Geometry Playground (Version 1.3)
Help on Commands

File: The tools in the File menu are:

- **New:** Erases everything in the current geometry. Shortcut is *Control+N*.
- **Open:** Loads a construction in the current geometry. To open a file on your computer, simply locate and select it. To open a file from the web, type the entire URL, beginning in http://. Shortcut is *Control+O*.
- **Save File:** Saves the current construction. Shortcut is *Control+S*. Saved files can be altered in numerous ways. However, three such alterations are useful enough to bear notice.

  1. One type of point, FIXedPT, cannot be created from the standard interface. It is mostly of use by teachers who wish to create a structure that cannot be moved except by isometry. To create a fixed point, simply change the type of any POINT in the saved file to FIXedPT.
  2. One type of measure, CONSTANT, cannot be created from the standard interface. It is mostly of use by teachers who wish to allow students to compare measures to some prearranged value. To create a CONSTANT, change the type of a POINT in the saved file to CONSTANT, and replace its z-coordinate with the constant's value. You may want to add an alternate string with the constant's name, see (3) below.
  3. Alternate strings for measured quantities can be added by appending a (string without commas or semicolons) after the item in the save file. For example, here is an entry in a saved file:

     ```
     21;RATIO;<0.0;-0.93;0.0>;{17;18};true;false;true
     ```

     The standard string appearing with the ratio will be something like: [whatever(17)means:whatever(18)means]. But perhaps I know that this ratio is the sine of the angle at vertex F0. I could append the string "sin(F0)" as follows:

     ```
     21;RATIO;<0.0;-0.93;0.0>;{17;18};true;false;true;sin(F0)
     ```

     so when I open the file, the string appearing with the ratio is just: sin(F0).

- **Save .jpg:** Saves the current image as a .jpg file. Shortcut is *Control+J*.
- **Undo:** Erases the most recent construction. There is no "redo", so undo cannot be undone. Shortcut is *Control+Z*.
- **Quit:** Ends Geometry Playground. Shortcut is *Control+Q*.

Construct: The tools in the Construct menu construct certain objects. Most objects can be constructed by simply choosing what you want to construct and then selecting pre-created points or "clicking" to create both points and the object desired.

- **Points:** This tool allows the user to construct points by clicking in the geometry. Alternately, the user may click on an object in the geometry, such as a circle or line, in order to construct a point on that object. Shortcut is “. ”, that is, the period (which looks like a point in a sketch).
- **Midpoints:** This tool constructs midpoints. To use it, simply click on two points; their midpoint will be created. Shortcut is *m*.
- **Intersections:** This tool constructs points of intersection. To use it, simply click on two objects, for example, a line and a circle; their point or points of intersection will be created. Note that the points of intersection are created even if they are non-real or otherwise invalid, so that if they later become valid, they will appear. For example, if two circles do not *apparently* intersect, they can be considered to intersect at points with *complex* coordinates. If the circles are later manipulated so that they apparently intersect, their points of intersection will appear. Shortcut is *i*.

- **Lines:** Constructs lines. Simply select two points to construct the line they determine. Shortcut is *l*.
- **Segments:** Constructs segments. In spherical and projective geometries, we have defined a segment as the shorter of the two arcs determined by subdividing the "line" using its determining points. We note that there is a difficulty if the two points are as far from each other as possible. In fact, in spherical geometry, such points do not even determine a line. In Geometry Playground, we avoid these difficulties by not allowing lines or segments to be constructed between such points. Shortcut is *s*.
- **Rays:** Constructs rays, the order of points selected is important. In spherical and projective geometries, rays and lines are synonymous. In toroidal geometry, rays are synonymous with lines so long as the slope is rational...which is always true in Geometry Playground, but not on a true torus. Shortcut is *r*.
- **Perpendiculars:** Constructs the line perpendicular to a line and through a point; the user selects first the line and then the point. Shortcut is *p*.
- **Parallels:** Constructs the line (or lines in hyperbolic) parallel to a line and through a point; the user selects first the line and then the point. Not valid in spherical and projective geometries. Shortcut is *q*.
- **Bisectors:** Constructs the line bisecting an angle. The user chooses the angle by selecting three points. The second of the three represents the vertex of the angle. Shortcut is *b*.

- **Circles:** To construct a circle, select two points. The first point will be the center of the circle, the second will be a point on the circle itself. Shortcut is *c*.

Measure:

The tools in the Measure menu measure certain values. The measurement appears as a black square with an attached approximate value. It can be moved by dragging the black square using the transform tool in the Manipulate menu.

- **Distance:** Measures the distance between two selected points. The measurement is the distance between the points *as measured in the geometry*. Shortcut is *d*.
- **Angle:** Measures the angle determined by three selected points, the second being the vertex of the angle. Shortcut is *a*.
- **Circumference:** Measures the circumference of the selected circle. Shortcut is *Shift+C*.
- **Circle Area:** Measures the area of the selected circle. The measurement is attached to the circle. Shortcut is *Shift+O*.
- **Triangle Area:** Measures the area of the triangle determined by three points selected. In toroidal geometry, some sets of three points do not determine a triangle. In this case, area is given as zero. Shortcut is *Shift+T*.
- **Sum:** Measures the sum of two chosen measures. Note that it is possible for one of the measures to be a sum, so that the sum of multiple measures can be found. Shortcut is +.
- **Difference:** Measures the difference (subtraction) between two chosen measures. Shortcut is -.
- **Product:** Measures the product (multiplication) of two chosen measures. Shortcut is *.
- **Ratio:** Measures the ratio (quotient, division) of two chosen measures. Note that it is possible for one of the measures to be a ratio, so that measuring the ratio of a value to another squared is possible. Shortcut is /.
- **Digits:** This allows the user to set the number of digits of accuracy to display, with a range of 2-8; the number of digits chosen is the shortcut

Some geometries (S,P,T,H) have a natural unit of distance, and hence area as well. For example, in spherical (and projective) geometry, all distances are given in terms of the radius of the sphere, which is assumed to be 1 unit. For others (E,M), the unit is arbitrary.

Manipulate:

The tools in the Manipulate menu allow the user to move things around in the geometry.

- **Transform:** Allows the user to drag a free object, usually a point that was constructed without constraints. All objects constructed using the dragged point are then updated. Alternately, the user may drag the entire space by pressing and dragging where there is no free object. Shortcut is *t*.
- **Fix:** This tool allows the user to fix an object in its position in space. The fixed object will appear in gray. Subsequent transformations will move everything within the constraint that the fixed object does not move. To "unfix" an object, use the tool and click on the screen where there is no fixable object. Shortcut is *f*.
- **Track Point:** Leaves a trace of the selected point following any transformations.

- **Reflect:** The user chooses a line and a point: a new point is created which is the image of the chosen point by reflection in the line. Invalid in toroidal geometry. Shortcut is *!*.
- **Rotate:** The user chooses an angle measure and a point: a new point is created which is the image of the chosen point by rotation in the angle. Invalid in toroidal geometry, valid but not an isometry in Manhattan geometry. Shortcut is *@*.
- **Translate:** The user chooses a distance measure and a point: a new point is created which is the image of the chosen point by translation in the distance (considered as a vector). Valid but equivalent to a rotation in spherical and projective geometries. Shortcut is *#*.
- **Invert:** The user chooses a circle and a point: a new point is created which is the image of the chosen point by inversion in the circle. Inversion is not generally defined in spherical (projective) geometry, however, we have defined it as the composition of stereo- (mono-)graphic projection onto the Euclidean plane, inversion of the image of the point in the image of the circle, and reverse projection. Not valid in toroidal geometry. Shortcut is *$*.

Display:

The tools in the Display menu allow the user to alter how objects appear on the screen in various ways.

- **Label:** This tool allows the user to display or hide an object's label. Labels are assigned in order of construction, so the first object constructed has label A0, the second B0, and so on up to Z0, followed by A1, etc. Labels can be edited by right-clicking on them. Labels cannot be displayed on measures. Shortcut is *Shift+L*.
- **Hide:** Hides selected objects. For example, there are several steps to construct the circumcircle of a given triangle. The **hide** tool can then be used to hide all of the constructions in the intermediate steps, so that only the triangle and its circumcircle appear. Shortcut is *h*.
- **Show All:** Shows hidden objects. To show all hidden objects, the shortcut is *Shift+U*. Otherwise, use the sub-menu to show all objects of a particular type, such as all lines.
- **Model:** Most geometries have more than one model, and in Geometry Playground, the user can choose the model they want to explore. The models, with their shortcuts (w, x, y, or z), appear in the following list.
    - Spherical Geometry:
        - *w* Sphere Model: This is essentially the "globe" model of spherical geometry. The back side of the globe is invisible. This model is accurate everywhere, but only half of the model can be seen at any time. It is also difficult to use, since it cannot be set on a flat surface.
        - *x* Plane Model: Imagine a plane through the equator, and an invisible line connecting the south pole to some other point on the sphere. That line hits the plane at some point, and the map taking points on the sphere to the corresponding points on the plane is called the stereographic projection of the sphere onto the plane. The south pole is said to map to the "point at

infinity" on the plane. This model is fairly accurate near the north pole, but horrible near the south pole.
    - *y* Mercator Model: This is the standard Mercator map that we learn about in geography class. Imagine that the sphere is inside an infinite cylinder so that they touch one another only at the equator of the sphere. An invisible ray beginning at the center of the sphere and hitting some point on the sphere (other than the poles) extends out to hit the cylinder at some point. The map that takes the points on the sphere to points on the cylinder as described is not the Mercator projection, but is similar enough that we will pretend for the moment that they are the same. Now just cut the cylinder vertically and "unroll" it to get a map of the sphere, missing only the poles. The back side of the sphere (model) is visible in the Mercator model, and is shown slightly darker for emphasis. This model is fairly accurate near the equator, but horrible near the poles.
- Projective Geometry:
    - *w* Half-Sphere Model: This model is the unit sphere modulo the equivalence relation identifying antipodal points, i.e., a half sphere with opposite points on the boundary identified. Angles appear correctly in this model, but lines appear curved.
    - *x* Plane Model: Imagine balancing a plane atop the top half of the unit sphere, and drawing a ray from the origin to a point on the half-sphere. So long as that point is not on the boundary, the ray intersects the plane at a point. The map that takes points on the half-sphere to points in the plane gives us the plane model of projective geometry. In this model, lines appear correctly, but angles appear distorted. This model has the advantage of representing what projective geometry looks like to someone living inside projective geometry.
- Euclidean Geometry:
    - *w* Plane Model: This is the well-known Cartesian model of Euclidean geometry.
    - *x* Projective Model: Imagine that the plane is balanced atop the top half of the unit sphere. Then a line segment joining the origin to a point in the plane intersects the top half of the sphere. That map gives us the projective model. Note the connection to projective geometry.
    - *y* Inverted Model: If a Euclidean point has polar coordinates (r,θ), then in this model it appears as $(1/r,\theta)$. The origin doesn't exist, while the "point at infinity" does.
- Toroidal Geometry:
    - *w* Square Model: Back in the 1970's, a video game called "Asteroids" was set on a finite (rectangular) screen, but to give the illusion of an infinite realm, objects exiting the top of the screen returned on the bottom, and objects exiting on the left returned on the right. If we glue the edges of a rectangle together like this, we obtain a torus: the surface of a donut. We have chosen to use the 1x1 square as the fundamental domain of our torus, and to make calculations concerning distance on that square. Note that the

conformal structure of the torus would change considerably if we used a rectangle or hexagon as the fundamental domain.
    - *x* Tiled Plane Model – **New in Version 1.3**
    - *y* Donut Model: In order to glue the edges of the square in the above model, we need to stretch the space. Hence lines and circles appear stretched in this model. Note that we could have done the distance calculations in this model and stretched things to place them into the square model. If so, lines and circles would appear quite differently in both models.
  - Hyperbolic Geometry:
    - *w* Poincaré Model: This is the most famous of the models of hyperbolic geometry. Angles appear correctly in this model, but lines appear distorted.
    - *x* Beltrami-Klein Model: In this model, lines appear correctly, but angles appear distorted. This model has the advantage of representing what hyperbolic geometry looks like to someone living inside hyperbolic geometry.
    - *y* Half-Plane Model: Similar to the Poincaré model, angles appear correctly, but lines appear distorted. This model has the distinct advantage of having a metric that is simple to calculate.
    - *z* Minkowski-Weierstrass Model: This model, sometimes called the Minkowski and sometimes the Weierstrass model, we have opted to call the Minkowski-Weierstrass model. It corresponds to the upper half of the hyperboloid of 2 sheets with the Lorentzian metric. Calculations in this model are simplified, and resemble calculations in spherical or projective geometries with the standard metric replaced with the Lorentzian. We project points in the hyperboloid onto the *xy*-plane by zeroing their *z*-coordinate, allowing us to graph objects on a flat screen.
  - Manhattan Geometry:
    - *w* Plane Model: Essentially the same as the Plane Model of Euclidean Geometry.
  - **New in Version 1.3**: Conical Geometry, with Plane (*w*), Tilted Plane (*x*) and Conical (*y*) Models.
- **Angle:** Angles can be measured in degrees or radians, and this menu allows the user to choose between the two. Shortcuts are *o* for degrees and *SHIFT+R* for radians.
- **Scale:** in the plane model of Spherical, Projective, and Euclidean geometries and the Minkowski-Weierstrass model of Hyperbolic Geometry, the scale can be adjusted up or down by factors of 2, allowing the user to view objects up close or from a distance. The shortcuts are > to scale up, and < to scale down.