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THIS CALENDAR lists meetings of the Society which have been approved by the Council at which papers may be presented. Programs of Annual Meetings appear in the Notices and on the AMS website; programs for sectional meetings appear on the AMS Web pages in the Meetings & Conferences section, and are electronically archived in the Notices section on the AMS website.

MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
976	May 3{ 5, 2002	Montreal, Quebec, Canada	March 12	Vol. 23, No. 3
977	June 12{ 16, 2002	Pisa, Italy	Expired	None
978	June 20{ 22, 2002	Portland, Oregon	Expired	Vol. 23, No. 3
979	October 5{ 6, 2002	Boston, Massachusetts	August 13	Vol. 23, No. 4
980	October 12{ 13, 2002	Madison, Wisconsin	August 20	Vol. 23, No. 4
981	October 26{ 27, 2002	Salt Lake City, Utah	September 4	Vol. 23, No. 4
982	November 9{ 10, 2002	Orlando, Florida	September 17	Vol. 23, No. 4
983	January 15{ 18, 2003 (109th Annual Meeting)	Baltimore, Maryland	October 1	Vol. 24, No. 1

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Abstracts of the 5003rd Meeting.

Invited Addresses .....	2
The Art/ Science of Using Mathematics in Applications .....	3
Innovations in the Teaching of Calculus .....	6
Creative Use of Technology in Teaching Mathematics .....	8
Getting Students to Explore Concepts Through Writing in Mathematics .....	10
Advances in Recreational Mathematics .....	13
E-Learning in Mathematics .....	14
Assessment of Student Learning in Undergraduate Mathematics .....	15
The Special Interest Group of the MAA on Research in Undergraduate Mathematics Education (Research-to-Practice) .....	18
Technology Innovations in Mathematics Education for Elementary and Secondary Teachers .....	19
Innovative Approaches in Quantitative Literacy .....	21
General Contributed Paper Session .....	23
Innovations in Teaching Upper Division Mathematics Courses .....	31
Modeling the Environment .....	33
Mathematics and the Visual Arts .....	34

The presenter of each talk is indicated by an asterisk (\*) in the abstract.

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5003-A0-1 Henri Rene Darmon\*, McGill University. Rational Points on Modular Elliptic Curves: Elliptic Curves.

Elliptic curves arise when considering some of the most classical questions of number theory. Some instances, such as the congruent number problem, date back to the early Greeks, and specific cases of elliptic curve equations were also considered by Fermat in studying his famous "Last Theorem" for exponents 3 and 4. The question of finding an algorithm for solving all elliptic curve equations - intimately connected to the celebrated Birch and Swinnerton-Dyer conjecture - has remained at the forefront of number theory since the time of Fermat. It is now, along with the Riemann hypothesis and the Poincaré conjecture, among the Clay Institute "millennium prize problems". (MAA Earle Raymond Hedrick Lecture: Lecture I) (Received February 27, 2003)

5003-A0-2 Henri Rene Darmon\*, McGill University. Rational Points on Modular Elliptic Curves: Modular Forms.

The work of Wiles, as extended and completed by Breuil, Conrad, Diamond and Taylor, shows that all elliptic curves over  $\mathbb{Q}$  are modular. This result caused a sensation when it was announced because it implies Fermat's Last Theorem. Equally exciting are its numerous applications to the theory of elliptic curves. For instance, when combined with the classical theory of complex multiplication, modularity allows the construction, by analytic means, of a plentiful and well-behaved supply of algebraic solutions to elliptic curve equations. The study of this rich arithmetic structure, initiated by the German mathematician Kurt Heegner and taken up by Bryan Birch in the late 70's and early 80's, is the basis for much of the progress on the Birch and Swinnerton-Dyer conjecture that has been made in the last 20 years, primarily through the seminal works of Gross-Zagier and Kolyvagin. (MAA Earle Raymond Hedrick Lecture: Lecture II) (Received February 27, 2003)

5003-A0-3 Henri Rene Darmon\*, McGill University. Rational Points on Modular Elliptic Curves: Complex Multiplication and Beyond.

The "Heegner points" arising from the theory of complex multiplication give rise to a rich arithmetic structure which forms the basis for the work of Gross-Zagier and Kolyvagin on the Birch and Swinnerton-Dyer conjecture. I will report on some (largely experimental) work undertaken in the last few years in collaboration with Bertolini, Green, and Logan, supporting the belief that the theory of Heegner points is but a special case of an emerging, and for the time being almost entirely conjectural, picture allowing the construction of algebraic points on elliptic curves from periods of associated modular forms. This has led to computer calculations of algebraic points on elliptic curves which, tantalizingly, cannot be justified theoretically. Beneath these calculations should lie a theory whose discovery would shed new light on the Birch and Swinnerton-Dyer conjecture. (MAA Earle Raymond Hedrick Lecture: Lecture III) (Received February 27, 2003)

5003-A0-4 Joan R. Leitzel\*, University of New Hampshire (Retired). Mathematics' Response to Issues in Higher Education.

Some of the most difficult issues that will confront higher education over the next decade and the unique roles that mathematics departments, mathematicians, and mathematics educators will need to play in addressing those issues on behalf of their institutions will be discussed from the perspective of an emerita university president. How mathematics departments can succeed in this environment, some of the areas where our profession has had considerable success over the last 10-15 years, what we have learned along the way, and what requires further serious attention will be highlighted. (MAA James R.C. Leitzel Lecture) (Received February 27, 2003)

5003-A0-5 James H. Curry\*, University of Colorado. Cars, Faces, and Flowers 22.5 Degrees of Separation, What Can Matrix Factorization Tell You?

Matrix factorization has a long history and has led to significant applications. The most recognized factorization method may be the LU decomposition that is a topic in many beginning numerical analysis courses. In this talk I will discuss a recent factorization, Non negative Matrix Factorization (NMF), that may have application to feature extraction. (MAA-NAM David Blackwell Lecture) (Received February 27, 2003)

5003-A0-6 Katherine P. Layton\*, Beverly Hills High School. What I Learned in Forty Years in Beverly Hills 90210.

Reflections by a high school mathematics teacher on the changes in mathematics education over the years 1960-2000. (Received February 27, 2003)

5003-A0-7 Jennifer J. Quinn\*, Occidental College. Proofs that really count. Every proof in this talk reduces to a counting problem { typically enumerated in two different ways. Counting leads to beautiful, often elementary, and very concrete proofs. While not necessarily the simplest approach, it offers another method to gain understanding of mathematical truths. To a combinatorialist, this kind of proof is the only right one. I have selected some favorite identities using Fibonacci numbers, binomial coefficients, Stirling numbers, and more. Hopefully when you encounter identities in the future, the first question to pop into your mind will not be "Why is this true?" but "What does this count?" (MAA Invited Address) (Received February 28, 2003)

5003-A0-8 Donald G. Saari\*, University of California at Irvine. Geometry of Departmental Discussions.

A frustrating aspect of departmental discussions is that no matter how well prepared a proposal may be, almost always a majority will want to "improve it." Why? When described in simple geometric terms the explanation becomes clear while displaying mathematical phenomena ranging from game theoretic concepts, geometric symmetries, to even singularity theory. (MAA Invited Address) (Received February 28, 2003)

5003-A0-9 Tandy J. Warnow\*, University of Texas at Austin. Computational Problems in Inferring Large Evolutionary Trees.

Evolutionary tree reconstruction is a fundamental part of much biological research, with applications in drug design, human migrations, gene function prediction, etc. From a computational perspective, however, inferring large evolutionary trees (on more than about 100 sequences) is enormously difficult: the favored approaches involve attempts to solve NP-hard optimization problems, and standard methods have poor performance under realistic conditions involving large datasets. In this talk, I will present a survey of the basic optimization problems in the area, discuss some current approaches that seem promising for reconstructing large trees, and present open problems. The talk should be accessible to graduate students; no background in biology is required. (MAA Invited Address) (Received February 28, 2003)

5003-A0-10 Robert L. Devaney\*, Boston University. Chaos Games and Fractal Images.

In this lecture we will describe some of the beautiful images that arise from the "Chaos Game." We will show how the simple steps of this game produce, when iterated millions of times, the intricate images known as fractals. We will describe some of the applications of this technique used in data compression as well as in Hollywood. We will also challenge students present to "Beat the Professor" at the chaos game and maybe win his computer. (Pi Mu Epsilon J. Sutherland Frame Lecture) (Received February 28, 2003)

5003-A0-11 Arthur T. Benjamin\*, Harvey Mudd College. The Art of Mental Calculation.

Art Benjamin will demonstrate and explain how to perform rapid mental calculations. (MAA Student Lecture) (Received February 28, 2003)

5003-A1-19 Jennifer M. Switkes\* (jmswitkes@supomona.edu), Mathematics Department, California State Polytechnic University, 3801 W. Temple Ave., Pomona, CA 91768. A Simple Application of Perturbation Theory to Projectile Motion.

The mathematics of perturbation theory provides beautiful insight into the solution of a very well known problem, that of vertical projectile motion under the force of gravity. The material presented here allows undergraduates to experience the power of the usually graduate-level mathematical concepts of non-dimensionalization, rescaling, and perturbation theory. A familiar first model assumes constant acceleration due to gravity. This model and its predictions are then modified to account for the inverse square law weakening of the acceleration due to gravity with increased elevation above the Earth's surface. Beginning differential equations students can follow the techniques and will understand the intuitive implications of the results. (Received May 09, 2003)

5003-A1-23 Majid M. Asso\* (m.asso@mu.edu), 8068 Stonewall Brigade Ct. # 201, Manassas, VA 20109. A Statistical Geometry Approach to the Study of Protein Structure.

This talk begins with an overview of proteins and their amino acid (AA) residue building blocks. Next, Voronoi and Delaunay tessellations are illustrated, as well as a description of how this is performed on a protein structure by using its set of alpha-carbon (Ca) coordinates. A 1-1 correspondence exists between the AA's and Ca's of a protein. Delaunay tessellation of a protein structure yields an aggregate of space-filling tetrahedra. Sets of four AA's in a protein are defined to be nearest neighbors if they form the vertices of one of the tetrahedra obtained

after performing Delaunay tessellation. A counting argument yields 8855 possible quadruplets of AA's, and a log-likelihood score is calculated for each quadruplet indicating the non-random preference for AA quadruplets to be neighbors. Scores are based on data from a training set of protein structures that form a basis for the space of known structures currently deposited in the Protein Data Bank ([www.pdb.org](http://www.pdb.org)). Finally, these scores are used to analyze the enzyme HIV-1 protease and all its possible mutants. By comparing scores associated with protease mutants to experimental results in which mutant enzymes were synthesized and their activity was measured, a correlation between mutant structure and function is elucidated. (Received May 13, 2003)

5003-A1-30 Donald A. Teets\* ([donald.teets@dsmt.edu](mailto:donald.teets@dsmt.edu)), 501 E. St. Joseph St., Rapid City, SD 57701. Predicting Opportunities for Viewing the International Space Station.

Orbital mechanics is one of the great applications of elementary mathematics. This topic can be woven into the standard freshman-sophomore mathematics curriculum in a wide variety of places, from freshman trigonometry to third semester calculus courses. Students find it very exciting to use ideas right out of their textbooks to compute orbit parameters found on NASA web sites. This talk will culminate by describing a recently published UMAP module that guides students through the entire process of predicting when and in what part of the sky to look for the International Space Station as it circles overhead, starting with nothing but initial position and velocity vectors. (Received May 15, 2003)

5003-A1-36 Kenneth R. Driessel\* ([driessel@ath.colostate.edu](mailto:driessel@ath.colostate.edu)), Kenneth R. Driessel, 1703 Palmer Drive, Laramie, WY 82070. On a mathematical model of ski skating.

I shall discuss the dynamics of ski skating. In particular, I shall discuss the following optimization problem: Maximize average speed for a given power. I first develop a mathematical model of ski skating. I limit my attention to steady periodic skiing with no poles on a level plane. I also limit my attention to the physics of the motion (that is, I ignore most biomechanical considerations). In preparation for this lecture, I suggest that you think about the following questions: What is a typical average speed for an intermediate skier? What is a typical power for an amateur athlete? What are the important forces in ski skating? What are typical parameters associated with these forces? You can find a copy of the paper 'On the dynamics of ski skating' at [www.berpipe.net/driessel](http://www.berpipe.net/driessel). (Received May 19, 2003)

5003-A1-38 Crista Coles\* ([ccoles@elon.edu](mailto:ccoles@elon.edu)), Elon University, Campus Box 2230, Elon, NC 27244. If You Accept Them, Will They Come?: Predicting the size of the incoming freshman class.

Each year, as applications for acceptance pour in, admissions offices across the nation wonder how to predict which students, when accepted, will actually choose to attend their college or university. Predicting the size of the incoming class is a difficult problem and can take into account a variety of student characteristics such as a student's high school GPA, SAT score, financial aid package, distance from home, and whether or not the student has come for a campus visit. This modeling project can engage students from a variety of statistical backgrounds. It entails data mining, hypothesis testing, log linear modeling, as well as comparing and validating models. (Received May 19, 2003)

5003-A1-47 Erin L. Landguth\* ([elandguth@ushmore.com](mailto:elandguth@ushmore.com)), 505 37th Street, Rapid City, South Dakota 57702. Space Shuttle Maneuvers and Orbital Transfers.

Each time NASA launches a shuttle to a destination such as the International Space Station or the Hubble Telescope, they publish (on the NASA website) data describing the initial shuttle orbit, the maneuvers (fuel burns), and the resulting new orbit. The problem will then be to describe how NASA's data of the initial shuttle orbit combined with the orbit's corresponding fuel burns produces the new orbit. (Received May 27, 2003)

5003-A1-56 Gary W. De Young\* ([gdeyoung@ingsu.ab.ca](mailto:gdeyoung@ingsu.ab.ca)), The King's University College, 9125 50th Street, Edmonton, Alberta T6B 2H3, Canada. Constrained Optimization with Implicit Differentiation.

The presentation will present an approach to constrained optimization problems that avoids much of the algebraic difficulties associated with the standard method of solving constrained optimization problems. The approach often leads to relations, not immediately available from the standard method, that give deeper insight into solutions.

The approach differs from the standard approach by using implicit differentiation in place of substitution and by finding relations that holds between variables.

In problems with one constraint, the solution can often be visualized by graphing the constraint and the newly found relation. Problems from single and multi-variable Calculus will be used to illustrate the method. The

emphasis will be on finding and understanding equations as relations that give general principles for optimization. (Received May 28, 2003)

5003-A1-57 William L. Briggs\* (wbriggs@at h. cudenver . edu), Mathematics Department, Box 170, University of Colorado at Denver, P.O. Box 173364, Denver, CO 80217-3364. Domino Chain Reactions.

Imagine standing, say, 10 meters from a wall with a bag full of dominoes. As everyone knows, if you build a line of dominoes, all standing on edge, between you and the wall, it will undergo a chain reaction when the first domino is tipped. How should you arrange the dominoes in order to minimize the travel time of the chain reaction (a) when the dominoes have a uniform spacing and (b) when the dominoes may have arbitrary spacing? While these problems may sound frivolous, their solutions involve an instructive combination of mathematical modeling, numerical calculations, and symbolic analysis - and they lead to some surprising results. (Received May 29, 2003)

5003-A1-60 Stacey Levine\* (sel @at hcs. duq. edu), 440 College Hall, Dept. of Mathematics and Computer Science, Duquesne University, Pittsburgh, PA 15282, and Eric Rawdon (rawdon@at hcs. duq. edu), 440 College Hall, Dept. of Mathematics and Computer Science, Duquesne University, Pittsburgh, PA 15282. A module-based course on current mathematics for science majors.

In fall 2002, the mathematics and science faculty at Duquesne University developed a new math course required of all for science majors. The purpose of the course is to discuss current problems in the sciences and then explore the mathematics behind them. In this talk, we discuss two of the modules: 'The Mathematics of Image Processing' and 'Knot Theory'. (Received May 30, 2003)

5003-A1-70 Charlotte A. Knotts-Zides\* (knot t szi deca@of f or d. edu), Wo@ord College, 429 North Church Street, Spartanburg, SC 29303. Launching a Single-stage Rocket into Space.

In this presentation, we will discuss a project which models the problem of launching a single-stage rocket into space. This project takes into account the combustion of fuel during flight and uses a more general formulation of Newton's second law to determine the first-order differential equation which governs the rocket's behavior. Students are asked to find the solution of the resulting initial value problem as well as calculate the velocity of the rocket at burnout. The students also determine the minimum value of the velocity which allows the rocket to escape the earth's gravitational field and, using a specific example of a single-stage rocket, we show that this rocket cannot attain the necessary escape velocity. This project can be used in a differential equations course or a math modeling course. The students get an opportunity to apply separable techniques, the more general form of the chain rule, and the Euler's method to approximate the displacement of the rocket, which makes this project a great review of many of the concepts that the students learn separately while discussing techniques for solving first-order differential equations. (Received June 02, 2003)

5003-A1-115 Florence S. Gordon\*, Mathematics Department, New York Institute of Technology, Old Westbury, NY 11568. The Power of Power Functions: Mathematical Applications in Biology.

Traditional college algebra courses focus almost exclusively on power functions such as  $y = x^2$  and  $y = x^3$  rather than the more general  $y = x^p$ : However, it is the more general form that is the basis of the mathematical models that arise throughout the natural sciences in a host of unexpected, but highly interesting, applications. This presentation will demonstrate a variety of applications drawn from biology that lead to power functions and some of the kinds of questions that can truly motivate students to find value in the mathematics they are learning. For instance: Why can't a turkey fly? How much did a pterodactyl weigh? How many different species can the island of Puerto Rico support? How large does an island have to be to have 100 species inhabit it? How does the size of an organism relate to how fast it can run? Or swim? Or fly? (Received June 04, 2003)

5003-A1-120 Laurie J. Heyer\* (l ahey e r @a vi dson. edu), Math Department, PO Box 6959, Davidson, NC 28036. Dynamic Programming and DNA.

Biological macromolecules such as DNA and proteins can be represented by sequences of single letters. In this talk I will describe a well-known method for comparing biological sequences and determining the optimal alignment of the two sequences. Alignment refers to the process of placing one sequence under the other, and accepting mismatches, or placing gaps in the sequences at just the right locations, with the objective of matching a large number of letters in one sequence with letters in the other sequence (relative to the number of mismatches and gaps used). By assigning positive scores to matches, and negative scores to mismatches and gaps, sequence alignment is formulated as a maximization problem. The optimal alignment of two sequences can be found exactly

(in quadratic time) using Dynamic Programming. Dynamic Programming has many applications in addition to sequence alignment, and is appropriate in courses such as Mathematical Modeling, Scientific Computing, Operations Research, and Optimization. (Received June 04, 2003)

5003-B1-15 Michael R. Huber\* (Michael.Huber@usma.edu), Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996, David A. Smith (David.A.Smith@usma.edu), Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996, and Brian J. Lunday (Brian.Lunday@usma.edu), Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996. More than semantics: a course in Modeling using Calculus instead of a Calculus Course focused on Modeling.

Every calculus text claims to have real-world problems and applications. By introducing realistic, applied problems at the start of each block of material, the faculty motivates students to solve real problems that are not trivial. This presentation will outline our success in having a group problem-solving lab to introduce the types of problems before the students have been taught the material, followed by another problem-solving lab at the end of the block and an assessment with a Modeling and Inquiry Problem (MIP). Students model and solve problems every day, in the areas of related rates, optimization, accumulation, harmonic oscillation, with applications to physics, engineering, and economics. The calculus is introduced as a vehicle to set up the models, and Mathematica is the computer algebra system to solve the models. (Received May 09, 2003)

5003-B1-16 P. Gavin LaRose\* (glarose@umich.edu), Department of Mathematics, University of Michigan, East Hall, 525 E. University, Ann Arbor, MI 48109-1109. An On-line Tutorial Supporting Skill Acquisition in Calculus I and II.

On-line gateway testing provides a means of assuring that students in reformed precalculus and calculus courses acquire the algebraic and computational skills needed in courses following these. Having the tests on-line allows students to easily practice them from their dorm rooms or computer labs before going to a proctored testing environment to take the test for a grade. However, in this case we need a mechanism for students to get feedback on the correct solution to problems that they miss, so that they do not simply repeat the same errors multiple times. This instructional feedback role is filled by an on-line tutorial system. The tutorial system is entered when a student completes a gateway test and has problems he or she has gotten wrong. Each of the missed problems includes a short explanation of the rule required to correctly obtain the answer and links to the tutorial system's "explanation engine," which generates a page showing the student the correct method for solving the problem. Depending on the gateway test topic, this page may link to further explanation pages showing how to complete sub-parts of the problem. In this paper we report on the details of this tutorial system and our assessment of its effectiveness. (Received May 09, 2003)

5003-B1-18 Jerry F. Dwyer\* (dwyer@ath.utk.edu), Department of Mathematics, University of Tennessee, Knoxville, TN 37996, and Katherine E. Hitchcox (kathy2h@ahoo.com), Department of Mathematics, University of Tennessee, Knoxville, TN 37996. Models for two dimensional surfaces and gradients.

This presentation describes a method of teaching the notions of two-dimensional calculus using actual physical surfaces constructed from playdoh or similar material. This was part of a new course on science models in the K-12 classroom offered by the authors at the University of Tennessee. The course is offered as an elective to students taking a master of mathematics for teachers degree. Most of the participants are K-12 teachers, with a focus on middle school math and science teachers. These students have had limited exposure to calculus and none of them had any previous two-dimensional calculus. This new understanding of theory will enable these teachers to develop lesson plans for their own middle school classrooms involving contour lines on topographic maps. The presentations begin with an explanation of the concepts of surfaces and their equations. Students are then asked to construct these surfaces from the playdoh. The interactions of  $x$  and  $y$  coordinates proved to be troublesome and most students failed to construct a surface for at the first attempt. The second stage is the visual presentation of paths at constant height on the surface. These are marked in a different color of play-doh. It is then demonstrated that the projection of these paths down onto the  $z$ -plane represents the contour lines. The partial derivatives in the  $x$ - and  $y$ - directions respectively are displayed by laying toothpicks parallel to the  $x$ - and  $y$ - axes and tangential to the surface. The lengths of the toothpicks are adjusted to match the magnitude of the partial derivatives in these two directions. The gradient vector is constructed from these

two components. These students have had no prior experience with vectors and we called on the experience of an earlier class on x- and y- components of velocity to illustrate the ideas. Finally the gradient vector is projected onto two perpendicular unit vectors to produce a directional derivative. Since these teachers had no prior understanding of surfaces we felt it was critical to give them visualization through the models. Abstract concepts became familiar when examined in the context of 'hills and valleys'. Homework did not involve solving a bunch of computational problems - rather, the students had to create physical surfaces and construct gradient vectors at various points on the surface - truly a hands-on calculus class. Students have definitely learned from this approach. It is difficult to teach these concepts to people that have had little experience of one-dimensional calculus. It is clear that the increased visualization has contributed to their understanding. It is unlikely that the subject would have been grasped at all without the models. In our presentation we will construct models of surfaces and corresponding gradient vectors and point out some interesting concepts that are difficult to grasp in an abstract manner but are clear from the models. (Received May 09, 2003)

5003-B1-24 Rebecca E. Hill\* (rehsma@iitvax.rit.edu), Department of Mathematics and Statistics, Rochester Institute of Technology, 85 Lomb Memorial Drive, Rochester, NY 14623. MSG for Multivariable Calculus.

I deliver the message (M), the students practice it in group learning exercises (G) and then it is further enhanced with laboratory projects using a symbolic algebra system (S). This summarizes my experimentation with our multivariable calculus during this past term. This course is the fourth course in a traditional calculus sequence taken by all engineering, chemistry, physics, biomedical computing, bioinformatics, imaging science, mathematics, statistics, computer science, and economics majors. The workshop calculus project that our department began two years ago with the first year calculus sequence does not extend to multivariable calculus. RIT's quarter system does not allow much time for exploring new ways to deliver a course, but the group work and the Mathematica labs have been well received by the students and I feel that they are partly responsible for the extremely low failure rate that I have experienced this term. (Received May 13, 2003)

5003-B1-41 Charles Redmond\* (chad@mercyrhurst.edu), Department of Mathematics, Mercyhurst College, 501 E. 38th St., Erie, PA 16546. Circuit Tournaments, Calculus I, and the NFL.

I will discuss how the mathematics behind the ranking of sports teams leads to a fun and unusual interpretation of Calculus I basics such as the Mean Value Theorem, the Mean Value Theorem for Integrals, and the Fundamental Theorem of Calculus. (Received May 20, 2003)

5003-B1-61 Maria Shea Terrell\* (mst1@cornell.edu), Mathematics Department, Malott Hall, Cornell University, Ithaca, NY 14853. Good Questions at Cornell University.

The GoodQuestions project seeks to improve mathematics instruction by adapting two methods developed in physics instruction, ConcepTests (<http://galileo.harvard.edu/>) and Just-in-Time-Teaching (JiTT) (<http://webphysics.iupui.edu/jitt/jitt.html>).

The essence of both approaches is good questions. JiTT questions are web based pre-class warm up questions. ConcepTests are in-class discussion questions. Both types of questions help instructors monitor student understanding through frequent formative assessments of what students are learning, and both provide instructors with tools to experiment with new ways to spend time in class.

The questions the project will address are: Will calculus instructors, many of whom are advanced graduate students and postdocs, use either or both of these approaches as they teach their classes? Will students demonstrate enhanced conceptual and computational ability if they spend more class time discussing key concepts with their instructor and their peers, and less time on specific problem solving?

Support for the Good Questions at Cornell project is provided by the National Science Foundation's Course, Curriculum, and Laboratory Improvement Program (grant DUE-0231154). (Received May 30, 2003)

5003-B1-71 Tevian Dray\* (teviand@math.orst.edu), Department of Mathematics, Oregon State University, Corvallis, OR 97331, and Corinne A. Manogue (corinne@physics.orst.edu), Department of Physics, Oregon State University, Corvallis, OR 97331. Bridging the Vector Calculus Gap.

For the last several years, we have led an NSF-supported effort to "bridge the vector calculus gap" between mathematics and physics. This project originated in our own failure to communicate with each other, despite years of collaboration in mathematical physics. Somewhat to our surprise, we have come to the conclusion that mathematicians tend to teach calculus algebraically, whereas physicists tend to use calculus geometrically. (Received June 03, 2003)



5003-B1-74 Sarah L. M abrouk\* (smabrouk@rc.mass.edu), Mathematics Department, Framingham State College, 100 State Street, P.O. Box 9101, Framingham, MA 01701-9101. A Laptop Enhanced Calculus III Course.

Through Framingham State College's wireless laptop program, I have had the opportunity to use wireless laptops in sections, eight in all, of 43.095 General Mathematics, 43.123 College Algebra, 43.221 Calculus III, and 43.222 Differential Equations. Using the laptops as a tool for teaching and learning provides many opportunities and advantages including email communication for additional instruction, access to information, course materials, and mathematics posted on the WWW, access to mathematics software such as Maple, and access to interactive tools that I have created using MS Excel. In this presentation I will share some of my experiences using laptops as a teaching tool in 43.221 Calculus III for the past two years as well as some of my plans for using the laptops during Fall 2003. I will share my experiences in teaching students to use Maple, demonstrate some of the Maple worksheets that I have created, demonstrate some of the interactive MS Excel tools that I have used for in-class demonstrations, discuss some of the ways in which I have used the laptops in course assignments and projects, and share some of the "adventures", surprises, and challenges that arise when using technology during class. (Received June 03, 2003)

5003-B1-148 James S Rolf\* (jimrolf@saf.a.edu), Department of Mathematical Sciences, Fairchild Hall, USAF, CO 80840, and Michael A Brillleslyper (mike.brillleslyper@saf.a.edu), Department of Mathematical Sciences, Fairchild Hall, USAF, CO 80840. Laptops and Calculus: Do they mix?

Beginning in the Fall of 2002, all students at the Air Force Academy were issued laptop computers. We will give an overview of how the incorporation of laptops along with Java applets, Excel, and Mathematica has affected the teaching of Calculus 1 and 2. In particular we will examine the impact these have had on the content of the course, the day-to-day classroom environment, and the varied forms of assessment utilized. Early comparison data will be presented. (Received June 05, 2003)

5003-C1-20 Douglas E. Ensley\* (deensl@hip.edu), Department of Mathematics, Shippensburg University, Shippensburg, PA 17257. Flash applications for teaching mathematical proof. Preliminary report.

This talk is a preliminary report on the NSF-DUE grant, "Development of Interactive Material for the Constructivist Teaching of Mathematical Proof to Future Teachers." Phase I of this project is the development of Flash movies that engage students in the interactive nature of mathematical proof. Phase II of the project will involve assessment of student learning objectives and refinement of the material through beta testing. Through the use of computer software in this project, we emphasize the roles of both the proof's author and the proof's reader in the proof process. By stressing this process, we believe students better understand the point-of-view from which proofs are written, and consequently are able to "find their voices" as proof authors. The specific audience for this material is the group of freshmen in our discrete mathematics class, many of whom are pursuing secondary mathematics teaching certification but most of whom are computer science majors. We also have a significant number of elementary education students minoring in mathematics in this course. (Received May 12, 2003)

5003-C1-25 Catherine A. Gorini\* (cgorini@um.edu), Maharishi University of Management, Fairfeld, IA 52557-1066. Using JavaSketchpad Applets in Teaching College Geometry.

This talk will describe computer labs for a college geometry course. Each lab consists of a web page with one or more JavaSketchpad applets together with questions for students. The labs require only a web browser and little or no experience with computers on the part of the students. Examples of labs for Ceva's theorem, circle geometry, and symmetry transformations will be given. (Received May 14, 2003)

5003-C1-32 Andrew S. Leahy\* (aleahy@nox.edu), Department of Mathematics, Knox College, 2 East South Street, Galesburg, IL 61401, and Dennis M. Schneider (dschneid@nox.edu), Department of Mathematics, Knox College, 2 East South Street, Galesburg, IL 61401. Turning Your Computing Lab into a Supercomputer.

This presentation will describe two mathematics courses developed as a part of an NSF-DUE CCLI project which are together designed to take advantage of the parallel computing capabilities of a distributed computing cluster. The first course is designed to introduce students to numerical mathematics as early in the undergraduate mathematics curriculum as possible. It is a replacement for a second semester calculus course and emphasizes

programming in Mathematica and numerical techniques for solving calculus-level problems. The second course is a replacement for a traditional numerical analysis course which emphasizes parallel algorithms, their application to numerical linear algebra, and numerical solutions to partial differential equations. Links to course material developed as a part of this project will be presented. We will also describe the distributed computing cluster we have developed and the other ways it has been used in our mathematics curriculum. (Received May 15, 2003)

5003-C1-33 Valerie L. Watts\* (watts@math.arizona.edu), Department of Mathematics, University of Arizona, Tucson, AZ, 85721. EXCElating Mathematics.

Technology, and in particular simulations, can be used as an effective tool to demonstrate abstract mathematical concepts that students find difficult to understand. Spreadsheets can handle large simulations easily and efficiently. We will use simulations in Excel to illustrate the following three concepts: the convergence of a power series at a point, the probability distribution of a large sample, and the expected value of a random variable. We have used these simulations, and others, in the classroom, and we have found them to be useful in explaining these mathematical concepts. Participants, who are interested in using these simulations in their classroom, will have the opportunity to leave the talk with a classroom-ready set of simulations. (Received May 16, 2003)

5003-C1-46 Mihaela Teodora Matache\* (matache@math.unomaha.edu), University of Nebraska at Omaha, Mathematics, DSC 203, Omaha, NE 68182, and Janice Rech (jrech@math.unomaha.edu), University of Nebraska at Omaha, Mathematics, DSC 203, Omaha, NE 68182. On-line Testing of Intermediate Algebra and Calculus.

To enhance the teaching effectiveness of lower level courses, such as Intermediate Algebra and Calculus, the Mathematics Department at the University of Nebraska at Omaha has attempted to offer the students an opportunity to be successful through a special trial algebra program called "Pathways to Success", and internet testing of basic algebra and calculus skills. The online tests consist each of 10 questions randomly selected from testbanks with 10 different topics and approximately 18-20 problems in each topic. The students are allowed to take an unlimited number of practice tests before taking the tests under the supervision of a proctor. The system provides immediate feedback on each test, so the students may find where their difficulties lie and address them prior to taking the tests for credit. We provide details on the requirements of the algebra program, as well as the online test implementation, including the test design, administration, grading policy, and the assessment of the effectiveness of the system based on student evaluations collected at the end of semester, containing students' opinion on the tests, as well as the online system. We also report on future plans to create similar tests for courses such as differential equations, and probability and statistics. (Received May 23, 2003)

5003-C1-52 Revathi Narasimhan\* (rnarasim@kean.edu), Dept. of Mathematics, Kean University, Union, NJ 07083. Creating interactive spreadsheet modules for precalculus and beyond.

The spreadsheet software is widely available and easy to learn, but is an under-utilized resource for teaching mathematics.

In this presentation, we will illustrate how powerful a spreadsheet can be for creating interactive learning materials. Topics will include graphing, exploring rates of change, exploring the Fundamental Theorem of Calculus, applications of calculus, and probability simulations. We will particularly focus on how the nature of the spreadsheet lends itself easily to demonstrating the symbolic, numerical and graphical representation of a function.

All examples given in the presentation will be available from the presenter's web site. Participants can use these examples as a base to create their own interactive materials. Web site: [www.collegemath.info](http://www.collegemath.info). (Received May 28, 2003)

5003-C1-73 Sarah L. Mabrourk\* (smabrourk@rc.mass.edu), Mathematics Department, Framingham State College, 100 State Street, P.O. Box 9101, Framingham, MA 01701-9101. Tools For Exploring Graphs Of Functions.

Many students taking courses such as General Mathematics, College Algebra, and Precalculus have difficulty graphing functions and understanding the connection among the function equation, the function coefficients, and the graph of the function. Graphing functions by hand does not allow the student to compare a variety of functions simultaneously and graphing calculators, while helpful for exploration, do not allow students to examine the change in the function coefficients on a "sliding scale", that is, the immediate changes in the graph produced by changing the function coefficients from one value to another. To meet this need and to provide a means for exploration and comparison of a variety of functions, I created a suite of interactive tools using MS Excel that can be used during class for demonstrations and discussions as well as outside of class for further exploration and student assignments. In this presentation, I will demonstrate this suite of interactive graphing

tools and discuss how I have used these tools to help students to explore lines as well as polynomial, exponential, and logarithmic functions. I will discuss student reaction to the use of these tools as well as some "adventures" involved in having students use technology in class and outside of class. (Received June 03, 2003)

5003-C1-147 Janet M McShane\* (Janet.McShane@au.edu), Box 5717, Department of Mathematics and Statistics, Flagsta@, AZ 86011-5717, and Michael I. Ratliff (Michael.Ratliff@au.edu), Box 5717, Department of Mathematics and Statistics, Flagsta@, AZ 86011-5717. Technology Illustrations of Five Solutions to the Problem of Apollonius.

Apollonius' (ca. 260-170 B.C.) provided us with one of the most famous classical construction problems. This problem, known as the Problem of Apollonius, calls for constructing a circle tangent to three given circles. Several mathematicians were attracted to this problem including Vieta, Newton, and Descartes. For instance, Vieta (1540-1603) proposed the problem to Romanus, who solved it by finding that the center of the tangent circle is the intersection of two parabolas. Vieta instead solved it very elegantly using ordinary geometry, while Newton reduced the intersection of the two loci, which Romanus had constructed, to the intersection of two straight lines. Descartes on his part approached the problem algebraically and Gow (1968) comments that "of the two solutions which he found, he admits that one gave an expression so complicated that he would not undertake to construct it in a month".

Several solutions to the Problem of Apollonius can be investigated and illustrated with the aid of technology, which allows us to easily make constructions, and to manipulate symbolic expressions. We discuss five methods of solution to this famous problem, four of which use dynamic software and one that uses analytic geometry. Each method initially discusses the problem when three (non-overlapping) circles are given, but reference is made to cases having overlapping circles, points, and lines. All the methods involve the use of technology; we will use Mathematica and Geometer's Sketchpad to solve the problem. (Received June 05, 2003)

5003-C1-159 Brian Hopkins\* (hopkins\_b@pc.edu), Department of Mathematics, 2641 Kennedy Blvd., Jersey City NJ 07306. Internet Tool for Modern Algebra.

A hands-on, discovery approach to modern algebra is hampered by the work of computing multiplication tables for groups. Groups32, maintained by John Wavrik at UCSD, is a freely accessible program that includes these tables for all groups up to order 32, and much more information. The talk will include a brief summary of what the program can do, but will focus on how it was used in a spring 2003 course. Other internet tools will be mentioned, including applets for visualizing symmetries. (Received June 05, 2003)

5003-D1-130 Janet Heine Barnett\* (janet.barnett@colostate-pueblo.edu), Department of Mathematics and Physics, Colorado State University - Pueblo, 2200 Bonforte Boulevard, Pueblo, CO 81001 - 4901. Forming, Firming and Fitting-in-Place: Using First (and second) draft writing to teach mathematics.

Having used student writing assignments as an integral part of my teaching for over a (baker's) dozen years now, I have increasingly come to realize the value of first draft writing. By (temporarily) relaxing our usually strict standards of 'mathematical correctness', as well as formal standards for grammar, spelling and composition, first draft writing places an emphasis on making sense of the mathematics in question. In turn, students are provided with an opportunity both 'to form' and 'to form' their conceptual understanding of new ideas, as well as to 'fit' these new concepts into place among previously understood mathematical concepts and structures. This talk will focus on my experience with two quite different uses of first - draft writing: (1) journal writing in a course for pre-service elementary teachers and (2) Course Reading Notebooks in upper division courses such as Linear Algebra and Advanced Calculus. Time permitting, I will also share examples of assignments for first year calculus students in which revision and audience play a role analogous to that played by first draft writing in these two assignments. (Received June 04, 2003)

5003-D1-134 Michael A Brilleslyper\* (mike.brilleslyper@safa.edu), Department of Mathematical Sciences, U. S. Air Force Academy, CO 80840. Specialized Grading Rubrics for Short Writing Assignments.

One fear of using writing assignments in the math classroom is the increased grading burden. Using assignment-specific grading rubrics greatly cuts down on grading time and provides a fair assessment of student work. In

addition, a good rubric accurately conveys the criteria upon which the assignment was judged. The use of these rubrics has essentially eliminated all student complaints concerning grading fairness. In this talk, I will present several examples of short writing assignments and the rubrics used to assess them. (Received June 04, 2003)

5003-D1-135 Pam Crawford\* (pcrawfo@u.edu), Department of Mathematics, Jacksonville University, 2800 University Boulevard North, Jacksonville, FL 32211. Two Writing Assignments in Proofs Class.

All mathematics majors and minors at Jacksonville University are required to complete the writing intensive course MS 220WI Mathematical Reasoning in which we examine methods of proof and mathematical writing through study of the fundamentals of mathematical logic, set theory, relations, functions and their limits, and cardinalities of sets. This talk will report on two course writing assignments. For the first developed from an idea of Margaret Robinson that was reported in a draft version of the MAA CUPM Curriculum Guide, students, after working with me in-class to develop the proof of the irrationality of the square root of two, were asked to write out and to explain the proof of the irrationality of the square root of two to another person, anyone other than a faculty member or someone who had previously taken MS 220WI - and to submit a signed statement from the other person stating that (s)he understood the proof. For the second assignment, I used Annie Seldon's idea of asking students to write about their favorite proof, including a discussion of what makes this proof their favorite. In this presentation, I will report on these assignments as well as the results, and I will discuss student reaction to these assignments. (Received June 04, 2003)

5003-D1-136 Paul J Karapol\* (karapol@payton.cps.k12.il.us), Walter Payton College Prep H.S., 1034 N. Wells St., Chicago, IL 60610. Geometry Papers that Count.

The author describes a series of short writing assignments for students that nurture geometric reasoning while requiring only basic numerical familiarity. The results are that students become comfortable discussing symmetries, congruence, and the use of systematic reasoning without being overwhelmed with the demand for a rigorous, axiom-based proof. While the author has used these writing assignments most extensively in high school geometry classes, they could also be used as non-frightening introductory writing assignments in upper-level courses. The same assignments are also open to extensions and generalizations that spark a student's interest in self-motivated mathematical research. (Received June 04, 2003)

5003-D1-137 Alexander Khait\* (khait@ever.jce.ac.il), Jerusalem College of Engineering, Ramat Beit HaKerem, POB 3566, Jerusalem 91035, Israel. Invoking Creativity In Conceptualization.

The purpose of this paper is to demonstrate work on concept creating capabilities of "regular mortals" as opposed to those of great mathematicians. After several stages of preparation students are presented with examples that "cry for definition" and asked to invent concepts by writing down suitable definitions. Some of these are actually standard concepts and some are new presented for practice of creative mathematical writing. (Received June 04, 2003)

5003-D1-138 Julianne M. Labbiento\* (jlabbiento@ccc.edu), 1334 South Fairview Road, Allentown, PA 18103. Exploring Mathematical Concepts Through Writing in a Liberal Arts Mathematics Course.

Writing papers, as an alternative to traditional exams, has proven an invaluable tool in my liberal arts mathematics classes. Using writing as a means of assessing a student's knowledge of a particular mathematical concept allows for a more in-depth probe into that student's mastery of the topic. The student can provide greater detail in illustrating what he or she does and does not know about a mathematical topic. In the papers that my students write, they have the option to choose their own examples to illustrate in the mathematical analysis. This enables a student to discuss something that he or she finds interesting, which keeps him or her focused on the task at hand. By applying the required mathematical concepts in the analysis of the example, students see the connection between something they enjoy and the mathematics course. Mathematics throughout the curriculum can also be fostered while utilizing writing as an assessment method by encouraging these papers to encompass a broad array of subject areas. Finally, in addition to learning mathematics in the course, incorporating this assessment format also reinforces the art of written communication as a means of comprehensively yet concisely conveying a thought to another person. (Received June 04, 2003)

5003-D1-139 Linda McGuire\* (lmcguire@uhlenberg.edu), Department of Mathematical Sciences, Muhlenberg College, Allentown, PA 18104. Writing For Your Audience.

This presentation will outline a project through which students may address how to tailor a mathematical argument to the audience that will read it. At four different points in the semester, students were given problems

## 12 GETTING STUDENTS TO EXPLORE CONCEPTS THROUGH WRITING IN MATHEMATICS

to solve using whatever proof technique (direct proof, indirect proof, proof by contradiction, induction, etc.) was applicable. They then had to write not one, but three, different proofs of the same problem. The first was to be written like an op/ed piece in the newspaper, with the assumption that the reader is literate, but may have little or no formal mathematical training. The second proof was to address an audience of their peers assumed to have a background akin to their own. The final version was to be written as if trained mathematicians were to read it. The results improved dramatically throughout the term and some important points were raised and discussed. Students were encouraged to have others read their work, both their mathematical peers and other people around the campus with varying mathematical backgrounds; many excellent conversations about writing ensued. This presentation will highlight the problems given, show student solutions to these problems, and to share the many positive questions and comments that the projects stimulated. (Received June 04, 2003)

5003-D1-140 Mike Pinter\* (pinterm@i .belmont.edu), Dept. of Mathematics and Comp. Science, Belmont University, Nashville, TN 37212. Response Essays for Mathematics Classes.

I incorporate response essays into courses that I teach, including the Basic Concepts of Mathematics and Analytics courses. The Basic Concepts course develops ideas and topics which do not require algebra, such as logic, mathematical problem solving, modular arithmetic, and coding. The readings for Basic Concepts are for a general audience and are very accessible to all of the students; sources include Math Horizons, the Humanistic Mathematics Network Journal, and "popular" magazines like Discover. The Analytics course is required of all students in Belmont's Honors Program. Because of its audience, in addition to readings described above for the Basic Concepts course I require some more challenging readings for response from the Analytics students. For example, one of the response essays is tied to readings about Godel's Theorem. I also generally require more related readings that help to enrich the essays. In particular, for Spring 2003 the students were required to read Hofmann's The Man Who Loved Only Numbers and Cole's The Universe and the Teacup: The Mathematics of Truth and Beauty. All in all, I've found that the response essays help make mathematics seem more relevant to the students because they see applications and connections to other things in their lives. (Received June 04, 2003)

5003-D1-141 Debasree Raychaudhuri\* (draycha@alstela.edu), Department of Mathematics, 5151 State University Drive, Los Angeles, CA 90032-8204. Exploring, Writing, Presenting, And Defending Mathematical Concepts: A Project For Elementary Teachers.

The content courses for elementary teachers seek to instill a deeper understanding of the crucial mathematical concepts surface-learned at earlier stages, typically during the learner's own elementary or middle school-years. Yet many a times prospective teachers depart with similar capacities they entered the course with, i.e., possession of surface knowledge and little confidence to present and defend mathematical concepts. In this presentation we will describe a group project that attempts to strengthen the tie between the subject mathematics and its future teachers, by demanding that the students explore mathematical concepts from different angles, write a report on their findings, present it to their peers and defend their point of views. This project offers pre-service elementary teachers a rare opportunity to talk mathematics in a safe and encouraging setting, receive constructive critique from peers and instructor, and in doing so gain valuable insight about mathematics as well as their own strengths and weaknesses as teachers of the subject. (Received June 04, 2003)

5003-D1-142 Peter D Schumer\* (schumer@middlebury.edu), Department of Mathematics, Middlebury College, Middlebury, Vermont, 05753. The Value of Writing Assignments in History of Mathematics and Combinatorics Courses.

Writing about mathematics is an important way to "do" mathematics. Naturally, writing mathematics includes theorem proving and problem solving, but writing can also incorporate many other key aspects of learning. In my History of Mathematics course writing is an essential ingredient in the course incorporating weekly response papers, two research papers and presentations on a mathematician or area of mathematics, and a final timeline project. In my Combinatorics class students work together in small groups, present their work to the class, and then carefully write up all the results of the semester in a comprehensive portfolio. In both classes, writing is an essential ingredient which forces students to learn the material well, think cogently, express themselves clearly, and helps tremendously in their retention of the material. In this talk I will include some samples of my students' work. (Received June 04, 2003)

- 5003-D1-143 Martha Ellen Waggoner\* (waggoner@simpson.edu), Mathematics Department, Simpson College, Indianola, IA 50125. An Evaluation of Portfolios Used as an Assessment in Mathematics Courses.

How do we know if a writing assignment met our goals or if the students simply saw the writing as a hoop to jump through? I collected qualitative and quantitative data about a reflective portfolio assignment used in two different mathematics courses to determine if the students found the writing useful or not. For the reflective portfolio, students gathered work from the mathematical writing for that semester that represented how they had grown academically and wrote an essay to reflect on their learning. The students found the portfolio to be a useful tool for reviewing and synthesizing the contents of the course and for reflecting on their accomplishments. They considered the writing an essential part of the course and valued the experience of writing the reflective essay. On the other hand, the students felt that writing in other disciplines did not prepare them adequately for mathematical writing. After a brief summary of the portfolio assignment, I will discuss the data collected and the student's perceptions of using writing in these specific courses. (Received June 04, 2003)

- 5003-D1-145 Valerie L Watts\* (watts@math.arizona.edu), University of Arizona, Department of Mathematics, PO Box 210089, 617 N. Santa Rita, Tucson, AZ 85721, and Bin Lu (binlu@math.arizona.edu), University of Arizona, Department of Mathematics, PO Box 210089, 617 N. Santa Rita, Tucson, AZ 85721. Using Lattice Theory to Promote Effective Communication of Mathematics.

A modified independent study is an effective way to help students learn the art of communicating mathematics. With lattice theory as a backdrop, students develop problem-solving and communication skills by giving lectures to their fellow students, working exercises and preparing a presentation and written report on a special topic. Lattice theory is an excellent topic for this type of study, in part because of its accessibility to students who may not have an extensive background in mathematics. (Received June 04, 2003)

- 5003-E1-67 Christopher N. Swanson\* (cswanson@shland.edu), Department of Mathematics, Ashland University, 401 College Avenue, Ashland, OH 44805. Derangements, Door Prizes, and Secret Santas. Preliminary report.

A derangement is a permutation of objects that leaves no object in its original position. The author of this contributed paper recently encountered two problems related to derangements, the first of which involves door prizes. Suppose  $n$  people attend an event that has  $k$  door prizes drawn by the attendees. What is the probability that no one draws his/her own name? When  $n = k$ , this is precisely the derangement problem. The second problem relates to a "Secret Santa" gift exchange. The basic idea behind such an exchange is that the organizer "randomly" assigns each participant the name of another participant for whom to buy gifts. Since no one wants to buy gifts for herself, the organizer must find a derangement of the participants. If the same group of people participate for two consecutive years, what is the probability that no one buys gifts for the same person two years in a row? In terms of derangements, what is the probability that two (possibly the same) randomly selected derangements of  $n$  objects have no object appearing in the same position in both derangements. The author will present a combinatorial analysis of the answers to these questions in his talk, as well as the results of simulating these problems using MAPLE, if time allows. (Received June 02, 2003)

- 5003-E1-85 David Einstein\* (einstein@dcorp.net), Structured Decisions Corporation, 1105 Washington St, West Newton MA, 02465. The amicable pairs less than  $10^{14}$ .

Pairs of numbers where the sum of divisors of both the numbers equal the sum of the two numbers are called amicable. Armed with little more than the Fundamental Theorem of Arithmetic and a few simple properties of the sum of divisors function, we compute all the amicable pairs whose smaller member is less than  $10^{14}$ . (Received June 03, 2003)

- 5003-E1-96 Sebastian Martin Ruiz (smruiz@telefonica.net), Avda. De Regla, 43, Chipiona 11550 (Cadiz), Spain, and Minh Lou Perez\*, Rehoboth, Box 141, NM 87301, USA. Properties and Problems Related to the Smarandache Type Functions.

In this paper we present the definitions and some properties of several Smarandache type functions that are involved in many proposed solved and unsolved problems and conjectures in number theory and recreational mathematics. Examples are also provided. Interesting solved problems related to them are attached as addenda to this article. (Received June 03, 2003)

5003-E1-98 Paul R. Coe\* (coepaul@maill.dom.edu), Dominican University, 7900 W. Division St., River Forest, IL 60305, Theresa Meshes (theramesh@otmaill.com), Dominican University, 7900 W. Division St., River Forest, IL 60305, and William T. Butterworth (wbutterw@arad.edu), Barat College of DePaul University, 700 E. Westleigh Road, Lake Forest, IL 60045. Banking on 'The Weakest Link': The American Syndicated Version.

On the syndicated game show "The Weakest Link" a decreasing number of contestants answer a series of questions trying to make money for their "team." However, only one contestant actually leaves with any money at the end of the show. The intrigue of the show is based on the contestants' voting on their "Weakest Link" after each round. The money won by the final contestant, however, is based on answering questions correctly and, in the earlier rounds, banking that money wisely.

In the first five rounds, the contestants take turns answering questions with each successive correct answer compounding the money that can be won. One correct answer is worth \$250, two correct answers in a row are worth \$500, three \$1,000, four \$2,500, five \$5,000, and six correct answers in a row are worth \$12,500. An incorrect answer restarts the chain at \$250. To save any of this money, however, it must be banked by a contestant saying "Bank" before hearing his or her next question. This restarts the chain at \$250, but the previously earned money is saved. Only banked money is preserved and won at the end of the show.

In this paper we will present the optimal strategy for banking by the contestants. It is based on probability models and data collected from observing the show. (Received June 04, 2003)

5003-E1-127 Darci L. Kracht\* (dkracht@kent.edu), Department of Mathematical Sciences, Kent State University, Kent, OH 44242. Simpson's Paradox in College Basketball Shooting Statistics.

In 2001, Kent State University's men's basketball team surprised the nation by upsetting Indiana in the first round of the NCAA tournament. At the time, a member of the team was in my "Explorations in Modern Mathematics" class. Thus, it was natural to look for examples in basketball statistics when we discussed Simpson's Paradox, an apparently rare phenomenon where the direction of a statistical association is reversed when a third variable is considered.

I found several examples when statistics for two-point and three-point field goals are considered separately and combined. This makes sense because three-pointers are considerably more difficult than two-pointers and some players shoot many more three-pointers than others. This provided me with a nice question for the final exam.

However, even after the exams were marked and the grades turned in, I continued to wonder about the frequency of Simpson's Paradox. I partitioned the 18 conference games into two nonempty sets in all possible ways and then computed each player's averages in each set and then in all games combined. Here, the rate of occurrence of Simpson's Paradox varies with the size of the sets in the partition.

(In 2002, KSU made it to the fourth round of the NCAA tournament only to be beaten by Indiana!) (Received June 04, 2003)

5003-E1-164 Hossein Behforooz\* (gbehforooz@utica.edu), Utica College of Syracuse University, Department of Mathematics, Utica, NY 13502. Historical Notes with Interesting properties on Some Famous Magic Squares.

A short historical notes on Albrecht Durer and Benjamin Franklin magic squares will be discussed. Some rare and unpublished properties of these squares will be presented. A review of knight's move magic squares and impossibility of a closed knight tour will be discussed. (Received June 05, 2003)

5003-F1-26 Richard Kuntz\* (kuntz@monmouth.edu), Department of Mathematics, Monmouth University, W. Long Branch, NJ 08720. A WEB-Based Practice and Testing System.

Over the past four years, the author has been developing and enhancing a web-based practice and testing system, called MUTester. The system has evolved into a "production" system that is currently supporting several courses at Monmouth University in the PSI (Keller Plan) format.

In addition to demonstrating the basic features, the presentation will discuss the distribution, setup and administration of the software package. The software will be available as a download from Monmouth University.

The presentation will conclude with a discussion of the recently created Consortium of MUTester users. The Goal of the Consortium is to provide a no-cost alternative to commercially distributed, web-based testing and tracking software. The MUTester software would be freely available to Consortium members as a shareware, open systems product. Subject of appropriate resources and skills, Consortium members would assist in the

development of problem set databases and software enhancements that would in turn be made freely available to Consortium members. (Received May 14, 2003)

5003-F1-28 Monty G. Fickel\* (mfickel@sc.edu), Department of Mathematics, Chadron State College, Chadron, Nebraska 69337. The ENTIRE Math Program Online? Yes!

The learning experiences of a small college mathematics department during the three-year migration of the entire program to online status have been significant. As faculty examine courses and write online versions, content selection, explanation, and assessment methods have been considered. Migration of 21 courses has made it possible to complete the major, minors, and teaching endorsements, as well as the content portion of the MS in Mathematics Education online. (Received May 15, 2003)

5003-F1-29 Ananda Gunawardena\* (guna@s.cmu.edu), School of Computer Science, 4000 Forbes Avenue, Pittsburgh, PA 15213. Adaptive Learning Technologies for Math Education.

In this presentation, I will discuss a new way to deliver a Math course using Adaptive Book technology. Adaptive book is a textbook management system that allows instructors to connect various components of the course (lectures, assignments, quizzes etc) to the digital textbook. Adaptive Book also allows instructors to deliver personalized content to students and students to markup content using highlights, annotations, web and file links. Furthermore, markups can be indexed, archived, searched or shared with other users. I will also discuss how to make a digital "cheat sheet" using Adaptive Book tool kit. (Received May 15, 2003)

5003-F1-86 Pramod Kanwar\*, Department of Mathematics, Ohio University at Zanesville. E-Learning and 'Explore and Learn Approach' in Math Courses.

The presenter will share his experiences in using technology in the 'Explore and Learn Approach' to teach Math courses. Among other things, the presenter will focus on use of computer algebra systems and other tools in teaching of Math courses in a way that can help the students understand mathematical concepts and also see the connections between these concepts and other sciences. The strengths and weaknesses of the approach will also be shared. (Received June 03, 2003)

5003-G1-31 Michael R. Huber\* (michael-huber@usma.edu), Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996, and Alex J. Heidenberg (alex-heidenberg@usma.edu), Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996. Assessing the Value of Laptop Computers to Student Learning in a Large Core Mathematics Program.

Since January 2002, USMA has participated in the MAA's NSF project SAUM, studying assessment of new technology in the freshman curriculum. Recently, focus has been shifted away from conventional assessment techniques because of the introduction of laptop computers. Every student in the Class of 2006 has a laptop with a computer algebra system, enabling him or her to more deeply explore and then discover mathematics and scientific concepts. The Department of Mathematical Sciences fosters a curriculum environment that creates problem solvers, and this presentation will outline our 18-month efforts in assessing the success of this new innovative curriculum. In particular, current assessment data will be presented, which include two-day (with and without technology) exams and modeling and inquiry problems (MIPs). (Received May 15, 2003)

5003-G1-40 Kevin W. Dennis\* (kdennis@umn.edu), 700 Terrace Heights, 59, Winona, MN 55987. Assessing Written and Oral Communication of Senior Projects.

In an evolving process to assess its majors, the mathematics department in a small liberal arts college decided to use senior project presentations and reports as instruments to assess the department's goal of developing the skill of formally presenting mathematics, in both oral and written form. The rubrics used to assess the presentations and reports will be presented. Findings from a recent class of graduates using these rubrics and the recommendations that came from the findings will be discussed. (Received May 19, 2003)

5003-G1-44 Barbara M. Morskala\* (bmorskala@mines.edu), Mathematical and Computer Sciences Department, Colorado School of Mines, Golden, CO 80401. Assessing the Engineering Calculus Sequence.

This paper will describe in detail one segment of the Colorado School of Mines (CSM) Mathematical and Computer Sciences Department's assessment system: the proposed evaluation of the Engineering Calculus Sequence.



All students at CSM are required to complete the coordinated courses that comprise the Engineering Calculus Sequence. Coordinated courses have multiple sections, which are taught by different instructors. A lead faculty member organizes these sections by holding regular meetings at which instructors have the opportunity to share instructional strategies and to create common assignments and/or exams. The lead faculty member has the additional responsibility of ensuring that the designated program objectives are assessed through common assignments and/or exams.

In the fall of 2002, members of the faculty began to question the appropriateness of the calculus sequence for supporting students in attaining the departmental goals. This stimulated a review and revision of the curriculum. The new curriculum will be implemented in the fall of 2003. In order to examine the effectiveness of the new curriculum, baseline assessment data will be collected in the spring of 2003. The proposed paper will describe the calculus sequence at CSM, and the instruments and the methods that will be used to evaluate the effectiveness of the revised curriculum. (Received May 21, 2003)

5003-G1-48 Betty S. Travis\* (btravis@utsa.edu), College of Sciences, University of Texas at San Antonio, San Antonio, TX 78249. Assessing Developmental Mathematics at the University of Texas at San Antonio.

The effectiveness of Developmental Mathematics instruction at the University of Texas at San Antonio (UTSA) was assessed in three ways: 1) by examining the passing rates in developmental mathematics courses; 2) by examining the passing rates on the state-mandated developmental mathematics test TASP (Texas Academic Skills Program); and 3) by comparing the passing rates of students in their first non-developmental mathematics course with students who did not require a developmental mathematics course. Results indicated that 70% of students in the developmental mathematics courses received CR for the course. Additionally, an adjusted rate of 59% of students passed the TASP test. Finally, the pass rates of students who had taken a TASP mathematics course were compared to a sample of non-TASP students in a subsequent non-developmental mathematics course. Although there were no discernible differences in the pass rates between the two groups, the grades received by the TASP-obligated students were lower. Only 4% of students who enter UTSA needing to take developmental mathematics ever finish an undergraduate degree at UTSA. Efforts are underway to improve all these rates by reducing class sizes, incorporating technology, and training faculty who teach this type of course. (Received May 27, 2003)

5003-G1-51 Daniel C. Kemp (daniel\_kemp@dstate.edu), South Dakota State University, Department of Mathematics and Statistics, Box 2220, Brookings, SD 57007, and Donna L. Flint\* (donna\_flint@dstate.edu), South Dakota State University, Department of Mathematics and Statistics, Box 2220, Brookings, SD 57007. Assessment of the Mathematics Major.

At South Dakota State University, assessment of Mathematics majors is done through a senior seminar (capstone) course. The course requirements include a portfolio, a major paper and a PowerPoint presentation. We will discuss these requirements and their assessment. Data from the previous year will be given. (Received May 28, 2003)

5003-G1-77 Jim Fulmer\* (jrfulmer@alr.edu), Dept of Mathematics & Statistics, Univ of Ark at Little Rock, Little Rock, AR 72204-1099. A Model for Assessing An Undergraduate Major in Mathematics.

Assessment of degree programs at the University of Arkansas at Little Rock has been an ongoing campus-wide project in all colleges and departments for several years. Assessment of the major in the Department of Mathematics and Statistics involves two undergraduate degree programs. For the last year, we have participated in an MAA workshop, "Strengthening Undergraduate Assessment of Mathematics (SAUM)". This talk will describe how we have adapted our assessment process to the assessment cycle developed at this workshop.

A primary concern in the development of our assessment plan has been: "To make it work, keep it simple." The steps in developing and adapting our assessment plan included developing a program mission statement consistent with the university's mission; articulating program goals and relating them to specific student learning objectives; and developing assessment criteria and methods for measuring the student learning objectives. After applying our assessment tools that measure how well we are meeting the student learning objectives, we complete the feedback loop by examining the assessment results and making determinations on how the program (or assessment process) can be improved.

Our plan involves a cycle of assessment activity to find out: What should our students learn, how well are they learning, and what should we change so that future students will learn more and understand it better? Our

talk will provide details of our assessment plan, its implementation, and the process by which we developed it. (Received June 03, 2003)

5003-G1-78 Jim Fulmer\* (jrfulmer@alr.edu), Dept of Mathematics & Statistics, Univ of Arkansas at Little Rock, Little Rock, AR 72204-1099. Reforming the Senior Seminar/ Capstone Course as an Assessment Tool.

This session will focus on a discussion of how the Senior Seminar/ Capstone course can be restructured or reformed to provide data to be used in assessment of degree programs. Several questions relative to the senior seminar/ capstone course for mathematics majors will be a focus point for discussion. What should be the structure for the senior seminar/ capstone course? What are its components? Number of credit hours? What rubrics can be used to assess writing, oral communication, and technology? How can the ETS-MFT be a part of the course? What about problem solving and cooperative group work? Can exit interviews and exit surveys be incorporated? What about helping students prepare a paper for student presentations at professional meetings? What assessment criteria and assessment methods can be used so as to provide evidence to be used in program/ outcomes assessment? This session will focus on a discussion of these issues and how they relate to program/ outcomes assessment. (Received June 03, 2003)

5003-G1-91 Dick Jardine\* (rjardine@eene.edu), Keene State College, 229 Main Street MS2001, Keene, NH 03431-2001. Getting a Department Involved with Assessment.

For program assessment to be truly successful, engaging as many faculty as possible in the effort is critical. This presentation will describe how one department has gained faculty participation in the assessment of three program goals during the past academic year. Successes and difficulties in accomplishing the ongoing assessment will be presented. Faculty "buy-in" is an important element of a successful program assessment, and strategies to get the entire department involved will be discussed. (Received June 03, 2003)

5003-G1-95 Jeffrey V. Berg\* (jeff.berg@arapahoe.edu), Arapahoe Community College, 5900 S. Santa Fe Dr., P.O. Box 9002, Littleton, CO 80160-9002, Erica M. Johnson (erica.johnson@arapahoe.edu), Arapahoe Community College, 5900 S. Santa Fe Dr., P.O. Box 9002, Littleton, CO 80160-9002, and David Heddens (david.heddens@arapahoe.edu), Arapahoe Community College, 5900 S. Santa Fe Dr., P.O. Box 9002, Littleton, CO 80160-9002. Assessing College Algebra at a Community College.

During the 2001-2002 academic year, the Arapahoe Community College Mathematics Department participated in a college-wide effort to expand assessment activities to the program and discipline level. The department chose to document and focus on data from a College Algebra common final to fulfill its role in the college-wide effort. This talk will detail department accomplishments, assessment results, and future direction contained in a report at <http://www.arapahoe.edu/custom/SAmathematics.html>. Improvements to the department assessment efforts made possible by involvement in the MAA CUPM Supporting Assessment in Undergraduate Mathematics (SAUM) Workshop series will also be presented. (Received June 03, 2003)

5003-G1-97 Stephanie Fitchett\* (sfitchett@au.edu), Honors College, Florida Atlantic University, 5353 Parkside Drive, Jupiter, FL 33458. Learning in a Discovery-Based Science and Mathematics Setting.

Several scientists and mathematicians at Florida Atlantic University's Honors College are in the final stages of a curriculum development project funded by the National Science Foundation. The project has sought to develop a science and mathematics curriculum that emphasizes the development of discovery-based undergraduate science and the creation of links between and among the sciences and math. In particular, the program seeks to provide science students with an early introduction to research in the undergraduate curriculum, and multiple opportunities for research during the undergraduate career. As part of the evaluation of the project, student learning is being assessed in science and mathematics courses that are aimed primarily at science majors, with particular emphasis on lower division courses. This report describes the assessment procedures and the findings thus far. (Received June 03, 2003)

5003-G1-99 Edward C. Keppelmann\* (keppelma@nr.edu), Department of Math and Stat AB601 MS084, University of Nevada Reno, Reno, NV 89557. Mathematics Assessment at the University of Nevada Reno: The Story of Reluctant and Skeptical Faculty Who Are Actually Doing a Great Deal. Preliminary report.

Like many schools UNR has recently been confronting the mandate of assessment. Unlike many schools, our administration has decided that this means all departments must assess their own degree programs. In Math, this is far from the best choice because (1) The number of students involved is relatively small compared to our

overall teaching mission and (2) The diverse interests of faculty in pure, applied, math ed, and statistics makes it impossible for us to agree on an approach. So the chairman has settled on his own work to study exit interviews and track students through our key analysis (proof based) sequence. There are significant differences between instructors in these courses but it is totally unclear how to present this to faculty and make lasting changes that everyone can agree on.

Concurrently assessment has taken place in other areas: (A) Considerations surrounding WEBCT for our lower level courses; (B) Discussions with Engineering to do curriculum reorganization; (C) Evaluation of math competency within our core curriculum; (D) System wide evaluation of remedial education.

As a subject, math is unique in its ability to interact with the university in so many ways. Will the administration embrace these difference or continue to push us in less effective directions? (Received June 04, 2003)

5003-G1-163 Richard P Vaughn\* (rick.vaughn@maricopa.edu), Paradise Valley Community College, Room G102, 18401 North 32nd Street, Phoenix, AZ 85032. Assessing Paradise: A Comprehensive Assessment Program at a Two-Year College.

At Paradise Valley Community College, a part of the Maricopa Community College District, a four-faceted assessment plan was developed and implemented in less than a year with help from the Supporting Assessment in Undergraduate Mathematics (SAUM) project. Strong leadership and a shared commitment to a meaningful, non-intrusive process has led to tremendous faculty buy-in, including adjunct faculty. Plan components include instructor "focus groups", rubrics, database reports, and a student surveys. (Received June 05, 2003)

5003-H1-50 John B. Urenko\* (jbu2@su.edu), School of Science, Engineering, & Technology, Penn State Capital College, 200 University Drive, Schuylkill Haven, PA 17972. Elucidating The Average Rate of Change Metaphor.

The purpose of this presentation is to provide an illustration of the manner in which Mathematical Idea Analysis informs introductory calculus instruction. This new discipline, introduced by G. Lakoff and R. Nunez, comprises the scientific study of the nature and structure of mathematical ideas from a cognitive perspective. Its fundamental principle is that a cognitive mechanism, known as conceptual metaphor, structures mathematics as human beings conceptualize it. Therefore, the meaning and significance of a particular mathematical concept inhere in its characterization vis-à-vis conceptual metaphor, not in its most facile representation. This is demonstrated by characterizing the average rate of change of a continuously differentiable function as the average value of the instantaneous rate of change of the function. This characterization demands twin instantiations of what Lakoff and Nunez refer to as the Basic Metaphor of Infinity. The deep conceptualization resulting from this characterization reduces, by means of the Fundamental Theorem, to the facile representation typically given early in introductory calculus courses. (Received May 27, 2003)

5003-H1-75 Premalatha Junius\* (pjuni@msfi.edu), Premalatha Junius, P.O. Box 124, Mansfield, PA 16933, (570) 662-3305. The Interplay between Motion, Analogy and Individual Experience.

The study investigated the complex cognitive process involved in integrating the concept of 'straightness' on plane and sphere in an upper-level modern geometry class for prospective mathematics teachers. The study was conducted over a period of seven semesters, utilizing an emergent design of mixed methods. The research supports the assertions of Poincare (1913) - motor space is the space and motion is the basis of geometry, and Lakoff's (1987) notion that reasoning is 'embodied and imaginative'. Samples of students' work will be used to show how motion, analogy and individual experience played a major role in integrating the concept of 'straightness' on plane and sphere.

Results from the research were used in an undergraduate calculus class that explored the idea of slope of a tangent line at a point on a curve and the relationships between a function, its first derivative and second derivative. Students' work that will be presented reflect the understanding of these relationships, through the use of motion, analogy and individual experience. (Received June 03, 2003)

5003-H1-94 Shandy Hauk\* (hauk@nco.edu), Department of Mathematical Sciences, Campus Box 122, University of Northern Colorado, Greeley, Colorado 80639, Robert A. Powers (rapower@nco.edu), Department of Mathematical Sciences, Campus Box 122, University of Northern Colorado, Greeley, Colorado 80639, April D. Judd (judd8691@nco.edu), Department of Mathematical Sciences, Campus Box 122, University of Northern Colorado, Greeley, Colorado 80639, and Jenq-Jong T say (tsay7018@nco.edu), Department of Mathematical Sciences, Campus Box 122, University of Northern Colorado, Greeley, Colorado 80639. Improving student performance and sense-making in Liberal Arts Mathematics..

We report on a continuing research and evaluation study of the Mathematics and Liberal Arts course at the University of Northern Colorado. The study is focused on ways to foster the development, in tandem, of flexibility of thought (cognition) and sense-making drive (affect). Towards that end, we are examining student conceptions of mathematics and problem-solving and their self-perceptions regarding mathematical task efficacy. A Conceptions of Mathematics survey was administered in Fall 2002 to 10 sections, pre- and post-course. The instrument attempts to measure five constructs: felt algebra needs of students, felt geometry needs of students, felt purpose of Math 120, conceptual perceptions of problem solving, and procedural perceptions of problem solving. Cronbach's alpha, correlations, and tests of significance indicate the instrument is reliable with high internal consistency. In Fall 2002, four of the 10 sections used one textbook, cooperative learning, and writing; six sections used another text with no regular cooperative learning or writing. Survey statistics, analysis of task-based interviews of students and teachers, and comparisons of sense-making and articulation efforts on short answer exam questions will be presented. (Received June 03, 2003)

5003-H1-100 Marilyn P. Carlson\* (marilyn.carlson@ox.net), 477 N. Mondel Dr., Gilbert, AZ, 85233. A Research-Based Instrument for Assessing Students' Readiness for Calculus: The Precalculus Concept Assessment Instrument.

The Precalculus Concept Assessment (PCA) instrument is a multiple choice exam that assesses major abilities and understandings (as described in the PCA taxonomy) that have been found to be foundational for beginning calculus. Each version of the exam contains 25 items. Each item has five answer choices that includes the correct answer and common incorrect responses that have been consistently provided by students. Each of the 125 items in the PCA pool provides information about students' strengths and weaknesses relative to specific components of the PCA taxonomy. Each PCA item has undergone multiple cycles of revision to assure that the: i) wording of the question is clear and consistently interpreted according to the design intent; ii) selection of the correct answer consistently reflects the understanding assessed; and iii) selection of each of the incorrect answers represents a common incorrect response consistently provided by students. The taxonomy components and associated PCA items will be described. Student interview excerpts will be presented to illustrate the power of individual items in assessing specific understandings. Data that correlates student scores on PCA with course grades in beginning calculus will be discussed. (Received June 04, 2003)

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5003-I1-13 Anthony L. Peressini\* (peressini@uiuc.edu), Department of Mathematics, University of Illinois, 273 Altgeld Hall, 1409 West Green St., Urbana, IL 61801. Developing and Delivering On-line Credit Short Courses For High School Mathematics Teachers Through Math Teacher Link.

The Department of Mathematics at the University of Illinois at Urbana-Champaign has developed and is currently delivering a series of credit short courses (course modules) for teachers at the 9 - 14 grade levels through its Math Teacher Link (MTL) web site at the following URL: <http://MTL.math.uiuc.edu/>.

The twelve MTL course modules currently available deal primarily with the integration of technology into the teaching of mathematics including software packages such as Mathematica and Geometer's Sketchpad, graphing calculators, and the creation of effective web-based materials for mathematics classes.

Several new MTL course modules are now under development that will focus on the mathematical content that is taught in high school but from the advanced perspective of college level mathematics courses and the many mathematical connections that are present within the content of high school mathematics itself. Some of these new modules are based on the recent text *Mathematics for High School Teachers - An Advanced Perspective* by Usiskin, Peressini, Marchisotto, and Stanley (Prentice-Hall, 2003). Although these modules rely on this text as

a reading and problem resource, they also extend the text by making use of interactive web-based materials and facilities to enhance and augment the learning experience.

This session will describe one of these new course modules in some detail and also describe how Math Teacher Link delivers and provides instructional support to teachers enrolled in these course modules. (Received April 21, 2003)

5003-I1-17 Carol Vobach\* (vobachc@hd.edu), University of Houston-Downtown, 1 N Main St, Houston, Texas 77002, Elias Deeba (deebae@hd.edu), University of Houston-Downtown, 1 N Main St, Houston, Texas 77002, and Ron Barnes (barnesr@hd.edu), University of Houston-Downtown, 1 N Main St, Houston, Texas 77002. Math Courses for an MAT (Master of Arts in Teaching) Program.

The MAT program is a 3-year evening/summer track designed for working teachers. Of the 39 hours coursework, only 9 hours are dedicated to mathematics or another chosen area. Mathematical backgrounds are not strong and students with varied expertise are teaching over a range of grades. Three courses in probability/statistics/combinatorics, mathematical structures and geometry were developed and implemented in summers 2002 and 2003. All classes met in a computer lab and technology was emphasized via SPSS, Maple and Geometer's Sketchpad software. Conclusions and concerns about this type of graduate program will be discussed. (Received May 09, 2003)

5003-I1-21 Kira Hylton Hamman\* (hamman@hood.edu), Hood College, 401 Rosemont Avenue, Frederick, Maryland 21701. Should Hood Go Co-Ed? In<sup>o</sup>mmatory Questions, Spreadsheets and Statistics.

At many institutions, pre-service teachers are required to study a certain amount of statistics as part of the mathematics credit they need for graduation. But what do they take away from these courses? Do they see statistics as an exciting resource or a dry bore? Do they see the big picture or spend too much time bogged down by computational details? In this talk we will argue that the second issue can be addressed by the use of spreadsheet software in teaching statistics, and that the first can be addressed by the strategic use of in<sup>o</sup>mmatory questions. (Received May 12, 2003)

5003-I1-22 Timothy W. Flood\* (tflood@ittstae.edu), Mathematics Department, Pittsburg State University, Pittsburg, KS 66762. An overview of three, 1-hour courses designed to prepare secondary mathematics teachers to use technology in the classroom.

Several years ago Pittsburg State University began offering three, 1-hour courses that are required for math teachers seeking secondary certification and recommended for those seeking middle school certification. One course is on the use of manipulatives including Geoboards, Algebra Tiles, Algeblocks, and Miras and is geared to middle school and early high school topics. Another course covers topics on graphing calculators appropriate for middle school and high school. This course provides hands-on experience using the TI-82, TI-92, and TI-CBL units. The third course is on the use of computer software including Excel, Geometer's Sketchpad, and Maple. It is primarily for high school teachers but can be modified for middle school teachers.

A brief history of the development and implementation of these three courses will be presented. Most of the presentation will emphasize the latter two courses, since they are more related to the topic of this session. A variety of the specific activities typically covered in each of these two courses will be highlighted. The relation of these topics to the state licensure requirements will be included. Also, details on how the courses have been modified and offered as in-service workshops will be provided. (Received May 12, 2003)

5003-I1-59 John F Hamman\* (jhamman@acc.edu), 101 College Parkway, Arnold, MD 21783. Bringing Euclid into the 21st Century.

The construction of regular polygons may seem only interesting has an historical note, but I will discuss why I think this is an important activity for all pre-service teachers. Furthermore, the use of dynamic geometric software such as Geometer's Sketchpad is essential in helping the pre-service teachers truly grasp the ideas of constructions. I will show several examples of student assignments and discuss ways to make the experience meaningful to both math students and future teachers. (Received May 30, 2003)

5003-I1-62 Maria G. Fung\* (fungm@ou.edu), Dept of Mathematics, Western Oregon University, 345 N. Monmouth Ave, Monmouth, OR 97361, and Leon Roland (roland@ou.edu), Dept of Mathematics, Western Oregon University, 345 N. Monmouth Ave, Monmouth, OR 97361. The MathForum Online Mentoring Project.

In this talk we will describe the history, structure and function of the Online Mentoring Project at Western Oregon University. We will discuss how this project constitutes a capstone experience for all pre-service K-8

teachers at WOU. In conclusion we will share some of the reactions of students who have been through this project. (Received June 02, 2003)

5003-I1-65 Timothy D. Comar\* (tcomar@en.edu), Department of Mathematics, Benedictine University, Lisle, IL 60532. A Technology-Enhanced Hyperbolic Geometry Course for Pre-Service and In-Service Secondary School Teachers.

We describe a technology-driven hyperbolic geometry course for pre-service and in-service high school teachers. Familiarity with hyperbolic geometry can deepen a teacher's understanding of Euclidean geometry and of relationships between various areas of mathematics. By using Derive as the primary environment, the students explore geometric phenomena and related mathematical concepts. (Received June 02, 2003)

5003-I1-79 Jim Fulmer\* (jrfulmer@alr.edu), Dept of Mathematics & Statistics, Univ of Arkansas at Little Rock, Little Rock, AR 72204-1099. Using History to Teach Mathematics through a Multimedia Approach.

The history of mathematics can be a valuable tool in teaching many standard topics in the mathematics curriculum. Restructuring courses to include the integration of historical perspectives into the curriculum is a way to include history in many courses. This session will describe a project on using history to teach mathematics through a multimedia approach. The multimedia involves a CD-Rom resource, several Internet resources, and the use of historical posters. The goal is to use the history of mathematics in teaching many standards topics in the mathematics curriculum. The CD-ROM was developed as a Mathematical Association of America (MAA) project with NSF funding. The resource involves ten different modules on various mathematics topics: Archimedes; Combinatorics; Exponentials and Logarithms; Functions; Geometric Proof; Lengths, Areas, and Volumes; Linear Equations; Negative Numbers; Statistics; and Trigonometry. Each module was written by a team of college and secondary school teachers and was field-tested around the country. A module may be reached by clicking on its title and an individual activity of the module may be reached by clicking on the section in the module's table of contents. (Received June 03, 2003)

5003-J1-12 Richard Alan Gillman\* (rick.gillman@alpo.edu), Department of Mathematics & Computer Science, 219 Gellersen Hall, Valparaiso University, Valparaiso IN 46383. Assessment Results from A QL Foundations Course.

In response to calls for improved mathematical skills from client disciplines, the Valparaiso University Department of Mathematics established a course in Quantitative Problem Solving. First offered in the fall of 1996, this course filled the "remedial" tier of the QL program described in the 1995 MAA Report on QL. Classroom sessions focused on improving students' problem solving skills while a CAS was used to review arithmetic and algebra skills. A series of assessment tools have been used in subsequent years to determine the effectiveness of the course at achieving its goals. This presentation is a survey of these assessment tools and the somewhat surprising conclusions that can be obtained from them. (Received April 08, 2003)

5003-J1-14 Richard H. Elderkin\* (relderkin@pomona.edu), Department of Mathematics, Pomona College, Claremont, CA 91711. Environmental Decision Making as a Path to Quantitative Literacy.

Many math-phobic students are completely unaware that math has anything to do with social goals that they value, such as making sound environmental decisions. The purposes of this course are first to offer math in a context that students value, and second to develop useful and intellectually coherent mathematics within that context. The course is at an immediately precalculus level, but is not calculus preparatory. We will discuss some particulars of the curriculum, as well as some challenges of teaching it. (Received May 06, 2003)

5003-J1-42 Mary E. Searcy\* (searcyme@ppstae.edu), Appalachian State University, Dept. of Mathematical Sciences, Walker Hall { Bodenheimer Dr., Boone, NC 28608. Learning to Teach and Teaching to Learn: A Quantitative Literacy Focus in Teacher Education.

"Why" is a small word, but it has the potential for striking terror into the hearts of mathematics students. This is particularly true when the students are elementary and middle school preservice teachers who suddenly realize they will be responsible for helping children learn mathematics that they themselves do not truly understand. To address this issue a two-course sequence for these preservice teachers has been created where "why" is an everyday occurrence and learning mathematics is more than knowing how to do mathematics. Instead of presenting familiar content in a context where students work problems like the ones given in the book, there are

no textbooks and a given problem set will often draw from multiple content areas. The cooperative learning dynamics of the course force students to shift their locus of authority in the classroom from the teacher to themselves. This environment provides opportunities for students to draw out important mathematics concepts from problems, justify solution processes to themselves and others, and make connections between mathematical topics and the world around them. The courses emphasize the fact that it is possible for mathematics to make sense and that "why" can be changed from a threat to a tool for opening the doors to learning. (Received May 21, 2003)

5003-J1-45 Carrie Muir\* (carriem@uclid.colorado.edu), 395 UCB, Boulder, CO 80309-0395. But WHICH Math Class Should I Take?

Many colleges and universities now offer students a wide variety of courses with which to fulfill the mathematics/quantitative literacy portion of their general education requirements. For science, mathematics, and engineering students, choosing a math class is straightforward and simple; their major dictates which course(s) they will take.

For many liberal arts students, the choice is far more difficult. Their majors are unlikely to have a specific course requirement. They may not have enough of a mathematics background to distinguish between courses based on the course descriptions. If they have academic advisors, those advisors may have as weak of a mathematics background as the students.

This session will highlight some of the methods and tools I have developed to help liberal arts students with course selection for our school's quantitative reasoning requirement: assessment and course suggestion tools, reference materials for both students and their advisors, and training for advisors. (Received May 21, 2003)

5003-J1-49 Patricia Clark Kenschaft\* (kenschaft@verizon.net), 56 Gordonhurst Ave., Montclair, NJ 07043. How Much is a Billion?

Comprehension of large numbers is difficult at best, but it is increasingly important for Americans to try. Mathematical techniques include time, graphs, proportions, and examining larger and smaller horizons. Budgets, oil consumption, and driving provide many opportunities for better understanding billions. How much gasoline could be saved if Americans kept their automobile tires properly inflated? If their average gasoline mileage was increased by three miles a gallon? (Received May 27, 2003)

5003-J1-54 Johannes\* (johannes@ember.ams.org), 1 College Circle, Geneseo, NY 14454-1401. Honesty is the Best Philosophy. Preliminary report.

The general population's impression of mathematics depends on how we present mathematics. A Platonist philosophy of mathematics that mathematics has existed forever in some mystical place that we can only access through ascension by experts further serves to separate the laity from comfort with mathematics.

On the other hand, embracing the fact that mathematics is a human creation helps to create an image of mathematics as something that can be understood. This empowers students rather than alienating them.

I have incorporated several discussions and readings on the philosophy of mathematics into my general audience mathematics course and I am impressed and pleased with the eagerness of my students to struggle with these questions.

Our endeavour to improve the mathematical literacy of this country will be improved by understanding and discussing how mathematics is similar and different from other human pursuits. In particular, mathematics is a human creation, not a realm of absolute truths. And so mathematics is nothing more nor less than the discipline that has seen almost universal agreement among humanity across time and cultures. (Received May 28, 2003)

5003-J1-58 William L. Briggs\* (wbriggs@math.cudenver.edu), Mathematics Department, Box 170, University of Colorado at Denver, P.O. Box 173364, Denver, CO 80217-3364. Quantitative Reasoning and Liberal Arts Mathematics.

As individuals and as a society in the 21st century, we face challenges and choices that could affect our survival as a species. Understanding those issues and making informed decisions require fundamental quantitative skills that all college and university students should possess. From decisions about personal finance and voting issues, to choices of food, lottery tickets and computers; from understanding the federal debt to appreciating the mathematics of pollution and deforestation, we are all called to higher levels of quantitative reasoning if we are to be effective citizens.

In this talk, I briefly survey the rationale for developing a quantitative reasoning (QR) course for liberal arts students and discussing the issues that arise in teaching such a course. The remainder of the talk consists of classroom activities, internet projects, and presentations that are used in the University of Colorado course that I teach. (Received May 29, 2003)

5003-J1-82 Jason G. Douma\* (jason.douma@siouxfalls.edu), Department of Mathematics, University of Sioux Falls, 1101 W. 22nd Street, Sioux Falls, SD 57105. Beyond Practicality: Creating a Course that strives to be Profound.

When our liberal arts college began developing an alternative to college algebra that will meet the general education mathematics requirement, concerns were expressed (within the mathematics department and beyond) about depth and level of rigor. Fears of some sort of watered-down "consumer math" course were quickly dispelled as the newly developed quantitative literacy course featured not only applications of mathematics, but also a substantial component oriented toward philosophical matters, such as epistemology, aesthetics, and the nature of analytic reasoning. This talk will highlight the successes and challenges encountered in pursuit of a proper balance between the practical and the impractical. (Received June 03, 2003)

5003-J1-90 Deann A. Leoni\* (dleoni@dcc.edu), Edmonds Community College, 20000 68th Avenue West, Lynnwood, WA 98036, and Rebecca T. Hartzler (rhartzle@dcc.edu), Edmonds Community College, 20000 68th Avenue West, Lynnwood, WA 98036. Addressing Quantitative Literacy Through Math Across the Curriculum.

The Mathematics Across the Curriculum Project was started at Edmonds Community College (Lynnwood, WA) in 1997 to address the Quantitative Skills Assessment Outcome. Now partially funded by the National Science Foundation (DUE # 0088149), the project has grown to include over 70 faculty members from high schools, two-year colleges, and four-year colleges and universities around Washington State. The project supports faculty who wish to integrate mathematics and/or quantitative reasoning into their courses by providing math mentors, guest lectures by math faculty, sample projects and curriculum, and summer institutes for faculty to work on designing curriculum. MAC participants have implemented projects incorporating mathematics or quantitative reasoning into more than twenty different disciplines, including many in the humanities, social science, business, and science divisions. A list of current MAC projects and sample assignments are available on the MAC Web site at <http://mac.edcc.edu>. (Received June 03, 2003)

5003-K1-55 Je® Johannes\* (johannes@ember.ams.org), 1 College Circle, Geneseo, NY 14454-1401. Days are Numbers: Mathematics and the Calendar. Preliminary report.

In this talk we will explore the many historical influences between the calendar and mathematics. As time allows we will discuss some of the following topics and perhaps others: Days in year, lunar v. solar traditions, Julian v. Gregorian calendar, Mayan calendar, French revolutionary calendar. We will also examine some modern proposed alternative calendars. (Received May 28, 2003)

5003-K1-64 Timothy D. Comar\* (tcomar@ben.edu), Department of Mathematics, Benedictine University, Lisle, IL 60532. Geometry and Matrix Groups in Linear Algebra.

We present several activities using the software Derive to integrate geometric and group theoretic notions into a linear algebra course via matrix actions on vector spaces and matrix multiplication. These activities provide additional geometric content to the mathematics major and ease the transition to more abstract courses in algebra. (Received June 02, 2003)

5003-K1-69 Therese N. Shelton\* (shelton@outhwestern.edu), 1001 E. University Ave., Georgetown, TX 78626. Animations Make Mathematics Come Alive.

Animations make mathematics come alive for many students. Even a calculator can do some simple animations, but a computer algebra system allows for more complex and helpful animations. Mathematica was used, but these modules can be easily adapted to Maple. Visualize the concepts of speed and arclength with parametric functions. See the vectors in a vector-valued function trace out a curve. Watch the tangent and normal vectors change along a curve. Technology can allow students to more fully understand that calculus describes motion as well as the effect of parameter changes. See the improvement of a normal  $\bar{t}$  to a binomial distribution as the parameters change. Make your own shows for dynamical systems. The author will share a variety of modules. (Received June 02, 2003)

5003-K1-72 Sarah L. Mabrouk\* (smabrouk@rc.mass.edu), Mathematics Department, Framingham State College, 100 State Street, P.O. Box 9101, Framingham, MA 01701-9101. Interactive MS Excel Tools For Exploring Applications of Quadratic Functions.

In studying quadratic functions, a standard topic in College Algebra and Precalculus courses, examining the function equation, the role of the function coefficients, and a variety of graphs alone is not sufficient for providing



students with a good understanding of quadratic functions. Exploring applications to real situations provides students with an understanding of how the mathematics that they study can be used in real life as well as motivation for learning about these functions. To help students to explore how changes in the function coefficients affect the nature of the problem and its solution, I created interactive MS Excel workbooks that allow students to change problem parameters and to examine how these changes affect the quadratic function associated with the application. These workbooks allow the student to manipulate problem parameters using scrollbars and to explore questions associated with the problem using radio buttons. In this presentation, I will discuss and demonstrate some of these MS Excel workbooks, in particular, projectile motion (a human cannonball problem) and revenue (various). I will discuss design considerations as well as comment on "adventures" that can arise when using technology in and outside of class. (Received June 03, 2003)

5003-K-1-81 Joe Thomas Harris\* (harri sjt@apc.edu), St. Andrews Presbyterian College, 1700 Dogwood Mile, Laurinburg, NC 28352. Applied Numerical Differentiation, Within the Context of Designing An Undergraduate Mathematics Project.

The modeling of physical systems often requires the computation of values of the derivative of a function which is presented as a table of data. The question of how best to do this arose as part of work to design a laboratory project, centered around Newton's law of heating and cooling. Data is collected on the temperature of a heating or cooling object Every thirty seconds for two to three hours. The data is then transferred to MAPLE 8 for analysis. The objective is to gather experimental evidence for an algebraic relationship between the derivative of the temperature, and the temperature. In order to do this, it is necessary to find a reliable way of computing this derivative. The obvious method of taking difference quotients between thirty second intervals does not work, as the measurement apparatus will not measure temperature changes that accurately. A method of numerically computing the derivative, similar to Simpson's rule for integrals, is studied. This method makes it possible to compute accurate values for the derivative; a statistical analysis of the variation in the probe gives a bound on the accuracy of these values. This makes it possible to find a relationship between the derivative of the temperature change, and the temperature itself. (Received June 03, 2003)

5003-K-1-87 David A. Hartenstine\* (hartenst@math.utah.edu), Department of Mathematics, University of Utah, 155 South 1400 East, JWB 233, Salt Lake City, UT 84112-0090, and Renzo Cavalieri (renzo@math.utah.edu), Department of Mathematics, University of Utah, 155 South 1400 East, JWB 233, Salt Lake City, UT 84112-0090. Math Circle, An Outreach Program at the University of Utah.

Math Circle at the University of Utah is a weekly program for high school students. One purpose of the program is to cultivate interest in mathematics in inclined students. Another benefit is that students become familiar with our department and university and the program establishes a link between our department and the high schools. We expose students to interesting, often advanced, topics not normally seen in high school or even in an undergraduate major's curriculum. Another component of Math Circle is giving the students the opportunity to work on challenging problems and to explain their solutions to the rest of the group. We strive to avoid a lecture format, preferring a looser, more fun atmosphere.

I will discuss the format and objectives of our Math Circle and give examples of the topics which led to the best sessions. I will also discuss challenges that we faced and how we attempted to overcome them. We believe that our Math Circle has been a positive experience and that similar programs could be set up elsewhere. With an eye toward this goal, I will conclude the talk with a discussion of what is needed to set up and run the program, and also ideas for adapting our version of Math Circle to different types of institutions. (Received June 03, 2003)

5003-K-1-88 Robert R. Molina\* (molina@ina.edu), Myles F. McNally (mcnally@ina.edu) and Ken W. Smith (smithkw@mich.edu). Characterizing Randomly  $P_k$ -Decomposable Graphs.

The main goal of this paper is to characterize randomly  $P_k$ -decomposable graphs for  $k \geq 10$ . This extends previous results which only went up to  $k = 5$  (L. Beineke, P. Hamburger, W. Goddard, Random Packings of Graphs, Discrete Math., 125 (1994) 45{54). Note that by  $P_k$  we mean a path with  $k$  edges.

A graph  $G$  is randomly  $H$ -decomposable if every family of edge disjoint subgraphs of  $G$ , each subgraph isomorphic to  $H$ , can be extended to an  $H$ -decomposition of  $G$ . Note that if  $G$  is randomly  $H$ -decomposable, then we can select any  $H$ -subgraph of  $G$  and remove its edges, then find any remaining  $H$ -subgraph and remove its edges, and continue in this manner until all of the edges of  $G$  have been removed. For example, the complete bipartite graph  $K_{1,6}$  is randomly  $P_2$ -decomposable since removal of any  $P_2$ -subgraph yields a

$K_{1;4}$  and removal of any remaining  $P_2$ -subgraph leaves a  $P_2$ . The graph  $P_6$ , which is  $P_2$ -decomposable, is not randomly- $P_2$  decomposable.

In this paper we characterize randomly  $P_k$ -decomposable graphs for  $k \leq 10$ , and discuss the basic approach to generating these graphs. Two structural results on randomly  $P_k$ -decomposable graphs which made generating these graphs possible will also be presented. Finally, a number of infinite families of randomly  $P_k$ -decomposable graphs are presented, and some open questions regarding randomly  $P_k$ -decomposable graphs are suggested. (Received June 03, 2003)

5003-K1-93 Mike Pinter\* (pinterm@ai.belmont.edu), Dept. of Mathematics and Computer Science, Belmont University, Nashville, TN 37212. Mathematical Musings and Munchings at Belmont University.

Since 1998, Belmont faculty and students have contributed presentations in the Mathematical Musings and Munchings (MM&M) series on our campus. Faculty presentations include a problem or two from a research area of interest, expository topics of general interest to students across campus, demonstrations of a classroom technique, or historical frameworks for mathematics. Student presentations generally spring from their independent work or undergraduate research. The MM&M series has been generally very successful. Overall, the series helps to promote scholarly efforts (both in the traditional sense of research/scholarship and in the scholarship of teaching and learning). All undergraduates at Belmont are required to accumulate a specified number of "convocation credits" (in addition to academic credits) in order to graduate. Consequently, academic departments across campus are expected to provide convocation opportunities for a wide range of students; the MM&M series serves that purpose. (Received June 03, 2003)

5003-K1-113 Jerry F. Dwyer (dwyer@ath.utk.edu), Department of Mathematics, University of Tennessee, Knoxville, TN 37996, and Katherine E. Hitchcox\* (kathy2h@yahoo.com), Department of Mathematics, University of Tennessee, Knoxville, TN 37996. Science Models in the K-12 classroom.

This paper describes a new course on science models in the K-12 classroom offered by the authors at the University of Tennessee. The course is offered as an elective to students taking a master of mathematics for teachers degree. A set of models are developed and described in the classroom. Students learn some of the mathematics and physical principles behind each model, together with lesson plans for the use of each model in K-12 classrooms. The course is team taught by a mathematics professor, an engineer and a district curriculum specialist. Most of the participants are K-12 teachers, with a focus on middle school math and science teachers.

Each class focuses on an individual scientific model. The mathematical and engineering aspects of the model are explained in detail. The material is presented at a level that is accessible to middle school teachers. Their detailed knowledge enables them to be more innovative in teaching their own students. Experiments on the models take place in class in order to demonstrate how they can be used in the classroom. After the class the teachers are in a position to use the models and explain them to their own students. The teacher knowledge is assessed through standard examinations and supervised hands on activities.

Participating teachers develop lesson plans for the models. The plans include a correlation to the local and state curriculum, and the NCTM Standards. A manual is being prepared, which includes the engineering and mathematical descriptions of the models as well as lesson plans for each model.

This presentation provides an overview of the math and science background of the models and their use in math lessons. These include

- a. A model glacier simulating the movement of an actual glacier and using Measurement, Scales, Proportion, and Water Properties
- b. The basics of composting modeled using a container containing red worms that will compost fruit and vegetable wastes from lunches and using Ratios, Rate, and Ecology
- c. Step-by-step process used to filter water and using Rate, Surface Area, and Density
- d. Looking at the complex circuitry used in computers and other machines, and using Fractions, Units, and Problem Solving
- e. An actual spider web used as the basis for observation and discussion and using Angles, Geometry, and Material Properties
- f. Geographic features displayed and mapped topographically and using 3-D Visualization, Slope, and Co-ordinate Systems
- g. Computer modeling to see the image development of fractal displays and using Geometry and Pattern Recognition.

Results of teacher assessment and follow up are also discussed. (Received June 04, 2003)

5003-K1-114 John E. Beam\* (beam@uwosh.edu), University of Wisconsin Oshkosh, Mathematics Dept., 800 Algoma Blvd., Oshkosh, WI 54901-8631. Probability, Gambling and de Finetti.

In the 1930's, around the same time that Kolmogorov developed the standard axioms for probability theory, de Finetti proposed an alternative model, consistent with Kolmogorov's but more general. He interpreted a

probability as an assignment of fair odds for a bet { an assignment of odds with respect to which it is impossible for a clever gambler to ensure himself a victory against an unwitting opponent. I will illustrate these ideas with some dice-rolling examples, and I will indicate some of the advantages of de Finetti's model over Kolmogorov's. (Received June 04, 2003)

5003-K1-121 William Wardlaw\* (wardlaw@omcast.net), Mathematics Department, United States Naval Academy, 572 Holloway Road, Annapolis MD 21402-5002. A Short Proof That Row Rank Equals Column Rank.

The purpose of this note is to present a short (perhaps shortest?) proof that the row rank of a matrix is equal to its column rank. The proof is elementary and accessible to students in a beginning linear algebra course. It requires only the definition of matrix multiplication and the fact that a minimal spanning set is a basis, and it is valid over any field of scalars. Let  $A$  be an  $m \times n$  matrix. If  $A = 0$ , then the row and column rank of  $A$  are both 0; otherwise, let  $r$  be the smallest positive integer such that there is an  $m \times r$  matrix  $B$  and an  $r \times n$  matrix  $C$  such that  $A = BC$ . Then it follows from the definition of matrix multiplication that the  $r$  rows of  $C$  form a minimal spanning set of the row space of  $A$  and the  $r$  columns of  $B$  form a minimal spanning set of the column space of  $A$ . Hence the row rank  $r$  of  $A$  is equal to the column rank  $r$  of  $A$ . Let  $A$  be an  $m \times n$  matrix. If  $A = 0$ , then the row and column rank of  $A$  are both 0; otherwise, let  $r$  be the smallest positive integer such that there is an  $m \times r$  matrix  $B$  and an  $r \times n$  matrix  $C$  such that  $A = BC$ . Then it follows from the definition of matrix multiplication that the  $r$  rows of  $C$  form a minimal spanning set of the row space of  $A$  and the  $r$  columns of  $B$  form a minimal spanning set of the column space of  $A$ . Hence the row rank  $r$  of  $A$  is equal to the column rank  $r$  of  $A$ . A few more comments quickly show that the rank is less than or equal to the minimum of  $m$  and  $n$ , and that the rank of a matrix is equal to the rank of its transpose. We can extend all of these notions to matrices over commutative rings by adopting the following definition: Definition: Let  $R$  be a commutative ring with identity and let  $A$  be an  $m \times n$  matrix over  $R$ . Then the spanning rank of  $A$  is 0 if  $A$  is 0, and is otherwise the smallest positive integer  $r$  such that there is an  $m \times r$  matrix  $B$  over  $R$  and an  $r \times n$  matrix  $C$  over  $R$  such that  $A = BC$ . This definition also facilitates a short proof that if  $A$  has spanning rank  $r$  then it satisfies the equation  $m(A) = 0$  for a monic polynomial  $m(x)$  in  $R[x]$  of degree less than or equal to  $r + 1$ . (Received June 04, 2003)

5003-K1-122 Lenny Jones\* (lkjone@hip.edu), Department of Mathematics, Shippensburg University, Shippensburg, PA 17257, and Jedd Beall (lkjone@hip.edu), Davis, Renn and Associates, Inc., 16808 Blake Road, Hagerstown, MD 21740. Sequences of Weighted Fibonacci Sums.

Let  $F_1 = F_2 = 1$  and  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 3$ , so that  $F_n$  denotes the  $n$ -th Fibonacci number. In this note we investigate divisibility properties of sequences  $a_n$ , where

$$a_n = \sum_{k=1}^n c_k F_k$$

with the  $c_k$  positive integers. In particular, letting  $f(n)$  denote a polynomial in  $n$ , we show that there exists a constant  $d$ , such that whenever  $f(n)$  is prime,

$$a_n \equiv d \pmod{f(n)};$$

for suitable choices of  $c_k$  and  $f(n)$ . We also investigate the converse as a possible primality test. (Received June 04, 2003)

5003-K1-123 Patricia Kiihne\* (pkiihne@illtop.ic.edu), Department of Mathematics, Illinois College, 1101 West College Avenue, Jacksonville, IL 62650. Reflections on Teaching Quantitative Methods for Business.

This talk will involve sharing ideas used in teaching Quantitative Methods in Business, a class which combines finite mathematics for business and business calculus. I will discuss my experiences in teaching business mathematics classes; this experience ranges from teaching large business calculus classes and large finite mathematics for business classes to teaching smaller classes in quantitative methods in business. Some teaching strategies used in these classes include assessment cards, cooperative learning, and projects. This talk will examine some of the advantages and disadvantages of each strategy and include ways I plan to modify the use of these strategies in my future classes. The intended audience is people who are looking for ideas to use in their business mathematics classes and who want to hear how those ideas worked in some specific situations. The talk will include a brief time at the end to invite audience members to share some strategies they have used in similar classes. (Received June 04, 2003)

5003-K1-124 Barbara L. Bendl\* (b.bendl@si.p.edu), Department of Mathematics, Physics, & Computer Science, University of the Sciences in Philadelphia, 600 South 43rd Street, Philadelphia, PA 19104, and Amy L. Kimchuk (a.kimchu@si.p.edu), Department of Mathematics, Physics, & Computer Science, University of the Sciences in Philadelphia, 600 South 43rd Street, Philadelphia, PA 19104. Mathematics and Biology Integrated Yet Separate.

Barbara Bendl and Amy Kimchuk participated in the MAA/NSF Partnership Workshop for Life Sciences and Mathematics in 2000. The philosophy and materials developed there, linking mathematics and the biological sciences, were incorporated into a two-semester math course (pre-calc and calc I). Two major topics of overlap were selected to model the connections between the two disciplines, representing material in both courses - scientific method (including scientific notation and graphing) and Hardy-Weinberg Equilibrium (a population biology topic using a mathematical equation). Short and long teaching examples were used to illustrate how these topics apply to both disciplines. This integrated yet separate approach was piloted in 2001-2002 with a control and an experimental group. At the end of the pilot project, all students completed a 25 item multiple-choice survey, asking about their perceived interests and abilities in biology and mathematics, how well they understood specific topics (overlap topics and a few topics covered in each discipline alone), and their perceptions of the relationships between these disciplines. Focus groups consisting of students in the control and experimental groups were asked seven questions that more specifically explored their perceptions. (Received June 04, 2003)

5003-K1-125 Kenneth Price\* (pri cek@axa.ci.s.uwosh.edu), UW-Oshkosh, Department of Mathematics, 800 Algoma Boulevard, Oshkosh WI 54901. A Meeting of Majors. Last semester I taught business calculus and linear algebra. I decided to link the two classes. My linear algebra students constructed posters in groups on applications of linear algebra in computer graphics, economics, regression, interpolation, traffic flow, and chemical equations. These posters were constructed for my business calculus students, who critically evaluated the posters. I will discuss the merits of this activity. (Received June 04, 2003)

5003-K1-126 Michelle Ghrist\* (michel le.ghrist@safa.af.mil), Department of Mathematical Sciences, United States Air Force Academy, HQ USAFA/DFMS, U.S.A.F. Academy, CO 80840, U.S.A.. Mathematics as an Art Form: My Journey Inside and Outside the Classroom.

During Spring 2002, I taught an honors seminar class titled, "Mathematics in Prose and Poetry" at Belmont University (in Nashville, Tennessee). When I tell people about this class, I usually get very strange looks; most people find it astonishing that anyone could connect such disparate topics. However, I view this class as a step on my journey towards discovering connections between mathematics and every other topic I learn. In this sense, I view myself as a true applied mathematician. While studying physics, engineering, biology, and fluid dynamics, it was very easy to find links between these topics and mathematics. However, I longed for more connections, especially non-scientific connections. As an autistic, I was fascinated by how many of the mathematicians I met were accomplished musicians. As I searched for connections between mathematics and art, philosophy, music, literature, and poetry, I started to see that mathematics is so much more than its logical, algorithmic foundations. I began to see beauty in mathematics in many different ways and contexts; I started to see it as an art form.

Then, I started thinking about a harder issue: if I truly believe that mathematics is an art form and that "applied mathematics" can be so much more than many people think it is, how do I translate this to a classroom environment? How can I begin to convey this beauty and these connections between mathematics and the rest of the world to my students? What began as a creative "side" project in a College Algebra class ended up culminating itself as the "Mathematics in Prose and Poetry" honors seminar. I began by allowing my students to explore connections between mathematics and creativity. Through the years, I have watched as students created mathematical paintings, wrote poetry, stories, plays, and songs about mathematics, and in the process, opened their minds to the possibility that mathematics is more than all of the algorithmic formulas that they have been led to believe. When my students began to create art about mathematics (and write about the connections between their work and mathematics), they began to view the applied nature of mathematics differently than they had before. They began to experience mathematics as an art form.

In this talk, I hope to convey some of the excitement I have experienced along my journey from mathematical scientist to mathematical artist (and back again). I also will show how I have helped guide students in their journey to discover connections between mathematics and their lives; this will include presenting quite a few creative works that my students have produced. I will also share how I have incorporated these ideas into my technical classes at the Air Force Academy. While I do not pretend to have all of the answers, I hope that each

attendee will come away with practical ways to incorporate "applied mathematics" in their own classrooms and in the world. (Received June 04, 2003)

5003-K-1-129 Janet Heine Barnett\* (janet.barnett@colostate-pueblo.edu), Department of Mathematics and Physics, Colorado State University - Pueblo, 2200 Bonforte Boulevard, Pueblo, CO 81001 - 4901. *Mathematicians in War and Peace: The World Wars.*

Like all other persons, mathematicians must live in human society and respond to the questions of their times. We examine the very different responses of two British mathematicians, Bertrand Russell and Alan Turing, to the profound social and political events of the first half of the twentieth century, both in their mathematical work and in their personal lives. (Received June 04, 2003)

5003-K-1-146 George Winfield Heine\* (gheine@atnhmaps.com), Math and Maps, 200 Sunset Lane, Pueblo, CO 81005. *Mathematicians in War and Peace: The French Revolution.*

Like all other persons, mathematicians must live in human society and respond to the questions of their times. We examine very different responses of two French mathematicians, the Marquis de Condorcet and Lazare Carnot, in their mathematical work and in their lives, to the profound philosophical and political events of the late 1700s. (Received June 04, 2003)

5003-K-1-153 Rob Harger\* (rharger@ihp.us.hi.gov), Mathematics Department, High Point University, 833 Montieu Avenue, High Point NC 27262. *A Generalization of Fermat's Little Theorem.*

Fermat's Little Theorem states that if  $p$  is a prime number and  $\gcd(x; p) = 1$ , then  $x^{p-1} \equiv 1 \pmod{p}$ . If the requirement that  $\gcd(x; p) = 1$  is dropped, we can say  $x^p \equiv x \pmod{p}$  for any integer  $x$ . Euler generalized Fermat's Theorem in the following way: If  $\gcd(x; n) = 1$  then  $x^{\phi(n)} \equiv 1 \pmod{n}$ , where  $\phi$  is the Euler phi-function. It is clear that Euler's result cannot be extended to all integers  $x$  in the same way Fermat's Theorem can; that is, the congruence  $x^{\phi(n)+1} \equiv x \pmod{n}$  is not always valid. In this talk we show exactly when the congruence  $x^{\phi(n)+1} \equiv x \pmod{n}$  is valid. (Received June 05, 2003)

5003-K-1-154 Joseph Corneli\* (jcorneli@mail.nm.utexas.edu), Mathematics Department, University of Texas, RLM 8.100, Austin, TX 78712. *Update on Double Bubbles in Spaces of Constant Curvature.*

We present the latest findings on least-area enclosures of two given volumes in the classical spaces of constant curvature. Our work is inspired by the proof by Hutchings, Morgan, Ritoré and Ros appearing in the March 2002 *Annals of Mathematics* which showed that the least-area two-volume enclosure in  $R^3$  is the standard soap bubble-like double bubble. We consider  $H^3$  and  $S^3$ , where the conjectured minimizer is similar but where the necessary computations are considerably more difficult. We also present a special least-area division of Gauss space of general dimension ( $R^n$  with measure weighted by the Gaussian) that leads us to results for higher dimensional spheres. (Received June 05, 2003)

5003-K-1-155 Kathleen Ambruso\* (Katie.Ambruso@abri.ni.edu), Cabrini College, 610 King of Prussia Road, Radnor, PA 19087, and Robert McGee (brucegee@abri.ni.edu), Cabrini College, 610 King of Prussia Road, Radnor, PA 19087. *Theorems in Color.*

Crockett Johnson patterned a lifelong career path from his interest in art. He worked as an art editor, political cartoonist, a writer and illustrator of books for children, and self-taught mathematician. He is perhaps best known for the children's book, *Harold and the Purple Crayon*.

This talk examines the mathematics of Crockett Johnson. His attraction to the subject inspired the creation of about one hundred mathematical paintings. His work depicted such theorems as: the Pythagorean Theorem, Morely's Theorem, Desargue's Theorem, Pascal's Hexagon Theorem and many more. He was particularly intrigued by the classical Greek Problems and devoted 20-25 published two notes in the *Mathematical Gazette* and these motivated several of his paintings in this area.

Many examples of Crockett Johnson's work will be shown and the corresponding theorems illustrated in each painting will be highlighted. (Received June 05, 2003)

5003-K-1-156 Craig Haile\* (haile@ofo.edu), College of the Ozarks, Mathematics Department, P.O. Box 17, Point Lookout, MO 65726. *Cryptology in the General Education Curriculum.*

For the past two years, a colleague in the Physics Department and I have taught a General Education Capstone course in Cryptology. The Capstone course at our college is meant to be an interdisciplinary course, open to all majors, that requires students to demonstrate the critical thinking, problem solving, and writing skills that they

have developed in the process of their education in the GE curriculum. We believe that Cryptology makes an interesting and challenging Capstone course. Many mathematics teachers are probably aware of how Number Theory and Modern Algebra form the underpinnings of modern cryptosystems, but may not be as familiar with the often sophisticated mathematics underlying the ciphering, deciphering, and cryptanalysis of more "classical" codes and ciphers. In this talk I will highlight some cryptographic schemes and how both paper and pencil and computer algebra systems like Mathematica can be used in the teaching of undergraduates. These activities can be used for those with a minimum of mathematical background, not only in a Cryptology course but also in the GE mathematics curriculum. Specifically, I will comment on the various ciphers and codes which we present in our course, and as time allows I will illustrate our approach in teaching them by focusing on three in particular: Hill's Method, the Enigma, and XOR. Hill's method is a straightforward encryption scheme using matrices, and (optionally) Mod 26 arithmetic. It can be used to demonstrate matrix multiplication, inverses, and applications of the solution of  $n \times n$  linear systems of equations. Next we will look at the famous World War II cipher Enigma. The Enigma can be thought of as a composition of Linear mappings which change with the coding of each letter, and which have their own interesting properties depending upon whether they are rotors, reflectors, or plugboards. We will see how to simulate both its workings and the workings of the machine that ultimately defeated it: the Turing Bombe. The Turing Bombe is the electro-mechanical device that was the brainchild of brilliant mathematician Alan Turing, and is the forerunner to the modern computer. Finally I will look at the XOR cipher, a modern form of the Vigenere square. This cipher represents the bridge between classical cryptosystems and their modern counterparts, and it remains the basic building block of private key systems like DES and AES. In understanding XOR, students learn to map keyboard characters to decimal and then binary representations. They also learn Binary arithmetic and the "group" properties of XOR that prevents multiple encipherment from increasing its security. The cryptanalysis of XOR provides an excellent opportunity for discussion of randomness, statistical probability, and how even the most difficult looking problems are not always as intractable as they may first seem. Time permitting, the encipherment, decipherment, and cryptanalysis of all three algorithms will be demonstrated using Mathematica. We have found Mathematica to be an excellent tool in the teaching of Cryptology, as it gives the necessary computing power for even the most advanced cryptosystems like RSA but also can be very interactive, even when the students have only the most basic familiarity with it. I would be happy to make the Mathematica notebooks for Hill's method, the Enigma, Turing Bombe, and XOR, as well as any other cipher which I have used in the course, available to anyone who desires them. (Received June 05, 2003)

5003-K1-157 Sean Forman\* (sean.forman@sj.u.edu), Math and Computer Science Department, Saint Joseph's University, 5600 Second Avenue, Philadelphia PA 19131. On-line Calculus Readiness Testing.

In an effort to streamline the administration of our annual calculus readiness exam, Saint Joseph's has implemented an on-line exam. Incoming freshmen receive a mailing in May alerting them to the exam and asking them to complete the exam prior to the rostering of classes. They visit the department's website and take the exam through their web browser. The test is a standard algebra and trigonometry exam and typically takes an hour to complete. The test infrastructure was designed in-house (written in perl using MySQL) and has allowed us to implement a number of features. The advantages are obvious. Tests are graded automatically, we can easily generate reports showing the distribution of scores by proposed major, and we can pick out students who previously would have been placed in a less challenging math course. In addition, we often run special topics courses for incoming freshmen with particular needs or interests, and this on-line test allows us to reach these students more easily and gauge their interest. On the negative side, the test is not proctored which can lead to some issues as students misunderstand the point of a placement exam, and technical glitches can lead to a flood of student e-mails. Another goal of the testing program that we have yet to realize is increased enrollment in our calculus courses. As a comprehensive university, only a small percentage of our incoming students take the calculus sequence required of math majors. We are hopeful that identifying prospective calculus students could lead to higher enrollment in these courses. I will discuss some of the mistakes we've made and some of the considerations that other departments may want to consider prior to implementing such a test. (Received June 05, 2003)

5003-K1-158 David J Ashe\* (ashedav@escape.net), UTC, Department of Mathematics, 615 McCallie Avenue, Chattanooga, TN 37403-2598. Proof of a Theorem in Graph Theory.

Given the constraints on the length of this summary, I will write in general terms and omit all but necessary specifics. The theorem being proven is stated below.

Theorem: Let  $F$  be a forest in the complete graph  $K_n$  with  $|E(F)| \equiv 1 \pmod{6}$  where  $E$  is a set of edges. There exists a 6-cycle system of  $G = K_n - E(F)$  if and only if: 1) all vertices in  $F$  have odd degree (so  $F$  is spanning), 2)  $|E(K_n - E(F))|$  is divisible by 6, and 3)  $n$  is even.

Showing that the 3 conditions are necessary is very straightforward, however showing that they are sufficient is much more involved. The approach taken is inductive. The small (base) cases that is when  $n < 10$  are solved by hand, trial and error, brute force, etc Solving the larger cases, required the following lemma.

Lemma: Let  $n$  be even and let  $F$  be a spanning forest of the complete graph  $K_n$  with  $(F)$  components. The number of edges in  $K_n - E(F)$  is divisible by 6 if and only if  $n$  and  $(F)$  are related as shown below.

$$n \equiv 12k \pmod{6} \iff 12k + 2 \pmod{6} \iff 12k + 4 \pmod{6} \iff 12k + 6 \pmod{6} \iff 12k + 8 \pmod{6} \iff 12k + 10 \pmod{6} \iff 0 \pmod{6}$$

Vertices are selected wisely and partitioned from  $K_n$  such that  $(F)$  and the number of vertices remaining in this modified graph call it  $H$  (where  $F \cap H$  is a general summary of the proof, however there are many cases that had to be considered and this process was applied to each of them. In actuality, these 6-cycles (the widget) must be found first keeping in mind that Sotteau forest in  $H$ ) fall into the same category in the table above. The lemma is here is a more general version. It is stated here for 6-cycle 1)  $a$  and  $b$  are even 2) 6 divides  $a$  or  $b$ , 3) minimum  $a, b \geq 4$

Unfortunately, Sotteaus specifically.

Sotteaus then used to show the number of edges in  $H$  that are not in  $F$  are divisible by 6. It must also be shown that all of the vertices in  $F$  have odd degree, and that the number of vertices in  $H$  are even. It can now be shown inductively that the edges in  $H$  not in  $F$  can be partitioned into 6-cycles.

The next step is to partition the edges incident with  $H$  and incident with the removed vertices from  $K_n$  (those not in  $H$ ) into 6-cycles. Sotteau (Received June 05, 2003)

5003-K-1-160 Ronald C. Mullin, Department of Mathematical Sciences, Florida Atlantic University, Boca Raton University 33486, and Ayan C. Mahalanobis\* (amah8857@au.edu), Department of Mathematical Sciences, Florida Atlantic University, Boca Raton University 33486. An Alternative Representation of Finite Fields.

In this paper we are looking at finite fields  $F_{q^n}$  of order  $q^n$  which are extensions of a field  $F_q$  of order  $q$  where  $q = p^k$  for some prime  $p$  and positive number  $k$ . One classical way of defining  $F_{q^n}$  over  $F_q$  is to take a monic irreducible polynomial  $\hat{A}(x)$  of degree  $n$  in  $F_q[x]$  and then define  $F_{q^n}$  as the

quotient  $F_q[x]/(\hat{A}(x))$ . In other words  $F_{q^n}$  is the set of all polynomials of degree less than  $n$ , that is  $F_{q^n}$  can be defined as a vector space over  $F_q$  of dimension  $n$  where  $\{1; x; x^2; \dots; x^{n-1}\}$  is the basis and multiplication is polynomial multiplication modulo  $\hat{A}(x)$ . For further information on elementary theory on finite field see [3]. In this paper we take an alternate view of the situation described above. We first define a free vector space over  $F_q$  and then make it an algebra with a multiplication scheme which is different from polynomial multiplication. Then we show that this algebra is isomorphic to  $F_q[x]$ . Though the isomorphism is not central to our work, it is invaluable in providing directions, as the theory of polynomial ring is well studied. (Received June 05, 2003)

5003-K-1-161 Javad Namazi\* (namazi@du.edu), Math/CS, Fairleigh Dickinson University, Madison NJ 07950. A Problem on Optimizing of Composite Membranes.

The physical problem is: Build a body of prescribed shape out of given materials (of varying densities) so that the body has a prescribed mass and the basic frequency of the resulting membranes (with fixed boundary) is as small as possible. This problem in more general mathematical terms can be stated as: Let  $\Omega \subset \mathbb{R}^n$  be a bounded set with Lipschitz boundary. Let  $\mu > 0$  and  $A \in [0; j]$  be given numbers. For a measurable set  $D \subset \Omega$  let  $\lambda_1(\mu; D)$  be the smallest eigenvalue of the problem

$$-\Delta u + \mu \hat{A}_D u = \lambda_1 u \quad \text{on } \Omega; \quad u = 0 \quad \text{on } \partial \Omega$$

Here the derivatives are understood to be in the weak sense. Define

$$\mu_1(\mu; A) = \inf_{D \subset \Omega} \lambda_1(\mu; D); \quad (1)$$

where  $\hat{A}_D$  is the indicator function of  $D$ . Any minimizer  $(u; D)$  of equation (1) is called an optimal configuration. We will discuss existence, uniqueness, and qualitative properties of this equation. It is known that there is a solution in  $C^{1;\pm}$  for some  $0 < \pm < 1$  that is not unique except for certain nice domains such as a ball. Also  $\mu_1(\mu; A)$  depends continuously on the parameters  $\mu$  and  $A$ . We will list conditions where convexity of  $\Omega$  implies the convexity of  $D^c$ , the complement of  $D$ . Also an interesting feature of equation (1) is its symmetry-breaking properties where for certain range of values of  $\mu$  and  $A$ , the optimal configuration  $D$  fails to be symmetric even when  $\Omega$  itself is. We will also explore these results for a more general differential equation when  $\Delta$  in (1) is replaced by a more general uniformly elliptic differential operator. (Received June 05, 2003)

5003-K1-162 Christopher Aubuchon\* (aubuchoc@adger.jsc.vsc.edu), Department of Mathematics, Johnson State College, Johnson VT 05656. Using Familiar Number Sequences to Help Students Learn to Recognize and Articulate Algebraic Patterns.

Quite often in the teaching of abstract mathematics (particularly number theory) we instruct our students to: (1) List empirical evidence, (2) identify a pattern and generate a hypothesis, and (3) prove this hypothesis true in the general case. While recently teaching an undergraduate course in number theory, I discovered that the above outline under-emphasizes the importance of not only identifying the pattern, but being able to communicate it in mathematical terms. This talk will briefly discuss examples that challenge the student to produce a careful mathematical formulation of patterns that seem obvious to the naked eye, and yet elude immediate precise description. In particular, we will look at interrelationships among polygonal numbers, as well as Fibonacci numbers and Pascal's triangle. (Received June 05, 2003)

5003-L1-27 Ioana Mihaila\* (imihaila@supomona.edu), Mathematics Dept., California State Poly Univ., Pomona, 3801 W. Temple Ave., Pomona, CA 91768. Proofs and Presentations: The Perfect Match.

This talk will discuss the use of student presentations in upper level classes. In the past, the speaker has asked students to prepare and present proofs of theorems, complete with examples and counterexample. The talk will focus on how to get the most out of this technique. Some of the issues discussed are: - time management - how to pick good topics - group presentations vs. individual presentations. - content of presentations - grading the presentations Handouts with examples used in real analysis, advanced calculus and partial differential equations will be available. (Received May 15, 2003)

5003-L1-34 Nancy L. Hagelgans\* (Nagelgans@ursinus.edu), Mathematics and Computer Science Department, Ursinus College, Collegeville, PA 19426. Guided Studying in Upper Level Mathematics Courses.

Study guides for each test have helped my upper level mathematics students focus on learning rather than on wondering what might possibly be asked. Several days before each test, I send the students a study guide via email. The study guide has three parts: lists of vocabulary words, sample questions, and forms of questions. Essentially I ask the students to learn every idea that we have encountered. I view the process of preparing the study guides as a first step in devising the examinations. Students report that they spend much more time studying for tests with these guides, and test scores are usually quite high. (Received May 16, 2003)

5003-L1-35 Mariel Vazquez\* (mariel@ath.berkeley.edu), Mathematics Department, University of California at Berkeley, Berkeley CA 94720-3840, and Javier Arsuaga (jarsuaga@ath.berkeley.edu), Mathematics Department, University of California at Berkeley, Berkeley, CA 94720-3840. DNA knots in the classroom.

DNA topology studies coiling, knotting and linking of circular DNA molecules, as well as their interaction with enzymes. Certain enzymes, such as site-specific recombinases, can cut DNA thus inducing topological changes. Oftentimes recombination yields a spectrum of knotted and/or linked molecules, which can then be analyzed using the Tangle Model. The Tangle Model is a topological method used to shed some light on the enzymatic mechanism.

The tangle model is a very good example of how mathematics can be used to understand biological phenomena. In this example an enzymatic action is translated into the language of knots and tangles. Within the mathematical frame the problem can be reduced to a system of tangle equations that can be further reduced to a system of basic algebraic equations (involving integers and absolute values). The whole process, from stating the biological problem, to solving the tangle equations, and interpreting the solutions in the biological context, is amenable for an undergraduate-level mathematics class. We will also present TangleSolve, a java applet that solves tangle equations and can also be useful as a teaching aid. (Received May 16, 2003)

5003-L1-39 Kyle Riley\* (kyle.riley@dsmt.edu), 501 East Saint Joseph Street, Dept. of Math and Computer Science, South Dakota School of Mines and Technology, Rapid City, SD 57701-3995. Concepts Inventory Assessment in Probability and Statistics.

The innovation of a concepts inventory assessment originated with the Force Concepts Inventory test in 1992 by Hestenes, et al. In our project, we endeavor to apply this assessment technique to measure the ability of



students to comprehend and retain basic concepts from a two-semester sequence of probability and statistics. The development of this type of assessment will provide a quantitative measure on how the curriculum imparts conceptual knowledge. We will review some sample questions and the initial results from students that have recently finished the probability and statistics sequence, as well as results from students that have completed an upper division Industrial Engineering course. (Received May 19, 2003)

5003-L 1-53 Sylvia Forman\* (syforman@ju.edu), Saint Joseph's University, Dept. of Math and Computer Science, 5600 City Ave., Philadelphia, PA 19131. Keeping Abstract Algebra Students on Task.

Even when teaching upper level courses, students may need some motivation and guidance for how to best prepare for class and to absorb the material. Some issues I have had are: students not reading the textbook carefully, students saving weekly problem sets until the night before the due date, and students not preparing for daily in-class presentations of problems and proofs or examples from the textbook. In this talk, I will discuss techniques I used in a one-semester abstract algebra course to alleviate some of the problems described above. (Received May 28, 2003)

5003-L 1-66 David B. Streid\* (dstreid@um.edu), Mathematics Department, Maharishi University of Management, Fairfield, IA 52557. A Mathematics Course for Math and Computer Science Majors.

This talk describes a course on the mathematics of computer graphics taught to upper division mathematics and computer science students. The main topics of the course were (a) computer projections utilizing linear algebra and projective geometry, and (b) curves, using the tools of calculus, linear algebra, and analytic geometry. Since the CS students didn't know proofs, proofs were taught using a programming model of inputs, code, and outputs. Programming projects were sometimes assigned in lieu of proofs. The main outcomes were: (i) Both Math and CS students appreciated the integration of many diverse concepts and techniques of mathematics in a single application, (ii) CS students gained a greater appreciation of mathematics as a practical tool for their profession, and (iii) CS students have become more interested in mathematics, signing up for more math courses and becoming double majors in math and CS. (Received June 02, 2003)

5003-L 1-68 Melvin A. Nyman\* (Nyman@lma.edu), Department of Mathematics & Computer Science, Alma College, 614 W. Superior St., Alma, MI 48801, and Timothy A. Sipka (sipka@lma.edu), Department of Mathematics & Computer Science, Alma College, 614 W. Superior St., Alma, MI 48801. Getting Students to Read the Text in a History of Math Course.

We will discuss a course presentation method that we used recently in a team-taught History of Mathematics course. Probably all mathematics instructors have noticed that students seldom read the textbook for mathematics courses except for the assigned homework. In particular, very few students seem to read the textbook before the material is presented in class by the instructor. As a means of fostering student reading of the textbook material before class discussion we give the students a set of "reading questions" on the material. At the end of each class session we pass out the set of questions on the material to be read for the next class session. This is a single sheet with five to eight questions on factual material throughout the textbook chapter. The questions are discussed and collected at the beginning of the next class session. We found that opening the class session with a review of the questions fosters better student discussion in the classroom. We believe that this method is readily transferable to most undergraduate mathematics courses. (Received June 02, 2003)

5003-L 1-80 Jason J. Moliterno\* (jason\_moliterno@yahoo.com), Department of Mathematics, Sacred Heart University, 5151 Park Avenue, Fairfield, CT 06825-1000. Making Abstract Concepts in Multivariable Calculus Less Abstract.

Multivariable Calculus has many concepts that can be abstract and difficult to visualize using the two-dimensional printed page. The aim of this talk is to give examples of projects in which the students were able to visualize such concepts. Concepts that will be discussed include Quadric Surfaces, Max/Min Problems, Lagrange Multipliers, and the Jacobian. In each of these projects, students were required to use technology (usually MAPLE) to graph certain functions. After constructing such graphs, students were asked to answer questions in which they needed to look at their graphs, think about them, and come to logical conclusions. If time permits, I will also mention ideas I have for when I teach Multivariable Calculus in the future. This will include the positive aspects of the existing projects as well as the aspects of the existing projects that need to be altered - or entirely scrapped! (Received June 03, 2003)

5003-L1-89 G. Daniel Callon\* (dcallon@franklincollege.edu), 501 E. Monroe Street, Franklin IN 46131. Translating Learning into Action: Two Complementary Project-Based Capstone Courses.

Two project-based courses together serve as capstones to the applied mathematics and quantitative analysis tracks for mathematics majors at Franklin College. These courses are designed to provide students a bridge to the professional workplace by simulating the nature and structure of job assignments in business, industry, and government. Besides the appropriate mathematics content, students are exposed to concepts of project management, the consulting process, and giving and receiving feedback.

The course in Numerical Analysis consists of five major team projects. No tests are given. In Regression Analysis and Experimental Design, which is cross-listed as an elective for students in economics, sociology, and psychology, students work as a team to complete a semester-long consulting project for a local non-profit agency. Students in this course have completed one to two semesters of probability and statistics before enrolling in the class.

Examples of projects from both courses will be provided. There will also be a discussion of course organization and assessment techniques and an explanation of the role of these courses in the overall mathematics curriculum. (Received June 03, 2003)

5003-L1-131 Janet Heine Barnett\* (janet.barnett@colostate-pueblo.edu), Department of Mathematics and Physics, Colorado State University - Pueblo, 2200 Bonforte Boulevard, Pueblo, CO 81001 - 4901. Instilling Rigor with the Four R's: Reading, wRiting, Reunions and Revisions.

This talk will provide an overview of two assignments (a Course Reading Notebook and a Personal Homework File) employed by the author in upper division "proof" courses such as Advanced Calculus and Linear Algebra. The primary objective of these assignments is to enhance students' ability to communicate mathematics formally. The Course Reading Notebook does this by supporting student efforts to critically read and comprehend a formal mathematics text, while the Homework File employs a revision policy as a means to support student efforts to write logically valid formal proofs. Additionally, "reunions" (in the form of study groups and required interviews with the instructor) provide students with further learning support, while providing the instructor with assessment data on their progress in meeting course objectives. This talk will present complete logistical details on these assignments, and consider the pros and cons of the instructional approach for both students and the instructor. (Received June 04, 2003)

5003-M1-116 Robert S. Cole\* (RSCole@evergreen.edu), The Evergreen State College, Olympia, WA 98505. Modeling with Stella Software.

Stella software affords a straight-forward entry to systems modeling notions for students from the pre-calculus level to the upper division level. The concepts involved (time-evolution of systems, response to perturbations, resilience, etc.) are relatively sophisticated compared to some of the quantitative topics in environmental studies. The visual nature of Stella diagrams enables students to focus on system behavior and interconnections, rather than on the mathematical syntax. That is an asset for teaching environmental, ecological, or biological concepts. Stella is an ideal tool for teaching interdisciplinary topics, or for teaching mathematical applications in the environmental arena. (Received June 04, 2003)

5003-M1-117 Roland H. Lamberson\* (rhl1@umboldt.edu), Department of Mathematics, Humboldt State University, Arcata, CA 95521-8299. The Management of Endangered Species: Bird Habitat and Salmon Genetics.

Because Field data is usually not available, mathematical models frequently play a crucial role in the study and management of vulnerable fish and wildlife populations. I will present two models that demonstrate the relevance of mathematical results to species preservation. One model demonstrates a threshold effect in density of habitat for territorial bird populations. The second addresses the impact that hatchery management policies have on the genetic structure of a salmon population. (Received June 04, 2003)

5003-M1-118 William D. Stone\* (wstone@mt.edu), Department of Mathematics & Computer Science, New Mexico Institute of Mining & Technology, Socorro, NM 87801. Age-Structured Population Models.

Age-structured models can be used to describe many populations. These models result in systems of iterative equations. We begin with a linear model, modify it with non-linear effects, and then add random effects. A simple model will be developed, and several different analysis approaches will be discussed. (Received June 04, 2003)

5003-M1-119 Albert A. Bartlett\* (Albert.Bartlett@colorado.edu), Department of Physics, University of Colorado at Boulder, Boulder, Colorado 80309-0390. Arithmetic, Population and Energy.

Most people have no idea of the arithmetic and consequences of steady growth of quantities such as populations or rates of consumption of resources. These consequences will be explored. To illustrate people's lack of understanding, quotations from experts, journalists, and scientists will be given. (Received June 04, 2003)

5003-N1-149 Carla Farsi\* (farsi@uclid.colorado.edu), Department of Mathematics, University of Colorado, 395 UCB Boulder, Colorado 80309-0395. Two in One: Mathematics and the Visual; Arts.

In this talk I will detail some aspects of the relationship between visual arts and mathematics by looking at test cases. (Received June 05, 2003)

5003-N1-150 Clifford Singer\*, 510 Broome Street, New York, New York 10013. Geometrical Art as an Applied Science.

Is geometrical art an applied science? There reigns a certain pre-established harmony in geometry, what is required in one line of thought is supplied in another line, so that there appears to be a logical necessity, independent of our individual disposition. It is not in the formalist's mode of thought to deal mainly with the skilful formal treatment of a formal question to devise an algorithm. But, it is rather for the intuitionists who give particular emphasis and stress on geometrical intuition in general and not to give a complete account of a specific subject area. An additive construction as a supplement to a range of mathematical views that find prevalent through geometrical thinking as to gain insight into the shape of curves as far as general classification and an enumeration of all fundamental forms are concerned.

All this suggests, is the question whether it would not be possible to create an abridged system of mathematics that is adapted to the needs of the applied sciences, without passing through the whole realm of abstract mathematics. It is my general aim to gain, in the course of time, a complete view of the whole field of mathematics with particular regard to the intuitional or in the highest sense of the term: geometrical standpoint.

What is the distinction between a naive and refined intuition? It must be stated that the root of the matter is that naive intuition is not exact. While the refining of intuition is not properly intuition at all, however, arises through a rational development of axioms considered as being perfectly correct. To further describe our naive intuition, for example when thinking of a point we do not picture in our mind an abstract mathematical point, but rather substitute something visual and concrete in its place. In imagining a line, we do not picture in our minds a length without breadth, but however a strip of a particular width. We can always imagine a tangent as a straight strip having a small portion or point in common with a curved strip; similarly with respect to an osculating circle or variable curvature.

Professor Joel Castellanos at Rice University, Department of Mathematics, Director of NonEuclid has stated, "I am especially drawn to [Clifford Singer's painting entitled] The Blue Rider. The lines move me in and around and are suggestive of so many different things and actions, yet never settle anywhere."

The tendencies thus characterized have exerted a wide-felt influence, and give a distinctive character to a large part of our present geometrical investigations into visual art. (Received June 05, 2003)

5003-N1-151 James L. Mai\*, School of Art, Campus Box 5620, Illinois State University, Normal, IL 61790-5620. "The Serious Work of Visual Play : Systematic and Geometric Processes in the Generation of Visual Form".

If mathematics may be described (with extreme generalization) as the search for orderly relationships and patterns of magnitude, then in this sense my work in the visual arts may be described as mathematical. My paintings involve formal organizations and processes that share some categorical similarities with mathematics:

arithmetic and geometric progressions; "planar topologies"; symmetries; proportional translations; combinatorial and permutational sets; and "golden section" ( $\phi$ ) divisions. However, the language by which I examine such relationships and patterns is not the mathematician's conceptual vocabulary of numbers and symbols, but rather the painter's perceptual vocabulary of shapes and colors| and this fundamental difference in languages yields some consequent differences in the kinds of questions to be asked and problems to be posed in the two disciplines. My paintings do not propose new (or even illustrate old) mathematical concepts; instead, rational and systematic processes are employed in order to generate new and unanticipated visual forms, to establish objective structures that may be understood by an independent viewer, and to provide the organizational "spirit" that permeates and integrates the "body" of the formal elements in my painting. (Received June 05, 2003)

5003-N1-152        George W. Hart\*, Dept. Computer Science, Stony Brook University, Stony Brook, NY  
11794-4400. Constructive Geometric Sculpture.

Geometric sculpture can be a potent tool for communicating mathematical ideas in a visual and tactile manner. While all art should be enriching and thought provoking, mathematically based art displays additional internal resonance through underlying relationships which appeal to one's sense of system and logic. It draws upon and celebrates the visual and structural modes of thinking which bind art with mathematics. As a mathematician and constructive sculptor, I create works that follow in a centuries-long tradition of mathematically informed art. My motivation, in part, is that I try to convey a sense of what I call the geometric aesthetic. I have the artistic conviction that the patterns and relations found in the classical geometry of three-dimensional structures can form a solid foundation for art that is beautiful, personally affecting, and visually engaging. Slides of geometric sculpture constructed from various materials will be presented. For additional insight and mathematical descriptions of their underlying structure, see the detailed references given in my web pages, <http://www.georgehart.com/> (Received June 05, 2003)

5003-N1-165        Michael J Field\* (mf@h.edu), Department of Mathematics, University of Houston,  
Houston, TX 77204-3008. Illuminating Chaos.

In this illustrated and very visual talk, we show how ideas using chaos and symmetry can be used as a tool for art and design. We briefly indicate some of the ways we have used these ideas in teaching (mathematics to art students in a junior/senior level course in the art department at UH, and to mathematics teachers in the Houston ISD.) (Received June 05, 2003)