introduce histograms and boxplots as part of the graphical representation of functions. A basic five-number summary is visualized in Figure 5.14.

Note that real world data sets can be massive—in this USAID dataset there are 74,369 observations and 757 variables. We can see that there are 5,141 missing values (or NA's). The code shown and the output come from the statistical software package R. Our students need practice with smaller, pre-cleaned data sets in spreadsheets before grappling with such messy real world data in command line driven environments like R.

Histograms are a fundamental way to visualize the distribution of our data. The percentage of people driving to work will be left skewed as shown in Figure 5.15, and we can imagine the distribution of the percentage of people walking to work will be right skewed.

A normal distribution is seen in the distribution of people per household in Figure 5.16, but there is a subtle shift in the vertical axis. Now instead of simple counts (or frequency) we have density, which means the area of each rectangle gives us the percentage of data values in a given bin. This is crucial to understanding the idea of area under the normal curve.

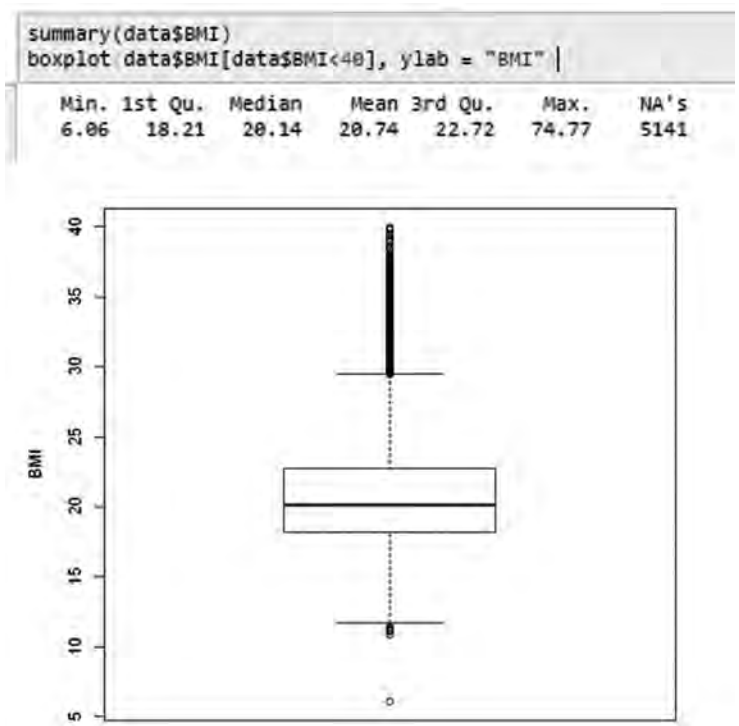


Figure 5.14. Summary and boxplot of USAID data showing BMI values. Source: `www.usaid.gov/results-and-data`.

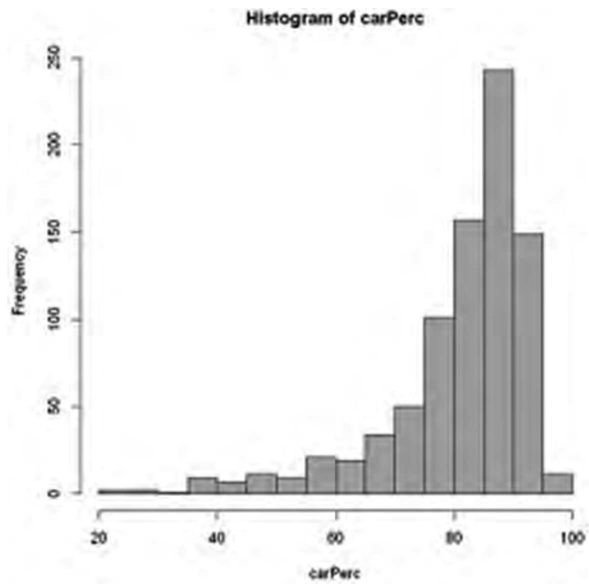


Figure 5.15. Histogram of percentage of people in OR census tracts driving car to work. Adapted from *Humanities Data in R: Exploring Networks, Geospatial Data, Images and Text* by Taylor Arnold and Lauren Tilton, Springer 2015.

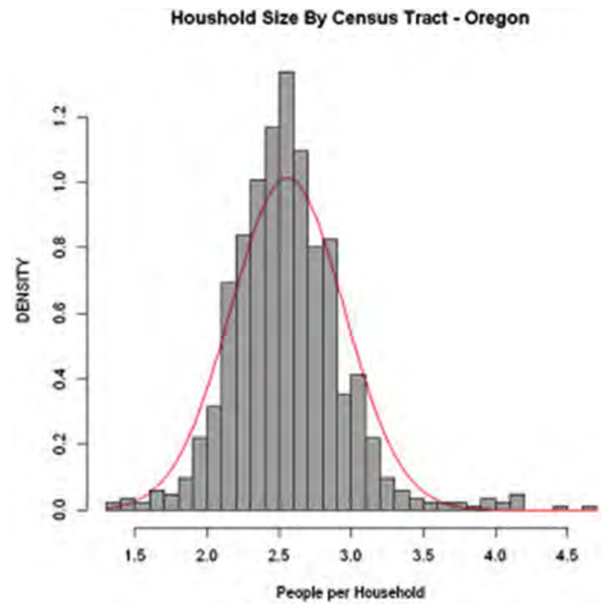


Figure 5.16. Histogram of people per household as density plot.

We can see a more obvious density plot by binning age data for an obese population data set by deciles. This forces the rectangles to each have an area of 10% as seen in Figure 5.17.

Normal distributions require a deep understanding of z-scores, so this topic should be found in QR courses. It fits perfectly within the proportional reasoning section, especially related to the notion of scales and units. All of these basic topics need to be rigorously developed in order for our students to understand more sophisticated graphics like the one shown in Figure 5.18.

In Figure 5.18, we see the medians for each univariate data set drawn in as dashed lines with the best fit trendline shown for the bivariate data. Schools in Maine are highlighted as dark circles. Interpreting the slope (0.48) and coeffi-

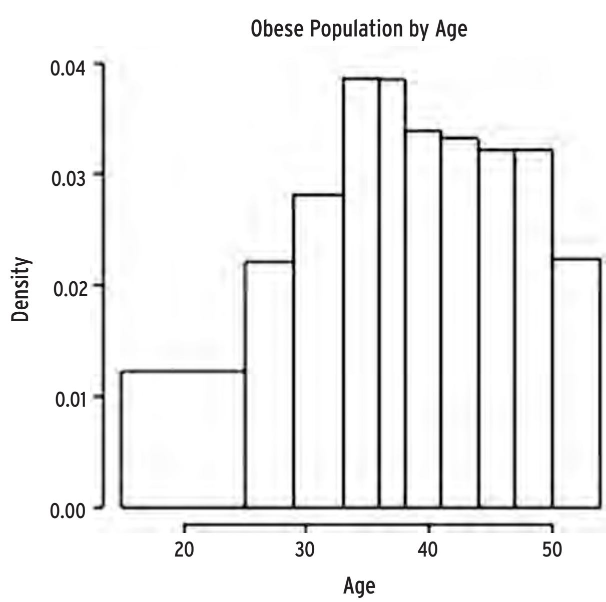


Figure 5.17. Histogram of age for an obese population subset.

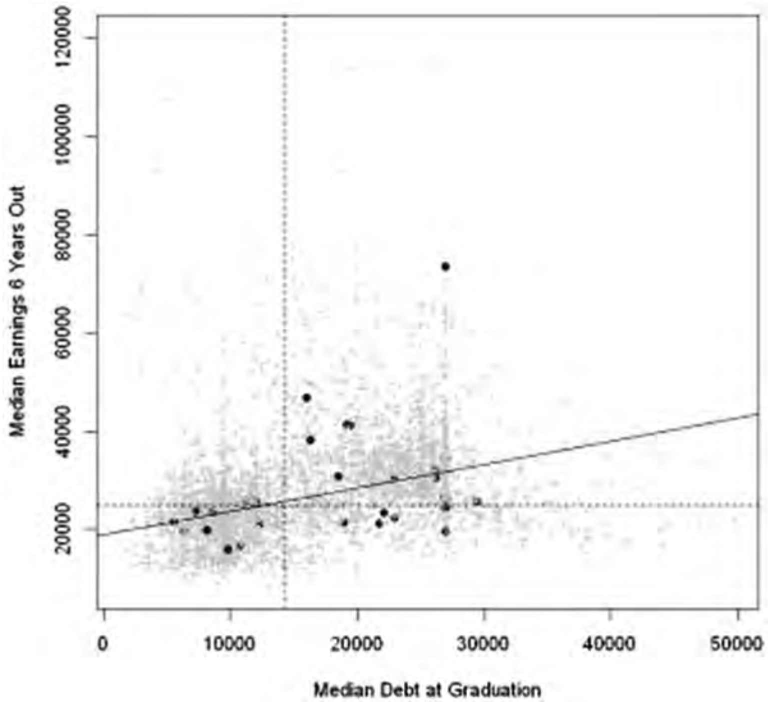


Figure 5.18. Scatterplot of earnings by debt for college. Source: ScoreCard data, available at `collegescorecard.ed.gov/data/`.

cient of linear determination ($r^2 = 0.16$) fall under basic statistical literacy. The data set here is large, containing 7,703 observations (post-secondary institutions) with 1,743 variables! The graphic was also created in R, and our students need to develop spreadsheet fluency in QR before graduating to such advanced data analytics. Inferential statistics can be explored in QR, but are more advanced and would require cutting back on other topics.

5.5 Conclusion

I have covered a myriad of topics in this chapter to give a sense of the mathematical richness of a QR course. Depending on the background preparation of your students you are going to most likely have to cover fewer topics and judiciously choose among many options. Covering the basics of proportional reasoning, with spreadsheet applications and some statistics mixed in, makes for a solid QR course. Covering linear functions is typical for many community college QR courses, while QR courses at many four-year institutions will additionally cover exponential functions along with some applications. Given that a QR course can be defined as a math class in which the context drives the content, our assessments need to be rethought from the traditional skills drills. Weekly in-class skills quizzes are important. As discussed above for the homework, students need to know how to find the equation of a line and compute percent change. But if our goal is to empower students to think for themselves and ask intelligent questions of experts, we need to give them practice in critical reading and analysis of arguments. A nice resource for such activities is found in the blog[2] pictured in Figure 5.19.

The Quantitative Literacy and Reasoning Assessment (QLRA) [9] instrument, pictured in Figure 5.20, is freely available.[3] There are over 100 schools as of 2016 administering this test, with many institutions assessing their entire incoming class over the summer before they arrive. This provides a nice "pre" data point for any outcomes "post" assessment of QR.

Finally, some concrete recommendations as you begin thinking about creating and teaching a QR course:

1. Carefully consider your comfort level with the different topics discussed here. Have you ever taught statistics?

[2] See `thinkingquantitatively.wordpress.com/`.
[3] Readers may contact Eric Gaze for more information: egaze@bowdoin.edu.

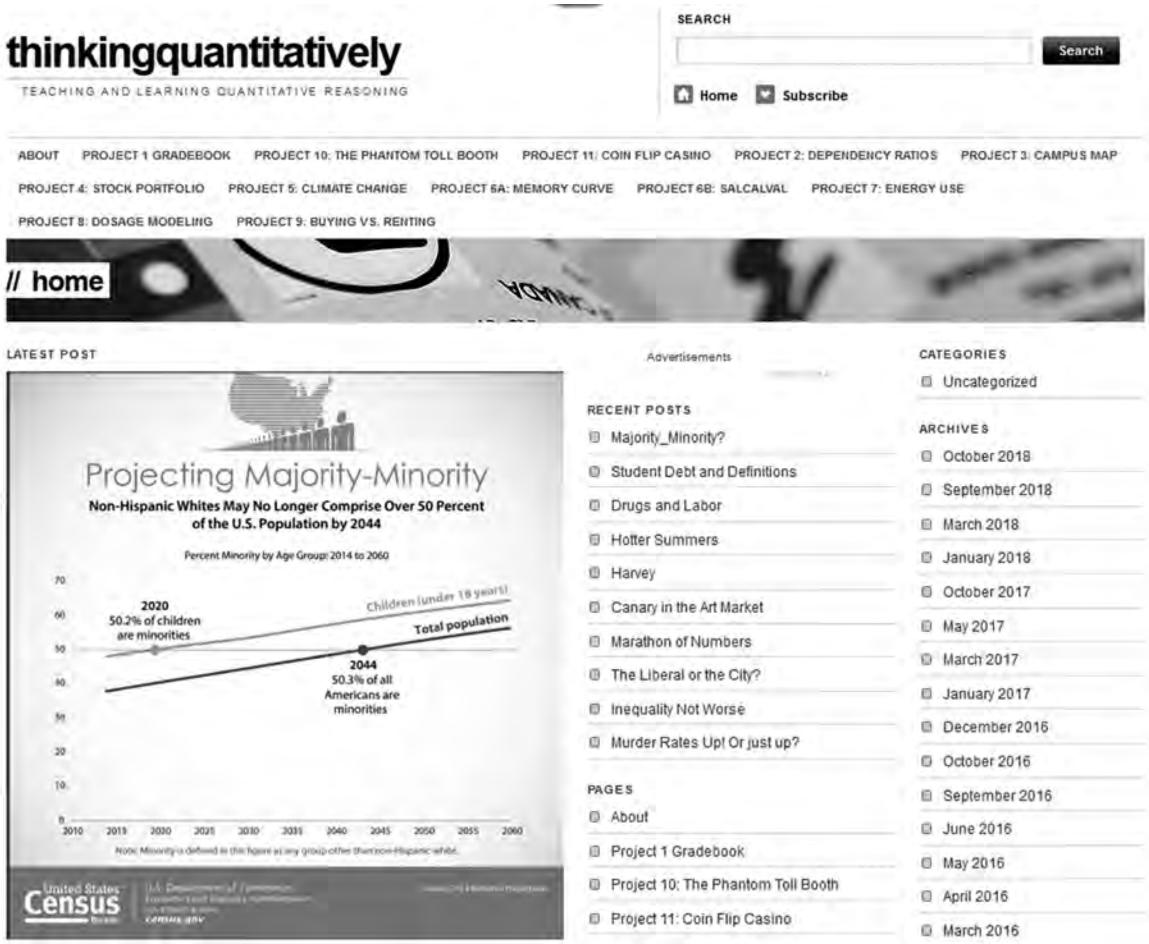
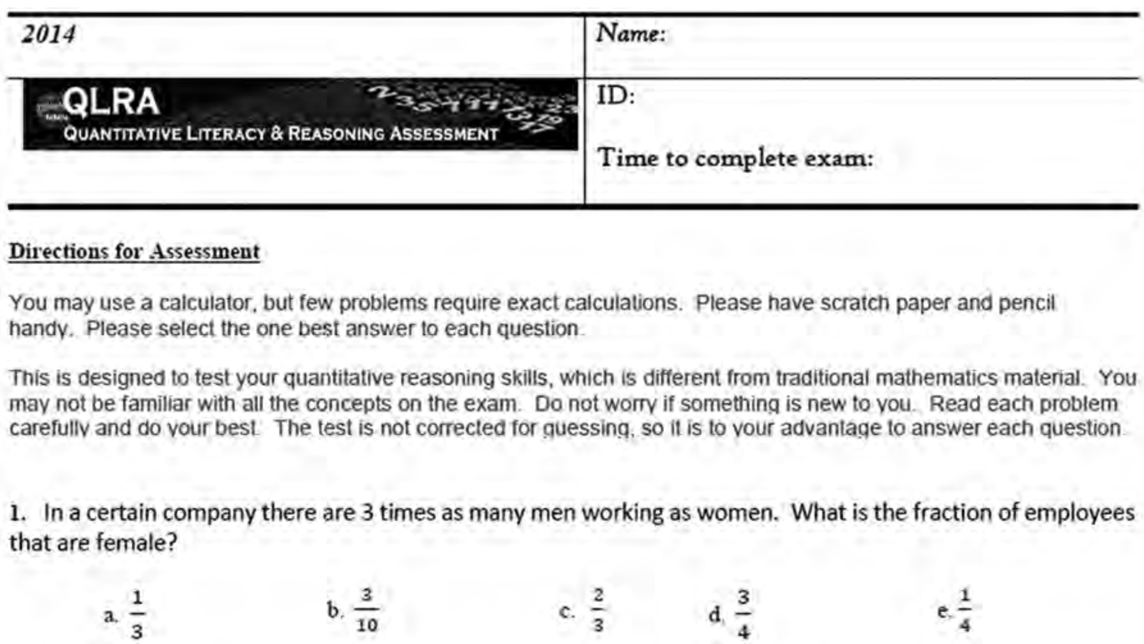


Figure 5.19. Thinking Quantitatively course blog.



2014	*Name:*
QLRA QUANTITATIVE LITERACY & REASONING ASSESSMENT	ID: Time to complete exam:

Directions for Assessment

You may use a calculator, but few problems require exact calculations. Please have scratch paper and pencil handy. Please select the one best answer to each question.

This is designed to test your quantitative reasoning skills, which is different from traditional mathematics material. You may not be familiar with all the concepts on the exam. Do not worry if something is new to you. Read each problem carefully and do your best. The test is not corrected for guessing, so it is to your advantage to answer each question.

1. In a certain company there are 3 times as many men working as women. What is the fraction of employees that are female?

a. $\frac{1}{3}$ b. $\frac{3}{10}$ c. $\frac{2}{3}$ d. $\frac{3}{4}$ e. $\frac{1}{4}$

Figure 5.20. QLRA instrument screenshot.

How savvy are you with financial information? Choose a manageable amount of material to cover and slowly build up your expertise.

2. Find topics that you are interested in and passionate about. Then craft new assignments using the given ones as templates. It is very important for you to be connected with the context, your enthusiasm will be contagious for your students! For example, you may not be interested in finance, and so focus on another topic instead.
3. Assess your comfort level with Excel. There is no need to assign huge amounts of Excel work, again choose manageable amounts for YOU, and then slowly build up your expertise and comfort level.
4. Be mindful of your student population. You may need to spend several weeks on ratio and proportion. Meet your students where they are! QR should empower our students.
5. Less is more. The homework problems and contextual spreadsheet activities take much more time than traditional homework. Cut back on what you assign. One well-run engaging project is worth far more than 4 hasty sloppy projects that students rush to complete with little understanding.
6. Enjoy your work! QR should be equally engaging for the instructor as it is for the students.

Best of luck.

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6

Quantitative Literacy in a First-Year Liberal Arts Seminar

Maria Fung
Worcester State University

6.1 Introduction

Based on a review of extant documents concerning quantitative literacy (QL), Karaali proposed a comprehensive definition of quantitative literacy as "Competence in interacting with myriad mathematical and statistical representations of the real world, in the contexts of daily life, work situations, and the civic life" [11, p. 25]. This framework positions quantitative literacy in an interdisciplinary realm, as an across-the-curriculum effort. Steen [20] calls on mathematics faculty to relate quantitative literacy to other "reform" movements such as first-year seminars and teacher preparation programs, the former of which I focus on here. In this article I discuss a way to incorporate quantitative literacy learning into first-year seminars through a series of projects and activities. This framework also agrees with Fisher's definition of QL presented in the opening chapter of this volume. These student explorations could easily be adapted by other faculty, both as a part of first-year seminars and as introductory or general education courses.

First-year seminars are courses designed to help new students adjust to college life both academically and socially. According to Goodman and Pascarella [8], first-year seminars are a high-impact practice—meaning they have been shown to be highly beneficial to students of all backgrounds and especially to underrepresented groups—that helps students' persistence and retention. Because nearly all four-year institutions in the United States now offer first-year seminars, there are different models for implementation of first-year seminars at different institutions, both in terms of structure and focus. Seminars vary in number of credits, content, pedagogical approaches and duration of class meetings, as well as in whether they are electives or requirements. Some seminars focus more on college adjustment issues (such as time management, study skills, academic planning and goals, campus resources), but a substantial number of seminars center around academic topics, ranging from interdisciplinary courses—for example focusing on Politics and Music—or discipline-specific ones, such as a first-year seminar for biology majors [12]. Those seminars in this latter group often have the goal of improving students' ability of critical inquiry, writing, and information literacy [13]. First-year seminars share the idea of instructor and students learning from each other in the context of a strong course and campus community. This interactive character and the goal of improving student literacy and communication position first-year seminars as an ideal venue for exploring topics in quantitative literacy. In this chapter, I describe student projects focused on quantitative literacy from three interdisciplinary first-year seminars that I designed and taught.

6.2 First-year Seminars at Worcester State University

Worcester State University (WSU), a comprehensive public urban university of about 5,500 undergraduates in New England with the mission of producing engaged and globally aware citizens, revised its general education curriculum about a decade ago. As part of this effort, all incoming students are now required to complete a three-credit first-year seminar in their first twenty-four college credits. These seminars are academic courses on a variety of topics, and they are typically more specific in scope and more integrative than general education or introductory major courses. A significant number of these seminars take on the liberal arts LC prefix (as opposed to department prefixes such as MA, EN, BIO for mathematics, English or biology courses, respectively) to reflect their unique place in the general education curriculum, where they do not fall within a departmental purview.

The first-year seminars at WSU are taught primarily by permanent faculty and staff from different disciplines. Like the Core 100 seminars offered at Xavier University and described by Schwab-McCoy in this volume, faculty from all departments are encouraged to design and to teach seminars on topics of interest that would engage first-year students in connecting academic content with college skills. There are approximately three-dozen first-year seminars offered every fall semester, spanning a variety of topics from the humanities, social sciences, and STEM fields. Some of the regular offerings include seminars such as Food in America, Holocaust through Literature and Film, A Healthy Body, Wild in New England, the Promise of the United Nations, Global Warming and Energy Sources, and The Color of Water (based on a book by this title).

The unifying feature of these WSU first-year courses is that they are all discussion-based, writing-intensive, and student-centered classes that often are interdisciplinary in nature, allowing for students to engage with the theme of the seminar from multiple perspectives. According to the general curriculum guidelines, these seminars, all capped at 20 students, share a three-part common goal of improving the students' critical thinking, communication skills, and information literacy. As pointed out in [3], there is significant overlap in student learning between these three goals and quantitative literacy. At WSU, a focus on quantitative literacy can meet all of the three-part learning goals of a first-year seminar. One way to ensure this is to have the first-year students collect, organize, and analyze statistical or numerical information from reputable sources (information literacy); next, critically examine the significance of their data (critical thinking); and then finally communicate their results (communication). This could be done on a limited basis or it could be a defining feature of each seminar. It seems reasonable that quantitative literacy could fit seamlessly into the first-year program at many institutions with a similar first-year seminar model or with an institutional structure that allows faculty freedom and flexibility in designing similar courses.

6.3 Three Seminars with a Quantitative Literacy Focus

I have offered three different interdisciplinary LC-prefixed first-year seminars with varying quantitative literacy emphasis at WSU since the fall of 2011, and I hope that others could use the ideas presented here to design their own quantitative literacy activities or courses. The first seminar was called Disturbing Times in Worcester and the World. The second was titled The Nature of Climate Change: A Quantitative Approach. The third one was What the Numbers Say. All three involved explorations of data sources (often using spreadsheets), bias and reliability of data collection, and the manipulation of results (including Excel spreadsheet work)—central topics for QL [7], among other possible approaches. These seminars had small groups of students—12, 19, and 20 respectively. A typical seminar class started with discussion of assigned readings, both in small groups and as a whole, often in a way similar to that described by Boersma, Diefenderfer, Dingman, and Madison in this volume. Then students worked on short projects and assignments, wrote about their findings and presented their ideas, both individually and in small groups. Every class concluded with a short debrief of the ideas from the day's explorations. Occasionally, I gave mini-lectures on topics from descriptive statistics, measurement, and probability. The mathematics topics in all three of the seminars were limited to descriptive statistics, elementary probability, units, percentages, proportions, lines of best fit, and normal distributions. In all three seminars students had to get familiar with new contexts from different disciplines, such as sociology, environmental science, or politics. That is, to use the ideas of Fisher from this volume, students engaged with ideas at the intersections of different communities of practice as they relate to quantitative thinking. This facet of the seminars might present an initial challenge for mathematics faculty anxious to cross boundaries into different disciplines. I believe an open mind, personal interest or passion for the theme in the seminar, collaboration with colleagues, and careful preparation is sufficient for navigating the interdisciplinary aspect of these seminars.

The Disturbing Times course had several themes running through it—globalization, population growth, human rights, international security, and environmental issues. (Using environmental issues is not new to QL—for instance, Baird, Nikbakht, Marland, and Palmer in this volume focus on the mathematics of sustainability as a driving force for quantitative literacy.) All of these themes were examined both locally (in Worcester) and globally (in the world). This model aligned well with the general education theme for that academic semester—Worcester in the World. The required texts for the course included *Disturbing Times: the State of the Planet and Its Possible Future* [5] , *What the Numbers Say* [15], and *Now or Never: Why We Must Act Now to End Climate Change and Create a Sustainable Future* [6].

The Climate Change course formed a learning community together with an honors English composition course. The students enrolled in both courses in consecutive time slots, and both classes shared the theme of climate change, a model similar to the one described in [17]. Students worked collaboratively in both courses on projects and presentations, engaging in problem-solving and peer-assessment frequently. This first-year seminar had two required readings: *The End of the Long Summer: Why We Must Remake Our Civilization to Survive on a Volatile Earth* [4], (The author is a local writer who met the students in person at the end of the semester), and *Climate Myths: The Campaign Against Climate Science* [1]. The English composition course used *Moral Ground: Ethical Action for a Planet in Peril* [14]. Several times during the semester, students attended lectures on environmental science and watched scientific documentaries related to climate change during the combined time blocks for the learning community. A joint final problem-and-solution essay on an environmental topic chosen by the students served as the capstone assignment for both the first-year seminar and the writing course.

The third seminar, What the Numbers Say, more narrowly focused on quantitative literacy content that follows the exposition in the two required readings: *What the Numbers Say: A Field Guide to Mastering Our Numerical World* [15] and *The Numbers Game: The Commonsense Guide to Understanding Numbers in the News, in Politics, and in Life* [2]. These two texts are recommended as supplementary by Madison in this volume, but in a first-year seminar centered on discussion and student projects, these texts provide common background knowledge and a starting point to allow students to begin explorations based on the main themes presented in the books.

The quantitative literacy focus in each of the three courses started with specific tasks, questions, or larger projects that the students considered in the context of the main themes in the seminar. For example, in the Disturbing Times in Worcester seminar, the students considered and quantified the problem of human trafficking both globally and in their own city. In the Climate Change seminar, students studied temperature changes in Worcester in the past 50 years. In the What the Numbers Say seminar students explored the impact of going meatless one day per week. These and similar problems arose from the readings, news sources, campus talks, YouTube videos, and student interest. In all cases, students worked collaboratively and individually to present their findings or solutions, both orally and in written form. Their work included collecting, organizing and summarizing data, discussing the reliability and potential bias of data sources, calculating and estimating with percentages and proportions, engaging with elementary probability theory (including conditional probability), and using modeling to make predictions. A basic overview of these assignments is in Table 6.1.

Below I first give suggestions how to use the seminar structure of these courses as a way to engage students with QL tasks, and then expand on some of the activities and projects from the What the Numbers Say seminar that could easily be adapted to other first-year seminars or general education mathematics courses.

6.4 Reading, Writing, and Discussion Ideas with a QL Emphasis

First-year seminars at WSU come with the expectation that students use reading, writing, and discussion as their primary learning tools. This is predicated on the assumption that comprehension, reflection, and communication skills are universal across disciplines and thus essential for college success. The texts used in my seminars are very different from mathematics books. It might be intimidating at first to lead class discussions and to create meaningful activities on topics that lie beyond the immediate expertise of mathematics faculty. However, attending training sessions for first-year seminar faculty and observing experienced colleagues are great first steps in getting prepared for this challenge. In addition, having a curious mind and deep interest in the themes of the seminar can translate into the creation of meaningful contextual QL projects. I share here some ideas that have worked successfully for me in facilitating a

Table 6.1. Overview of core assignments in the three first-year seminars.

Course	Assignment Overview
Disturbing Times in Worcester and the World	For the activity on human trafficking, students focus on the acts, the means, and the purpose of trafficking, and then work to compare world statistics to what is happening in Massachusetts.[1] Students ultimately consider the question, What could we do to prevent human trafficking? In taking on that question, there is an expectation that students will use quantitative reasoning in their writing.
Climate Change	For the Temperatures in Worcester Activity, students (among other things) • collect average temperature data from the last 50 years, • compare the increase in the first 25 versus the last 25 years, and • compare averages and standard deviation in average temperature. They then use their data and findings to write a memo to a climate-change doubter—a task that truly highlights the notion of intersecting communities in Fisher's QL definition.
What the Numbers Say	For the Meatless Mondays Project, students start out by watching *Meat the Truth* [19]. They then write an essay about the environmental impact of eating meat, considering prompts such as: • Estimate the saving of money and resources if one person decided to eat a vegetarian diet once a week • Estimate how—if everyone in the world did this—the changes would impact our entire planet.

first-year seminar, through reading, writing, and discussion assignments, with an emphasis on QL.

In What the Numbers Say, I assigned students daily readings from the course texts. I found it important to vary the mechanism of ensuring these readings were completed in a thorough and efficient manner, since they were essential for mastering the concepts of the course and as a preparation for more complex in-class activities and projects. Writing is a central part of the WSU first-year seminars and it must be included in as many ways and contexts as possible. Students write in a variety of styles and for different audiences, both as a way to learn or reflect on the material and to prepare for all types of college writing. Discussion of ideas was one the main pedagogical tools used in my first-year seminars.

Below are some suggestions that I found successful in helping students develop their reading, writing skills, and discussion skills as related to QL.

Motivate Reading a Chapter for Homework with an In-class Activity

For example, I had my students spend some time thinking individually about what what could be counted (e.g., manageable sets of things such as the number of pair of shoes one owns, the number of times one sneezes per day) and what could be counted precisely (e.g., the population of the world at any given moment or the U.S. debt), as well as how to estimate a quantity with a certain accuracy. Then they shared their thoughts in small groups. Finally we created a list of all of their ideas as a lead-in to the chapter in *The Numbers Game* on counting and estimation. In that same class period, students completed a warm-up activity related to counting how many donuts fit in a large box, using only several partly obstructed images of the box [23]. My students also estimated how many melons will fill the White House. The discussion involved an emphasis on assumptions (e.g., the volume of an "average" melon; how furniture is accounted for; means of slicing the melons for fitting in tight spaces) and results (e.g., how do we know our estimate is close enough?). The benefit of this approach was to put the assigned reading in perspective, so students see it as an essential part of their learning in the course.

[1] Students visit `www.unodc.org/unodc/en/human-trafficking/global-report-on-trafficking-in-persons.html` for information about human trafficking, and `humantraffickinghotline.org/state/massachusetts` for Massachusetts data.

Designing Appropriate Follow-up Activities to Assigned Readings

For instance, I had students develop their own examples of how frequentist thinking could be detrimental in probability theory. For a different chapter, students had to create their own data sets where one of the measures of center is superior to the rest, and then describe what an "average" WSU student might look like. On another occasion, I had my students create their own estimation problems involving small and large numbers, with the caveat that they needed to present careful solutions, stating each and every one of their assumptions, and paying close attention to units. Occasionally, my students had to create their own discussion and follow-up questions and find outside sources related to the reading; these questions were then used heavily in class as a starting point of the in-class group discussion. These strategies ensured my students were doing the readings with a careful eye and a critical mind and they came to class prepared to take part in the activities for the day. These strategies also provided me with a solid organizational structure for my classes.

Create Interesting Writing Assignments Based on the Readings

At the beginning of the course, students typically summarized their readings using bulleted outlines and created discussion questions. Later, they had to write a letter to a relative who is not versed in elementary probability, or an opinion piece on using numbers without a perspective for the *New York Times*, or a memo to the Provost arguing that every student at WSU must read a chapter in the text dealing with normal curves, randomness and the difference between correlation and causation. For the final chapter in *What the Numbers Say: A Field Guide to Mastering Our Numerical World* my students wrote an entry for a mathematics education blog summarizing the Math Wars [18] and expressing their opinion on the role of QL relative to mathematics education today.

Tap Into Campus Events and Talks

For example, at the 2016 Sustainability Fair, students wanted a series of short films on food from all over the world. Each of these films raised interesting questions about numeric data associated with food production. As part of both class time and homework, my students attended talks at the annual fall Sustainability Fair, and a lecture by a prominent Latina educator. These events are critical in introducing first-year students to college life and to the university community. I have found that many general campus events could—in some way or another—relate to our course focus on quantitative literacy. In deciding on the appropriateness of a given event, I considered questions like: Was there quantitative information presented at these talks? Where did the numbers come from? What purpose did they serve? Could we find connection to the course readings? My students had to answer those questions after attending these events in short reflection papers.

Create Discussion Opportunities through Group Work

Occasionally, my students struggled with some of the QL ideas presented in the text. They also needed continued practice in communicating quantitatively. Group discussion activities worked well for improving their understanding of the main concepts in the course. For example, students were tasked to work in groups to prepare a short presentation on a particular topic such as Bayes' theorem and became experts on it. Then, all but one moved to another group and shared their knowledge with the new group, taking turns. In a different session, students made posters on specific topics in descriptive statistics and then assessed their classmates' work with a rubric they themselves designed. In yet another class, students used their knowledge of percentages and their limitations and presented their thoughts in a very short PechaKucha-style presentation [16]. At the end of the course, students even created their own videos for the next incoming group of first-year students encouraging them to consider What the Numbers Say, focusing on the main QL topics they learned from the readings. On several occasions throughout, students took part in a seminar (one such example was a seminar on surveys), structured as a whole class discussion without a script [9].

The ideas presented here for developing students' reading, writing, and discussion skills mainly pertain to seminar-style courses, but they could also be adapted to other mathematics or statistics courses that use contextual learning for developing quantitative literacy skills. Next I focus on three general contexts for QL that allow for more complex projects and easily extend beyond the scope of first-year seminars into the general mathematics curriculum—risk and choices, population, and environmental topics.

6.5 Risk and Choices

Creating arguments that use quantitative information could form a significant part of any first-year seminar like the ones offered at WSU. In particular, the topics of risk and choices are general enough to be used in many different contexts. In What the Numbers Say, my students were introduced to the concept of conditional probability and relative risk through their readings. As a follow-up, they had to find and to read at least two popular and two research articles on the risks of eating red meat relative to cancer risk. Their goal was to create arguments in favor of and against eating red meat that are informed by research and supported by data (see also [21]). Some questions they discussed in small groups were: What is the differences between these two types of sources? How can we tell if a study is conclusive? What are the limitations of these studies? How do we make an informed decision regarding red meat consumption? For homework, students used a similar procedure to decide whether using cell phones increases their risk of cancer and by how much, or whether drinking more than a certain number of cups of coffee per day is detrimental to their health. Students shared their essays with each other and several students reported out to the entire class. This was followed by a whole-class discussion of an article discussing the truthfulness of findings in many research articles [10]. One of the recurring themes in the course, presented in the second chapter of Niederman and Boyum's book, was the idea that quantitatively literate citizens both only trust the numbers and never trust the numbers, depending on context and details—these risk activities were instrumental in illustrating this dualist idea, and are common in other QL courses such as those at Michigan State University (see [22]).

6.6 Explorations Related to Population

As Gaze discusses in this volume, modeling with spreadsheets is an important way of engaging students with real-life data. In What the Numbers Say, looking at problems related to population and its growth worked well for this goal. Groups of students considered different estimates for the population of Worcester, Massachusetts, the United States, and the world. They modeled population change using different trend functions in Excel (e.g., linear, exponential, logarithmic) and forecasting how the population would change according to the different models. Which models appear more accurate? What are features that these simplistic models miss? Interestingly, Worcester's population has not recovered from a sharp decline in the early 80s. There were always lively discussions about the difficulties in accounting for every person in initiatives such as the U.S. Census, and in calculating the precise birth and death rates. It is intriguing to note the discrepancies in numbers for the world population even from reputable sources.[2] Students think, discuss, and write about how these differences can be explained and accounted for.

Another population project involved looking at U.S. Census data from Worcester in 2000 and 2010. The first part of this assignment required an explicit discussion of any changes that could be observed in this time period, with the appropriate numerical measures such as percentage growth. Students had to use appropriate graphs and charts to show their comparisons. The second part of the assignment asked students to find data about racial and ethnic distribution of students in the Worcester public schools from the Massachusetts Department of Education website. I posed a number of questions, including: Is there a discrepancy between the two sets of population data? How can the discrepancy be explained or resolved? Can you support your claims with some further research? At the heart of this project was the fact that hispanics, a large group in Worcester, often self-report as caucasian on the Census, thus skewing the numbers of whites.

Population considerations most definitely could include a global component. Nigeria, for instance, is the seventh largest nation in the world in terms of population, and the largest nation in Africa. The reasoning that accompanies QL often requires familiarity with a specific social context (as described by Fisher in this volume). Thus a project focused on creating a plan for reducing the population growth of Nigeria provided students with the opportunity to learn about the history, culture, government and tax structure of this populous country. This prerequisite knowledge helped students analyze the hypothetical impact that initiatives such as improved education, better tax policies, and ample donations of birth control from the United States might bring within a decade, fifty years, and even a century. This topic was also important for my students to consider since it naturally led to questions about whether the U.S. should interfere in other countries' affairs and under what circumstances, whether birth control is an acceptable means

[2] For example two such sources are `www.census.gov/popclock/` and `worldpopulationhistory.org/map/2017/mercator/1/0/25/`.

for reducing population growth given one's religious beliefs, and whether knowing the state of the world's population might change people's minds about how many children they choose to have. I believe we should not be afraid to wade in political, ethical, and sociological waters while providing QL context for our students—in fact these moments give us opportunities to show students how to respect different perspectives and look for a way to work together in spite of our disagreements. All the essays on the Nigerian population reduction programs focused on different measures, but shared good predictive numerical calculations about the cost of their plans and the resulting change in the population growth.

Topics in population growth and change (whether the population as a whole, or specific groups) is a topic that could generate an inexhaustible series of projects and questions for students in the QL classroom. In addition to the practice with modeling, calculating, and predicting, students can enhance their perspective on the world and its many cultures.

6.7 Environmental Impact Projects

Similarly to population projects, the environmental impact of climate change could provide instructors with many options for QL projects. Below are three examples that only begin to scratch the surface on the possibilities of student exploration, which is discussed in detail by Baird, Nikbakht, Marland, and Palmer in this volume.

One of the more significant data projects in The Nature of Climate Change and What the Numbers Say involved collecting and analyzing climate data on Worcester, MA. Students used a weather website[3] to collect the average monthly temperatures in Worcester (gathered at the regional airport) from the past 50 years. Then they obtained yearly average temperatures for Worcester from the past 50 years. After entering their data in an Excel spreadsheet, students generated a time-series graph that they then analyzed for trends. The next step was to construct a scatterplot with straight lines and markers of the average yearly temperatures, and then to find the line of best fit for their scatterplot. They observed that the slope of the trend-line was positive, so the temperature has been steadily increasing. The second part of the project required the entire data to be split up into two separate groups: one representing the first 25 years, and one for the last 25 years. Then the analysis was repeated. Students discovered that the increase in the last 25 years is much more significant. The third part of the project involved calculating the standard deviations in all three cases, the goal being to observe that in recent years, there is more spread in the temperatures. The implication of such increased standard deviation—as students noted—was that the frequency of more extreme temperatures observed in the more recent time period was higher. The final part of the project required the students to summarize their results in a letter written to a climate change doubter.

Just as in the case of population projects, environmental projects can be considered globally. China, the most populous nation on earth, has a huge environmental impact through its use of fossil fuels and other materials that produce greenhouse gases, economic development, water/food resources, green energy, and population change. Students researched and collected data related to all of these environmental aspects for an outside of class project. A starting resource, for example, is the think tank, Council on Foreign Relations. [4] Then students were asked to produce different charts and graphs that make the Chinese impact first seem non-essential, especially given their new policies and leadership in renewable energy, and then very significant. This easily leads to a discussion of deliberate misinformation in the media.

As pointed out by Schwab-McCoy in this volume, YouTube videos can serve as an excellent means of incorporating context in the QL classroom. Based on an in-class discussion on food and sustainability, one of my students recommended the documentary *Meat the Truth* [19]. After watching the film outside of class, my students completed essays titled "Meatless Fridays: Are They Worth It?" In their essays, many showed—using quantitative reasoning—the impact a certain proportion of the population choosing not to eat meal once a week would have on the environment, as well as on the meat industry.

These environmental impact projects support the common goal of many U.S. universities of producing informed and competent citizens of the world, readily prepared to navigate today's complex problems with thoughtfulness. I am happy to share any of these projects, as well as course syllabi, with interested readers.[5]

[3]Students visit `www.wunderground.com/history` for historical weather information.
[4]See `www.cfr.org/china/chinas-environmental-crisis/p12608`.
[5]The interested reader may contact me at mfung@worcester.edu.

6.8 A Quantitative Literacy Module in Each Seminar or Introductory General Education Mathematics Course

During the fall of 2016, there were forty-four first-year seminars offered at Worcester State University on over twenty different topics (several seminars offered more than one section). Each of these topics was general enough to allow for a quantitative literacy module (a series of projects or activities aimed at supporting QL)—one that could focus on any of the three threads discussed in this paper—or on some additional topics. For example, there were several seminars at WSU focused on global issues such as health, poverty, and literacy. All of these lend themselves beautifully to exploring data and its limitations. Moving beyond first-year seminars, general education courses from the mathematics department, including introductory statistics or college algebra, can—with advanced planning and thought—accommodate several of the activities described in this manuscript.

Typically, institutions with a first-year program offer yearly training sessions for first-year seminar faculty instructors. These sessions would be a suitable venue for experienced QL mathematics instructors to aid beginners in creating these modules. Nearly all seminars and all general education introductory mathematics courses operate in complex contexts that allow for a quantitative perspective. Having students explore quantitative data, probability, measurement and estimation, and model creation and prediction, helps them to develop the invaluable quantitative habits of mind to navigate today's world.

First-year liberal arts seminars provide a natural opportunity for developing students' quantitative literacy skills. Topics related to number, statistics, probability or modeling, either embedded as stand-alone modules or as a thematic feature of an entire course, can enhance first-year students' understanding of quantitative information, while simultaneously developing the necessary literacy skills for the 21st century.

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7

"Life in the Data Deluge"

Developing Quantitative Fluency in a First-Year Seminar Course

Aimee Schwab-McCoy
Creighton University

7.1 Introduction

The presence of data science, mathematics, and statistics in mainstream media and popular culture is skyrocketing. In 2009, Hal Varian famously wrote in *The McKinsey Quarterly*, "I keep saying the sexy job in the next ten years will be statisticians. People think I'm joking, but who would've guessed that computer engineers would've been the sexy job of the 1990s?" The growth in popularity of statistics and data science—from late night talk shows to YouTube videos—has allowed statistics and the broader mathematical sciences to enjoy some lighthearted exposure. Today's students are constantly surrounded by data and numbers, and the growing exposure means a growing demand for quantitative skills.

The explosion of attention to data, numbers, and mathematics in traditional and online media presents new opportunities and challenges to quantitative literacy instructors. Specifically, how can increased media attention to issues of scientific research and quantitative literacy be leveraged to generate student interest and make connections beyond the classroom? Moreover, what skills are really necessary beyond the quantitative literacy classroom? Many quantitative literacy courses often stop just short of "conversational" quantitative literacy—developing the skills and competencies required to speak and communicate effectively using quantitative evidence. Like learning a new language, the skills required to be quantitatively literate can be viewed as part of a hierarchy. Conversational quantitative literacy, or quantitative fluency, may be viewed as a next step in this progression.

In this chapter, I will describe the importance of developing quantitative fluency in undergraduate students and the growing inclusion of mainstream media in mathematics courses, followed by the exploration of a first-year seminar course designed to address quantitative fluency and statistical literacy in the age of "big data." The course, Core 100: Life in the Data Deluge, focuses on readings and classroom discussions rather than traditional lecture, and eschews traditional learning materials such as the textbook. The course also introduces students to complex issues of data generation, data analysis and computation, and the building of effective quantitative arguments through active class discussion and readings selected from non-traditional outlets. This chapter will also highlight possible techniques for encouraging quantitative discussions in our courses. While the seminar format may not be transferrable to other settings, many of the lessons and strategies discussed can be incorporated into or adapted for a quantitative literacy classroom.

7.2 Quantitative Fluency and Quantitative Literacy

Earlier in this book, Fisher proposed the following quantitative literacy (QL) definition: "Quantitative literacy is the facility to participate in the intersecting quantitative practices of many different communities (each with its own patterns of discourse)." In language learning, literacy leads to fluency—an innate comfort with a language and with social constructs surrounding it. Students may learn to engage with numbers, graphs, charts, etc. as they develop quantitative skills in different communities, but for many there is still some inherent discomfort. As students become more confident and comfortable reasoning with quantitative evidence, they move from being quantitatively literate to quantitatively fluent. Quantitative fluency, the ability to speak naturally, conversationally, and extemporaneously about quantitative evidence, has two core elements. First, in order for a student to be quantitatively fluent, they must be also quantitatively literate. They must be able to perform and understand the essential quantitative mechanics, read and understand a piece of quantitative evidence, and describe that quantitative evidence clearly and cohesively. To achieve quantitative fluency, a student should be inherently comfortable with quantitative evidence, just as a language learner must achieve a level of comfort and confidence with their skills in the language to speak freely. Students should be able to use quantitative evidence in regular conversation and explain such evidence verbally to their colleagues and peers. Additionally, the truly quantitatively fluent can speak about quantitative evidence freely and quickly, without much rehearsal or refinement. Students who are quantitatively literate can speak about numbers "on the spot," in a clear and conversational manner. Quantitative fluency requires a degree of comfort with mathematical skills that is even higher than quantitative literacy—for spoken communication there is rarely a template or a list of questions for students to answer.

Quantitative fluency may be considered the next step in developing quantitatively literate students. The development of quantitative literacy skills can be viewed as a cycle, as in Figure 7.1. First, students learn the "grammar" of a new quantitative skill, and develop the mathematical competencies necessary to handle a new calculation or problem. Once they know the mechanics, they can develop the "reading" skills necessary to recognize a particular class of mathematical problem and identify the necessary components of the calculation. The next step in the cycle is "writing": this is when students begin to explain their solution using written mathematical evidence, or begin to use quantitative evidence in an argument. In a typical quantitative literacy course, this often happens through summative or formative assessments such as quizzes, homework assignments, projects, and exams. The final step to developing quantitative fluency is the "speaking" stage. At this point, students have developed the prerequisite skills and knowledge to per-

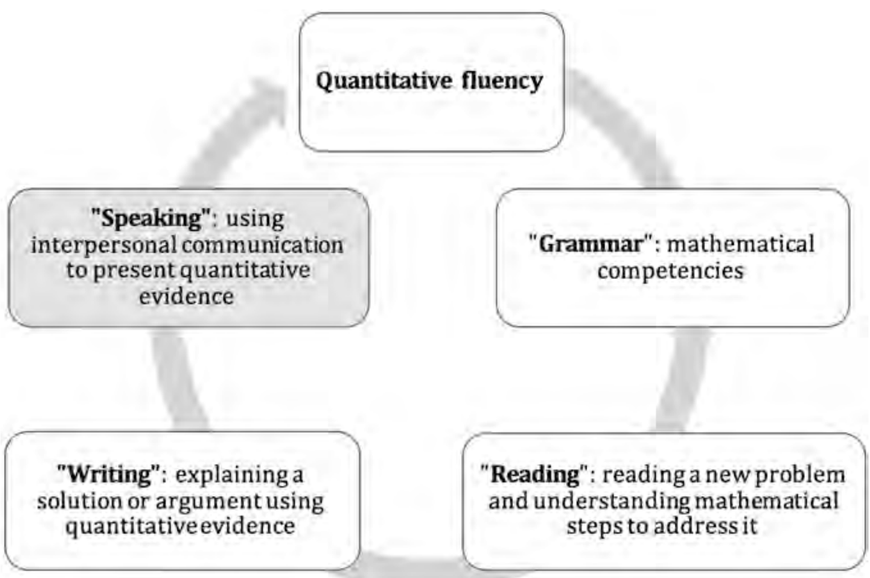

Figure 7.1. The quantitative fluency cycle. Like learning a language, there are a series of steps to developing quantitative communication skills. Quantitative fluency can be thought of as the "speaking" phase of learning a new language: at this point, students become comfortable using interpersonal, informal communication to present and discuss quantitative evidence.

form the quantitative exercise and write about it, and are now beginning to gain informal communication skills using quantitative evidence.

The essential idea of quantitative fluency is not new. In a 2014 paper, Madison [14] outlined six learning objectives on a Quantitative Literacy Assessment Rubric for evaluating student progress in quantitative literacy. Those learning objectives were:

1. Interpretation: Ability to glean and explain mathematical information presented in various forms (e.g., equations, graphs, diagrams, tables, words).
2. Representation: Ability to convert information from one mathematical form (e.g., equations, graphs, diagrams, tables, words) into another.
3. Calculation: Ability to perform arithmetical and mathematical calculations.
4. Analysis/Synthesis: Ability to make and draw conclusions based on quantitative analysis.
5. Assumptions: Ability to make and evaluate important assumptions in estimation, modeling, and data analysis.
6. Communication: Ability to explain thoughts and processes in terms of what evidence is used, how it is organized, presented, and contextualized.

In a typical quantitative literacy course, these learning objectives might be assessed through in-class activities, homework assignments, quizzes, or exams. However, informal communications and discussions are still rare in many mathematics classrooms. A grounding in quantitative skills is vital for creating informed citizens in today's politically charged climate [28], and data provides an unbiased lens for separating fact from fiction. Interactions with quantitative material after a college education do not come in the form of structured activities or designed assessments, but through media reports, Facebook posts, and conversations with friends and colleagues (see Craig, Mehta, and Howard in this volume for a more detailed discussion of quantitative literacies in the digital age). In order to equip students to discuss quantitative evidence comfortably, we should begin to practice those skills in the classroom.

7.3 Mainstream Media in the Quantitative Literacy Classroom

As a community, educators of quantitative literacy have made strong progress in incorporating examples from the mainstream media and popular culture into our classrooms (see Boersma, Diefenderfer, Dingman, and Madison in this volume). The rationale for doing so is clear: mainstream media and popular culture are ubiquitous in the lives of our students, and by including them in our classes we can help them develop the necessary quantitative literacy and fluency skills in a more natural environment. Zillmann, Callison, and Gibson showed that individuals with low quantitative literacy skills were less likely to accurately recall or interpret numbers in printed news reports [30]. In New Zealand, Budgett and Pfannkuch advocated for using media reports to motivate statistical concepts in undergraduate statistical literacy classes [3]. In fact, in response to the call for increased media representation, Madison, Boersma, Diefenderfer, and Dingman have written a text, *Case Studies for Quantitative Reasoning: A Casebook of Media Articles*, specifically designed for teaching quantitative literacy courses [12].

News reports are not the only media resource being used in our classrooms. The *New York Times* bestseller *Freakanomics*, for example, has been used in statistics courses to illustrate concepts and help strengthen statistical literacy skills through the use of online discussion assignments [27]. A course at Whittier College, "Numb3rs in Lett3rs and Fi1ms," uses popular movies such as *A Beautiful Mind* or novels like *The Oxford Murders* to teach mathematical concepts such as logic, geometry, and modular arithmetic in an interdisciplinary setting [4]. Even popular cartoons like *Futurama* have been used in mathematics courses with positive student response [7]. Clearly, mainstream media is not new to the mathematics classroom.

Media resources such as news articles, video clips, and blog posts can add an extra dimension to context in the quantitative literacy classroom. Social context is an important consideration in quantitative literacy courses; a discussion of the source and quality of quantitative evidence should go hand in hand with the evidence itself. For example, the implications of choice in data collection—what variable to measure, how to record the data—reach beyond just analysis of data, but into interpretation and the consequences of any statistical or quantitative study [2, 21]. This extra dimension can be leveraged into increased student learning. Henrich explored service learning in a quantitative literacy course, "Quantitative Literacy and Social Justice," and found evidence that the service-learning component improved student attitudes toward mathematics [9]. Issues of social construction are particularly important in our politically contentious atmosphere, and can lead to heated debates in the classroom. Struggling with the social construction of quantitative evidence is an important part of the learning process [21], and challenges students to be more comfortable using quantitative evidence in conversation to support a potentially contentious argument. This struggle is key to becoming quantitatively fluent.

In many cases, students have shown a favorable reaction to using examples from mainstream media and popular culture in a traditional quantitative literacy or statistical literacy classroom [27, 4, 8]. A recent trend in undergraduate education has been the movement toward integrating seminar-style courses into the undergraduate curriculum [10]. Freshmen seminars are perceived by many institutions, especially small liberal arts colleges, as a method to build deeper communities within the student body, encourage closer relationships between students and faculty, and help students develop stronger critical thinking skills early in their academic careers. Seminar-style courses have shown some promise, especially in encouraging student persistence and pursuit of science, technology, engineering, and mathematics degrees [5, 29], and have been designated as a "high-impact" practice by the American Association of Colleges and Universities. Seminar-style courses are intentionally very different from the traditional quantitative literacy classroom. Students are expected to actively contribute to daily discussion of challenging texts, and take an ownership role in their own learning and progress. The emphasis on spoken communication makes the seminar course environment an ideal space to begin developing quantitative fluency skills.

7.4 Core 100: Life in the Data Deluge

Xavier University, a Jesuit Catholic university in the midwestern United States, introduced a first-year seminar (FYS) program in 2015. In the previous chapter, Fung discusses specific advantages of first-year seminar programs in more detail and in her context at Worcester State University. All first-year and transfer students attending Xavier are required to take an FYS course, designated Core 100, during their first two semesters at the university. The course is designed to encourage deeper thought and connection to the "greater good" through careful reading and discussion of challenging texts. All Xavier University faculty are welcome and encouraged to submit course proposals for Core 100, and may choose any topic regardless of their academic field of study (similar to the first-year seminar program at Worcester State University). Life in the Data Deluge was designed to expose students to both the quantitative side and the social side of "big data" through the seminar course format [22]. The course first ran in fall 2016 with a cohort of 14 students: 13 freshmen and 1 sophomore.

Life in the Data Deluge was designed with pedagogical values from quantitative literacy, statistical literacy, and quantitative fluency in mind. Students were not exposed to quantitative material through lecture at any point in the course, unless they specifically asked for clarification on a mathematical topic. This does not mean that the course did not address quantitative concepts—it did. Rather, quantitative material was addressed through sample problems, reading questions, and the course readings.

This course also differed from a traditional quantitative literacy course in the readings and supplemental materials assigned. Students read chapters from three "pop-statistics" books about applications of big data and its future implications: *Big Data* [16], T*he Signal and the Noise* [23], and *Dataclysm* [19]. Chapters from these books were supplemented with data-driven news articles and blog posts, interviews, TED Talks, and YouTube clips to engage students in the quantitative content. Course readings were carefully selected to discuss a variety of quantitative literacy skills such as interpreting proportional data, reading prediction maps, and describing information from complex survey tables. Important statistical ideas were also emphasized at a conceptual level, such as algorithms, prediction, testing, and interval estimation through the lens of big data.

Course readings and other learning materials were primarily selected from popular books, blog posts, and news articles. Special attention was paid to the quantitative content of the reading material. All readings chosen made significant use of quantitative or statistical evidence. Readings were also selected to expose students to a variety of contexts and quantitative applications. Topics discussed during the course included racism and stereotypes in dating, interpersonal communication, sports analytics, gambling, data privacy and safety, and the role of "big data" techniques in retail and advertising. When possible, readings were chosen that were written by data scientists or quantitative analysts in order to provide an appropriate technical grounding. These chapters and articles modeled what effective quantitative communication should look like. For example, supporting a quantitative argument with tables, figures, or other evidence. Technical concepts in these texts were communicated in plain language, an essential element of quantitatively fluent communication. The primary goal for this course was to strengthen students' quantitative literacy and fluency through examples. Students learned what a strong quantitative argument looks like and how to effectively support their conclusions using quantitative material.

Many of today's college students come from the "YouTube Generation" [6] and have spent their entire lives with short videos available to them immediately just about anywhere in the world, and have come to expect such media in their classrooms. While videos are certainly not necessary for incorporating context into the quantitative literacy classroom, they are an important communication venue in today's world. Moreover, the target audience for the typical YouTube video is not a mathematician (although there certainly are such YouTube channels), but our students! These videos typically assume a basic quantitative background, and provide a great model for quantitatively fluent conversation. Due to the nature of the first-year seminar, short YouTube clips and other videos often supplemented the quantitative material in the reading, either to provide more context or further illustrate a quantitative idea in a more engaging way. Interested readers are directed to the appendix for a list of sample video resources.[1]

7.5 Lessons in Quantitative Fluency Using Popular Media

The incorporation of media resources into a quantitative literacy classroom, and the development of quantitative fluency skills, should be thoughtful and deliberate. Students often enjoy reading popular books or watching videos in the classroom, but instructors must take care not to use popular media just for the sake of entertainment. Book chapters, videos, and news articles used in class should be targeted toward a particular lesson plan or learning objective. In today's pop culture landscape, this is a much easier task than ever before. Online media has exploded, and quantitative literacy instructors have countless possible candidates for inclusion in their courses. The question is not where to find media resources, but how to develop lesson plans that effectively use mainstream media to deepen student engagement and develop quantitative literacy and fluency skills. Two possible strategies are to encourage active discussion and writing based on the media resource, and to explicitly connect the media resource to classroom material and student learning outcomes.

7.5.1 Encouraging Active Discussion and Writing

In 1999, Steen wrote: "The test of quantitative literacy, as of verbal literacy, is whether a person naturally uses appropriate skills in many contexts" [28]. Reading news articles or book chapters that use quantitative skills to explore an interesting topic is just part of the equation. Using active discussion and writing assessments can engage students more deeply in the course material and expose them to different ideas or interpretations. More importantly, students gain practice supporting their ideas through discussions using quantitative arguments or evidence. Active discussion helps students become more quantitatively fluent and writing assessments help develop both informal and formal communication skills using quantitative ideas.

Active Discussion in a Seminar Course

In this course, active discussion was the biggest component of class time. Most mathematics and statistics instructors have probably tried to encourage discussion in their courses at one time or another, and would agree that this can be a lofty goal. Class discussions were typically based on a set of pre-discussion questions or an in-class activity. Pre-discussion questions were assigned before every course meeting, and took the place of traditional homework assignments. The most effective discussion questions were open-ended, and encouraged students to take notes on what they had read and reflect on possible points for discussion. Discussion questions often included a short task such as gathering some data from their peers, collecting data from political polls, or interpreting a new quantitative artifact such as a map or table connected to the reading. These prompts also included a reminder that opinions should be supported with some quantitative figures from the reading assignment. Table 7.1 includes some examples of discussion questions that were used in the fall 2016 course.

[1] For more information about the course or assignments, readers may contact me at aimeeschwab-mccoy@creighton.edu.

Table 7.1. Sample pre-discussion questions and reading assignments from Core 100: Life in the Data Deluge.

Reading assignment	Pre-discussion questions
Signal and the Noise, "Introduction"	Nate Silver writes "we can never make perfectly objective predictions. They will always be tainted by our subjective point of view." Do you agree or disagree? Why? Use an example from the chapter to motivate your answer.
"Laptop Multitasking Hinders Classroom Learning for Both Users and Nearby Peers" [20]	What do you think of "Experiment 1" as described in the paper? Would you change anything to improve the experiment? Do you agree with the researchers' conclusions? How was mathematical evidence used to support their conclusions?
The Signal and the Noise, "Desperately Seeking Signal"	In this chapter, the author mentions the United States Geological Survey's online earthquake forecast. A map in a *New York Times* article shows the 2016 Continental United States forecast map for earthquake-related damage.[2] What do you think of our chances in the Cincinnati area? Use appropriate mathematical evidence to support your reasoning.
"The Beauty of Data Visualization" [17]	Which of the data visualizations in the video was the easiest to understand? Which was the hardest? What makes a data visualization clear? What makes a data visualization confusing?
"How math helps fight epidemics like Zika" [18]	What is the difference between a statistical model and a mathematical model? Think of two examples we've discussed in class or in a previous reading—what type of modeling is being used?

In class, students used their pre-discussion questions as a starting point for small group peer discussions. Peer discussions encouraged student participation in class, particularly early in the semester, since the peer group is considered by many students to be a "safer" space for discussion [24]. After the small group discussion of the assigned questions, students regrouped for an entire class discussion. In my experience, small-group peer discussion became less important as the semester progressed and students grew more comfortable with the instructor and each other. It should be noted that "large group" and "small group" are relative terms, and that a large lecture class using discussion activities may never progress to a full group discussion.

During the full group discussion students are asked follow-up questions, and encouraged to ask their own questions. This gave me the opportunity to discover which quantitative concepts in the reading students may have struggled with, and provide clarification on a topic. Students often make larger connections during the entire group discussion. For example, during the course a student noted that the models described for making earthquake predictions in *The Signal and the Noise* were similar to information they had read a few weeks earlier about Google's Flu Trends model, and asked if they were "related". Recognizing that the same quantitative methods can be used in different contexts is an important indicator of student growth in quantitative literacy, and students in this course began making those connections solely through exposure to popular media resources.

Another possible strategy to encourage active discussion during class time is by assigning discussion leaders. Students were assigned to groups of two or three to prepare a discussion plan for an upcoming class. The reading assignments for the day's meeting were assigned by the instructor, however the class discussion was completely up to the student team leading the day's discussion. Student discussion leaders were encouraged to incorporate ice breakers and activities into the class session. Some groups created online quizzes in various online platforms to test reading comprehension, others used the "four corners" exercise to get students to share their responses to a prompt by standing in one of the corners of the room [11]. Students informally reported that the discussion leading assignment introduced a fun and competitive element to the classroom; discussion leaders continually tried to "one up" the other leaders to make their class session more fun and engaging!

[2] See `www.nytimes.com/2016/03/30/us/considering-earthquake-threats-in-california-and-oklahoma.html` for the map.

Writing Assignments

As a seminar-style course, writing is another important component in Core 100. Writing assignments can be formative and reflective, such as a written reflection on what the student found most challenging in the assigned reading. Writing assignments may also be summative, and incorporate quantitative literacy in an extended written project. During the course students wrote annotated bibliographies on a topic of their choosing related to "big data," such as election polling or sports analytics in baseball and soccer. Their annotated bibliographies included entries from at least five academic journal articles and at least five news articles or blog posts related to their topic that make a quantitative argument in some way. Their annotated bibliographies summarized the major conclusions of the article and critiqued the use of quantitative information to support the argument. Students were asked to consider questions such as, "Do you believe the authors included enough information to completely understand the data-based evidence? If so, what was particularly effective? If not, what components of the article needed revision?" The assignment prompts were written in a way to encourage analytical writing and identify specific parts of the articles in need of additional attention. By critiquing statistical and quantitative evidence in written work, students then identified important things to consider in their own communication, both written and spoken, about numbers and data.

7.5.2 Making an Explicit Connection between Classroom Material and Student Learning Objectives

Encouraging active learning through discussion and writing in a quantitative literacy seminar is relatively easy to do through course assignments. However, it is significantly harder to design a lesson plan in which students make the connection between the context of the learning materials and the quantitative learning objectives. In any course where interesting applications of data appear, there is always the danger that students will focus in on the context itself, and not the quantitative arguments [13]. In this case, responsibility falls on the instructor to explicitly connect the resources to the student learning objectives.

This should be done from the very beginning with thoughtful course design. One potential criticism of the seminar-style classroom is that there is no clear progression or roadmap for topics that should be covered [12]. However, in courses designed around developing quantitative fluency skills, this may be viewed as a strength. Students are significantly more comfortable talking about quantitative evidence when the context is familiar to them, and the seminar course gives instructors the freedom to pull interesting examples from nearly any context.

Some media resources will have a clearer connection to course material. For example, a chapter in *The Signal and the Noise*, "Role Models," explores predictions in disease models such as the H1N1 flu virus and the AIDS epidemic. This particular chapter explores statistical topics such as extrapolation and modeling assumptions, as well as more quantitative topics such as how to calculate a predicted value. For students reading this chapter within a more traditional quantitative literacy course or those with some statistics content in their previous course work, the connection to linear equations and regression should be immediately clear. This chapter also describes an advanced modeling technique known as agent-based modeling, which seeks to predict an entire population's behavior by representing individuals with digital agents. In a classical quantitative literacy course, such a topic certainly would not be on the syllabus. However, once a student is familiar with the basic ideas of a statistical model, the conceptual framework for a more advanced model is not difficult to develop. In a course targeted toward challenging readings, active discussion, and developing quantitative fluency, like Core 100, these connections can be explored and discussed.

Many of Madison's quantitative literacy learning objectives were assessed during the course. However, calculation may be the hardest of the six objectives presented to incorporate into a seminar-style course. Seminar courses are built and designed around reading, writing, and discussion, and do not typically require calculation or quantitative exercises. Active learning activities are a good method to connect the practical skills of calculation with the softer skills of discussion. For example, an in-class activity in Core 100 required students to choose any Twitter account, and then keep track of the appearance of certain words in the tweets. Students collected data from the 100 most recent Tweets—depending on the account this spanned anywhere from days to years—and recorded how often emotional or sentimental words appeared, such as "love" or "hate". One goal of this learning activity was to identify emotional trends over time in the accounts, while illustrating the necessity of computer-based algorithms when working with large data sets. However, calculation was also an important part of this activity, as students recorded data by hand, estimated the relative frequencies of certain terms, and graphed the occurrence of each term before discussing their results as a class. With mindful intent, quantitative learning objectives such as calculation can be "snuck into" the seminar-style classroom and enhance student learning.

7.6 Conclusion

The increased popularity of quantitative applications in mainstream media presents a new opportunity for enhancing quantitative literacy and fluency skills beyond the classroom. The inclusion of popular media presents quantitative reasoning in a way that students can relate to easily, and illustrates the need for quantitative skills in everyday life. The increasing need for quantitative literacy and fluency in our students necessitates an increased exposure to quantitative applications throughout the college curriculum.

Core 100: Life in the Data Deluge is one example of how quantitative fluency lessons can be implemented in a seminar-style classroom. By using engaging examples and contexts from popular media resources, encouraging active discussion and communication, writing about quantitative evidence, and mindfully designing course activities, quantitative fluency can be addressed in the seminar course. Designing new courses or revamping existing courses to address quantitative literacy requirements involves thoughtful consideration and flexibility on the part of the instructor. However, techniques from the seminar classroom, such as reading data-driven selections, illustrating quantitative concepts through videos and articles, and designing classroom activities and discussion questions can be used in any existing mathematics or statistics course. Using exciting and engaging media resources to teach quantitative literacy skills, and active discussion to increase students' quantitative fluency, may help develop lasting quantitative skills in today's students.

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7.8 Appendix: Selected Video Resources

The list below is by no means exhaustive, but represents a sampling of potential learning materials for a quantitative literacy or fluency lesson using popular media.

1. **TED-Ed. Ideas Worth Sharing**. TED talks are fun and engaging "mini-lectures" on a variety of topics. Videos posted to TED-Ed include both TED talks and original animated shorts, searchable by topic, length, and language. Many videos available on TED-Ed have sample lesson plans or learning objectives available. The "Mathematics" series is especially worth a look, and features a number of suitable videos for a quantitative literacy course. (`ed.ted.com`)

2. **Against All Odds: Inside Statistics**. The Against All Odds video series is a great companion resource for media articles using a specific statistical technique. Explanations are clear without being overly mathematical and emphasize "real people, working on real problems" [1]. Consider assigning these videos for out of class viewing along with the reading assignment to better prepare students for class discussion. Mathematicians interested in incorporating more "data science" into their quantitative literacy classrooms may also be interested!

3. **The Human Face of Big Data**. First published in 2012, this book [25] features ten short essays on human-centered applications of big data. A PBS documentary [26] based on the book premiered in 2016. The documentary features dynamic data visualizations and animations, as well as interviews with data scientists and subjects of the various studies presented.

8

Teaching Quantitative Reasoning with the News

Stuart Boersma, Caren L. Diefenderfer, Shannon W. Dingman, Bernard L. Madison
Central Washington University, Hollins University, University of Arkansas, University of Arkansas

8.1 Introduction

Current events have special appeal to college students and teachers alike, and news reports of current events offer a variety of informative, intriguing, and sometimes puzzling accounts that increase this appeal. Many news reports contain quantitative information that may or may not be correct, carefully explained, or complete. Interpreting, extending, validating, and describing these news reports provides a myriad of opportunities to reason quantitatively and communicate the results of such reasoning.

Quantitative reasoning (QR) is a habit of mind, and habits are developed and sharpened by practice over time. News articles provide a natural and ever-present vehicle for such practice. Further, the development of QR does not result from one course or even several courses, which means that it is critical to continue this practice after completing an undergraduate degree. News articles provide a venue for this lifelong practice, because former students will likely be reading news articles years after they stop reading textbooks or any other form of academic writing.

The learning opportunities afforded by news reports vary in both content and complexity. The content of these reports ranges over all areas of social interactions—politics, economics, sports and entertainment, science, medicine and health, education, and personal finance, among others. Additionally, the complexity of these reports ranges from information derived by simple arithmetic and proportional reasoning to rigorous data analysis and modeling of national and global financial situations. Such variation allows for capturing divergent students' interests as well as finding appropriate material to challenge students of differing backgrounds and mathematical abilities. For example, Deborah Hughes-Hallett leads a QR course at the Kennedy School at Harvard University for masters-level students in public policy and uses media articles as part of her curriculum. Alternatively, for entry-level college students, Bolker and Mast's book *Common Sense Mathematics* [5] uses excerpts from media sources as prompts for hundreds of exercises, and our book *Case Studies for Quantitative Reasoning* [11] contains thirty case studies of media articles. Additionally, Eric Gaze's *Thinking Quantitatively* [11] offers a more technologically attuned study of information gleaned from media reports. There are other books that use media articles as resources. A more complete list is discussed by Madison in this book. Because of the variety of subject matter contained in media articles, the possibilities for interdisciplinary learning—a theme throughout this volume—are virtually unlimited. Current news articles offer subject matter that is ever fresh and part of the students' everyday world. As a consequence, since students are likely aware of many contemporary issues, incorporating news articles into the curriculum enlivens the classroom and facilitates active student learning.

Understanding the world around us constitutes a critical component of education and this understanding extends to informed citizenship and the workplace in several ways. Nowhere, and at no previous time in history, has this been more important than in the U.S. today. Current events are complicated and news reports are instantaneous and numerous. Sorting and parsing the information into some understandable form requires QR that is informed by efficient analysis of past events and future prospects. Additionally, as pointed out by Craig, Mehta, and Howard in this book, the ever changing multimodal nature of media requires consumers of digital and social media to develop and master a wider array of quantitative literacies. Analyzing news in the classroom can aid enormously in dealing effectively with the spate of information that threatens to swamp us all. The knowledge and skills required to accommodate the convergence of "social niches" (as noted by Fisher in this book) is likely beyond the range of general education. Consequently, as Erickson argues [6], QL depends on the reasoned dependence on the knowledge of others. The various social niches and the need for accessing and assessing positions of possible experts arise frequently when analyzing media articles.

Using news in the QR classroom can take different forms, from using entire articles to using brief excerpts that draw some quantitative conclusion. Often, original news articles do not contain all the quantitative information one might think is relevant to the topic at hand. Completely understanding an issue and reaching a satisfying conclusion may require seeking out additional information. Consequently, learning what information to access and how to do so provides additional QR and extended educational opportunities. This may involve students working closely with librarians to learn proper ways to search for and discover reliable information.

Finally, using news articles in the classroom encourages, indeed requires, critical reading, interpretation, and communication (see, for example, [4]). These complex skills not only reveal understanding but also sharpen students' abilities to reason in contextual settings (see, for example, [12] and [13]). Such a venue for combining reading, reasoning and writing occurs rarely in today's highly compartmentalized educational system but offers rich learning opportunities when it does.

In this chapter, we provide the reader with several ways in which instructors can effectively incorporate media articles into their courses. We also offer several points instructors should consider in their quest for appropriate media articles, and we also discuss the challenge as well as potential solutions as to how instructors can properly assess complex and multifaceted student work stemming from analysis of media articles.

8.2 Integrating Media Articles into Your QR Course

Instructors have many options for integrating media articles into a QR course. There are a number of variables to consider as one plans for implementation, ranging from the time allotted for investigation to the purpose of such an investigation. The following sections give a few choices to consider (ranging from the easiest to the most challenging to implement).

8.2.1 Headlines

Occasionally a single headline can provide content for a lengthy student investigation. For example, [7] offers the following headline accompanying an article on the ranking of health care facilities in the U.S.: "A Web site reports that 38 of 4,000 facilities in the United States are better than average". At first glance, the number of "better than average" health care facilities does not seem to correlate with common conceptions of average or mean. Thus, this headline can spark discussion regarding the difference between mean and median, as well as how data can be skewed. Another example [5, p. 52] from *The Boston Globe* proclaims, "Solar use will push energy costs up in Mass. 20-year rise put at $1 billion." To analyze this headline one can find additional information online and see if the headline makes sense. Why would solar use increase costs? How much per month would this add to a customer's monthly bill? Interested students (or instructors) could also search for the original article for more details.

8.2.2 Advertisements

Many advertisements found in newspapers, magazines, or online media contain quantitative assertions. Clearly the quantitative information in advertisements is intended to persuade, or, some may argue, mislead, a reader. For example, an ad in *The New York Times* on October 1, 2008 contained the following two quantitative statements:

odds of surviving airline crash: 24%
odds of surviving pancreatic cancer: 4%

Such an ad can be used in a QR classroom in a variety of ways. One may use it to initiate a lengthy classroom discussion on public perceptions of odds and dispositions towards catastrophic events. It can form the basis for student research projects to investigate the "facts" and comment on their individual validity as well as whether it makes sense to make such a comparison. Additionally, these short quantitative statements can prompt discussion regarding how odds and probabilities are calculated.

8.2.3 Short News Articles

Instructors can use short news articles as motivating examples to introduce a QR topic. Conversely, instructors might wish to base a homework problem on a short news article. If one wishes to try this approach, consider either selecting a textbook that provides the short news articles (both [5] and [36] take this approach) or supplementing from other sources (short case studies such as those found in [11] or articles that you find). The Bolker and Mast model builds exercises around short excerpts from various sources, including news articles. The most common of these exercises make quantitative assertions that students are asked to analyze. Each exercise requires composing a narrative that answers specific questions. For example, the very first exercise in [5] uses an excerpt from *The Boston Globe* that states that Warren Buffet is so rich that he could invest his money at 1% and earn more in one hour than the average annual U.S. salary. Students appreciate receiving the news article and assignment before class. This practice allows students to read and digest the article on their own schedule. This is particularly important for slow readers and students who are not native English speakers.

8.2.4 News of the Day Presentations

Consider reserving time for a daily (or weekly) ten or fifteen minute class segment for "News of the Day" presentations. During these sessions, individual students present short news articles and explain how the articles are connected to course topics, or students may interpret and present the quantitative information contained in the article. Depending on the size of the class, students may be able to give several such presentations in a course. One of the obvious benefits of this practice is allowing students to connect the course material with subjects that they find interesting and engaging.

8.2.5 Formal Projects and Presentations

Some instructors may require that students prepare a formal written document, citing their sources, while others may prefer to have students give oral in-class presentations. These assignments might require students to use a spreadsheet in the analysis.

A successful *QR in the News Assignment* at Hollins University was based on a poster that the Environmental Advisory Board placed in the lobby of the Dana Science Building. The poster stated that the Hollins campus uses 616,308 gallons of water per week and spends $212,000 per year on water. The assignment required students to verify whether the following statements were true.

1. Hollins uses 87,803 gallons of water per day.
2. Hollins uses 3,658 gallons of water per hour.
3. The amount the water costs Hollins per year is equivalent to 4.7 "full rides" to Hollins.
4. The amount the water costs Hollins per year is equivalent to 141 round trip flights to London.
5. The amount the water costs Hollins per year is equivalent to 67,700 gallons of gas.
6. Does an average showerhead put out 10 gallons of water per minute? How long is your daily shower? How much water do you use showering in a month?
7. Does an average load of laundry use 60 gallons of water? How many loads do you do in a month?
8. Estimate how much water you use in a month based only on six and seven.

Students could earn extra credit by determining a better estimate for eight by considering the amount of water used to wash hands, flush a toilet, brush teeth, etc. Instructions for the assignment indicated that students needed to include at least one calculation for each statement, an explanation of all work (in complete sentences), and a final conclusion to determine if the information given on the poster was reasonable and reliable.

8.2.6 Case Studies

A well-chosen media article can serve as a basis for a case study for students to investigate. A collection of 20–30 of these case studies can provide the entire content for a QR course. Examples of such an approach can be found in *Case Studies for Quantitative Reasoning* [11]. For us, a case study consists of a series of warm-up exercises, a media article, and a set of study questions. The warm-up exercises introduce the mathematical methods and concepts that students need to complete the case study. They also provide the student and instructor with immediate feedback on certain computational competencies. After completing the warm-up exercises and reading the article, students can come to class prepared to work individually, or in groups, on the study questions. Instructors have a lot of flexibility on how students may present their final work: oral presentations, short paragraphs, or a longer written report.

8.2.7 QR Project for Your Campus

Such a project would allow students to choose a topic of importance to them that they wish to share with others. Students would become the campus experts on the topic and share the importance and significance of the topic with the campus community. Instructors could give prizes for the level of the research, the creativity of the approach, and the connection to campus events.

8.2.8 Open-ended Investigations

As mentioned earlier, Deborah Hughes-Hallett uses issues from the media in a graduate course at the Harvard Kennedy School of Government [10]. Students are given topics (e.g., Roadside police stops and racial profiling) with data and suggested readings. Hughes-Hallett's syllabus instructs students to "… focus on explaining results in non-technical language, suitable for a news article or parliamentary briefing." In this way students have to develop an understanding of data, policy, and practices to an extent that the essence of the case can be presented to a much larger public. In addition, Schwab-McCoy in this book describes the importance of open-ended study questions for encouraging discussion in a seminar style QR course.

All of the approaches described above allow students to demonstrate skills they may not associate with a mathematics course. Certain students who may have felt constrained by thinking that mathematics questions always have a single correct answers now have a chance to thrive when confronted with open-ended questions. By integrating news articles into a class, many students see, for the first time, that learning mathematics is an important life skill.

8.3 Finding the Right Media Article

As discussed earlier, content from the media can take a variety of forms: entire articles, brief excerpts, headlines, graphics, or advertisements, among other possibilities. Additionally, the subject matter of media articles varies greatly, from, for example, the financial analysis of national budgets, to the odds associated with sporting events. And, of course, the underlying mathematics found in the articles can vary from simple arithmetic and proportional reasoning to more advanced data analysis and mathematical modeling. Thus, there are many factors that go into choosing appropriate articles for a QR course. For example, an instructor may wish to assemble a series of articles to help in assessing some specified set of learning outcomes. Or, instructors may wish to focus on content that is immediately meaningful and interesting to their students. If one is not using a textbook or reference manual of previously collected material, one needs to look at local newspapers and identify appropriate articles for his or her course. After a little practice, one finds that this is neither very difficult nor time consuming. As Boersma describes in [2], articles that are especially rich are those that allow students to:

- Ask "What if"
- Interpret the magnitude of a quantity and place it in a personal context

- Check the accuracy of stated facts or conduct some research on a topic
- Discuss how quantities were measured, who did the measuring, and any inaccuracies or biases that may have resulted
- Perform a quick calculation or engage in mental estimation to check the author's claims
- Build mathematical models
- Convert an absolute change into a relative change, or vice versa, and reflect on the different measures
- Compare numerical information in the article with the information presented in an accompanying graphic
- Become familiar with language used to represent and compare quantities.

Two examples are given below. Example 1 illustrates how a newspaper article on a community college initiative could be used as a student activity since it has many of the above elements. The article from Example 1 could be used to assess students' abilities in the following areas: working with percents, comparing absolute and relative change, and proportional reasoning. The material from Example 1 could also be used as the basis for a longer student research project. Example 2 describes a newspaper article and gives an example of detailed study questions that have been used in our QR courses [11]. These study questions assess students' abilities in understanding quantitative arguments, making reasonable assumptions, and creating, using, and analyzing linear and exponential models.

Example 1: Consider a Washington Post article headlined "Obama Announces $12 Billion Community College Initiative" [15]. This article contains numerous quantitative statements and the subject matter may resonate in a classroom composed of students with community college experiences. One could easily design student study questions that focus on several of the elements listed above. Specific examples include:

1. *Interpret the magnitude of a quantity and placing it in a personal context*: How large is $12 billion? If this money is evenly distributed over all the nation's community colleges, how much would each college receive? How would this amount compare to the school's overall budget and what types of activities could be funded with this amount of money?
2. *Perform a quick calculation*: The article mentioned that this program would also "add 5 million new graduates by 2020." What is $12 billion divided among 5 million students?
3. *Absolute change and relative change*: The article states that "the funds would be used to support … a 40 percent to 50 percent increase in the number of people who graduate from a community college…" while "currently, about 1 million students graduate from community college each year." How do these two statements compare with the "5 million new graduates by 2020" statement?
4. *Student research opportunities*: Students could choose a local community college and collect information such as its recent budget history and its recent enrollment history. Students could then estimate how much money it would take to increase enrollment by 40 to 50 percent and compare with the dollar amounts in the article.
5. *Discuss how quantities are measured*: How do colleges compute graduation rates? Is the number of graduates a reasonable measure of student success at a community college? Are other measures more robust?

Example 2: In an August 21, 2001 *Lincoln Journal-Star* article titled "Forcing Fuel Efficiency on Consumers Doesn't Work" [42], author Jerry Taylor makes the following assertions:

1. Economists have discovered over the long run, a 20 percent increase in gasoline costs, for instance, will result in a 20 percent decline in gasoline consumption.
2. A recent report from the National Academy of Sciences, for instance, notes that the fuel efficiency of a large pickup could be increased from 18.1 miles per gallon to 26.7 miles per gallon at a cost to automakers of $1,466.
3. It would take the typical driver 14 years before he would save enough in gasoline costs to pay for the mandated up-front expenditure [$1,466].

4. A similar calculation for getting a large SUV up to 25.1 miles per gallon leads to a $1,348 expenditure and, similarly, more than a decade before buyers would break even.
5. You could take that $1,466, for instance, put it in a checking account yielding 5 percent interest, and make a heck of a lot more money than you could by investing it in automobile fuel efficiency.

In [11] students are given the entire article to read and the following set of questions:

1. (a) Which of the assertions can be checked without considerable research?
   (b) What assumptions would need to be made in checking assertion 3?
   (c) What assumptions would need to be made in checking assertion 4?
   (d) What assumptions would need to be made in checking assertion 5?
2. (a) Is assertion 3 reasonable? Explain why or why not.
   (b) What would be the effect of increased costs of gasoline on assertion 3?
   (c) What would be the effect of increased miles driven per year on assertion 3?
   (d) Assume the cost of gasoline in 2001 was $1.40 per gallon and that it would take 14 years for the "typical driver" to recover the $1466 through savings in gasoline costs. How many miles per year would the "typical driver" drive?
3. (a) Is assertion 4 reasonable? Why or why not?
   (b) How would the savings be affected if the current miles per gallon of large SUVs were lower than 18.1?
4. (a) Is assertion 5 reasonable? Why or why not?
   (b) If the $1466 is placed in one account at 5% interest and the annual savings from gasoline is deposited in a second account earning 5% interest, compounded annually, how do the amounts in the two accounts compare?

These examples provide a snapshot of the possibilities one might be able to create given a standard media article. Other examples that might spur further insight into what one can do can be found in the various textbooks referred to throughout this chapter.

8.4 Assessing Student Work Stemming from Media Articles

One of the major challenges for instructors using media articles as curricular sources is properly assessing complex student work. In many instances, students use a diverse set of knowledge and skills to analyze the situation and provide calculations toward answering a question. This process requires more than providing a correct solution. Thus, the feedback and grade provided by an instructor should reflect the complexity of the solution process. As argued by Zerr in this book, assessment therefore should include multiple instruments and provide both quantitative and qualitative methods to properly measure students' developing habit of mind.

In an effort to provide guidance to instructors working to assess such work, we designed and tested the Quantitative Literacy Assessment Rubric (QLAR), [4]. This rubric, which builds from the Association of American Colleges and University QL VALUE rubric [1], outlines six core competencies that highlight the diverse skill set required to solve QL tasks arising from media articles. These competencies include:

- Interpretation: the ability to glean and explain mathematical information presented in various forms (e.g., equations, graphs, diagrams, tables, words)
- Representation: the ability to convert information from one mathematical form into another (e.g., converting between equations, graphs, diagrams, tables, and/or words)
- Calculation: the ability to perform arithmetical and mathematical calculations
- Analysis/Synthesis: the ability to make and draw conclusions based on quantitative analysis
- Assumptions: the ability to make and evaluate important assumptions in estimation, modeling, and data analysis

- Communication: the ability to explain thoughts and processes in terms of what evidence is used, how it is organized, presented, and contextualized.

The rubric provides the user with a four-point scale (0-3) that can be used to provide feedback on student work, depending upon the competencies demonstrated in the process. As part of this work, we mapped the competencies in the QLAR to the case studies in [11], to provide instructors with a guide as to how student work could be scored.

For example, in Example 2 (above) students are given a series of questions that require them to analyze the quantitative assertions made by the author in order to determine the reasonableness of the author's claims. The question set provides students opportunities to demonstrate each of the competencies listed in the QLAR and the questions range from ones that require just one competency for solution (such as questions 1b, 1c, and 1d, which only require the student to make assumptions in order to check a claim) to ones that require students to use multiple competencies (such as 2a, which expects the student to generate a representation to model the situation, perform calculations based on the model to examine the author's claims, analyze/synthesize the information obtained from the calculations to test the accuracy of the author's assertions, and then communicate those findings in a written explanation). Therefore, in using the QLAR to assess student work, an instructor could then provide feedback to the student regarding various aspects of their work, thereby honoring the complexity of the student solution and highlighting areas where the student succeeded and where further improvement is needed. We found the application of the scoring rubric to be highly reliable in terms of the score given to student work (see [4] for additional information on testing the QLAR).

In addition to using the QLAR to assess student responses to specific prompts in a QL assignment, the QLAR has also been successful in measuring students' habits of mind to employ QL skills in situations that do not explicitly solicit quantitative information. As Boersma and Klyve explain in [3], media articles provide a rich resource to study and assess students' predispositions to seek out quantitative information, critique quantitative evidence, and use quantitative skills.

However, there are aspects of QL that are not addressed in either the QLAR or the VALUE rubric, namely information literacy and critical thinking. Nevertheless, using a tool such as the QLAR can assist instructors in other ways. The QLAR can, for example, aid instructors in the process of assessing curricular materials to understand how the various competencies are present in student tasks. To that end, we used the QLAR to map the case studies in our casebook [11] to the skills outlined in the QLAR to better understand how the competencies are dispersed across various lessons. We found that certain skills and competencies were more prevalent than others, which allows one to understand the amount of student exposure to expect. Additionally, we posit that one of the key benefits of the QLAR may actually be in creating questions in addition to assessing student work. In revisiting the written question sets in [11], we found that, with slight changes to how certain questions were asked, tasks could be rewritten to encompass a greater number of QLAR competencies and thus provide a richer activity for students. This conclusion underscores the close relationship between choosing quality media articles with which students can engage, and assessing such work. Assessment can not only provide feedback and meaning to student work, but also guide the work of curriculum development as instructors consider how to pose rich questions stemming from media articles.

8.5 Conclusion

One of the challenges that has historically confronted mathematics teachers at all levels is how to contextualize the mathematics under study. Students often have wondered "When will we ever use this?" as they struggle with abstract concepts that permeate the middle, secondary, and post-secondary mathematics curriculum. In this chapter, we have offered the use of media articles as a way not only to contextualize mathematics, but more importantly to allow students to reason and solve problems in settings aligned with their own interests.

One should also take note that using media articles in your course is not easy. First of all, it takes time to find and gather a collection that can be used in specific courses, though, as discussed above, materials and online sources do exist that can lessen this challenge. In addition, media articles can take students and instructors into unfamiliar contexts that can challenge one's own knowledge and understanding. Students and instructors alike may also find themselves discussing polarizing topics. At first, instructors may find themselves uncomfortable facilitating such discussions, but by keeping the focus on the quantitative arguments, one may achieve success. Students also have come to expect template problems and examples that can be followed to assist them as they begin to work independently. With media articles, template problems are antithetical to QR, as different situations may require different methods for solving problems and concepts can occur unpredictably.

However, as alluded to earlier, QR is a habit of mind, and habits are formed over time and through continued practice. By embedding this practice in familiar circumstances—ones that often confront newspaper readers—instructors can provide students authentic situations in which to hone their skills in reasoning quantitatively and solving real-world problems in a manner consistent with how they may solve similar problems when their course-taking days are over. This "lifelong" skill will serve students well as they become part of an educated and informed citizenry.

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9

Revealing the Mathematics of Sustainability

Alana Baird, Sherareh Nikbakht, Eric Marland, and Katrina M. Palmer
Appalachian State University

9.1 Interest in Sustainability

Interest in sustainability and environmental stewardship has experienced a rapid growth in the past few years and this interest has not been restricted to traditionally scientific fields. The separation between science, business, and policy has never been closer than in efforts to understand and mitigate the effects of climate change and to reduce the harmful effects of pollution and other human activity. We have repeatedly seen the influence of pop culture on views related to the environment, from themed music and art to celebrity endorsements. The combined effect of these disparate fields' involvement is that environmental issues are constantly surrounding us and filling our thoughts. Keeping mathematics relevant to the interests of our students motivates them and keeps them engaged.

But in fact, their increase in interest is perhaps dwarfed by the tangible need for our students to understand the key issues in sustainability and appreciate their role in addressing the pressing needs of our society in its struggles to deal with a changing global environment.

In addition, every region has its own unique set of challenges and barriers that need to be overcome. In Boone NC, recent efforts to secure enough water for the growing town has created a great deal of controversy. In some areas of the world, there is plenty of water but the water quality is below healthful levels. In other areas, the challenges might be securing sufficient energy, improving air quality, or managing waste. As communities and regions grope for solutions to these systemic issues, individuals struggle with issues that, although addressed on a smaller scale, have large-scale impacts. Convincing people to recycle, to compost their wastes, or to reduce their use of paper might be issues in other areas.

At the heart of any attempt to strategize or develop policies for sustainability is the ability to quantify. How much water can a typical roof collect in my town and how does this collection affect the surrounding environment? How can we better predict broader consequences of well-intentioned policies? Does building with wood pollute more or less than building with steel? How much paper can I save in my classroom by moving to electronic submissions and by taking notes on tablets?

Mathematics and computation are integral to these efforts. The depth of mathematics needed to understand and quantify important sustainability ideas varies greatly, making it a rich area to incorporate into the classroom at many different levels. In fact, some of the ideas can be revisited repeatedly as more skills and techniques are incorporated into students' repertoire and more details about sustainability are introduced. It also furthers our awareness of the mathematics in the world around us, filling our minds with numerical possibilities with a sweep of the landscape.

In this chapter, we focus on concepts, activities, and ideas that demonstrate the role that mathematics plays in sustainability in the contexts of our students, our local communities, and our global society. We provide a few examples to highlight where the current conversation lies with sustainability and invite readers to share additional ideas to help drive sustainability forward in both our minds and our practices. Whether it be in a beginning or advanced mathematics course or a course in a related discipline, we hope that these examples will provide a starting point toward using sustainability to enhance quantitative literacy and quantitative reasoning for our students.

9.1.1 Defining Sustainability

Sustainability has become very popular in the last few years as a way for both individuals and groups to help mitigate the effects of climate change and to generally help preserve or rejuvenate the environment.

Sustainability is not a well-defined concept and may, like the term "being green," mean different things to different people. This lack of a single definition does not come from lack of effort [9]. Sometimes we find that two mutually exclusive activities might both be considered sustainable for entirely different reasons. For example, how do we balance the visibility and effectiveness of paper posters advertising sustainable practices with the desire to move to reduce our reliance on paper as one of those practices? Bamboo floors are made with a renewable resource, but the glue used to make them may come from fossil fuels and the transportation of the bamboo from overseas may have used much more fossil fuel than a locally grown wood that does not regrow as quickly.

However in some ways, the abstract nature of the term has been deemed a positive and intentional strategy. More people can feel like they are participating if they can construe sustainability to suit their particular situation. Increased buy-in is considered a necessary step in mitigation efforts against global climate change. A constrained definition might confine activities to only those that a small group has outlined, while excluding new and innovative ideas that come along later. If everyone feels that they can participate and contribute, general policies on the community and higher level may become easier to implement.

Here, we will provide a general working definition of sustainability, built from a variety of sources, that is solely intended to guide our own ideas and not restrict alternative, and equally valid definitions.

> Sustainability is an effort to sustain or nurture the natural environment. This includes direct and indirect activities that impact the natural environment and efforts to better understand interactions with the natural environment in order to inform decision-making.

At Appalachian State, sustainability has even made its way into the mission statement of the university: "... prepares students to lead purposeful lives as engaged global citizens who understand their responsibilities in creating a sustainable future for all" [1]. Throughout various facets of the university, efforts are being made to encourage sustainability, from groundskeeping to instruction, and from commutes to instructional technology. We now have "Zero Waste" events where recycling and composting bins are monitored during breaks and meals [25]. A simple internet search shows that many other universities and organizations are moving in the same direction.

9.1.2 Quantitative Literacy in Education

Reflecting on the various definitions of quantitative literacy presented by Forest Fisher in this volume, we recognize that quantitative literacy does not just reside in the first mathematics course taken at a university. Quantitative literacy is not restricted to students whose primary interests lie outside of mathematical or data driven fields. We also realize that this literacy may well mean something different to students in different fields, because our backgrounds are not the same and how we view the connections to the world around us vary as much as the variation in the connections themselves.

What is considered an elementary tool to a chemist is not the same as an elementary tool for a physicist or a historian. However, the world that we live in is the same, regardless of our backgrounds, and the habits of mind that we develop are parallel even if they do not coincide. We consider here a range of ideas that connect different niches of the community with sustainable ideas.

For students outside of STEM (Science, Technology, Engineering, and Mathematics) related disciplines, mathematical and quantitative ideas take a back seat to other important issues, and some students in these fields may find quantitative ideas less relevant to their interests than more humanistic or social challenges. In these contexts, quantitative literacy helps students become engaged and interested in learning mathematics because it becomes a tool with which they can explore topics in other disciplines that are more closely aligned to their interests. The "real-world" nature of topics centered around sustainability provides students with a reason to learn the mathematics involved because it provides natural connections between social, ethical, political, technical, and scientific issues that face us in increasingly important ways every day.

Students in STEM disciplines understand the need for quantification in their own fields of study and they see the connections between their scientific and technical fields and the tools of mathematics, data science, and visualization. However, these STEM students rarely see the humanistic and social aspects of their work and making these connections is equally important in this context. For these students, quantitative literacy must still bridge the same connections, but begins on the other side. Sustainability is an area where the classic divide between the humanities and the sciences is narrow, and the value of reaching across is fundamentally important both in our daily lives and in our society as a whole.

Learning the mathematics and using relevant data and calculations can help to better understand the social and scientific issues in a sustainable area, possibly helping us to make better currently important policy decisions. Also, it allows students to leverage and reinforce the related topics they are learning in other courses. They can see how ideas work together in real and timely applications, which will engage students and help them to see the benefits of learning the mathematics.

9.1.3 Sustainability in Quantitative Literacy

What role does quantitative literacy play in understanding and communicating sustainability? In addition to having a department on our campus with the term "sustainability" in the title, the biology department and the Environmental Science Program both teach sustainable practices and study the consequences of sustainable (and non-sustainable) living. It would be easy to assume that these groups are better suited to teaching sustainability concepts to our students. Why not leave those topics to those departments? What role does mathematics have to play that is not already being handled? Here is a short list of reasons:

1. Students are interested in these topics, which helps them engage their minds and distract them from the fact that they are learning material (mathematics) that they may have struggled with in the past. That is, it helps teach students mathematics. Sustainability is important to students right now [2].
2. The field of sustainability is full of rich examples that use many areas of mathematics at a number of levels making it appropriate in many classes. See, for example, modules from DIMACS [7].
3. We need it. On January 1, 2016, the 17 Sustainable Development Goals of the United Nations Summit came into force. The goals task all of us globally to address the linked issues of poverty, inequalities, and climate change [14].
4. Learning about important issues like sustainability (and mathematics) must take place in multiple courses, in multiple settings, repeatedly, and at spaced intervals [8, 24].

Due to the growing popularity and the global importance of sustainability [14], there are good resources that make incorporating this topic into the mathematics curriculum easy to accomplish and worthwhile. Since sustainability is such a broad idea, finding topics to incorporate into lessons is relatively easy. Topics can range from local to global issues in sustainability. Also, students can begin to research topics they are personally passionate about, which will help everyone involved in the class learn about new and interesting topics and approaches to sustainability.

Sustainability topics address some of the 21st century themes promoted by P21 (Partnership Framework for 21st Century Learning [22]). Specifically in the P21 Framework Definitions publication, P21 calls for schools to begin "weaving 21st century interdisciplinary themes into key subjects" including "Global Awareness" and "Environmental Literacy." Globally aware students should be able "to understand and address global issues," and students with environmental literacy should be able to:

- Demonstrate knowledge and understanding of the environment and the circumstances and conditions affecting it, particularly as relates to air, climate, land, food, energy, water, and ecosystems.
- Demonstrate knowledge and understanding of society's impact on the natural world (e.g., population growth, population development, resource consumption rate, etc.).

- Investigate and analyze environmental issues, and make accurate conclusions about effective solutions.
- Take individual and collective action towards addressing environmental challenges (e.g., participating in global actions, designing solutions that inspire action on environmental issues).

Sustainability problems provide students with the opportunity to make a contribution or impact the world in some way. If students become aware of situations and discuss ways to correct or alleviate problems, students will begin to learn the steps necessary to bring about change.

Students often say they do not know how to apply the mathematical topics to their own areas of interest, or they do not see the relevance of the mathematical processes in their lives. Students will learn new ways in which mathematics can become relevant to their lives by using sustainability problems that addresses several of the Basic Principles of Beyond Crossroads such as [15]:

- "**Broadening.** Mathematics courses and programs in the first two years of college should broaden students' options in educational and career choices. The mathematical content, reasoning skills, and communication skills developed in mathematics courses should open doors for students to pursue future work in a variety of fields" [15, p. 10].
- "**Inquiry.** Effective mathematics instruction should require students to be active participants. Students learn through investigation. Advances in neuroscience confirm that students' active involvement in learning mathematics is important in the process of building understanding and modifying the structure of the mind" [15, p. 10].
- "**Relevance.** The mathematics that students study should be meaningful and foster their appreciation of the discipline. Mathematics should be presented in the context of realistic, understandable, applied problems that help students develop an appreciation of the nature, history, and usefulness of the discipline" [15, p. 10].

Improving students' understanding of the connections of mathematics to other subject areas can further be promoted by incorporating quantitative literacy tasks into other courses. Furthermore, the use of such tasks in other courses will help to show students how mathematics can be applied in across disciplinary interests while also improving students' computational skills. According to the MAA's Curriculum Foundations Project:

> Interdisciplinary cooperation can help students overcome the transfer problem from mathematics courses to partner discipline courses. Specifically, students often have difficulty seeing the relationships between problems in non-mathematics disciplines and material studied in mathematics courses. Colleagues in partner disciplines believe that exposing students in mathematics courses to discipline-specific contexts for various mathematical topics will have a positive effect on their ability to transfer knowledge between courses [16].

Using sustainability topics to promote quantitative literacy will also improve critical thinking skills because it will require students to analyze current events, assess concerns, and create solutions to complex real-world problems. These ideas fall under the "Part I: Recommendations for departments, programs and all courses" of the CUPM Curriculum Guide 2004 [8]. In the second section about developing mathematical thinking and communication skills, it states:

> Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to:
>
> - State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;
> - Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;
> - Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking [8, p. 13].

The types of problems suggested by the CUPM guide provide students with the opportunity to research, calculate, and make decisions using mathematics. Students can assess statements and determine for themselves if the conclusions are true.

9.2 Incorporating Sustainability into Quantitative Literacy

In this section we provide four examples: water flow, salt marshes, passive solar shading, and plastic bottles. Each example represents a module that provides an overview and statement of importance with some lesson specifics. In Section 9.2.2, we provide ten additional brief examples to be used in variety of classes.

9.2.1 Module Examples

Water Flow

In this lesson students calculate the discharge rate for a water fountain, a leaky faucet, and a stream. Since the discharge rate is a volumetric flow, the units for a discharge rate are units of volume per unit of time.
Importance: Understanding the discharge rate is important for many reasons. One is to estimate the volume of runoff to help with storm water management. Another is to decide how much water towns are allowed to take out of nearby rivers, leaving enough for towns downstream. Another is to calculate the flow of a regular faucet versus a low-flow faucet to calculate both the financial and water savings.
Mathematical Content: This activity guides students through two methods of calculating discharge rates: the bucket method and the float method. Students use rates and Riemann sums to calculate the discharge rate.
Lesson Specifics: The bucket method uses a container (of known volume) and a stopwatch to time how long it takes for the container to fill up. For better accuracy, this step is repeated. Then students calculate the discharge rate by dividing the volume by the time. This method is best for calculating the discharge rate of water faucets, showers, and drink machines. The float method, for calculating stream discharge rates, finds the area of a cross section and the velocity of the stream to then calculate the discharge rate. To estimate the area of the cross section, students first measure the width of the stream, then measure the depth at different distances going across. For example, if the stream is 5 feet across, they may measure the depth at 1, 2, 3 and 4 feet (see Figure 9.1). Then they estimate the cross sectional area with Riemann sums.

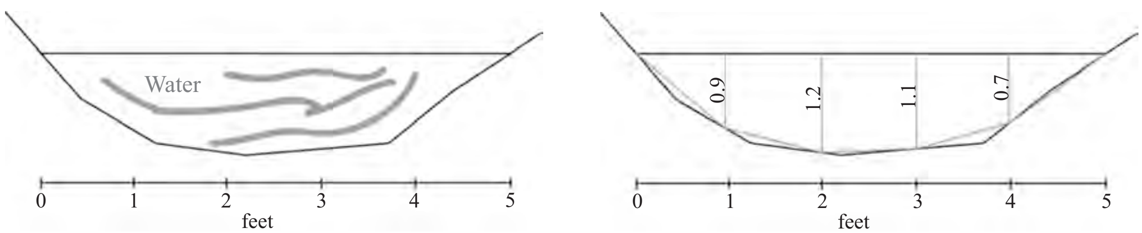

Figure 9.1. Diagram of the cross sectional area of a river.

To approximate the speed, students time how long it takes for an object to float along a measured distance of the stream. Finally students multiply the cross-sectional area with the speed to obtain the discharge rate. There are a number of possible extensions to this module, some of which the students might want to develop into independent projects. Leaving some of the ideas open ended allows students to increase their level of ownership of the material.

Salt Marshes

In this lesson, students start by researching salt marshes and their important role in the ecosystem [18] as well as the role of fiddler crabs on the health of a salt marsh ecosystem [23].
Importance: Salt marshes are among the most productive ecosystems on earth and exist in many coastal areas around the globe. They provide real and measurable environmental, social, and economic benefits. Salt marshes export carbon and energy into the water column, store carbon in their root systems and sediments, and filter nutrients, pollutants, and pathogens from water along the coastlines. Salt marshes also help to protect against storm damage, flooding, and erosion. Human activities such as oil spills, development, and agricultural drainage can destroy salt marsh habitats.
Mathematical Content: The Salt Marshes activity uses basic geometry and algebra skills. Students calculate the area and volume of regular shapes, as well as percentages, ratios, and averages.
Lesson Specifics: For the first part of this lesson, students conduct online research about the importance of salt marshes and their crucial role in the environmental health of the coastal areas. They summarize their results in a two-column

table that compares the advantages of draining and removing salt marshes versus protecting and maintaining them. Students use one class period to present their comparison tables to their peers and initiate conversations about the health of salt marshes along with other related issues. The next part of this activity provides data that allows students to do statistical analyses, geometrical calculations, and basic algebraic manipulations in learning about the important role of fiddler crabs in the health of salt marshes [11]. At the end, students discuss the results and how those results help justify the need for salt marshes. They also discuss the fiddler crabs' contribution to the environment.

This example may not entice students who live away from a coastal area. Instead, students might select other environmental issues that they feel passionate about and identify reasons why people should really care about the chosen issue. Students can then research and calculate facts, data, and statistics about their issue. Convincing arguments can then be presented to their classmates/peers to make a commitment to address the issue. Students might develop a television, radio, newspaper, or internet public service announcement about the issue as a part of an assignment. This approach can inform, increase awareness, and call to action their peers on this issue while integrating mathematical ideas. Student examples can be found in the "Salt Marshes Secrets" activity [11].

Passive Solar Shading

Passive solar shading is a building design element that uses more windows on a south facing wall to gain more heat in the winter when the sun is lower in the sky, and in warmer months block the higher summer sun with overhangs (see Figure 9.2).

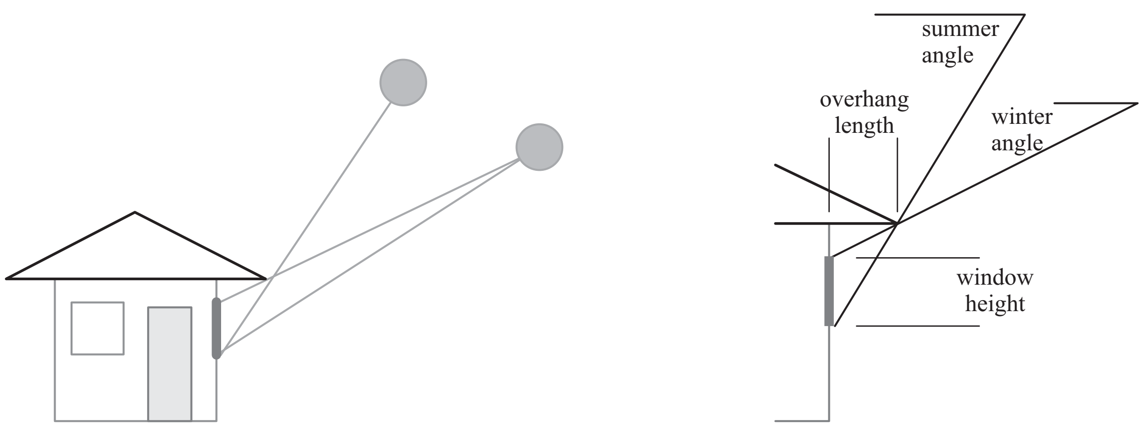

Figure 9.2. This simple diagram shows that the summer sun is much higher in the sky than the winter sun, allowing a shade to fully block the summer sun while allowing the window full exposure to the winter sun. The angle of the sun depends on location, day of the year, and time of day.

Importance: Building with passive solar designs reduces our dependence on fossil fuels by reducing the need for winter heat and summer air conditioning since the design prevents solar gain in the summer, and increases solar gain in the cooler months. In addition, the cost of building a home with passive solar design elements does not add much more cost. Increased insulation in walls and windows is the largest monetary factor.

Mathematical Content: Basic passive solar design uses right triangle trigonometry for various calculations such as the length of the overhang or the placement and size of the windows. Questions related to passive solar design can vary in complexity. For example, instead of just thinking about the sunrays at noon, we might ask "What time frame is optimal for letting the sun in?". This requires the use of additional ideas about the motion of the sun during the day.

Lesson Specifics: A module about passive solar design can be found at [7]. The module can be used over several class days, or you can pull a few problems from it. For example: Given the window placement for a house in Vilas, NC shown in Figure 9.3, determine the shortest overhang length that would guarantee no direct sun-rays enter the window at noon on June 21st. The angle of the sun at noon on June 21st in Vilas NC can be found at [10].

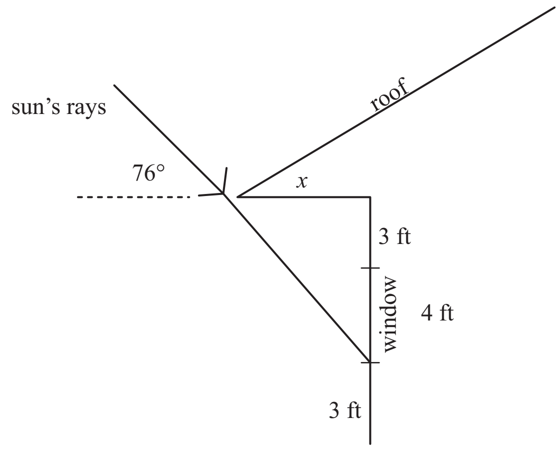

Figure 9.3. Diagram showing the sun's angle at noon at June 21st in Vilas NC where x is the overhang length.

Plastic Bottles

In this lesson, students calculate how many times water bottles produced in the United States during one year would wrap around the world and how many of those bottles would be saved by using a water filter.

Importance: Plastics are everywhere in our society. However, they are not biodegradable and because of that, plastic pollution, especially in our oceans, has become a major problem; but one that many people are not aware of.

Mathematical Content: This activity guides students through the mathematical modeling cycle, helping to build their critical thinking skills. The students must think of questions with problems, ask for the tools and information needed to solve the problems, identify the assumptions they are working under, calculate answers, and then analyze the results for reasonableness. According to Gaze [17], mathematical modeling is one of the three main components of quantitative reasoning (QR) courses.

Students encounter estimation, scientific notation, unit conversions, and basic geometry during this activity. Also, the activity could be altered or adapted in many ways. For instance, students might be asked to calculate the circumference of the earth, incorporating more geometry into the lesson. The mathematics used is simple arithmetic operations; however, students are required to think about the necessary information and the procedures for calculating instead of simply being provided with all the tools, which is often the case in normal textbook problems.

Lesson Specifics: Students can be initially engaged with videos about the ocean gyres filling up with plastics. After showing students some background information about the problem, the problem can be discussed mathematically. On the activity sheet, which can be found at NCDocs, students begin the lesson by guessing how many times water bottles wrap around the earth [5]. Next, show students a video, "Brita" Part 1 from Dan Meyer's 3 Act Math Tasks, to help students determine which problems need to be solved [19]. After determining the goals, students think about the tools they need to solve the problems. Once students ask for particular information (like the circumference of the earth and the size of the water bottles) information can be provided. Using the requested information students can then solve the problem about how many times all the bottles would wrap around the world. The last step of the modeling cycle has students assess the model they created, so the activity sheet guides students through questions to help them evaluate their assumptions and answer. Finally, students calculate how many bottles would be saved by using one water filter, which can lead to a discussion about finding other solutions to the current plastic problem. To wrap up the activity, students can watch "Brita" Part 3 from Dan Meyer's 3 Act Math Tasks to confirm their mathematical calculations were done correctly [19].

Extensions: Plastics discussion could also lead to developing a model for how much plastic is actually out in the environment or in the ocean. Following the "Modeling Orbital Debris Problems" by Illuminations [20], another lesson, "A Plethora of Plastics" was created to have students build a mathematical formula for calculating the total plastics on earth [6]. Students first look at the linear growth of plastic creation, then think about how the nature of plastics (being non-biodegradable) would actually lead to an exponential growth formula over a period of time. Students walk through the process of evaluating different mathematical models to fit the data. A further supplement, or homework problem, to the in-class activity has students analyze the exponential decay of plastics in the oceans.

The examples presented here are designed to give some insight into the types of activities that might engage students in sustainable ideas. We have also shown how basic ideas give rise to interesting mathematical problems and how student driven problems might develop with even more student "ownership" of projects.

9.2.2 Shorter Examples

Rather than trying to incorporate full-day (or longer) sustainability lessons, there are other ways to start by including shorter examples. Sustainability examples can be incorporated in more subtle ways or in small pieces that contribute to a larger scale, more pervasive effort in increasing a general awareness of sustainability. These could vary from a single example or problem on a homework assignment or exam, but might also be expanded to something much larger like a project, or the driving theme in a class. The examples outlined here are intended to demonstrate smaller ways to increase awareness. Referenced resources provide additional ideas and data.

Going paperless, does it really save?

How much paper is saved and what is the cost of the paper? How much money would be spent in order to go paperless and how long does it take to save that much money?

Analyzing data from campus water dispensers

At many institutions and public facilities, we now have water dispensers that monitor how many gallons of water are saved by filling water bottles rather than using the standard water fountain. Students can gain access to and investigate the data and see how closely their estimates agree with the display on the dispenser.

Antarctica ice melt—How much would sea levels rise due to melting glaciers?

How much sea levels would rise vertically if the glaciers in Antarctica were to melt? Students investigate the relationship between surface area and volume and then think critically about the assumptions made to make the calculations and mathematics feasible [4].

Melting sea ice

Sea ice extent can be thought of as the area (in square kilometers) that is covered at least partially by ice. Using a graph of sea ice extent from the National Snow and Ice Data Center (NSIDC) [21], students can interpret a graph from one year or analyze a graph with data over several years. These graphs can be modeled with sinusoidal functions.

A Bright idea—How much energy and money different light bulbs can save us

Students explore an easy way of conserving energy while they do some comparison between two types of light bulbs. Students calculate the cost advantage of replacing an incandescent bulb with a compact florescent bulb [12].

Cost of heating a house with natural gas in Eastern North Carolina

Students learn about natural gas and its usage while they calculate the cost of heating a house. Also, students discuss environmental impacts of using natural gas as a source of energy and possible actions we could take to conserve heat energy [13].

Daily water Use

The United States Geological Society (USGS) keeps track of water use in the U.S. and has posted state by state data since 1950. With data on their home state, students can derive a statistical fit of that data and debate the possibility for the trend to continue into the future. The data can give insight into both the amount of water used per person per day and changes in overall water use. In addition, the derivative of the functional fit of the data provides information on the rate of increase in the water use.

Pollution per mile

Personal automobiles are a primary source of carbon emissions in the United States. Can we determine just how much? There are several online calculators that estimate the change in mileage with driving speed. Here students can calculate the emissions from their own vehicle sand determine the carbon output from a long car trip. They can also determine the changes of carbon emissions as a consequence of changing their average driving speed.

Solar panels

The solar panels installed in a parking lot on a business or college campus can produce energy that adds to the total energy available to the campus. The total energy bank might then be used to charge electric cars. Given the solar potential of their region, students can calculate the square footage of solar arrays needed to produce power to recharge electric cars. As a followup, they can think about the variation in production and how that might influence the successful charging of the cars.

Volume of runoff

Runoff is the draining away of water (or substances carried in it) from the surface of an area of land, a building or structure, etc. Runoff can contribute to an increased amount of contaminants in streams. Students can measure the inches of rain from a storm and then use Google Maps to calculate roof areas to then calculate the volume of runoff from a particular building on campus.

There are other developed resources available from a variety of sources; however some of the most powerful materials that engage students are those that are developed by the students themselves and are based on the happenings on the individual campus or in the local communities.

9.3 Conclusions

Living sustainably is a crucial step in caring for our environment and mitigating the effects of climate change. There are many ways to incorporate sustainability into our lives and at least as many ways to incorporate sustainability into the classroom, whether it be a mathematics, statistics, or partner discipline classroom. This provides a great deal of flexibility and opportunity to provide students with real and practical knowledge and furthers their sustainable habits of mind.

One of the ways to increase buy-in to sustainable practices is to quantify the costs and benefits of various measures we can take. When we see how little we need to give up and how much we have to gain, it is difficult to justify any other approach. Mathematics is the language of quantification and the range and scope of examples provides a great opportunity.

Furthermore, as students become more engaged in their courses, they can see opportunities for taking their interests beyond the classroom. Research studies and community projects are natural extensions of these simple classroom exercises and there are many ideas to pursue where the mathematics is accessible to students with little technical background.

Realizing the goals of Quantitative Literacy to make connections between social niches (as described by Fisher in this volume) can create a more well-rounded student. Students will not only be more engaged in the classroom, but more engaged in their communities. Sustainability provides a vehicle for that engagement, while at the same time addressing one of the most pressing challenges we have ever faced.

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10

It's a Question of Money

Financial Mathematics Projects that Build Students' Quantitative Literacy

Suzanne Dorée and Eleonore Balbach
Augsburg University, Saint Catherine University

Part of the fun and challenge of teaching quantitative literacy and quantitative reasoning (QL/QR)[1] is creating engaging, realistic projects. Not surprisingly, one particularly engaging context is personal finance. After all, who is not interested in money? In this chapter we present four projects that build college students' QL/QR in the context of personal finance. We describe where we have used these projects, include the instructions we give students, offer notes on grading, and reflect on student learning. Since our projects are more realistic than standard textbook problems, they require a little deeper mathematics. Recognizing that some instructors might not have seen this material, or have not seen it in a way that they can present to students with a minimal background in algebra, we also include a brief overview of the prerequisite concepts and skills in the Appendix.

10.1 The Context of Personal Finance

In *Mathematics and Democracy* [12], Lynn Steen wrote: "Managing money well is probably the most common context in which ordinary people are faced with sophisticated quantitative issues. It is also an area greatly neglected in the traditional academic track of the mathematics curriculum" [12, p. 13].

How do we remedy this neglect and help college students develop financial literacy, equipping them to navigate major milestones such as repaying their student debt, buying a house, and saving for retirement? We believe asking students to set up and solve real-world problems using real-world data is key. After all, we would like our students to be able to answer real-world questions such as

> *Why does your credit card company advertise 16% interest but also say the APR is 17.23%? How much faster can you pay off your student loan if you add an extra $10 per month to your payment? How much do you need to save each month for retirement if you start right now, and by how much would that increase if*

[1] In [11], Powell and Leveson define quantitative literacy as "a basic familiarity with numbers, arithmetic and graphs" and quantitative reasoning as "the application of logic to problems and the ability to understand the real-world meaning of numbers and mathematical statements," noting that these terms refer to "end-members of a continuous spectrum of quantitative concepts." In this chapter we use the term QL/QR to refer to either "quantitative reasoning" or "quantitative literacy." See Fisher's work in this volume for a thoughtful discussion of the definition of "quantitative literacy" as a societal construct.

you start in five years instead? What are the questions you should ask if you are considering refinancing the mortgage on your house?

There is a great deal of public conversation about students' inability to understand such questions and the importance of educating college students about finance. This conversation is fueled, in part, by the historic increase in student loan debt, which increased from $600 billion to $1.1 trillion from 2008 to 2014, and recently surpassed $1.4 trillion [4, 6].

There is hope of reversing this trend, as de Bassa Scheresberg explains in [3]: "Financial literacy is shown to increase young adults likelihood of having precautionary savings and planning for retirement, while it decreases the likelihood of using high-cost borrowing methods" (p. 2). These findings are echoed in [10] where researchers Nye and Hillyard found "more quantitatively-literate consumers make more forward-looking financial choices" (p. 16). The financial industry has responded with some consumer education programs. Some of these recent financial literacy projects include Khan Academy's partnership with VISA called "Money Skills for Life,"[2], Decision Partner's "Financial Literacy 101,"[3] and The Actuarial Foundation's high school level "Building Your Future."[4]

Although financial literacy programs are a reasonable place to start, understanding debt requires much more than knowing the vocabulary and having facility with an online financial calculator, many of which typically only handle a single set of regular payments (one annuity). Students need to understand the key concept behind the calculations—the "time value of money"—meaning that they can only compare monetary values when viewed at the same (focal) date. They ought to be able to model the exchange of money using a time diagram and a couple of key formulas. Students would benefit from experience identifying and finding any missing information and asking critical questions. That is, we want students to understand the *mathematics* that drives personal finance decisions.

As mathematicians, we are uniquely positioned to help students develop these skills. As Steen argues in [12, p. 2]

> Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world.

We designed projects that expect students to examine their own personal finances through the lens of mathematics, reason carefully about choices and consequences of personal financial decision, and pose intelligent questions. These projects ask students to work with their own data—what they owe in student loans, where they might want to buy a house, how old they are, and how much retirement income they would like to have, among other possible personal connections. We have found that students are very interested in understanding their own personal finances, they take ownership of their projects and are excited to learn the necessary mathematics. We believe that this high level of engagement bolsters students' persistence in the course, especially as the calculations, concepts, and analysis get increasingly complicated.

Not surprisingly, when students are personally vested in the problems they are solving, this engagement translates into increased effort, willingness to persist through multistep processes, and an appreciation of the value of communicating their results. The idea of using contextualized problems to increase student engagement is not new. See, for example, Berns and Erickson's work in [2] and Frymier and Shulman's findings in [7].

There is another reason we chose the context of personal finance. In our experience, personal finance is a highly effective context in which to teach the deeper skills of QL/QR. One secret of the success of teaching QL/QR through the lens of students' personal finance will probably strike a familiar chord. Perhaps you have tried helping a struggling student understand percents or decimals. You suggest thinking about the mathematics in terms of money. Where $2.15 + 4.5$ seemed alien, now $\$2.15 + \$4.50$ is recognizable. Where 20% of 34 was a mystery, 20% of \$34 is last night's tip at a restaurant. Adding the dollar sign establishes a context, a reality against which students can evaluate their answers.

[2] See `www.practicalmoneyskills.com/personalfinance/experts/khanacademy/`.
[3] See `www.financialliteracy101.org/about.cfm`.
[4] See `www.actuarialfoundation.org/programs/youth/buildingyourfuture.shtml`.

10.2 History and Purpose

We first developed our projects for an introductory level QL/QR course focused entirely on financial mathematics that was taught by mathematics faculty. The course, called *Math of Interest* (and, later, *Introduction to Financial Mathematics*), was developed by our former colleague, Dr. Kenneth Kaminsky, who had experience with actuarial science. The course centers on compound interest, annuities, and life contingencies (such as life insurance and life annuities) and is based on Kaminsky's textbook *Financial Literacy: Introduction to the Mathematics of Interest, Annuities, and Insurance* [9].

We have also used these projects in a QL/QR course taught out of Bennett and Briggs' classic *Using and Understanding Mathematics* [1], a mathematics for liberal arts course taught out of Tannebaum's *Excursions in Modern Mathematics* [13], and we believe these projects would be suitable for a Finite Mathematics or Mathematics for Business course, for example taught out of Goldstein, Schneider, and Siegel's *Finite Mathematics & Its Applications* [8]. Those texts include financial projects but are limited in scope because they rarely venture beyond the present and future value formulas for annuities. To include our more realistic projects, some additional class time is needed to introduce modeling with time diagrams, the concept of focal date, and the general annuity formula.

In all of these courses we incorporate daily activities where students work with their neighbor or in small groups solving problems. We designed the projects to be unique to each individual student to ensure that each student demonstrates QL/QR. The financial projects we describe in this chapter are designed to help reach our key learning objectives from that course syllabus, which are to:

- Understand the **time value of money** through topics such as compound interest, nominal vs. effective rates, present and future values, focal dates, annuities including both loans (borrowing) and investment (saving), and
- Strengthen their **quantitative literacy and reasoning skills** including evaluating the reasonableness of answers, explaining their reasoning, and posing good questions.

Research suggests several key ingredients for teaching QL/QR. A summary by the Numeracy Infusion Course for Higher Education [14], for instance, recommends that courses should:

- Include real-world applications
- Use active learning and discovery methods
- Pair QR instruction with writing and critical reading
- Use technology, including computers
- Incorporate collaborative instruction and group work
- Use pedagogy sensitive to differences in students' culture and learning styles and
- Scaffold the learning process and provide rich feedback and opportunities for revision.

Our projects align with the recommendations. First, they are based on real-world scenarios. To complete each project, students must model fairly complicated situations and creatively combine strategies they have learned. We expect students to explain their calculations and results in writing. They need to use computational technology, address rounding errors, and evaluate information from internet sources, among other things. We spend some class time having students meet in small groups to compare progress and share results. Each project has two parts, allowing us to give feedback midway through the process and giving students the opportunity to revise their work. We believe all of these factors contribute to students strengthening their quantitative skills.

10.3 The Money Projects

In this section we describe the structure of the projects and their content. Each project has two parts—Part A where students find the relevant data and pose questions, and Part B where students answer the questions and reflect on their findings. Students submit and get feedback on Part A before they begin Part B. We use the same set of instructions for all of the projects, which have been revised over the years.

Part A Instructions: Your assignment is to write the story for your project by changing all of the underlined names and numbers in the story you are given to describe you and your family (real or fictitious). Update the footnotes to show the sources you used to obtain realistic data: credible, up-to-date websites are fine. You are not asked to solve the problem, but rather just state it. Include your version of the story and the instructions and questions, changing the names and numbers there too. Feel free to be creative, but be sure to use realistic numbers. Include at least one additional question that you would like to answer. The goal of Part A is to investigate current interest rates and other realistic data, as well as to develop your ability to pose clear quantitative questions.

Part B Instructions: Your assignment is to first fix Part A based on instructor feedback, if necessary. Next, answer each question, showing step-by-step solutions, justifying your models, and explaining how you chose the appropriate formulas in each step. Provide a synopsis of your findings, explained so that a friend who is not in this class could understand. Add at least one additional question that you would like answered. You are not asked to answer it, and it is okay if you do not know how to answer it. Conclude your paper by reflecting on your findings. For example, are your answers reasonable? Did anything surprise you? What was the most interesting or useful part of the project? Is there anything you have done differently? What did you learn? The goal of Part B is to give you practice solving realistic problems and to develop your intuition for real costs.

Over the years we have created many projects; we present four favorites here. Project topics we have used that are not listed here include managing credit card debt, deciding between buying and leasing a car, and questions about life insurance and retirement annuities. The last topics require additional mathematics beyond what those presented here since they involve "life contingent" payments (meaning some payments are made only if the person is living or, in the case of life insurance, only upon the death of the insured).

As a reminder, each student replaces all underlined information with their own data. Students are permitted to use realistic fictitious data if they prefer to maintain privacy.

Project 1: Paying Back Student Loans

Tydan is now a junior in college and hopes to graduate next year on May 1, 2019. He borrowed $10,000 in his first year, $8,500 his second year, and $9,800 this year, each on September 1. He anticipates needing to borrow only $5,000 next year. He is paying 5.94% APR interest, which accrues immediately. His first payment is not due until 6 months after he graduates.

(a) Draw a time diagram showing the amounts Tydan borrowed.

(b) How much will Tydan need to pay each month, assuming he takes 15 years to pay back the loan?

(c) If Tydan pays an extra $50 per month, how long will it take him to pay off the loan?

Sources cited: Student loan interest and terms: `/www.wellsfargo.com/student/loans/#undergraduates`

Project 2: Saving for Retirement

Elena just got her first job and is beginning to think about saving for retirement. She's 27 years old and expects to retire at age 67. She expects to live until age 86, but would like to purchase an annuity when she retires that will cover her expenses until age 96. Right now she has saved $0. She would like the retirement annuity to provide $1,500 a month for 30 years based on a 2% APR and she expects to receive another $2,000 a month in Social Security payments. (So, together she will have a retirement income of $42,000 a year.)

(a) According to a retirement calculator, how much money will Elena need at the time she retires?

(b) How much will Elena need to save each month, assuming she starts now? Experts suggest using an anticipated interest rate of 5% on a retirement savings account.

(c) If Elena waits and starts saving in 5 years instead, how much will she need to save each month?

Sources cited:

Life Expectancy calculator: www.ssa.gov/cgi-bin/longevity.cgi

Standards & Poor's retirement calculator: fc.standardandpoors.com/sites/client/tda/tdap/calculator.vm?siteContent=5196&topic=5035

Project 3: Buying a House

Rami and Kaab are hoping to buy a house. Rami works as a lab technician and earns $35,000 per year. Kaab works 35 hours a week at a clothing store and earns $9.79 per hour. Interest rates on a 30-year mortgage are low, around 3.821% APR. [Do not change 30-year term.] Their family has given them $15,000 to put towards a down payment on a house.

(a) Assuming that Rami and Kaab spend one third of their monthly salary on a house payment, and that 25% of that payment goes to taxes and insurance (so that leaves 75% to pay the principal and interest of the mortgage), how much can they afford to pay each month in mortgage?

(b) What price house can Rami and Kaab afford? Do not forget to account for the down payment.

(c) The are interested in a house on the Eastside of Saint Paul near Kaab's sister, where the median price of a house is around $145,000. Can they afford that price? If they cannot afford it, how much more a month would they need to be able to pay in order to afford it? If they can afford it, could they afford to have a 15-year mortgage instead? [Include an estimate of current 15-year mortgage interest rates, with citation.]

Sources cited:

Lab tech salary: www.glassdoor.com/Salaries/lab-technician-salary-SRCH_KO0,14.htm

Retail clothing salary: www.payscale.com/research/US/Job=Retail_Sales_Associate/Hourly_Rate

30-year mortgage interest rates: www.usbank.com/home-loans/mortgage/mortgage-rates.aspx

Payne-Phalen median price: www.zillow.com/payne-phalen-saint-paul-mn/home-values/

Project 4: Refinancing a House

Rami and Kaab bought a townhome in White Bear Lake, much closer to Rami's work, for $213,900 and put $15,000 down on a 30-year mortgage at 3.821% APR. [Use same interest rate as Project 3.]

(a) What is their monthly payment for principal and interest? Adding 33% for taxes and insurance, what is their total monthly payment?

(b) That was 5 years ago. What do they currently owe on their mortgage?

(c) Rami and Kaab are considering refinancing their mortgage. They are offered a new 30-year mortgage at 3.321% APR [Use your story's rate $-.500\%$]. The refi costs 3% of their current mortgage (i.e., they will borrow 103% of what they currently owe). What would their new monthly payment for principal and interest be?

(d) Should they refinance? As part of your work, calculate the monthly payment (for principal and interest) if they were to pay off the refinanced loan in 25 years (instead of 30), which is how long much longer it would take to pay off their current mortgage.

Sources cited:

White Bear Lake median price: www.zillow.com/white-bear-lake-mn/home-values/

Cost of refi: www.federalreserve.gov/pubs/refinancings/default.htm

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In Part A of each project, students write the questions they will answer later. While we want students to create a project that is relevant to them, keeping the projects within a common framework means we can realistically grade all different projects. Students need to research current interest rates, loan terms, housing prices, savings plans, and other realistic financial data. This touchstone with real world numbers balances textbook examples which understandably cannot be updated frequently enough to stay current.

We grade Part A, giving feedback and suggestions if any of the questions they posed are too vague or missing information, which allows students to revise their questions before they attempt to answer their questions in Part B of the project. Another advantage to assigning Part A separately and early in the course is that students tend to write questions they are naturally curious about, not limiting themselves to questions they have seen how to solve in class. Students report that each Part A typically takes under 30 minutes per project, so we are comfortable assigning it alongside regular homework during the first month of the semester.

In Part B of each project, students apply the mathematics they have learned to answer their own questions and interpret their results in the context of the original problem, including justifying the reasonableness of their answers. We expect them to clearly communicate their reasoning and to offer a clear summary of their findings. Students report that Part B takes up to four hours, depending on the project, so we normally give students a couple of weeks to complete each Part B.

In between Part A and Part B of each project, we introduce the mathematics necessary to solve Part B and ask students to work in pairs on sample problems that mirror some of the key steps they will need to complete their project. We reserve 10–15 minutes in class for students to compare their stories in Part A and to discuss common strategies for solving Part B. During the class before Part B is due, we often pair students for another 10-15 minutes to share their preliminary results and to discuss the reasonableness of their answers. We find taking this time in class helps students know if their work is on the right track, or to correct it.

10.4 Notes on Grading

With each of our twenty-five to thirty students submitting a different version of each project, it takes some time to grade. To speed up the process, we set up a spreadsheet template with the original story information and relevant calculations for each step of the given questions. That way we can enter and check each student's calculations. It also helps to use a rubric, such as one like the following.

Part A Sample Grading Scale:

The story includes new names, numbers, and other requested information.	____ /5
The new numbers are realistic and detailed.	____ /10
Information comes from credible sources and proper citations are included.	____ /10
The story includes at least one additional question that is clearly stated and includes all necessary information. *Bonus points for adding additional/creative questions.*	____ /10
The paper is neatly typeset.	____ /5

We allow students to submit Part A late, or to revise and resubmit a corrected version with a small penalty if there were serious errors in order to make sure that students have Part A sufficiently complete and realistic before beginning Part B. While this step adds a little time to grading, that time is recouped by having more sensible papers to grade in Part B.

Part B Sample Grading Scale:

Part A is attached as the cover page. *Any concerns raised by the instructor have been corrected.*	____ /5
Each question is answered correctly with detailed calculations and explanations. (We use a more detailed project-specific grading scale for this portion of Part B.)	____ /30
There is a clear summary of findings and thoughtful reflection.	____ /10
The reflection includes at least one additional question that is clearly stated and includes all necessary information.	____ /10
The paper is neatly presented. Part A and the reflection are typeset. The solutions are typeset or neatly handwritten.	____ /5

On occasion a student submits Part B without having adequately addressed concerns raised about their Part A, or they begin their analysis with a flawed model or serious miscalculations. In such circumstances, rather than grade the project we have found it simpler to return the project ungraded to the student with an opportunity to revise and resubmit.

10.5 Student Learning

Students like the projects and recognize the value of the material. In their own words (taken from representative comments in our end-of-term course evaluations):

> "The material is extremely relevant to any college student. It should be a required class."
> "The projects gave us a chance to do work that wasn't just quizzes and exams and forced us to apply our knowledge to real-life situations."

We believe that students have much a much deeper understanding of the mathematical concepts and are much more accurate in their calculations because of the time they invest in these projects and their personal interest in the results. Moreover, their analysis of their findings nearly always show solid QL/QR; they are able to tackle complicated problems, articulate their reasoning, interpret and evaluate their results, and pose good questions. As Edwards, Melfi, and Satyam in this volume explain: "By critically reasoning about quantitative information, we mean that students in both courses move beyond solving procedural problems and spend significant time engaged with tasks that are more conceptual." In this way, the projects are a more rigorous experience than more procedural projects typically found in textbooks.

One challenge in teaching QL/QR courses is that students tend to focus on getting the right answer. We stress the importance of explaining how they calculated their answer and why they believe their answer is reasonable and correct. Since students are working with different data than their neighbor, they will have different answers, and so this sort of explanation is natural for them. Some students benefit from extra encouragement to slow down and deliberately think through their process—both *what* they are doing and *why* they are doing it. For example, a project might include interest described as an annual percentage rate while the model uses a monthly rate, so they need to convert appropriately before proceeding. Students are apt to miss that level of detail if they are rushing to an answer without thoughtful deliberation.

Another significant challenge in teaching about money is acknowledging the fundamental middle-class bias of most materials, including these projects. We both teach at schools that serve students from a wide range of socioeconomic classes and have struggled with making sure that our perspective does not exclude students from less (or more) affluent backgrounds. While such conversations about financial privilege can be uncomfortable, our experience shows students appreciate the conversation. For more insight, see Esmonde's work on how affluent students made sense of social justice issues embedded in mathematical activities [5].

With less affluent students, it is exciting to see a student who has never lived in a home their family owns learn how a mortgage works and to figure out how they might become homeowners. In this way financial literacy sits within the broader context of social mobility, as discussed by Craig, Guzmán and Harper in this volume.

> Learning mathematics is a means to a larger end—enabling groups of people, particularly those socially and historically marginalized in their societies, to thrive in an oppressive and unjust society (including mathematical spaces) while simultaneously providing them with the means to (use mathematics to) impact and reshape the world in which they live (p. 7).

10.6 Looking Ahead

In addition to using these projects as presented, instructors might use this format to create relevant projects in other topics. The fill-in-the-blanks story format and Part A/Part B structure could be adapted for applied measurement and geometry or comparing linear and exponential growth, for example. Another way we have used the projects is to assign only Part A of all the projects, with additional emphasis on creating more questions; we call that assignment "It's a Question of Money"—hence the title of this chapter. This strategy might be a reasonable way to test-drive the projects for a new instructor and can be followed by working through the solutions in class. Alternatively, one might ask future teachers to adapt this structure to create lessons for their future students.

It is deeply gratifying to see students making sense of the world using mathematics. We encourage you to make the mathematics relevant and hope you will incorporate these or similar projects into your courses.

Acknowledgements

A special thank you to Ken Kaminksy, Professor Emeritus at Augsburg University, for his persistent humor and encouragement. The mathematical primer included in the Appendix is largely paraphrasing his work. He taught us how to captivate student interest and develop students' QL/QR using the mathematics of finance.

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Appendix: a Quick Introduction to Financial Mathematics

Directing these student projects requires faculty to understand a little more about the time value of money than is normally presented in a short section of a Finite Mathematics or Quantitative Reasoning book. We offer a quick introduction here. For more in-depth explanation, see [9].

The value of money we borrow or invest changes over time based on the interest rate and terms. For example, if we invest \$100 at 3% interest compounded annually then in 5 years we would have $100(1.03)^5 \approx \$115.93$. Notice that if we borrow \$100 at 3% interest compounded annually then in 5 years we would owe \$115.93. To a mathematician, whether an exchange of money is called a loan or investment only depends on your point of view—the equations are identical.

Interest is often compounded monthly, but other terms such as quarterly are possible. That \$100 invested at 3% interest compounded monthly is worth $100\left(1+\frac{.03}{12}\right)^{5\times 12} \approx \$116.16$ in 5 years. If we write j_m for the **nominal interest rate** compounded m time periods a year, then its **effective annual rate** (or Annual Percentage Rate) is j_1 in the sense that paying j_m interest for m time periods a year is equivalent to paying j_1 annually. If we equate the cost of a \$1 loan for 1 year we get

$$j_1 = \left(1 + \frac{j_m}{m}\right)^m - 1.$$

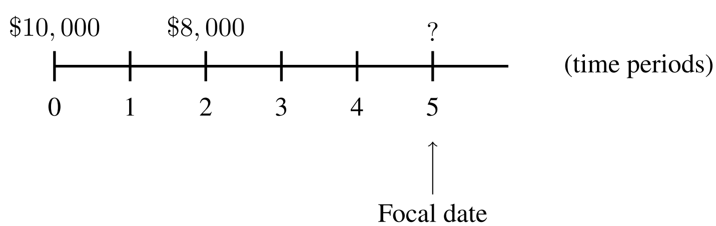
So, for example, $j_{12} = 3\% = .03$ has effective rate $j_1 = \left(1+\frac{.03}{12}\right)^{12} - 1 \approx 0.0304 = 3.04\%$.

When the compounded period is understood, we write $i = j_m/m$. The key idea is that $\$P$ at a particular time is equivalent to $\$A$ after n time periods as given by the equation $A = P(1+i)^n$. To reconcile several (or many) different payments, we need to set a fixed **focal date** at which all payments are valued.

For example, suppose we borrowed \$10,000 and then two years later we borrowed \$8,000, both at 3% interest compounded monthly. If three years later we want to repay the debt in full we would owe

$$10{,}000\left(1+\frac{.03}{12}\right)^{5\times 12} + 8{,}000\left(1+\frac{.03}{12}\right)^{3\times 12} \approx \$20{,}368.58.$$

It is often helpful to draw a **time diagram**, especially if several quantities are involved.



Many applications involve an **annuity**, a series of n repeated payment of a fixed amount of money $\$P$. At the focal date of the first payment, this series is worth (the geometric sum)

$$P + P(1+i)^{-1} + P(1+i)^{-2} + \ldots + P(1+i)^{-(n-1)} = P\frac{(1+i)^{-n} - 1}{(1+i)^{-1} - 1} = P\frac{1-(1+i)^{-n}}{i}(1+i).$$

At the focal date of d time periods after the first payment, its value changes by a factor of $(1+i)^d$, so the value of that series of payments $\$S$ is given by the formula

$$S(P, i, n, d) = P\frac{1-(1+i)^{-n}}{i}(1+i)^{d+1}.$$

This formula works wherever the focal date is placed, since a focal date that occurs before the first payment can be represented with a negative value for d. This formula works in place of both the usual present value and future value of annuities formulas and is due to Ken Kaminsky.

For example, suppose we deposit \$500 per month at the start of each month for 30 years at 3% interest compounded monthly so $i = .03/12 = .0025$. Ten years later (meaning 40 years after the first deposit), we cash out the account at

$$S = 500\frac{1-(1.0025)^{-360}}{.0025}(1.0025)^{481} \approx \$394{,}141.94.$$

11

Yes, But Is It Rigorous? Similarities Between A Quantitative Literacy Course and Transitions to Formal Mathematics

Richard Abraham Edwards, Vincent Melfi, and Visala Rani Satyam
Michigan State University

11.1 Framing Our Work

Mathematics compares the most unique phenomena and discovers the secret analogies that unite them.

—Fourier [7, p. 7]

In this chapter, we discuss the quantitative skills and habits of mind that students develop in a Quantitative Literacy (QL) course at a large research university, and compare the student experiences in this QL course with the experiences of mathematics majors in a standard transitions to formal mathematics (transitions) course. The transitions course is offered to students who have completed some of the calculus sequence and who are preparing to take upper-division courses such as abstract algebra and real analysis. The QL course we discuss—like those offered at many universities—is intended for students who do not plan to major in a quantitative subject. Many of these students have not had successful experiences in advanced high school and beginning collegiate-level mathematics courses. In one sense, the population of students who take the transitions course is very different from the population of students who use the QL course to fulfill a general education requirement in mathematics. As Fisher (from this volume) might say, they are different communities, and their patterns of discourse with respect to quantities are quite different. Yet, as we will argue in the coming pages, there are shared characteristics related to thinking and reasoning that appear in both courses. We believe that awareness of these shared characteristics can inform further research on both QL and transitions to formal Mathematics courses, and also provides an argument for the importance of QL to science, technology, engineering, and mathematics (STEM) majors.

Many of us who are involved with QL/QR instruction grapple with the question of whether or not the content of these courses represents authentic, rigorous mathematics. The answer to that question depends largely on what we mean by rigor. In this chapter we argue that the shared characteristics between a QL course and a transitions course imply that students in our QL course (and by extension, students in many other QL/QR courses) have a genuinely rigorous mathematical experience.

In this chapter, we discuss and analyze tasks from the QL and transitions courses as they exist at our institution (see [24] for a more specific description of the particular QL course). We chose examples from these courses because we were significant contributors to the design of the QL course, and two of us have taught the transitions course.

We expect, based on our knowledge of QL/QR and transitions courses at other universities, that our conclusions are generalizable to many institutional contexts.

Our motivation for attending to the parallels between QL and transitions to formal Mathematics is three-fold. First, we wish to add another counterargument to those who suggest that QL represents little more than watered-down mathematics for students who cannot handle a "real" mathematics course. Second, we wish to identify a few powerful ideas related to quantitative literacy that challenge students in both courses, thus indicating the importance of QL for all students, including STEM majors. Finally, we believe that by recognizing shared characteristics between a QL course and a transitions course, that mathematics education researchers could begin adapting research frames and methods related to reasoning and proof (a well-researched field) into QL (which has been less systematically explored).

11.2 Three Strands

We have identified three strands that characterize student experiences in both the QL course and in the transitions course. These strands describe ways in which students might engage with quantities: *critical reasoning and analysis*, *application of ideas to novel contexts*, and *effective communication*. In this section, we ground these strands in existing theories from mathematics education. In subsequent sections, we will show how these strands allow us to find common ground between courses, at which point we will be able to discuss future research that could enhance our understanding of quantitative literacy for all undergraduates.

11.2.1 Critical Reasoning and Analysis

By critically reasoning about quantitative information, we mean that students in both courses move beyond solving procedural problems and spend significant time engaged with tasks that are more conceptual. In their foundational work on mathematical tasks, Hiebert and Lefevre [13] noted that there are important differences between academic tasks that push students to make use of relationships and connections between different quantitative ideas, and tasks that require skill and proficiency with symbols and routines. Both types of thinking are valuable, and recent authors [15] have noted that procedural knowledge and conceptual knowledge support each other. Nevertheless, Hiebert and Lefevre [13] indicated that mathematics instructors tend to err on the side of implementing tasks that are heavily procedural. Few, if any, academic tasks are purely procedural or purely conceptual, but we believe that many classroom tasks lean toward one of those ends. For example, a task from the transitions course that is largely procedural is proving that if x and y are odd integers, then their sum $x + y$ is an even integer.

Teaching students to produce mechanistic proofs is not the end goal of the transitions course at our university. A major focus of the course is on understanding relationships and connections between different mathematical ideas. For this reason, we claim that the transitions course is a place where students engage in *critical reasoning and analysis*—a potentially sharp turn from the more procedural thinking many students employ in calculus. An example of a task that pushes student thinking into a more conceptual realm is:

Find the largest domain and codomain in $\mathbb{R}$ such that the function $f(x) = \cos(2x + \frac{\pi}{2})$ is bijective.

Ideally, completing the above task requires students to make connections among abstract mathematical ideas such as function, mapping, domain, codomain, and range. Students think about the graph of the function, the definitions of injective and surjective, and what the upper and lower bounds on the domain could be. This task is meant to engage students with more conceptual thinking than might arise in the previous example. Most students enter the course with solid understanding of the mathematical notions of even and odd, but few of them have thought deeply about surjective, injective, or bijective functions.

In practice, students may still approach this task in very procedural way. Upon completion of the problem, some will admit that while they know how to write the proof, they do not know what the proof *means*. Another goal of the transitions course is to motivate students to critically analyze their proofs, and not just rely on the kinds of procedural techniques that might have been successful for them in their calculus courses.

In comparison, very few of the tasks in our QL course focus solely on testing students' computational skills; instead, they favor just-in-time remediation of common trouble spots ranging from arithmetic to algebra. We want the student experience in the QL course to be significantly different from those in traditional mathematics courses such as college algebra, where the emphasis is typically on procedural routines.

One task from the QL course that exemplifies the push for critical reasoning involves a discussion of public health policies related to mandatory mammograms for women aged 35 and older. Students are given data related to the sensitivity and specificity of the mammograms, and use tree diagrams to determine the number of women out of a given population whose lives might be saved by mandatory screenings. This activity is not simply a lesson in calculating false positives—we intend students to wrestle with factors such as cultural norms, ethical obligations, and cost to the health-care system. Simply computing a theoretical number of lives saved by screenings is not the goal of the task, just as mechanically producing a proof about bijective functions is not the goal of the transitions task. Both tasks represent places where students can engage in critical reasoning and analysis related to mathematics.

11.2.2 Application to Novel Contexts

Another goal of both courses is for students to apply existing knowledge to solve problems in *novel contexts*. Part of operationalizing this goal means that instructors assign tasks that are (at least in their conception) characterized by high levels of cognitive demand [19, 12, 20]. Students often find such tasks to be highly ambiguous [3] because they cannot rely on known formulas for generating solutions.

We recognize that the cognitive demands of a task are not inherent properties of the task itself, but are determined by how students interact with the task [4]. For example, the cognitive demand placed on students in the transitions course to prove that a function is surjective depends on when the students confront the task. Early in the semester, when their ideas about functions are not well developed, the task may have a high level of cognitive demand. After they have successfully completed the course, however, the proof has a relatively low level of cognitive demand for most students.

As mathematics teachers, we tend to spend significant time exposing students to an increasingly wide variety of problems that they "learn to solve." As others have made clear [23, 11, 22], typical calculus courses can focus too much on finding and applying solution techniques, with little space left for other kinds of mathematical thinking. In contrast, both the transitions course and the QL course engage students with tasks marked by high levels of content-demand that cannot be lessened simply by learning a new solution method. In both courses, we aim to give students daily opportunities for rich mathematical thinking, characterized by making conjectures, generating their own examples, finding counterexamples, and engaging in effective, logical argumentation. Importantly, we want students to be able to take knowledge learned in one context, and apply it in a novel one. In particular, we want students to see how similar mathematical ideas are often woven throughout many different contexts. Few problems in either course have straightforward computational approaches that, once learned, might lower the cognitive demand of future tasks.

The notion of cognitive demand as described above is helpful in considering similarities between our QL course and our transitions course because it helps us move past thinking of mathematical tasks as easy or hard. Moving past such binary distinctions is important if we wish to think about the presence of mathematical rigor in a course. We contend that in both courses, students spend relatively little time mimicking previously encountered solution techniques, and that they spend significant time engaged with tasks that Selden and Selden [18] describe as very non-routine. Such tasks stand in stark contrast to the majority of very routine exercises that many of our students encountered in their previous mathematics courses.

11.2.3 Effective Communication

For many students, mathematical communication is itself a high-content demand activity. Both the QL course and the transitions course place serious demands on students in terms of making mathematical conjectures, reflecting and evaluating, and using appropriate language to describe their results. One framework that helps us classify these kinds of demands is Frith and Prince's [10] taxonomy of quantitative literacy demands of classroom tasks (Table 11.1).

This framework has been used by international scholars to analyze the QL demand of tasks from a variety of disciplines, including civil engineering, finance, and medicine [9, 14, 16], and was used by Edwards [6] to interpret the connections that students make between tasks in a QL course and tasks in subsequent general education science courses. A central idea behind the framework is that most quantitative tasks, regardless of discipline, require students to engage one or more levels of quantitative literacy when they interact with the task.

Table 11.1. Framework for analyzing the quantitative literacy demands in a quantitative literacy event. Adapted from [10, p. 89].

Knowing	• Knowing the meanings of quantitative terms and phrases (verbal representations) • Knowing the conventions for the symbolic representation of numbers, measurements, variables, and operations • Knowing the conventions for the representation of quantitative information in tables, charts, graphs, diagrams, and objects (visual representations)
Identifying and distinguishing	• Identifying connections and distinction between different representations of quantitative concepts • Identifying the mathematics to be done and strategies to do it • Identifying relevant and irrelevant information in representations
Deriving meaning	• Understanding a verbal description of a quantitative concept/situation/process • Deriving meaning from representations of data in context • Deriving meaning from graphical representations of relationships • Deriving meaning from diagrammatic representations of spatial entities • Translating between different representations
Applying mathematical techniques	• Using mathematical techniques to solve a problem or clarify understanding—for example: calculating, estimating, measuring, ordering, modeling, applying algebraic techniques, etc.
Higher order thinking	• Synthesising information or ideas from more than one source • Logical reasoning • Conjecturing • Interpreting, reflecting, and evaluating
Expressing quantitative concepts	• Representing quantitative information using appropriate representational conventions and language • Describing quantitative ideas and relationships using appropriate language

In the QL-demand framework, tasks are evaluated in terms of a taxonomy ranging from simple knowing (e.g., knowing the definition of words or the conventions of notation in a problem) through applying mathematical techniques, and then to higher-order thinking (reflecting, conjecturing, evaluating) and expressing quantitative concepts. As an example, the bijective function task referenced earlier asks students to understand the meaning of several terms, to apply mathematical techniques, and to use logical reasoning to complete the task. Depending on how the task is presented, it even allows for students to make their own conjectures related to the appropriate codomain, and to represent their conclusions using appropriate conventions and language.

Similarly, the aforementioned mammography example could require higher-order thinking, as students synthesize information from a variety of health reports, interpret the results of tree diagrams, and reflect critically on claims related to mandatory screenings. The task can also push students to express quantitative concepts related to absolute and relative risk using appropriate mathematical conventions and language. Throughout the transitions course, we push students to make their own conjectures and prove them (e.g., conjecturing a closed-form expression for a recursive relationship and then proving it using induction), which can also elicit higher-order thinking.

One might reasonably expect students in a QL/QR course to engage with multiple levels of QL-demand, and indeed this is the case. However, students in our transitions course regularly engage with all six forms of QL-demand that appear in the Frith and Prince taxonomy. The amount of QL-demand that appears in a transitions to formal mathematics course suggests that students in STEM majors may actually benefit from more direct instruction in QL.

11.3 Two Representative Tasks

To illustrate how the three stands of *critical reasoning and analysis, application to novel contexts*, and *effective communication* characterize tasks in both courses, we first present a representative task from a single lesson in the QL course, followed by a task of similar length from the transitions course. These examples were chosen because they make all three strands easily apparent, and are typical of the kinds of thinking we ask students to engage with in both courses.

Table 11.2. Fairness criteria. Adapted from [1, p. 651].

Criterion 1	If a candidate receives a majority of the first-place votes, that candidate should be the winner.
Criterion 2	If a candidate is favored over every other candidate in pairwise races, that candidate should be declared the winner.
Criterion 3	Suppose that Candidate X is declared the winner of an election, and then a second election is held; If some voters rank X higher in the second election that in the first election (without changing the order of the other candidates), then X should also win the second election.
Criterion 4	Suppose that Candidate X is declared the winner of an election, and then a second election is held; If voters do not change their preferences but one (or more) of the losing candidates drops out, then X should also win the second election.

11.3.1 The Alternative Voting Task

During a module entitled *Politics and Voting*, our QL students study alternative voting systems (e.g., Plurality, Single Runoff, and Borda Count) and then use those systems to model election results. In one rendition of the module, a task asked them to consider fairness criteria (mathematical tenets that are supposed to hold true for a voting system to be considered fair), and then assess the results of one election using different voting systems. In particular, the task asks students to consider a major question: Is our method of conducting elections in the United States fair?

We present students with the results of four simple elections involving a small number of candidates. Using the number of votes, students compute the winner of the election using plurality, single runoff, and other methods. Most students come to understand that, once a voting system has been selected, there will always exist the possibility of an election result that violates one or more of the fairness criteria (see Table 11.2).

One goal of the task is to challenge students' thinking about what fairness means, specifically from a mathematical standpoint. The fact that plurality voting may satisfy all four fairness criteria in one election, but violate the second criterion in a different election, is a source of cognitive conflict for many students. Students typically ask, is there any voting system that can always be sure to uphold all four tenets of fairness? This leads to a discussion of Arrow's impossibility theorem.

In the task described above, students *critically analyze* quantitative information, including tables of data as well as the rules for determining the winner of an election. They *apply* existing quantitative skills learned earlier in the course (e.g., proportional reasoning and the logic of conditional statements) to a novel context. Finally, by extending their thinking to a discussion of whether elections in the United States are fair, students must effectively *communicate* their ideas to help others understand the relative appropriateness of different voting systems.

This task can be the source of powerful learning opportunities as students begin to see rather deep complexities related to elections, and realize that any "solution" actually creates other problems. They begin to understand how viewing the world through a quantitative lens can be a useful way of approaching a wide variety of complex issues. In classroom discussions, deeper issues often emerge related to power, justice, and culture. The task is meant to make students more capable of reading and writing their words through mathematics [8].

Having seen how the three strands of *critical reasoning, application,* and *effective communication* weave together through a QL task, we now turn to a similar presentation of these strands in the transitions course.

11.3.2 The Defective Chessboard Task

The transitions to formal Mathematics course is meant to saturate students in abstraction, problem solving, and proof. Our goal is for students to learn to think independently, develop abstract concepts and tools related to higher mathematics, and express themselves clearly through mathematical proof. Tasks in the course emphasize the strands of *critically analyzing information*, *applying ideas to novel contexts*, and *effectively communication*. Early in the semester, students work through a standard proof:

For every positive integer n, $1+2+3+\cdots+n=\frac{n(n+1)}{2}$.

They struggle with the phrase "for every," decide what value of n should be used for the basis case, learn to phrase the inductive hypothesis, and proceed to follow a straightforward, formulaic proof. After students have seen similar

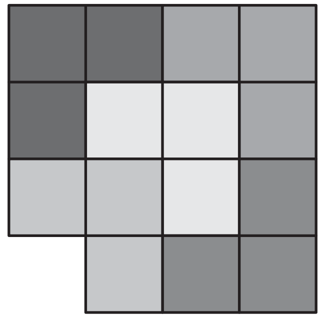


Figure 11.1. Example of a defective chessboard of size 2^n by 2^n for $n = 2$, tiled with five colored triominoes.

inductive proofs, they are presented with a novel (to them) problem such as the following, which is illustrated in Figure 11.1:

A 2^n by 2^n square grid is called a chessboard. Such a grid with exactly one of its squares removed is called a defective chessboard. For any natural number $n > 1$, prove that any defective chessboard can be tiled completely by L-shaped triominoes, each of which covers exactly three squares.

On the surface, the defective chessboard task may not seem to share much in common with the alternative voting task described above. When we consider the tasks in light of our three strands, however, interesting parallels appear. The defective chessboard task requires that students *critically analyze* the quantitative information in the problem. Many students engage with the task by generating examples of defective chessboards, realize that a wide variety of defects exist, and begin to tile the squares in a variety of ways. Among other things, they confront the idea of an abstract $2^n \times 2^n$ chessboard, interpret what the task is asking them to actually *do*, and choose an appropriate technique for proving the claim. Students must then *apply existing knowledge* in a *novel context*. They know they need to show the basis case and write the inductive hypothesis, but what is the basis case? Developing the inductive hypothesis is especially challenging, and once students have written it, they still struggle to see how to *use* it in the proof. Even students who are adept in writing inductive proofs are challenged when it comes to *effectively communicating* their ideas through this proof.

The task can move students into mental spaces that can be messy, ill-defined, and creative. Students deal with complexities related to induction, consider how the "solution" to previous tasks might help (or hinder) their solutions to the new task, and begin to understand how mathematical induction can help us make sense of a wide variety of phenomena. Upon completion of the task, many students recognize that they are becoming capable of independent mathematical thinking, and some students begin to think of themselves as budding mathematicians.

The appearance of the three strands, *critically analyzing information*, *applying ideas to novel contexts*, and *effectively communicating* in both courses suggests that there are important shared characteristics between the QL and transitions courses. In the following section, we consider a brief set of ideas that flow from the presence of such similarities. Our goal is to help teachers and researchers involved with both types of courses begin to symbiotically adapt pedagogy and research.

11.4 QL and transitions: Similarities and Differences

11.4.1 Three Strands

One aspect of *critically analyzing* quantitative information in both courses involves the use of new mathematical vocabulary. Occasionally, mathematical words have meanings that are quite different from their everyday definitions [5]. Examples of such words that appeared in the tasks discussed above include *fairness, arbitrary, any*, and *every*. In other tasks from the QL course, students interpret phrases such as "Participants who took [Drug X] were twice as likely to exhibit negative side effects than those who took a placebo." Many students interpret the phrase *twice as likely* in terms of absolute risk, thinking that if someone is twice as likely to experience negative side effects, then that person's chances of experiencing negative side effects must be high. On the contrary, a large relative risk may not be dangerous, provided the absolute risk is very low. Helping students understand distinctions between terms such as *absolute* and *relative* is an important strand in the QL course.

One challenge of *applying existing knowledge to new contexts* that arises for students in both courses is a tendency to focus on aspects of the task that are not salient. For instance, when students take their existing knowledge of mathematical induction and try to apply it to proving the defective chessboard proposition, many will search for an equation that they can manipulate, simply because this technique was useful in an earlier proof about the sum of the first n natural numbers. In the QL course, students are asked to determine whether or not mandatory mammography for all women should start at age 35. Many of the students will use their existing knowledge of tree diagrams, sensitivity, and specificity to claim that early mammography saves lives (a strong, but not infallible claim). In a sense, however, the claim is incomplete. In their rush to focus on the numbers at the bottom of a tree diagram, many students fail to focus on the larger question of whether mandatory mammography for relatively young women is a sound policy.

In terms of *communicating* their reasoning to others, both courses make students think about style as a new criterion for correctness. By style we mean the effective, efficient communication of information. In the transitions course, students rarely know how much (or how little) to include in a proof, aiming to include enough information so as not to mystify the reader, but not wishing to include extraneous commentary or examples. Furthermore, we frequently find that many students do not know when their proofs are finished, and begin to retread previous ground. In our QL course, we expect students to develop a knack for making reasonable arguments based on data, to attend to underlying assumptions, and to convince others of their reasoning. Stated differently, we want QL students to think about how much data to include in an argument, who their intended audience is, and when to appropriately conclude their argument.

11.4.2 Instructional Contexts

Many college students have been conditioned by years of mathematical experiences to think in particular ways about mathematics. Students in the transitions course are often very skilled at solving computational calculus problems, especially when the problems have straightforward solution techniques. Students in the QL course are also used to thinking about mathematics in specific ways. Many of them see mathematics as a set of rules, procedures, and formulas and many view problem solving as a search for the right rule. These ideas about mathematics are disrupted by the tasks in both the QL and transitions courses.

Said differently, the instructional context [4] of both courses is different from the instructional context with which many students in the courses are familiar. Instructional context refers to aspects of the teacher-student relationship, including the set of expectations and norms communicated by the instructor, which shape students' interpretation of a task. A common norm in freshman calculus courses is that the problems have straightforward computational solutions. If a student gets stuck, they can expect help from the instructor, and such help often takes the form of a mathematical technique, a reminder of a derivative rule, or possibly the discovery of an arithmetic error. The expectations and norms in the transitions class represent a potentially sharp break from the students' previous experiences. Similarly, the instructional context of the QL course differs from most students' previous experiences in a mathematics course. For many QL students, that course is the first time that they are expected to see and make their own connections between multiple quantitative ideas, and to begin viewing the world through a quantitative lens.

11.4.3 Differences

We recognize that there are differences between tasks in the QL course and those in the transitions course. For one, tasks in the transitions course place more emphasis on techniques than on deep understanding of specific content such as algebra or number theory. Our hope is that students become comfortable with proof techniques so that they can better focus on the *ideas* in their upper-division courses without having to learn proof techniques simultaneously. In the QL course, contexts are foregrounded (e.g., fair voting) and quantitative techniques are developed to understand the context. Furthermore, in the QL course, a significant emphasis is placed on questions that are either open-ended, or have more than one promising solution. For example, students might learn a technique for modeling rising ocean temperatures, but the technique by itself does not tell them what society should do to combat global climate change. The students must take the additional step of discussing how a quantitative result might affect public policy. In the transitions course, students generally do not wrestle with problems of that type. In one sense, solutions to tasks in the transitions course are more definite. Nevertheless, in both courses, students transition away from computationally-driven mathematics into experiences that require reflection, conjecturing, abstraction, and evaluation, hallmarks of higher-order quantitative thinking [10]. They learn to analyze quantitative information, and apply their knowledge and skills to solve problems in novel contexts.

Having said all this, one might ask: Does it matter that we can find and describe similarities between a transitions course and a QL course? We argue yes. One reason for showing that two objects are similar is so that we might apply what we know about one object to the other. In the final section, we suggest how the similarities we have uncovered could allow researchers and educators to draw from methodologies related to reasoning and proof in order to improve QL courses.

11.5 Implications for Research

As the number of QL courses in colleges and universities rises, so too will research questions surrounding teaching and learning with respect to those courses. In coming years, we expect to see the continued development of exciting and innovative theoretical lenses that help us make sense of a myriad of aspects of QL instruction. While developing totally new frames may be important for conducting QL research, we suggest that it may also be fruitful to adapt existing frameworks from related areas. Perhaps some frameworks could be translated directly, while others could serve as skeletons on which to build new frameworks.

We believe that there are enough similarities between transitions courses and QL courses to justify borrowing existing frameworks currently used to make sense of issues related to the former in order to help us conduct QL research. Of course, there will be aspects of those frameworks that will not translate neatly to research on QL. That challenge is part of what makes our work enjoyable. Nevertheless, existing frameworks have the advantage of being well-established, field-tested, and refined through use.

There are dozens of excellent frameworks that may serve as examples. One framework that might be adopted for analyzing aspects of QL instruction is Stylianides' [21] analytic framework for reasoning-and-proving, conceived in three components: the mathematical, the psychological, and the pedagogical. Looking at this framework in Table 11.3 raises several questions for us as QL researchers: Under what conditions do we see students identifying patterns, making conjectures, providing examples, or making an empirical argument? What types of rationale do students in a QL course tend to give for their arguments, and what changes can we detect in types of rationale used from the beginning of a course to its end? What are students' perceptions of the mathematical nature of a pattern they see in real-life data? In what ways are QL students convinced (or not) by mathematical or statistical arguments, and which arguments do they find more convincing, and why? How can instructors make the mathematical nature of a real-life situation more transparent to students?

Table 11.3. Analytic framework for reasoning-and-proving. Adapted from [21, p. 10].

Mathematical Component	Making Mathematical Generalizations		Providing Support to Mathematical Claims	
	Identifying a Pattern	Making a Conjecture	Providing a Proof	Providing a Non-Proof Argument
	• Plausible Pattern • Definite Pattern	• Conjecture	• Generic Example • Demonstration	• Empirical Argument • Rationale
Psychological Component	What is the solver's perception of the mathematical nature of a pattern, conjecture, proof, or non-proof argument?			
Pedagogical Component	How does the mathematical nature of a pattern, conjecture, proof, or non-proof argumen becomet transparent to the solver?			

A second framework that we find intriguing for conducting QL research is the multidimensional problem solving framework described by Carlson and Bloom [2]. An underlying assumption of this framework is that the level of difficulty of a problem is both a function of the the task variables as well as a function of the problem solver (e.g., the student). This notion seems to fit well with the notion that the cognitive demand of a task is a function of how students interact with it. In the Carlson and Bloom framework, there are four phases in a cyclical problem solving process: orienting, planning, executing, and checking. The erienting phase is characterized by behaviors such as sense-making, organizing, and drawing pictures. The planning phase is characterized by making conjectures, accessing mathematical tools, and evaluating the appropriateness of different techniques. Next, the executing phase is characterized by computations, constructions, and drawing on conceptual knowledge, facts, or algorithms. Finally, the checking phase includes verifying and reflecting on the solution, which may result in a return to the planning phase. Adapting this framework to QL tasks could allow us to consider questions such as: What resources and heuristics do students rely on when solving QL tasks? How much time do students spend in the orienting or planning phases of a QL task, as opposed to the phase in which they check their solutions? What affective elements (e.g., beliefs, motivations, frustrations) surface during a QL task?

Finally, Savic [17] has argued that some problem solving frameworks, such as that proposed by Carlson and Bloom [2], should themselves be considered in light of Wallas's [25] four stages of mathematical creativity. Broadly speaking, Wallas suggested four aspects to creativity in problem solving: preparation, in which students try to understand the problem, incubation, a mental process where the problem solver mediates subconsciously on the problem, illumination, or the "AHA! Moment" and verification, in which students determine whether or not their solution is correct. Under the impression that we desire QL students to develop their creative capacities in terms of problem solving, we might attend to whether or not they are given tasks that allow for all four of these components. In particular, how do we provide students in a QL course with opportunities for creating their own ideas or for taking risks in terms of applying a quantitative lens to their world?

11.6 Conclusion

Uniting several theoretical frames around the ideas of *critical reasoning and analysis*, *application of ideas to novel contexts*, and *effective communication*, we have demonstrated shared characterstics between two mathematics courses that are typically thought of as quite different. Identifying shared characteristics helps to dispel the notion that quantitative literacy courses are not appropriately rigorous, and simultaneously helps to illustrate ideas that challenge students in both QL courses and transition to formal mathematics courses. Additionally, research frameworks that were developed for, and utilized in, proof-based courses may be useful in understanding QL courses. One of our goals is to advance a conversation about adapting research on proof and proving to research related to QL. We recognize that the relationship may in fact be bidirectional, and that that research surrounding QL may help us better understand phenomena related to other undergraduate mathematics courses.

Finally, we suggest that many of the major ideas in the QL course—quantitative skills, critical reasoning, developing a quantitative habit of mind, application of ideas to novel contexts, and communication exist in the transitions course. These core ideas likely arise in Calculus, Differential Equations, and many other courses. Thus, the value of quantitative literacy seems to be not solely for the non-STEM students in our colleges and universities. The connections we have suggested between QL and transitions courses may help us shift ideas about QL into many new contexts.

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Part III

Quantitative Literacy in an Institutional Context

After getting a sense of what QL/QR looks like in a variety of classrooms, we now explore QL/QR in the institutional context. Part III of this volume brings together several articles that display in plain view the range of issues and opportunities that accompany the institutional decision to develop and support a QL/QR program.

Amber Parsons, Matthew Salomone, and Benjamin Smith, in their chapter "Going Public: What Institutional Moments Bring Everyone to the Table?" review how quantitative literacy requirements and programs have sprung up at larger public institutions over the past few decades. They identify commonalities in the experiences of a wide range of institutions, offering Hamilton College as a successful case study. After this optimistic perspective we move on to "Reflections on Sustaining QL Course Innovations," by Jacqueline Dewar, Suzanne Larson, and Thomas Zachariah, where we consider some of the myriad obstacles that come up in the long term in creating a QL/QR course at a particular institution. The authors draw upon their own experiences as they explore why some innovations take hold and others do not, making it clear that setting up a QL/QR program is only half of the work: to create a sustainable program might require attention to many moving parts.

We next shift gears a bit and briefly consider QL/QR courses in the wider framework of college core curricula. In their chapter "Expanding Access to Quantitative Reasoning Courses," Amy Getz, Connie Richardson, Rebecca Hartzler, and Francesca Fraga Leahy argue that QL/QR courses could and should become the main gateway mathematics courses in the community college setting. They also address the role QL/QR courses could play in four-year colleges; all throughout, they frame the issue as one about access and equity.

With an institutional commitment to QL/QR comes the need for assessment. To this end, the last three chapters in this part address various aspects of assessment. We begin with "Reflections on the Assessment of Quantitative Reasoning," a theoretical article by Richard Shavelson, Julián Mariño von Hildebrand, Olga Zlatkin-Troitschanskaia, and Susanne Schmidt. The authors briefly reflect upon the nuances of various definitions of QL and QR, and propose ways to assess the construct based on their reflections. Ryan Zerr, in his article "Assessing Quantitative Literacy as a Cumulatively-Acquired Intellectual Skill," frames QL/QR as a cumulatively-acquired intellectual skill and concludes that its assessment must take this into consideration. In the context of postsecondary education, this means that assessment should be done near graduation, and can be done through the use of a performance task process, which Zerr carefully describes. In the last article in this section, "Assessing Quantitative Literacy: Challenges and Opportunities," by Semra Kiliç-Bahi and Andrew Cahoon, we get a broader perspective on the status quo with respect to QL/QR assessment. The authors share with us a list of eight possible paths for the future of QL/QR assessment, broad and significant challenges which will keep QL/QR practitioners busy for many years to come.

12

Going Public: What Institutional Moments Bring Everyone to the Table?

Amber Parsons, Matthew Salomone, and Benjamin Smith
University of Washington–Bothell, Bridgewater State University, Hamilton College

12.1 Introduction

Quantitative literacy (QL), as distinguished from mathematical literacy, has been part of the educational landscape since at least 1968 [31, 35], and the public mandate for teaching skills that we now identify as QL is at least as old as the 1970s [46, 29]. But the growth of programs, curricula, and support for QL in higher education has been especially dramatic in the 21st century, accelerated by both a more ready access to information about existing programs and by professional networks of support. Indeed, just as we see that QL itself entails engagement with the "intersecting quantitative practices of many different communities" (as argued by Fisher in this volume), we also see the convergence of practices in the diverse communities of educators engaged in the teaching and support of QL skills.

Despite the intellectual and professional leadership of organizations such as the Mathematical Association of America and the National Numeracy Network in defining what QL is and how it *may* be taught, supported, and assessed, until recently there have been few mandates that specify what, if any, QL *must* be present at a given institution. With little national, state, or system-level coordination, programs for quantitative literacy in higher education have mostly emerged organically with a wide diversity of scopes and sizes across institutions. Each institution has discovered and responded to its need for QL programming on its own timetable and in its own way [12]. Yet, like many processes of wide-scale institutional change, this has not been a methodical, incremental process on many campuses. Instead, QL programs tend to emerge from particular moments of crisis in an institution's life cycle. Because of the broad scope of quantitative literacy, the broad engagement of stakeholders from across disciplines and divisions required to initiate and sustain the work can be difficult to achieve in the absence of exigent circumstances.

The history of emergent programs in quantitative literacy seems to have been one of "going public," both on the long timescale in which programs that first emerged at small private institutions have been spread into larger sectors of higher education, and in the sense that at each institution, campus change agents have been required to engage many "publics" in the work. In this chapter, we identify and explore some of the more common types of exigencies that have led to the growth of QL programs at institutions and trace their prevalence through the literature, offering a case study of Hamilton College's program in order to illustrate the complex dynamics at play when all parties are brought to the table. We conclude with four questions for proponents of QL to consider when attempting to establish a QL program of their own that marries the "stable core" of quantitative literacy with the unique features of their institution's "shifting context."

12.2 Defining a “QL Program”

There are as many ways to design and implement a QL program as there are institutions who have or will do so. However, *Quantitative Reasoning for College Graduates: A Complement to the Standards* [39] defines a “QL Program” as having the following characteristics:

1) Explicit quantitative requirements for college entry or for entry into courses or experiences that can be credited toward the baccalaureate degree
2) Placement testing intended to help determine appropriate entry into the quantitative literacy program
3) Foundation experiences to be accomplished ordinarily within the first year of the student’s college work
4) Further quantitative experiences in diverse contexts to be accomplished during a student’s sophomore, junior, and senior college years so as to be interspersed throughout the work of these years.

In our examination of the literature, we did not find many explicit references to entrance exams or placement testing, though we assume these to be in place as they are standard at most institutions. Instead, we examined institutions that placed an emphasis on providing “quantitative experiences in diverse contexts” by expanding QL beyond the discipline of traditional mathematics.

12.3 Institutional Calls to Action around QL

Professional organizations such as the Association of American Colleges and Universities, the Mathematical Association of America (MAA), the National Numeracy Network (NNN), and the National Council of Teachers of Mathematics (NCTM) have each offered definitions of quantitative literacy that, because of the essential role played by contextual reasoning skills, emphasize its cross-disciplinary nature. Balancing the various needs and incentives of faculty (especially across disciplines), students, administrators, accreditors, and alumni is a difficult and often slow process that necessitates a “cultural approach” to institutional change that is responsive to the individual context in which each campus operates [25].

Overcoming institutional inertia to establish a QL program requires campus engagement from faculty and staff across various and somewhat disparate disciplines, a task that would theoretically prove more challenging at larger institutions with more requisite participants. Perhaps for this reason it is widely believed that the earliest QL programs appeared mostly in small, private colleges and that the movement grew to larger, public institutions later. Indeed, five private institutions are consistently cited as the inspirations for the development of QL programs at other schools [12]: St. Olaf College, Mount Holyoke College, Dartmouth College, Wellesley College, and Alverno College. Whether this reflects an actual trend of QL moving from private to public institutions, however, has not previously been investigated in the literature.

To test the hypothesis that QL programs were overrepresented in private institutions, the authors recorded the Carnegie classification of every institution whose QL program was referenced in the literature up to the 2006 publication of *Current Practices in Quantitative Literacy* (Table 12.1), and compared the proportion of public and private institutions with the overall national proportion using a chi-square goodness of fit test. Private, for-profit institutions were omitted as there were no descriptions of any QL programs in the literature from such institutions. Few community colleges were found in the literature prior to 2006 (see [18] for an exception), but Getz, Richardson, Hartzler, and Leahy in this volume offer further insights on more recent developments in two-year institutions. Private institutions comprised the majority (26 of 41, 63.4%) of institutions with QL programs in the literature reviewed. Compared to the national proportion (1731 of 3377, 51.3%), the difference in the proportion of public versus private, not-for-profit institutions in the literature from that in the country on the whole was not significant ($\chi^2 = 2.02$, $p = 0.1194$). Thus it seems that despite the widespread attribution of QL programs to work that originated in private institutions, the prevalence of implemented QL programs was representative of the national public/private distribution as of the 2006 publication of *Current Practices*. This finding shows that the end of the 20th century was a time of great growth that resulted in the propagation of QL programs in institutions of all types, but especially in those with public funding. In this period of growth, what conditions or moments in an institution’s life cycle have most readily given rise to the formal development of QL?

Table 12.1. Carnegie classifications of institutions with QL programs described or referred to in [12, 40, 31, 41, 43].

	Public	**Private Not-for-Profit**
Small	Farmingdale State University	Alverno College Babson College Clark University DePauw University Dickinson College Dominican University Hamilton College Hollins University Juniata College Kalamazoo College Lawrence University Macalester College Moravian College Mount Holyoke College Mount Mary College Point Loma Nazarene University Skidmore College Trinity College Wellesley College
Medium	The Evergreen State College University of Washington—Bothell U.S. Military Academy—West Point	Dartmouth College Rensselaer Polytechnic Institute St. Olaf College
Large	Appalachian State University Edmonds Community College Indiana University Northern Illinois University Oklahoma State University Sam Houston State University State University of New York—Stony Brook University of Massachusetts—Boston University of Nevada—Reno University of Tennessee—Knoxville Virginia Commonwealth University	DePaul University Massachusetts Institute of Technology New York University University of Pennsylvania

While conversations around QL may begin at a smaller scale as, for example, with the collaboration of faculty who teach statistics courses in order to improve student success [20], there are some institutional moments that inherently trigger the kinds of broader conversations necessary for large-scale change. Chief among these are proposals to revise requirements for graduation or general education. In nearly every case in the literature where a rationale for change was documented, the implementation of a QL program was preceded by a faculty review of the general education requirements. These reviews were precipitated by a variety of different events, from major changes in institutional identity, as in the cases of Farmingdale University [14] and Dominican University [6], to the ripple effects of smaller faculty conversations around graduation requirements, such as the former college algebra requirement at Virginia Commonwealth University [9]. Indeed, rarely has an institution brought a new QL program into being without eventually confronting extensive revisions to its general education requirements, with the program at the University of Nevada—Reno being an apparent exception [22].

In recent years, institutional accreditation has had an increasing role in necessitating work around quantitative literacy in general education. The Association of American Colleges and Universities' LEAP project has been highly influential in identifying QL as an essential competency for liberal education [44], and some multi-campus accrediting organizations have since identified quantitative literacy or quantitative reasoning as a core expectation for all undergraduate students [45]. Since reaccreditation is an ongoing institution-level priority that relies on partnerships between faculty and administrators, the role of the emergence of QL in the accreditation context cannot be understated in its ability to bring wide groups of stakeholders to the table.

However, accreditation has rarely been singled out in the literature as a primary motivator of curricular change. More often, institutions credit their faculty with having made a decision regarding the curriculum [8, 10, 4]. Given the growing prominence of QL in accreditation standards, we suspect that this external pressure may become increasingly responsible for precipitating such faculty decision-making in the future.

Additional sources of exigencies in the literature that have lead to the development of QL programs include those changes in the makeup of the student body of an institution. While this can occur gradually over time, as in with Hamilton College in the 1980s, these changes can also arise suddenly. Whether due to an institutional reclassification as with Dominican University [6], or a merger of institutions as Hamilton and Kirkland Colleges underwent in the 1970s [38], these moments necessitate a review of the purpose and mission of an institution. Consideration of curricular demands and needs follows from this and can provide an opportunity for the development of a QL program of some form.

12.4 How Institutions Fit their Response to their Needs

Pressure for stronger quantitative reasoning skills among college graduates has mounted in recent years, exerted on higher education by the needs of the skilled workforce [5], accreditors [45], and economic development organizations [32]. An institution's response to these needs, however, may originate in many corners of the campus. In some cases, administrators want external accountability for students' QL skills. In other cases, academic support centers face an unmet demand for tutors to work with students in QL-rich courses. Sometimes faculty members themselves—particularly outside of the mathematics department—lament students' apparent QL skill deficits or inappropriate requirements in hallway conversations [9]. Perhaps because of the variety in the sources of initiatives to develop new QL programs, the forms of institutional responses vary as well. Across the diversity of institutional responses, most will involve addressing graduation requirements, new QL courses and course components, learning outcomes assessment, support for student learning, and support for faculty professional development.

12.4.1 Graduation Requirements

While most schools have a standard for competency in mathematical skills as an entrance requirement, such as a minimum score on the SAT, many also require mathematical reasoning as a general education requirement. Such requirements are typically satisfied by a combination of exam scores or courses taken for credit at the institution. However, mathematical reasoning per se is an insufficient proxy for quantitative literacy [41].

The simplest course of action seems to be replacing a previously existing "mathematics" requirement with a "quantitative reasoning" requirement. As with mathematical reasoning, these requirements may be met by passing an exam, as at Juniata College [4], University of Massachusetts, Boston [28], or historically at Hamilton College [23]. Increasingly, however, institutions have developed a course or courses specifically to meet that end. Examples abound, like the "Problem Solving" course at Point Loma Nazarene University [21] or the "Quantitative Reasoning and Informed Citizenship" course at Moravian University [36]. These courses have supplemented and, in some cases, supplanted courses such as college algebra that traditionally focus to a greater extent on mathematical reasoning.

12.4.2 Development of QL "Overlay" Courses

While many examples of courses addressing foundational QL skills exist, on their own they are not sufficient for what Gillman [12] describes as a full QL program. A full QL program includes various implementations of QL embedded in context throughout the curriculum. To this end, many successful QL programs have developed courses on a myriad of other topics that infuse, interlace, or embed quantitative reasoning skills into those courses. These can be existing courses that get retooled or entirely new courses. A model originating at Wellesley College that has proven to persist is to offer a "little q" course oriented around basic mathematical and quantitative reasoning skills, and "Big Q" or "overlay" courses [42]. The Big Q courses are about topics within majors and disciplines, but have an emphasis on application of quantitative reasoning to these other disciplines. This model has been the inspiration for other institutions like Hollins University [8] and the University of Washington, Bothell [20]. This model has also been adapted to the needs of Skidmore College [41] and Lawrence University [16], with Big Q course following a traditional mathematics-type first course at Babson College [41].

12.4.3 Assessment

Wherever QL skills occur in a student's experience, there is a need to evaluate the extent of their success in those skills. Evidence-driven assessment of student learning outcomes is not present in the MAA's earliest definitions of the components of a QL program. Indeed, assessment culture in higher education is largely a 21st century phenomenon [30], and increasing expectations from accrediting organizations and state funding agencies have spurred assessment efforts from the course level to the institution level and beyond. The recent literature on assessment of quantitative literacy is rich and continues to grow. Efforts have been developed to measure these outcomes through skills tests [11], within QL courses and programs [43], within multidisciplinary portfolios of student writing [15], and within a "state of nature" where no quantitative prompt is given [2]. Assessment data can both inform campus conversations around QL skill attainment, as well as lead them, providing evidence of need for new supports, programming, and even new courses [23].

12.4.4 Support for Student Learning

Whether triggered by an external assessment, as found during reaccreditation or during a top-down review by a central state agency, or developed independently within an institution, acknowledgement of a need for developing student quantitative reasoning skills can arise. To address the need to support students' quantitative reasoning skills in a wide range of courses across the disciplines, many institutions have established campus-wide "quantitative support centers" (QSCs). These centers typically provide individual and small-group tutoring, with the operation of the center and the recruitment and training of its tutors typically overseen by a professional staff or faculty director.

The campus-wide nature of the offered support usually situates QSCs outside of academic departments and in proximity to other campus-wide learning supports such as writing centers. This autonomous model typified the structure of the quantitative reasoning programs in a survey of twelve small, selective private colleges [24]. QSC staff may also partner with academic departments in the coordination of quantitative reasoning curricula and course offerings, especially in cases where a faculty liaison or faculty director oversees the center, and in these cases may engage in leading faculty development as well [34].

12.4.5 Support for Faculty Development

Certainly any change in curriculum requires support from some or all of an institution's faculty, often requiring support for the faculty. Faculty support is offered in a variety of ways, typically depending on the changes being implemented at the institution. For the development of new courses and the overlaying of existing courses, stipends may be offered, frequently provided through National Science Foundation grants, such as at NYU, Sam Houston State University, and University of Nevada, Reno [22]. The Washington Center at Evergreen State College offers faculty-driven workshops, and has found bringing respected speakers from other campuses can help faculty across disciplines to see the value of quantitative literacy [20].

At the University of Washington, Bothell currently, the director of the Quantitative Skills Center meets individually with Interdisciplinary Arts and Sciences faculty to develop quantitative assignments that bolster and augment the instructor's qualitative goals. In one such assignment, students analyze the quantitative evidence from a scientific paper about gender and are encouraged to try different ways to visually represent the data. In a different series of workshops, students play an academic version of "The Hunger Games" [7] (a recent popular book series aimed at young adults) to get an intuitive feel for probability and risk. The students are assigned aliases from different "districts," in which in-class performance affects one's entries into a public quiz on course content. As another example, the director of the QSR Center at Hamilton College, with the support of a faculty advisory committee, has recently begun to assist faculty in the social sciences and humanities to develop and incorporate relevant QR content into pre-existing courses.

Bridgewater State University began its support for quantitative reasoning with a faculty development initiative called Quantity Across the Curriculum (QuAC). QuAC has given support to individual faculty who are teaching and developing courses with quantitative content in the disciplines as well as to institutional research and assessment efforts around QR. An absence of learning outcomes behind the QR requirement, however, has necessitated much work to create an institutional conversation around the nature of quantitative literacy and its complementary relationship to mathematical reasoning. Having this shared understanding will help inform future efforts to revise general education requirements.

12.5 A Complex Interplay of Responses: The Case of Hamilton College

While what has been described above is indicative of the most common practices, things are rarely so straightforward. We offer a bit of insight into the long history of QL at Hamilton College to illustrate how incorporating a truly cross-curricular program required multiple campus-wide conversations initiated by the types of exigencies discussed above.

Hamilton's early explorations into QL began as a response to factors both at the grassroots and the institutional level. Faculty members who were concerned about students' poor performance with quantitative skills, primarily in the economics department sought a grant in 1978 from the IBM Corporation to fund a multi-year skills assessment and remediation project [23]. This assessment-first approach is notably different from later programs at Hollins and Wellesley that redesigned course content prior to institutional assessment efforts.

Meanwhile, in the same year, previously all-male Hamilton College merged with neighboring all-female Kirkland College to form a coeducational institution. This change in the makeup of the student body was one motivator for examining QL according to those involved with the initial project [38]. As Hamilton's more traditional focus on the liberal arts met Kirkland's more contemporary emphasis on humanities, the merger of colleges necessitated a new integration of educational philosophies that led to the development of formal educational goals in the mid-1980s. These goals were implemented alongside the introduction of a "QSkills" exam in 1984, one of the recommendations arising out of the Quantitative Literacy Project funded by the IBM Grant. As such, this change was intrinsically motivated and faculty-driven, even in the presence of external catalysts.

Another recommendation of the QL Project was the establishment of a Quantitative Literacy Center, which was ultimately established in 1990. Its original mission was to support students preparing to meet the QSkills exam requirement, though it soon began to provide tutoring for students taking courses with quantitative content. By 1996, students could satisfy the QL graduation requirement by successfully achieving a minimum score of 50% on the QSkills exam, successfully completing a series of QLC tutorial sessions, or completing one of the credit-bearing courses for which the QLC provided tutoring support [23]. Starting with the class of 2005, the faculty voted to eschew distribution requirements in favor of an open curriculum, where students only need to achieve a certain number of credits in addition to the demands of their disciplinary concentration. The only exceptions were the writing and QL requirements. In 2004 the QSkills exam became fully optional, and in an attempt to embrace the open curriculum model adopted earlier in the decade, the exam was abandoned, along with the QL requirement, in favor of a Quantitative and Symbolic Reasoning (QSR) course requirement, which is still in place as of this writing.

The QSR designation expanded the notion of quantitative literacy by allowing courses containing adequate amounts of formal symbolic manipulation, graphical representation, as well as mathematical and statistical analysis to qualify as QSR. Students must now complete one such course before the end of their second year. These courses are found in a variety of disciplines that include philosophy, theatre, sociology, and dance and movement studies in addition to the traditional math, science, and economics. The rebirth of QL as QSR at Hamilton occurred just prior to another revision of the campus educational goals in 2011, which now include: "Analytic Discernment—analyzing information, patterns, connections, arguments, ideas, and views *quantitatively and symbolically*" [17, emphasis added]. The QL Center was rebranded as the Quantitative and Symbolic Reasoning Center, a source for tutoring students in designated courses and independent statistical research projects, as well as for helping faculty design quantitative course content.

Hamilton's own transition from assessment, to skills testing as a requirement, to course requirements reflects some of the more common exigent circumstances leading to the implementation of QL content, but with significant overlap in models of execution and a blurring of the lines between different models. The evolution of QL/QSR at Hamilton can serve as a reminder for those seeking to develop content at their own institutions that while it is useful to consider the most relevant factors contributing to and influencing the incorporation of quantitative literacy, the path in developing such a program can be long and far from clearly defined. Ambiguity notwithstanding, there are certain considerations worth taking when embarking on effecting change for the development and implementation of QL programming.

12.6 Considerations for QL Program Implementation

While descriptions of, and suggestions for, QL programs abound (e.g., [13, 39], and chapters in this volume), those aspiring to initiate a QL program may consider the following four questions:

1) What conversations currently, or could, occur at my institution that would bring together stakeholders from multiple disciplines and campus roles?

The revision of graduation requirements or a general education curriculum is overwhelmingly cited as the exigency giving rise to new or revised QL programs. Because graduation requirements affect the educational experience of all of its students, these requirements encode an institution's most deeply-held cultural values, such as what knowledge and skills it views as worth having, and therefore what social and practical value is assigned to the credentials it awards. These requirements also drive every aspect of the educational experience at an institution, from the courses offered by academic departments to the content taught by faculty members in those courses and the means by which the institution assesses that content and, by extension, assesses the faculty and the departments who teach that content. Revision of these requirements poses a unique opportunity to bring together stakeholders across all divisions of an institution.

Other conversations that bring people from multiple campus roles to the table may include those surrounding institutional image or branding, shifts in student demographics or demands, revised models of developmental math education, and how to meet accreditation standards, among others.

2) What role could or should our mathematics department play in the development and implementation of our QL program?

While QL requirements and programs are, by nature, defined, designed, and delivered as shared responsibilities across departments [13], mathematics faculty are crucial stakeholders in developing and maintaining such programs. The institutional momentum of a large mathematics department, and the expertise and engagement of its faculty, can provide a powerful push for QL program implementation. Finding champions within the department and clearly defining their role in the wider campus conversation are necessary steps to ensure this push occurs in a forward direction.

As a rule, mathematicians are keenly aware of the distinction between mathematical reasoning and quantitative reasoning, and an initiative to elevate one may be received as a threat to the other. At least one faculty champion within the department is essential to speaking to his or her colleagues' apprehensions on the one hand, and ensuring on the other hand that the voice of the department is present in the design of QL courses and programs. These liaisons are sometimes, but not always, found among mathematicians with disciplinary specialties in applied mathematics or statistics. They may also be faculty members in any subfield of math who are highly engaged with the institution's general education program.

Defining the role of this liaison with respect to the wider campus conversation is critical. Recent sentiment in the literature favors a model of partnership across disciplines as opposed to ownership. Neil Lutsky [22] has suggested that mathematicians function more as "librarians" than leaders: as a nexus of information and resources that brings together a faculty conversation across many departments. Even in the majority of institutions surveyed by the MAA's SIGMAA-QL group whose quantitative reasoning requirements necessarily included one or more courses taught by a math department [37], the work of math faculty across disciplines is critical to ensure these skills are appropriately supported and reinforced in a wider variety of contexts in other courses.

3) How can I liaison with existing support services for students?

Regardless of the form a QL program might take, it is necessary to consider how to provide student support. As most schools provide some form of academic support, whether peer or professional, there is no need to reinvent the wheel when developing support for QL.

For institutions that have pre-existing academic support for mathematics, there will likely be overlap in the goals between established support centers and any new initiatives. Rather than duplicating services or generating competition (real or perceived), consider expanding the function of the existing center to include intentional support for QL. Even established non-quantitative support centers can serve as allies through shared staff training, data management and usage tracking, as well is in marketing and referrals. Maintaining positive ties with all academic support programs can help build a cohesive student experience and a collaborative culture on campus, in addition to providing QL support.

4) How can I leverage existing faculty support structures?

Support both from and for faculty in the implementation of a QL program is essential to its long-term success. In particular, securing buy-in from faculty, particularly those who would be involved in the ultimate delivery of QL programming, is essential to successful implementation tactic. In another chapter in this volume, Dewar, Larson, & Zachariah describe factors that promote faculty buy-in for sustainable instructional innovation.

Where structures already exist to support faculty professional development, such as teaching and learning centers, these can be a nexus for invaluable partnerships that both increase buy-in and provide faculty a vehicle for ongoing support. In a personal interview, Cinnamon Hillyard said that when she became Director of the QSC at the University of Washington, Bothell, social sciences faculty were already accustomed to working with subject-specific librarians to support their curricular development. Dr. Hillyard worked alongside faculty specifically for quantitative curricular development and believes her approach was one of the reasons for the successful adoption of QL at UW Bothell [33].

QuAC at Bridgewater State University had similar origins. It originated as partnership between BSU's Office of Teaching and Learning on the one hand, and its Math Services academic support center on the other, with the faculty director of the latter coordinating the initiative [34]. While the university's 2006 Core Curriculum provided for separate graduation requirements for mathematical reasoning and quantitative reasoning [3], learning outcomes for the QR requirement had not been developed and students were permitted to use a second course in mathematics to meet the QR requirement. The QuAC initiative has, through these faculty support channels, helped better inform faculty of the distinction and complementary importance of mathematical and quantitative reasoning.

12.7 Conclusion

Existing literature demonstrates that a QL program can be successfully and sustainably implemented in academic institutions of any size or mission class. The flavor of implementation most likely to succeed at an institution will be as varied as the factors motivating the shift from a narrower "private" focus strictly on mathematics to a broader "public" application of quantitative reasoning across the curriculum. Those wanting to implement a new quantitative literacy program may wish to introduce the idea in tandem with conversations around institutional identity and general graduation requirements, partnership with mathematics faculty, and to leverage existing support structures for both students and faculty for implementation.

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13

Reflections on Sustaining QL Course Innovations
A Cautionary Tale

Jacqueline Dewar, Suzanne Larson, and Thomas Zachariah
Loyola Marymount University

13.1 Introduction

As this volume and its predecessors [9, 24, 27] attest, attention to Quantitative Literacy (QL) is expanding, both within the mathematics community and more widely across higher education in the U.S. [19, 25, 20]. Many individuals have developed innovative pedagogical approaches to QL, documented and assessed the results, and made such work public for others to build on. Anyone interested in exploring or teaching QL now has access to supportive QL communities, for example the National Numeracy Network[1] and the Mathematical Association of America's Special Interest Group on QL.[2] As Madison notes within this volume, support for QL has grown tremendously, if haphazardly, during the last decade and he provides an extensive list of other resources and supportive organizations. Even so, innovative and effective approaches can fizzle out. As three faculty members who collaborated on incorporating civic engagement into a general education QL course by introducing group projects on local community issues [5], we will draw upon our experience as a case study to explore why some innovations take hold and others do not. Our observations align fairly well with recent research on what factors promote and sustain an innovation [10, 11, 12, 13, 16, 21]. Our reflections may serve to inform both long-term and new members of the QL community as well as those interested in promoting innovations in other areas of undergraduate mathematics instruction.

13.2 Background

Prior to 1998, Loyola Marymount University (LMU) required students to demonstrate mathematics proficiency and complete two courses in mathematics, science, or technology. At that time, mathematics proficiency was defined as a student being able to demonstrate the algebra skills necessary to succeed in subsequent mathematics or science classes at LMU. On entrance to the University, students took a mathematics placement exam that primarily tested algebra and precalculus skills. Students who scored sufficiently high on the placement exam were deemed to have demonstrated mathematics proficiency and were only required to take two mathematics, science, or technology courses. Students choosing to take a mathematics course to (partially) fulfill this two-course requirement chose from a variety of courses specifically designed as general education mathematics courses or any standard mathematics course beyond the intermediate algebra level. Any student whose test score did not demonstrate mathematics proficiency was

[1] See serc.carleton.edu/nnn/index.html.
[2] See sigmaa.maa.org/ql.

required to demonstrate proficiency by completing an intermediate algebra course followed by two additional courses in mathematics, science, or technology. For more than a decade, the intermediate algebra course had been offered in a mastery-approach format (allowing multiple attempts to pass each chapter test) that was augmented by in-class lectures and quizzes on solving typical algebra word problems.

In 1998, the mathematics department reconsidered what counted as mathematics proficiency and recommended that the University adopt a new definition, namely, possessing the quantitative and analytic skills needed to function as an informed citizen in an increasingly technological world. This aligned with a stated goal of the University core curriculum to prepare students "for the life they will live after LMU" [17, p. 5]. The University Core Curriculum Committee unanimously supported the recommendation.

To implement requirements based on the updated definition of mathematics proficiency, a QL course (Math 102: Quantitative Skills for the Modern World) was introduced that would be required for all undergraduate students except those with majors that required other mathematics courses (e.g., science, mathematics, engineering, business, psychology, economics, and elementary education). The general goals of the QL course were (and still are) to

- prepare students for general education science classes (related course content: number sense, percents, significant digits, and elementary statistics)
- provide students with quantitative and analytical skills useful in day-to-day living (related course content: number sense, elementary probability and statistics, use of computer spreadsheets, financial mathematics, compound interest, loans, credit cards, annuities, taxes, and theory of voting).

In addition to three hours of lecture, students attend a weekly computerlab session where they learn to use the Excel spreadsheet application to create and modify spreadsheet projects that perform a variety of information organization and analytical tasks.[3]

The department unanimously agreed to this restructuring of mathematics proficiency, even though that meant courses specifically designed as general education mathematics courses lost their audience (e.g., cryptography, mathematics of symmetry, women and mathematics). Institutional support for the restructuring included a commitment of financial resources for LMU's Learning Resource Center staff to support the associated Excel lab and one course remission per year for a full-time faculty member to act as the QL course coordinator.

In essence, the introduction of QL at LMU followed one of the more common models described by Parsons, Salomone, and Smith in this volume: a proposal "to revise requirements for graduation or general education … precipitated by … smaller faculty conversations around graduation requirements."

13.3 Incorporating Civic Engagement into the QL Course

Between 1997 and 2004, The College Board, the Mathematical Association of America, and the National Council for Education and the Disciplines produced four major reports addressing various aspects of QL [19, 25, 20, 27]. Around this same time, journals and publications of the Association of American Colleges and Universities (AAC&U) began to give significant attention to engaging students by connecting ideas with action (for example, see [2] or [14]). Richard Freeland, president emeritus of Northeastern University, wrote in AAC&U's *Liberal Education*, "The goal is to enrich liberal learning by connecting it more strongly with the lives students will actually live after college" [7, p. 9]. Carol Geary Schneider, president emerita of AAC&U, declared that civic engagement was almost becoming an imperative for higher education due to public pressure to demonstrate the value of a liberal education [22]. Richard Vaz, then Associate Dean, Interdisciplinary and Global Studies Division, Worcester Polytechnic Institute, noted that a common response to these calls, community service by students, often failed to link with the curriculum [31]. According to Burns [1], a better model would involve an open-ended inquiry in response to community needs. Such an approach would connect theory and practice by bringing knowledge to bear on real-world problems. It also has the potential to produce new knowledge and place it in the service of moral aims.

In 2004, we decided to develop a new version of the standard QL course, one based on Burn's model of incorporating open-ended inquiry in response to community needs. We approached our course redesign with the aid of a grant from SENCER (Science Education for New Civic Engagements and Responsibilities). SENCER is a comprehensive

[3] Much of the description of the development of the QL course in this and the next section is derived from [5].

faculty development and science education reform project funded by the National Science Foundation. SENCER aims to engage student interest in mathematics and science by supporting the development of academic programs and undergraduate courses that teach *to* basic science and mathematics *through* complex, capacious, and unsolved public issues. SENCER's philosophy aligns well with the description of numeracy by Steen et al.: "The numerate individual ... seeks out the world and quantitative skills to come to grips with its varied settings and concrete peculiarity" [26, p. 6].

Our redesigned course involved students in semester-long group projects concerning local community or campus issues that they could investigate using the relatively modest set of mathematical tools developed in the QL course. We hoped the project experience would better prepare these students to play active roles in understanding and addressing the problems and challenges of the world in which they live, using mathematics. Each group of students was required to formulate and carry out an action plan based on their findings, for example, making a recommendation to other students, writing a letter to the campus newspaper, or delivering a report to a client. We can now see that this combination of collaborative teamwork and action in the world seems to fit the frame of Intercultural Citizenship as described by Cardetti, Wagner, and Byram in this volume.

The revised QL course needed to meet the same general goals of the standard QL course (Math 102), as outlined in Section 13.2, but we also wanted it to provide all students, regardless of their previous mathematics background, with a new, challenging, and yet feasible experience researching an open-ended question; to demonstrate the power of mathematics as a tool for analyzing complex issues; to encourage students to think and act critically with regard to environmental or civic issues; and to provide students and faculty a venue for collaborative research. So, we adopted three more specific student learning outcomes:

I. (Awareness/Attitude) Students will be aware of the usefulness of mathematics in addressing real-world problems, and will have greater confidence in using mathematics.

II. (Performance) Students will be able to describe, analyze, and make recommendations about local community or environmental problems using appropriate mathematical tools.

III. (Engagement) Students will be engaged in a community issue during the course, and more likely to be engaged in civic issues in the future.

Our decision to use semester-long group projects to engage students with local community or campus questions offered several structural advantages. It allowed for flexibility in terms of the project topics, the mathematical content of the course sufficiently conformed to the approved QL course, and the approach meshed with the standard QL course computer lab. During our initial planning we learned by surveying students that they would prefer to investigate local issues rather than explore larger issues. Incorporating projects on local issues was consistent with Steen et al.'s perspective: "Only by encountering the elements and expressions of numeracy in real contexts that are meaningful to them will students develop the habits of mind of a numerate citizen" [26, p. 18].

Fortunately, campus and the surrounding area offered a wealth of project topics. Examples of questions that our students investigated included: Is there sufficient parking on campus and is it safe? Which of the coffee venues on campus is "best"? Is it better to live on or off campus? Who uses the Student Health Center, who does not, and why? The Health Center incorporated results from the latter study into their reaccreditation report, and requested student assistance with additional data gathering the following semester. Students also worked with local naturalists to undertake an insect population study in a nearby wetlands area.

To assess whether the projects approach was successful, we gathered data from the SENCER Student Assessment of their Learning Gains instrument,[4] three focus groups, course surveys, and pre- and post-tests. On the post-test, compared to students in the standard QL course, the students in the projects-based course performed similarly on all questions except one, and significantly better on that one question. That question—explaining the meaning of margin of error—was the only one that had potential to show a difference between the two approaches, since all the other questions were straightforward computational questions (for example, finding the monthly payment on a car loan). Certainly, students in both versions of the course had computed margins of error. But working on projects that involved survey data that they had collected and writing reports and making presentations required students to explain in their own words what the margin of error that they computed actually signified. Students in the project-based course

[4]See `salgsite.net`.

also showed increased awareness of community issues, gained confidence in their ability to respond to mathematical situations using course material, and reported learning non-mathematical skills. For more information on the course, how the projects were introduced and organized, how students were graded, and an assessment of the outcomes, see [5] and [30].

We note that the projects approach to teaching QL reflected several of the pedagogical implications that emerged from Fisher's definition of QL in this volume. For example, the projects provide ample opportunities for students to engage in discussions about numbers, and focused on the social contexts in which numbers arise. We concur with Fisher that QL is "as much about the communities that practice quantitative skills as it is about the skills themselves."

13.4 Favorable Structural Factors

Both our department and institution view instructional innovation favorably, especially if it does not require too many additional resources. We were also fortunate to find an approach that meshed well with the existing QL structure. But the plethora of approaches to QL at institutions of all types and sizes (e.g., [8], [18], [28], [**?**]) suggests that QL courses can flourish in a variety of institutional cultures.

We found the value of having three developers to be significant. Of course, having multiple developers made it possible to share certain tasks and allowed for wider informal dissemination of our progress and results. Having three different individuals involved in the development also helped to ensure that a measure of flexibility was built into the course that would allow each instructor some ability to adapt the approach to his or her individual situation.

We had previously collaborated on developing another course so we knew we worked well together. Moreover, one of us (Larson) was the QL course coordinator. This meant that coordinating with faculty teaching the standard QL course and obtaining information needed to make comparisons between our version of the course and the standard QL course was somewhat easier because of her position and her familiarity with both versions of the course.

Being part of SENCER was invaluable as it provided not only a level of funding support for the course development work, but also provided a larger network of support and professional development through SENCER workshops, access to an assessment tool, and opportunities for dissemination and feedback for our work throughout the development.

13.5 Dissemination

13.5.1 Local Dissemination Efforts

In mid-September 2006, the final report to the NSF for our SENCER (sub)grant described our progress on three goals: (1) to develop an alternate version of our current mathematics core class (Math 102: Quantitative Skills for the Modern World) that is accessible to students with only a high school mathematics background and in which students learn and apply mathematics to address problems in the local Los Angeles area, (2) have each of the three mathematics faculty on the team teach at least one section of the course during the academic year 2005-06, and (3) recruit other faculty to teach the SENCER version of the course. In that report we noted that we had clearly accomplished goals (1) and (2), but the outcome of goal (3) was, at that time, still pending.

To assist other instructors to include projects when teaching QL, we wrote *Using Group Projects on Community Issues to Develop Quantitative Literacy*, a 99-page course manual "as a guide to anyone who wishes to adopt this course or apply a projects-based approach to another course" [30, p. 6]. We also encouraged anyone with questions about this approach to contact us. The preface and a separate introduction to the manual provided an overview of the course and described the specific SENCER-inspired student learning outcomes (see Section 13.3).

The manual included a sample syllabus, project topic ideas and resources, a demonstration project on student loan debt for the instructor to model each stage of work on the project, advice on implementation of group projects in a course, and assessment and grading tools. The first five chapters of the manual each began with *Introductory Notes to the Instructor* to further explain the approach and the resources contained in that chapter. The sixth and final chapter reported the results of our assessment of the student learning outcomes and contained reflections and advice on a variety of pedagogical issues, including selecting project topics, facilitating group work, responding to student reactions to the open-ended nature of the projects, providing opportunities for students to revise their work, and more.

Chapter 3 of the manual provided a detailed rationale in support of this pedagogical approach using Chickering and Gamson's Seven Principles for Good Practice in Undergraduate Education [3] as a frame. It presented a description of each principle and then discussed how our approach using semester-long group projects conformed to that principle. As an example, here is our discussion of the fourth principle:

> *Principle: Good Practice Gives Prompt Feedback*
> *Knowing what you know and don't know focuses learning. Students need appropriate feedback on performance to benefit from courses. In getting started, students need help in assessing existing knowledge and competence. In classes, students need frequent opportunities to perform and receive suggestions for improvement. At various points during college, and at the end, students need chances to reflect on what they have learned, what they still need to know, and how to assess themselves* [3, p. 4].
>
> Commentary: The division of the project into stages with evaluation rubrics guiding assessment of each stage provides numerous opportunities for feedback. Because revision is possible, even requested, at each stage, opportunities abound for improvement. Several times students are asked for a self-assessment of their group participation. The mathematical topics list encourages students to reflect on the mathematical topics considered in the course and requires them to select which ones to apply to the project. By communicating a summary of peer evaluation forms including the written comments, we provide students with an alternative assessment of their project from their peers [30, p. 53].

In addition to the manual, we undertook several dissemination and recruitment efforts that involved outreach to department faculty. To familiarize potential instructors of the course with the approach and materials we had developed, we first described our projects approach to 15 full-time faculty in a departmental seminar on April 10, 2006. Then, just after the end of the semester, on May 10, 2006, to further encourage our colleagues to use projects when they were assigned to teach Math 102, we presented a half-day workshop open to all mathematics faculty members. Nine colleagues—two part-time, one full-time non-tenure track, and six full-time tenure track faculty members—chose to attend. Lunch and a small stipend were provided to attendees. The goal was to familiarize potential instructors of the course with the use of projects and with the materials we had developed. We evaluated the outcomes of the workshop with a short survey.

Local Dissemination Outcomes

In the post-workshop survey, 100% of the nine participants "agreed" or "strongly agreed" with all three of the following statements:

As a result of attending this workshop I have a better idea about:

- *The rationale for teaching Math 102 with projects*
- *The materials that are available to teach Math 102 with projects*
- *How to teach Math 102 with projects.*

When asked if they would willingly volunteer to teach the projects-based version of the QL course, 56% said "Yes" and 44% said they were "Not Sure." Every person who responded "Not Sure" went on to comment that it was the extra time commitment that made him or her hesitate. Those who responded positively commented:

- *"It would be fun."*
- *"It sounds cool."*
- *"Great way to engage students with material—gives students a valuable experience."*
- *"Seems rewarding, but wouldn't do it with other new preparations."*

One of us again taught the course in fall 2006. All departmental faculty with experience teaching the course were provided access to the final version of the course manual and invited to adopt the projects approach. However, to date, no one else has used the approach of having group projects on community issues. Furthermore, because there has been a greater departmental need to cover other courses, and one of us has since retired, this version has not been taught within the past decade. So, from the perspective of the course being taught in our own department, in essence, this innovation has fizzled out.

Looking Back

In retrospect, there are several other recruitment efforts we could have used within our department. We could have actively sought out a colleague to try our version of QL and mentored them by offering to teach another section along with them in the same semester. We were rather reluctant to approach part-time instructors to adopt the projects approach, recognizing the special time constraints many of them had due to teaching courses at multiple institutions, and the relatively low wages they received. We thought that students in a course using projects would want an instructor who was more accessible outside class than part-time instructors' schedules would allow. Had we been aware of Virginia Commonwealth University's reported success in recruiting and renewing a cadre of part-time, temporary instructors for a QL course that involved an independent study project [6], perhaps we would have been more willing to attempt to recruit part-time instructors.

To capitalize on the one or two occasions when faculty expressed an interest in mission-related innovations, we could have done more than have just a brief conversation and then send them a copy of our manual. Our inadequate response resulted from several factors. The usual time pressures from other commitments were partially responsible. We had also had a previous experience developing an unusual problem-solving course for beginning mathematics majors, writing a manual for it, and seeing it successfully institutionalized without undertaking a recruitment effort [4]. But in that case, the development of the course had been commissioned by the department in response to a concern about retention of majors after the first year. While the standard QL course had full departmental support, our particular approach was just an innovation on it undertaken by individuals.

However, our projects approach did influence a subsequent local innovation (which also did not persist). In an early part of a five-year long revision of our institution's general education core, one of the developers (Larson) collaborated with another mathematics faculty member (Dr. Curtis Bennett) to pilot another version of the QL course in fall 2008. They too adopted the SENCER philosophy of teaching mathematics through real-world situations, but used shorter individual and group assignments rather than semester-long group projects. At the time, Larson and Bennett envisioned these materials and assignments as being more easily adopted by other faculty teaching these courses.

13.5.2 Dissemination Efforts Beyond Our Institution

Our projects approach to incorporating civic engagement into QL was more broadly disseminated than just on our own campus. It was selected as a 2006 model course by SENCER.[5] We presented papers at the Joint Mathematics Meetings (2006), the International Society for the Scholarship of Teaching and Learning conference (2006), the Lilly West Conference (2006), and at SENCER meetings held at Chapman College (2007) and Woodbury College (2010). We also made a poster presentation at the 2007 SENCER Symposium on Capitol Hill in Washington, DC. In 2013, we were invited to present a workshop on using group projects on local issues to incorporate civic engagement into a QL course at the SENCER-WEST conference held at St. Mary's College in Moraga, CA.

Impact Beyond Our Institution

Although our innovation failed to take hold locally, we do know that it had an impact on at least one other institution: Metropolitan State University in St. Paul, MN. There, Dr. Cindy Kaus incorporated community-based projects into a general education statistics course so students would "see the importance of statistics in their disciplines and personal lives" [15, p. 100]. Kaus acknowledged that our SENCER model course, Quantitative Literacy Through Community-Based Projects, had provided the idea for her innovation. Given the number of presentations we made, it is possible that some of the ideas we espoused influenced others that we are unaware of. But the disappointing fact remains that the innovation did not take hold on our campus.

[5] See `ncsce.net/quantitative-literacy-through-community-based-group-projects`.

13.6 Factors that Promote and Sustain Innovation in STEM Instruction

After one of us heard Charles Henderson's opening plenary, "The challenges of spreading and sustaining research-based instruction in undergraduate STEM," at the 2015 Conference on Research in Undergraduate Mathematics held in Pittsburgh, PA, we became more aware of research and findings on promoting change in instructional practices in STEM courses ([10], [11], [13], [16]). The most striking of these findings about promoting change in instructional practices came from Henderson, Beach, and Finkelstein [10, p. 952]: "Two commonly used change strategies are clearly not effective: developing and testing 'best practice' curricular materials and then making these materials available to other faculty and 'top-down' policy-making meant to influence instructional practices." Stanford, Cole, Froyd, Friedrichsen, Khatri, and Henderson [23] argued that an important reason for the lack of adoption is that developers primarily focus their efforts on dissemination (spreading the word) instead of propagation (promoting successful adoption). This certainly rings true in the case of our QL course. In addition, Henderson and Dancy described the importance of identifying and attending to situational factors that can act as barriers in [12]. We highlight four key points that we synthesized from this literature and describe how each of these played a role in our own course development and dissemination efforts.

i. *Collegiate teaching occurs within a complex multilevel system (individuals, departmental, institutional, and extra-institutional). At the individual level, it is important to recognize that instructors already have experience or ideas about what content should be taught, how it should be presented, and how students learn. Some of these ideas may be in conflict with the proposed innovation and may require modification for successful implementation.*

We discovered that full-time instructors interested in innovation in Math 102 at LMU had their own ideas about how to engage students in QL and wanted to pursue those rather than follow our lead by using group projects on local civic issues. Generally, it is good when faculty have the latitude to pursue their own ideas and teaching innovations. But we had not anticipated that faculty interest in other ways to engage students in mathematics would be a factor that kept some of our colleagues from adopting our projects approach to QL.

ii. *A variety of structural barriers to adoption of innovations may exist at the individual, departmental, institutional, or extra-institutional levels. The need for special technology, a particular type of classroom layout, or access to community or financial resources are examples. Lack of instructor time, student expectations of the instructor, or departmental norms for teaching are other possible barriers.*

In our case, needing access to technology was not a problem, because we had designed our course to use the same computer lab as that used by the standard QL course; in fact, students from both versions of the course could enroll in the same lab section together. Also, our department accepted and encouraged teaching innovations. The University did too, subject to the availability of additional resources, if needed. We tried to anticipate student concerns and lack of experience or prior negative experiences working in groups on projects and attempted to address those during the development process. The course manual also provided potential adopters with advice on these matters.

A significant structural barrier for us was that only about 20% of the 20 sections of the Math 102 QL course typically offered each year were taught by tenure-track instructors. As mentioned in the *Looking Back* discussion of Section 13.5.1, we were reluctant to approach anyone who was not a tenure-track instructor to incorporate projects. That self-imposed constraint severely limited our pool of potential adopters locally. Our post-workshop survey (see Section 13.5.1, *Local Dissemination Outcomes*) suggested that time constraints would be a hindrance to converting even a full-time instructor's interest into adoption.

iii. *Potential adopters are almost always presented with a finished product, rather than having been part of the development process. If instead they could provide feedback early in the development process, developers would gain a better understanding of needs and barriers. In fact, the final materials and approaches will almost certainly be modified by the instructors to fit their particular situations. Therefore, dissemination information should include the how and the why an innovation works so that an instructor's customization is more likely to be effective.*

As noted in Section 13.4, having three developers meant we did have three different perspectives represented. But, we did not engage colleagues in the development phase other than by having a few sporadic, informal hallway conversations. They received the first significant details after the materials were in their final form during a departmental seminar, and later in a half-day workshop. The revision effort undertaken by Bennett and Larson noted in the *Looking Back* discussion of Section 13.5.1 presents an example of a modification of an innovation taking place locally. This modification benefited from one of us being involved. Even if that had not been the case, the course manual contained quite a bit of detail on both how and why the innovation worked (see Section 13.5.1).

iv. *Effecting change takes time (not a one-day workshop), is assisted by having ongoing support and feedback for implementing change, and is facilitated by interactive dissemination and personal communication.*

We were guilty of trying to disseminate the new approach with just a half-day workshop. As previously noted, offering to teach a section of the course while one or more colleagues tried out our version would have been a better strategy. As described in the *Looking Back* discussion of Section 13.5.1, in what could be considered our one successful local dissemination effort, Larson did engage with a colleague in developing and simultaneously teaching sections of a greatly scaled-down version of the projects approach.

Our SENCER grant began with our attending a three-day summer institute to learn about the SENCER approach to teaching mathematics and science through complex public issues. This experience and the ongoing support provided by being part of the SENCER community helped our course development efforts tremendously. As described in Section 13.5.2, the existence of the SENCER community was a key factor in the spread of the community-projects approach to another institution. Once again, our own situation developing and disseminating innovations related to QL reflected what we later found in the literature about promoting change in instructional practices.

There are many ways to think about how individuals and departments can foster and support change. For example, the recent work of Reinholz and Abkarian [21], which draws on some of the same literature as we have, offers a model with four frames (structures, symbols, people, and power) "to support and study sustainable systemic change" (p. 8). We hope that readers will consult the sources cited in this section for additional material on promoting and sustaining instructional innovations.

13.7 Closing Remarks

Promoting not just QL but other innovations (such as active learning methods) in undergraduate mathematics instruction has captured the attention of the mathematics community. Examples of this are readily seen in the Mathematical Association of America's 2015 document *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*[6] and its *MAA Instructional Practices Guide*,[7] the Conference Board of Mathematical Sciences' statement in support of active learning,[8] and the Transforming Post-Secondary Education in Mathematics initiative,[9] among others. We hope this brief discussion and reflection on promoting and sustaining successful adoption of innovations will be of use to those who work toward improving the quantitative literacy of undergraduates and the teaching of undergraduate mathematics across the board.

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14

Understanding the Problem:

The Need to Expand Access to Quantitative Reasoning Courses

Amy Getz, Connie Richardson, Rebecca Hartzler, and Francesca Fraga Leahy
The Charles A. Dana Center at The University of Texas at Austin

14.1 Introduction

All students should engage in the mathematical experiences that will best serve their professional, personal, and civic lives. For many, those experiences could and should be provided through a high-quality quantitative reasoning (QR) or quantitative literacy (QL) course. Access to such courses varies greatly, and in some systems, is especially limited in the community college sector. This disproportionately affects low-income and underrepresented minority students which creates concerns that these students are not being given equitable opportunities to attain valuable skills that enable economic and social mobility.

In this chapter, we call for the expanded use of QL/QR courses as the default gateway mathematics course for many programs of study, meaning that these courses meet general education and program of study requirements for mathematics and have no college-level prerequisites. QL/QR courses meet the needs of many people who increasingly need to navigate non-routine situations and make decisions based on quantitative concepts, such as rates of change, data visualization, and descriptive and inferential statistics, rather than algebra skills.

In the next section, we present data that help illustrate the problem. We then examine reasons for the variability in the use of QL/QR as a gateway course, as well as obstacles that have prevented QL/QR enrollments from growing, ending with a discussion on how QL/QR educators can mobilize for change.

14.2 Understanding the Problem: The Need to Expand Access

We often assume that students have access to opportunities simply if there is a not a visible barrier restricting their participation. True access, though, means more than the lack of restriction. In the context of this discussion, we use *access* to signify that QL/QR courses are available, that students are aware of the courses and understand how the courses can benefit them, and that students are actively encouraged to enroll in the courses.

Over the last few decades, QL/QR educators have tirelessly made their case for the value of QL/QR courses, to great effect. As shown in Table 14.1, national data from the Conference Board of Mathematical Sciences (CBMS) show a steady increase in student enrollment in mathematics for the liberal arts courses in two-year colleges. In 1995, there were 38,000 students enrolled, increasing to 59,000 in 2005 and totaling 91,000 by 2010 [3, p. 139]. While the mathematics for the liberal arts classification is not a perfect fit for describing QL/QR courses, it is the closest ap-

Table 14.1. Enrollment and percentage in thousands in the three highest enrollment gateway mathematics courses at two-year colleges, 1995–2010. From [3].

	1995	**2000**	**2005**	**2010**
College Algebra and College Algebra with Trigonometry	193 (64%)	189 (62%)	224 (57%)	243 (52%)
Elementary Statistics	69 (23%)	71 (23%)	111 (28%)	134 (29%)
Mathematics for the Liberal Arts	38 (13%)	43 (14%)	59 (15%)	91 (19%)

proximation available in many large data sets. See the chapters from Madison and from Parsons, Salomone, and Smith in this volume for discussions of evidence on the growth of QL/QR courses. The data also show an overall increase of enrollment in mathematics, with small shifts in the proportions of students in different courses. The percentage of students in college algebra decreased from 1995 to 2010, but still represented a majority in 2010. Elementary statistics increased from 23% to 29% of enrollment and mathematics for the liberal arts increased from 13% to 19% over the 15-year period.

This increase in enrollment in QL/QR courses is a testament to the passion of educators from multiple disciplines who have championed QL/QR on their local campuses, as well as through conference presentations, publications, and activity in professional associations. Yet, there is still much to be done. The CBMS survey also shows that, in 2010, college algebra courses had the largest enrollment of all college mathematics in two-year colleges.

The CBMS survey does not use the same course classifications for enrollments at four-year institutions, but from our observations to work across a dozen states we expect that the upward trends in enrollment in mathematics for the liberal arts would be similar. If anything, we expect the growth to be even greater in four-year institutions since they are able to make independent changes more easily than community colleges can, given that the latter have to consider implications for transfer to multiple institutions. We discuss this more fully below.

The CBMS data provides a high-level view of trends across states, but we know that QL/QR offerings vary widely within different states. Some states, such as Montana and Texas, have long had a classification for transferable QL/QR courses. This means that a student is guaranteed that the course will transfer for credit as an elective, but there is no guarantee that the credit will count for the mathematics requirement at the receiving institution. The institutional requirements tend to vary greatly. In other states, the entire classification for QL/QR courses is relatively new. For example, Michigan added QL/QR to the Michigan Transfer Agreement in 2012, and Ohio added QL/QR to the Ohio Transfer Module in 2016. Other states have dramatically increased enrollment in QL/QR through systemic change. In the last four years, North Carolina and Indiana's community college systems both made QL/QR the default mathematics course for all students who were not enrolled in a program requiring some form of calculus.

A review of state-level data provides insights into the variation of the use of QL/QR courses across institutions. Table 14.2 shows the enrollment patterns across the three gateway mathematics courses at four-year institutions in Colorado.[1] In six out of ten institutions, fewer than 50% of the total students in these courses were enrolled in college algebra. In contrast, four universities enrolled more than two-thirds of their students in college algebra. The percentage of students enrolled in mathematics for the liberal arts ranges from 7% to 48%, with seven of the ten institutions enrolling more than 20%.

We see similar variability between institutions within sectors in other states. For example, the University of Texas at Arlington had 26% more students in its QL/QR course, contemporary mathematics, than in college algebra in fall 2016 [2]. At the same time, the University of Houston–Downtown was just beginning a process to add QL/QR to its general education core.

Returning to the Colorado data, important observations can be made by comparing the four-year institutions to two-year colleges, the latter of which are in Table 14.3. There is less variation within the two-year sector, but there is a sharp contrast between the two-year and the four-year sectors. Every two-year college in the state enrolled more than

[1] These data include all public four-year institutions in Colorado except the Colorado School of Mines, which is devoted to engineering and applied science and requires all students to take calculus so is not relevant to this discussion.

Table 14.2. Enrollment in gateway mathematics courses at all Colorado public four-year institutions in fall 2013 (excluding the Colorado School of Mines). From [10].

Institution Name	% Enrolled college algebra	% Enrolled Math for the Liberal Arts	% Enrolled Introduction to Statistics
Adams State University	89	7	4
Colorado Mesa University	67	24	9
Colorado State University	73	27	0
Colorado State University—Pueblo	44	23	33
Fort Lewis College	45	12	43
Metropolitan State University of Denver	25	43	32
University of Colorado Boulder	35	48	17
University of Colorado Denver	39	35	26
University of Northern Colorado	27	21	51
Western State Colorado University	85	15	0

Table 14.3. Enrollment in gateway mathematics courses at Colorado two-year institutions in fall 2013. From [10].

Institution Name	% Enrolled college algebra	% Enrolled Math for the Liberal Arts	% Enrolled Introduction to Statistics
Aims Community College	66	12	22
Arapahoe Community College	66	12	22
Colorado Mountain College	74	9	17
Colorado Northwestern Community College	65	20	15
Community College of Aurora	64	15	21
Community College of Denver	62	18	20
Front Range Community College	73	7	20
Lamar Community College	70	23	7
Morgan Community College	60	3	37
Northeastern Junior College	79	1	19
Otero Junior College	70	18	11
Pikes Peak Community College	75	12	12
Pueblo Community College	54	22	25
Red Rocks Community College	69	13	18
Trinidad State Junior College	75	7	18

half (54% to 79%) of its students in college algebra, and only three had 20% or more of their students in mathematics for the liberal arts.

This case study demonstrates that growth in QL/QR course enrollments has not been evenly distributed, even within states in which QL/QR has been long established. Of special concern is the situation demonstrated in this data in which the two-year sector lags the four-year in QL/QR enrollment. This raises serious questions about equitable access to QL/QR courses. Indeed, The National Center for Public Policy and Higher Education [24] notes that "Students who enroll in community colleges are more likely to be low-income, the first in their families to go to college, and from underrepresented racial or ethnic groups" (p. 4). In fall 2015, over half of the Hispanic undergraduates at all degreegranting institutions were enrolled at two-year colleges, along with 44% of black students, 40% of Asian students, and 39% of white students [22].

Faculty report that the disconnect between sectors demonstrated in Colorado is not uncommon. However, there are cases in which the positions are reversed. In Indiana, for example, the statewide community college system has dramatically increased enrollments in its QL/QR course. We do not have enrollment data from the four-year institutions, but as of early 2017, only seven out of 12 of the public four-year universities had submitted a QL/QR course for state transfer. We will further explore the status of QL/QR offerings in Indiana later.

14.2.1 How Many Students Should Be in QL/QR Courses?

The variation in QL/QR enrollments across institutions and states raises the question: What proportion of students might we expect to see in QL/QR if enrollments were aligned to the needs of programs of study? A definitive answer to this question depends on many factors. Here we offer evidence that the current preponderance of enrollments in college algebra and related courses is inappropriate.

Historically, the role of college algebra was to prepare students for calculus [19, 29]. Over time, however, the course evolved into a catch-all that serves as a terminal, general education mathematics course for many students, while simultaneously serving as preparation for advanced mathematics courses. Many mathematics educators we have spoken to would argue that the result is, all too often, a course that does not fulfill either function well.

Indeed, the Mathematical Association of America's Committee on the Undergraduate Mathematics Program (CUPM) made this point in 2004. The Committee's Curriculum Guide noted, "Unfortunately, there is often a serious mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course," and recommended careful review of the effectiveness of college algebra in meeting students' needs [11, p. 27]. The precept that college algebra should be used to prepare students for calculus or other courses that require strong algebraic skills provides a starting point for considering appropriate enrollment levels across gateway mathematics courses.

Studies have found that only 20% of students in community colleges are in programs requiring calculus [4]. The percentage increases to 28% in four-year institutions [8]. This does not mean that 72% to 80% of students should be taking QL/QR courses. Introductory statistics, for example, is appropriate for many social sciences and allied health programs, and some systems offer specialized courses for programs such as elementary education and business. We propose that a range of 20% to 50% would be appropriate at most institutions, depending on their program offerings and state policies.

This range is a rough estimate. The appropriate percentage of students in any mathematics pathway at a given institution depends on the state and local context. Factors to be considered include: the gateway mathematics courses available through state policy or local offerings, the programs offered at an institution, and the proportion of students in those programs, among others. For example, Ivy Tech Community College, the statewide community college system of Indiana, has made a commitment to advising students into an appropriate mathematics pathway. They offer three pathways: college algebra (as a pathway to calculus), technical mathematics for certificate programs, and QL/QR for all other students. In fall 2015, 45% of Ivy Tech students in the gateway mathematics courses were enrolled in QR. As we will discuss later, this percentage could increase with changes in the state Transfer Library. In another example, the University of Colorado–Boulder enrolls 48% of students in mathematics for the liberal arts, as shown in Table 14.2. These examples demonstrate that QL/QR enrollments can approach 50%. At the other end of the range are institutions that use introductory statistics as a requirement for programs in the social sciences and allied health, thus lowering the number of students in QR. In this case, we estimate the appropriate QL/QR enrollment could be as low as 20%.

Many people have understandable concerns that efforts to expand access to QL/QR courses have the potential to reinforce existing gaps in minority and underrepresented populations' participation in science, technology, engineering and mathematics (STEM) fields. For many years, people concerned about these gaps have advocated that all students follow the algebraic-intensive path that opens doors to STEM programs. However, when applied at the postsecondary level, this approach has not narrowed achievement gaps and has instead contributed to high failure rates in traditional algebra-based remedial course sequences, likely decreasing overall college completion rates for underrepresented and underserved populations [18]. Evidence of continuing achievement gaps is all too prevalent. A study that followed students from 1995 to 2001 found that black and hispanic students were as likely to enter STEM fields as white students, but less likely to complete a STEM degree or certificate [9]. From 2008 to 2013, there was not a significant increase in the percentage of black or hispanic STEM degree or certificate completers, although both groups increased as a percentage of the overall college population [31, 32]. The percentage of women completing STEM programs also continues to lag their overall representation in higher education [25]. We make the argument that all students should first be supported to understand their course options and opportunities as a part of their goal-setting process and, secondly, should then have access to high-quality, rigorous mathematics courses aligned to those goals.[2]

14.2.2 Why the Mismatch?

The first step in correcting the enrollment distribution in QL/QR courses is understanding the reasons for this mismatch. A number of factors contribute to the relatively low enrollment in QL/QR courses at many institutions. In this chapter, we focus on systemic barriers preventing mathematics departments from increasing their QL/QR offerings.

The Charles A. Dana Center works in and across a number of states to support the implementation of multiple mathematics pathways. This work includes advocating for QL/QR courses, supporting faculty seeking to design new QL/QR courses, and helping institutions increase offerings of existing courses. A common problem is that departments offer QL/QR courses but cannot get students to enroll. Why do so many institutions struggle with this issue?

We observed three factors that contribute to low enrollments in QR courses:

1. Limited understanding of and misperceptions about QL/QR courses
2. Ambiguity and lack of consistency across QL/QR courses
3. Challenges of enacting systemic change.

14.2.3 Limited Understanding and Misperceptions

One factor contributing to low enrollment is simply a lack of general understanding about QL/QR courses. Many people have no experience with QL/QR courses, including faculty in other disciplines (and sometimes within mathematics), advisers, and students. Even if they have experienced such a course, they may not associate it with the name "quantitative reasoning" or "quantitative literacy." Common course titles include "mathematics for the liberal arts," "contemporary mathematics," or "college mathematics." This highlights one problem in promoting QL/QR: the challenge in helping people understand something that does not have a consistent name. Many of us, including the authors, have enjoyed passionate discussions about the nuances of the names "quantitative literacy" and "quantitative reasoning," and which name should be used in different circumstances. These debates have played an important role in the evolving definition of this classification of courses, but have not helped build understanding among external audiences, including students. The Dana Center has settled on the term *quantitative reasoning*, which we acknowledge is more useful when communicating to fellow QL/QR proponents than to students or other colleagues.

In fact, very few college catalogs list "quantitative reasoning" as a course. If there is a standard title, it is likely to be "nathematics for the liberal arts," "contemporary mathematics," or simply "college mathematics." Also, courses are offered under these general headings that most QL/QR faculty would not consider as true quantitative reasoning or quantitative literacy courses since they include survey courses that do not delve deeply into problem-solving and critical thinking or, on the other end of the spectrum, focus on very narrow topics with little practical application. This tends to occur because of a lack of common, well-defined standards across this classification of courses. To further complicate matters, many departments have local course names that may reflect the context of the course. It is no wonder that students, advisers, and faculty are often unsure about these courses.

Contrast this with the ubiquitous "college algebra." The name is familiar and used fairly consistently across institutions. Many students, especially those classified as low-income, first-generation, and underrepresented minorities enter college with insufficient understanding of the system and limited support and guidance [16]. Advisors trying to help these students have large case-loads and in many cases, students are allowed to self-advise [6]. All this makes it more likely that students will either be directed to or self-select the most familiar option.

[2] How students make decisions about goals is more complex than can be adequately discussed here. A myriad of issues spanning cultural and individual identity and mindsets, social supports, and institutional structures are involved. We do not wish to appear to oversimplify or ignore these issues while we also make the case that access to QL/QR courses is, in itself, an issue of equity.

Beyond the name, all parties need an understanding of what a quantitative reasoning course actually is: What are the standard learning outcomes? What topics are addressed? How are QL/QR courses different from traditional algebra courses? Faculty in partner disciplines who determine the mathematics requirements for their programs need to understand how the course will prepare students for future coursework, the workplace, and in their lives as informed citizens and consumers. Advisors need to understand how students will benefit from the different content and pedagogy. All of these parties want what is best for their students, and they need sufficient, appropriate information in order to provide it.

Addressing misperceptions about QL/QR courses is perhaps more challenging than building new understanding of QR. The most common misperception is that these courses are not rigorous and are simply an easy way to get a mathematics credit. One of the authors once heard a QL/QR course inaccurately described as "mathematics for people who don't need mathematics." QL/QR advocates can play into this misperception when arguing that more students will be successful in a QL/QR course than in college algebra. The statement is true in many cases, but it can lead to a superficial interpretation that QL/QR is easier than college algebra, an interpretation that Edwards, Melfi, and Satyam in this volume would challenge.

QR faculty can address others' limited understanding by:

- Finding compromise for a common naming convention, at least within systems.
- Conducting outreach to stakeholder groups to help them learn about QL/QR courses and understand how they benefit students, and to address questions about rigor.

A key step in helping external groups understand QR is coming to consensus on core content for gateway QL/QR courses, as we will explore in the next section.

14.2.4 Ambiguity and Lack of Consistency

Quantitative reasoning courses employ a rich diversity of mathematical concepts, meaningful contexts, and pedagogical techniques. This diversity is often a part of the attraction for both faculty and students; systems that offer QL/QR courses are often careful not to impinge on the ability of faculty to build innovative courses. For example, until 2016, the description for *Contemporary Mathematics* in the Texas Academic Course Guide Manual was "Topics *may include* [emphasis added] introductory treatments of sets, logic, number systems, number theory, relations, functions, probability and statistics. Appropriate applications are included" [30, p. 117]. One might argue that almost any mathematics course could fall under this definition. Such ambiguity is consistent with one of the author's experiences when assigned to teach a College Mathematics course for the first time. She was handed a textbook and told: "Pick three or four chapters on the topics you want to teach." This casual attitude toward content contributes to the problem of building common understanding of the courses. How do you explain QL/QR to someone if the classification can cover everything from an algebraic-intensive modeling course to a statistics course, to a survey course, to a logic course?

Many states, including Indiana, North Carolina, Ohio, Oregon, Texas, and Virginia, have begun to address this issue by developing more definitive student learning outcomes for QL/QR. Faculty leading these efforts seek to create better coherence across their systems without limiting the flexibility that makes many QL/QR courses so effective. One approach is to define a list of mathematical skills and concepts that can be taught within a variety of contexts. A danger of this approach is that it can lead to an extensive list of outcomes, essentially forcing faculty into covering topics superficially in order to meet the requirements that many see as contrary to the goals of QR [17]. Oregon's approach to this challenge was to define 70% of the course. Faculty can then select from a list of topics for the remaining 30% of the course. Virginia began with a similar approach but encountered transfer concerns, eventually settling on a standardized course description. Ohio's approach was to draw heavily from recommendations from the CUPM 2004 and 2015 curriculum guides to describe the overall purpose of the course, and then to define learning outcomes under three topics: numeracy, mathematical modeling, and probability and statistics.

Some people may question the need for any level of standardization in a field that has flourished in large part because of its diversity. Is it not enough for a QL/QR course to be well understood and established within a college or university? Several leaders note that establishing common benchmarks and definitions will aid assessment and increase the quality of courses [17]. From the standpoint of access, an equally important issue is ensuring transferability of courses. Our higher education system reflects the growing mobility of our society. A national study found that 64%

of students who earned a bachelor's degree in 2014-2015 attended more than one institution [28]. This increase in mobility requires that institutions ensure that courses will both transfer *and* apply to programs of study reliably. As long as there is any doubt or ambiguity about QL/QR, advisers and students will likely default to the "safe bet" for mathematics requirements, which is usually college algebra.

The Colorado data in Tables 2 and 3 demonstrate this tendency; it is no accident that the community colleges have the highest enrollments in college algebra. Part of a community college's mission is to ensure that its students will be prepared to seek a higher degree at another institution. For example, an adviser at Trinidad Junior College might believe that a QL/QR course would be most beneficial for a student's career interests and future needs, but if there is a chance that the student will transfer to Adams State University, the adviser is likely to recommend college algebra, the predominant course at that institution.

Only a careful analysis can determine why any particular institution may have high enrollments in QL/QR. Some institutions still require that all students take college algebra or a higher level mathematics course. This is the case, for example, at the University of Missouri and some Texas A&M campuses at the time of writing this chapter. The University of Houston–Downtown changed their college algebra requirement in 2016, but did not begin offering QL/QR courses when the change went into effect. In other cases, an institution might offer QL/QR but still have a number of programs of study requiring college algebra, even though QL/QR might be a better choice. All of this inconsistency creates ambiguity and confusion, especially for community college students who intend to transfer but are not sure where.

Programs that allow too many choices also contribute to the problem. Many programs, especially in the liberal arts, accept any college-level mathematics course. On the surface, this approach would appear to allow students great flexibility, but the ambiguity of "any general education mathematics course" can be quite intimidating. Without guidance about which mathematics course is the most appropriate, students and advisers tend to default to the most familiar one.

Bailey, Smith Jaggars, and Jenkins [1] make the argument that faculty, as experts in their fields, should make clear recommendations about which general education courses students should take. This guidance does not have to be a rigid requirement, but it should send a clear message about which course is preferred. QL/QR faculty can support this effort by explaining why it is important to establish a default requirement and making the case for why QL/QR is the best course for students in a given program.

In particular, QL/QR faculty can provide consistency and clarity around QL/QR courses by:

- Promoting and supporting efforts to establish high quality student learning outcomes at the state level.
- Working with colleagues in partner disciplines to establish unambiguous mathematics requirements for all programs and requirements for QL/QR where appropriate.
- Working across institutions to establish common requirements for similar programs.

14.2.5 Challenges of Systemic Change

So far, we have discussed the core work that QL/QR faculty must do together to better define QL/QR courses and to communicate that understanding to other groups. Now we examine the role of institutional systems.

Changing hearts and minds can be simple compared to changing complex systems. Even with the best intentions, individuals may struggle to enact change if attention is not paid to building systemic supports. This is especially true in large systems that grapple with communication challenges across large numbers of people, personnel turnover, and initiative fatigue.

The term *system* in this context can refer to a single college or university, a system of colleges, or a state system. Enacting systemic change requires understanding how the system works: What creates momentum for change within a system? What creates barriers? What is the hierarchy and relationship between stakeholder groups? Input and support from people from across the system is needed to answer these questions.

Advising provides an example of the complexity of change within an institution. Advisors have a critical role in expanding access to QL/QR courses. Yet convincing advisers of the value of QL/QR is necessary but not sufficient. Advisors must act upon departmental requirements and often must look beyond their own institutions to the practices and requirements of others. Advisors also have to develop processes to help students who have not declared a major decide on the most appropriate mathematics course to take. Changing advising practice requires work on multiple dimensions, including the development of information resources, the collection of data on different programs, outreach to and collaboration with partner disciplines within the institution and across transfer partners, coordination with student services leaders to offer training and support for advisers, support for advisers to develop effective processes, and tools to support students in selecting the right gateway mathematics course. One of the most effective practices is to establish a default mathematics requirement for groups of programs, sometimes called meta-majors. A default or recommended requirement communicates clearly to students and advisers the preferred mathematics pathways and is more effective than vague options such as "any general education mathematics course."

Removing barriers creates opportunity for change, but it does not ensure that change will be enacted. The goal is to make enacting the change as easy as possible, which requires support for the stakeholders involved, typically very busy people. To return to the example of advising, it is important to make it easy for students and advisers to identify quickly and confidently the appropriate mathematics course. If that course is QL/QR, student-friendly materials should be made available to help students understand the course they will be taking.

The chapter from Dewar, Larson, and Zachariah in this volume offers an example of the challenges of enacting meaningful, long-term change. The authors describe their unsuccessful attempt to implement a problem-based QL/QR course at their university. In their reflection, they note that the course was the brainchild of a few individuals and not a priority for the department, the nature of the course limited the instructor pool, and the time and effort required of faculty was too great. This is an excellent example of why good ideas championed by committed and passionate people fail to lead to change. Sustainable change requires a strong driver who creates broad engagement across stakeholders, planning with the end in mind, e.g., designing a course that many people can teach, and support or structures that enable a wide range of people to implement effectively.

In the case of increasing access to QL/QR courses, the challenge is greatly increased because full implementation requires change across institutions. Indiana provides a case study highlighting the importance of this cross-institutional cooperation.

As noted above, Indiana has one statewide community college system, Ivy Tech Community College. In 2012, Ivy Tech began a concerted effort to increase the success of students in gateway mathematics courses through two major strategies: increasing access to QL/QR courses and supporting underprepared students to go directly into a college-level course. Ivy Tech did succeed in increasing the percentage of students in QL/QR, but they still faced challenges for students who planned to transfer. Students needed to know that the course would transfer and apply predictably and consistently to programs across institutions. In 2014, Indiana convened a Mathematics Innovation Council comprised of mathematics faculty leaders from two- and four-year institutions across the state. In 2015, the Council recommended QL/QR as the preferred gateway course for three meta-majors: Arts and Humanities, Social and Behavioral Sciences, and Health [21]. This does not guarantee that all institutions will follow the recommendations, but it is a step towards coherence across the system. The Council also established a working group to develop new state-level learning outcomes for QL/QR.

The Mathematics Innovation Council's report noted that simply offering QL/QR courses was not sufficient: "If quantitative reasoning coursework is truly the *best* option for many students and majors, preferable to college algebra or calculus, strategies must be put in place to make it more likely that students register for that course" [21, p. 14]. As institutions begin to align their programs with these recommendations, many more students across all higher education institutions will benefit from high-quality, rigorous QR courses. A milestone in this process was achieved in December, 2017 when the Statewide Transfer and Articulation Committee recommended the addition of QL/QR to the Core Transfer Library, thus ensuring that Ivy Tech's QL/QR course would transfer.

In sum, we argue that QL/QR faculty can mobilize to enact systemic change by:

- Collaborating with colleagues in mathematics, partner disciplines, student support services administration, and policy agencies to plan for enacting systemic change within and across institutions.
- Providing or helping develop communication materials (e.g., to be disseminated in advising offices) with student-friendly language.
- Establishing structures to train and support full-time and adjunct faculty.

Lastly, we examine a final factor in access, the pipeline leading into QL/QR courses.

14.3 A Final Dimension of Expanding Access

Thus far, the discussion has focused on directing more students who are eligible to enter gateway mathematics courses towards a QL/QR option, as appropriate, for their programs of study. A final issue in expanding access is to expand the pool of students eligible for a college-level QL/QR course.

There are two problems to address here. The first is how eligibility, or college readiness, is defined and determined. The second is how students who are not college ready can be supported to succeed relatively quickly in their gateway college-level mathematics course.

These two problems have strong equity implications, as low-income and minority students are less likely to be determined to be college ready based on traditional placement metrics. Different studies have found that referrals to remediation among blacks and hispanics in two-year colleges range from 11 to 21 percentage points higher than white students [7, 13]. The gap between hispanic and white students at four-year institutions is much smaller—five to seven percentage points—but larger for black students with a range of 26 to 30 percentage points. In both two-year and four-year institutions, referral rates for remediation are 16 to 19 percentage points higher for students in the bottom quartile of income compared to those in the top quartile . Therefore, any gains that can be made in expanding access for students to enter directly into a college-level QL/QR course will disproportionately benefit students of color and those from low-income families.

College readiness in mathematics has typically been defined as readiness for college algebra, and the placement tests used to determine readiness are largely focused on algebraic manipulation skills. There is no evidence that these same skills are essential for success in a QL/QR or a statistics course, which raises the question: "Readiness for what?" The Dana Center advocates for differentiated placement for different gateway mathematics courses [14]. Texas and Colorado are examples of states that have instituted policy changes to allow for differentiated placement.

There is a danger that this approach reinforces the misperception that QL/QR courses are less rigorous than college algebra. Three key points address this issue:

- The mismatch between the content of placement tests and QL/QR content makes placement tests an inaccurate indicator of readiness.
- Traditional definitions of rigor emphasize algebraic fluency; some mathematics educators advocate for a more expansive definition. The Common Core State Standards for Mathematics describe three components of rigor: procedural fluency balanced with conceptual understanding and authentic context [26]. This perspective is widely accepted and referenced.
- One reason that more students are successful in QL/QR courses is not that the content is easier, but because it is relevant and engaging. Research shows that people learn better when they understand how the content will be useful to them [33].

We return to the question of "Readiness for what?" in addressing the second problem of supporting underprepared students for success. Students who are not college ready should receive content instruction that directly supports success in the college-level course rather than remediating skills or topics that they are unlikely to use in their future courses, careers, and lives. For example, factoring polynomials may be a necessary prerequisite skill for an algebraically-intensive course such as college algebra, but is unlikely to be essential for success in a QL/QR course. "Readiness" for college algebra is different than readiness for QL/QR, so the supports for underprepared students should also be different. Such supports can be provided through two promising structures that have emerged in the last few years.

The first structure is a one-semester corequisite model in which underprepared students enter directly into a college-level mathematics course and are provided with supports to address gaps in their preparation. Ivy Tech Community College is one of many systems that have implemented a corequisite model with supports directly aligned to a QL/QR course. Prior to the implementation of this model, 29% of students earned college-level mathematics credit in *three years*. With a corequisite model, 64% earned credit in a QL/QR course in *one semester* [12].

The second promising practice involves offering a developmental mathematics course focused on quantitative reasoning rather than the traditional algebra sequence. Typically, this model is implemented as a one-year pathway with a single one-semester developmental course followed by a college-level mathematics course. It provides another option for students who are not likely to succeed in the corequisite model described above. This structure has been promoted by the American Mathematical Association of Two-Year Colleges' New Life Project and has been demonstrated to improve student success in a number of settings, including the Dana Center's New Mathways Project, now called the Dana Center Mathematics Pathways [33], and the Carnegie Foundation's Quantway [20].[3] It is important to acknowledge a critical concern that mathematics pathways will create obstacles for students who change their majors, particularly for those students seeking to switch from non-STEM to STEM majors. However, a substantial body of evidence indicates that the vast majority of students who start in a non-STEM field will remain in a similar field [15]. Therefore, the Dana Center recommends that institutions should design mathematics pathways to serve the needs of the greatest number of students possible and ensure that appropriate options exist for students who change to STEM majors.

14.4 Preparing for Increased Access to QR

While we recommend optimistically to enroll 20% to 50% of college students in QL/QR courses, we also need to prepare our systems to handle this shift. Institutions that dramatically increase the number of students in QL/QR courses must address a number of challenges:

Faculty capacity: Many mathematics faculty need ongoing professional learning to prepare to teach quantitative reasoning, including understanding the content, using different pedagogies and assessment techniques, and gaining confidence in teaching mathematics within complex and ill-defined contexts [5, 17]. This is especially true for adjunct faculty who often do not have the same opportunities for professional learning as full-time faculty. In addition to training, faculty need high quality curricular materials and ongoing support to improve their practice.

Facility and institutional capacity: Many departments have lower student enrollment capacity for QL/QR courses and use classrooms that support collaborative learning. Additional challenges include access to technology and adapting QL/QR courses for online delivery. Increasing demand in these courses can place a strain on the staff capacity, facilities, and budget.

Student supports: Tutoring services are often staffed by individuals who have never taken a QL/QR course. Like faculty, tutors need professional learning to be prepared to best support students in QL/QR courses.

See the chapter from Dewar, Larson, and Zachariah in this book for an in-depth discussion of Loyola Marymount University's efforts to address many of these structural challenges.

14.5 Conclusion: Is It Worth It?

The Mathematical Association of America (MAA) published *Achieving Quantitative Literacy: An Urgent Challenge for Higher Education* in 2004 [29]. At that time, the challenge addressed in the publication was to expand the understanding of the basic concept of QL and make the case for its importance in the modern mathematics curriculum. Since 2004, there has been remarkable growth in both the acceptance of QL/QR as legitimate and valuable mathematics and the offerings of courses in colleges and universities.

Early work in this field was largely done at the grassroots level. Professional associations, including the MAA, the American Mathematical Association of Two-Year Colleges, and the National Numeracy Network, have helped promote and disseminate ideas and resources. Leaders like Lynn Steen inspired people such as the authors of this chapter to innovate. And faculty designed courses and inspired other faculty and, eventually, whole departments and institutions, to embrace QL/QR. Yet as we have seen, the progress has not been consistent across institutions and systems.

So now we move on to an urgent challenge: fully integrating QL/QR as a normative feature of modern higher education entry-level mathematics programs. This requires coordinated action and a willingness to compromise as we move towards a common understanding of the gateway QL/QR course. We must communicate what this means to other faculty, staff, and students and work across institutions to ensure that students can use their earned credits to progress towards a degree. It requires that we think expansively about who can succeed in these courses. And finally, it requires innovation to address the challenges of offering high quality QL/QR courses to a larger number of students.

[3] More information on the Dana Center Mathematics Pathways and the Carnegie Math Pathways is available at `dcmathpathways.org/` and `pathways.carnegiehub.org/`.

The work outlined in this chapter may seem daunting. Yet we also see evidence that making QL/QR a viable option for all students is an achievable goal. The data we have presented from CBMS, Colorado, and Ivy Tech demonstrates that diverse institutions have found solutions to the many challenges. We have an opportunity to strategically build on this momentum and dramatically increase the number of students who graduate with an associate's or bachelor's degree, *and* strong quantitative reasoning skills.

Is it worth it? Do we believe that QL/QR courses present a valuable learning opportunity to students? Do we believe that QL/QR courses will help more students succeed in college mathematics and, ultimately, be better prepared to face the quantitative challenges of their future lives? If the answer is yes, then it is worth the work, the compromises, and the occasional frustration to ensure that every student who can benefit from a QL/QR course has access and an opportunity to succeed.

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15

Reflections on the Assessment of Quantitative Reasoning

Richard J. Shavelson, Julián P. Mariño von Hildebrand, Olga Zlatkin-Troitschanskaia, Susanne Schmidt

Stanford University, University of Los Andes, Johannes Gutenberg University of Mainz, Johannes Gutenberg University of Mainz

15.1 Introduction

Although quantitative reasoning (QR) is recognized as a fundamental 21st century skill, many educational systems worldwide have not adequately fostered QR in their students (e.g., [2, 22]).[1] Hence, QR has garnered considerable attention from educators, policy makers, and the public (as noted by Madison in this volume). However, there is by no means full agreement on what QR is.[2]

In this chapter, we begin by examining definitions of QR, settling on a definition reflective of a "situated" or "socio-cultural" approach (from now on, "situated"). Karaali, Hernandez, and Taylor's [5, p. 25] "common thread" definition of QR reflects this approach: "competence in interacting with myriad mathematical and statistical representations of the *real world in contexts of daily life, work situations and the civic life.*" We then present a principled approach to developing situated QR assessment tasks (Evidence-Centered Design, or ECD) that provides a reasonable trade-off between cost and authenticity. While other papers in this volume focus on various means of assessing QR (i.e., those by Zerr and by Kiliç-Bahi and Cahoon), our focus is on the *construction* of both new innovative assessments and the adaptation of existing assessments in higher education. We draw on three examples. The first is from the country of Colombia's assessment of QR as part of its national SABER testing programs [25]. This is a case of front-end use of ECD to create a QR test. The second example is taken from the Collegiate Learning Assessment (CLA; [23]) where performance assessments of critical thinking incorporated QR as one component. While the CLA was not front-end ECD built, we use ECD here to describe its development. The third example is drawn from business and economics education and we give a case where QR is measured in an achievement test that test developers did not view as tapping QR.

[1] QR and quantitative literacy have been used interchangeably; we do not make a distinction in this chapter. We note that Kiliç-Bahi and Cahoon (this volume) distinguish numeracy (at the elementary school level), quantitative literacy (middle school) and QR (advanced); our focus is on higher education and QR.

[2] Its definition varies (e.g., chapters in this volume, as well as [2, 4, 7, 9, 11, 5, 17, 29]).

15.2 Definition of Quantitative Reasoning

Definitions matter, especially when it comes to assessing QR. One such definition is closely tied to mathematics and the corresponding skills. In contrast, another definition is closely linked to applying mathematical skills as well as other skills to solving real world problems, skills that professionals and responsible citizens are expected to have. Other definitions include both. The majority of definitions, however, lean toward application with an emphasis on skills involved in dealing with messy, complex, real world, everyday challenges with to a greater or lesser extent quantities and their various representations (e.g., [5, 22]).

Shavelson [22] focused on QR assessment because, on the one hand, it signals what QR is and what aspects of QR are important and, on the other hand, its assessment provides a very powerful policy instrument for change if wielded properly [23]. He argued that assessments of QR varied systematically reflecting underlying beliefs about the nature and definition of QR. He noted that three different beliefs (or philosophical positions) underlie the various QR definitions. These beliefs lead to different types of QR assessments: behavioral (psychometric-response driven), cognitive (process-driven), and situated (context-driven) (see Table 15.1).

Behavioral Approach

The behavioral approach leads to an assessment that focuses on the observed, independent performance or actions in response to a set of similar test questions. It assumes that QR "requires reasoning based on mathematical properties and relations. The reasoning process may be either inductive or deductive, or some combination of them" [7, p. 239]. QR items, perhaps in contrast to typical mathematics items, do not place a high demand on computation but rather focus on reasoning with numbers, operations, and patterns. To elicit QR, typically word problems, number series, and tasks requiring an appropriate arithmetic operation constitute an assessment of QR (Table 15.1a).

Cognitive Approach

The cognitive approach assumes that observed behavior can be divided into component input, cognitive, and response processes and seeks to identify the cognitive operations that underlie performance. The goal of the approach is to extract sets of elementary cognitive processes that account for a wide range of performances. In general, this approach probes the types of elementary cognitive processes that are exhibited when solving a problem involving QR. A classic QR problem in this tradition is DONALD + GERALD (Table 15.1b). This approach begins with an analysis of a single task, seeking out its affordances and constraints. The task is analyzed logically and mathematically for possible solution paths. Then an extensive analysis moves from the particular to the general set of rules for solving problems in the same domain. The test-takers are asked to "think aloud" as they perform tasks and solve problems (e.g., [10]) in order to capture cognitive processes in action (working memory). Findings from the think-aloud protocols are then used to validate or revise QR tasks. Think alouds are ultimately used to establish a step-by-step process of problem solution and characterize common errors. Cognitive models often are tested by programming computers to simulate human information processing; if the computer behaves in the same way as the human, then this provides validity evidence for the claim that the cognitive model is useful. The cognitive approach's main contribution to the assessment of QR lies in its analysis of the cognitive processes using think alouds as a means of providing evidence that the assessment items have, indeed, evoked reasoning with quantities.

Situated Approach

The situated approach views QR as context dependent in attempting to understand how QR manifests itself in a particular situation, context, or culture (Table 15.1c). The approach views performance, in our case QR, as influenced in part by what the individual brings to the situation and in part by how the situation supports ("affords") and constrains performance. The situated approach is reflected in Madison's [17] characterization of QR as opposed to mathematics. Following Madison ([17]; see also [5]), the main focus of QR in contrast to mathematics is that it:

(a) operates in real, authentic situations in contrast to seeking powerful abstraction
(b) operates in specific, particular applications in contrast to seeking generality (e.g., proofs)
(c) is heavily context dependent in contrast to being lightly context dependent
(d) is society dependent in contrast to independent

Table 15.1. Three approaches and corresponding QR tasks.

<table>
<tr><th>Approach</th><th>Example Task</th></tr>
<tr><td>(a) Behavioral (Psychometric)[3]</td><td>Question 4
A power station is located on the boundary of a square region that measures 10 miles on each side. Three substations are located inside the square region.
Quantity A: The sum of the distances from the power station to each of the substations
Quantity B: 30 miles
A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.</td></tr>
<tr><td>(b) Cognitive (Process) [26, p. 143]</td><td>DONALD D = 5
+GERALD
ROBERT
Here each letter represents a digit (0, 1, …, 9) and you know D = 5; no other letter equals 5. What digits should be assigned to the letters such that, when the letters are replaced by their corresponding digits, the sum is satisfied?</td></tr>
<tr><td>(c) Situated (Context) [22, p. 35]</td><td>One of the drugs in the Coronary Drug Project was nicotinic acid. Suppose the results on nicotinic acid were as reproduced below. Something looks wrong. What, and why?
<table>
<tr><th></th><th colspan="2">Nicotinic Acid</th><th colspan="2">Placebo</th></tr>
<tr><td>Group</td><td>Number</td><td>Deaths</td><td>Number</td><td>Deaths</td></tr>
<tr><td>Adherers</td><td>558</td><td>13%</td><td>1813</td><td>15%</td></tr>
<tr><td>Non-Adherers</td><td>487</td><td>26%</td><td>882</td><td>28%</td></tr>
<tr><td>Total</td><td>1045</td><td>19%</td><td>2695</td><td>19%</td></tr>
</table></td></tr>
</table>

(e) is political in contrast to apolitical

(f) employs ad hoc methods in contrast to focusing on methods and algorithms

(g) is evidenced in response to ill-defined problems in contrast to well defined problems

(h) employs estimation in contrast to employing approximation

(i) is interdisciplinary in contrast to being heavily disciplinary

(j) focuses on problem descriptions in contrast to focusing on solutions

(k) focuses on practical situations outside the classroom in contrast to primarily classroom-type tasks

(l) is unpredictable in contrast to being predictable.

The situated approach comes closest to what many academic scholars and teachers refer to as QR. For example, consistent with Fisher (this volume), it emphasizes social contexts and communication (patterns of discourse in numerals, algebraic expressions, graphs, charts, and computers); consistent with Kiliç-Bahi and Cahoon (this volume), it focuses on the display of competence in analyzing, appreciating, deciding, understanding, and using QR in a real-world context. It creates a rapprochement for the many definitions of QR.

There are, of course, alternative definitions. Synthesizing Fisher (this volume), Kiliç-Bahi and Cahoon (this volume), and Karaali, Hernandez, and Taylor [5], we note that QR involves competence in using mathematical, statistical, spatial, verbal (and other, e.g., kinesthetic) representations of the real world ("cross-disciplinary"), drawing on knowledge of statistics, data representation, arithmetic, mathematics, and logic. It involves *habits of mind* (e.g., [12]) such that given a wide variety of information, those competent in QR draw on quantitative information as one basis for justifying a claim or an argument without being prompted to do so.

[3]See p. 12 of GRE Practice Test 1, available at `www.ets.org/s/gre/accessible/GRE_Practice_Test_1_Quant.doc`.

We agree with Karaali, Hernandez, and Taylor [5, p. 10] that there is a common thread to the definition of QR as indicated here. We believe the situated approach reflects this common thread definition. The various facets of the definition lead to the development of different types of tasks or test items that evoke QR and lead to the multidimensional measurement of QR (if supported empirically). However, we believe that scholars will continue to debate the definition of QR. As research and definitions progress, the field will grow in knowledge about QR. However, a final definition is not necessary to be able to proceed with assessing QR. Rather we need a working definition and test framework which is itself the subject of research and scholarship.

In the following, we present an approach to developing or adapting measures of QR along the facets of the situated approach.

15.3 Principled Assessment Design

Science and art converge in building assessments. Here we briefly sketch an approach to building an assessment of QR. We introduce the "assessment triangle" to provide a macro view of the assessment development and validation process. We then look a bit deeper in order to provide a principled approach to generating a QR assessment: Evidence-Centered Design. This deeper look put us in a position to illustrate the ECD approach with examples from Colombia's assessment system (SABER) and for the performance task in Figure 15.2 (see below). We also exemplify a post-hoc approach to repurposing existing tests of economic and business knowledge, the Test of Understanding College Economics (TUCE; [31]), and Examen General para el Egreso de la Licenciatura (EGEL; [8]), to examine its quantitative-literacy demands.

Assessment Triangle

The assessment triangle (Figure 15.1) describes an approach to building assessments [18]. Originally the vertices were cognition, observation, and interpretation. We replaced cognition with the more generic term, construct (QR here), because the focus of measurement may not always be on cognition. We have also embellished the vertices and sides to show how evidence can be brought to bear on the assessment's claim to measure QR. The double-headed arrows show that assessment development is an iterative process of construction-tryout-revision-tryout-etc.

In building an assessment (of QR) whether for large-scale, institutional or even classroom use, the first and essential step is to define what is to be measured, the construct definition (construct vertex in Figure 15.1). In building a working definition we sometimes begin with a vague idea and create tasks or test items that are expected to elicit evidence of QR. After looking at what we produced we are in a position to build an initial working definition. As assessment development proceeds, that definition is often revised and increasingly formalized (especially in larger-scale assessment). The definition is always open to revision as conceptual, logical, and empirical evidence amasses in favor of revision. It is for this reason that we have spent so much time above discussing definitions of QR.

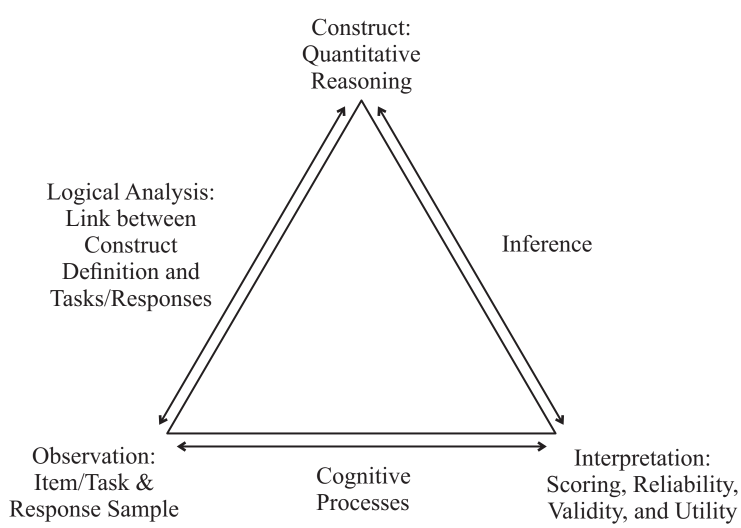

Figure 15.1. Assessment triangle.

Based on the situated QR definition, tasks and test items are generated (the observation vertex). The intent is to use these tasks to elicit evidence of test-takers' QR abilities. The tasks must fall within the boundary of the QR definition and represent the full bounded space ("content representativeness" or "content validity"). More specifically, the working definition of QR bounds the nature of tasks or items that might probe QR and those that do not. For example, from a situated perspective, an abstract number series task might fall outside of the bounds of QR, but a task of interpreting a graph of data showing the relationship between the number of police officers and the number of crimes in various cities would fall within the bounds. One way of evaluating the representativeness of the content is by logically analyzing both the tasks and the QR definition to see if they are representative of the QR domain.

The tasks alone are only necessary conditions for the observation vertex. Without a definition of how responses to those tasks will be scored, the observation vertex is incomplete. In the case of multiple-choice items, this type of specification is readily apparent. However, for complex, open-ended performance tasks, a scoring rubric needs to be specified.

With a set of tasks and items and their corresponding scores, we are in a position to interpret them (the interpretation vertex). Are the scores reliable? Are they valid? Do they reflect the types of QR facets that are the intent of the assessment? Do they provide useful, not redundant, or already-available-information?

An analysis of students' think alouds provides evidence bearing on whether the tasks and items evoked the type of reasoning that fits with the QR definition (cognitive processes in Figure 15.1). An analysis of item scores and their composites provides evidence on the reliability or consistency of the measure of QR. An analysis of content representativeness (content validity), the think alouds (cognitive validity), the structure of the QR test (does it fit empirically to the structure (facets) of QR in the definition used?), and its relation to other measures of QR bear on validity and utility [1].

In summary, on the basis of these analyses of item responses and score composites, we are in a position to interpret those scores as to whether they are reliable, valid, and useful measures of QR. Based on this process the QR definition, tasks and items might be modified and new data collected in an iterative process until the evidence for a QR interpretation of scores is warranted. Here, we include the development of QR assessments for the classroom as well as larger-scale use. While we do not believe all the formal steps will be followed in practice, classroom assessment would be improved by taking into account the (often implicit) instructor QR definition, whether the test items actually fit that definition, and evidence from students (e.g., think aloud, test scores) supporting the claim that the test reflects the nature of QR intended to be assessed.

Evidence-centered Design

ECD [19, 20, 18] provides a way of formalizing ideas in the assessment triangle. ECD grew out of extensive assessment development experience and validation theory (e.g., [15]). ECD formalizes steps in supporting the interpretive claim that an assessment measures QR based on conceptual evidence and empirical evidence from students' responses to tasks and items. ECD forces us to be quite clear about the interpretations or claims we intend to make on the basis of the assessment; it pushes us further to describe the evidence we would need to support such claims, as well as to specify the types of tasks or items that would provide data that could be summarized as evidence.

ECD begins with an analysis of the construct domain (in our case, the QR domain), asking about what is important in the domain, as well as what activities, knowledge representations, and skills are central in the domain. It then moves to a model of the domain representing key aspects of content that can be assessed [21]. Next, a conceptual assessment framework is built to specify a student model, or the (complex) knowledge and skills to be assessed; evidence, or the data (think alouds, item responses) to be collected; and task models, or the nature of the tasks that elicit the QR responses. The next step is assessment implementation where a student model (knowledge and skills), a task model (item types), and scoring and statistical models (data becoming evidence) are put forward. Finally, assessment delivery involves students interacting with tasks, and the evaluation of performances [3].

For our purpose, the construct definition of QR including the domain analysis and model is important. This is where QR is defined and the different facets of QR are set forth. For each facet, ECD would specify the: claim or claims to be made about students' QR (e.g., students can represent data graphically and schematically), the evidence that could be collected to support or refute the claim (e.g., performance on a set of tasks drawn from the domain), and exemplar tasks that provide response evidence of QR. In the next section, we provide a concrete example of the application of ECD to the construction and evaluation of an assessment of QR.

15.4 Assessment of Quantitative Reasoning: SABER PRO

The country of Colombia assesses student achievement from elementary school through college. All students in higher education institutions approaching graduation are required to take a government-run exam, called SABER PRO [25], that includes both generic and domain-specific tests. The development of all SABER PRO tests, including a QR test, was done between 2010 and 2012 following ECD.

Application of the ECD Approach to Item Construction

Initially, a definition of the QR construct was established, based on a review of literature and examples of existing tests (discussed below). The test's generic and situated character was crucial in that test definition: it was intended to assess competencies that all finishing undergraduate students, independently of their career path, should possess to appropriately engage in activities expected of college-educated citizens.

Next, specifications for the test were set in three hierarchical layers. In the first layer are distinct assertions about students' development of competencies intended to be verified with the test. The second layer contains specific evidence that should be collected with the test to support the intended assertions. A third level, which is kept undisclosed, contains the different tasks test-takers are asked to do to answer the items and thus to provide the required evidence.

After a first version of specifications was completed, framework and item development followed an iterative process with rounds of item construction and validation, pilot testing and adjustment of the specifications and the construct definition. Item validation was (mainly) done using a rather economical setting of student focus groups. Students would first answer individually a set of items and then discuss the difficulties they had encountered, the process they had followed, and the reasons they had considered to select an answer. Validation leads to rejection, adjustment, or approval of items to go on trial in pilot tests.

Pilot testing was carried out under real testing conditions with thousands of students taking the (compulsory) SABER PRO exam. Psychometric analysis of pilot results was used to calibrate or reject items, examine reliability, and identify item differential functioning across groups of students. The round ended with eventual adjustments of specifications (and more rarely of construct) that might be due to either the addition of new tasks developed in the construction process, the need to change wording to facilitate item builders' understanding of the framework, difficulties in getting enough items of some particular type out of the construction and validation process, or systematic psychometric problems (most commonly low correlation with the test or too high/low difficulty) of items of a particular type. It is worth noticing that these were practical and psychometric considerations that shaped the construct and the specifications.

The SABER PRO QR Framework divides QR into three general competencies [13]:

1. Data Interpretation. This is the ability to understand and manipulate representations of quantitative or schematic information presented in different formats (texts, tables, graphs, diagrams, schemas). This involves, among other things, extracting local and global information, comparing representations from a communicative perspective (e.g., which figure represents something in a clearer or more appropriate way), representing information graphically, and tabulating functions and relationships. It may require simple calculations or estimates, such as adding, subtracting, multiplying, averaging, or rounding numbers.

2. Formulation and Execution. This is the ability to establish, execute, and evaluate strategies to analyze or solve problems involving quantitative or schematic information. It involves, among other things, selecting relevant information and identifying variables to abstractly model concrete situations; analyzing model assumptions and evaluating their usefulness; selecting and executing mathematical procedures such as algebraic manipulations and calculations; and evaluating the outcome of a mathematical procedure.

3. Argumentation. This is the ability to justify or give reasons for statements or judgments about situations involving quantitative information or mathematical objects (statements and judgments can refer to representations, models, procedures, results, etc.). Faced with a problem or an argument involving quantitative or schematic information, students must propose or identify valid reasons, correctly use examples and counterexamples, distinguish facts from assumptions, and recognize fallacies.

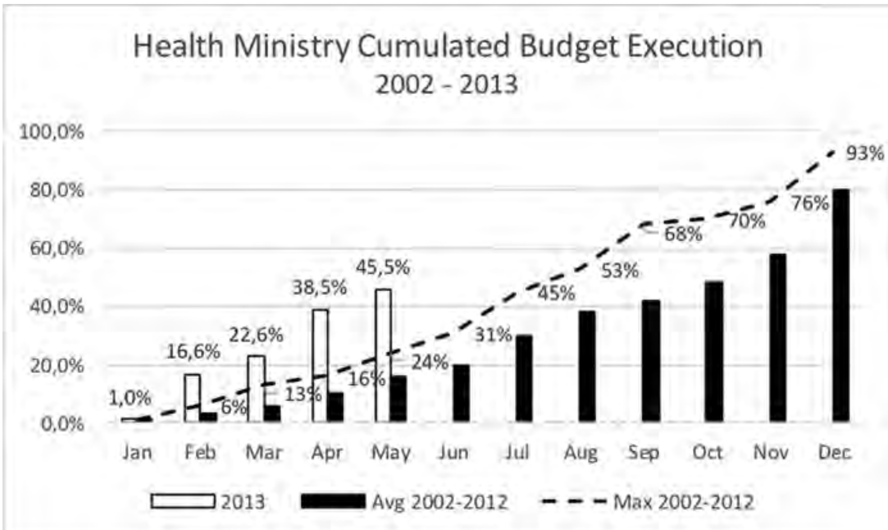

Figure 15.2. SABER QR example.

Examples of Items Testing Data Interpretation and Argumentation.

In what follows, we provide examples of test tasks/items that probe the QR facet of data interpretation and argumentation. Our intent is to make concrete the definition of the facet and the nature of the evidence to be used to justify interpretation of the SABER QR test as a test of QR.

Test-takers are presented the graph in Figure 15.2 and told it shows the cumulated monthly execution of the governmental budget for the health ministry with data corresponding to the first five months of 2013 (white bars), the average of the period 2002–2012 (black bars), and the maximum in that same period (line).

In a first item, corresponding to data interpretation, test-takers are asked to identify among the four options (Figure 15.3) which one represents the average monthly expenses for the 2002–2012 period.

In a second item, corresponding to argumentation, test-takers are asked to identify among the next four options the

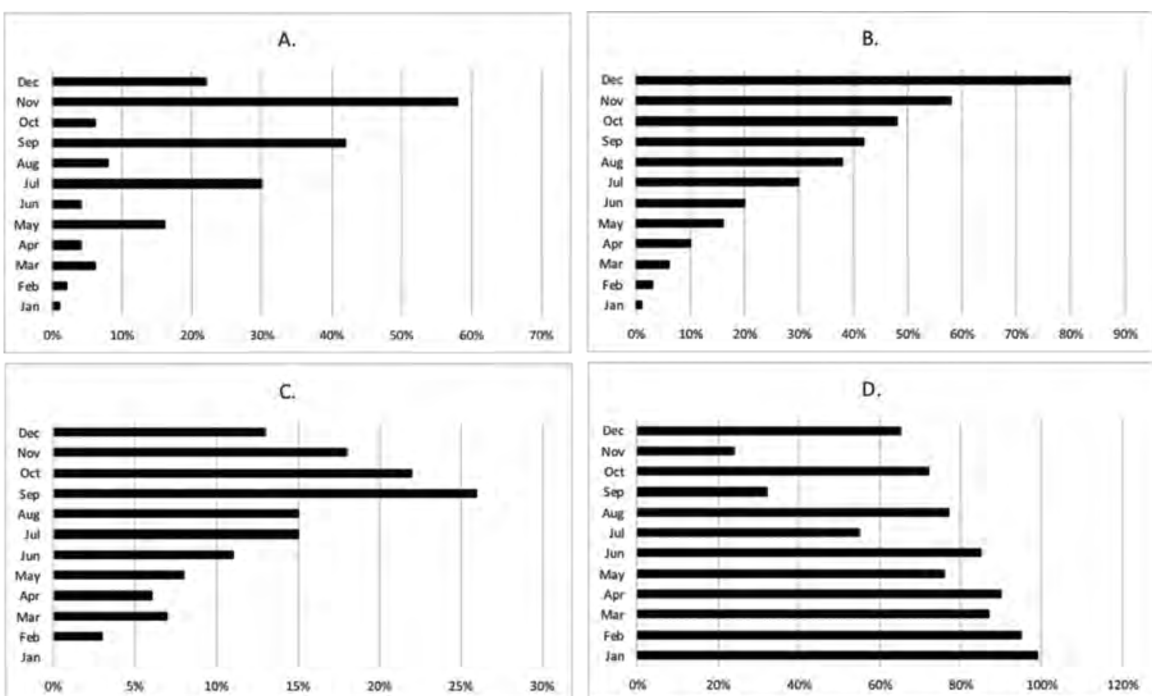

Figure 15.3. Options for interpretation item.

reason why values for 2013 are always increasing from one month to the next:

1. The graph shows the execution percentage for each month is always bigger than 2002–2012 average.
2. Monthly execution percentages in 2013 are in each case bigger than 2002–2012 averages.
3. The cumulated execution of one month is obtained by adding that month's execution to the value of the previous month.
4. The execution percentage of a given month is always bigger than that of the previous month.

Having provided an example of front-end development of a measure of QR, we now turn our attention to using ECD to describe a subtask of a performance of critical thinking that taps QR. The intent is to provide another example of how ECD can be used to create QR tasks.

15.5 Assessment of Quantitative Reasoning—Performance Tasks

The Collegiate Learning Assessment (CLA and CLA+) is an assessment of college students' critical thinking, analytic reasoning, problem solving, and communication when confronted with tasks drawn from the situated perspective (e.g., [23]). Performance-assessment tasks from the CLA may come close to the situated vision of QR described above. For example, one task taken from the performance assessment, Airplane [22], asks test-takers to decide whether the aircraft in question, the SwiftAir 235, is indeed accident prone. The information provides the test-takers with two panels of data (Figure 15.4). One panel provides information on aircraft sales and the second panel provides data on accidents of the SwiftAir 235 and competitor aircraft. With this information, the person is in a position to determine whether the SwiftAir 235 is accident prone. If the person focuses on the panel showing the number of accidents—information directly related to the accident prone question—the conclusion is that the aircraft is accident prone. However, if the person stops a minute and considers the sales data in conjunction with the number of accidents, the accident rate rather than the number of accidents becomes available; this information leads to the conclusion that the SwiftAir 235's accident rate turns out to be the lowest.

In building a performance assessment of critical thinking, a real-world event or problem-to-be solved is presented along with information more or less relevant to the event or problem [24]. The problem might be similar to that in Figure 15.4, for example, which requires critical thinking to combine both pieces of data, and some elementary QR to solve it. In another case, the problem might be visual-spatial, for example, and involve creating an art exhibition that revolves around tension between technological progress and its potentially positive and negative impact on the environment. The performance task might also be verbal, in which case varying sides to a proposed civic project—for example, where to situate a prominent movie mogul's museum, if at all—are presented to the test-taker and an understanding of these various perspectives is needed to make progress. Regardless, in each case quantitative data and representations would be incorporated into the task. The data and the information representations may be reliable and valid, or not. At least in one case, they might admit to erroneous judgment based on thinking quickly and interpreting. For example, in a scatterplot of the relationship between the number of police officers and the number of crimes across a sample of cities, the positive slope could be erroneously interpreted as the police causing crime.

As noted above, there are many definitions of QR. Drawing from the situated perspective, at its core is the analysis or synthesis and evaluation of quantitative information embedded in real-world situations to form and act on a warranted judgment based on reliable and valid evidence (information). The situation (or problem or event) is typically complex and ill-defined with no clear, single resolution, as in real life. In order to assess QR, four elements need to be introduced into the issue, problem, or event that in part involves quantitative information and is likely to evoke QR: the reliability of the information source (reliable/unreliable), the validity of the information (relevant or irrelevant to the issue), the information's susceptibility to judgmental errors when thinking too quickly, and communicating a decision or recommended course of action providing the evidentiary base for the judgment.

From an ECD perspective, the *claim* is that the assessment task taps QR on everyday complex issues, events, problems, and the like. The *evidence* comes from evaluating test-takers' responses to the assessment tasks. The *tasks* in the assessment manipulate the reliability, validity, and judgmental aspects of the information (minimally quantitative) that confront a person when thinking critically about important events. What follows is an explication of how tasks are sampled and transformed into assessments of QR.

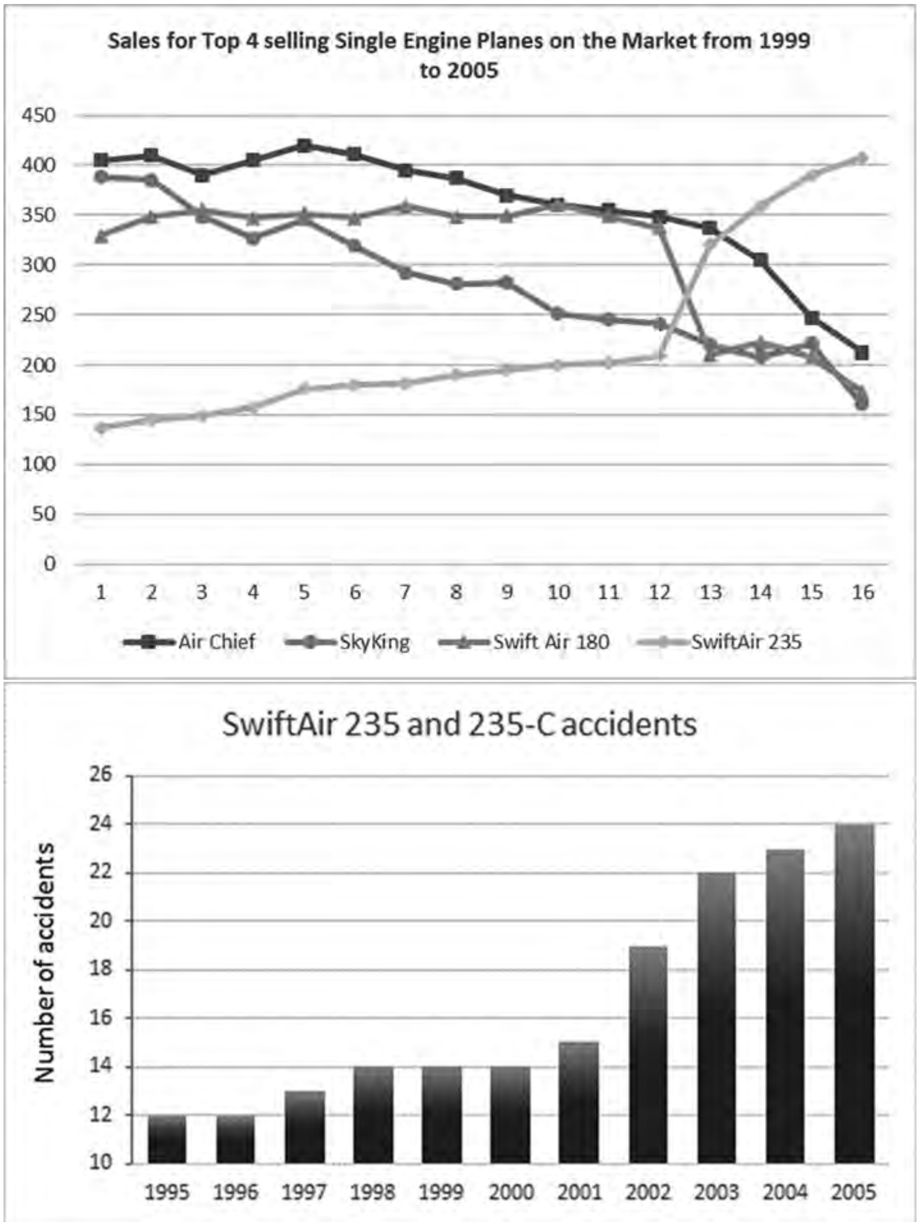

Figure 15.4. Quantitative information provided in a task drawn from the Airplane performance assessment [22, p. 36]. Note: The test taker sees not only the data in the bottom panel but also additional charts for each of the four aircraft shown in the top panel.

The universe of tasks demanding generic QR skills comprises the myriad everyday complex life situations. A prime source of situations may be found readily in newspapers (e.g., politics, environment, sports, education, business, fashion, arts, and science). The Airplane task described above, for example, was inspired by a *Los Angeles Times* report about an aircraft accident at the Van Nuys Airport in Southern California.

These tasks involve quantitative (and other) information and are complex, often without a clear path toward a solution, decision, or action. Instead, there are tradeoffs. There is often more than one solution; however, when incorporated into an assessment, there are better and worse solutions or decisions, actions, etc. The tasks are compelling in the sense that they represent current everyday challenges that test-takers face or might be expected to face as college graduates and citizens more generally.

For the CLA, assessment tasks are developed to include certain elements that invite students to think critically using quantitative (and other) information. These elements are [23]:

Information-source Sampling. Material such as newspaper articles, YouTube videos, and government reports are sampled from real-world domains. The information provided may be manipulated to be either:

1. Reliable or trustworthy quantitative information such as the Federal Aviation Report in the Airplane task; in contrast, an amateur aviator's opinion article would be considered to be less reliable or unreliable
2. Valid or directly relevant to the issue at hand (FAA Report) or tangential or unrelated to the task (photos of the SwiftAir 135 and 235).

Judgmental and Decision Heuristic Sampling. When using information to make judgments and decisions, people often take shortcuts or use heuristics to make judgments or reach a decision using quantities. The work of Tversky and Kahneman [30] opened up a field that has become known as rational thought (e.g., [14, 28]). These heuristics are usually applicable in the real world, where quick judgments or decisions must be made and deliberative thought might be dangerous (e.g., get out of the crosswalk because the car is not going to stop). Heuristics have been quite useful throughout human evolution. However, they can interfere with rationality, or reasoning quantitatively, when the situation is important enough to demand a rational decision (e.g., buying a house). In this case, deliberative thought is needed to simulate alternatives and their consequences. Since Tversky and Kahneman's initial research, the list of judgmental and decision making heuristics has grown exponentially (e.g., [27]) and can now be easily researched on the internet. Consequently, irrational (when the situation demands otherwise) quantitative thinking heuristics are built into performance tasks to assess the judgmental aspect of QR.

The Airplane task (Figure 15.4) uses one of those heuristics where baseline conditions (number of aircrafts sold) is ignored and unadjusted data are used to make decisions. If we ignore sales, we must conclude (problematically) that the SwiftAir 235 is, indeed, more accident prone than its competitors.

Communicating. The ability to communicate clearly, concisely, accurately, and compellingly is part of our conception of generic skills including QR. The communication might be in writing (e.g., a memo to the president of a company or an op-ed piece), orally with visuals, or both (e.g., PowerPoint presentation with notes). This type of communication would

- use reliable information and avoid less than reliable information
- use relevant information and avoid peripheral information
- avoid judgmental and decision making "traps"
- consider alternative courses of action to the one proposed and, based on data indicate why the recommendation is given
- use concise and compelling arguments, evidence, and conclusions to rhetorically establish a position, decision, course of action, or recommendation.

Scoring. For the extended constructed responses, analytic (dimensional) and holistic scoring rubrics were developed based on the construct definition. The rubrics take into account the test-takers' use of reliable or unreliable and valid or invalid information as well as their avoidance of heuristics that lead to errors in judgment and decision making. The rubrics reflect the use of quantitative information in justifying decisions, problem solutions and/or recommendations for action. Moreover, they can evaluate argumentation, the use of evidence to support claims, and clarity of communication. While detailed concerns regarding scoring are often related to large-scale assessment, classroom instructors would do well to explicitly define their scoring criteria to themselves and their students before the exam. This is a tack that we have seen lead to improved teaching, learning, and assessment.

15.6 Assessment of Quantitative Reasoning—Economic Knowledge Tests

Often QR is assessed unintentionally. This happens because QR is embedded in various disciplines and professions as well as in teaching-learning-assessing and the knowledge-practice domain. Consequently, QR is often not labeled as such in the discipline or profession or in achievement tests. Separating out QR seems reasonable as it provides feedback to teachers and students on how QR contributes to solving domain-specific tasks and problems. Here, we provide an

Quantitative Reasoning Item

In Sunshine City, one local ice cream company operates in a competitive labor market and product market. It can hire workers for $45 a day and sell ice cream cones for $1 each. The table below shows the relationship between the number of workers hired and the number of ice cream cones produced and sold.

Number of Workers Hired	Number of Ice Cream Cones Sold
4	340
5	400
6	450
7	490
8	520

As long as the company stays in business, how many workers will it hire to maximize profits or minimize losses?

A. 5
B. 6
C. 7
D. 8

Verbal Reasoning Item

At the profit-maximizing level of output, a perfectly competitive firm will:

A. Produce the quantity of output at which marginal cost equals price.
B. Produce the quantity of output at which marginal cost is minimized.
C. Keep marginal cost lower than price, so profits will be greater than zero.
D. Try to sell all the output it can produce, to spread fixed costs across the largest possible number of units.

Figure 15.5. Example quantitative- and verbal-reasoning Items.

example from the domain of business and economics. Tests of students' knowledge and understanding of business and economics (IGEL and TUCE) include items and tasks that fall within our definition of QR. We explore techniques suitable to identify such embedded test-facets and examine the reliability and interpretability of their measurement.

Tasks in business and economics can be divided into those demanding more or less QR and those requiring more or less verbal reasoning (VR) (see [5, p. 514] for TUCE items).[3] Consequently, we analyzed the items in TUCE and EGEL in order to identify quantitative and verbal content demands. With a specific focus on subdomains for which QR is a particular requirement in business and economics, we used items from TUCE dealing with microeconomics and items from EGEL dealing with managerial accounting and business finance in these analyses. Items that required students to mainly apply their mathematical abilities were called QR items while items that did not require numerical skills were grouped together as VR items.

From an ECD perspective, the claim is that QR involves various forms of numeracy and can be differentiated from VR. The evidence is derived from data generated by students' responses to the items of the TUCE and EGEL (see Table 15.2).

Table 15.2. Distribution of quantitative reasoning (QR) and verbal reasoning (VR) items from the three content domains of the TUCE and EGEL.

Area/Construct	QR	VR	Total
Microeconomics	4	26	30
Managerial Accounting	13	3	16
Business Finance	11	4	15
Total	28	33	61

[3] Due to a lack of testing time, general QR could not be assessed in this study. In the ILLEV project tasks based on numerical sequences (scale from the intelligence test) served as indicators for the participants' numerical abilities; a positive correlation with business and economics knowledge was found [32, p. 76].

The data were collected in a sub-study of the WiWiKom project. The sub-sample included 1,492 students from 40 German universities.[4]

In order to collect initial reliability and validity evidence to support a QR interpretation of the TUCE and EGEL test item responses, we examined the underlying structure of item responses. More specifically, we used confirmatory factor analysis (CFAs), and specified a two-factor structure underlying the item responses reflecting QR and VR (see Table 15.2). We tested this model against a general business-economics single-factor model. The difference test shows that model 1 fits the data better than model 2. The moderately high latent correlation between the QR factor and the VR factor of 0.76 ($p = 0.000$) shows that QR and VR share much in common, but also that they can be interpreted separately.

In order to estimate the reliability, reliability of the items corresponding to the two factors was calculated. The factor reliability of the QR scores was 0.70, and of the VR scores was 0.75, which can be interpreted as indicating sufficient internal consistency for both of the theoretically derived scales from the three content domains of business and economics. The evidence consequently supports an interpretation that the TUCE and EGEL items provide reliable measures of QR and VR. The findings of the quantitative analyses presented here were also confirmed in think aloud interviews with the test-takers (see [6]).

15.7 Conclusion

QR has been defined in many different ways. Some definitions focus on mathematics while others focus on the application of quantitative tools to every-day, complex real-world problems where solutions go beyond (simple) mathematics. We identified three approaches to the definition: a behavioral approach (correctly responding), a cognitive approach (reasoning mentally), and a situated approach (balancing evidence and reaching a warranted course of action). We prefer the situated approach and the examples we provide exemplify this approach with assessment tasks ranging from constrained, correct-answer multiple-choice tests to open-ended, ill-defined problems or situations in a performance assessment. Our preferred type of assessment is the latter one; however, due to time, cost, and technical know-how, we recognize the need for multiple test formats.

To develop measures of QR, we recommend using the assessment triangle with its vertices of: QR definition, QR tasks within the definitional domain, and interpretation of responses to those tasks as an indicator of the level of QR. Building on the triangle, we recommend the use of an evidence-centered design where the following facets are made clear and are conceptually and logically linked: (a) the *claim* that the test measures the construct, QR; (b) the *evidence*, as in the aggregation of test-takers' responses to a set of tasks from the QR domain; and (c) the *tasks* that purportedly tap some aspects of QR. By following this chain of reasoning and assessment development, the chances of constructing measures that withstand validity scrutiny is greatly enhanced compared to simply generating multiple-choice and short-answer items.

QR has attracted considerable attention from educators, policy makers, and the public, which increases the importance of the development and use of valid measurements of QR so as to promote the teaching and learning of QR. While we can identify alternative approaches to the definition and consequently also the measurement of QR—behavioral, cognitive, and situative—the last approach is most consistent with the definitions of QR in the community as reflected in, for example, the chapters from Fisher and from Zerr in this volume. Moreover, given the signaling function of assessment, the situative approach is the most accepted by the public. The assessment triangle [18] that links the QR definition with the construction of QR tasks and the validity of the interpretation of QR scores provides a useful tool to start developing QR measures. By combining the assessment triangle with the tools provided by evidence-centered design [19], we believe reliable and valid measures of QR can be produced. We supported this claim with three different approaches to the development of QR measures—the development of a new QR measure, describing existing QR measures, and revising a test of economics so a subset of the test focuses on QR. It behooves all of us to continue to strive to examine and improve current measurements of QR. In doing so, we will make progress in providing methods not only for tracking QR learning but also in improving our understanding and definition of QR.

[4] For further information on the WiwiKom test instrument and the study design see [33].

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16

Assessing Quantitative Literacy as a Cumulatively-Acquired Intellectual Skill

Ryan J. Zerr
University of North Dakota

16.1 Introduction

For over 20 years, and with increasing emphasis during the early part of the 21st century, explicit calls for baccalaureate quantitative literacy (QL) requirements have grown in parallel with the need for colleges and universities to assess their institution's learning outcomes [15, 16, 19, 21]. In both cases change has occurred. In fact, a recent survey demonstrates there is an upward trend in the number of institutions with QL learning outcomes, to the extent that a significant proportion (80%) of institutions now have such a requirement [1]. And with learning outcomes assessment a staple of regional accreditation requirements [15], the situation is one in which many colleges and universities have baccalaureate requirements involving quantitative literacy, along with a clear need to assess student learning relative to this requirement. How should this be done?

One variable having an effect on the answer to this question concerns the locus of interest for the learning, is it within a specific course or across the curriculum? If one is trying to assess students' quantitative literacy as a result of a single course, then the timing and the means for assessment may differ substantially from an approach whose interest is in the entire curriculum's impact. If course-based, then the assessment method can take advantage of the course context, such as by using student work from the course and situating the assessment process within the term of the course's offering. This certainly offers advantages related to consistency, ease of evaluation, and logistics.

If, however, the intent is to assess students' quantitative literacy based on the cumulative effect of the curriculum, the course-based approach loses much of its appeal. For instance, a course-based approach may not adequately take into account that students' QL is a cumulatively-acquired intellectual skill with contributions coming from multiple courses, not all of whose emphases are on quantitative literacy. Another problem is that of consistency. One challenge involves the fact that student work comes from multiple courses in a variety of departments. Another challenge for consistency is in finding a uniform process. For example, finding a rubric that faculty from across campus apply consistently is a challenge. Some faculty may not have the disciplinary background that allows them to apply the rubric to student work outside of their area [11]. Fundamentally this distinction is one of finding the most desirable timing for QL assessment. Should it be mid-stream and course-focused or cumulative and degree-level focused?

One can see in this distinction the larger theme of this volume manifest in different assessment approaches. The stable core of QL is faced with a need to assess students' abilities within a shifted context, one where QL once may have been viewed as primarily about mathematical ability (likely coming within a single course) toward a more nuanced and cumulatively-acquired intellectual skill.

I do not presume to answer the matter of ideal QL assessment timing in a definitive way, as its answer certainly depends on institutional factors and assessment goals. Instead, I offer reasons why doing QL assessment near graduation may be a good (or at least reasonable) choice, relating the matter of timing to the notion of QL as a multidisciplinary intellectual skill that is cumulatively acquired and perhaps most accurately demonstrated in student work that also involves intellectual skills such as effective writing and critical thinking. I end with an illustrative example of an assessment strategy timed to occur near graduation, one that grew out of the experience of trying a course-based approach, in multiple guises, with limited success. One of my assumptions is that students' quantitative literacy may develop over time and in courses that are not necessarily focused on a QL learning outcome per se; I contend it may be worthwhile to be able to measure this cumulative effect. Another virtue of this assessment method is that it may be used regardless of the nature of a given college's or university's curriculum, such as whether it has an explicit QL requirement or not.

16.2 Background

Excellent guides to recent efforts to assess QL can be found in Vacher [23] and in Kiliç-Bahi and Cahoon (in this volume), who demonstrate that significant attention has been paid to this aspect of the QL movement within the last decade. A survey of this work reveals examples that speak to the issue of timing. For instance, Steele and Kiliç-Bahi [20] describe a pre-test/post-test assessment design in which seniors' QL—operationalized as skills demonstrated on an exam—is compared with that of entering freshmen. Hathcoat, Sundre, and Johnston [10] offer a similar example, although in this case the post-assessment is completed by students nearer the mid-point of the curriculum. Other examples can be found where the concern is the assessment of students' QL based on the effect of a single class [3].

The matter of the timing for QL assessment is bound up with other important considerations. For instance, if one's intent is to assess QL near graduation as an institutional learning outcome, then students from QL and non-QL focused disciplines will necessarily be involved. This makes the desirability of partnering across disciplines and designing assessments outside of a specific QL course's context inherent to the process. Thus near-graduation timing, whatever its intrinsic virtues, also becomes a setting in which one might naturally (and perhaps in most cases unavoidably) situate other potentially desirable QL assessment strategies.

Taking for granted that any common conception for QL would involve some sense of being able to work with quantitative information in the context of daily, professional, or civic life, one can see additional advantages to an assessment approach that occurs late in a student's career. At this point students will have had their broadest exposure to information from other courses, and should be near their time of greatest proficiency in intellectual skills like critical thinking and effective communication. As such, it should be possible to assess QL in a context that will be much less contrived than when the assessment is bound to a single course or at an earlier point in the curriculum. The importance of such an attention to context is mentioned by Kiliç-Bahi and Cahoon in this volume, and extends beyond just considerations of content authenticity. It can also be seen to reasonably apply to the nature of the assessment task and to the mode in which students represent their work. In other words, asking students to do work which also calls upon "real world" intellectual skills like critical thinking and effective communication will bring any assessment task in better alignment with the type of work they would expect to do in other situations. Hence, timing QL assessment so that it can capitalize on students' senior-level proficiency in these other areas could be seen as an advantage to timing that comes at a point close to graduation.

This latter point touches on the cumulative effect on QL learning that can only come after a student has progressed through most of their curriculum. To this it would be reasonable to add that students almost certainly acquire varying amounts of QL residue from courses across the curriculum. The only place to gain some reasonable sense of how these different traces of QL learning come together as part of a student's curricular experience is once all of the varying components of that curriculum have had a chance to produce an effect.

For reasons such as these, it is unsurprising that the desire to assess students' cumulative progress toward institutional learning outcomes appears common [2], with many institutions reporting such an assessment approach, either within departments (85%) or as part of their general education program (67%). For those doing so for their general education program, the following approaches are reported to occur with varying levels of popularity [2]:

- Rubrics applied to examples of student work: 91%

- Culminating or capstone projects: 78%
- Student surveys and self-reports: 64%
- Locally developed common assignments in some courses: 62%
- Locally developed examinations: 46%
- Standardized national tests of general skills, such as critical thinking: 38%
- Standardized national tests of general knowledge, such as science or humanities: 33%

Examples like these specific to QL, and which can be found in Vacher [23], include self-report approaches [24] and locally developed examinations that, through the work of the developers, have effectively become standardized national-level tests [9, 22]. Near-graduation timing is possible with each of the strategies listed above. However, for the remainder of this chapter I discuss the advantages of an assessment strategy that combines elements such as those described in Boersma and Klyve [3], Hathcoat, Sundre, and Johnston [10], and Sundre and Thelk [22]. This strategy illustrates many of the key points mentioned above—partnering across disciplines, content authenticity, and acknowledging QL as cumulatively acquired—and helps concretely illustrate the importance of considering the best timing for the most effective QL assessment.

16.3 Performance Tasks

A QL assessment approach well suited to gathering information near graduation is predicated on the concept of a performance task. Performance tasks, along with approaches that include things as varied as inquiry-based learning and service learning, are meant to operationalize the idea that students need to experience or in some way authentically use what they are learning to fully internalize and understand it [6]. Pedagogical approaches incorporating this philosophy often stand in contrast to the lecture method, with students asked to work on "real-world problems" or to develop their own understanding based on a process of inquiry rather than simple transmission of content from teacher to student. This general notion has historical touchpoints that include the work of familiar names like John Dewey [7] and, more specific to mathematics, R. L. Moore [14].

As this bit of context is meant to illustrate, the performance task concept has been designed to serve functions beyond just assessment. As described fully in Chun [6], performance tasks consist of real-world scenarios that require the student to follow an authentic, complex process meant to approximate what would happen if they were actually facing that scenario. Higher-order thinking is built into the expectations of the task by requiring students to analyze information, and synthesize and apply evidence to arrive at a conclusion from which they must create a product that involves an authentic performance, such as by writing a memo, creating a presentation, or perhaps developing some type of written product meant for a public audience, such as a letter to the editor or an op-ed piece for a local newspaper. Finally, transparent evaluation criteria are seen as key, with the intended learning outcomes at the forefront during the task's creation, and with the assessment rubric made clear to students before they complete the task so that they can judge their own work while it is in progress. As an assessment tool, performance tasks are a component of the Collegiate Learning Assessment, a widely-used standardized national assessment [5].

Additional features of the use of performance tasks for QL assessment are described by Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt in this volume. They describe such tasks in terms of a situated-in-context approach to QL assessment. I provide a different focus, although one that still takes advantage of the situated-in-context approach. Performance tasks can provide an ideal mechanism for assessing QL in a way that is both situated in real world contexts and that is situated at an ideal time to account for students' accumulated educational experience.

16.3.1 Performance Task Development

As a QL assessment tool, the performance task (with features described above) can offer a favorable option for obtaining information near to graduation based on a highly consistent data source. Implementing this approach necessitates having an appropriate performance task and associated assessment criteria. Although one might imagine these components existing within the literature, the need to assess student learning according to an institution's local definition and criteria for QL make a locally-developed performance task, keyed to an existing QL rubric, highly desirable, if not a necessity. This point also touches on the importance of context-dependence noted by Fisher in his definition of QL.

Beyond even this, though, promoting faculty ownership of all aspects to a QL learning outcome—definition, teaching, assessment criteria as embodied in a rubric, and the performance task itself—illustrates advantages of a campus-wide effort focused on creating the right tool. This, in fact, is another advantageous aspect to the idea of partnering across disciplinary boundaries.

Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt (in this volume) provide a number of key observations regarding the nature of a good task. A concrete example is the one developed by Carmichael, Kelsch, Kubatova, Smart, and Zerr [4], with additional commentary on the task and on the related idea of authentic assessment provided in Hutchings, Jankowski, and Schultz [13]. The essential features of that task, shared generally by any performance task, include an authentic scenario, a document library that forms the source materials for the task, a task suitable to the scenario, and in this case, an assignment to write a letter in which quantitative information is brought to bear in addressing the issue outlined as part of the task's scenario.

16.3.2 Student Recruitment

With a focus being on near-graduation QL assessment, it becomes necessary to take into account strategies for collecting student work. For instance, inherent in such an assessment process is the fact that many students from which data will be collected may be majoring in non-STEM disciplines. If one's interest is in the effect of a curriculum, as would be the case for general education, then this is the nature of the population from which data must be collected. So, even after suitable performance task for collecting QL assessment data is developed, collecting the data in a way that provides a valid measure of students' quantitative literacy requires some care. For instance, there may be statistical considerations that come with the need to sample a population that is too large to assess in its entirety. Another factor, which is particularly relevant if the assessment occurs outside of the requirements for a course, is the level of motivation students may (or may not) feel as they complete an assignment that does not necessarily count for any part of a grade. Not surprisingly, questions may arise about the accuracy of the assessment results in situations like this [12].

Frankly, there may be no ideal solution. One reasonable approach is an informal partnership between the individuals leading the assessment effort and their faculty colleagues who happen to be teaching capstone (or any other senior-level) courses during the term in which the assessment will take place. The role of the assessment leaders is to handle the logistics of the process; the role of the capstone faculty is to recruit their students to complete the QL performance task even though it may have no obvious connection to their class, either in terms of content or course grade. The difficulties associated with devising an effective student recruitment process are certainly a familiar issue with any assessment tool [12].

A related issue concerns the quality of the student work, pertaining as it does to the validity of the corresponding assessment results. In other words, what level of motivation is it reasonable to expect students to bring to the performance task's completion? Here again, there are no simple answers (although Hosch [12] provides valuable insights). But, some of the same features that make this a good example of the near-graduation assessment process discussed above can also be seen as virtues for helping to obtain a high quality data set. To the extent that a task is genuinely authentic, and thereby representative of a meaningful situation for students, it becomes possible to develop some confidence in the level of seriousness students bring to the work of completing the task. In this context, it helps to capitalize on the likelihood that many students are about to enter a post-graduation world that involves considerations such as looking for employment, or any of the many other features of the more independent life that comes upon completion of a degree. Because performance tasks are generally situated in real world settings, students near graduation are more likely to be able to relate to the scenario presented to them, provided, that is, that it is well-designed to capitalize on this effect. Appealing to the way an authentic task can foreshadow life after graduation provides yet one more favorable consideration for QL assessment timed to the end of a student's curriculum.

Another complicating factor concerns self-selection bias. For some institutions, and in certain circumstances, mandating performance task completion by students may be feasible (see Hathcoat, Sundre, and Johnston [10] as a notable example). But, whenever it may not be, this method of QL assessment will suffer from the possibility that those who volunteer to complete the task are not representative of the student population, with a likely bias toward being more quantitatively literate than their peers who do not opt in. For anyone considering adopting this process, it will be important to consider effects such as these on assessment data accuracy, and to consider how to mitigate against biases that may make the resulting dataset unusable for whatever purposes it may be intended.

16.3.3 Scoring

Development and use of performance tasks requires clear and accurate knowledge of the intellectual skill being assessed through the task. As such information is usually embodied in a rubric, the rubric's criteria and scoring levels are key considerations throughout the performance task development process. And when the task is implemented, one expects that students will have the rubric information available to them so that they can craft a response that attends to expectations that will be present when their work is scored [6]. Of course, the rubric is also a necessary tool for turning student work into evidence of student achievement relative to QL.

As has been the case with other aspects of the assessment process being described here, certain features inherent to the near-graduation timing are advantages. For instance, in the same way that partnering across disciplinary boundaries is called for in terms of development of the assessment tool and student recruitment, having a wide range of disciplines represented when reading work against a rubric allows for the nuanced understanding that is practically inherent in the notion of QL. Put differently, even though a mathematician may be looking for a solution that cleverly and efficiently uses a certain technique, the fact that a student's solution demonstrates a proper line of reasoning, even if inefficient, may be seen as proficient by a social scientist. Thus, performance tasks create a good illustrative example in that they naturally embody many of the same features that were highlighted as part of the timing discussion above.

The use of qualitative evidence in assessing QL can find a place as part of a well-developed performance task scoring process (or, for that matter, a scoring process applied to other types of rubric-scored evidence). In particular, by gathering a group of faculty for a focused scoring session at which a large number of student work products are scored against a rubric, a post-scoring debriefing period serves to collect qualitative information about the QL work of students. In some ways this can be as valuable as the scores themselves, representing the well-informed sense of faculty engaged in the trenches of assessment work. This type of process also provides an excellent opportunity for faculty to see first-hand how their students are doing relative to QL (or any other learning objective of interest). This almost inevitably results in productive conversations about matters of importance concerning the curriculum, pedagogy, definitions of the objective, or any of a number of other elements that relate to how students may be doing as they are about to graduate.

16.4 An Illustrative Example

Imagine a situation in which an institution has historically collected QL assessment information by focusing on student work from a common 100-level university mathematics requirement. At the behest of a new provost, an exit survey is developed and the institution begins to collect information from students as they are about to graduate. The survey touches on many things, including items focused on institutional learning outcomes and the degree to which students feel they are "career ready" in relation to those outcomes, one of which is QL.

After a year or two the results are not encouraging. Despite consistent year-after-year results from the course-based assessment that show students achieving at a high level relative to QL, the exit survey paints a starkly different picture, one in which students do not feel equiped at all to successfully navigate quantitative situations they may find in the post-graduation world.

Campus discussions about how to interpret the results focus on three points: the information from the 100-level course-based assessment is direct information about student learning, and thus very credible in demonstrating students do have high-level QL abilities at that point in the curriculum; the information from the exit survey is indirect information, based only on students' self-reported sense of their QL ability, and thus may or may not be an accurate barometer of what they are capable of doing; and because there is no upper-level mathematics or QL-focused university requirement, there are no natural course-based approaches that could be used to collect direct information on students' QL abilities at a point near graduation for those in, say, humanities degree programs.

A solution involving performance tasks is developed, one which is able to account for the nuanced definition of QL present at this institution. By gathering a group of faculty from multiple disciplines, a real-world task requiring a sophisticated understanding and use of quantitative information is developed and tied clearly to the institution's QL definition and associated rubric. During the semester prior to graduation, students complete the task, and a group of multidisciplinary faculty gather in a post-semester scoring session to apply the rubric to student work products.

The conclusion? Students have been overly pessimistic about their QL abilities in their exit survey responses. But neither did the 100-level course-based assessment information paint an accurate picture, as it suggested high levels of QL ability, but abilities that in hindsight are too focused on purely mathematical skills and less so on the more nuanced QL definition the campus has set for itself. The performance task approach has helped gather more meaningful information on students' QL abilities, and has also highlighted an inconsistency between what the institution is doing to deliver QL and what it really wants its graduates to have in terms of QL learning.

16.5 Conclusion

The performance task process described here is one means for effectively collecting QL assessment data near graduation. It shines as an illustrative example because it exemplifies key virtues of such assessment timing by facilitating partnerships across disciplines to create and use an instrument that is easily situated in authentic contexts, and that is well matched to assess QL learning accumulated throughout the curriculum. However, the specific example aside, the main point should not be lost, that the advantages to assessing QL later rather than sooner should be considered as part of any assessment strategy.

Viewing QL as a proficiency that evolves with time and across multiple courses is certainly consistent with the views expressed in, for example, the work of Ellis, Jr. [8] and Schneider [18], where the point is made that promoting QL is an endeavor that must be shared across disciplinary boundaries. From this it follows that one may not be able to effectively assess a student's QL without taking into account the totality of their educational experience. Thus, as in many other things, for QL assessment "timing is everything" readily applies.

Without discounting the possible need to do QL assessments at the course level, the cumulative effect of QL learning necessitates having an approach that accounts for (if not emphasizes) timing in relation to graduation. Aside from an institution's need to know this type of information for purposes of assessing the effectiveness of its curriculum, adopting this assessment timing broadly may help identify highly effective QL curricular approaches that could be used as exemplars of how to more effectively advance QL learning in the 21st Century.

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17

Assessing Quantitative Literacy

Challenges and Opportunities

Andrew D. Cahoon and Semra Kiliç-Bahi
Colby-Sawyer College

17.1 Introduction

The 1994 Mathematical Association of America (MAA) publication of *Quantitative Reasoning for College Graduates: A Complement to the Standards* [13] was a call to action for quantitative literacy and quantitative reasoning (QL/QR) in higher education. The report concluded that every college graduate should be quantitatively literate and, to achieve that outcome, colleges and universities must establish QL/QR programs with full curriculum integration and assess those programs to ensure their success. During the decade that followed the MAA report, the QL/QR movement gained widespread acceptance. As evidence of this, in 2005, the Learning Education and America's Promise (LEAP) initiative of the Association of American College and Universities (AAC&U) included QL/QR among its essential skills to prepare for twenty-first century challenges. Today, most colleges and universities have established QL/QR learning outcomes, many have implemented specific QL/QR programs, and there are several collections of resources for integrating QL/QR into curricula.[1] Given the progress in acceptance and the work done towards curriculum integration, it is important to address the challenges of assessing QL/QR.

Of course, quality assessment relies on a precise definition of what is being assessed. Do we agree on what it means to be quantitatively literate? Definitions in the literature are plentiful, but not always consistent.[2] What is meant by QL/QR has changed with time and has gone by different names. Madison and Steen [15] traced the history in the inaugural issue of the journal *Numeracy*. Therein, the authors point to the 1959 publication of the *Crowther Report*, which introduced the term numeracy as a pair to reading and writing literacy, with emphasis on reasoning and communicating about real world issues. In fact, the *Crowther Report* description would not look out of place today. However, the higher aim for numeracy initially fell short and, in application, numeracy became the ability to do simple arithmetic. Decades later, following a 1982 report by Cockroft, the original conception of numeracy reemerged and the ability to use mathematics in real-life was re-emphasized. Thus, numeracy was viewed as a set of practical skills, which

[1] Some examples are: the Carnegie Foundation www.carnegiefoundation.org/in-action/carnegie-math-pathways/ Carnegie Math Pathways, the University of Texas at Austin Charles A. Dana Center www.utdanacenter.org/higher-education/new-mathways-project/new-mathways-project-curricular-materials/quantitative-reasoning-course/ Quantitative Reasoning Course, and Carleton College SERC serc.carleton.edu/NICHE/teaching_materials_qr.html NICHE teaching materials and serc.carleton.edu/sp/ssac/index.html Spreadsheets Across the Curriculum. Additional examples can be found in the journal *Numeracy*.

[2] This point is also raised in the separate chapters by Madison and by Fisher in this volume. Madison discusses the difficulty distinguishing QL/QR from mathematics and statistics, while Fisher analyzes the term quantitative literacy from a social linguistic perspective.

is often referred to as functional mathematics. It was in this setting that the 1994 MAA report was released. Hence, a quantitatively literate college graduate was defined as one who applies simple mathematical models to solve real-world problems and demonstrates five accordant capabilities: interpret mathematical models, represent mathematical information, use basic mathematical methods of arithmetic, algebra, geometry, and statistics, estimate and evaluate answers, and recognize the limits of mathematical and statistical methods.

Since then, the meaning of QL/QR has continued to evolve. Consider the way it is implicitly defined by the National Numeracy Network (NNN) in their vision statement: "The power and habit of mind to search for quantitative information, critique it, reflect upon it, and apply it in their public, personal and professional lives." With the NNN vision statement in mind, in 2009, the AAC&U developed a rubric for assessing QL/QR, which was later modified in 2011 by Boersma, Diefenderfer, Dingman, and Madison [1] for the Quantitative Literacy Assessment Rubric (QLAR). The QLAR is comprised of six core competencies: interpretation, representation, calculation, analysis and synthesis, assumptions, and communication. This rubric offers a precise, encompassing definition that can be applied equally to QL/QR tests and more holistic measures, like portfolios. Still, there remains a question of names. Are names such as numeracy, quantitative literacy, and quantitative reasoning synonymous? Does the QLAR, for example, apply equally to numeracy or quantitative reasoning?

In 2016, Karaali, Villafane-Hernandez, and Taylor [11] analyzed the usage of QL/QR-related names over time and developed a four-dimensional framework for comparison. Using their framework, the authors found that the various names are being used in overlapping but distinguishable ways. For its part, the characteristics of quantitative literacy in each dimension are:

1. Quality of desired outcome: ability and habit of mind (intermediate to advanced)
2. Knowledge domain: data, mathematics/mathematical, arithmetic/quantitative and logical
3. Display of expertise: analyze, appreciate, decide, understand, use (active, reactive)
4. Context: citizen, information, practical situations (daily life, work, civic life)

After applying their framework to recent publications, the authors observed that the usage of terms in the literature lends some support for the hierarchical description suggested by Vacher [24], wherein numeracy (foundational skills, primary and elementary school), quantitative literacy (intermediate skills, middle school), and quantitative reasoning (advanced skills, high school and higher education) represent increasing levels of skill and education. The fit of the hierarchy to the framework is not perfect, but offers insight regarding the present usage of terms.

While the meaning of QL/QR has evolved and its integration with curricula has spread, its assessment has not kept pace.[3] In 2009, the MAA's Special Interest Group in QL (SIGMAA-QL) conducted a survey of 1,554 institutions and found that while 87% of respondents have some form of quantitative requirement, only about 23% have pre-post assessment of quantitative skills [20]. Without assessment, it is impossible to know if QL/QR requirements are working or how to refine them. Additionally, none of the existing assessment tools is able to fully assess QL/QR as defined above.[4] Madison [14] asserts that testing in the U.S. and Europe is still dominated by the functional mathematics perspective. Kosko and Wilkins [12] examined open-ended questions in four large scale assessments and found that most questions require a single answer as a response. They saw a paucity of questions that require a description of procedures or strategies, explanations and justifications, or mathematical representations, and fewer still questions that require a combination of response types. None of this is surprising; functional mathematics abilities are comparatively easier to assess. A single answer is often sufficient. There are inherent challenges in assessing the full scope of what we expect from a quantitatively literate person. Amorphous, but essential, characteristics such as "habit of mind" cannot be assessed by a single answer response.

In the preface to the 2001 MAA review *Mathematics and Democracy*, Orril remarked that "if individuals lack the ability to think numerically, they cannot participate fully in civic life, thereby bringing into question the very basis of government of, by, and for the people" [16, p. xvi]. Seen in this light, the stakes are quite high. Being quantitatively literate has tremendous value in the real world and QL/QR assessment is a key step to securing a quantitatively literate society. In this chapter, we survey some recent QL/QR assessment efforts and, in doing so, identify significant

[3]The driving factors for QL/QR programs to be instituted, including assessment practices, are discussed in the chapter by Parsons, Salomone, and Smith in this volume.

[4]There are many available assessment instruments that either directly or indirectly assess quantitative skills. A comprehensive list can be found on the NICHE website: `serc.carleton.edu/NICHE/ex_qr_assessment.html`.

challenges and opportunities for the QL/QR research community. Our primary goal is to answer: What kind of QL/QR assessment is happening and, broadly, what have we learned from the results? Some key ideas highlighted herein are echoed and explored more thoroughly in the separate chapters in this volume by Zerr and by Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt. Zerr considers when QL/QR assessment should happen within a student's academic career and what kind of assessment would be most effective, given the cumulative nature of QL/QR. Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt distinguish types of assessment and present a framework for corresponding assessment tools. We direct the reader to both of these chapters as complements to the present chapter.

17.2 The PISA Test of Mathematics Literacy

Students entering college and university are the products of their grade school and high school experiences. To accurately assess their quantitative literacy improvement during college, it is important to know what skills they have at matriculation. The Programme for International Student Assessment (PISA) is a comprehensive ongoing assessment that, in 2000, began testing 15 year olds from 32 countries in reading, mathematics, and science literacy. Since then, the PISA test has been administered every three years, most recently in 2015 with 72 participating countries. In regards to mathematics literacy, the test closely aligns with the functional mathematics description of QL/QR. Questions focus on student abilities to "apply knowledge and skills to tasks that are relevant to their future life, rather than on the memorization of subject matter knowledge" [18, p. 3]. The results of PISA allow us to see how U.S. 15 year olds compare internationally and to parse their collective strengths and weaknesses on specific quantitative skills.

The relative performance of U.S. students has been stable over time in all three major PISA categories: about average in reading and science literacy, but notably below average in mathematics literacy. In 2012, the U.S. ranked 27th in overall mathematics score out of the 34 countries comprising the Organisation for Economic Co-operation and Development (OECD).[5] Based on their collective responses to test questions, students are assigned a proficiency level from 1-6, with 6 being the most proficient. Figure 17.1 shows the percentage of U.S. students at each proficiency level in comparison to the OECD averages. More U.S. students fall into the lower levels (52.2% at Level 2 and below) and fewer students fall into the higher levels (24.6% at Level 4 and above) than the OECD averages (45.5% and 30.7%, respectively). Thus, improvement is needed at both ends of the spectrum for the U.S. to be competitive globally.

Analysis of individual items reveals that U.S. students struggle with cognitively demanding [22] skills like using the number π in a calculation, generating a mathematical model, interpreting real-world aspects, reasoning spatially, and

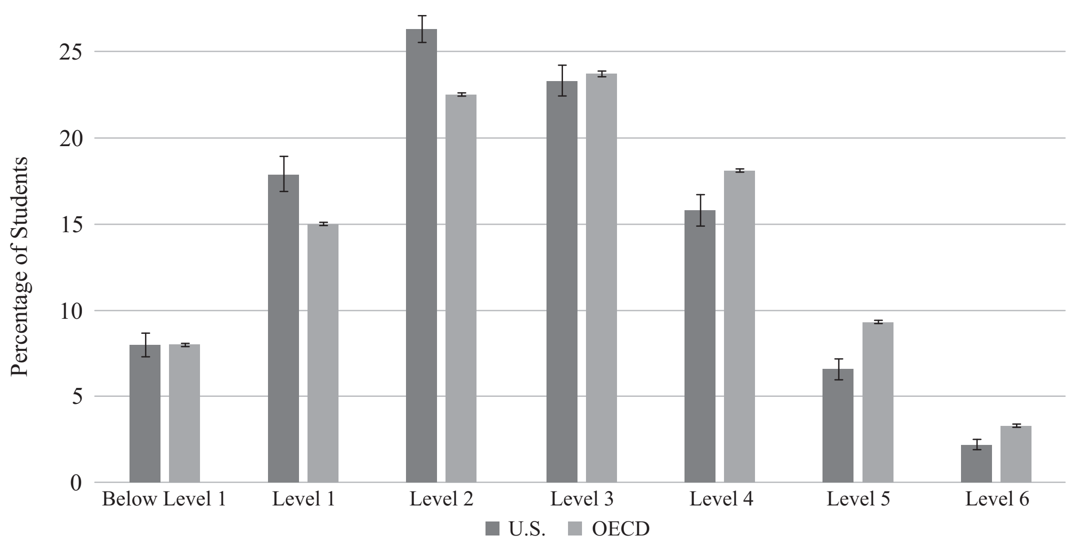

Figure 17.1. Percentage of students at each proficiency level in mathematics from PISA 2012 for the U.S. in comparison to the OECD averages.

[5]In the recently released 2015 test results, U.S. students ranked 31st out of 35 OECD countries. However, the 2015 test was focused on science literacy. 2012 is the most recent year in which mathematics literacy was the focus of the test.

applying basic skills in complex ways. However, they are able to do straightforward tasks with tables and diagrams and direct use of formulae. The 2012 PISA Country Note for the U.S. [19] speculates about the source of U.S. students' struggles:

> It may be that U.S. students seldom work on well-crafted tasks that situate algebra, proportional relationships and rational numbers within authentic contexts. More generally, perhaps the application problems that most students encounter today are the worst of all worlds: fake applications that strive to make the mathematics curriculum more palatable, yet do no justice either to modeling or to the pure mathematics involved.

This suggests that we think carefully about what "in context" truly means. It is plausible that contexts that appear fake or contrived prevent engagement and also obscure the underlying mathematics. Thus, the inauthentic context ultimately hampers development of both underlying mathematics skills and higher cognitive skills.

The development of Common Core State Standards for Mathematics (CCSSM) [17] has coincided with the growth of QL/QR as a movement. Accordingly, the CCSSM incorporate all five of the capabilities that were outlined in the 1994 MAA Report. It should be the case that implementing CCSSM improves QL/QR skills, which leads to improvement in PISA mathematics scores. The authors of PISA think so and believe that CCSSM will result in "significant performance gains" on the test [19]. However, Madison [14] points out that "the standards are weak in supporting reasoning and interpretation" and also expresses concern that applications "will not include many QL contextual situations." *How* the standards are implemented will determine the size of their effect on student performance.

There is also a question of how well PISA measures QL/QR. The aforementioned Kosko and Wilkins [12] study explored the questions on the PISA mathematics literacy test. As with the other tests they examined, the PISA test requires mostly simplistic responses, i.e., a single answer that is graded as correct or incorrect. The authors concluded: "While many items on these QL assessments may have required test-takers to think critically and deeply about the mathematics they were using, few items in these tests *assessed* such thinking" [12, p. 13]. By its nature, QL/QR is difficult to capture with a right or wrong answer. A student who gives a correct answer may be accurately measured, but it is the student who gives an incorrect answer that could provide valuable insight. At what link in the student's logic was the chain broken? On a continuum scale, how sound was the student's reasoning? Assessing reasoning and ability to communicate would allow for identification of how and why students arrive at an answer. PISA and other large scale assessments provide valuable data on the mathematical abilities of our students, but as a test of QL/QR, there are gaps in what they can measure. As such, PISA is not a complete measure of QL/QR.

17.3 Assessment at U.S. Colleges and Universities

At the college and university level, QL/QR assessment can be used in a variety of ways: measuring student learning on individual assignments, placing students into appropriate courses, and evaluating student progress within a course, a program of study, or the complete college experience. Thus, it can be used to put students in a position to succeed and to determine if curricula are properly designed for students to increase and improve their QL/QR skills. Assessment is necessary for faculty and administrators to know if stated goals and outcomes have been met and what curriculum modifications might be made. Here we discuss recent QL/QR assessments at a handful of colleges and universities, each with its own approach to QL/QR curriculum integration. These assessments show the relative impact of QL/QR curricula on QL/QR skills for different cohorts of students.

We begin with our home institution, Colby-Sawyer College, which received an NSF grant in 2007 titled "QL Across the Curriculum in a Small Liberal Arts College." This grant supported workshops for faculty, the hosting of conferences, and the development of an assessment plan for the college. Over the five-year duration of the grant, freshmen and seniors were each given two tests, one for basic mathematical skills and one for QL/QR skills, and an attitude survey. At the start, the tests were given to a small sample of students. After the second year, with support of the liberal education coordinator, the test was administered to all freshman and seniors. Thus, for one class, we were able to collect data for almost all students as both freshmen and seniors. The primary purpose of the tests was to see if students' basic mathematics skills improved along with their QL/QR skills while these basic skills were revisited through just-in-time style teaching in QL/QR-rich contexts. The basic mathematics and QL/QR skills tests each were comprised of 25 multiple-choice questions and QL/QR was defined, in the functional mathematics sense, as using simple mathematics concepts to solve real problems. The attitude survey had five categories: self-confidence, anxiety, value, enjoyment, and motivation. In addition, faculty were surveyed regarding use of QL/QR in their courses.

The results of the various assessments were analyzed and reported by Steele and Kiliç-Bahi [21]. Basic mathematics and QL/QR skills did increase, but seniors scored below 55% on both tests and only exceeded freshmen by 10 percentage points or less (+7% on basic mathematics skills and +10% on QL/QR skills). Prior to administering the test, our goal was an average of at least 70% for seniors. By this standard, most seniors did not demonstrate sufficient growth from freshman to senior year to attain the desired level of QL/QR skills. Without national norms for comparison, the authors could not interpret the scale of impact for the initiative. Faculty designing assessment tests can subjectively decide what students should or should not be able to do, but comparison data between institutions and gathered over time is necessary to measure amounts of progress and attach credit to particular interventions. It may be that students' QL/QR skills scores would improve by 10% without any intervention at all. The lack of a national assessment construct and interpretive comparative data led to an NSF supported collaboration between Colby-Sawyer, Bowdoin, and Wellesley Colleges. Through this collaboration, the QLRA instrument was developed and implemented in more than 30 academic institutions.[6]

The surveys revealed little positive change in student attitudes, but significant gains among faculty. On the student attitude survey, the only category that showed improvement was Value (+7%). The scores stayed the same or were lower for the other categories. It was also observed that the self-confidence scores significantly decreased for female students whereas scores stayed the same or increased for males. This happened despite the fact that there was no statistically significant difference between males and females in either basic mathematics or QL/QR skills scores. While the impact on students was disappointing, the impact on faculty was encouraging. In fact, the largest gains on any of the assessments were in faculty development and faculty attitude towards QL/QR. In 2007 and again in 2009, faculty were surveyed about the use of QL/QR in their classes. They were given a list of QL/QR skills and asked to choose how often each skill was used. Faculty responses of "never" decreased significantly for several skills, indicating that faculty were either integrating more QL/QR skills into their courses or better recognizing the QL/QR skills that were already present. The increased awareness and acceptance of the value of QL/QR among faculty led to permanent curriculum changes. Currently, Colby-Sawyer has implemented a QL/QR proficiency course requirement and an applied QL/QR requirement that is satisfied within each major.[7] Thus, the initiative has left a footprint of QL/QR on campus and a community more receptive to QL/QR integration.

At James Madison University (JMU), there is a long-standing effort to develop and implement assessment tools. This has resulted in commercial assessment tests that are continuously revised by the JMU Center for Assessment and Research Studies and are available through the independent Madison Assessment company. Their Quantitative Reasoning Test `www.madisonassessment.com/assessment-testing/quantitative-reasoning-test` has gone through several stages of content alignment, reliability, and validity tests [23]. As of 2015, they were on their ninth version. It is a 26-question multiple choice test with questions targeting two specific learning outcomes:

1. Use graphical, symbolic, and numerical methods to analyze, organize, and interpret natural phenomenon; and
2. Discriminate between association and causation, and identify the types of evidence used to establish causation.

Both of these outcomes apply to functional mathematics and, like the learning outcomes reported by Steele and Kiliç-Bahi [21], overlap with the QL/QR capabilities listed in the 1994 MAA report. The JMU assessment tests are biannually administered to randomly assigned JMU students on two designated assessment days, first in the fall to entering freshmen and then in the spring to students who have completed 45-70 credit hours.

Hathcoat, Sundre, and Johnston [9] recently analyzed the results of the JMU Quantitative Reasoning Test. At the freshman level, the authors compared the scores of students matriculating with and without relevant quantitative or science college credit. Surprisingly, students with transfer credit performed similarly to students without any credit. Only students entering with Advanced Placement (AP) or International Baccalaureate (IB) credit performed notably better. The evidence suggests that aptitude tests (students with AP or IB credit also had higher SAT scores in all categories) are correlated with Quantitative Reasoning Test performance whereas the impact of transfer coursework is questionable. Comparing freshman to sophomores/juniors, they found that Quantitative Reasoning Test scores increase

[6] The Quantitative Literacy and Reasoning Assessment (QLRA) is discussed in the chapter by Gaze.

[7] For the applied QL/QR requirement, every major of the college was asked to develop a QL/QR-focused course at the upper level within the major or map QL/QR skills through graded assignments in other courses required for the major. To meet the requirement, the QL/QR assignments had to amount to 20% of a four-credit course grade. For example, some majors created a two-credit QL/QR class wherein 40% of the assignments involved QL/QR skills.

on average and that the changes are attributable to relevant courses taken at JMU in the intervening years. At the sophomore/junior level, students who had taken specific quantitative and scientific reasoning courses at JMU showed higher scores and faculty assess a higher proportion of those students as meeting standards. However, the authors point out that the gains are relatively modest compared to what may be expected from students taking courses targeted to improve the specific skills that are tested. Additionally, the authors report faculty concern about the percentage of students surpassing the set standard of minimally competent, which in 2013 was only 58%. The authors also compared students who had completed their required quantitative and scientific reasoning courses, who averaged 72% on the Quantitative Reasoning Test, with the scores of graduating seniors in science majors (several science majors participate in a competition for graduating seniors wherein the Quantitative Reasoning Test is administered). They found that senior science majors scored much higher. For example, physics seniors averaged 91% on the Quantitative Reasoning Test. Based on their analysis, the authors conclude that substantive growth in QL/QR abilities "may require courses taught beyond a basic level" [9, p. 8].

The Quantitative Reasoning Test at JMU is clearly valued by the institution and established in the curriculum. Having designated days for assessment allows for consistent implementation and, through continuous analysis and revision, the Quantitative Reasoning Test can be used to assess the quantitative courses within their general education program. In many ways, they have built a model framework for assessment. The study by Hathcoat, Sundre, and Johnston [9] raises important questions about the impact of quantitative coursework on test scores. While not conclusive, it certainly seems that basic level coursework, at best, yields modest gains. This finding is in agreement with the Colby-Sawyer study. Given the proprietary nature of the JMU tests, they are not freely available to be implemented or studied by other institutions. Without analyzing the questions on the Quantitative Reasoning Test, it is difficult to evaluate how well the test addresses the full scope of QL/QR as it is defined by NNN or within the QLRA. Both the Colby-Sawyer and JMU tests are entirely comprised of multiple-choice questions, which are expedient, but certainly limit what can be learned about students' thinking and habits of mind. The following studies are aimed at these aspects of QL/QR.

The Quantitative Inquiry, Reasoning, and Knowledge (`apps.carleton.edu/quirk/`QuIRK) initiative at Carleton College has explored the effectiveness of quantitative courses by the use of quantitative evidence in student writing. As part of the initiative, there is annual evaluation of writing samples, ongoing faculty development, and regular curricular change. The stated learning objectives for students are strong quantitative thinking skills, competent implementation, thoughtful analysis and assessment, and effective communication. To assess the writing samples, they have developed a rating sheet and a rubric [8]. The rating sheet asks about the relevance of QR to the paper, the extent of numerical evidence and QR in the paper, an overall assessment of QR usage, problematic characteristics of QR presented, and whether or not the assignment explicitly calls for use of QR. The rubric can be used to assess the quality of QR used in the paper on a scale of 1-4. There are separate rubrics for papers where QR is centrally or peripherally relevant.

In successive studies by Grawe and co-authors [8, 6, 7], the QuIRK rubric was applied to papers taken from student portfolio submissions from their sophomore years. In 2010, Grawe, Lutsky, and Tassava [8] conducted reliability testing with a diverse set of 11 readers (readers varied in gender, faculty status, and academic area) and found that ratings of the same papers by different instructors are generally consistent, with strong agreement at low or high end ratings. There was more disagreement on middle categories, which can be attributed to the fact that middle categories can overlap with either the level below or above them. Overall, they concluded that readers from across disciplines can be trained to apply the rubric to writing samples. After confirming the viability of the rubric, in 2011 Grawe [6] investigated the relevance of QR in each major division of the college: Arts and Literature, Humanities, Natural Sciences, and Social Sciences. He found that "ample opportunities exist for students to experience and engage in QR across the curriculum" [6, p. 2], even in the Arts and Literature division where QR was centrally or peripherally relevant to about 20% of papers.[8] Then in 2013 Grawe [7] compared QR in writing among students taking various quantitative courses. He found that QR in writing improves significantly more for students who have taken a first-year seminar specifically designed to teach quantitative thinking and communication than it does among those who have only taken courses in statistics, economics, or social sciences.

The QuIRK rubric is a tool particularly geared towards assessing a difficult component of QL/QR, namely the habit

[8] The percentages of central or peripheral relevance were: Arts and Literature, 19.5%, Humanities, 29.3%, Natural Sciences, 92.5%, and Social Sciences, 73.4%.

of mind and ability to communicate with quantitative information. The above studies demonstrate its promise for the assessment of writing samples and reveal ways of branching QL/QR assessment from sciences and into arts, literature, and humanities. At present, the rubric is somewhat of a blunt instrument in that readers can most reliably make binary decisions about the quantitative content of writing samples and the rubric does not distinguish specific quantitative skills. With further refinement, the rubric could be tuned to target specific QL/QR skills within communication and yield more information on strengths and weaknesses of quantitative arguments used in student writing.

A related effort to investigate habit of mind was conducted by Boersma and Klyve [2] at Central Washington University (CWU). They set out to create a "prompt-less" instrument to work in conjunction with a modified version of the QLAR. With a prompt-less instrument, students would not be prompted to discuss or incorporate quantitative information. Rather, any usage of quantitative information would come naturally while making arguments within the writing assignment. The authors also aimed to design an instrument that could be administered quickly, by a single instructor, within a typical class period wherein pre- and post-assessments could be conducted. They noted that the QuIRK assessment tool is time-intensive and better suited to programmatic assessment. In the instrument developed by Boersma and Klyve, students are instructed to read a chosen newspaper article (that has a graph) and then to respond in written form to the questions:

1. Did you understand the article?
2. What was the main point(s) of the article?
3. What facts did the author use to support the main point(s)?
4. Were there any particular strengths or weaknesses in how these facts were reported?
5. Does the graph help interpret the numerical information found in the text? Explain your thoughts.

The authors concede that this is not truly a "prompt-less" instrument; perhaps it is more appropriate to call it a "less-prompt" instrument. By design, the instrument seeks open-ended responses to questions that do not refer to calculation in any way and includes a short amount of space for answers. Their hypothesis is that the prompts do not guide students to responses that would be identified by the rubric and students will focus their ideas to fill the short space and isolate what they think is most important. In the fall 2009 and fall 2011 semesters, Boersma and Klyve applied their instrument at the beginning and end of a few sections of an introductory quantitative reasoning course. At CWU, students are required to either take the quantitative reasoning course or a higher level mathematics course. Accordingly, the course is mostly taken by students in non-STEM majors. Their sample included 23 non-STEM majors from the general population of students and 40 non-STEM majors from the honors program. Both cohorts followed the same textbook and course structure. As expected, the honors cohort scored higher than the general population, but neither group showed statistically significant improvement on any question. The authors concluded that their instrument could measure students' habits of mind, but that the course was ultimately not able to change their habits.

Also with these same cohorts of students from fall 2009 and fall 2011, Boersma and Klyve [3] investigated the effectiveness of a media-article approach to learning basic quantitative reasoning skills. For this study, the authors gave pre- and post-assessments, each consisting of eight open-response word problems. The pre-assessment was given at the start of the semester. Then, the class covered a series of case studies and students were assigned to read newspaper articles and to practice basic calculations as homework. The following class involved small and large group discussion of a new case study. By design, students practiced calculation skills along with verbal and written communication using quantitative information, and they did so in the authentic contexts of case studies. As noted earlier, the authors of the PISA study speculated that fake or contrived applications might hamper learning [19]. Here, the contexts were neither fake nor contrived; they were taken from real life. Yet, when given the post-assessment three weeks later, the general population did not show any statistically significant improvement on any question. Only the honors cohort improved or stayed the same on every question, with statistically significant improvement on three questions. Why does this approach work (however modestly) with honors students, but not the general population of students? As one possible explanation, the authors noted that honors students were far more likely to seek help outside of class than the general population. This added instruction could be a contributing factor in their relative success. Regardless, it could be that authentic contexts are preferable to contrived, and yet authentic contexts do not appear to be a sufficient condition for learning QL/QR skills.

17.4 Future Directions

Taken together, the above studies bring to focus many of the challenges we face in teaching and assessing QL/QR. On the teaching side, we find that QL/QR-related curricula are mostly falling short of their desired impact. High aptitude students are succeeding on these measures, as are students in STEM majors, but what of the general population of students in non-STEM majors? How can we reach that group? Here we recommend the chapter by Craig, Guzmán, and Harper in this volume, who challenge the perceptions of who needs to be quantitatively literate. Perhaps to answer this question, we must entertain the approaches that are discussed by those authors. On the assessment side, we see tools and instruments in use that are designed to target various facets of QL/QR, but none that is all-encompassing. There is a chain linking the assessment to the student whom it should ultimately serve. To build thorough assessment, we must know what should be assessed. To know that, we must determine the skills that are essential. To determine the essential skills, we must uncover how individual success depends on QL/QR. With each link in the chain there is an opportunity for progress. Here we suggest some future directions.

1. **Settling the meaning of QL/QR and related terms**

   The timeline from the 1994 MAA Report to today is scattered with various assessment instruments for various facets of QL/QR. Nearly every assessment study begins with its description of QL/QR. The evolution in how we, as a community, think of QL/QR has made for a moving and ill-defined target. As we move forward in developing assessment instruments, it is essential to have consistent and precise language. We recommend building from the four-dimensional framework of Karaali, Villafane-Hernandez, and Taylor [11], along with the hierarchical structure of numeracy to quantitative literacy to quantitative reasoning that was suggested by Vacher [24]. We also note the contribution of Fisher in this volume, which reframes QL/QR from the perspective of social linguistics. With broader agreement on the language, assessment instruments can more easily be adapted, applied, and interpreted across age and education levels.

2. **Conducting cognitive research on learning and mindset**

   The psychological perspective is critical to developing quality assessment and it applies to defining QL/QR,[9] pinpointing when the mind is able to learn a skill, and exploring the mindset towards learning. We must refine the list of quantitative skills and how they are associated with progression (for example, from numeracy to quantitative literacy to quantitative reasoning) so that skills are taught at the optimal ages. Wilkins [25] introduced a measurement model of quantitative literacy that encompasses its multifaceted makeup. The results suggest that quantitative literacy is characterized by an interrelationship among mathematical cognition, mathematical beliefs, and mathematical disposition. How do we change the mindset of our students? How can we help them to move from a fixed mindset to a growth mindset? At which level of their education are they more receptive to such change? It may be necessary to answer these questions in order to effect real progress. Otherwise, our pedagogical efforts may be stifled by cognitive roadblocks.

3. **Assessing pre-college mathematics education**

   Perhaps the most significant QL/QR-related obstacle for instructors in higher education is the deficit in the development of basic procedural skills prior to college. As noted by Cremin [4], inert skills taught in K-8 education, like the ability to comprehend instructions and perform routine procedures, are needed to achieve the liberating literacy proficiency that is implicit in the NNN vision statement. PISA results show that U.S. 15 year olds lag well behind most of their international counterparts in mathematics, while they maintain average performance in reading and science. Thus, many U.S. students start college needing to learn basic skills and QL/QR skills in tandem. Honing assessment instruments for pre-college education is essential. A promising contribution was reported by Gittens [5], who developed a scale for measuring numeracy and applied it to a sample of 3rd to 8th grade students. The numeracy scale results showed strong positive correlations with scores on the Cognitive Abilities Test (CogAT) and the Iowa Test of Basic Skills Mathematics test. Despite some inherent limitations of the study—such as a small number of test subjects from each grade—the results warrant replication with larger samples and a wider range of school environments.

[9] See the chapter by Shavelson, Mariño von Hildebrand, Zlatkin-Troitschanskaia, and Schmidt in this volume for discussion of the three basic approaches: behavioral, cognitive, and situated.

4. **Creating international college-level assessments**

   To gain a global perspective, we require assessment instruments for the college level that can be applied on a large scale, both nationally and internationally. Efforts like PISA assess pre-college education relative to other countries, but there are no such efforts at the college level. Accordingly, there is no clear way to compare U.S. institutions of higher learning to one another or to international institutions. At a time when the value of college education is under scrutiny, particularly at small liberal arts colleges, it would be helpful to measure students' growth in QL/QR skills during their college years. Broad application of refined assessment instruments at colleges and universities may provide evidence that distinguishes their value and helps those institutions identify areas for improvement that will keep them competitive into the future. Studies like this would also help to identify successful practices and allow researchers to investigate correlations between pre-college experiences and performance in college.

5. **Assessing skills after leaving college**

   The ultimate goal is, of course, for students to be successful post-graduation. Yet, there is a lack of longitudinal study of skills as students leave college and enter the work force. It would be valuable to know what skills are retained, what skills are not, and how QL/QR skills relate to career success. As discussed in the chapter by Zerr in this volume, because QL/QR is cumulatively acquired, there may be advantages to assessing QL/QR skills later in a student's academic career. Presumably, students continue to process and learn some QL/QR skills post-graduation as well. We expect that student retention of course content degrades over time; the farther removed from the course, the fewer facts, ideas, and skills that they hold onto. However, this may not be true for some aspects of QL/QR. The accumulation of course, life, and job experiences may lead to continuous development of QL/QR skills over time. Longitudinal studies could answer important questions about the learning of and the need for QL/QR.

6. **Assessing the teaching of basic skills within QL/QR-rich contexts**

   Boersma and Klyve [3] and Steele and Kiliç-Bahi [21] both investigated teaching QL/QR using contextually rich problems and both found, at best, minor gains among their students. These studies prompt the following question: Should basic skills be taught before moving on to QL/QR-rich contexts or within QL/QR-rich contexts, or should QL/QR-rich contexts be used to motivate students to master these basic skills? Of course, it would be ideal if students master these basic skills before college. However, if students come to college without them, we must know how to teach basic and QL/QR skills in tandem. Boersma and Klyve conjectured that "students' prior knowledge and misconceptions about percentages are so ingrained in their habits of thinking that most direct instruction has little to no effect" [3, p. 9]. If so, what is the optimal strategy?

7. **Assessing QL/QR courses and programs**

   Many schools have a QL/QR requirement that is satisfied by either taking a specifically designed lower-level QL/QR course, any mathematics course, or a series of quantitative lower-level or upper-level courses. However, there has not been extensive research on the effectiveness of these different approaches. Although the study by Grawe [7] indicates that QR-in-writing benefits from a focused approach, we do not know how abilities differ if students take a QL/QR-focused course, a traditional mathematics course (such as precalculus or college algebra), or experience QL/QR in a series of courses. Similarly, we do not know if it is more effective to teach QL/QR within lower-level or upper-level courses, or online or traditional courses. The answers may vary by individual skill. With regard to quantitative courses, we should expect that students' QL/QR skills are positively correlated with their course grades. Might the same be true in non-quantitative courses? For example, it is reasonable to expect that the logical reasoning required for QL/QR is associated with the ability to construct written arguments. Thus, QL/QR skills should also be correlated with writing grades. If so, how much more does a quantitative course improve QL/QR skills as compared to a writing course? More research is needed to assess the value of QL/QR courses and programs.

8. **Connecting QL/QR to critical thinking and decision making**

   Hillyard [10] conducted a comparative study of the numeracy education and Writing Across the Curriculum (WAC) movements in terms of their growth and acceptance. The primary goal of the study was to learn from the

WAC movement and ensure similar success for numeracy. Among the suggestions made by Hillyard is to "document that numeracy education increases students' critical thinking and decision-making skills." Such qualities are always atop lists of what employers are looking for in job applicants and what faculty want to see from their students. Instructors of quantitative courses certainly think of critical thinking and decision-making as built-in to numeracy or QL/QR, but other faculty may not share that understanding. Making the connection explicit may ease integration across the curriculum, as it did for WAC. Therefore, we must study the correlation of QL/QR with critical thinking and decision-making and present it in such a way that general faculty can appreciate the value to their own disciplines.

It is our hope that the above list provides a starting point to think about specific ways of advancing QL/QR assessment and, consequentially, improving the teaching and learning of quantitative reasoning. This effort requires participation from all academic areas. The available evidence suggests that quantitative reasoning can and does happen outside of the mathematics and sciences, where it will often occur in ways that cannot only be assessed by multiple-choice questions and word problems. In fact, the students in non-STEM majors are the group in greatest need of our attention. For those students, their QL/QR development must be woven into the curriculum independent of their major. In the time since the last MAA report on QL/QR, there has been much progress in establishing QL/QR programs at colleges and universities. Along with continued growth on that front, by the time of the next MAA report on QL/QR, we should see how those programs can lead to successful outcomes for students across the college.

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Part IV

Perspectives from the Quantitative Literacy Community

Our campus visit to QLU is nearing its end. After exploring QL/QR on site, in all its nitty-gritty of implementation and assessment, we are finally ready to accompany to dinner a handful of QL/QR veterans, people who have dedicated many years of their careers to working on its various aspects, as they share with us reminiscences, hard-learned lessons, critiques, and provocations.

In “Four Adventures in Four Decades,” Ethan Bolker shares his evolution as an instructor and retells the story of how he found his way to quantitative reasoning. His is a personal and personable account, opinionated as well as colorful. We encourage the reader to examine connections between Bolker’s chapter and that from Madison; by the end of this campus visit, Bolker is in some sense providing us with a summary of how far the QL/QR community has come in the past decades, just as Madison did.

Next, Jeffrey Craig, Lynette Guzmán, and Frances Harper, in their chapter “A Citizenry with Quantitative Literacy: Efficient Functioning or Disruptive Action?” emphatically place quantitative literacy in the context of other literacies and point toward the limits and boundaries of its standard conceptualizations. In particular, they push back on various assumptions (such as rugged individualism) that situate it within a particular political stance and propose alternative political stances that value collectivist and activist approaches. If the reader is prepared for more provocation, this is amply provided in the next chapter, “Classrooms as Laboratories of Democracy: The Role of New Quantitative Literacies for Social Transformation” by Thomas Philip and Laurie Rubel. As with the Craig, Guzmán, and Harper chapter, we are faced with the inevitable truth that our classrooms cannot be apolitical places; Philip and Rubel push us just a bit further and provide us with possible pathways to make our classrooms into “laboratories of democracy.”

Finally, we wrap up this part with an interview with Len Vacher, in “On Animals, QL Converts, and Transfer: An Interview with Len Vacher.” A founding co-editor of *Numeracy*, the flagship journal of the National Numeracy Network and a popular outlet for QL/QR scholarship, Vacher came to quantitative literacy through computational geology, and in this interview he shares his views on how QL/QR has evolved over the years. As the single geologist in the contributors list for this volume, Vacher emphasizes the importance of interdisciplinary communication and collaboration as we look into the future for further growth of our practice and scholarship.

18

Four Adventures in Four Decades

Ethan D. Bolker
University of Massachusetts, Boston

18.1 Introduction

In this essay I reflect on four adventures teaching college mathematics to nonscientists: one decade each for liberal arts mathematics, elementary algebra meant to be useful, college algebra with applications, and, finally, contemporary quantitative literacy (QL) and quantiative reasoning (QR): common sense mathematics.

As a mathematician I tackled problems that intrigued me without first reading about them. I did the same in my teaching, so I must apologize for my ignorance of the excellent published material on mathematics for nonmajors. When I set out to write this essay even my own contributions surprised me. I found there glimmers of many of the ideas Maura Mast and I implemented in *Common Sense Mathematics* [6]. The other essays in this volume offer evidence that my development as a QL/QR teacher paralleled that in our profession.

18.2 The Seventies

In the seventies, years before QL/QR, UMass Boston students not majoring in a science studied Liberal Arts Mathematics, "math for poets" in the vernacular of the day. I had always proselytized for mathematics, so I taught the course several times. Fortunately, I found old class material in just the second box of the many unopened ones I took from my office when I retired. The faded dittoed assignments are still (just barely) readable.[1] They tell of a syllabus packed with my favorite gems, including Pythagorean triples and Fermat's last theorem, Königsberg's bridges, Platonic polyhedra, the fourth dimension, the uncountability of the continuum, and more.

I considered Sherman Stein's *Mathematics, the Man-Made Universe* [18] for the text, but found too much there I did not need and too little overlap with what I wanted.[2] I decided instead on Abbott's *Flatland* [1] and selections from Clifton Fadiman's *Fantasia Mathematica* [8]. I know we read Arthur Porges's "The Devil and Samuel Flagg" [15], in which Flagg bargains with Satan to settle Fermat's last Theorem,[3] Robert Heinlein's "And He Built a Crooked House" [10] featuring an eight room tesseract, Martin Gardner's "The Island of Five Colors" [9], and Kurd Lasswitz's "The Universal Library" [13]—sadly, not Borges's *The Library of Babel* [7]. Now I would skip Gardner[4] and assign both

[1] See `en.wikipedia.org/wiki/Spirit_duplicator` for readers too young to know about dittos.

[2] For a contemporary take on the book, see Underwood Dudley's 2011 review at `www.maa.org/press/maa-reviews/mathematics-the-man-made-universe-an-introduction-to-the-spirit-of-mathematics`.

[3] I probably read this when it first appeared in 1954 in *Fantasy and Science Fiction*.

[4] The only credit for this story in *Fantasia Mathematica* is "©1952 by Martin Gardner"—no information about whether he published it then. From `kasmana.people.cofc.edu/MATHFICT/mfview.php?callnumber=mf100`: "Gardner apparently no longer likes to see this story in print. His recent collection *The No-Sided Professor* does not include it. He explains that '(1) It was based on a confusion between the four-color map theorem and a simpler theorem, easily proved, which says that five regions on the plane cannot be mutually contiguous, (2) the

Lasswitz and Borges [14]. Rereading I find many of the stories (not Borges) annoyingly sexist. Times change.

Once I co-taught Liberal Arts Mathematics paired with a section of Freshman English. The English instructor and I hoped we could invent a kind of writing and mathematics across the curriculum. One adventure was playing Eleusis [2], which I learned about from Martin Gardner's "Mathematical Games" column in *Scientific American*. I think neither the game nor the paired course worked well.

I often assigned open-ended exercises. For example, when discussing the irrationality of the square root of 2, I asked the students to try to extend the sequence of good approximations beginning $3/2, 7/5, 17/12, 41/29$ and to explain why they expected the pattern to continue. I hoped for and often got correct guesses and perhaps a little algebra but not formal induction. I also required a term paper that embedded some mathematical content, in the spirit of the stories. They were fun to read. The best (which, sadly, I cannot find) was a riff on pot in the time of Nixon, dividing a half-kilo stash into smaller bags of ounces.

The fact that I taught unit calculations meant I was at least a little bit interested in applications. There are other hints. In my files I found a clipping from *The Boston Globe* discussing gasoline consumption month by month in 1972 and 1973. It would provide a lovely QL/QR question today. I have no evidence that I actually used it then.

I think the best summary of the course comes from one student's wrapup document,[5] typed (not on a computer) so I did not have to read their handwriting. That was the only technology that mattered then.

> I have been reviewing my notes and papers and assignments for this class and am surprised at the variety of different things we did and the new concepts I have learned. I've explored Flatland and outer space, built models of solid figures from cardboard and tape, written a poem, visited the Science Museum and learned how important it is to put "not" in the right place. I've learned about change ringing in England, found out what a google [sic] is, touched on permutations (wish we had time for more of that), learned about prime numbers, a non-existent universal library and begun to understand about one part per million. I've been annoyed at times, confused at times, and wondered what was the use of knowing some of the things we've learned, but I don't think I was ever bored or disinterested and I usually "saw the light" before long.
>
> I know my life has been enriched because last Sunday while attending Mass after an absence of many weeks, I noticed for the first time the beautiful geometric pattern of the ceiling over the Sanctuary. As I sat reflecting on the loveliness of the design and the creativity of the architect, I wondered that I had never seen it before. Then I realized I would still not be aware of it but for this Math course.

18.3 The Eighties

In the late seventies, high school algebra entered the college curriculum at UMass Boston and elsewhere, as Lynn Steen describes in *Achieving Quantitative Literacy* [17, p. 37]. Students who wanted to major in science who had been ill-served by their high schools deserved the chance. We started offering just a few sections of college algebra, so called so it would be eligible for graduation credit. It served its purpose for the few for whom it was appropriate—I remember one student in particular who went on to a successful major in computer science with a minor in mathematics. But Gresham's Law ("bad money drives out good") took over and college algebra soon became the default freshman math course. Sherman Stein discusses this explicitly in "Gresham's Law: Algorithm Drives Out Thought" [19].

Traditional college algebra is often a conspiracy. The teacher promises that the questions on the final will be just like those in the book, with different numbers. When the students answer they can pretend they learned something; the teacher can pretend to have taught something. The students check the mathematics requirement box and forget the experience immediately. The teacher may have to repeat it.

I wanted something better—a way to review high school algebra that was more interesting, more useful, and new and advanced enough to justify (in my mind) college credit. I designed a new course and wrote a text: *Using Algebra*.

From the Preface:

> The book consists entirely of problems in which mathematics models familiar phenomena, like automobile fuel consumption, depreciation, population growth, inflation, and the spread of epidemics. To investigate such problems requires algebra, a thorough study of linear equations, unit calculations, the estimation

true four-color theorem, unproved when I wrote my story, has since been established by computer programs, though not very elegantly. As science fiction, the tale is now as dated as a story about Martians or about the twilight zone of Mercury.' "

[5] Today I would need and ask for the student's permission to share this. I am sure they would grant it.

> of large numbers, logarithms, e, and the trigonometry of right triangles. Throughout the book the nature of the problems studied dictates the approach to the mathematics—setting up models and drawing inferences, but little formal manipulation and no proofs identified as such. I believe that the abstract truths that make mathematics beautiful and useful are best left implicit at this level. I have found that students tend to agree [4, p. *v*].

That suggested four sections, one each for linear, exponential, quadratic, and trigonometric models. They come in that order, since the first two are the most important. If an instructor were pressed for time then the last two could fall off the end of the semester. That often happened to me.

Designing the course I set myself this constraint: only "word problems," in both the text and the exercises, always with meaningful variable names, never using x or y. Here is a representative homework problem:

> The profit a movie theater makes in a week depends on the number of customers. In a week when there are 1372 customers the profit is $1790; 1115 customers yield a profit of $1310.
>
> 1. Name the variables and find a linear equation relating them.
> 2. Draw a graph. Identify the units and explain the meaning of the slope and the intercept.
> 3. Find the profit in a good week—one with 2000 customers.
> 4. What is the break-even point? That is, how many customers does the theater need to begin making a profit?
> 5. Find a formula expressing the number of customers in terms of the profit.

The good news is that you have to think a little to solve it. To draw the graph you must choose reasonable scales on the axes, which you would not label x and y. The bad news is that it is completely artificial. There is no such movie theater. I made up the numbers. The only reason a student might care about the problem is that it is homework.

A later exercise asked about a Hans Holbein sketch that sold for $53,000 in 1950 and for $86,000 four and a half years later. I asked for the equation for an exponential model. There was no suggestion that inflation should be taken into account. I provided no source for the data—I probably made them up.

At least some of the questions used real data, sometimes even with appropriate references. We studied a table with the history of first class mail rates. I wrote an exercise on percentages that started with the black population of the United States reported in the 1980 census.

The mathematics was occasionally pretty sophisticated. In the "thorough study of linear equations," I treated mixture problems as weighted averages. The section on compound interest ended with continuous compounding and an introduction to e; I never mentioned APR. I taught and used logarithms to solve exponential equations. When discussing population growth I introduced the logistic differential equation (not by name) and the logistic curve as its solution (with no derivation). In the discussion of free fall and the problem of instantaneous velocity, I found the slope of a parabola by calculating the limit of the slope of the secant.

Students needed a scientific calculator—four functions were not enough. That was new technology for most of them. Graphing calculators came into fashion in the late eighties. I am glad I never had to use one.

I had trouble finding a publisher; finally Little Brown offered. Very few schools adopted the book. At UMass we did (for a while), because I had enough clout to make that happen. When the book went of print I retrieved the copyright. Wyndham Hall republished *Using Algebra*, printing on demand from camera-ready copy. That meant they had essentially no up front costs, and, presumably, minimal overhead for a reasonably large list of titles—a business model ahead of its time. The modest reception did not surprise me. I knew when I was writing that my approach was too idiosyncratic to appeal to textbook adoption committees. I was delighted when my mentor Andy Gleason said the book inspired some ideas that he and Deb Hughes-Hallett used in their calculus text [11], part of the "reform movement" in beginning undergraduate instruction in mathematics.

18.4 The Nineties

By the early nineties, the course for which I wrote *Using Algebra* was long gone. One summer, I taught from an early version of colleague Linda Kime and Judy Clark's *Explorations in College Algebra* [12], another contribution to the reform movement.

Only two students enrolled, but the course ran anyway so we could pilot the text. All three of us hated the experience, mostly for reasons that had nothing to do with the book. The students came with very different preparations and goals; I was unsure of my goals for the course; the "class" was too small for discussion; I was teaching in the summer with great reluctance (so that I could afford an upcoming sabbatical); the computer lab venue was cramped and uncomfortable, and both the hardware and software were flaky.

That said, I was uncomfortable with the book. It seemed to me to be doubling down on the strategy I had tried in mine, using essentially artificial examples to teach algebra, while I was moving toward thinking that we should not be teaching much mathematics at all. Where I had been perhaps too spare and terse, I found *Explorations* voluminous and encyclopedic, almost cumbersome. Tastes vary—the ultimate success of the book (five editions in a decade and a half) suggests that many instructors liked it.

Its use of technology was part of the reform, up to date for its era, but I found it clumsy. Excel was in the syllabus, but with too little time for students to master a useful set of skills. At least I learned some: I remember practicing by building a graph to show that a catenary was not a parabola. Needless to say, I did not assign that exercise. The homegrown Mac software with sliders to set input parameter values for graphing lines and data for computing means, medians, and modes was beautiful but not particularly instructive. I remember watching the (two) students play with it, say "Wow!" when the output jumped around, but spend little time thinking about how input correlated with output. Applications like that are ubiquitous on the web now; I wonder whether they help students learn.

18.5 The New Century

My frustration in that summer course turned me off teaching nonmajors for a while. I was also learning and teaching computer science at the time. Then, one year a mathematics course on my schedule was underenrolled, so cancelled. I agreed to take on a section of the Quantitative Reasoning course that replaced College Algebra as one way for non-STEM students to meet the QL/QR requirement.

Instructors could choose between Kime and Clark and the popular Bennett and Briggs [3]. Since I disliked Kime and Clark (see above), I tried the other. I found it ponderous and boring (and outrageously expensive). I stumbled through the course somehow, picking a few topics and a few exercises that I thought made for good learning experiences, and decided to try to do better (by my lights). The following semester, Maura Mast and I convinced department chair, Dennis Wortman to let us coteach a section of the QL/QR course. Maura had been hired to design a college-wide quantitative reasoning requirement, hence this course. She and her colleague Mark Pawlak had developed new QL/QR materials, but no systematic reworking of the syllabus and no new full text. Now would be our chance. Maura brought her deep knowledge of the QL/QR literature and contemporary trends. I brought my varied experience and lots of frustration. We worked together to set some goals. The principle we acted on was to think about what we hoped our students might remember and find useful ten years after they had taken the course. That meant very little algebra, surely no logarithms—in fact, we wondered and worried about what would be left when we left out "real mathematics."

We started the first day with a copy of that day's edition of *The Boston Globe* and mined the pages for numbers that needed explaining. We were probably as apprehensive as our students—this was the ultimate in winging it since we saw the paper just a few hours before they did. We were excited, inventing classes and writing notes on the fly. Our syllabus ended up including (among other things) estimation skills, a review of percentage calculations (a perennial bugbear), unit calculations for fuel economy and grocery shopping, an electricity bill, a promotional flyer for a penny stock, credit card interest, mortgage calculations, the graduated income tax, and the house advantage at the casino and in the Massachusetts state lottery.

We called the text under construction just *Common Sense*. Reminding the students of what they thought of as "mathematics" would interfere with our ten year goal. We found no use for the quadratic formula. Even the linear equations that came up did not require forgotten algebra. For example, to figure out how well you must do in a semester to raise your GPA to 3.0, guess-and-check-and-iterate makes sense, provides insight, and works in many contexts where there is no closed form answer.

Two new themes emerged for me in this last decade teaching QL/QR. The first was the importance of common knowledge. When we asked students to read quotes from the daily paper or the web we found that many had trouble understanding and hence relating to the material, well before they began to struggle with the mathematical content. Sometimes their problem was missing background information, sometimes vocabulary, sometimes complex sentence structure, sometimes just attention span. For example, atomic physics came up several times in the semester. We stumbled on “megaton” while introducing the metric prefixes. We discussed “$e = mc^2$” when describing the source of the energy produced by a nuclear power plant. Reasoning about half-lives called for an understanding of radioactive decay. Each of these led to a digression into history, physics, economics, public health, or how to read carefully. From our ten-year perspective on what we hope students will remember, these are not digressions. We struggled to address them in print in *Common Sense Mathematics* in simple terms without dumbing them down. Talking about them in class was easier (though still hard).

The second theme was that moral, ethical, and political issues mattered when questions raised by current concerns drove the curriculum. Several examples:

- We used the distribution of family income in the United States to frame the different perceptions that emerged when you used the mean, median, or mode to summarize data. A histogram made the distinctions clear. Then a discussion of income inequality was inevitable.
- In the classes on credit card interest we pointed out that paying your bill on time meant borrowing interest free from the credit card company, so to make a profit they must get their income some other way. Most students were surprised to learn that the main source of that income was the fees charged to merchants. That meant you could save a local merchant money by paying in cash, at the cost to you of foregoing any rewards you might get for using your credit card.
- Many of the students who learned for the first time about the piecewise linearity of federal income tax responded “That’s not fair. Everyone should pay the same percentage.” They were shocked when we looked at historical tax brackets and rates. That prompted a class discussion of the marginal value of money.
- Students were OK with the house advantage at the casino—it is what you pay for the thrill of the play. But they were appalled by the estimate that (in Massachusetts) average per capita spending on the lottery was $500, with money coming disproportionally from the poor and from people who cannot figure out the fair value of a ticket.
- When discussing the parallel between gambling and insurance, the politics of Obamacare could not help but come up. The most compelling argument for single payer “medicare for all” I have ever seen is in George Bernard Shaw’s essay “The Vice of Gambling and the Virtue of Insurance” [16]. I would assign it if I were teaching now even though it was written a century ago by a socialist.

It is hard for an instructor to teach that kind of material and keep his or her personal politics out of the classroom. My students were smart enough to figure out mine. I tried to acknowledge that, while telling them explicitly that they did not have to agree with me to do well in the course.

Incorporating technology was critical. We spent class time teaching Excel for descriptive statistics and for graphing linear and exponential models. We encouraged students to use the web wisely whenever it might help finding data or answering a question. Exams were open notes, open book, open computer, open internet—like life. When we announced that policy students thought exams would be easy until we explained that “open everything” meant we could ask interesting questions rather than routine ones. Homework assignments called for answers in complete paragraphs (not just complete sentences); we encouraged students to type them, writing in the few equations by hand since typing mathematics in Microsoft Word is painful. The course met in a computer lab. The machines were arranged in rows, so it was hard for students to huddle to work together. They could all too easily hide behind their monitors checking facebook. In subsequent years the increasing fraction of students with tablets or smartphones made it possible to use a calculator and the internet in a regular classroom, so I taught the first third of the course in one, moving to the lab only when it was time to start Excel. Spreadsheets on the small devices did not work well!

After that joint first semester we each taught the course regularly but separately, refining the text. Maura managed our successful application for a National Science Foundation grant to support and evaluate that effort. I began blogging to record my thoughts about the task [5].

A publisher’s representative found my open office door and took notes on my complaints about texts and our work on a better one. Not long thereafter we heard from an acquisitions editor. He liked our draft manuscript so he sent it out for reviews. Some were enthusiastic and almost all were favorable. We met for coffee when he was in Boston and discussed important changes—in particular, moving estimation and Fermi problems from Chapter 3 to Chapter 1 and adding “Mathematics” to the title. We signed a contract.

Then the publishing industrial complex runaround began. Could we please create an online homework and exam system, so that students could submit work electronically and have it graded by machine? Would we add chapters on formal logic and on geometry, so that the book met statewide requirements in Florida and Texas? Could we cut back on the direct quotations from the news, so that we would not have to pay for all those permissions? And wait N years to bring this out until the competition releases their new edition, because that is when people switch texts—besides, it will take that long for our technical staff to produce the book. By the way, please use Microsoft Word; we do not do TeX.

We began to feel that although the editor signed our book because he found it excitingly different, the company then tried to turn it into one just like the ones that dominate the market. We soon realized that that the compromises we were willing to make could not bridge the gap between what they wanted and what we cared most about. They graciously arranged a peaceful parting of the ways. That done, we turned immediately to the MAA. Steve Kennedy was enthusiastic by return email.

The rest is recent history.

18.6 What Next?

I am very pleased with *Common Sense Mathematics* and its reception in the QL/QR community. Even some students seem to like it. I hope to see its long life. I'm still collecting exercises from the daily news so instructors can keep courses current. But writing this essay makes me hanker for liberal arts mathematics again, even if that does not directly speak to important useful QL/QR as it's defined in this volume. Mathematics just for fun is much of what I do at the elementary school where I volunteer, and with a group including my grandkids I have played mathematics with every month for years. Maybe I will come out of retirement to try it again at UMass.

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19

Quantitative Literacy Scholarship from Individualist, Collectivist, and Activist Perspectives

Jeffrey Craig, Lynette Guzmán, and Frances K. Harper
Michigan State University

19.1 Introduction

In the editors' early notes on writing chapters for this compilation, they offered writers extra space to engage our ideas about quantitative literacy (QL) with depth and nuance, asking that we "avoid an overabundance of statements describing the need for quantitative literacy." The space is greatly appreciated. As such, our chapter is not focused on the specific practice of quantitative literacy, but the scholarship involved in it; not how to achieve a particular vision for quantitative literacy, but the merits of different visions for it. Simultaneously, although we may collectively agree about the need for quantitative literacy, we should be wary of obscuring our theoretical differences when it comes to creating a vision for quantitative literacy. This chapter outlines our initial effort to focus on the limits of quantitative literacy scholarship in the ways it is currently constructed around identifying people currently innumerate in hopes of developing numeracy within them. The goal of this chapter is not to engage a pessimism about the important works being done by scholars in quantitative literacy. Rather, as Popkewitz said, "It is just the opposite. The strategy is of an optimism that to unthink what seems natural is to open other possibilities" [23, p. xv]. Popkewitz's method of unthinking is *historicizing*, and includes a deep inspection of the history of present discourses as they shape and constrain thinking and action. What we provide here is one sliver of the history of present discourse on quantitative literacy, significant directions of which have been outlined and advanced by Patricia Cline Cohen [3, 4]. Instead of tracing the current discourse about quantitative literacy as an individual need for informed citizenship, as Cohen did, we seek to reveal the implied ideology behind the language of some quantitative literacy scholarship, in order to open other possibilities for quantitative literacy. Specifically, we see quantitative literacy discourse situated within an individualist ideology—the idea that individuals are solely (or at least highly) responsible for protecting themselves and remedying their vulnerability. In this paper, we explore that idea and offer two alternative conceptualizations, *collectivism* and *activism*. Individualist, collectivist, and activist ideologies are often operating simultaneously, but we argue that quantitative literacy discourse emphasizes individualism, which limits the potential for achieving quantitative literacy by framing the "problem" of quantitative literacy in a particular way.

Discourse analysis inspects how common language and assumptions shape and constrain thinking about quantitative literacy. Discourse analysis is a wide methodological approach that can offer many ways to study quantitative literacy, for example Philip and Rubel [?] and Tunstall [32]. In this chapter, we engage in a particular method of discourse analysis, emulating Michel Foucault [8] when he argued that the very discourses uniting our work can prescribe our thinking, potentially limiting that work. An important aspect of Foucault's theory of discourse is to focus on "things said" and not on hidden intentions. This is particularly relevant to scholars who try to sell the ideas of quantitative literacy to people who deeply connect to individualist ideology, despite personally upholding social notions of quantitative literacy. For instance, when a scholar constructs an innumerate individual who needs to be changed to numerate, this rhetoric may not align well with the same scholar believing in quantitative ethics in media, business, and research (especially if both ideas are not stated). Stanic's historical account of mathematics curricular reform in the 1920s demonstrated how one advocate of a unified mathematics movement, William David Reeve, couched his argument in language that did not fully reflect his perspective. Stanic wrote:

> [The] rhetoric that Reeve had used to advocate his general mathematics program actually came—for whatever reason—to be reflected in the program itself. On the surface, this change may seem perfectly reasonable; perhaps it was not an unintended consequence at all. Reeve may have made a reasoned and conscious compromise in order to implement some form of his program. However, if one compares the rigorous mathematical roots of general mathematics with its ultimate connotation as remedial (recognizing that it may not have had this connotation for everyone who used the term), not all the consequences appear to have been those originally intended. Furthermore, one must consider the extent to which such a compromise may have had long-term consequences [27, p. 201].

Unifying our scholarly efforts around a conception of an easily sold quantitative literacy only partially represents our multiple shared and different goals for improving quantitative literacy. Rather than simplifying our discourse to communicate a narrow perspective on quantitative literacy, we hope to open up more breadth for our scholarly community. Our focus here, however, is on some qualities of existing quantitative literacy writings because "it is not possible to promote a conception of mathematical literacy without at the same time – implicitly or explicitly – promoting a particular social practice" [16, p. 75]. We argue that the social practice commonly promoted by quantitative literacy scholarship is characterized by notions of self-reliance, self-interest, and self-preservation while generally discounting ideas of social interdependence, collective good, and collective justice [15]. When this assumption operates within quantitative literacy discourses, it constrains the possibilities for effecting quantitative literacy. We analyze selected pieces of literature to propose further inspection and introspection. Although we have only just begun exploring ways individualism discourse frames the problems of quantitative literacy in ways that prescribe how we usually study it, we suggest some initial ideas about potentially alternative conceptions. We conclude the chapter by offering glimpses into how related domains of scholarship have connected and informed our work, and suggest them as potent for advancing quantitative literacy. We do not claim that individualism is an inappropriate lens to think about, study, and effect quantitative literacy. Instead, we are suggesting that having multiple perspectives on QL will diversify our findings and, ultimately, improve our practice-based efforts.

19.2 The Threat of/to Innumerate Citizens

A fundamental premise in much of the scholarship in quantitative literacy is that without quantitative literacy, people make sub-optimal and wrong decisions, and those people "suffer from innumeracy" [22, p. 8]. This premise positions our practice-based work as helping the innumerate other because "an innumerate citizen is as vulnerable as the illiterate peasant of Gutenberg's time" [28, p. xv]. The inefficient decisions that an innumerate person makes are viewed as contrary to the person's self-interest, and invade all aspects of the person's life: from work, to civic engagement, to finances, to politics [30]. Innumerate people are also positioned as a burden to others through the "harmful social effects of innumeracy" [22, p. 99], and that burden is at least partially borne by those people whom the scholarship considers numerate.

Interestingly, we also often describe quantitative literacy as an asset. Usually, this is done in an individualist way: a personal asset to help its owner survive and even thrive. At times, quantitative literacy is also framed as beneficial to society as a whole. For example, Steen [29, p. 10] claimed that quantitative literacy finds "its most profound value to society [to be] the role it plays in supporting informed citizenship and democratic government." Wiest, Higgins, and

Frost [33, p. 47] echoed the importance of quantitative literacy to full participation in society when they framed quantitative literacy as a matter of justice, saying quantitative literacy "is a matter of social justice in that it places numeric understanding in the hands of 'ordinary' citizens, preparing them to function—for example—as informed voters and consumers." Because innumeracy burdens individuals and numeracy empowers them, our work in quantitative literacy is fairly straightforward. We seek to move people from the former category into the latter, with the ultimate goal of making everyone quantitatively literate. In hopes of including all people in quantitative literacy, these discourses can reinscribe the connection between innumeracy and inefficiency. An image of the innumerate citizen is drawn quite explicitly in the preface of one of the predecessors of this volume, *Mathematics and Democracy* [20]: "Indeed, Dewey reminds us, a successful democracy is conceivable only when and where individuals are able to 'think for themselves,' 'judge independently,' and discriminate between good and bad information" [21, p. xiv]. Embedded into these storylines about a changing world come often explicit discussions about the relationships between quantitative literacy and informed civic participation, as an excerpt from Carnevale and Desrochers [2, p. 29] illustrates:

> The wall of ignorance between those who are mathematically and scientifically literate and those who are not can threaten democratic cultures. The scientifically and mathematically illiterate are outsiders in a society in which effective participation in public dialogue presumes a grasp of basic science and mathematics.

Thus, a major goal for informed citizenship within quantitative literacy scholarship has focused on combating innumeracy with efforts for every citizen to be quantitatively literate.

19.3 Goal for Informed Citizenship: Each Citizen Quantitatively Literate

We want to restate that wanting to help people practice quantitative literacy because, if they fail to do so, they are vulnerable to the power that quantification has in our society is reasonable. The quiet assumptions motivating calls for quantitative literacy, however, should be subjects of thought and study to open up new possibilities for quantitative literacy practices. Our goal for this section is to consider how the prevalence of the individualist ideology in our scholarships constrains what gets thought and said about quantitative literacy. Individualism seems most compatible with how we write about, and argue for, quantitative literacy as a need for informed citizenship because calls for quantitative literacy construct the image of an individual expressing quantitative literacy through their personal thinking, reasoning, and decision-making, in absolutely beneficial ways.

Steen et al. [30] outlined the many benefits of quantitative literacy: in citizenship, culture, education, professions, personal finance, personal health, and work. With this comprehensive list of potential ways to express quantitative literacy, the authors' fundamental idea is that a person who is quantitatively literate engages in particular practices and makes particular decisions. This fundamental image of a quantitatively literate person seems to unify us, despite the fact that people define their own self-interest differently. In other words, the form of an individual's decision is contingent on the way they think about the world, thus making it nearly impossible to universally assess quantitative literacy based on individual decision-making. We have formulated improving quantitative literacy as having each person make the right, or better, decisions when quantitative claims are involved. Other mathematics education scholarship also discusses the power of mathematics and statistics. As a consequence of becoming embedded in so much of the social world, mathematics and statistics learning is critical and imperative for all. As Ernest [7, p. 1-2] claimed,

> Social empowerment through mathematics concerns the ability to use mathematics to better one's life chances in study and work and to participate more fully in society through critical mathematical citizenship. Thus it involves the gaining of power over a broader social domain, including the worlds of work, life and social affairs.

What emerges from these calls is the agreeable image of a quantitatively literate citizen who functions efficiently in society. Efficient functioning involves making correct decisions (along the innumeracy/numeracy division) relative to self interest with quantitative information, improved access to quantitative information, the ability to correctly interpret quantitative information, and an improved likelihood of financial and social success. Given the roots of quantitative literacy scholarship as reacting to innumeracy, the description of a quantitatively literate citizen being able to function efficiently is a double gesture[1] that creates a category of citizens who are a burden to society, on whom

[1] Derrida uses the term "double gesture" to refer to differences of levels, development, characteristics, etc. of discourses that are seemingly contradictory. However, these disparities are not true contradictions because discourses can accommodate the existence of these differences/disparities.

educators and scholars should focus. In fact, this double gesture of wanting to include people as being quantitatively literate simultaneously describes people who are excluded from informed decision-making. This double gesture may be unavoidable when pushing for inclusion. In our push to make everyone quantitatively literate, our focus is almost singularly on those not yet included. This focus has its rewards: our scholarship and practice has likely improved some of our students' lives by giving them tools to better understand and analyze a more quantitative world. This is a worthy goal, and can be a laudable accomplishment. But, this focus also has a cost: our scholarship and practice has generally not paid attention to the responsibilities of those we would deem quantitatively literate. We have not often sought to understand how individuals interact together around quantitative issues and claims. We have also not attended to how people might use their quantitative literacy to help other people thrive in this world, instead focusing on how an individual should use their quantitative literacy to help themselves. The rest of this chapter begins some of this work by introducing collectivism and activism as additional lenses to study and effect quantitative literacy.

19.4 Alternatives and Complements to Individualism

At the heart of the individualist ideology is the idea of self-reliance, wherein it is considered weak to rely on others to help meet your needs [14, 15]. Each person holds a personal responsibility to become informed in order to not be a burden on society. In what follows, we outline the potential of collectivism to include more focus on the possible roles of the numerate person in a more quantitative society. We also outline activism as a way to emphasize the potential of quantitative literacy involved in creating and producing rather than only in functioning and consuming. The consequences of invoking a vulnerable innumerate individual in quantitative literacy scholarship include a lack of scholarship expressing a *collectivist* perspective that tries to understand the potential roles of quantitatively literate groups and quantitatively literate individuals within groups. After considering a collectivist perspective, we then discuss a third perspective, *activism*, to think about the potential of quantitatively literate individuals.

19.4.1 Collectivism

Individualism highlights the role of each person in affecting their own lives. An array of individualist perspectives helps clarify when and how quantitative literacy improves individuals' decision-making and social efficiency. The attention of quantitative literacy scholarship has generally been on creating quantitatively literate individuals capable of protecting and helping themselves. These are important and worthy goals, but have arguably been the singular focus of quantitative literacy scholarship. In this section we begin thinking about the implications of a collectivist perspective on our scholarship.

Collectivist ideology attends to the roles and rights of groups in society [6]. Collectivist approaches to quantitative literacy, then, attend to how groups might demonstrate and utilize their quantitative literacy which, in our view, provides a productive way to think about numerate practices rather than innumerate people. In particular, a collectivist ideology might center on a concept like quantitative responsibility. For example, some professional groups employ their quantitative expertise to comment on relevant social policies and decision-making, such as the American Statistical Association's [ASA] statement on using value-added models for educational assessment [1]. A collectivist ideology emphasizes the roles and responsibilities of expert groups, such as the ASA, as well as expert individuals within groups. It seems likely that in large enough groups there are quantitatively literate members. A collectivist perspective might focus on how those individuals contribute to their groups through their quantitative literacy. Quantitative literacy scholars might ask questions about trust and communication, in addition to knowledge and decision-making. Where might such a line of thinking take us? Well, trust seems unlikely to develop without experts communicating how they use quantification responsibly (and following through). Perhaps it is the responsibility of quantitative experts to question the pervasiveness of numbers. In other words, we argue that a collectivist perspective highlights the accountability of quantitative experts in creating a world where, as Steen [28, p. xxvi] claimed in 1997, "the relentless quantification of society [that] continues unabated." He continued by identifying innumerate individuals as a threat to democracy, noting that "limited numeracy can easily shift the balance to a technocracy." His line of thinking, therefore, flirted with thinking about the potential negative aspects of quantification (a technocracy), but still focused on the innumerate individual rather than the numerate ones. A collectivist perspective may help scholars acknowledge the responsibility of those technically savvy to prevent overreliance on quantification of problems in their fields.

We have reconsidered the idea of quantitative literacy for all citizens, and much of our discussion reflected back on Steen's claim about the vulnerability of innumerate citizens. Although the reasoning seems sound—that there is a problem because people without specific quantitative skills and dispositions are susceptible to making decisions not in their self-interest—arguably, "from such perspectives, the 'blame' for failure is often placed with the victims, and engenders a deficit model of thinking" [19, p. 27]. We recognize the idealism of hoping that our media reporters, businesspeople, and politicians will be responsible users of quantitative information; however, we believe that there is room to think about and explore the moral and ethical ways in which quantitative literacy might be practiced. With this focus, we might become better able to hold people accountable for how they use their quantitative abilities. Nevertheless, quantitative literacy scholars can work to ensure that our writings do not overemphasize individual vulnerabilities and obscure who and what creates and takes advantage of those vulnerabilities. We argue that it is possible for us to reconceptualize quantitative literacy to emphasize both individual goals and collective responsibilities, and discuss the relationships therein. Focusing on the roles and responsibilities of those members and groups of society with quantitative literacy yields the potential for a quantitative ethos and real attention to holding other groups, political and media, accountable for their quantitative rhetoric. Social engagement that demands accountability leads us to also consider how we might think about activism in quantitative literacy scholarship.

19.4.2 Activism

Besides considering collectivist perspectives on quantitative literacy, alternative individualist ideologies might also prove productive for our scholarship. One perspective on individualism emphases individuality, or the idea that each individual can do unique things by virtue of their uniqueness, as opposed to self-reliance. Individuality in quantitative literacy scholarship might move us from attending solely to vulnerability toward considering individual creative possibilities. Advocates of this perspective on individualism believe that people can and should act in their uniquely new and creative ways. We call this perspective *activism* because it can conjure images of creative resistance and actions toward just ends. This perspective also highlights individuals taking an active role in seeking out and creating quantitative arguments, rather than solely a reactive role to those arguments they encounter. Activism reorganizes the discourse around quantitative literacy by shifting focus from what people cannot do quantitatively to what they can (and already) do, circumventing the primary problems discussed in current quantitative literacy scholarship by reframing the notion that everyone needs the same quantitative literacy. Instead, a two-fold shift takes place. First, quantitative literacy is not necessary to recognize creative, imaginative, and just ends of quantitative expression. Second, quantitative literacy is not necessary to participate in creative, imaginative, or justice work. This reorganization of quantitative literacy also seems to open collective possibilities, wherein individuals, all of whom engage in practices of quantitative literacy, can and should act with others to greater ends.

The quantitative literacy community has focused a good deal of energy trying to help innumerate people become numerate. The transition between innumeracy and numeracy is signaled by a move from being vulnerable to being empowered. Quantitative literacy is the marker of someone who comfortably navigates a more quantitative world and uses their quantitative abilities to improve their life. The primary goal of our scholarship is to educate people to survive in a world full of systems and people who try to take advantage of their innumeracy. This is a worthy and important goal. An activist perspective exposes that the goal is not achievable without also creating a quantitative literacy that challenges and changes those systems and people who would take advantage of the innumerate. That is, it is not enough to educate people to survive the world without also educating them to improving the world. Just surviving existing problems and existing within unjust systems is incomplete without also working to dismantle those systems (e.g., patriarchy, white supremacy, heterosexism, colonialism). In contrast, the challenges the present holds for individuals and societies deserve creative action to correct injustices and humanize other people. We believe that quantitative expressions and quantitative literacy can have important roles in shaping the future. An activist quantitative literacy perspective emphasizes learning to use quantitative literacy to rethink current social and political structures in an effort to mitigate inequities and injustices.

In the 1980–1990s, scholars [9, 25] coined the term *critical mathematics education* and attempted to carve out a place in mathematics education for their critiques of society and their commitments to justice (see Powell [24] for a full discussion of the development). In the United States, as critical mathematics education trickled down from the college level [9, 10] into K–12 mathematics classrooms [12], it came to be known as *teaching mathematics for social* *justice*. For clarity, we refer to these various lines of scholarship as critical/social justice mathematics education, but we recognize that each distinct theoretical development relies on nuanced differences. Some of those differences arise from the diverse theoretical influences informing the work [13] and culturally relevant pedagogy [18] but most critical/social justice mathematics education scholars [9, 12, 25] draw shared inspiration from Freire's [11] conception of critical literacy. As a result, this scholarship is meaningful to quantitative literacy scholars in the way that it deconstructs the ideas of not being able to do mathematics (or innumeracy) as socially formatted through institutionalized power and historical discourses.

Critical/social justice mathematics education, for example, aims "to engage students, socially marginalized in their societies, in cognitively demanding mathematics in ways that help them succeed in learning that which dominant ideology positions them to believe they are incapable" [24, p. 27]. Although critical/social justice mathematics calls for mathematics learning, this call is not necessarily motivated by individualism. Instead, the primary motivating force is disruptive action (i.e., activism), which is necessarily collective. Learning mathematics (or becoming quantitatively literate) is primarily a means to a larger end, enabling groups of people, particularly those socially and historically marginalized in their societies, to survive the current society while simultaneously providing them with the means to use mathematics to impact and reshape the world in which they live.

Some mathematics education scholars argue that we face a moral and ethical imperative to transform mathematics from a tool for violence and colonialism to a tool for building on and supporting emerging understandings of systems of privilege and oppression for all groups of people [5, 17, 26, 31]. This moral imperative has a renewed collective urgency as social media and social networks connect people of all ages from many communities, and supports awareness of local, national, and global issues. Here, we have argued for a similar transformation in quantitative literacy education, and we believe that critical/social justice mathematics education offers a model for such transformation. Blurring the borders between critical/social justice mathematics education and quantitative literacy can bring new goals and emphases, reimagining an approach to quantitative literacy that helps those who are quantitatively literate (in the traditional sense) understand the moral and ethical uses of quantitative literacy towards a more just world. Simultaneously, quantitative literacy scholarship can contribute to critical/social justice mathematics education through its history of reimagining the goals of mathematics education and reformulating curricular subjects.

19.5 Crossing Borders: Where We Are Turning Next

This chapter is not intended to be an indictment of individualist ideologies, nor of current quantitative literacy scholarship. Instead, we wanted to describe the potential we see in expanding the ways quantitative literacy is framed, studied, and thereafter effected. Although we have only just begun the intellectual work of understanding the ways individualism discourse constrains our work, we have suggested some initial ideas about alternative conceptions, and are considering the ways that individualism can and should contribute to our collective thinking about quantitative literacy, in concert with other perspectives. In addition to alternative discourses, other mathematics education scholarship offers methods, theories, and concepts that may be useful for quantitative literacy researchers. In this volume, Philip and Rubel, and Cardetti, Wagner, and Byram provide excellent demonstrations of the potential of these alternative perspectives to open up possibilities for quantitative literacy. How else might we draw on these perspectives in quantitative literacy spaces? Do we consider our students' lived experiences with (and interests in) cultural, social, and political issues? How can we continue pursuing what Piercey described as quantitative ethics: the moral and ethical responsibilities involved when people enact quantification?

Our effort to open up some new spaces for quantitative literacy scholarship involved exploring how an individualism ideology might be promoted by quantitative literacy scholars' writings. We also proposed our early thoughts about two alternative (and related) ideological approaches to quantitative literacy: collectivism and activism. Throughout, we sought to blur boundaries between quantitative literacy and other educational scholarship within critical/social justice mathematics education to encourage new possibilities for quantitative literacy scholarship. These critical mathematics educators offer alternative discourses that quantitative literacy scholars should enact, especially if it more accurately portrays their complex beliefs about quantitative literacy. Our purpose in critiquing a density of quantitative literacy discourse that centers on individualism is to reimagine creating new systems and dismantling structures that marginalize people labeled as innumerate or quantitatively illiterate. Rather than placing all the burden on individuals to navigate an unjust system for efficient functioning, we call for more attention to efforts that serve as disruptive action. For scholars, this means an increased awareness of how in trying to “sell” quantitative literacy to various stakeholders, we make rhetorical choices. Understanding quantitative literacy from an individualist perspective is important and reasonable. We contend that overreliance on the simple idea of making innumerate people numerate can obscure our efforts to improve quantitative literacy and its education.

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20

Classroom as Laboratories of Democracy

The Role of New Quantitative Literacies for Social Transformation

Thomas M. Philip and Laurie Rubel[1]
University of California Los Angeles, Brooklyn College of the City University of New York

20.1 Toward a Collective Vision of Quantitative Literacies

Mathematics and Democracy [24] marked an important shift toward recognizing the role of quantitative literacies in a vibrant democracy. Steen and colleagues argued that people can become thrifty consumers, productive and employable workers, informed voters, and more deeply enjoy recreational and cultural activities through increased quantitative literacy. Underlying this orientation toward the individual, however, are assumptions that our political and economic systems are neutral and fair, that there is equitable access to quantitative literacy, and that empowering individuals through quantitative literacy will aggregate to an improved society.

The assumptions that our political and economic systems are impartial can be challenged on a host of fronts. For example, there is persistent racialized residential segregation in the U.S. that results in inequitable geographies of opportunity that impact consequential domains like health care, education, personal finance, transportation, and food [17]. Racism in the United States gives rise to unjust policing and criminal justice systems that result in the disproportionate mass incarceration of African Americans [1]. There is unyielding sexism that fuels violence against women and produces a gender wage gap in which women earn about 80% of men's earnings, a gap which is most pronounced for African American and Latina women [14]. Drastic inequalities in the U.S. derive from the neoliberal economic policies that have shaped the nation over the last three to four decades [13]. For example, a mere 20 Americans have more financial assets than the bottom half of the country—157 million people—combined [7]. The notion that increased quantitative literacy in individuals alone would ameliorate these inequalities suggests that a lack of quantitative literacy is a root case of the inequities. Such an attribution ignores the underlying systems that position few in domination of many. A presumption that increased quantitative literacy, spread across individuals, will lead to social justice further burdens those who are subjugated, rather than challenging and changing the unjust hierarchies on which the injustices rest. Imperative, but absent here, is an outline as to how people collectively adopt, adapt, transform, and judiciously select and reject quantitative literacies toward a vision of a more equitable and just society.

We recognize that the very construct of quantitative literacy has dramatically shifted since the publication of *Mathematics and Democracy*. New digital technologies and the ubiquity of digital data—pervasive today in ways that were perhaps unimaginable for Steen [24] and his collaborators just a decade and a half ago—highlight the need for new quantitative literacies (as described by Craig, Mehta, and Howard in this volume). With the preponderance of new

[1] The authors' names are listed alphabetically. Philip and Rubel contributed equally to the conceptualization and writing of this chapter.

forms of digital data, people are required to be quantitatively literate about a diversity of data that are messier, less complete, and more unstructured than in the past. In our view, new quantitative literacies share many mathematical and statistical practices outlined by Steen and colleagues, but incorporate practices from other disciplines (e.g., data representation and visualization practices common in computer science and mapping and spatial literacies from geography). Our use of the term emphasizes the multidisciplinary practices involved in "reading and writing" with new digital data.

Offering an alternative to the individual-centered view of quantitative literacy from the era preceding "the data deluge" [38], we re-envision classrooms as laboratories of democracy [11], or spaces of democratic participation where students purposefully use and repurpose new quantitative literacies as they engage in the complexities of collective deliberation (see also the chapter from Cardetti, Wagner, and Byram in this volume). We first outline our rationale for shifting from an individualistic lens that tends to highlight quantitative illiteracy to a collective lens that allows researchers and educators to better notice and build on situated forms of quantitative literacy (see also the chapter from Craig, Guzmán, and Harper in this volume). We then describe Guinier and Torres' [11] notion of laboratories of democracy through a brief discussion of "power-with democracy." We use these constructs to critically reexamine our prior efforts to incorporate new quantitative literacies into classrooms. To add nuance to our vision of classrooms as laboratories of democracy, we explore the potential role of new quantitative literacies in these spaces but also examine the risks of prioritizing quantitative literacies over democratic deliberation. We argue that classrooms should be laboratories of democracy, which include space for students to practice power-with democracy supported by new quantitative literacies.

20.2 Rethinking Our Starting Point: Noticing Literacy Rather Than Displaying Illiteracy

The choice of whether to use an individualistic or collective lens influences how researchers and educators see, study, and address quantitative literacy. Scholarship that focuses on the proficiency of individuals has documented extensive illiteracy across the populace, even among those successful in school mathematics and statistics [21, 22, 27]. Similarly, examples of individuals' lack of proficiency with respect to decision-making about civic issues abound [5, 6, 26, 40]. However, anthropological research that studies people as they interact with others in everyday activities has demonstrated that people use sophisticated quantitative literacy practices in and across these contexts [19, 37]. Building on this lens that focuses on people in collectives, we argue that studying genuine forms of democratic interaction will allow us to better understand how people authentically use quantitative literacies as they collectively engage with civic and political questions [4, 28].

Social transformation is clearly more complex than learning to apply quantitative reasoning to civic and political problems [6]. Conventional quantitative practices, which often gloss over the historical, social, political, and economic dimensions of power, will not be sufficient to ignite or facilitate authentic transformations. Rather, social transformation necessitates new ways of thinking, new tools, and new forms of quantitative literacies. Approaches to change sanctioned by existing systems of power, where dominant interests converge [2], may allow for certain temporary gains for oppressed and marginalized groups, but they will not fundamentally reorganize the inequitable and unjust systems themselves.

Within the constraints of contemporary schooling, it is difficult for classrooms to become genuine spaces of social transformation. But, they can certainly approximate these spaces and foster democratic deliberation [15, 16]. Toward this end, we argue that classrooms should not only be spaces where students master existing quantitative literacies that they can later apply as civic agents. Classrooms should also provide opportunities for students to purposefully use, repurpose, and innovate quantitative literacies as democratic participants.

20.3 Power-with Democracy and Laboratories of Democracy

We draw on Guinier and Torres' [11] conceptualization of democracy to further consider the relationship between quantitative literacies and social transformation, a relationship in which power plays an essential role. The purpose of democracy, according to Guinier and Torres, is to bring people "into the arena of public-decision making as participants, not as spectators" [11, p. 171]. Power, as seen in the dominant paradigm, is "power-over," that is, competition for power yields the strong who emerge as victorious and dominate the weak. From the perspective of a power-over paradigm, it is sensible to construct quantitative literacy as an avenue towards individual empowerment, as a means to bolster an individual's strength in a competition for power. However, we accept Guinier and Torres' proposition that political transformation demands more than individual empowerment, meaning that the underlying power-over paradigm needs to be challenged. As an alternative to the power-over paradigm, Guinier and Torres propose that we consider "power-with," which is relational, interactive, and participatory. From the power-with perspective, a focus on the individual is replaced with an emphasis on resistance and struggle in collective forms. A fundamental difference in the goal of empowerment in a power-with framework is to "change asymmetrical power relationships, rather than merely struggle for the right to participate in them" [11, p. 147].

Laboratories of democracy are essential building blocks for the realization of power-with democracy [11]. They are "prefigurative" spaces where participants embody the "forms of social relations, decision-making, culture, and human experience that are the ultimate goal [of an equtiable and just democracy]" [3, p. 100]. Laboratories of democracy are places where people come together to experiment with ideas, share information, reflect, and self-correct. These spaces must entail a commitment to attend to internal and external forms of power with respect to "the dynamics of group interaction" and "the content of inquiry" [11, p. 148]. A focus on internal processes of deliberation emphasizes the importance of how we engage in democratic deliberation, not simply the goal of coming to an informed decision or even achieving a shared goal. Guinier and Torres acknowledge that practices that are genuinely committed to a power-with participatory framework may appear messy and unstable and can be incredibly time-consuming, but are indispensable for genuine democracy. By confronting "embedded internal hierarchies" and engaging "the untidiness of conflict over time," laboratories of democracy allow people to "struggle against external challenges in ways they have not yet imagined [while they still] struggle with internal conflict" (p. 159).

20.4 Learning from Two Cases of New Quantitative Literacies in Classrooms

We reflect on the successes and struggles we encountered in two projects in which new quantitative literacies were incorporated in high school classrooms with the explicit goal of addressing issues of democratic engagement, equity, and social justice. Neither project was designed with laboratories of democracy as a central commitment, but the construct has provided a useful retrospective lens to consider the affordances and limitations of these projects, and has shaped our aspirational vision for new quantitative literacies in classrooms. We indicate the shortcomings of each project, with our hope that our candor will ultimately provide others a more advantageous point of departure.

The contexts of each project differed significantly. The first case, studied by Philip, was a part of a large scale implementation of an introductory data science curriculum across multiple school sites. While the curriculum attempted to use socially relevant issues as a motivational hook for students, it was not designed to support democratic deliberation about those issues. This case exemplifies the risks that emerge when curriculum developers assume that new forms of quantitative literacies necessarily lead to increased democratic literacies. It provides a cautionary note that the prevalent model of STEM reform can be counterproductive to the civic and democratic goals of education. The second case, co-designed and studied by Rubel, explores a curricular unit taught in one classroom. It was intentionally designed to create spaces for democratic deliberation, to leverage new quantitative literacies to examine inequities and injustices across themes like personal finance and the state lottery. The second case provides powerful glimpses of how new quantitative literacies can play a role in working toward classrooms as laboratories of democracy.

Our roles in these projects were different. Given the size, design, and unexpected changes in the organization and objectives of the project Philip worked with, research findings from Philip and his colleagues did not inform iterative designs of the curriculum. In contrast, Rubel led a smaller design team, and the team's research findings informed revisions to the curriculum described in the second case.

20.4.1 Case I: The Risks of Exclusive Fidelity to Quantitative Literacy

From 2010 to 2016, Philip researched a large scale high school reform project intended to introduce students to the increasingly influential field of data science. The year-long curriculum was developed by university-based researchers in statistics, computer science, and education in collaboration with school district personnel. Interspersed through the year-long curriculum were numerous opportunities for students to engage in new quantitative literacies. For instance, students were introduced to the ways in which corporations use big data to predict consumer behaviors for marketing [25]. They learned about highly publicized cases like Target's ability to use data analytics to predict pregnancies in order to increase their overall market share since people change shopping habits during major life events [8]. In addition, they learned about seemingly more innocuous examples such as using data from Facebook posts to find patterns in seasons when people are more likely to end romantic relationships. Students also collected, interpreted, and analyzed self-generated data so that they might develop more personal and nuanced understandings of data.

The focus on connecting data to contemporary issues and personal interests seeded powerful opportunities to engage in the civic dimension of new quantitative literacies. The curriculum encouraged students to develop the statistical and computational dimensions of working with data. However, socially significant questions arising from the data were often glossed over since they could not, ostensibly, be answered with the available data or analysis tools. The apparent fidelity to quantitative literacy and statistical thinking and the desire to engage students in personally and civically relevant issues consistently brought issues of race and racism to the fore, but without adequate lenses to examine systemic and institutional forms of racism. Without a commitment to address internal hierarchies and external forms of power, these potential opportunities instead became instances where deficit perspectives about students and about communities of color were reproduced [31, 32]. The assumption, by curriculum designers, that new quantitative literacies would translate into better understandings of civic issues not only failed to materialize, but in fact reinscribed practices that were fundamentally anti-democratic. A lens of classrooms as laboratories of democracy could have prompted greater attention to the contestations of power that would (and should) emerge in classroom conversations focused on data about students' lives.

The introduction of a seemingly benign data visualization about movie rental patterns was a particularly notable example of a cursory approach to race that backfired [29]. The curriculum developers included an example of a data visualization related to a popular blockbuster movie and a niche film with an all African American cast to illustrate how some visualizations can simply be proxies for race. The classroom discussion, however, became a heated and extensive contestation over what it can mean to be African American. The students and teachers in the class reproduced both internal hierarchies and external forms of power as some students drew on the visualization to associate African American neighborhoods with ghettos. The voices of African American students in the class were marginalized and alienated and the class lost out on a powerful opportunity to learn about new quantitative literacies and practice democratic deliberation. In a second example, an inquiry into students' snacking habits re-inscribed ideas that that low-income people make poor choices to eat unhealthy food. [30]. The curricular insistence to make statistically sound arguments based on the limited data students had collected about their snacking habits precluded analyses of systemic factors pertaining to differential access to healthy foods based on geography, income, transportation, work schedules, and other related factors. However, there was a great deal of latitude when the teacher and the students made generalizations about people of color based on trends they saw in the data they had collected. There were not similar expectations for students to make judicious arguments based in social theory when they made assertions that had to do with race. The demand for students to think like a data scientist (in a relatively narrow sense), but not necessarily like a social theorist, gave the air of scientific objectivity to problematic claims about the nutritional habits of people of color.

Philip's reflections about this project demonstrate that teaching new quantitative literacies with the assumption that students will then better understand issues that are at the intersection of race and racism, personal agency, and the distribution of resources in society is naive at best, and perhaps more accurately, negligent and deleterious. When addressing power in a highly racialized society in the course of teaching new quantitative literacies, it is essential that opportunities are made for students to become racially data literate: racially literate about data and data literate about race [29]. While closer attention to issues of race can undoubtedly address some of the shortcomings Philip documented, designing classrooms toward laboratories of democracy would allow for new quantitative literacies and racial literacies to productively co-construct each other.

20.4.2 Case II: The Potential of Putting Quantitatively-driven Narratives in Dialogue with Competing Analyses

From 2013 to 2017, Rubel and a team (consisting of educational researchers and mathematics teachers together with an interdisciplinary team of mappers and urban planners) developed and studied the mathematics curriculum around themes of spatial justice. The team designed and piloted [33, 35] two curricular modules to enable investigations about two spatial systems. One module focused on the state lottery (*Local Lotto*) and the second on a city's two-tiered personal finance system, marked by one tier of banks and a second tier of alternative financial institutions such as check-cashers and pawnshops (*Cash City*). The modules were designed for high school students to use mathematics to understand the underlying mechanics of each system. In *Local Lotto*, students used probability and combinatorics to understand the expected value of a lottery ticket. Students used percentages and ratios in *Cash City* to model loans. By examining the geographic distribution of lottery ticket sales or the locations of types of financial institutions, the spatial dimension afforded opportunities to use new quantitative literacies as part of a sociopolitical critique of these systems. In contrast to the examples described in Case I, mathematically-oriented arguments suggested by quantitative data were nuanced with multiple, and at times, competing narratives gathered by students from families, community members, and local businesses through participatory mapping [34].

We believe that spatial justice curricula, like *Local Lotto* and *Cash City,* are promising as seeds for laboratories of democracy in schools, and are especially conducive to interdisciplinary investigations that promote data and other quantitative literacies. For example, *Local Lotto* powerfully impacted many students. They used their probability calculations, new sense of number and spatial scaling, and new insights about the lottery's role as a state fundraiser to try to sway their parents towards minimal or no participation [36]. Analyses of the lottery quickly opened up spaces to consider people's motivations for playing the lottery that would have been otherwise missed if the lottery was viewed as a matter of individual choice. For instance, students considered the tensions low-income earners might experience in having to choose, in their words, between their "hunger for food" and their "hunger for hope," noting that lottery tickets and food are sold by the same stores [36]. Participation in *Local Lotto* enabled students to reflect on how "low-income people become inherently more vulnerable to the lottery because of the [...] day-to-day realities of living on a low income" [36, p. 19].

Along with students' mathematical progress and success in analyzing these systems as predatory, other students emphasized the value of individual freedoms, to choose to buy lottery tickets or to make use of alternative financial institutions like check-cashers and pawnshops instead of banks. Rubel, Hall-Wieckert, and Lim [33] reflect how these perspectives from students might be a reflection of libertarian or capitalistic values but also could represent an intentional and thoughtful contestation to the way that the modules' narratives, led by white teachers and a white and Asian design team, might have inadvertently positioned students and their families as quantitatively illiterate and powerless, instead of positioning these systems as unjust and how they might be reimagined to be fairer. Issues related to internal hierarchies of power in those classrooms were exacerbated by the omission of race from the associated data visualizations [35]. The lottery and the personal finance system as systems are directly related to money and seemed most connected to data related to income and income inequality. However, by focusing on income and omitting race, the visualizations not only left race implicit but also egregiously aggregated all of the city's diverse low-income neighborhoods into a single category. As in Case I, race was persistently raised by students (i.e., "the white man" in response to a teacher-posed question "who's making money off of this system?"), but was quickly rejected by other students ("you can't say that, there are poor white people too") or, was too volatile or risky to be picked up by this teacher [33]. Here too, race was not sufficiently attended to and deficit perspectives about residents of low-income communities were likely reproduced.

More broadly, *Local Lotto* and *Cash City* could have been ideal precursors into a classroom space as a laboratory of democracy in which students examine and address deeper issues of hope relative to themes like collective responsibility, community health, and taxation. To restate Guinier and Torres [11], the real issue for a vibrant democracy is not specific to any one industry or system, but rather how deliberation about the equity of state sponsored systems can support students to understand and address collective aspirations and desperations. Essential to this deliberation as part of a laboratory of democracy is for students to reimagine systems so to achieve greater justice.

20.5 The Risks of Prioritizing New Quantitative Literacies over Democratic Deliberation

There is certainly value in the new quantitative literacies activities documented in both cases. These approaches were opportunities for students to see instances in which new quantitative literacies might be relevant to everyday problems and to experience a range of ways in which new quantitative literacies can contribute to "reading and writing the world" [9, 12]. The more we engage with new quantitative literacies for democratic participation, however, the more we become concerned that these activities do not go far enough to create spaces where students can practice power-with democracy. While the second case came closer to realizing certain characteristics of a laboratory of democracy, both projects reached their limits before they opened up the full potential of democratic deliberation.

Classrooms as laboratories of democracy aspire to allow students to adopt, adapt, transform, and judiciously select and reject quantitative or data literacies within the multiple, intersecting, and often conflicting intricacies of social existence. Deep social struggles are not problems that can be solved with the help of equations, data visualizations, or computational simulations alone; they are intricate problems that involve people as complex actors. In the above cases, we have demonstrated how new quantitative literacies can contribute to the process of democratic deliberation. At the same time, we caution against overemphasizing the role of new quantitative literacies and below highlight several examples that explore the risks of an overemphasis on new quantitative literacies in the process of democratic deliberation. This list is by no means exhaustive; it is only meant to show how a singular focus on new quantitative literacies can have deleterious effects on democratic deliberation.

When new quantitative literacies are "powerless"

Statistics about policing policies such as "stop-and-frisk" in New York City and the killings of young men of color by police across the United States clearly show systemic racism [10, 39]. Even when movements have sought to bring racism to light by leveraging new quantitative literacy practices, the populace, particularly white Americans, often dismiss statistically sound reasoning about systemic racism and displace the responsibility on allegedly wayward youth of color [18]. It is naive and disingenuous to assume that fostering new quantitative literacy for individuals about stop-and-frisk and police killings is sufficient to transform the extant policing practices. Students must learn to analyze issues like policing practices, supported with a lens of new quantitative literacies, but framed in the context that includes systemic racism. Curriculum designers and educators must consciously design for and dynamically address these considerations of power that are notably absent in the seminal work on quantitative literacy. Students must practice engaging themselves and others in addressing complicity with the systemic racism that leads to these unjust policing practices. By highlighting social transformation towards justice as their goal, classrooms as laboratories of democracy would engage new uses and transformations of new quantitative literacies.

When "real" people choose to disregard new quantitative literacies in the name of other ways of knowing

Cash City, one of the curriculum examples described in [34, 35, 36], compared alternative financial institutions (AFIs) like pawnshops and check-cashers with banks. By virtue of their higher interest rates and density in certain spaces, the AFIs can quickly seem predatory and choosing AFIs over banks would seem to be contraindicated by these quantitative comparisons. And yet, during their data gathering through participatory mapping, several African American students experienced exclusion and racism in local banks and felt more welcomed in AFI spaces [34]. Pedestrians who were interviewed by students shared narratives that yielded additional complexities, such as the role of banks in gentrification and its impact on community life. These contrasts between the financial institutions were not otherwise captured in the quantitative datasets but add significant nuances to more accurately illuminate people's decision-making. We posit that valuing diversity in ways of knowing would better support authentic reimaginings of unfair systems. A classroom as a laboratory for democracy would allow students to engage in more realistic explorations of complex systems like these, which take into account factors broader than those suggested by new quantitative literacies alone.

When a narrow focus on new quantitative literacies excludes other perspectives

A focus on new quantitative literacies can detrimentally marginalize other important perspectives. For instance, in the class Philip studied, students analyzed self-generated data about their snacking habits. However, given the data they had collected, there was little room to contextualize their data within their larger sociopolitical context. Students largely explained their unhealthy snacking habits through a deficit-oriented lens [30] that focused on poor choices made by students, their parents, or their communities. The curriculum did not support an inquiry into how the lenses of immigration, urban food deserts, or colonialism could have complicated the simplistic and problematic quantitative argument that students produced. Classrooms as laboratories of democracy could provide a space for students to genuinely consider and address food inequities and injustices.

When our positionality allows us to see different things through the lens of new quantitative literacies

Coal plants have become a contentious political issue. Steeped in quantitative arguments, environmentalists have called for their closure; unions, on the other hand, have resisted the loss of well-paying, union represented jobs that the plant closures would lead to [23]. Both arguments might be sound from a new quantitative literacies perspective, but their resolution would require individuals to put their perspectives, as environmentalists and union laborers, in dialogue with each other in ways that address internal and external forms of power. Every region faces parallel tensions between different interests and goals. Conventional approaches to new quantitative literacies will most likely lead to prioritizing one perspective over the other or at best an effort to address one goal while mitigating the impact on the other. Classrooms as laboratories of democracy could provide a space where "differences in perspectives are examined out in the open to develop greater insight, stimulate constructive disagreement, and spark innovation" [11, p. 148].

A focus on new quantitative literacies, as we illustrate above, risks oversimplifying the historical, social, political, and economic dimensions of highly textured and complex problems in an imperfect democracy. Prioritizing new quantitative literacies in classrooms over the processes of democratic deliberation can effectively undermine democracy. There is most definitely the need for spaces and times when classrooms focus on new quantitative literacies skills and practices. And classrooms need not always be laboratories of democracy. But, if there is a genuine commitment to nurturing students' use of new quantitative literacies toward civic and political engagement, there must be opportunities for students to practice power-with democracy as they adopt, adapt, transform, and come to terms with the limits of new quantitative literacies.

20.6 Conclusions

The good intentions of creating opportunities for students to learn new quantitative literacies with the expectation that they will apply them to civic and political problems that they encounter in everyday life is unrealistic and perhaps even detrimental. It begins with and reproduces the assumption that individuals' quantitative illiteracy is a root cause of their powerlessness. We have shown how conventional approaches that prioritize new quantitative literacies over democratic deliberation are not reflective of the intricacies of the real world and ultimately erode the liberating possibilities of power-with democracy. We have argued for a radically different approach to new quantitative literacies that carves out spaces in classrooms for students to practice power-with democracy. Through a critical reflection on our prior work that attempted to incorporate new quantitative literacies into classrooms, we outlined the potential for classrooms to become laboratories of democracy where new quantitative literacies are part of a collective movement toward transformation and social justice. With transformation and social justice as a priority, new quantitative literacies can support students as they learn to "*change* asymmetrical power relationships (both internal and external), rather than merely struggle for the right to *participate* in them" [11, p. 147]. Such an approach to new quantitative literacies would open new avenues for social transformation, new possibilities for collectivism and mutual reciprocity between participants, and, at the same time, encourage the development of new quantitative literacies as tools that better support this vision.

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21

On Animals, QL Converts, and Transfer

An Interview with Len Vacher

Gizem Karaali
Pomona College

21.1 Introduction

While working on this book, I was delighted by the diversity of topics and contributors this volume would represent. However I also noted that some significant perspectives were missing. In particular, I believed strongly that the editorial team of *Numeracy*, the flagship journal of the National Numeracy Network, and the sole publication outlet exclusively dedicated to scholarship on quantitative literacy (QL), should somehow be a part of this endeavor.

To this end, I proposed to Luke and Victor that I interview Len Vacher, one of the editors of *Numeracy*, and possibly include Dorothy Wallace and Nathan Grawe, the other two editors. I assumed that in editing the journal, all three would be quite overbooked and would not be able to contribute a full paper, even if they wanted to; however, perhaps simply talking away for a little while, they could share some thoughts. This was not a merely benevolent proposal on my part—I was quite keen on having an opportunity to chat with Len, as we had met in person only once in addition to exchanging emails through the years. I genuinely wanted to pick his brain on a bunch of topics, as I knew that he was someone with strong opinions, on QL and many other things. But perhaps more relevant to this volume, I was simply curious to hear his thoughts on certain themes that repeatedly came up in this project. This idea was quite ambitious on my part, as I had never interviewed anyone before, nor (more importantly) transcribed an interview before. Fortunately, Luke, Victor, and Len were all on board.

After some initial discussion, we decided that I would send Len a list of questions ahead of time, and then we would talk on the phone, recording our conversation. With technical support from Vic Ricchezza of the University of South Florida (USF), and the Media Services team from Pomona College, Len and I were able to meet virtually on March 15, 2017. The conversation lasted over two and a half hours—we only stopped because it was past lunch time in Tampa, and I was getting worried that my recording device might fail somehow.

Below is the outcome, edited for clarity, grammar, and length (with Len's substantial input and explicit approval). We hope what remains interests you or even better challenges or provokes you.

21.2 The Interview

Gizem: OK, let's start. As you know, the first question is about you. How did you find your way to *Numeracy*? How did a geologist end up being the editor-in-chief of a scholarly publication in quantitative literacy?

Len: Well it's a very long answer. It evolved over a long period of time. But before getting into that, I just want to say that, to me, mathematics and what I know now to be quantitative literacy (QL) have always been a part of my geology. I've said many times that I really don't understand something in geology until I understand it mathematically. This is because we're dealing with metaphors and models all the time, and models are basically mathematical and logical (and I learned logic through math). That's always been the way that I've thought about mathematics, and I've found that that distinguished me from many of my colleagues, but less so as time has moved on. Now, it has been fifty years since I was in graduate school. I've seen quite a change in my field in some respects (math is increasingly more necessary), but not so much in other respects (still so many geology majors are put off by math, often from bad prior experiences). So it worries me for my field, especially with respect to QL (which is what math becomes, in my opinion, when it enters into geology).

But anyway, I've always had a bias toward understanding things in a mathematical way, although my core interest has always been in geology (since the fourth grade). I've been extremely fortunate in the people that I've met. When I was in college, I went to the University of Washington (UW) as part of their first honors class. And so I took honors mathematics courses. Although I was majoring in geology, I tried to take a mathematics course every semester (thinking it would be "good for me").

One of the semesters, I had Carl Allendoerfer. He was prominent in the MAA (president 1959-1960), especially in relation to mathematics teaching. He was chair of the UW mathematics department, so he promoted strong teaching in mathematics, especially in the honors track. I took two years of honors calculus, which was supposed to be equivalent to three years of "normal" calculus. I didn't understand (or even like) much of it, because it was so heavy on the proofs. But then I took a lot of logic courses over in the philosophy department, leading up to a graduate course, where mathematics really came in handy because everybody else was struggling with the logic proofs which came fairly easily for me—the undergraduate in this graduate course—because I had seen a pile of proofs in mathematics. Even though I really didn't understand the mathematics. But, you know, I didn't particularly want to understand that mathematics (I mean with all those proofs). I don't feel like I began to understand the mathematics until I began to teach hydrogeology, which came much later.

After my time at UW, I went to study geology for graduate work at Northwestern, which was very mathematically oriented, especially in statistical geology. Bill Krumbein, who revolutionized a lot of thinking about what counts as geology and founded the field of statistical geology, was at Northwestern at the time. In my time there, I was on an NSF graduate fellowship, and they all treated me quite gingerly. However, they immediately declared that I was mathematically deficient and that I had to take more mathematics and statistics. This had the effect of making me displeased with statistics ever since, or at least until recently when I learned about statistical literacy from Dick Scheaffer, Milo Schield, and Joel Best through my work with the National Numeracy Network (NNN).

So, at the time, Northwestern was at the center of the founding of low-temperature (sedimentary) geochemistry. This was from 1965 to 1970, and that was right at the beginning of aqueous geochemistry, as well as when geophysics was becoming entrenched in geology departments and ...

Gizem: And geophysics is very mathematical?

Len: Oh, geophysics is heavily mathematical, and geochemistry can be, to the extent it involves physical chemistry. So I took courses in chemical thermodynamics, and that sort of stuff, welcoming it because it was "just" (to me) calculus and graphs. And then what I found in my career ever since is that I can move from field to field within geology fairly easily.

What I've done mostly in my research career is in groundwater hydrogeology and physical hydrogeology, which are mathematical—it's the physics of flow in porous media. I never had a course in hydrogeology (not many existed when I was in grad school), although I ended up teaching it here at USF for over twenty-five years.

The reason I got into hydrogeology is an interesting story on its own. I was doing some stratigraphy work in Bermuda for my dissertation (completely nonmathematical) and I was the only geologist on the island. The Ministry of Public Works and Agriculture realized that they had fresh groundwater underground. They came to me and said: "You are the only geologist on the island; tell us something about it." Well, I told them, I'm interested in just about all aspects of geology except groundwater. I don't know anything about it. Well, they said, you've been mapping our rocks for some time; you can at least interpret what's coming up out of the well. So I agreed to do that, and one thing led to another. I was finishing up my dissertation when that happened, and that interaction with the Bermuda

Government sparked a curiosity about groundwater. Coincidentally, I had an opportunity for an NSF postdoc position at Binghamton University (SUNY–Binghamton back then). I could choose to study anything I wanted to, so I chose to focus on the groundwater of islands. How is it possible that fresh water can occur on some of those carbonate islands? I found some papers in journals of the American Geophysical Union, mathematical in nature, and so I gravitated to the subject. Near the end of the postdoc, I went back to the Bermuda Government and proposed: pay me for two years to come down and work with your engineers, with a budget to contract drillers, and we'll find fresh groundwater. And they did, and we did. I was in my twenties. The project was successful, mainly because I talked to the drillers and, from their experience, realized what was going on—it all involved patterns relating to the mathematics of water tables and their gradients. All that was possible because of my mathematics background (and the experience and savvy of the drillers and engineers). I knew how to proceed as a geologist. I was able to get into the literature, and see and use the mathematics on top of the geology. That basically led to my entire career as a geologist. What I've done, if you look into my geological work, includes geology and sea-level history of Bermuda, as well as groundwater on ocean islands—small carbonate islands at various tropical and semitropical places, and a few in Florida.

So, what does that have to do with QL? In the so-called "30,000-foot view" it's a "forestructure"[1] of my career-long appreciation of the applicability and usefulness of mathematics, specifically because of its utility through transfer between contexts. More "down on the ground," here's the next chapter: I came here to USF after having taught large general-education geology courses at Washington State University (WSU). The time I was teaching at WSU was in the late seventies; I was there for nine years. The reason I came here was my interest in the hydrogeology of carbonate islands (not many of those Washington), to be close to the Bahamas, Bermuda, and so on. So whereas I taught introductory geology and sedimentary rocks at WSU, when I came here, I fell into teaching the first physical hydrogeology course in a series of graduate-level courses. This course, Physical Principles of Groundwater Flow, includes a lot of material using mathematics covered in multivariable calculus. The course was for graduate students, and we had the requirement that, to be admitted to the program, you had to have taken a year of calculus, and so I was quite comfortable thinking the students would be able to handle Darcy's Law, the Continuity Equation, and various other mathematically expressed concepts. But, and here's the point, what I found was that they didn't understand the mathematics that they had taken, regardless of where they came from. There were students from highly selective liberal arts colleges, state universities, our own majors, and so on. They didn't have a clue (with a few exceptions, of course).

So I decided to teach a course that would address that problem, and we called it Mathematical Concepts in Geology. Although the course was more calculus-oriented than it is now, it was exactly what it said in the title; I reviewed mathematics concepts—things that I thought that grad students should be able to handle and have in their tool box. And I found the thing that worked for me was spreadsheets. We even got into finite-difference modeling using a grid on spreadsheets. So every lecture, every session, involved interacting in some way with spreadsheets that they would have to create by themselves. That eventually turned into a series of NSF grants, for about ten years, where the theme was teaching using spreadsheet modules (including Spreadsheets Across the Curriculum [14]).

More to the immediate point of QL: I'd done that (a mathematics concepts course) for five years or so when a couple of undergraduates caught me in the parking lot and asked if they could sit in on the course. I said sure, but it turned out, what they were really asking was to take the course as undergraduates. Now it's a graduate-level course, three times as expensive as undergraduate courses. They weren't interested in paying that, so they asked me to develop an undergraduate version of the course. I told them, I never thought I would see the day that undergraduates would ask for more math, at least not more mathematics in geology. They told me that they had heard about the spreadsheets and they thought that the course on math with spreadsheets would give them a competitive edge for employment. And so I made them a deal, saying I would develop the course if they got a petition that proves there is a demand for it. And they did. What they didn't know was that the chair (Mark Stewart) was a physical hydrogeologist; so, I would be teaching the core mathematics that he would use in his courses.

Gizem: So he was excited?

Len: Oh yeah, he was all for it! So that was the birth of the Computational Geology undergraduate course. (See [10] for more on this course.)

So, now switch to our (geology's) National Association of Geoscience Teachers (then, National Association of Geology Teachers) and its *Journal of Geoscience Education*. The editor, Jim Shea, was of the opinion that, well,

[1] The term is from hermeneutics, the theory of interpretation. For a personal account, see [13].

it's nonsense how introductory geology textbooks were devoid of quantitative content. So, unsurprisingly, his journal included a short column called "On the back of an envelope" by Don Triplehorn with the intent "to promote the use of mathematics in the undergraduate curriculum." Well, Don retired. At that same time, I had cancer, and I was on leave for a long time. I wanted something to do during the recovery period, so I volunteered to take over that column. I renamed it "Computational Geology," and that turned into a series of short papers, each one on a mathematical topic set in a geological context. That lasted for about six years (1998-2005) with a total of about thirty columns. Most of them are available online.[2] About a year or two after the beginning of that column, Project Kaleidoscope (PKAL) had a workshop at William and Mary College on increasing the quantitative skills in geology courses.

Gizem: Just perfect for you!

Len: Yeah! And the organizers, Cathy Manduca and Heather Macdonald (mainstays of NAGT), asked me to give a keynote. In the meantime, through writing that column, I had discovered George Pólya and his book *How to Solve It* [8], and so I gave a talk to the geology professors at the PKAL session on Pólya. It went over extremely well. Jeanne Narum, who was the founder of PKAL, was particularly interested and intrigued by it, as she had never heard of Pólya. She lived in Northfield, Minnesota, which is where Lynn Steen lived.

Gizem: Small world!

Len: Unbeknownst to me, after producing *Mathematics and Democracy: The Case for Quantitative Literacy* [7], Lynn Steen wanted the working group to reassemble and reassess where to go after publication. So Susan Ganter, a member of the design team, was charged with putting a new group together to consider the question: Where do we go from here? Lynn wanted to bring in people from outside of mathematics. I don't know this for sure, but the Jeanne Narum–Lynn Steen connection has to be the reason that I was invited. Susan was the one who invited me. She and I were talking about this linkage a couple of months ago (at the 2017 Joint Mathematics Meetings in Atlanta), and the only thing that makes sense to either of us is that she got my name from Lynn, and Lynn got my name from Jeanne. (Ah, the network!)

So Susan invited me to meet with this group in Philadelphia (July of 2001), and that's when I met Susan, Lynn, and Bernie Madison. Bernie was also new to the group at that time. That's where I met Dorothy Wallace, Wade Ellis, Jerry Johnson, Judy Moran, Janet Ray—many of the *Mathematics and Democracy* authors and the Design Team. Before that meeting, of course, I was given some reading material, which included *Mathematics and Democracy*, and I looked up a book from the National Council of Teachers of Mathematics, a book on mathematics for the twenty-first century [1]. In it, there was a paper by Keith Devlin as well as a paper by Dorothy Wallace.[3] That paper introduced me to Dorothy, which led to my reading of her "talking to the animals" [15]. I think [15] is a paper that everyone interested in QL ought to read, even now.

At the time, Dorothy was well into a big mathematics across the curriculum grant, which I believe was the first of its kind.[4] Math Across the Curriculum is a big movement that began at Dartmouth and has been emulated and built upon in a lot of projects ever since. I remember being on the plane reading Dorothy's papers on the way to Philadelphia, and my takeaway was, excitedly, "Hey! I'm an animal! And they're going to talk to me!"[5] That's how it all started for me—I became a part of the group. And we (meaning, really, Bernie and Lynn) had a forum at the National Academy of Sciences ("Quantitative Literacy: Why numeracy matters for schools and colleges"), as well as series of workshops in the summers. I was part of the traveling band, including Bernie, Caren Diefenderfer, Jerry Johnson, Judy Moran, Rebecca Hartzler, and Dorothy[6] to some extent. We went to various spaces to talk about quantitative literacy and mathematics across the curriculum. And the upshot of the 2001 meeting in Philadelphia was that we founded a network consisting of a consortium of four established programs at Dartmouth, Trinity College (CT), University of Nevada-Reno, and the Washington Center at Evergreen State College (WA), as described in the first issue of *Numeracy*, in a paper by Madison and Steen [6].

[2] See `nagt.org/nagt/jge/columns/compgeo.html`.

[3] These are "The Four Faces of Mathematics" [2] by Keith Devlin and "The Many Roads to Numeracy" [16] by Dorothy Wallace.

[4] See `math.dartmouth.edu/~matc/` for more information.

[5] Any erstwhile field geologist (wannabe or actual) old enough to remember geologists who personally recalled and told of when geology was all field boots, rock hammers, compasses, and burros is apt to consider "animal" as sort of an honorific.

[6] For future reference (in *Numeracy*), let the record show that Bernie, Jerry, Judy, and Dorothy are mathematicians, Rebecca is a physicist, and Len, of course, is a geologist. Caren, who passed away in 2017, was a mathematician.

At the time that those workshops were happening (circa 2001 to 2005), I was also involved in geological research. The groundwater interest here in Florida and prior work in Bermuda got me into the field of karst—cave-ridden limestone—and after a series of things, I ended up on the board of directors (2006-2008) of the new National Caves and Karst Research Institute (NCKRI). When the acting executive director of the NCKRI wanted to visit here in Tampa, she asked to meet with our librarian, Todd Chavez, because Todd was developing a karst portal of information and resources. I was invited to the meeting. Before the meeting, I made an appointment and went over to the library to have a sit-down talk with him. I opened the door to his office and I introduced myself. He said "Oh, I know who you are!" This surprised me. And he said: "You're the guy from the National Numeracy Network." I was thinking to myself: "Well, you must be the only person on campus who knows there is such a thing," and asked, "how did you know?" He said: "I've been investigating." So I tried to explain to him what the National Numeracy Network was. And he said: "Oh, stop. I've been investigating. What you guys need is a journal."

And so we got serious there. We decided that we would grab Dorothy, and she and I, backed by Todd, would make a proposal to the NNN that we would do an online open-access journal. Dorothy and I presented Todd's proposal to the NNN in 2006, and the board of directors of the NNN thought for about five minutes (as I recall it) and said, "Yes, please!"

Gizem: "Yes, please!" That's cool!

Len: So within six months, we had another meeting at Dartmouth, with several potential associate editors, to figure out things as basic as what we're going to call it, how often we would publish, what the various roles might be, and so on. So we sorted all that out. And I believe it was at Dartmouth's expense. Dartmouth through Dorothy's convincing, and her example—explaining and demonstrating the meaning and value of quantitative literacy—has been instrumental in the formation and the maintenance of a lot of good things, involving numeracy (the field), NNN (the organization) and now *Numeracy* (the journal). But anyway, Corri Taylor, an economist and director of Wellesley's Quantitative Reasoning Program, was on the NNN board at the time (and president soon after)—she's the one who came up with the name. We were throwing names up on the blackboard and she went up front and wrote "NUMERACY!" We said that's fine, except for the exclamation point. Then we came up with the subtitle "Advancing Education in Quantitative Literacy." And so then we set a target of a year out (January of 2008).

So it's been a long evolution. Geologist, Mathematical Concepts in Geology, Computational Geology, a series of columns, stumbling as one of Dorothy's animals into what became the NNN, and then becoming caught up in the mission (Todd's vision, really) of open-access publishing with *Numeracy*.

Gizem: In your story, you're talking about how as you went through your geology career, you kept observing how mathematics allowed you to move within fields—it allowed you to understand things. But transferability is very hard to teach, and part of why you needed to develop those courses is because what the USF mathematics department was teaching was not transferring somehow.

Len: Yes, exactly. And I think that's really the crux of the problem. Let's say: I think quantitative literacy does not belong only in a mathematics department.

Gizem: I agree.

Len: Quantitative literacy is too big of a field to leave to the mathematicians to develop. Now, I certainly welcome the fact that many mathematicians identify with quantitative literacy and see the need for it. I really appreciate them. And I don't envy them their task, because I can see that it can be a battle for them.

But on the other hand, quantitative literacy is not going to work unless the rest of the academic world is involved as well. At least an equal amount. One of the things that's very exciting to me about the journal *Numeracy* is—and I've tried to touch on it in my "Grassroots" editorial [12]—articles that are resonating with the readership are commonly written by folks other than mathematicians. To me there's a difference between quantitative literacy and mathematics, which are reflexively conflated by many. For starters, using a spreadsheet as a metaphor, one is a discipline (a row), and the other is a transdisciplinary ability (a column). For another, and this is commonly said by my geology colleagues, one (yours) is the world of idealization and abstractions, and the other (ours) is the real, messy world (dare I say, the world of animals?). To my mind, and very simply put, quantitative literacy is about quantities. To me, a "quantity" is a number with units attached. Once units are attached, it's no longer math; it jumps out of its row and into the column of its context—it's in the land of quantitative literacy. A crass way to look at it is: mathematics is what is taught in mathematics departments, and quantitative literacy is how mathematics transitions over to what's used outside of mathematics departments. (Mathematical literacy is something different, I believe, and I don't feel qualified to talk about it at the moment.)

Gizem: But that then seems to imply that mathematicians can't teach QL, right?

Len: Well, it's difficult. It's difficult because the context is so important. You see, outside of mathematics is where the context experts are. You mathematicians have situated QL to a large extent with problems that you are familiar with as citizens. And this is part of the heritage that comes out of *Mathematics and Democracy*—sort of a legacy thing. There's an association with some people, in some people's minds, of quantitative literacy with social justice, for example. And you know, there's certainly a lot of connections between the contexts in which QL is taught in mathematics departments, and things that people are familiar with, such as financial literacy, voting issues, social justice, and so on.

Gizem: I see. I think what you're saying is: mathematicians have focused on a certain subset of QL, and maybe they can teach QL in the context of things that they as citizens have exposure to, but then we are either ignoring or not able to address QL in other disciplines.

Len: Unless you're working with them! Now there's a lot of work going on in mathematical biology, for example. Dorothy, for example, teaches what could be considered a QL course involving mathematical modeling in biology, taken by biology majors [9]. I think that's what she does. It includes differential equations—nonlinear differential equations in fact—so she has them program. I would consider that, and I think she would too, at least quantitative reasoning. So where I differ with a lot of the QL community is that I see different levels of QL, wherein the level has to do with the level of mathematics. I guess I'm not as sold on the notion that mathematicians can't teach QL (which you tossed to me) as there's a lot of QL being taught outside of mathematics. That's what I wanted to say!

And when we in geology, for example, or in geophysics or geochemistry, use mathematics to teach our subject, that's QL. We're using the mathematics to understand our context.

Gizem: We need allies, right? We need converts in each department to be able to do that.

Len: Converts!

Gizem: Converts. Because you really came to it through your own personal experience with and exposure to mathematics . . . but it also amuses me that your mathematical experiences were the type that do not seem to invite transfer. You took proof-based calculus courses. That really does not invite questions like "How do I use the divergence theorem when it comes up in geology?"

Len: I learned mathematics through trying to teach geology.

Gizem: That's very interesting.

Len: What those proof-based calculus courses did for me while in college was to introduce me to logic. I was actually envious of the students who took regular calculus because they were applying it.

Gizem: They could solve problems!

Len: Yes, and I couldn't. Until I was sitting in a hotel room in Bermuda trying to solve a flow problem, and it worked!

Gizem: Let me now ask you a question about NNN and SIGMAA-QL (or perhaps more broadly, the MAA). These are all organizational structures we created around the concept of numeracy. Do you see them as playing different roles, competing roles, or having some other form of relationship? How can they work together? How can the MAA members, for instance, help support the journal?

Len: Certainly there are different roles, different realities, and different opportunities. Here is my prepared list of ways mathematicians could contribute:

1. join NNN (currently for $30/year),
2. submit papers to *Numeracy*,
3. cite *Numeracy* references.

So for example, if you have a QL-related NSF grant, you can disseminate your findings in *Numeracy*. If you are moved to argue with a paper in *Numeracy*, do write a discussion paper, or if you read a good book, you can write a book review (those tend to get a lot of downloads, by the way).

Now more pointedly, SIGMAA-QL is a group within a mathematics society. You have to be a member of the society to be a member of the SIGMAA-QL. And the SIGMAA-QL is almost exclusively mathematicians. I'm a member of it, and I believe Milo Schield is too. But I believe we non-mathematicians make up a near-zero fraction. In contrast, the NNN is completely independent.

Gizem: It is discipline- and association-independent; it has economists, sociologists, psychologists, geologists, as well as mathematicians.

Len: Yes! Thinking again of the SIGMAA-QL: it has a place to meet. It's a part of the MAA, so with that comes the opportunity to have a couple of annual meetings and build a network. In my experience, however, the QL sessions themselves are not particularly highly regarded by the bigger organization.

Gizem: Sometimes. And sometimes there's so much going on that there is not much attendance, I've found.

Len: Dare I say it: I feel like the SIGMAA-QL—though a fine bunch of dedicated people, and probably very good mathematicians—is just sort of tolerated by the MAA.

Gizem: I wouldn't say that. I think the society likes to have several SIGMAAs, and they don't necessarily value one over the other; there are much bigger ones, which bring much bigger audiences to their talks, and SIGMAA-QL doesn't seem to bring as much. I think they don't really even have a way to favor one SIGMAA over another. But you are right: within the bigger MAA community, it is a very small group of people.

Len: I didn't mean favoring one SIGMAA over another, or maybe I guess I might mean that. I don't know; I just don't think that QL itself is valued as much as it ought to be within the mathematics community. What's valued in mathematics, it seems to me, is mathematics research. And that's where the intellectual effort goes; that's what PhD students are trained to do; that's where the preferred activities go that count toward advancement and so on; and mathematicians interested in QL are just not equal.

Gizem: I guess it's viewed as a service activity, as opposed to research or scholarship. It's perhaps considered a service to the community, to the country, or to students, but it's not necessarily viewed as intellectually stimulating. It's not even intellectually taxing, so it's even easy, some people might think, or it's not as important or challenging as mathematics research. For those of us who're trying to figure these things out, I think it's pretty tough, actually. But I agree with what you say. I think there's a perception that those who can't do mathematics, teach, right? And those who can't teach, teach teachers, that is, think about pedagogy.[7,8]

Len: Yes! That's the SIGMAA-QL, underappreciated, in my opinion, but nevertheless advantaged on account of being attached to a major society. Now in contrast, the NNN is not attached to anything. The big thing that NNN has at the moment is a journal, of course. Its journal is completely paid for by the USF Libraries. It's open access, so there are no revenues coming in. The good news about *Numeracy*, the journal, is the open access—it's available to anyone who wants to read it: here, across the U.S., and around the world. The bad news is that there is no revenue stream.

Back to SIGMAA-QL. It has mathematicians in it, and these mathematicians have the opportunity to meet at the annual meetings. They tend to publish their QL work either in *Numeracy* or in *PRIMUS*. *PRIMUS* charges $500 for yearly subscriptions to libraries, $150 for yearly subscriptions to individuals, or $50 just to "borrow" a paper for a day.

Gizem: Very different models.

Len: Yeah. Back to the NNN. It has very low membership dues, $30, but you don't get anything for it. The big thing is that it provides the journal, and what's more, because we're completely open in publishing—that is, we don't charge any page charges—we're free all the way around, in every sense of the word — except for peer review, of course; that's pretty rigorous (although aimed at improving the manuscripts with the purpose of helping authors produce stronger papers, not in rejecting manuscripts with the purpose of jacking up and touting a metric of rigor).

Gizem: No friction then, but also no revenue.

[7] Apparently George Bernard Shaw actually said: "He who can, does. He who cannot, teaches."

[8] To clarify, all of these are what we have heard from others or read in between their dismissing comments. There are a lot of people who do not value pedagogical work, and who do not value scholarship that focuses on education. GK and LV both want to explicitly state here that we strongly disagree with this perspective.

Len: Yeah no source of revenue. In my opinion, though, NNN has to remain independent. It can't be a subsidiary of a mathematical association, for example, because it's not a mathematics group.

Gizem: How about something like the Association of American Colleges and Universities (AAC&U), for example?

Len: That's exactly the right sort of thing, or maybe a library group. Basically what it comes down to is transdisciplinary education. Who does transdisciplinary education? And what's the purpose of transdisciplinary education? That gets to the issue of transferability. So go back to this editorial that I wrote for *Numeracy* about LEAP (Liberal Education and America's Promise) [11]. It had to do with the AAC&U taking on QL as one of its learning goals, and I set up a spreadsheet. I fundamentally think of disciplines as rows in a spreadsheet (horizontal silos), and things like QL as cross-cutting columns. I said that today, many words ago. It's in that framework that I think every member of SIGMAA-QL belongs in NNN. But the converse, of course, is not true.

Gizem: I agree.

Len: You asked how MAA members can collaborate. Well, some of you do submit some papers, review for *Numeracy*, and serve on the editorial board, but I think all of you should be members of NNN. Even if you don't get anything for it. I think you should think of it in terms of a charitable contribution.

Gizem: I may have let my membership lapse this year. Maybe I will renew my membership after we finish this interview.

Len: Well, I think mine has probably lapsed, too.

Gizem: There you go. You and I should go and renew our NNN memberships!

Now I also asked you as part of the pre-interview preparation: "Are there articles from *Numeracy* that have changed or transformed how you view quantitative literacy? How so?" I like what you wrote: "Yes. They all have. Especially the ones that I have written." And of course, you're right, the peer review process helps you evolve even more. As I edit the *Journal of Humanistic Mathematics*, I am deeply involved with each individual paper, and each of them adds something to the way I think about mathematics. About mathematics for me, and about numeracy for you, right?

Len: Absolutely! I'm especially intrigued now by the international dimension. The more I read about what's going on in QL outside the United States, the more I'm thinking that it might turn out—from the outside looking in—that the emperor has no clothes, with regard to QL in the U.S. That is, it could be that more exciting—more enduring—advances are being made in QL outside the U.S.

Gizem: Well, that doesn't necessarily mean the emperor has no clothes. That might mean that we need a more diverse wardrobe, right? I too was thinking that both SIGMAA-QL and NNN are very much based here in North America. And there are organizations around the world and many researchers too, like in Australia and Israel, for instance, who care about QL. I was thinking about contemporary Israeli research on adult literacy and numeracy, for instance. So there's work going on that we may not be aware of. In this connected world, maybe we should be better connected.

Len: Exactly. I feel the same way. You asked what's been achieved in *Numeracy*. I think one thing is a steady stream of papers from outside the U.S. For example, counting the issue coming out this summer, here is the run of annual numbers of international papers, starting with volume 1 (2008):

$$0, 0, 0, 1, 3, 5, 2, 1, 4, 3.$$

And here's the provenance: South Africa (4 papers, 3 author sets), Australia (3 papers, 3 author sets), England (4 papers, 2 author sets), Canada (2 papers, 2 author sets), Israel, Scotland, Germany, France, Switzerland, Austria.

Gizem: Excellent!

Len: They consistently make me stop and think. We can learn from these people.

Gizem: Yes, maybe more international collaboration and cooperation. And for us to listen a little bit more, right?

Len: Exactly, exactly.

Gizem: We have to learn before we can possibly even contribute. Who knows?

Len: For us animals, it's a whole new, international ecosystem to understand.

Gizem: Yes, let's talk to them, or let them talk to us!

Len, I think *Numeracy* is unique. Even though there might be other organizations and other research groups and other practitioners thinking about these questions around the world, the journal *Numeracy* is unique in that it's the only outlet that I know of that is dedicated to scholarship in numeracy. Is there anyone else?

Len: Well, there isn't that I know of.

Gizem: So you will be flooded soon.

And we have a few more questions. So this was a question that Luke wanted us to talk about. Do you believe there is a core ethos of QL, and if so, do you believe that it's stable?

Len: Well, let me read you what I wrote. I start with "I'm not comfortable with the word 'ethos.' I have core values—rationality and competence—and I associate QL with both of them. And yes, that core is quite stable." But I don't think that's what you, or I guess Luke, wanted. Now that I know the question is coming from Luke, I think I can guess where he's coming from. But anyways, rationality and competence are what I think QL is all about. See, in my Computational Geology course, I've gone over entirely to word problems now. Word problems almost by definition involve higher-order thinking. You actually have to take apart a problem, so you're analyzing, working at the individual parts, putting them together, and synthesizing. It's not in my course to the point of the highest level of Bloom's taxonomy, where it's metacognition, but what I want to have my students do in Computational Geology, through all those word problems, is to operate at steps 4 and 5 in Bloom's Taxonomy. As necessary, they reach down to levels 1, 2, and 3, in their mathematics, and transfer that into the context where they're working at the higher levels. So if they can do that, if they can reach into their calculus, or into their algebra, or their discrete mathematics, if they can do that, then I consider them competent. That's what I'm looking for. That's a large part of operational quantitative literacy to me (the rest concerns communication).

Gizem: That's interesting. So you're thinking that the lower levels are skills that you can retain after successful completion of early undergraduate mathematics courses, possibly, but then in some sense, you're saying that the higher levels have to be addressed in context.

Len: You guys can do the higher-order thinking for your majors, within the mathematics context or the contexts you're familiar with, such as financial literacy or health numeracy, but I don't think there's many mathematicians out there that know enough geology, for example, to be able to do it within a context involving the real earth.

Gizem: Of course. We don't have the expertise. This comes back to the need for converts.

Len: Right. So I believe QL, as a course, ought to be taught in every department, including mathematics. And I would hate for you not to teach students algebra or calculus. We need for you all to do that. Even if it means that we have to follow you and do the extra work of demonstrating to the students why they should have learned it better. They learn it better the second time around, from someone else. You all are running interference for us.

Gizem: Well, I like teaching calculus. I insert some history into it—some human context. But of course my real context is the mathematics itself. So for me there are difficult things and interesting things, and even contextualizing the history gives me a deeper sense of mathematics, and why people did what they did. But then, of course, it is again within the mathematics department. It's not necessarily allowing, or helping even, the student to transfer that to their physics classes, for instance, unless my examples are a little bit related. It's hard. But then those problems are very hard to assign to students, as well, because they don't actually like the "real world application" problems.

Len: Well, not if it is in the mathematics department, because it doesn't have the same texture. In my experience, even reading Pólya's book on trying to apply mathematics to circumstances outside of mathematics, they're so contrived, or they're so out of real context. Where it really works for me in geology is when the students are so engaged in the geology that they pick up their mathematics and they don't know they're doing that. What's nicer still is when they don't have to stop to be taught the mathematics just-in-time, but in my course, I expect to do that (just-in-time teaching) and it works out fine (not many surprises anymore).

Gizem: That's kind of how I learned linear algebra. I didn't learn linear algebra in college. I did learn how to multiply matrices of course, and even how to diagonalize them, but I had no idea what I was doing or why. But when I started learning representation theory in grad school, I thought, oh my goodness, this is so beautiful! And then I had to go back and learn linear algebra to pass an exam, and then I realized that everything I liked in representation theory was linear algebra. So I learned linear algebra through representation theory It is an interesting phenomenon that you can sometimes feed people things without them noticing what's going into their diet.

So we talked quite a lot about some of these questions. We also just started talking a little bit about AAC&U's LEAP initiative and all their learning objectives. There are several literacies they consider in their rubrics: information literacy, digital literacy, media literacy, along with quantitative literacy and science literacy. Maybe these all come back to transdisciplinary education.

Len: Yes. Yes. I don't really know much about those other literacies though. But QL, especially logic and discrete mathematics, is a precursor or fellow traveler with digital literacy, I would think. And QL is necessary for media literacy, I would think. And I suppose I would situate QL as a subset within information literacy, but I'd have to consult Todd on that one. I associate information literacy with information science and information science with the library. I think the AAC&U is possibly a happy home for QL. I think the library, in general, is a happy home for everything that is transdisciplinary. I'm really disturbed by the silos, though I can see why the university needs its silos. On the other hand, it seems to me, especially when the university is thinking about what to do under the heading of promoting student success, it's those sturdy columns—the transdisciplinary columns—that lead to successful alumni, I believe, more so than those rows where one learns more and more about less and less, in my opinion. I guess then as a veteran of major research universities, I might be seeing benefits of a small liberal arts college.

Gizem: This is interesting too. I'm in a small college and we don't have a separate honors college. But I can see how the library works in my context. This may be very context-dependent, based on the institutional context. For instance at Pomona we have an Interdisciplinary Studies Program, which doesn't exist actually—it's just a code that we use for first year seminars. The first year seminars are supposed to center on critical inquiry; they're taught by specialists from across the disciplines. It's in fact pre-disciplinary, before you move into the disciplines, or above disciplines, across disciplines, so the word inter- is not quite right, and trans- is much better. We would never be able to pass one here, in our current climate, but it seems that a QL requirement, as part of a set of general literacies requirements would also belong, in my institutional context, in such a formation, in such a construct.[9]

The library's role does not always fall within education in all institutional contexts. A lot of places just see the library as a research resource. So the way you see the library as a transdisciplinary column won't make sense to some people. Now it does make sense to me, because I also work closely with some of our librarians here, and our librarians are very much interested in instructional support. But I don't know if that's really common. You're at a big research institution, so I'm pleased to hear that your library seems like a welcoming place for it, but then again it was Todd Chavez who was the person who said "You need a journal!"

Len: Yes. He is a unique person, of course. But we've got to find somebody. We've got to find some unit that crosses disciplines.

Gizem: QL is not a disciplinary issue, it is transdisciplinary. I totally agree with that.

So we've talked a bit about *Numeracy* contributors who are mathematicians, and contributors who are not mathematicians. We've talked a bit about mathematicians' blind spots. And we've talked about why we desperately need to have converts from other disciplines. Now I wonder if you have anything you can say about blind spots for non-mathematicians. Have you noticed any of those? Let me see what you wrote for that. You said "no comment on blind spots." Are there any things that you would like non-mathematicians to improve on?

Len: I would like them to recognize when they are doing QL. And to publish about it.

Gizem: So maybe be more aware of the reach of QL?

Len: Well, absolutely. I think one of the big problems is that—I see this in particular in sciences, in engineering—they've all had calculus, so *ipso facto* they all must be quantitatively literate. I think that's fallacious of course. So there are a lot of requirements in universities and colleges, something they call quantitative reasoning requirements, that are satisfied by things like a calculus course; that's a conflation I referred to many words ago in the context of a row vs. a column. I want people—including engineers and scientists—to recognize that reasoning quantitatively and QL are something separate from routine calculus and algebra. In other words scientists, engineers, economists, social scientists, many of them, are actually teaching quantitative reasoning in many of their courses. If they are consciously explaining, contextualizing and using the relevant mathematics, creating and using word problems, doing labs or research projects requiring the student to figure out the math to use (rather than plug into a given formulaic

[9] Claremont Graduate University has a transdisciplinary studies program: `www.cgu.edu/why-cgu/transdisciplinary-studies/`. GK has taught a course on humanistic mathematics there!

equation) and not talking around the mathematics or (falsely) assuming that the students already understand it just because they've had calculus, for example, they are teaching quantitative reasoning.

Gizem: That's kind of my first paper in *Numeracy* [4]. I was assigned the task of teaching this calculus course to students who actually did not want to take calculus, but the course was required, especially by the psychology department, to basically weed people out, because it was the most popular major at the University of California Santa Barbara. The person who had developed the course talked to client departments, figuring out that what they needed was transfer and skills with word problems. Since they were also possibly going to have premed students as well, they needed to have "calculus" on their transcript. So he developed a course that had calculus in its title but it was really about solving problems. And that's kind of how I got into all of this QL business. Because when I was teaching that course, I realized maybe this is what they need to learn—how to solve problems—and the mathematics content is not always the most significant. That was my first foray into QL.

Len: I think what you said underscores one of the big problems we do have in science. We have it in geology. There are a lot of departments that require calculus because it's a weed-out course. Simply because it's a way of establishing whether or not the students are degree-worthy, and then in their subsequent courses it's immaterial.

Gizem: They don't use it.

Len: So why take it?

Gizem: Very good point!

What shall we talk about next? How about: What do you see as the next steps for *Numeracy* and the QL movement? We talked a bit about it, but maybe we might want to focus on it. What has proved to be too difficult to overcome? You wrote here: "Travel to a meeting that's not already on the calendar." I like that one!

Len: Yeah, it gets back to the structure of NNN and SIGMAA-QL. I really don't know what the future of NNN is going to be. I think it has some difficult structural problems, but I think it's necessary that there be some sort of an organization that is not housed in a mathematics department.

Gizem: I agree.

Len: I don't see us as partners so much as fellow travelers. And I think we should be aware of each other's work, and inform each other, but there are some fundamental differences and in a sense barriers that can't be crossed. Barrier is not quite the word. Perhaps I mean boundary (and possibly a permeable one). One of the things I was stumbling around with is the beauty side of mathematics. And I think your position is that that belongs more in mathematical literacy. I'm OK with that. But I was stumbling around because I didn't realize that that was your position, but now I remember that was in that article that you wrote [5].

But I guess the point I want to make now is that I was in this game for about twenty years before I began to realize any of that. So that, what you're talking about, that side of mathematics, is really highly evolved, and yet, many of us who really value mathematics, and use it and want to know more about it, we never reach that level. Or we reach it only in our old age.

Gizem: Well, it's big, right—mathematics is big? The world is big. One needs a little bit more, I guess, perspective...

Len: Yes, but there's a corollary. You have to keep it in mind when you speak of us as converts. We're not mathematicians. We're not going to be converted to be mathematicians. We don't want to be mathematicians. It might be expecting too much of many of us to appreciate the "beauty" side of mathematics, or the many nuances, the way you all wish we would, or to the level that many of you might think necessary to be qualified to speak usefully about mathematics.

OK, so let's get back to Luke's question about the ethos of QL. What I said there.

Gizem: Values, you said, reason—the age of reason, that comes to my mind—OK you said: rationality and competence.

Len: Yeah. OK, next. You might mean purpose. In that case, I would say that I view QL as having a communitarian purpose (in the sense of Amitai Etzioni), and to the extent that it serves individual purposes, that's so that the individuals can better serve the common good. You know, communitarianism, that's where I think QL is.

Gizem: It's very interesting. In our volume we have a chapter (by Craig, Guzmán, and Harper) that argues that the QL literature has been very much steeped within the language of individual success and individualism, and they want to push back on that and bring out a community aspect of QL. And so it's interesting that you raise that point here, that you think for you internally QL is already communitarian. Could you open that up a little bit? That would be very interesting, because in particular we have that chapter that argues that QL literature has been often individualistic. Like individual success, like people will not be fooled by others trying to trick them by numbers, and then you will know who to vote for because you as an individual will know what is in your favor ... In other words, QL is promoted, according to these authors, in such a way that prioritizes individual outcomes (as in: a quantitatively literate individual would not be fooled by others trying to trick them with numbers).

Len: Actually, I would say that that position—not to be fooled by others trying to trick them by numbers—is a part of my communitarian take on QL. That's because to me, communication is a hallmark component of QL. For me, problems often are underpinned by semantic difficulties impeding communication of the kind necessary to build a community and have it prosper. Oftentimes, the miscommunication is not at all intentional; it's just that people are often lousy at it, especially when it comes to quantities (quantitative material). To me, communication is the "L" in QL. Communication is one of the corners of a triangle representing the QL triad. Calculation is another one, and logic is the third. You can see a portrayal of that in the paper [10] about my Computational Geology course. The QL triad is Figure 3 in that paper. One of the associated principles (see that same figure) is "words underperform our thoughts."

That reminds me of something else that I want to make sure to get in. Can I get it in?

Gizem: Sure.

Len: It's about how SIGMAA-QL people can help *Numeracy*. You know I am a member of SIGMAA-QL and I lurk on the listserv, and very rarely drop in. Very rarely. Because I don't want people to be aware of that, because I'm an outsider. But I'm very fascinated by the discussions. And I'm really annoyed that the discussions end there. In the stereotyping that routinely goes on about mathematicians, I think, OK, they do talk a lot.

Gizem: Oh yes we do. Too much talk and no work.

Len: Oh, I wouldn't say that, but how about writing something? You may remember that we had a paper a couple of issues ago by Ander Erickson questioning whether QL is one of those things to be left to the experts. Apparently this paper has created quite a stir in some departments, I've heard. Well, we have our first discussion and reply on this paper.

Gizem: I will need to check up on that, because the people who brought up this idea that QL has been too individualistic have also brought up the idea that, oh, maybe not everybody has to be QL experts. They were also espousing the same idea.

Len: Well, that's what made me think of it. A mathematics person, who is an exceptionally readable writer, wrote a discussion piece, and I accepted it. Just two days ago, I sent it to Ander and said I would like to publish a reply about the same length, two pages, and this would give you an opportunity to expand on your views. And he got back to me and he was thrilled. So we're going to have our first discussion and reply.[10]

So you know, I think I would like to—through you now and this interview—to take this opportunity to charge the people who are discussing issues on the SIGMAA-QL listserv ...

Gizem: To write them up!

Len: And, if it has something to do with a *Numeracy* paper that you're discussing, write a discussion and we'll offer the original author an opportunity to write a reply. We're in our tenth year, and no one's done that until now. And you know, I don't think people realize that they can do that. But it's there in the instructions to authors.

Gizem: I unfortunately should close this out now. Thank you, Len. I'm very honored that you took the time, and you've been pretty open. You were also quite gentle with me about mathematics and my feelings, and I appreciate that.

Len: Hey, I just want you to know that I've liked and admired every mathematician I've met. Working with first NNN, now *Numeracy*, and knowing many in the SIGMAA—that's been a pleasant final phase of my career. Mathematicians have all been nice to me. I especially have liked it when they talk to us animals. Or, shall I say, so long as they let us get a word in edgewise. So thank you for giving me this generous opportunity.

[10] Ander's reply [3] was published following this interview.

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About the Editors

Samuel Luke Tunstall

Luke is a PhD candidate in mathematics education and University Distinguished Fellow at Michigan State University. His research centers on the dynamic relationship between quantitative literacy practices and mathematics teaching and learning. These interests developed over time through teaching courses ranging from developmental mathematics and college algebra to statistics and advanced problem-solving for gifted middle-school students in Duke University's Talent Identification Program. He is currently a board member of the National Numeracy Network and chair-elect of the MAA's Special Interest Group on Quantitative Literacy (SIGMAA-QL).

Gizem Karaali

Gizem Karaali is originally from Istanbul, Turkey, where she received undergraduate degrees in electrical engineering and mathematics. She earned her mathematics PhD at the University of California, Berkeley. Today she is an associate professor of mathematics at Pomona College. Her recent scholarship involves humanistic mathematics, quantitative literacy, and social justice implications of mathematics and mathematics education. Karaali is a founding editor of the *Journal of Humanistic Mathematics* and an associate editor of the *Mathematical Intelligencer*. She also serves as an associate editor of *Numeracy*, the flagship journal of the National Numeracy Network (NNN) and the only academic journal focused solely on scholarship in quantitative literacy. Karaali has served as the secretary and treasurer for SIGMAA-QL from February 2010 to January 2016 (for two consecutive terms) and is currently the chair.

Victor Piercey

Victor is an associate professor of mathematics and the Director of General Education at Ferris State University. He received his PhD in mathematics from the University of Arizona in 2012. He also holds a BA in Humanities from Michigan State University, a law degree from Columbia University, and a MS in Mathematics from Michigan State University. He practiced law in the New York office of Weil, Gotshal, & Manges LLP for two years before returning to Michigan for a career in mathematics. Dr. Piercey is interested in using his legal experience to enhance his mathematics instruction and provide students with transformative experiences. His current project is an inquiry-based sequence of courses entitled Quantitative Reasoning for Professionals. Some sections of this course are linked with sections of freshman writing courses.