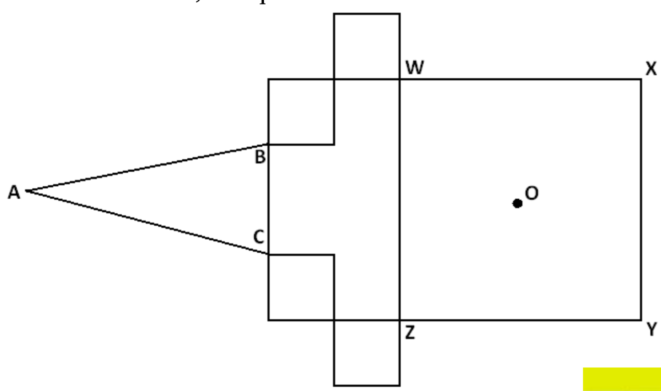
Curriculum Burst 121: A Complicated Area

By Dr. James Tanton, MAA Mathematician in Residence

In the figure, the area of square $WXYZ$ is $25 cm^2$. The four smaller squares have sides $1 cm$ long, either parallel to or coinciding with the sides of the large square. In ΔABC, $AB = AC$, and when ΔABC is folded over side $\overline{BC}$, point A coincides with O, the center of square $WXYZ$. What is the area of ΔABC, in square centimeters?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS: Geometry

COMMON CORE STATE STANDARDS

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: **PERSEVERANCE IS KEY**

SOURCE: This is question # 25 from the 2003 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

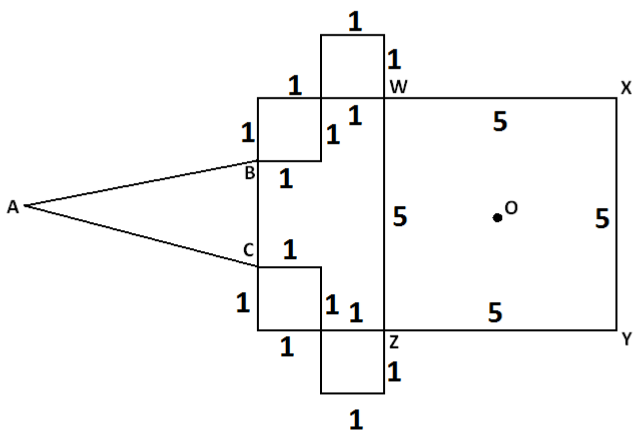
This question seems crazy! At first it looks like it is a bit overwhelming, but as I read it, I realize each piece of information given makes sense. Let me start just by adding to the diagram everything I am told.

In the figure, the area of square $WXYZ$ *is* $25cm^2$.

So we have several side-lengths of 5 cm.

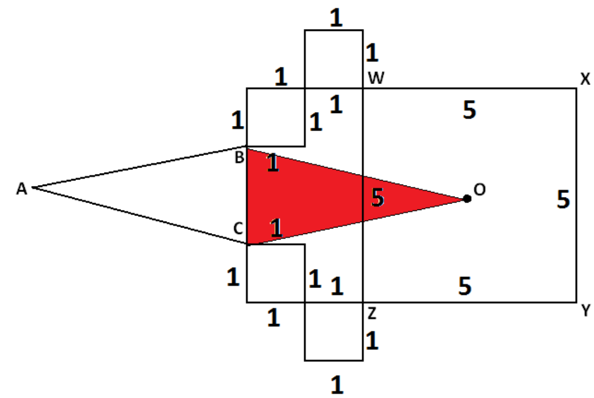
The four smaller squares have sides $1cm$ *long, either parallel to or coinciding with the sides of the large square.*

And we have some lengths of 1 cm. Here they all are:



In ΔABC, $AB = AC$, *and when* ΔABC *is folded over side* $\overline{BC}$, *point* A *coincides with* O, *the center of square* $WXYZ$.

Okay, we have an isosceles triangle that folds over like this:

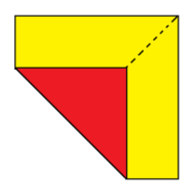


What is the area of ΔABC, *in square centimeters?*

Well, the area of ΔABC is the same as the area of the shaded triangle in my picture. I see that the base of this triangle, $\overline{BC}$, has length $5-1-1=3$ cm. The height of the triangle from this base is $1+1+2\frac{1}{2}=4\frac{1}{2}$ cm. So the area of the triangle, "half base times height," must be: $\frac{1}{2}\times 3\times 4\frac{1}{2}$. This is $\frac{1}{2}\times 3\times\frac{9}{2}=\frac{27}{4}$ square centimeters. Done!

Extension 1: We never used that fact that ΔABC is an isosceles triangle. Must this triangle be isosceles?

Extension 2: The corner of a square piece of paper is folded to a point somewhere along the diagonal of the square as shown:



If the area of the paper exposed to air on the top side of the paper equals the area of the paper of the bottom side of the paper exposed, where exactly along the diagonal is the corner of the paper?

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