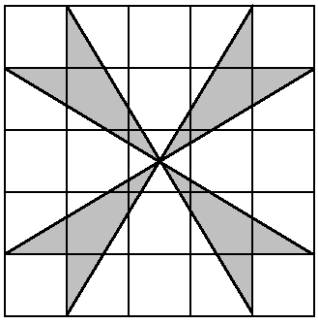
Curriculum Burst 65: Pinwheel Area

By Dr. James Tanton, MAA Mathematician in Residence

What is the area of the shaded pinwheel shown in the 5×5 grid?



QUICK STATS:

Click here for video

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grades.

MATHEMATICAL TOPICS

Geometry

COMMON CORE STATE STANDARDS

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: **AVOID HARD WORK**

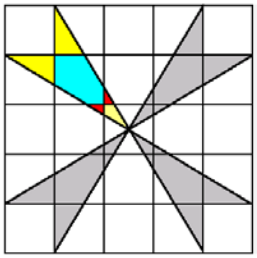
SOURCE: This is question # 23 from the 2007 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

As always, the best start is …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

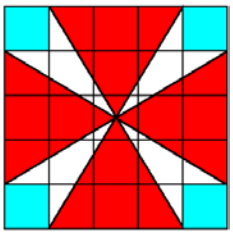
This question feels manageable. I can just divide the pinwheel into a whole bunch of separate pieces and work out the areas of those pieces in turn.



Actually, on second thought, that seems hard: some of the pieces have weird shapes! I suppose I could divide those shapes into triangles too, but now the problem is starting to feel like a lot of work.

Is there a way to avoid hard work? Hmm.

Oh …. Look at this!



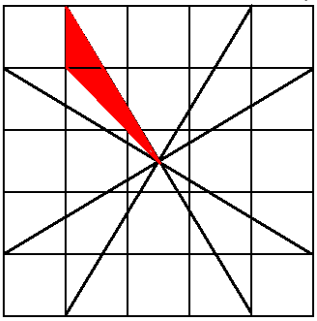
The region outside of the pinwheel is composed of four triangles of area $\frac{1}{2}\cdot 3\cdot 2\frac{1}{2}$ and four squares of area 1.

Thus the area of the pinwheel is:

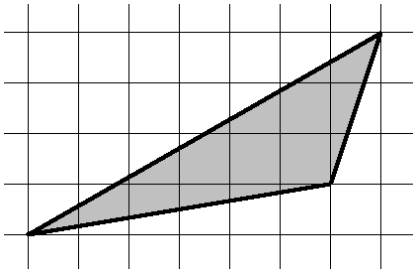
$$\begin{aligned} & 25-4\times\frac{1}{2}\cdot 3\cdot 2\frac{1}{2}-4\times 1 \\ &= 25-1\cdot 3\cdot 5-4 \\ &= 25-15-4 \\ &= 6. \end{aligned}$$

Done!

Extension 1: What is the area of half of a pinwheel spoke?



Extension 2: A "lattice triangle" is a triangle drawn on a grid of unit squares with each corner of the triangle lying at an intersection point of the grid.



Prove that the area of a lattice triangle is sure to be an integer or a half integer.

Is it possible to draw a lattice equilateral triangle?

Comment: For the answer to this question – and more about lattice triangles and lattice polygons – see http://www.jamestanton.com/wp-content/uploads/2012/03/Cool-Math-Newsletter_December2013_LATTICE-POLYGONS.pdf

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