

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 21: Pencil Prices

By Dr. James Tanton, MAA Mathematician in Residence

A majority of the 30 students in Ms. Demeanor's class bought pencils at the bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was \$17.71. What was the cost of a pencil in cents?

SOURCE: This is question # 10 from the 2011 MAA AMC 10a Competition.

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the 10th grade level.

MATHEMATICAL TOPICS

Algebra

COMMON CORE STATE STANDARDS

A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGIES

ESSAY 8: [SECOND-GUESS THE AUTHOR](#)

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THE PROBLEM-SOLVING PROCESS:

The first step ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question feels as though it was very carefully constructed: there is a lot of very specific special information in this piece. And it feels as though it is going to take a lot of work just to understand the question – let alone solve it.

I bet there is something special about the amount \$17.71. I doubt that number is a random choice.

Since the question asks about a cost in cents, let's think of this amount as 1771 cents.

Now to the question. Let's read it again slowly, putting things into "math speak" as we go along.

** Students in the class bought pencils.*

Let S = the number of students who bought pencils. As this is the majority of 30 students we have $S > 15$.

** Each student bought the same number of pencils.*

Let N = the number of pencils each student bought. We're told $N > 1$.

** Each pencil has a cost in cents that is greater than the number of pencils each student bought.*

Let C = the cost in cents of each pencil. We have $C > N$.

** The total cost of all the pencils is 1771.*

There are S students each buying N pencils, so that's a total of SN pencils. Each pencil costs C cents, so the total cost is:

$$SNC = 1771.$$

Okay .. so what's special about 1771? I can't see anything of note about it right away.

Does the number factor? Well, it's not divisible by 2 (or any even number) and it is not divisibly by 5.

Is it divisible by 3? Well, 1800 is a multiple of three, and 1771 is 29 less than this. But since 29 is not a multiple of three, 1771 fails to be one as well.

Okay ... 7? (Oh heavens!) Well, 1400 is a multiple of seven:

$$1771 = 1400 + 371$$

Oh, let's pull out 350, another multiple of seven.

$$1771 = 1400 + 350 + 21$$

Yes! We have: $1771 = 7 \times (200 + 50 + 3) = 7 \cdot 253$.

Does 253 factor? Well the outer two digits add to the middle, which means $253 = 11 \times 23$. (Do you know this property of multiples of 11? Have a look at www.jamestanton.com/?p=1287.)

Okay, so $1771 = 7 \times 11 \times 23$. And this is as far 1771 factors since each of 7, 11 and 23 are prime.

Umm. Does this prime factorization help? We have:

$$SNC = 1771$$

with $S > 15$ and $C > N > 1$. This asks as to consider 1771 as a product of three numbers (each different from 1). But as 1771 factors into three primes and so the factors S , N and C must be the primes 7, 11, and 23 in some order. In fact, the inequalities force:

$$S = 23, C = 11 \text{ and } N = 7.$$

The cost of each pencil is 11 cents.

Extension: i) Suppose p , q and r are three different prime numbers. How many factors, in total, does $A = pqr$ possess? ii) Suppose p and q are two different primes and $B = p^2q$. How many factors does B possess. If $SNC = B$, do you still know what S , N and C must be (up to some order)? iii) How many different factors does $D = p^3q^5r^{107}$ possess?

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