

Almost solved!!!



Write Your Own Recipe for Rubik's Cube

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Rubik's cube is back! After years of relative quiet it is once again common to see people twisting away at their cubes. There have also been some exciting new developments, such as complex solving methods, new cube designs, and streamlined ways of manipulating the cube that allow experts to solve the cube in less than 10 seconds. And, on its return, the cube has been accompanied by hundreds of related twisty puzzles demanding to be unscrambled.

You like mathematics, so chances are that at some point you've played around with Rubik's cube. How far did you get? Could you figure out the whole thing? Or

did you just manage to fix one layer, but then were not able to progress any further?

If you got through only one layer, then you're in the good company of millions of others. You may then have gone on to "solve" the cube by following one of the ready-made recipes available in books and on the Internet. But, of course, this really is "solving," not *solving*, and it's not nearly as satisfying as the real thing. Moreover, you've possibly now forgotten almost everything about that recipe, leaving you with little chance of unscrambling the next cube that confronts you.

What I want to do here is to share with you a simple but very powerful trick. Provided you know how to solve the first layer, this trick will allow you to derive

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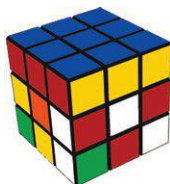
your very own recipes for solving the cube and many other twisty puzzles. Even if you haven't solved that first layer yet, you'll be able to understand how the trick works and will be able to remember and apply it: ideal for the next time you're stranded on a desert island with only a Rubik's cube to keep you occupied.

This probably all sounds too good to be true, but the trick is easy to learn and it really works. It's well-known and used by many genuine twisty-puzzle solvers. And it's not that these experts are attempting to protect a treasured trade secret. It seems more that the trick tends to get lost in the fiddly details of particular solutions.

I'd like to stress that my goal is to give you a good chance of solving it all by yourself, any time you choose and with no help from others. However, actually finding a solution to the cube does not come for free. You'll still need to do some work to put the trick into practice—perhaps about the same amount of work as it took to figure out how that first layer of the cube works.

SOLVING THE FIRST LAYER

To make sure we're all on the same page, let's start by stating the (almost) obvious. By "solving the first layer" I mean twisting the cube until all nine pieces with stickers of one color are arranged exactly as in a solved cube. To repeat, to apply the trick I am about to describe, you have to be comfortable solving the first layer. However, to understand how the trick works in principle, there are no prerequisites.



FLIPPING EDGES

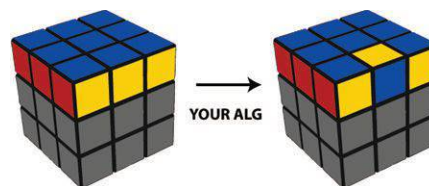
It's no mystery what is tricky about solving a cube beyond the first layer: nearly anything you try next will destroy parts of the layer that you've just painstakingly sorted out. What you'd really like to find are special sequences of twists of the cube that will allow you to manipulate just a few of the cube's pieces while simultaneously leaving the rest of the pieces, including your precious solved layer, undisturbed.

Cube experts refer to useful sequences of twists as *algorithms* (or *algs*). Finding algorithms that move only a few of the cube's pieces is exactly the purpose of the trick we are about to discuss.

Consider the practice cube at right, where all the stickers below the top layer have been removed.



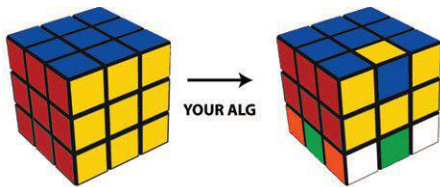
Since you can solve the top layer of the full cube, you already know how to solve a scrambled version of this practice cube. Now, your first challenge is to flip just one of the top edges of the simplified cube: You can use any algorithm you want, but the result is one top edge flipped, with the rest of the top layer unchanged. Many different algorithms will do this, and you would have discovered and employed such algorithms when solving the cube's top layer.



There's no need to go into details here: We'll just assume you've found YOUR ALG, or you can consider it

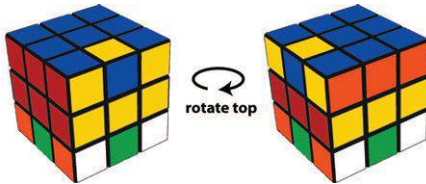
your homework for later.

Let's now consider the effect of YOUR ALG on a full, nonpractice cube. YOUR ALG will leave the top layer unchanged, except for flipping that one edge (the exact purpose of the algorithm), AND it will mess up some of the previously grayed-out lower layers. Of course, the way the lower layers are messed up will depend upon the specifics of YOUR ALG, but that doesn't matter. For MY ALG, the cube will end up as in the picture below.

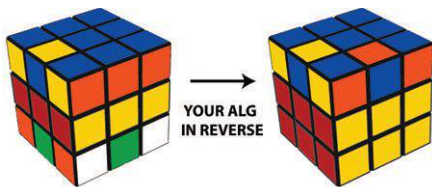


It is also very easy to restore the cube: simply run YOUR ALG IN REVERSE. That is, the effect of the reverse algorithm is to flip that one piece, leave the top layer otherwise unchanged, and restore the lower layers.

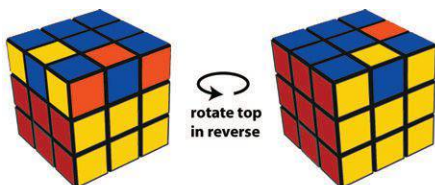
Now here comes the magic. Instead of immediately performing YOUR ALG IN REVERSE, first rotate the top layer a quarter turn:



Only after this quarter turn do you perform YOUR ALG IN REVERSE. What is the effect? Since rotating the top layer has no effect on the lower layers, the reverse move will still restore the lower layers. And, in the top layer, just one more edge gets flipped, with the rest left unchanged.



Then, rotating back the quarter turn, the result will be a solved cube with exactly two flipped edges.



In summary, we find that the combination *your alg - 1/4 twist - your alg in reverse - reverse 1/4 twist* leaves the whole cube unchanged except for flipping two of the top edges.

That's it! I hope you are impressed. However, in case you're not, let's stop to consider what we've just achieved. We've found an extremely simple method that allows anybody with some basic cube skills to design, by themselves, a very powerful algorithm that otherwise would be completely out of their reach, an algorithm that allows them to miraculously manipulate any two adjacent edge pieces in isolation. In fact, this algorithm is all you need to correct the orientations of all the edges in a scrambled cube.

Also, once you've designed your miracle algorithm, it is clear why it does what it does. You really know what you are doing when you use your algorithm, unlike the vast majority of other "solvers," including many champion speed solvers who can do the cube in less than 10 seconds.

CONQUERING THE REST OF THE CUBE

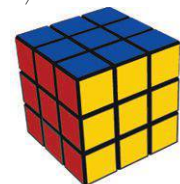
A simple but very powerful trick? You bet! But that's only the beginning. Using the same basic idea, you can produce three more miracle algorithms of the form *your alg - 1/4 twist - your alg in reverse - reverse 1/4 twist*: one allows you to twist corner pieces; another allows you to move edges; and the final miracle algorithm allows you to move corners. Together, your four miracle algorithms (patent pending) are all you need to solve the cube.

Did I promise too much? One simple trick (and a little bit of brain and perseverance) is all you need to conquer the cube. If you're up for the challenge, try to figure out the rest for yourself.

However, if you'd like a few more hints, like to watch some demo movie clips, find out how all this applies to other twisty puzzles, and much more, have a look at the following webpage:

http://www.qedcat.com/rubiks_cube/.

HAPPY PUZZLING!



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