$3x+1$ and $6x+1$ are perfect squares. Let $6x+1=X^2$. Then the original equation becomes

$$X^6 - X^2 = 12ky^2 \qquad k = 1, 2, 3, 4, 5, 6.$$

Let a be the product of the square-free factors of $12k$. Then the equation reduces to the form $x^4-1=aZ^2$, where a is given by

Case	I	II	IV	V	VI	X
a	3	6	1	3	15	2

Now it is known that $X^4-Y^4=aZ^2$ has no integral solutions (other than $X=Y$, $Z=0$) when $a=1, 2, 3$. Therefore cases I, IV, X fail. The only integral solution for $X^4-Y^4=15Z^2$ with $Y=1$ is (2, 1, 1) which is clearly inadmissible here. In the remaining case II, $X^4-Y^4=6Z^2$ has only one solution when $Y=1$, namely (7, 1, 20). Thus the given equations have no solutions unless $k=2$, and for this value, the unique solution $x=8$.

Also solved (partially) by D. C. B. Marsh and R. Venkatachalam Iyer.

RECENT PUBLICATIONS

EDITED BY E. P. VANCE, Oberlin College

All books for review should be sent directly to E. P. Vance, Oberlin College, Oberlin, Ohio

Universal Mathematics. Part I. Functions and Limits.* Preliminary Edition. By the 1954 Summer Writing Group of the University of Kansas. Lawrence, Kansas, Student Union Book Store. 1954. 10+310 pages. Planographed. $2.75.

This book is based upon an outline prepared by a committee of the Mathematical Association of America which was appointed to study the undergraduate program in mathematics and to make recommendations relating thereto. *Part I, Functions and Limits*, encompasses only the first semester's work of the freshman year and is intended for a course meeting three times per week. It is to be followed by *Part II, Structures in Sets*, which is to constitute the second semester's work. As the name implies, *Universal Mathematics* is intended to be used in a basic course in mathematics for all students, including engineering students. It is designed for students with normal high school preparation consisting of at least two years of high school mathematics, including intermediate algebra.

The book deviates sharply from other books on first year college mathematics both in its organization and in its content. In organizing their material the

* *Editorial Note:* In connection with this review the reader should refer to the following account by Professor Duren of his experiences in teaching this book at Tulane.

authors have given parallel presentations of almost every topic covered, one treatment being essentially intuitive, and the other a formal and more difficult treatment. It is not intended that the formal treatment be followed continuously but that appropriate selections of theorems and proofs be made by the instructor.

More emphasis is placed upon concepts and ideas than is customary in books at this level and the authors recognize that the average student with only normal preparation will not acquire from this book alone the manipulative techniques which are necessary for the further study of calculus and analytic geometry. The authors suggest that this situation be met by adding a "Technical Laboratory" course to the curriculum of the freshman year. This course would meet two hours per week throughout the year and would be intended for engineering students and other students who plan to take calculus and analytic geometry during their sophomore year.

The many departures from tradition in this book are not suggested by the chapter headings, which are as follows: I Coordinate Systems. II Scientific Measurements. III Functions. IV Limits. V Derivatives. VI Integrals. VII The Logarithmic and Exponential Functions. VIII Summary. In addition there is a fifteen page introduction and an eight page appendix. The Introduction contains an interesting discussion of the relationship of mathematical theories to the natural universe. The difference between empirical and mathematical proofs, the role of intuitive reasoning in mathematics, and the need for further mathematical theories are also discussed in the Introduction. Some of the ideas in the Introduction are followed up in the second chapter, where it is explained how real numbers and vectors are used for the measurement of physical quantities.

Throughout the book a serious effort has been made to present real mathematics at an elementary level. This is exemplified in the first chapter, where a careful discussion of the real number line by means of postulates is given. The real numbers are represented as infinite decimals and the postulate for a least upper bound is invoked to insure their existence. Further properties of real numbers, based upon the postulates for a complete ordered field, are given in the appendix. The treatment of real numbers paves the way for a discussion of approximate measurements and intervals, and for a discussion of the plane and its Cartesian coordinates. Vectors are introduced by means of parallel displacements and play a prominent role in the mathematical description of the plane. The usual analytic Euclidean geometry of the plane is by-passed in favor of a geometry of graphs (in which distances not measured along the axes, angles, circles, and arc length are undefined concepts). A relation is defined as a subset of the pairs which form the Cartesian product of two sets, and graphs of various kinds of relations are constructed.

Functions are introduced as special (single-valued) relations. A careful distinction is made between a function f and its functional value $f[a]$ at a. Brackets are used to indicate functional values whereas parentheses designate composite

functions. The letter x, when placed in parentheses after another function, is used to represent the identity function on a set X, so that if a is an element of X we have $f(x)[a]=f[x[a]]=f[a]$, and there is no logical distinction between f and $f(x)$. Polynomial functions and the algebra of these functions are discussed with reasonable thoroughness. A minor criticism might be made that it is neither proved nor stated that a polynomial function is expressible uniquely, and that without such uniqueness the concept of the degree of a polynomial is not well-defined. Convex sets on a line and in the plane, as well as convex functions, are also discussed in the chapter on functions.

The theory of limits which is used is based upon the Moore-Smith definition of a limit, which depends upon the properties of partially ordered sets. One might believe that these concepts would prove to be bitter pills for most beginning students. However this may be, and the reviewer is not too sanguine, the pills are sugar-coated with good illustrative examples and meticulous explanations. The more usual epsilon definition of a limit is given in the guise of a theorem. Limits of functions and continuous functions are defined and discussed for functions whose domains are convex sets. The theory of limits which is adopted leads to an elegant theory of the derivative and the definite integral.

The divided difference of a function over a neighborhood in its domain, which in a sense is a generalization of the familiar difference quotient, is fundamental to the authors' definition of a derivative. This divided difference is in fact an interval in which each point designates a difference quotient. The derivative is then defined as the final residue which is contained in all of the divided differences which are generated by a certain convergent limit process. If the limit process does not converge the final residue is a proper interval rather than a point, in which case the derivative does not exist. Even in this case the divided difference is a useful concept for the study of functions. These ideas are of course expressed in a precise mathematical form in the book. Standard topics such as the mean value theorem, maxima and minima, rates, motion, and inverse functions are in no way neglected.

The chapter on integration begins with a discussion of partitions, step functions, areas bounded by step functions, and Riemann sums. The Riemann sum which is defined, and which is fundamental for the definition of the definite integral, is not the classical Riemann sum, but is defined as a certain interval in which each point designates a classical Riemann sum. This makes possible the definition of a definite integral as a Moore-Smith limit of Riemann sums. Proofs of the fundamental theorem of the integral calculus, the substitution theorem, and the theorem for the existence of a definite integral of a continuous function are then given with an economy of effort. Existence theorems for definite integrals of monotone functions and sectionally integrable functions are seldom or never discussed in elementary courses, but they are proved in this book. The logarithmic function is defined by means of a definite integral and e is defined by the equation $\ln[e]=1$. It is shown that e^x is the inverse function of $\ln(x)$ and the properties of $\ln(x)$ and e^x are derived.

The authors have indicated on the cover of the book that it is "a book of experimental text materials." Trial and experiment will determine how much of the material can be effectively incorporated into courses designed for a large number of beginning students. The formal exposition, which is in a fairly concise mathematical style, will probably require, for the most part, more intellectual effort and innate mathematical ability than can reasonably be expected of most beginning students. But under the guidance of a experienced teacher the more able and ambitious students can study much of this material profitably. A larger number will be able to profit from the more discursive intuitive treatment upon which most of the class work is supposed to be based. This treatment makes use of diagrams, figures, and good illustrative examples wherever these aids are helpful. Both the formal and intuitive treatments include adequate lists of well chosen exercises. In the opinion of the reviewer the book would be easier to read if the formal parts were placed in appendices at the ends of the various chapters instead of being interspersed with the intuitive parts.

In spite of the general excellence of the exposition the reviewer believes that even the intuitive treatment will be difficult for most students to follow and that the instructor will not always find it easy to present the material effectively. But, as Professor Alfred Whitehead once wrote: "Whenever a textbook is written of real educational worth, you may be quite certain that some reviewer will say that it will be too difficult to teach from it. Of course it will be difficult to teach from it. If it were easy the book ought to be burned; for it cannot be educational."

Whether or not he expects to experiment with a course organized along the lines of this book the teacher of mathematics will do well to examine the book carefully. Its treatment of classical subject matter from a fresh and modern viewpoint will give him a deeper insight into his subject which cannot fail to have an impact upon his teaching.

H. P. Evans
University of Wisconsin

TULANE EXPERIENCE WITH UNIVERSAL MATHEMATICS, PART I

The University of Kansas and the Social Science Research Council supported in 1954 a Kansas Summer Writing Group which wrote a book of experimental text materials called: *Universal Mathematics, Part I*, which was an effort to put into concrete form some of the results of the studies of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America. This hastily written preliminary edition was given a mass trial at Tulane University as a universal mathematics course for all first year students who do not have to take intermediate algebra. Some 750 students in engineering, liberal arts, and business administration were involved. About 28 instructors, ranging from graduate teaching assistants to full professors, took part in the trial. Recently, a staff meeting was held to discuss the results.