Curriculum Burst 109: A Weird Calculator

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A certain calculator has only two keys $[+1]$ and $[\times 2]$. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed $[+1]$, it would display "10." If you then pressed $[\times 2]$, it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "200"?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number Sense

COMMON CORE STATE STANDARDS

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 2: **DO SOMETHING**

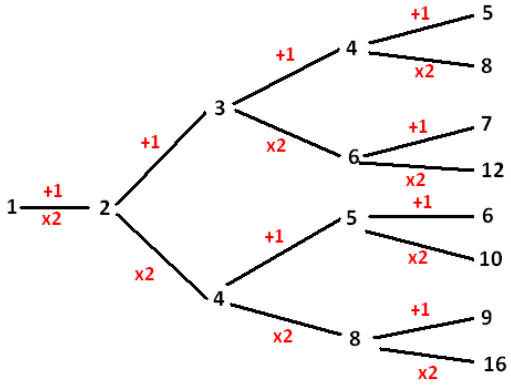
SOURCE: This is question # 24 from the 2005 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

You know, I am just going to try going through all the possibilities! I need to get a feel for it. How about a diagram like this?



Okay. This is getting pretty overwhelming pretty quickly! But I do see that the $[\times 2]$ operation gets to bigger numbers more quickly. So maybe we can do lots of $[\times 2]$s and get to 200?

Well, we can try to get to 100 and then do $[\times 2]$. Actually to 50 and then do $[\times 2]$ and $[\times 2]$. Actually to 25 and then do three $[\times 2]$s.

Hmm. To get 25 we have to do a $[+1]$ from 24. But thinking backwards this way is good and shows me how to get to 200 in nine moves:

1 [x2] 2 [+1] 3 [x2] 6 [x2] 12 [x2] 24 [+1] 25 [x2] 50 [x2] 100 [x2] 200

Now the question is: Is this the best number? Could we get to 200 in just eight moves?

Hmm.

Well, 200 could come from 100 with $[\times 2]$, or it could come from 198 from $[+1]$ and $[+1]$. So we have two options to explore:

1. Can we reach 100 in seven moves?
2. Can we reach 198 in six moves?

Actually, the answer to the second question is no. Since $2^6 = 64$ and the $[\times 2]$ gets us to big numbers the quickest, we can't reach 198 in six moves.

Option 1? It splits into two possibilities:

3. Can we reach 50 in six moves?
4. Can we reach 98 in five moves?

Again, the second option is out because 2^5 is only 32.

Option 3 splits:

5. Can we reach 25 in five moves?
6. Can we reach 48 in four moves?

The second is out (as $2^4 = 16$ is not large enough.)

Option five becomes:

7. Can we reach 24 in four moves?

And the answer is no because, again, $2^4 = 16$ is not large enough. Phew!

It is impossible to reach 200 in eight moves, but it can be obtained in nine. The answer to the question is 9.

Extension 1: What is the minimum number of keystrokes needed to get to one million?

Extension 2: How many different numbers can you obtain if each button $[+1]$ and $[\times 2]$ is pressed once in some order? Each pressed twice? Each pressed five times?

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.

MAA acknowledges with gratitude the generous contributions of
the following donors to the Curriculum Inspirations Project:

The TBL and Akamai Foundations
for providing continuing support

The Mary P. Dolciani Halloran Foundation for providing seed
funding by supporting the Dolciani Visiting
Mathematician Program during fall 2012

MathWorks for its support at the Winner's Circle Level