Once upon a time, calculus was the first college-level mathematics course taken by mathematically talented students. The students in first-semester calculus were mathematically motivated, generally well prepared, and they were seeing these ideas for the very first time. This is no longer true. Most of our best-prepared mathematics students arrive in college with credit for at least the first semester of calculus, many of them with credit for both semesters.

Despite steady growth in majors in science and engineering, enrollment in first-semester calculus has been flat or slightly declining at both two- and four-year undergraduate programs. It is the College Board’s Advanced Placement Calculus Program that has been growing steadily at 7–8% per year (see figure 1).

In 2004 over 225,000 high school students took the AP calculus exam. This number is far larger than the number of students who took mainstream first-semester calculus in all four-year undergraduate programs in the Fall of 2000. By the time of the next CBMS survey in 2005–06, we can expect that more students will take an AP Calculus exam than will take mainstream Calculus I in the Fall of 2005 in all 2-year and 4-year institutions combined.

First-semester calculus has become a high school topic for most of our strongest students. This has several implications:

1. We should ensure that students who take calculus in high school are prepared for the further study of mathematics.

2. We should address the particular needs of those students who arrive in college with credit for calculus.

3. We should recognize that the students who take first-semester calculus in college may need more support and be less likely to continue with further mathematics than those of a generation ago.

This article will address the implications for calculus taught in high schools. A second article, “The Changing Face of Calculus: First- and Second-Semester Calculus as College Courses,” will look at the implications for how we teach calculus in colleges and universities.

Recommendations for High School Calculus

The pressure to take calculus in high school is understandable. Competition for admission to the best colleges and universities is fierce. It has helped to create strong growth in AP programs across the board. Many mathematicians deplore this movement of calculus from the college to the high school curriculum, but the pressures are too strong to stop or even substantially slow it. What we can hope is to shape it.

With this in mind, the presidents of the MAA and NCTM issued a joint statement in 1986 [3] with two strong recommendations which I paraphrase here:
1. In spite of the pressures to take calculus while still in high school, students should never short-change their mathematical preparation in subjects such as algebra, geometry, or trigonometry. Solid mathematical preparation is far more important than exposure to calculus.

2. When calculus is taught in high school, it should be a college-level course. This means that the goal of the course should be to give students the same breadth of topics and mastery of calculus obtained by students taking such a course in college. It means that the course should be taught with the expectation that students who perform satisfactorily will be able to place into the succeeding college calculus course.

I believe that these recommendations need to be repeated and re-emphasized. One of the inevitable weaknesses of the AP program is that student enrollment in an AP class appears on the transcript that is reviewed for college admission, but the test that evaluates whether or not the student has learned this material at a college level is not administered until after college acceptances have been sent out. This is why many students enroll in AP courses but do not take the examinations. While schools are under pressure to offer a course that is nominally an AP Calculus course, even if they expect few students will be able to pass the AP exam. These recommendations are intended to back up the teachers who are trying to resist rushing students into calculus before they are properly prepared.

It is particularly important that the calculus taught in high school should be a substantive course that prepares students for further work in mathematics. A weak overview of calculus does little to reinforce student knowledge of algebra, geometry, or trigonometry. In fact, it may encourage the neglect of these subjects in order to get into the calculus course that will improve the appearance of one’s transcript. On the other hand, a solid calculus course should require and help develop a level of mastery of these core subjects that is essential for any further work in mathematics.

Finally, these recommendations recognize that the students who take calculus in high school are among our best students. They must be prepared for college-level mathematics. Once they are ready for and are studying calculus, they should be learning it in a course that is comparable to what they would see in a mainstream college course.

The Responsibilities of Mathematicians

How calculus is taught is important. As I argued in 1992 [2], calculus is not only essential for building mathematical models of the world around us and thus informing disciplines such as physics, economics, and biology, its creation/discovery was the defining moment in the birth of modern mathematics. It has shaped our modern conception of and expectations for mathematics. Calculus should not be the only pillar supporting the undergraduate curriculum in mathematics. Discrete mathematics, geometry, and data analysis have equally important if very different roles to play. But calculus must remain one of those pillars. To ensure that it remains so, mathematicians must be concerned about how it is taught both in colleges and in high schools.

Calculus can be and is being taught well in high schools, but as the number of high school calculus courses expands, so does the number of high school teachers who must teach these courses without much more preparation than the undergraduate course they themselves took, often many years before. At many high schools, only one person teaches calculus, and so peer support may be lacking. The purpose of the AP Calculus examinations is to provide a common standard against which to measure students from all of these classes, but it can only accomplish so much. Ultimately, the way to ensure that what is taught in high school calculus really is a college-level course is through the preparation and support of the teachers who will lead these classes.

The College Board runs many workshops for AP Calculus teachers. NCTM meetings include well-attended sessions that address their needs. The MAA is beginning to realize its own potential in this area. But there still remain far too few university-level mathematicians who are willing to assist in the task of preparing and supporting high school teachers. At the very least, all mathematicians have a responsibility to be aware of the AP Calculus program: its course expectations and the nature of its examinations. Every department should encourage at least one individual to attend the annual AP Reading (the grading of the free response questions), to work with local AP Calculus teachers, or to help prepare and support those who will teach calculus in high school.

Calculus II as a High School Class

The same pressures that are pushing Calculus I into the high school curriculum are doing the same for Calculus II. Traditionally, it was a very elite group of students who took BC Calculus, covering the entire two-semester college syllabus. That group of students also grew by 6-8% per year until the mid-1990s. Over the period 1995–98, the rate of growth of BC calculus accelerated to 10–11% per year, a rate that has held up since then. In 2004, the number of students taking the BC Calculus exam exceeded 50,000. It will likely exceed 60,000 by 2005–06, the year of the next CBM survey.

In 2002, 23% of the students who took BC Calculus did so before their senior year [7]. These high school students are not necessarily well served by taking classes in linear algebra, several variable calculus, or differential equations at a local college. Picking up additional college credits is far less useful for them than deepening and broadening the mathematics they already think they know. These students need to be challenged, but they also need to be prepared for and enticed into a deep study of further mathematics in the company of their peers.

There are many local programs that recognize this. In Minnesota, we have the University of Minnesota Talented Youth Math Program (UMTYMP). At the North Carolina School of Science and Mathematics, the mathematics department is developing courses that return to calculus, using several variables, differential equations, and modeling to
explore its topics in greater depth. But not enough students have access to these kinds of programs. There is a need for a substantial national effort to create materials that can be used with these students and to help teachers learn how to use them.

The movement of calculus into the high schools is not necessarily bad, but it does require the efforts of the mathematical community—individuals, departments, and professional associations—to prepare and support those who will teach it and to resist the pressures that would weaken it.

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Bibliography


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Notes

1. For 1980 and 1985, the CBMS Survey only reports total numbers of students taking all calculus classes in the fall. The numbers of students taking Calculus I were estimated by taking 55% of this total for 4-year institutions and 60% for 2-year institutions (the approximate percentages in the years 1990, 1995, and 2000).

2. The total number of high school students taking calculus each year is unknown. Numbers range from 300,000, the NCES figure for 2000 ([6], table 141), to 500,000 or more. The larger number is based on the College Board estimate that 60% of AP Calculus students take the examination and the observation that many students take a high school calculus course that is not an AP course. This includes students in the International Baccalaureate program and students in joint programs between high schools and community colleges.

3. Daniel Teague, Benjamin Klein, and I are in the process of establishing a SIGMAA for high school teachers that will focus on support for teaching college-level mathematics courses in high schools.

4. The increased rate was almost certainly helped by the fact that an AB subscore was made available for the BC exam beginning in 1998.