

Curriculum Inspirations

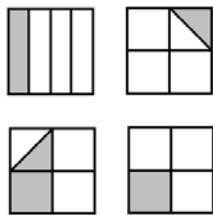
Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 88: Percent Shaded

By Dr. James Tanton, MAA Mathematician in Residence

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry: Area; Number Sense: Percentages

COMMON CORE STATE STANDARDS

- 6.G.A1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems
- 6.RP.A3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: [AVOID HARD WORK](#)

SOURCE: This is question # 7 from the 2011 MAA AMC 8 Competition.

[Click here for video](#)



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

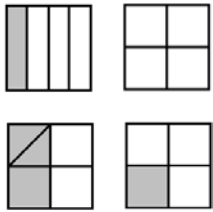
I am a little confused by this question. There are four pictures, and I could work out the percentage shaded in each. So is this really a four-part question with four separate answers?

When confused, reread the question!

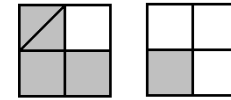
The question asks for the percentage of the total area that is shaded. I guess this means this is one picture.

Okay, but do I want to work out four percentages and add them up? I suppose I could but that seems like hard work.

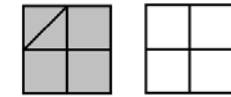
But look ... put the two triangles together, and we can look at the shaded parts of this diagram instead.



In fact, the top left square is divided into four parts and the shaded region in it is equivalent to the area of one sub-square. So we can work with this picture:

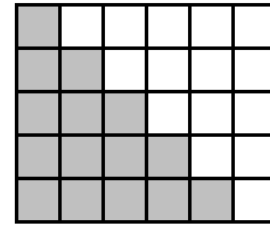


We might as well put all the shaded regions together!



So one quarter of the entire diagram is shaded. That's 25% . Cool!

Extension: What's $1 + 2 + 3 + 4 + 5$? Explain how the following figure gives the answer:



What's $1 + 2 + 3 + 4 + \dots + 56 + 57$?

Draw an enlightening picture for $3 + 6 + 9 + 12 + 15$!

Curriculum Inspirations is brought to you by the [Mathematical Association of America](http://www.maa.org) and the [MAA American Mathematics Competitions](http://www.maa.org).

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