

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 83: Biking Towards Each Other

By Dr. James Tanton, MAA Mathematician in Residence

Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grade levels.

MATHEMATICAL TOPICS

Velocity and Rates of Change

COMMON CORE STATE STANDARDS

A-CED.1 Create equations and inequalities in one variable and use them to solve problems.

MATHEMATICAL PRACTICE STANDARDS

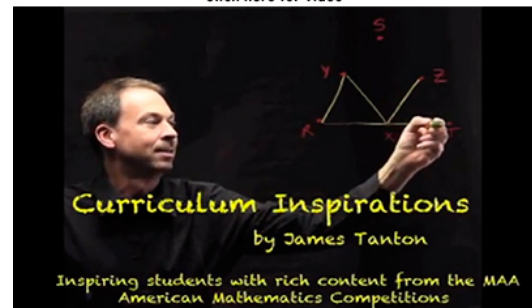
- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 4: [DRAW A PICTURE](#)

SOURCE: This is question # 20 from the 2009 MAA AMC 10A Competition.

[Click here for video](#)



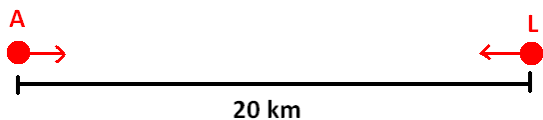
THE PROBLEM-SOLVING PROCESS:

As always, the best start is ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I have to draw a picture! I really can't take in this question without a sketch.

Andrea and Lauren are 20 kilometers apart. They bike toward one another...



Andrea traveling three times as fast as Lauren ...

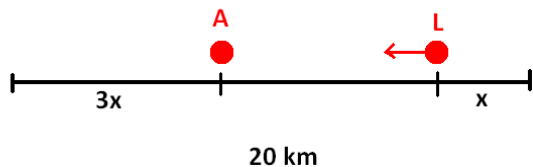
Okay. Not sure how to sketch that!

... and the distance between them decreasing at a rate of 1 kilometer per minute.

Also don't know how to sketch this!

After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren.

Five minutes



Okay. At the five minute mark we have something like the picture. Since Andrea is traveling three times as fast, she traveled three times the distance.

After how many minutes from the time they started to bike does Lauren reach Andrea?

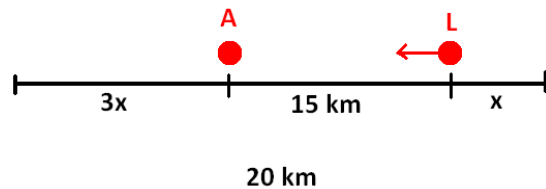
This seems hard to answer. So... let's not answer it! (At least not yet!)

Is there anything else I can add to the picture? The only piece of information I haven't yet sketched is "the distance between them decreasing at a rate of 1 kilometer per minute."

So what does this mean for this picture at the 5 minute mark?

Well... the distance between them decreased one kilometer each minute for five minutes. The gap between them was initially 20 km, so now it is 15 km.

Five minutes



From the picture we have $4x + 15 = 20$, giving $x = \frac{5}{4}$. I

guess that $\frac{5}{4}$ km actually. So Lauren traveled $\frac{5}{4}$ km in five

minutes. Her speed is $\frac{5/4}{5} = \frac{1}{4}$ km/min.

Okay. That feels helpful. What was the question?

After how many minutes from the time they started to bike does Lauren reach Andrea?

So Lauren is going to ride a total distance of $15 + \frac{5}{4}$ km at a

speed of $\frac{1}{4}$ km/min. It is going to take her

$15 + \frac{5}{4} \div \frac{1}{4} = 60 + 5 = 65$ minutes to do this. Wow. Done!

Extension: A fly starts at the eastern-most point of Andrea's front wheel and flies straight towards Andrea's front wheel. When it reaches the wheel it instantly turns around and heads back towards Lauren's front wheel. When it reaches it, it instantly turns around and heads back to Andrea's wheel, and so on, doing this back and forth until the fly, sadly, is squished between the two front wheels when Andrea reaches Lauren. What is the total distance the fly traveled if it flew at a constant speed of a) 1 km/min? b) 1000 km/min? (Dare I ask how many times the fly touched Andrea's front wheel in each case?)

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