

# Curriculum Inspirations

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MAA American Mathematics Competitions



## Curriculum Burst 84: Repeated Visits

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Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?

### QUICK STATS:

#### MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grade levels.

#### MATHEMATICAL TOPICS

Double Counting.

#### COMMON CORE STATE STANDARDS

**S-CP.B7** Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model

#### MATHEMATICAL PRACTICE STANDARDS

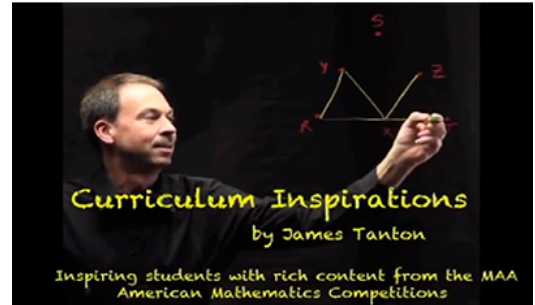
- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

#### PROBLEM SOLVING STRATEGY

ESSAY 2: [DO SOMETHING](#)

**SOURCE:** This is question # 17 from the 2013 MAA AMC 10A Competition.

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## THE PROBLEM-SOLVING PROCESS:

As always ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I need to get a feel for this problem.

If today is day 1, then Alice will visit on days 3, 6, 9, 12, ..., and Beatrice on days 4, 8, 12, 16, ..., and Claire on days 5, 10, 15, 20, .... These day numbers are nice multiples!

Is it possible for two people to visit on the same day? Sure: For example, on day 12 both Alice and Beatrice visit, and on day 20 both Beatrice and Claire visit.

Actually, on days that are both multiples of 3 and of 4, that is multiples of 12, both Alice and Beatrice visit. Similarly on days that are both multiples of 4 and 5, that is, multiples of 20, both Beatrice and Claire visit. And on days that are multiples of  $3 \times 5 = 15$ , both Alice and Claire appear.

Alos, on days that are multiples of 3, 4, and 5, all three friends appear.

Okay, here's a table summarizing visit information:

A days:	3, 6, 9, 12, 15, ..., 363.
B days:	4, 8, 12, 16, 20, ..., 364.
C days:	5, 10, 15, 20, ..., 365.
AB days:	12, 24, 36, 48, ..., 360.
BC days:	20, 40, 60, 80, ..., 360.
AC days:	15, 30, 45, 60, ..., 360.
ABC days:	60, 120, 180, ..., 360.

Notice that  $363 = 121 \times 3$  so there are 121 "A days."  
There are 91 B days, and 73 C days.

Let's keep going. There are:

$$\frac{360}{12} = 30 \quad \text{AB days,}$$

$$\frac{360}{20} = 18 \quad \text{BC days,}$$

$$\frac{360}{15} = 24 \quad \text{AC days,}$$

and

$$\frac{360}{60} = 6 \quad \text{ABC days.}$$

So there are  $30 + 18 + 24 = 72$  days where two people visit?

I am pausing because I am worried about my counts overlapping. For example, day 60 is counted as an AB day, and as an BC day, and as an AC day, and the question doesn't want to count it at all! (There is that word *exactly* in the question.) It got counted three times when it should be counted zero times. Ditto for days 120, 180, 240, 300, and 360. So our answer is an over-count of three six times. The correct answer is thus:

$$72 - 3 \times 6 = 54.$$

Tricky!

**Extension:** Daphne keeps a notebook. Each evening she writes down a triple of numbers  $(a, b, c)$  where  $a$  is the number of days since she last saw Alice (with  $a = 0$  meaning Alice came that day),  $b$  is the number of days since seeing Beatrice, and  $c$  the number of days since seeing the Claire. Will there be a day of the year she writes the triple  $(2, 1, 4)$ ? Which day of the year will be the first day she repeats a triple previously written?

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