Introduction

Mathematics is universal: it underlies modern technology, informs public policy, plays an essential role in many disciplines, and enchants the mind. At the start of the twenty-first century, the undergraduate study of mathematics can and should be a vital and engaging part of preparation for many careers and for well-informed citizenship. In the *CUPM Guide 2004*, the term ‘mathematics’ is generally synonymous with ‘mathematical sciences’ and refers to a collection of mathematics-related disciplines, including, but not necessarily limited to, pure and applied mathematics, mathematics education, computational mathematics, operations research, and statistics. Departments of mathematical sciences can and should play a central role in their institutions’ undergraduate programs. The *CUPM Guide 2004*[3] calls on mathematicians and mathematics departments to rethink the full range of their undergraduate curriculum and co-curriculum to ensure the best possible mathematical education for all their students, from liberal arts students taking just one course to students majoring in the mathematical sciences.

The need for action

Over the past one hundred years mathematics has become more important to more disciplines than ever before. At the same time both the number and diversity of students in post-secondary education and the variety of their mathematical backgrounds have increased dramatically. Additionally, computer technology has forever altered the way mathematics is used in the workplace; from retail store registers to financial institutions to laboratories doing advanced scientific research.

These developments present unprecedented curricular challenges to departments of mathematical sciences—challenges many departments and individual faculty members are engaged in meeting. During the past twenty years there has been an explosive increase in the number of presentations and publications on issues and innovations in the teaching of post-secondary mathematics.[4] This activity reflects a growing movement to address the undergraduate mathematics curriculum conscientiously and creatively.

Yet at many institutions, students and faculty outside mathematics perceive mathematics departments as uninterested in adapting instruction to new circumstances, especially to the needs of non-majors and those in entry-level courses. Indeed, many view the formal study of mathematics as irrelevant or tangential to the needs of today’s society. They see mathematics departments as disconnected from other disciplines except through a service component that they believe is accepted only reluctantly and executed without inspiration or effectiveness. Such views were expressed by a majority of the academic deans at the

[3] The MAA publication *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*, MAA, 2001, available at `www.maa.org/guidelines/guidelines.html`, complements the *CUPM Guide 2004* and other curricular reports by presenting a set of recommendations that deal with a broad range of structural issues that face mathematical sciences departments and their institutional administrations.

[4] For example, at the January 2003 Joint Mathematics Meetings, more than one third of the talks concerned mathematics education.

research universities sampled for the American Mathematical Society (AMS) study *Towards Excellence*: "The prevalent theme in every discussion [with deans] was the insularity of mathematics. Mathematicians do not interact with other departments or with faculty outside mathematics, many deans claimed. … The deans … seemed to view mathematics departments as excessively inward looking."[5]

This perception is often due more to poor communication than to a lack of effort or good intention. At the least, it points to the need for better communication. But there are other indicators that all is not well. National data provide clear evidence that undergraduate mathematics programs are under serious pressure, with decreasing numbers of mathematics majors and declining enrollment in advanced mathematics courses.[6] From 1985 to 2000 the total number of bachelor's degrees awarded annually in the U.S. rose 25% and the number of science and technology degrees grew 20%. However, data collected by the Conference Board of the Mathematical Sciences (CBMS)[7] show that the total number of degrees awarded annually by mathematics and statistics departments, including those in secondary mathematics education, stayed essentially flat during this 15-year period. In fact, the annual total in these departments fell 4% between 1995 and 2000, and the number of annual degrees in mathematics fell 19% in the 1990s. The drop in mathematics degrees occurred at the same time as the need for new teachers of secondary mathematics grew more acute.

One might expect the increase in science and technology degrees to translate into higher enrollment in advanced mathematics courses as allied subjects. In fact, the opposite has occurred: enrollment in advanced courses taught in mathematics departments has fallen, dropping 25% from 1985 to 2000; an increase from 1995 to 2000, while encouraging, has not returned enrollment to 1990 levels. Further, CBMS data show that even the *availability* of advanced courses has declined in the past five years, as the percentages of departments offering several typical courses[8] has decreased, in some cases by more than 20%. This trend is unfortunate, not only for the health of the mathematical sciences major but also because the health of disciplines that use mathematics—and by extension the health of society—is enhanced when a significant number of students are knowledgeable about the advanced mathematics that is relevant to their fields.

Striking successes at a number of colleges and universities demonstrate that these perceptions can be changed and these trends reversed. For instance, the MAA volume *Models That Work: Case Studies in Effective Undergraduate Mathematics Programs,*[9] summarizes effective practices at a set of mathematics departments that have excelled in (i) attracting and training large numbers of mathematics majors, or (ii) preparing students to pursue advanced study in mathematics, or (iii) preparing future school mathematics teachers, or (iv) attracting and training underrepresented groups in mathematics. Site-visits to ten departments and information on a number of others revealed "no single key to a successful undergraduate program in mathematics." However, there were common features. "What was a bit unexpected was the common attitude in effective programs that the faculty are not satisfied with the current program. They are constantly trying innovations and looking for improvement."

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[5] *Towards Excellence: Leading a Doctoral Mathematics Department in the 21st Century*, American Mathematical Society Task Force on Excellence, J. Ewing editor, AMS, 1999, p. 65.

[6] See Appendix 3 for further analysis of data on numbers of majors and the supply of secondary teachers of mathematics, and Appendix 4 for data on enrollment in and availability of advanced courses.

[7] *CBMS 2000: Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*, D. Lutzer, J. Maxwell and S. Rodi, AMS, 2002.

[8] Including algebra, analysis, geometry, mathematical modeling and applied mathematics; see Table 4-3 in Appendix 4.

[9] MAA Notes **38** (1995), Alan C. Tucker, editor.

Areas for attention and action

Mathematics departments need to serve *all* students well—not only those who major in the mathematical or physical sciences. The following steps will help departments reach this goal.

- Design undergraduate programs to address the broad array of problems in the diverse disciplines that are making increasing use of mathematics.
- Guide students to learn mathematics in a way that helps them to better understand its place in society: its meaning, its history, and its uses. Such understanding is often lacking even among students who major in mathematics.
- Employ a broad range of instructional techniques, and require students to confront, explore, and communicate important ideas of modern mathematics and the uses of mathematics in society. Students need more classroom experiences in which they learn to think, to do, to analyze—not just to memorize and reproduce theories or algorithms.
- Understand and respond to the impact of computer technology on course content and instructional techniques.
- Encourage and support faculty in this work—a task both for departments and for administrations.

The *CUPM Guide 2004* presents six general recommendations to assist mathematics departments in the design and teaching of all of their courses and programs. It also contains supplementary recommendations for particular student audiences.

1. Understand the student population and evaluate courses and programs

In summarizing the common features of the programs described in *Models That Work,* the authors wrote that one of the "states of mind that underlie faculty attitudes in effective programs" is "teaching for the students one has, not the students one wished one had." *Towards Excellence* echoes this theme: "Mathematics departments should position themselves to receive new or reallocated resources by meeting the needs of their institutions. That does not mean sacrificing the intellectual integrity of an academic program, nor does it mean relegating mathematics to a mere service role. It *does* mean fulfilling a bargain with the institution in which one lives, and for most departments a major part of that bargain involves instruction."[10]

Recommendation 1: Mathematical sciences departments should

- Understand the strengths, weaknesses, career plans, fields of study, and aspirations of the students enrolled in mathematics courses;
- Determine the extent to which the goals of courses and programs offered are aligned with the needs of students as well as the extent to which these goals are achieved;
- Continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts.

2. Develop mathematical thinking and communication skills

The power of mathematical thinking—pattern recognition, generalization, abstraction, problem solving, careful analysis, and rigorous argument—is important for every citizen. It is highly valued by employers and by other disciplines but widely misunderstood and undervalued by students.

[10] *Towards Excellence*, p. xiii.

Communication is integral to learning and using mathematics, and skill in communicating is commonly listed as the most important quality employers seek in a prospective employee.[11] However, many students expect mathematics classes to be wordless islands where they won't be asked to read, write, or discuss ideas.

Appropriate instructional approaches to reasoning and proof have been passionately debated among mathematicians for decades, but with a greater sense of urgency during the last twenty years. While much remains to be learned about how best to teach reasoning and proof skills—as well as how best to improve communication skills—a variety of strategies can help students progress. Mathematics faculty should deliver an unambiguous message concerning the importance of mathematical reasoning and communication skills and adopt instructional methods and curriculum content that develop these skills. Designing a curriculum that develops these skills effectively and at appropriate levels for all students is one of the biggest and most important challenges for mathematics departments.

Recommendation 2: Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to

- State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;
- Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;
- Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.

3. Communicate the breadth and interconnections of the mathematical sciences

Many students do not see the connections between mathematics and other disciplines or between mathematics and the world in which they live. Too often they leave mathematics courses with a superficial mastery of skills that they are unable to apply in non-routine settings and whose importance to their future careers is unrecognized. Conceptual understanding of mathematical ideas and facility in mathematical thinking are essential for both applications and further study of mathematics, yet they are often lost in a long list of required topics and computational techniques. Even when students successfully apply mathematical techniques to problems, they are often unable to interpret their results effectively or communicate them with clarity.

The beauty, creativity, and intellectual power of mathematics and its contemporary challenges and discoveries, are often unknown and unappreciated. The interplay between differing perspectives—continuous and discrete, deterministic and stochastic, algebraic and geometric, exact and approximate—is appreciated by very few students, even though flexible use of these varying perspectives is critical for applications and for learning new mathematics.

Recommendation 3: Every course should strive to

- Present key ideas and concepts from a variety of perspectives;
- Employ a broad range of examples and applications to motivate and illustrate the material;

[11] See, for instance, surveys by the National Association of Colleges and Employers, `www.naceweb.org`.

- Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects;
- Introduce contemporary topics from the mathematical sciences and their applications, and enhance student perceptions of the vitality and importance of mathematics in the modern world.

4. Promote interdisciplinary cooperation

Mathematics programs have traditionally drawn heavily from the physical sciences for applications. In recent years, mathematics has come to play a significant role in far more disciplines, but many mathematics programs have not adjusted to this new reality.

Mathematics departments should seize the opportunity to harness the growing awareness in other disciplines of the power and importance of mathematical methods. A curriculum developed in consultation with other disciplines that includes a variety of courses and degree options can attract more students, help them learn important mathematical ideas, retain more students for intermediate and advanced coursework, strengthen their ability to apply mathematics to other areas, and improve the quantity and quality of the mathematics majors and minors.

Recommendation 4: Mathematical sciences departments should encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses, create joint or cooperative majors, devise undergraduate research projects, and possibly team-teach courses or units within courses.

5. Use computer technology to support problem solving and to promote understanding

Recent advances in desktop and handheld computer technology can be used to improve the pedagogy and content of mathematics courses at all levels. Some mathematical ideas and procedures have become less important because of these emerging technological tools; others have gained importance. The 2001 *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences* recommended that departments "should employ technology in ways that foster teaching and learning, increase the students' understanding of mathematical concepts, and prepare students for the use of technology in their careers or in their graduate study." [12] However, much remains to be done to effectively meet the challenges posed by the growth of technology.

Recommendation 5: At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology

- Appropriately and effectively as a tool for solving problems;
- As an aid to understanding mathematical ideas.

6. Provide faculty support for curricular and instructional improvement

Many of the recommendations in the *CUPM Guide 2004*, including collaborating with colleagues in other disciplines, adapting material from other parts of mathematics or from other disciplines for use in teaching, evaluating student writing, and making effective use of technology, require time and effort from faculty beyond what they might ordinarily devote to the revision and creation of courses. Departments and administrators need to acknowledge that meeting these recommendations makes substantial demands on faculty (and, in some cases, on graduate teaching assistants and other temporary or part-time instructors).

[12] *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*, MAA, 2001.

Recommendation 6: Mathematical sciences departments and institutional administrators should encourage, support and reward faculty efforts to improve the efficacy of teaching and strengthen curricula.

Using the CUPM Guide 2004

Part I of the *CUPM Guide 2004* elaborates on and specifies the meaning of the six general recommendations as well as suggesting ways that a department can evaluate progress in meeting them. Part II contains supplementary recommendations concerning particular student audiences.

Some students major in fields that do not require specific mathematical preparation. They may take one course in mathematics, perhaps to satisfy a general education requirement of their institution or major program. Section A of Part II addresses the needs of these students, many of whom—especially among the hundreds of thousands enrolled each semester in courses called College Algebra—are not optimally served by the mathematics courses they take.

Partner disciplines are those whose majors are required to take one or more specific mathematics courses. These disciplines vary by institution but usually include the physical sciences, the life sciences, computer science, engineering, economics, business, education, and often several social sciences. Recommendations concerning these students are in Section B of Part II, including those for pre-service K–8 teachers.

Section C of Part II contains recommendations concerning students majoring in the mathematical sciences. The recommendations urge departments to learn the probable career paths and needs of their majors and offer them a flexible program that provides appropriate breadth and depth. Section D contains further recommendations for mathematical sciences majors preparing to teach secondary school mathematics, planning for non-academic employment, or intending post-baccalaureate study.

There are many ways to carry out each recommendation, and different choices will be appropriate in different institutional settings. Consequently, these recommendations rarely specify particular courses or syllabi. That doesn't mean "anything goes." Indeed, each recommendation is accompanied by measures to help a department gauge its effectiveness.[13] As stated in the *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*, "These measures will, of necessity, be multi-dimensional since no single statistic can adequately represent departmental performance with respect to most departmental goals. Measures of student learning and other student outcomes should be included."[14] Course syllabi and sample assignments, along with their contribution to students' grades, are other valuable measures.

Although no specific methods for implementation are outlined, the hyper-linked web document *Illustrative Resources for CUPM Guide 2004* at `www.maa.org/cupm/` is designed to help departments implement and improve practices to satisfy the recommendations. It is organized and numbered the same way as the recommendations in Parts I and II. A variety of examples, including assignments, courses (with suggested syllabi and texts), and programs are provided for each recommendation. The examples range along a continuum, from modest first steps and small changes that can be easily effected to more ambitious efforts. Pointers to additional resources, such as websites and publications, are also given, with live links where appropriate.

These recommendations have been reduced to a core judged essential for building and supporting department strength and effectively meeting department obligations. They are not a wish list for an ideal future department. Indeed, the reality is that departments at many institutions are coping with diminished

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[13] Also see Appendix 6, Sample questions for department self-study.

[14] MAA 2001, available at `www.maa.org/guidelines/guidelines.html`.

human and financial resources and conflicting and escalating demands on faculty time. Moreover, meaningful change is never easy. Nonetheless, the use of the word "should" in a recommendation means that departments are expected to make a conscientious effort to achieve steady improvement until they are able to satisfy it.

Background for the recommendations

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments in designing curricula for their undergraduate students. CUPM began issuing reports in 1953, updating them at roughly 10-year intervals. In 1999 work began on the current recommendations. CUPM solicited position papers from prominent mathematicians and conducted panel discussions and focus groups at national meetings to obtain reactions to preliminary drafts of these recommendations. There has been extensive consultation with other professional societies in the mathematical sciences. From 1999 to 2002 CUPM's subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY) conducted a series of workshops on the mathematics curriculum with participants from a broad range of partner disciplines.[15]

The MAA Board of Governors has endorsed the six fundamental recommendations for the design and teaching of all courses and programs. CUPM unanimously approved the supplementary recommendations in Part II of the *CUPM Guide 2004*. Many of the current recommendations echo those in previous CUPM reports, but some are new. In particular, previous reports focused on the undergraduate program for mathematics majors, although with a steadily broadening definition of the major in the 1981 and 1991 reports. The *CUPM Guide 2004*, in contrast, addresses the *entire* college-level mathematics curriculum, for *all* students, even those who take just one course.[16]

[15] See Appendices 1 and 2 for detailed accounts of CUPM's activities and of the CRAFTY workshops collectively known as the Curriculum Foundations project. The results of the project are contained in the MAA publication *The Curriculum Foundations Project: Voices of the Partner Disciplines,* edited and with an introduction "A Collective Vision: Voices of the Partner Disciplines" by Susan Ganter and William Barker (MAA, 2004).

[16] While attempting to address the college-level curriculum in mathematics more comprehensively, the *CUPM Guide 2004* does not discuss a number of important issues, including non-credit or developmental courses and articulation between institutions.