

Geometry Playground Activity

Lines: Families of Lines

Tools: Point, Lines, Perpendicular lines, Distance, Move lines or the space.

The queen ant has just read a little book about Descartes and Cartesian coordinates. She now wants to apply her new found knowledge and asks Lieutenant Maps to implement the idea of a coordinate system made of equally spaced north-south lines and equally spaced east-west lines and a numbering system so that locations can be easily identified.

Choose your geometry below. As you work, play around with the different model options and moving things around if needed for increased visibility.

Spherical Geometry – In search of a coordinate grid

Lieutenant Maps begins. Choose a line to be the main east-west line, call it the x-axis. Choose a line to be the main north-south line, call it the y-axis; it needs to be perpendicular to the x-axis.

1. Lieutenant Maps takes a large squadron and spaces them equidistant apart along the x-axis with himself on the y-axis. Assume he has enough to make it all the way around the x-axis. The instructions are to walk a straight line going North at a march tempo they all hear through their helmets. What happens?

Draw the lines of march on the sphere as follows. Place points equally spaced along the x-axis using the moving and distance tools as needed, construct the perpendiculars to the x-axis through those points. Describe what the ants experience. Consider spacing of the marched lines, will the ants know when they've reached North? How should they proceed?

2. How would you suggest Lieutenant Maps number the march lines so that there is no confusion? Move between models and move the sphere to change your point of view to explore this answer.
3. Not to be daunted by the whole equal spacing dilemma, Lieutenant Maps proceeds with creating the East-West lines. He again lines up his squadron this time on the y-axis and equally spaced, with himself on the x-axis. In march tempo they all march East. What happens? Do this on your sphere just as in 1.
4. Is there a labeling for both axes that will allow you to easily pinpoint the locations? Place 4 points in 4 of the 8 different quadrants created by the x and y axes. Using your labeling, label the 4 points. Verify that your labeling really does pinpoint all location uniquely – use more test points if you need to.
5. Conclusions:
 - a. Can you draw equally spaced lines on the sphere? Why or why not?
 - b. Does a family of lines that are all perpendicular to one line have a second common perpendicular? What's your example to show this? Why is your example sufficient?

- c. If five lines are all perpendicular to the same line, what must be true about the intersection points of the five lines? Offer an explanation or proof of your statement.
- d. In Euclidean geometry, parallel lines are also described as being everywhere equidistant. Is there a similar notion in spherical geometry?
- e. On the globe we use latitude and longitude. How would you describe these in terms Lieutenant Maps could use to have his squadron march along either set?
- f. Which is better, Lieutenant Maps' coordinate system as requested by the queen or latitude and longitude? Why?

Hyperbolic Geometry – In search of a coordinate grid

Remember to play around between the different models and move things for a fuller grasp of what's going on.

Lieutenant Maps chooses a main north-south line, call it the y-axis and a main east-west line, call it the x-axis that is perpendicular to the y-axis. Draw yours in.

1. Lieutenant Maps takes a squadron of 10 soldiers and spaces them equidistant apart along the x-axis with himself on the y-axis. He places 5 on his right and 5 on his left. The instructions are to walk a straight line going North at a march tempo they all hear through their helmets. What happens?

Draw in the points along the x-axis and use the move and distance tool to get them roughly equally spaced – at least to the nearest tenth. Then draw the perpendiculars to the x-axis going through those points. Describe what the ants experience.

- a. Consider spacing of the march lines – do they remain equally spaced?
 - b. Is there a North? Why or why not? How do you get there if there is one? If there isn't is there a similar notion that's usable?
 - c. Is it possible to reach the North end of the marching line? Why or why not?
 - d. Are there any differences between marching to the North end versus marching to the South end?
2. Is there a numbering system Lieutenant Maps can use to uniquely identify each line as in when we draw an xy-coordinate system? Do you foresee any problems?
 3. Lieutenant Maps proceeds with creating the East-West lines. He again lines up his squadron of 10 men this time on the y-axis and equally spaced, with himself on the x-axis. In march tempo they all march East. What happens? Do this on your model of hyperbolic space proceeding as in 1.
 4. Is there a labeling for both axes that will allow you to easily pinpoint the locations? Place 4 points, one in each of the four different quadrants created by the x and y axes. Have at least one point out near the boundary in the Poincaré model. Using your labeling, label the 4 points or give a range in between which they can be found.
 - a. How many points do you need to have labeled along the x and y axes to be able to locate the point out near the boundary assuming random placement of the point?

Discuss this in terms of amount of space and in comparison to Euclidean geometry.

- b. Look at the upper half space model. Does your coordinate system seem effective for labeling locations? If so, why do you think that – again consider points very near the boundary. If not, do you have an alternative suggestion?
5. Conclusions:
- a. Can you draw equally spaced lines in hyperbolic space? Describe how or describe why you can't.
 - b. Does a family of lines that are all perpendicular to one line have a second common perpendicular? What's your example to show this? Why is your example sufficient?
 - c. Go to the Poincaré model. Five lines are all perpendicular to the same line. Label them in left to right order along the perpendicular line - what must be true about the sequence of their north end points on the boundary and their south end points on the boundary?
 - d. In Euclidean geometry parallel lines never meet and are everywhere equidistant. Would either of these be effective descriptions of parallel in hyperbolic space? Discuss both ideas. How would you measure distance between two lines in hyperbolic space? Show an example and what the onscreen measurement was. Is your example reproducible with different lines?
 - e. In Euclidean space, two lines that have a common perpendicular are parallel. In hyperbolic space, if two lines have a common perpendicular what conclusions can you draw? Draw an example in any model, study it in all models, drag things around a bit as you experiment.
 - f. Can you draw a rectangle in hyperbolic space? Explain.