

Contents

Introduction	vii
I Classroom-tested Projects	
The Game of “Take Away”	3
Mark MacLean	
Students discover and clearly justify winning strategies for two simple combinatorial games in this project that introduces proof writing. One to two class periods, perhaps at the beginning of the course.	
Pile Splitting Problem: Introducing Strong Induction	7
Bill Marion	
After students conjecture an invariant for a problem, they analyze a strong induction proof and apply the technique to related problems. The project also reinforces the connection between strong induction and recurrence relations. One to two class periods.	
Generalizing Pascal: The Euler Triangles	11
Sandy Norman and Betty Travis	
Students explore an interpretation of binomial coefficients in terms of paths and generalize this to coefficients of trinomial and higher order polynomials, along with number patterns in the associated triangles. Two to three class periods, with project extensions.	
Coloring and Counting Rectangles on the Board	19
Michael A. Jones and Mika Munakata	
In a rich game of using partial information to determine how a covered rectangular board is colored, students use counting, logical reasoning, and geometry. One to three periods.	
Fun and Games with Squares and Planes	31
Maureen T. Carroll and Steven T. Dougherty	
Students explore Latin squares and then come to understand the geometry of affine planes by playing Tic-Tac-Toe on them. Two to three class periods.	
Exploring Recursion with the Josephus Problem: (Or how to play “One Potato, Two Potato” for keeps)	45
Douglas E. Ensley and James E. Hamblin	
Students are introduced to recursion in this exploration of the Josephus problem. This project can be used early in the semester, and the authors outline extensions for returning to this problem when covering induction, binary numbers, and modular arithmetic. One to two class periods.	
Using Trains to Model Recurrence Relations	55
Benjamin Sinwell	
Using manipulatives, students builds “trains” to explore Fibonacci numbers and modify the model to consider several related recurrence relations. One to two class periods.	
Codon Classes	61
Brian Hopkins	
Using various equivalence relations, students explore classes of the 64 codons from genetics and then determine which is the best model for the “standard code” found in nature. One to two class periods.	

How to change coins, M&M's, or chicken nuggets: The linear Diophantine problem of Frobenius	65
Matthias Beck	
Working from the Euclidean algorithm, students explore the problem of what numbers can be represented as a non-negative linear combination of fixed positive integers. Some elementary number theory is required for further questions and potential research projects. Two or more class periods.	
Calculator Activities for a Discrete Mathematics Course	75
Jean M. Horn and Toni T. Robertson	
Students use graphing calculators and computer algebra systems to explore modular arithmetic, the floor and ceiling functions, the growth of functions, and how technology can help with proofs. Less than one class period per worksheet.	
Bulgarian Solitaire	83
Suzanne Dorée	
This project introduces graph theory from an operation on integer partitions which highlights the triangular numbers. One class period, or two half-periods.	
Can you make the geodesic dome?	93
Andrew Felt and Linda Lesniak	
In this follow-up activity to Eulerian cycles and paths, students construct a geodesic dome from rope and plastic pipes and then determine how many edges must be repeated to make a path. Two class periods.	
Exploring Polyhedra and Discovering Euler's Formula	97
Leah Wrenn Berman and Gordon Williams	
This major project includes four activities on polyhedra and the derivation and extensions of $V - E + F = 2$. The extensive notes for the instructor and two appendices serve as a primer for the subject and also address connections to discrete mathematics courses. One or more class periods.	
Further Explorations with the Towers of Hanoi	117
Jon Stadler	
Students use graph theory, Hamiltonian cycles, modular arithmetic, binary and ternary numbers to discover more about the popular Towers of Hanoi puzzle. Each of the four worksheets takes one class period; they may be spaced throughout the class.	
The Two Color Theorem	125
David Hunter	
The Four Color Theorem has an easier analog if countries are determined by infinite straight lines: in this project students prove that two colors suffice, comparing two proof techniques. A follow-up activity introduces topology and knot theory. One class period for the initial activity, more for both.	
Counting Perfect Matchings and Benzenoids	131
Fred J. Rispoli	
The stability of certain hydrocarbons is related to the number of particular subgraphs of the corresponding molecular graphs. The mathematics in this application to chemistry also includes Fibonacci numbers and matrix determinants. Two or more class periods.	
Exploring Data Compression via Binary Trees	143
Mark Daniel Ward	
Students use a particular binary tree as a retrieval structure for binary strings, and go on to study a popular data compression algorithm. Two or three class periods, with possible coding extensions.	
A Problem in Typography	151
Larry E. Thomas	
The typesetting program $\text{T}_{\text{E}}\text{X}$ dynamically chooses a minimal path in a weighted graph to determine line breaks. In this project, students use a simplified version of the Knuth-Pless algorithm starting from graphs derived from $\text{T}_{\text{E}}\text{X}$ output. One or two class periods.	
Graph Complexity	159
Michael Orrison	
After students have been exposed to graph theory, this very open project calls on them to define and explore a notion of graph complexity. Two or more class periods.	

II Historical Projects in Discrete Mathematics and Computer Science

Introduction	165
Janet Barnett, Guram Bezhaniashvili, Hing Leung, Jerry Lodder, David Pengelley, Desh Ranjan	
This brief discussion motivates the use of original sources, explains how these projects can be incorporated into a course, and provides students with suggestions for these modules.	
Binary Arithmetic: From Leibniz to von Neumann	169
Jerry M. Lodder	
This introduction to binary arithmetic draws on Leibniz’ 1703 work inspired by evidence of binary counting in ancient Chinese texts and von Neumann’s 1945 report on the EDVAC, an early computer.	
Arithmetic Backwards from Shannon to the Chinese Abacus	179
Jerry M. Lodder	
The examination of binary arithmetic continues with Shannon’s 1938 article on circuits and concludes with explorations of the abacus.	
Pascal’s Treatise on the Arithmetical Triangle: Mathematical Induction, Combinations, the Binomial Theorem and Fermat’s Theorem	185
David Pengelley	
This project allows students to learn mathematical induction from its first recorded use in Pascal’s 1654 treatise about the famed arithmetic triangle. Subsequent parts explore combinations and extensions into number theory.	
Early Writings on Graph Theory: Euler Circuits and The Königsberg Bridge Problem	197
Janet Heine Barnett	
Euler’s 1736 article serves as an introduction to graph theory. Students work through Euler’s reasoning of this famous problem and compare the modern proofs.	
Counting Triangulations of a Convex Polygon	209
Desh Ranjan	
Students use mathematical exercises and dynamic programming to study Lamé’s 1838 work on what we call Catalan numbers.	
Early Writings on Graph Theory: Hamiltonian Circuits and The Icosian Game	217
Janet Heine Barnett	
Using the pamphlet that accompanied Hamilton’s 1859 game, students explore Hamilton circuits and the associated non-commutative “icosian calculus.”	
Are All Infinities Created Equal?	225
Guram Bezhaniashvili	
Students explore set theory from Cantor’s seminal 1895 and 1897 work, with emphasis on 1-to-1 correspondences and the famous diagonalization argument.	
Early Writings on Graph Theory: Topological Connections	231
Janet Heine Barnett	
Students familiar with graph theory use Veblen’s 1922 work to explore connections to topology. The project includes extensions for those familiar with linear algebra.	
A Study of Logic and Programming via Turing Machines	241
Jerry M. Lodder	
In this four-part project, students explore Turing Machines with the original 1936 article. The first two sections enrich the study of set theory and recursion, respectively, while the remaining two sections lead students through Turing’s main results.	
Church’s Thesis	253
Guram Bezhaniashvili	
More advanced students consider Turing machines in relation to Gödel’s notion of recursive function from the original sources and subsequent work of Kleene.	
Two-Way Deterministic Finite Automata	267
Hing Leung	
More advanced students follow Shepherdson’s construction showing the equivalence of certain finite automata; early questions can be answered working by hand, while later ones require programming.	

III Articles Extending Discrete Mathematics Content

A Rabbi, Three Sums, and Three Problems277

Shai Simonson

Starting with an account of 14th century work by Levi ben Gershon, this article solves three counting problems, working from data to discover the formulas to be proved. This historically informed example of exploration could easily be adapted for classroom use.

Storing Graphs in Computer Memory 287

Larry E. Thomas

This article explores the use of adjacency lists to store nonlinear data (such as matrices and graphs) in computer memory, using the “treasure hunt game” to explain pointers. An introduction to data structures can help students connect the standard discrete mathematics content to computer applications.

Inclusion-Exclusion and the Topology of Partially Ordered Sets 293

Eric Gottlieb

This article explores the connection between the well known principle of inclusion-exclusion and Möbius inversion on certain lattices. Thorough examples of Eulers totient function and the lattice of divisors lead to topology of partially ordered sets.

IV Articles on Discrete Mathematics Pedagogy

Guided Group Discovery in a Discrete Mathematics Course for Mathematics Majors 305

Mary E. Flahive

This article summarizes the group discovery methods and materials developed for introductory combinatorics by the late Kenneth Bogart and their adaptation for larger discrete mathematics courses. An appendix includes sample material on labeled trees and Prüfer codes.

The Use of Logic in Teaching Proof 313

Susanna S. Epp

Drawing on her own experience and the work of many others, the author discusses ideas for helping students learn to construct simple proofs. This inspiring article includes sections on building from students’ knowledge of logic, encouraging their early efforts, and motivating the need for proof.

About the Editor323