

Cheat sheet for “Bovino-Weierstrass and Other Fractured Theorems,” by Matt Koetz, Heather A. Lewis, and Mark McKinzie, *Math Horizons*, April 2014, pp. 26–27

[Find “More Fractured Theorems” on the [Math Horizons website](#). The cheat sheet for those theorems is include below.]

Calculus

Sandwich theorem: If the crusts of your sandwich touch, you’ve run out of filling.

Based on the Sandwich theorem (also known as the Squeeze theorem): Let I be an interval containing the point a , and let f , g , and h be functions such that $g(x) \leq f(x) \leq h(x)$ for all $x \in I$ (except possibly $x = a$). If $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} f(x) = L$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Rolls theorem: If you have a pile of spaghetti on your plate, there is at least one place you can put your meatball so that it doesn’t roll away.

Based on Rolle’s theorem: Let f be differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$. If $f(a) = f(b)$, there is at least one point c in (a, b) where $f'(c) = 0$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Submarine value theorem: While your submersible descends, at some point its instantaneous rate of descent matches its average rate of descent.

Based on the mean value theorem: Let f be differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$. Then there is at least one point c in (a, b) such that the instantaneous rate of change, $f'(c)$, is equal to the average rate of change, $\frac{f(b)-f(a)}{b-a}$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Extreme homework theorem: You put the most effort into your homework right when it’s assigned, right before it’s due, or at a point just before you start slacking off.

Based on the extreme value theorem: A continuous function on a closed interval has both a maximum and minimum value. These values occur either at an endpoint or at a critical point.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Hospital theorem: As the fluids in your intravenous drip run dry, the ratio of antibiotic to saline solution in your IV is equal to the ratio of the flow rate of antibiotics to the flow rate of saline.

Based on L'Hôpital's rule: Suppose that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ are both zero or both $\pm\infty$. Then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

(as long as $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ exists).

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Fundamental theorem of calculus class: Regardless of when you study the material, or how much you cram, all that matters is how much you've learned at the end of the semester.

Based on the fundamental theorem of calculus: If f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Leibniz's alternating series test: If you put a box of Leibniz cookies on a table, take some of the cookies away, then return some of the ones you took, then take away some of the cookies you just returned, then put back some of those, etc. . . , in the end there'll be some cookies on the table.

Based on Leibniz's alternating series test: An alternating series $\sum (-1)^k a_k$ converges if $a_1 \geq a_2 \geq \dots$ and $\lim_{k \rightarrow \infty} a_k = 0$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

National Geographic series test: If each pile of magazines is 1 percent larger than the previous pile, it's time to cancel your subscription.

Based on the geometric series test: $\sum_{k=0}^{\infty} x^k$ diverges when $|x| \geq 1$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Cauchy condensation test: You can get a sense of the opinion of a group by dismissing the opinions of everyone except the 1st, 2nd, 4th, 8th, . . . people.

Based on the Cauchy condensation test: Let $\{a_n\}$ be a sequence of positive terms with $a_{n+1} \leq a_n$. Then $\sum_{n=1}^{\infty} a_n$ converges if, and only if, $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Taylor's theorem: Starting with a rough approximation of a suit, you can get a better fit by sewing on and cutting away smaller and smaller pieces of material.

Based on Taylor's theorem: Any k -times differentiable function f can be approximated by a Taylor polynomial of degree k :

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^k}{k!}f^{(k)}(0) + R_k(x).$$

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Conservative vector field theorem: All Republicans have straight hair (no curl).

Based on conservative vector fields: In a conservative vector field, the curl is always zero; that is, $\nabla \times \vec{F} = \mathbf{0}$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Differential Equations

Piano existence theorem: If you are told what key to play in, and you know the sequence of musical intervals, then you can play the song on a piano.

Based on Peano's existence theorem: If $D \subset \mathbb{R}^2$ is an open set, and $f : D \rightarrow \mathbb{R}$ is continuous, then for every $(x_0, y_0) \in D$, the initial value problem $y(x_0) = y_0$, $y'(x) = f(x, y(x))$ has a solution in some neighborhood of x_0 .

Link:

- [Wikipedia](#)

Oiler's method: You can approximate the net loss of engine oil by measuring the rate oil is leaking at various times and using interpolation.

Based on Euler's method: Euler's method is an implicit method for solving an ordinary differential equation that uses an initial condition and the rate of change at various points.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Analysis and Topology

Bovino-Weierstrass theorem: In any bounded pasture containing an infinite number of cow pies, you can stand in one (rather unpleasant) location where no matter how close you look, there are an infinite number of cow pies near your feet.

Based on the Bolzano-Weierstrass theorem: Every bounded infinite set in \mathbb{R}^n has an accumulation point.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Seinfeld's completeness theorem: Not bounded above? No sup for you!

Based on the completeness of \mathbb{R} : The real numbers are complete, and thus every nonempty subset $S \subset \mathbb{R}$ that is bounded above has a (unique) supremum, or sup.

Links:

- [Wikipedia](#)
- [Wikipedia](#)

Banach-Starsky and Hutch theorem: Given any TV show, you can create two more TV shows, each with the same content as the original.

Based on the Banach-Tarski theorem: It is possible to decompose a ball into six pieces that can be reassembled by rigid motions to form two balls the same size as the original.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Piano space-filling curve: If you overtighten one of the strings, it will break, and there'll be piano wire everywhere inside the soundbox.

Based on the Peano space-filling curve: This space-filling curve (a bijection between the unit interval $[0, 1]$ and the unit square $[0, 1] \times [0, 1]$) was discovered by Giuseppe Peano.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Tickng off theorem: You can verify that a direct product of topological spaces is compact by checking, one at a time, that each of the individual spaces is compact. (Is K_1 compact? CHECK! Is K_2 compact? CHECK! ...)

Based on Tychonoff's theorem: The topological product of any number of compact spaces is compact.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Jordan curve theorem: No matter how complex international borders become, at any given time you are either inside Jordan or outside Jordan.

Based on the Jordan curve theorem, also known as the Jordan-Brouwer theorem: If J is a simple closed curve in \mathbb{R}^2 , then $\mathbb{R}^2 \setminus J$ has two components: the inside and the outside.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Logic and Set Theory

Law of excluded middle: Your teacher will or will not tell jokes in class.

Based on the law of the excluded middle: For any statement A , either A or $\sim A$ must be true.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Girdle incompleteness theorem: No matter how well-designed your girdle is, there is at least one person for whom it will not work.

Based on Gödel's incompleteness theorem: A consistent, effectively generated formal theory of arithmetic cannot also be complete; that is, there is an arithmetical statement that is true, but not provable in the theory.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Well-ordering theorem: If you're eating out with friends, someone will have the cheapest meal.

Based on the well ordering of the natural numbers: Any nonempty subset of \mathbb{N} has a least element.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Axiom of choice: You can illustrate the utility of an abstract concept by giving an example of how it is used from each course in which it arises.

Based on the axiom of choice: Given any set of mutually disjoint nonempty sets, there exists at least one set that contains exactly one element in common with each of the nonempty sets.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Arrow's theorem: It is not possible to make this theorem both funny and interesting.

Arrows' theorem: An archery contest is just as fair as any other method for determining the winner of an election with at least three candidates.

Based on Arrow's theorem: There is no nondictatorial voting method that satisfies both the Pareto condition and independence of irrelevant alternatives with three or more candidates.

Link:

- [Wikipedia](#)

Graph Theory

Oiler's theorem: If there are more than two T intersections in your town, then a highway maintenance crew cannot spray a single layer of fresh oil on each road without turning off the sprayer.

Yuler's theorem: If your neighborhood has houses on both sides of every street, then it is always possible for a group of holiday carolers, singing continuously, to sing exactly once at every house, even if they mispronounce some of the words.

Based on Euler's theorem: An Eulerian path is a walk on the edges of a graph that uses each edge exactly once. A connected graph has an Eulerian path if, and only if, it has at most two vertices of odd degree.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Hamilton Bulldogs theorem: You can go on a road trip with the Hamilton Bulldogs and visit every team in the American Hockey League exactly once before returning to Hamilton.

Based on Hamiltonian paths: A Hamiltonian path is a walk on the edges of a graph that visits each vertex exactly once.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Current off-ski theorem: If your circuit has a subdivision isomorphic to $K_{3,3}$ or K_5 , then burning the circuit to a chip will result in a short.

Based on Kuratowski's theorem: Every nonplanar graph contains either the complete bipartite graph $K_{3,3}$ or the complete graph K_5 as a graph minor.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Probability and Statistics

Tschubby chefs theorem: At least $3/4$ of all cooks have a body mass index within two standard deviations of the population mean.

Based on Tschebyscheff's inequality (also known as Chebychev's inequality): In every probability distribution, at least $1 - 1/k^2$ proportion of the data is within k standard deviations of its mean.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Central heating theorem: If there are enough heating vents in your house, the average temperature throughout your house will more closely match the setting on the thermostat.

Based on the central limit theorem: For any distribution, the distribution of sample means approaches a normal distribution (with the same mean as the original).

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

San Francisco Bay's theorem: The probability that you are a Giants fan, given that you live in Oakland, can be computed using the probability that you live in Oakland, the probability that you are a Giants fan, and the probability that you live in Oakland, given that you are a Giants fan.

Based on Bayes's theorem: $P(A | B) = \frac{P(A)P(B|A)}{P(B)}$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Cramér vs. Cramér's theorem: If independent parents have a normal child, then they are normal too.

Based on Cramér's theorem: If X and Y are independent variables and $X + Y$ is a normal distribution, then both X and Y have normal distributions.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Abstract Algebra

Galois-lapalooza theorem: If you're at a music festival, there is a one-to-one correspondence between the groups playing and the fields on which they play. The larger the group, the smaller the (unoccupied) field.

Based on Galois theory: For fields $K \subset F$, let $\text{Aut}(F/K)$ denote the group of automorphisms of F that leave K fixed. There is a one-to-one correspondence between intermediate fields H with $K \subset H \subset F$ and the subgroups of $\text{Aut}(F/K)$. This correspondence is inclusion-reversing: if $H_1 \subset H_2$, then $\text{Aut}(F/H_2) \subset \text{Aut}(F/H_1)$.

Link:

- [Wikipedia](#)

See-low theorem: You can find subgroups whose order is small.

Based on the first Sylow theorem: For any prime p , if p^n divides the order of a group G , then G has a subgroup of size p^k for $k = 1, \dots, n$.

Link:

- [Wikipedia](#)

LaGrunge band theorem: If you form a subgroup of a Seattle musical group, you will divide it.

Based on Lagrange's theorem: The order of a subgroup divides the order of the group.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Number Theory

Fundamental theorem of a limerick:

Any number you pick, I dare say,
When factored in any old way,
Results in some primes
Together with times,
Unique up to order. Hooray!

Based on the fundamental theorem of arithmetic: Every positive integer (except the number 1) can be represented uniquely (up to order) as a product of prime powers.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Num3rs sieve: Netflix recommends shows by eliminating ones that don't fit your preferences.

Based on number sieve (also known as the sieve of Eratosthenes): This is a process for finding prime numbers by eliminating all the composite numbers.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Chinese remainder theorem: You can figure out how much Chinese takeout you ordered if you remember how many leftover boxes there were when you grouped the boxes by twos, threes, and fives.

Based on the Chinese remainder theorem: It is always possible to find a positive integer n from the remainders when n is divided by a sequence of relatively prime numbers, although n is not unique.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Format's last theorem: If you reformat your hard drive, the margin of this paper IS big enough to contain all of your recoverable data.

Based on Fermat's last theorem: No three positive integers a , b , and c can satisfy the equation $a^n + b^n = c^n$ if n is an integer greater than two. It is famous because Pierre Fermat wrote in the margin of a textbook that he had a proof of the theorem, but the margin was too small to contain it. More than 300 passed before a proof was published.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Cheat sheet for “More Fractured Theorems”

Calculus

Extreme grading theorem: If your professor is continuous on a closed vacation interval, then she will grade the homework at a critical point (when it’s due) or on the first or last day of the vacation.

Based on the extreme value theorem: A continuous function on a closed interval has both a maximum and minimum value. These values occur either at an endpoint or at a critical point.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Fig Newton method: While eating Fig Newtons, you can approximate when you’ll run out by assuming that you’ll keep eating at your current rate.

Based on Newton’s method: Newton’s method is an iterative algorithm for finding the root of an equation by using a current approximation and rate of change.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Divergent series test: If each book in Veronica Roth’s series has at least one page, her fans will never run out of reading material.

Based on the divergent series test (also known as the nth term test): If the terms of an infinite sequence do not approach zero, then the sum of the sequence must diverge.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Comparison test: If you have a higher score than Pat on every question on the exam, then your overall score will be higher than Pat’s.

Based on the comparison test: If $a_1 \leq b_1$, $a_2 \leq b_2$, \dots then $\sum a_k \leq \sum b_k$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Chia Pet trick series test: If your Chia pet learns 1/2 of a trick, then the next 1/4 of the trick, then the next 1/8 of the trick, then eventually it will know the trick sufficiently well.

Based on the geometric series test: $\sum_{k=0}^{\infty} x^k$ converges to $1/(1-x)$ when $|x| < 1$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Green's theorem: You can measure the size of the target on a golf hole by integrating $x dy$ around the perimeter of the target.

Based on Green's theorem: The value of a line integral around a simple closed curve is equal to an integral over the region inside the simple closed curve. In particular,

$$\oint_{\partial D} f(x, y) dx + g(x, y) dy = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA.$$

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Analysis and Topology

A housed-off space: You can build separate houses at distinct addresses on the street.

Based on a Hausdorff space: A topological space is Hausdorff if any two points have disjoint neighborhoods.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Zorn's lemma: Since every list of mathematicians can be alphabetized, there must be some mathematician whose name occurs at the end of the alphabet.

Based on Zorn's lemma: If S is any nonempty partially ordered set in which every chain has an upper bound, then S has a maximal element.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Banana fixed-point theorem: If you squish a banana without breaking the peel, there is a unique point of banana that is in the same place it was before.

Based on the Banach fixed-point theorem: Let f be a contraction mapping from a closed subset F of a Banach space E into F . Then there exists a unique $z \in F$ such that $f(z) = z$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Jean-Luc Picard's theorem: As the *Enterprise* passes through a wormhole, Patrick Stewart will perform every Shakespearean role (except Hamlet) infinitely often in his ready-room.

Based on Picard's theorem: Every analytic function assumes every complex value, with possibly one exception, infinitely often in any neighborhood of an essential singularity.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Graph Theory

Puck's theorem: If you create a polygonal hockey arena on a lattice and put hockey pucks at each of the grid intersections, the area of the arena will be one less than the number of hockey pucks in the middle plus half the number of pucks on the boundary.

Based on Pick's theorem: Let A be the area of a simply closed lattice polygon. Let B denote the number of lattice points on the polygon's edges and I the number of points in the interior of the polygon. Then $A = I + \frac{1}{2}B - 1$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Abstract Algebra and Number Theory

Filbert's basis theorem: If there is a largest nut in a can of mixed nuts, then any handful of nuts will contain finitely many filberts.

Based on Hilbert's basis theorem: Every ideal in the ring of multivariate polynomials over a Noetherian ring is finitely generated.

Links:

- [Wikipedia](#)

Bailey's theorem: Every drink is isomorphic to one made at a well-stocked bar.

Based on Cayley's theorem: Every group is isomorphic to a subgroup of a permutation group.

Links:

- [Wikipedia](#)

Gesundheit's theorem: If the trajectory of Chris's sneeze is a degree m polynomial and the trajectory of Pat's sneeze is a degree n polynomial, then there are mn places where you really don't want to stand.

Based on Bézout's theorem: Two algebraic curves of degrees m and n intersect in $m \cdot n$ points.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Buy no meal coefficient: If you and Pat eat at a restaurant 7 times this week, $\binom{7}{n}$ computes the number of ways you can be treated to dinner n of those times.

Based on binomial coefficients: The number of ways to pick n items out of 7 is written $\binom{7}{n}$.

Links:

- [Wolfram|Alpha](#)
- [Wikipedia](#)

Probability and Statistics

Bouffant hairdo theorem: If you randomly toss hairpins at a well-coiffed woman from the 1950s, you can predict the proportion that will stick in her hairdo based on the length of the hairpin.

Based on the Buffon needle experiment: If a floor is made of parallel strips of wood, you can compute the probability that a needle will lie between two strips based on the length of the needle.

Links:

- [Wikipedia](#)

Mark-off chain: If you mark one link in a chain, you know what to put on the next link without looking at all the previous links.

Based on Markov chains: A Markov chain is a mathematical system in which the next state depends only on the current state, and not on any previous states.

Links:

- [Wikipedia](#)