Preface

Calls for Change

For over 35 years the Committee on the Undergraduate Program in Mathematics (CUPM) has helped provide coherence to the mathematics major by monitoring practice, advocating goals, and suggesting model curricula. This volume brings together various curriculum reports issued during the decade of the 1980's. It provides a convenient reference for the mathematical community as it begins to reshape college mathematics in response to mounting demands for change.

Many of the calls for reform have been expressed in published reports, for example:

1989 *Curriculum and Evaluation Standards for School Mathematics*, published by the National Council of Teachers of Mathematics.

1989 *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, published by the National Research Council.

1988 *Changing America: The New Face of Science and Engineering*, published by the White House Task Force on Women, Minorities, and the Handicapped in Science and Technology.

1987 *The Underachieving Curriculum*, published by the Second International Mathematics Study.

1986 *Towards a Lean and Lively Calculus*, published by the Mathematical Association of America.

1985 *Integrity in the College Curriculum: A Report to the Academic Community*, published by the Association of American Colleges.

1984 *Renewing U.S. Mathematics: Critical Resource for the Future*, published by the National Research Council.

Other pressures for change are expressed in articles diffused throughout the literature on a wide range of issues, from remediation (too much) through Ph.D. production (too little), from students (greater diversity) to mathematics (greater applicability), and from technology (under-utilized) to pedagogy (too passive). What all reports have in common is the case they make for significant change in undergraduate mathematics to serve better the needs of students who will live and work in the twenty-first century.

Background

CUPM was established in 1953 to "modernize and upgrade" the mathematics curriculum and to halt what was even then decried as "the pessimistic retreat to remedial mathematics." At that time total enrollment in college mathematics courses in the United States was approximately 800,000; each year about 4,000 students received a bachelor's degree in mathematics, and about 200 received Ph.D. degrees.

In its early years CUPM concentrated on proposals to strengthen undergraduate preparation for graduate study in mathematics. Spurred on by Sputnik and assisted by significant support from the National Science Foundation, the mathematical community in the United States matured rapidly from a servant discipline indentured to science and engineering to vigorous world leadership. By 1970 U.S. mathematics departments produced 24,000 bachelor's and 1,200 doctoral degrees—a six-fold increase in less than twenty years.

Then the bubble burst. As student interest shifted from personal goals to financial security, and as computer science began to attract increasing numbers of students who in earlier years might have studied mathematics, the numbers of mathematics bachelor's degrees dropped by over 50% in ten years, as did the number of U.S. students who went on to a Ph.D. in mathematics. In 1981, at the nadir of B.A. productivity, CUPM published a comprehensive report entitled *Recommendations for a General Mathematical Sciences Program.* Significantly, this report advocated not a strengthened program in traditional (pre-doctoral) mathematics, but a broad, innovative program in *mathematical sciences.*

Although CUPM did not create the movement towards mathematical sciences, its 1981 report helped legitimize a process that was well under way. As a consequence, mathematics programs in U.S. colleges and universities are now dominated by variations on two paradigms that reflect the two phases of CUPM activity. The first, a fading image of the CUPM recommendations of the 1960's, focuses on core mathematics as preparation for graduate study in mathematics. The second, reflecting the broader objectives of CUPM's 1981 mathematical sciences report, focuses on mathematical tools needed for a "life-long series of different jobs." Most campuses support a mixed model representing a locally devised compromise between these two standards.

Issues of the 1980's

During the 1980's several issues emerged that had great bearing on the conduct of undergraduate mathematics. Pressure from the computer science community created a demand for a freshman or sophomore course in discrete mathematics. This posed issues of definition (what was to be included?), level (how much maturity was required?), and articulation (where could it fit into the ubiquitous calculus sequence?). No single answer emerged, and experiments continue to determine locally optimal strategies for meeting this important new need.

As American society moved towards greater concern with material well-being, pressure from many sources—not the least being from parents and schools in affluent districts—created enormous demand for high school calculus. Suddenly large numbers of students came to college with uneven preparation that partially overlapped standard introductory college courses. Problems of articulation between high school leaving and college entering became quite intense. The new *Standards* for school mathematics of NCTM promise to increase the diversity of student preparation in years ahead, as some districts adopt new programs and others retain old habits.

From a different source—mostly from parents and public officials concerned with the quality of higher education—came calls for assessment and evaluation to ensure that all students receive certain minimal skills from their college study. Quantitative literacy (or "numeracy") joined the litany of demands generated by discussions of "cultural literacy" and "competitiveness." Quantitative competence and mathematical appreciation of students who do not study mathematics for professional reasons became—and still is—a major concern on college campuses.

The Mathematical Association of America responded to each of these issues—discrete mathematics, school articulation, quantitative literacy—by a variety of studies, some under CUPM, some in cooperation with other organizations. Issued at different times throughout the past decade, these studies supplement CUPM's comprehensive recommendations for an undergraduate program with specific recommendations in areas of timely concern. They are all gathered in this volume, where together they provide a thorough airing of issues pertinent to reshaping college mathematics.

The More Things Change ...

Despite the many new issues that have arisen in recent years (e.g., desk-top workstations, changing demographics, calculus reform), curriculum reports in this volume—which were generally written three to ten years ago—align remarkably well with contemporary calls for change that one hears at every professional meeting. Today's advice, by and large, is no different than yesterday's. It is just being said with greater urgency.

Here, for example, is a sample of recommendations to be found among the reports in this volume:

ON GOALS:

- A mathematical sciences major should develop a student's capacity to undertake intellectually demanding mathematical tasks.
- A major in mathematical sciences should emphasize general mathematical reasoning as much as mastery of various subject matter.
- The instructor's central goal should be to teach students how to learn mathematics, expecting that students will correctly retain only a tiny portion of what was taught.
- A mathematical sciences curriculum should be designed around the abilities and academic needs of the average student, with supplementary work available to attract and challenge more advanced students.
- College students must understand the historical and contemporary role of mathematics and be able to place the discipline properly in the context of other human intellectual achievement.
- Students should gain an ability to read and learn mathematics on their own. Such maturity is as much a function of *how* mathematics is learned as *what* is learned.

ON TEACHING:

- Students should be led to discover mathematics for themselves, rather than merely being presented with the results of concise, polished theories.
- The approach to most topics should involve an interplay of applications, problem-solving, and theory. Applications should motivate theory so that theory is seen by students as useful and enlightening.
- Freshman courses in mathematics should be designed to appeal to as broad an audience as is academically reasonable.
- In the first two years, theorems should be *used* rather than *proved*. The place for theoretical rigor is in later upper-level courses.
- The greatest challenge is that students enter college with much less mathematics than they used to, but they expect to leave with much more.

ON COMPUTING:

- Students should make full use of calculators and computers in all mathematics courses.
- All mathematical sciences students must be given an introduction to the basic concepts of computer science.
- Computing assignments should be used in most mathematics courses.

ON MODELING:

- Applications and modeling should be included in a nontrivial way in most college-level mathematical sciences courses.
- All students majoring in mathematics should undertake some real-world mathematical modelling project.

ON WRITING:

- Explaining a mathematical result in terms of a real-world setting involves the need to communicate in a precise and lucid manner. This aspect of a mathematical scientist's training should not be left to courses in other sciences or to on-the-job learning after graduation.
- Teachers of mathematics should employ strategies that encourage student reading, writing, and reflection.
- Students should be asked to make formal oral and written presentations. A (non-original) paper serves the dual purpose of developing communication skills and introducing pedagogical flexibility.

ON STATISTICS:

- New knowledge has rendered a course devoted solely to the theory of classical parametric procedures out of date.
- The traditional undergraduate course in statistical theory has little contact with statistics as it is practiced and is not a suitable introduction to the subject.

ON DISCRETE MATHEMATICS:

- Discrete mathematics at the intellectual level of calculus should be part of the standard mathematics curriculum in the first two years.
- Topics covered are less important than acquiring mathematical maturity and skills in using abstraction and generalization.
- Mathematics majors should be required to take at least one course in discrete mathematics.

ON CALCULUS:

- Students should learn the content of the full four years of high school mathematics before enrolling in calculus.
- Calculus in high school should be taught with the expectation that successful graduates would not repeat calculus in college.
- Colleges need to provide individualized placement for students who have studied calculus in high school.

ON MINIMAL EXPECTATIONS:

- All college graduates should be expected to demonstrate reasonable proficiency in the mathematical sciences.
- College remedial courses should not be a rehash—and certainly not an accelerated rehash—of traditional school courses. Students should find even remedial courses fresh, interesting, and significant.

A Context for Reform

One must wonder, after reviewing all the arguments produced by the various committees whose reports are contained in this volume, why so little has changed. Why did it take seven years from the time that CUPM urged in 1981 that all beginning courses must be taught in an effective and attractive manner for the community to take a hard look at calculus? Why is it only now rather than in 1981 that mathematicians are beginning to realize the importance of writing assignments—both to learn to write and to write to learn?

Momentum may be one reason. In the early 1980's there was very little support for educational change. The political agenda of the nation at that time was not supportive of issues in science and mathematics education. All educational activities at the National Science Foundation were eliminated in 1980, only to be restored several years later. MAA and NCTM released separate reports (*Prime 80, An Agenda for Action*) into this thin atmosphere of uncertainty about science and mathematics education. It should not be surprising to find, ten years later, that most of the problems identified in these reports are still evident today.

Today, in contrast to 1980, many different organizations are working together for the improvement of mathematics education. The National Academy of Sciences, the National Science Foundation, and many private foundations have joined with the several mathematical societies to work on a common plan for revitalizing mathematics education. Now, after a decade of talking, everyone is finally moving in the same direction.

A partial list of current activities that relate to college mathematics reveals clearly the breadth of current support for reshaping college mathematics:

- The Undergraduate Curriculum Initiative at the National Science Foundation featured calculus reform in its first wave of proposal solicitations.
- The Mathematical Sciences Education Board and the Board on Mathematical Sciences at the National Research Council have jointly established the Committee on Mathematical Sciences in the Year 2000 to analyze collegiate mathematics and make recommendations for improvement.
- The Division of Mathematical Sciences at the National Science Foundation is now supporting research experiences for undergraduates, and is planning to add educational dimensions to many of its new initiatives.
- The MAA Committee on the Mathematical Education of Teachers is working with NCTM and with the National Board for Professional Teaching Standards to revise the recommendations for the mathematical education of teachers of mathematics in a manner consistent with the new NCTM *Standards.*
- Both AMS and SIAM now have committees dealing with education, as well as liaison members on CUPM.
- The MAA Committee on the Undergraduate Program in Mathematics has subcommittees working on recommendations for calculus and other courses in the first two years, on the mathematical sciences major, on service courses, and on the role of symbolic computer systems.
- MAA has recently published two volumes of papers dealing with calculus, one report on the role of computers in undergraduate mathematics, and one report on the continuing mathematical education of teachers. A report on discrete mathematics is forthcoming.

The present volume provides wisdom to support these efforts. It reflects the best thinking of many experienced mathematicians and teachers who have struggled with curricular questions facing college mathematics as part of their work for CUPM and other MAA committees. Although certain sections are obviously dated (e.g., discussions of computing, reference lists), the central message of this volume provides a philosophy of instruction that is as valuable now as when it was written.

We don't need to look far for sound goals and objectives for college mathematics. Most of what we need can be found in this volume. What remains to be done—as much now as ever—is to find effective means of turning ideals into practice.

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