# Teacher Preparation:
# K–12 Mathematics


## CRAFTY Curriculum Foundations Project
## Michigan State University, Division of Science and Mathematics Education (DSME) November 1–3, 2000

Sharon L. Senk, Brian Keller, and Joan Ferrini-Mundy, Report Editors
Sharon L. Senk, Workshop Organizer


## Summary


As noted in the Mathematical Education of Teachers (CBMS, 2001, p. 7), "teachers need to understand the fundamental principles that underlie school mathematics, so that they can teach it to diverse groups of students as a coherent, reasoned activity and communicate an appreciation of the elegance and power of the subject." Although the mathematical needs of teachers depend upon the grade level at which they intend to teach, substantial knowledge of content is needed by all teachers. For instance, elementary school teachers need a solid understanding of place value and the distributive property to help their students make sense of operations with whole numbers and decimals. High school mathematics teachers need to understand that same content to help students make sense of polynomial arithmetic.

Below are five principles concerning the mathematical preparation of teachers of all grade levels that emerged during this CRAFTY Curriculum Foundations Workshop. Though not officially adopted at the workshop, these principles reflect the workshop discussions and the views expressed by most of the workshop participants.

1. Mathematics courses for future teachers should develop “deep understanding” of mathematics, particularly of the mathematics taught in schools at their chosen grade level.
2. Tools for teaching and learning, such as calculators, computers, and physical objects, including manipulatives commonly found in schools, should be available for problem solving in mathematics courses taken by prospective teachers.
3 Mathematics courses for future teachers should provide opportunities for students to learn mathematics using a variety of instructional methods, including many we would like them to use in their teaching.
4. Faculty involved in the preparation of teachers of mathematics should engage in study and discussion of how people learn mathematics.
5. Greater communication and cooperation is necessary among all stakeholders in the mathematics preparation of teachers.

The mathematical preparation of future teachers is a complex and often controversial enterprise. Given this, as well as the diverse nature of the workshop participants, it should come as no surprise that consensus was not always achieved in our discussions. However, the following narrative represents the editors'

best attempt to summarize the dominant opinions expressed during the workshop sessions and provide further details in support of the five principles stated above.

# Narrative

## Introduction and Background

According to data collected by the National Center for Education Statistics, mathematics teacher preparation is the second largest of the specialty areas considered by the Curriculum Foundations Project. Only business has more undergraduate majors.

Mathematics teacher preparation is not only a large endeavor it is also quite complex. Teacher preparation is influenced by certification requirements set by the states and recommendations provided by several professional societies. In some states elementary teachers are not required to take any courses about the mathematics they will teach in schools. In others, they are required to take several courses that focus on deepening their understanding of number systems and geometry. In addition, teacher education programs in universities often participate in state or national accreditation processes that also impose specific standards for student learning.

Regardless of certification regulations or accreditation expectations, the mathematical requirements of future teachers vary considerably with the level at which the student intends to teach. Elementary teachers are usually prepared to be generalists who take courses in many departments. Consequently, they are required to take relatively few courses in mathematics. In contrast, teachers of high school mathematics are expected to be specialists, and often must complete a full mathematics major. In states where special certification for mathematics teachers of middle grades exists, the mathematical requirements for such teachers are generally between those of future elementary and high school teachers.

Consistent with the charge from CRAFTY, most participants in this workshop represented the discipline of mathematics education. Invited participants included classroom teachers with elementary, middle and high school teaching experience, college professors who teach methods courses and supervise field placements of prospective teachers, and college professors who teach courses in the mathematical content needed by teachers. Among the latter were both mathematicians and mathematics educators. In addition, professors and graduate students from the departments of mathematics, teacher education, and educational psychology at Michigan State University also attended parts of the workshop.

In order to give guidance to colleges and universities, professional societies have issued recommendations for the mathematical preparation of future teachers. Two of the most influential were *A Call for Change* (MAA, 1991) and *Professional Standards for Teaching Mathematics* (NCTM, 1992). Most recently, the CBMS member societies collaborated to produce a report called the *Mathematical Education of Teachers* [MET] (CBMS, 2001). The MET report recognizes the mathematical needs of teachers preparing to work at the elementary, middle grades, and secondary school levels.

A revised draft of the MET report was available on the web in September 2000. Because the recommendations in that draft overlap with the issues identified by the CRAFTY Curriculum Foundations Project, the planning committee for this CF workshop decided to use the MET report as a basis for discussion at the workshop, and chose to interweave the themes provided by the CF project within that framework. The workshop opened with a plenary address about the MET report. During the next day and a half workshop participants heard a series of presentations on specific issues related to the preparation of teachers at elementary, middle, and high school levels. Speakers at these plenary sessions included both mathematicians and mathematics educators. Following each presentation participants met in breakout groups to discuss issues in the preparation of teachers at that level. Our goal was for participants to discuss the MET report, and to provide reactions and perspectives based on these discussions to CRAFTY regarding the specific mathematical needs of future school mathematics teachers during their first two years in college.

What constitutes the first two years of college mathematics for a future teacher varies widely in institutions across the country. Future high school teachers usually take calculus during their first two years, while future elementary teachers seldom do. When mathematics courses about number or geometry are offered for future elementary teachers, in some universities they are taken during the first two years, but in others they are taken later. In our discussions, the conference participants leaned intentionally on the side of breadth.

Participants were assigned to breakout groups to ensure that various levels of teacher preparation and different academic departments were represented. On the last morning of the workshop, following a presentation about the role of community colleges in the preparation of teachers, participants chose one of elementary, middle or high school teacher preparation and developed initial recommendations to CRAFTY about mathematics during the first two years of college for that level. Recommendations from each of these final breakout groups were presented to the whole group. Because of this structure no one at the workshop heard all the discussions taking place. Because of the size and complexity of the teacher preparation enterprise in the United States and the diversity of the workshop's participants, many perspectives on issues were presented in the breakout sessions and spirited debates often ensued. Although there was some time for whole group discussion at the end of the workshop, there was no time to even begin to develop consensus; rather, we were able to raise the issues as they emerged from the subgroups.

What follows is drawn from the presentations and discussions that took place at the workshop. Space does not permit us to report all of the comments and examples made at the conference. The people whose names are identified were among those who spoke at the plenary sessions. Remarks set off in italics indicate ideas presented to the whole group and recorded in papers provided to the organizers after the workshop. The five principles noted in the summary and in bold on the following pages reflect syntheses by the authors of this report of ideas that seemed to be accepted by most of the workshop participants.

One caveat: The report that follows contains many suggestions about what conference participants thought might be added to courses that are either part of the general education requirements for all college students or for specific courses for future teachers. Given the time available at the workshop participants did not discuss what they might delete from the college curriculum, or how it might be reorganized, to make room for these newer areas of emphasis. Nor did participants discuss how they might address situations in courses, e.g. calculus, in which needs of future teachers might be somewhat different than needs of future engineers or others.

## Understanding and Content

**Mathematics courses should enable future teachers to develop "deep understanding" of mathematics, particularly of the mathematics taught in schools at their chosen grade level.**

"Deep understanding" is a term used without definition in the MET report. Workshop participants discussed the ambiguity of the term "deep understanding" and did not come to resolution about an exact meaning. Participants' usage seemed to include conceptual understanding that goes beyond procedural fluency, including being able to represent concepts in multiple ways, explain why procedures work or recognize how two ideas are related. It also involves being able to solve problems and to make connections among mathematical topics or between mathematics and other disciplines.

All students benefit from studying connections within a topic, e.g., how graphs, tables, words, and symbols may all represent the same function, and between topics e.g., how algebra and geometry are related. In addition, prospective teachers benefit from knowing about the connections between the mathematics they are studying in college and the mathematics they will have to teach. Thus, deep understanding should be a goal of all mathematics courses taken by future teachers, whether they are courses satisfying general requirements for graduation or specialized courses for teachers.

Deborah Ball and Hyman Bass argued in their presentation that looking at the actual practice of teaching provides insights into what mathematical understanding is needed by future elementary teachers. Videos of a 3rd grade class taught by Ball suggest that a teacher with a deep understanding of number theory and the role of definitions, conjectures and proofs is able to engage young children in mathematical study that emphasizes meaning. This suggests that mathematics courses taken by prospective teachers should develop understanding of both mathematical content and mathematical processes such as defining, conjecturing and proving.

Courses in algebra or college algebra often satisfy general education requirements and serve as prerequisites for specialized content courses for many elementary and secondary teachers. The content and methodologies of these pre-calculus courses should be examined by mathematics departments and committees of the MAA such as CRAFTY and the CUPM. Algebra courses should be redesigned to reflect the deep understanding recommended in the MET report. In general, there should be less symbolic manipulation, and more modeling and problem solving from both a discrete and continuous perspective. Emphasizing connections between algebra, arithmetic, and geometry is one way to develop understanding in these courses. An example of connections that could be explored involves examining the relation between the FOIL method of multiplying binomials, strategies used for mental multiplication, and areas of rectangles. Such revised algebra courses may also better serve students in other majors.

Calculus often serves as the entrance requirement into a major in mathematics. The desire for fluency in finding derivatives and integrals symbolically should be balanced with questions asking about the implications of the mathematics being studied. Problems requiring students to explain or justify their thinking should become more prominent in calculus courses. Curt Bennett, a conference participant, suggested that having students discuss issues such as the potential equality of .9999... = 1.0 or how round-off error affects calculations might deepen their understanding of both topics in the school curriculum and topics in calculus.

Bennett also suggested that calculus courses spend more time on infinite sequences and series. He claimed that at present

> *students learn a plethora of techniques for proving whether or not a series converges, but they gain little understanding of why the techniques work, or even why you might want to know them. Spending more time on this topic would allow students to explore various important series, and connect them more directly to the idea of number. In particular, infinite decimals could be explored in more depth beyond repeating decimals, something all our students would benefit from. For example, students could explore the series.*
>
> $$\sum_{n=1}^{\infty} \frac{1}{n!}.$$
>
> *Similarly, they could explore other interesting series that converge to well known numbers preparing them for the study of Taylor's series at the same time.*

To be able to spend more time on sequences and series, Bennett would spend less time on integration techniques. Several workshop participants mentioned that some pre-calculus and calculus courses seem like collections of miscellaneous techniques to students. They suggested that each mathematics course should have a theme. Mike Lehman, a secondary school teacher, said that every mathematics course "should be about *something*." Others suggested that every course should emphasize a few *big ideas*, and that the teacher should explicitly refer to the development of those ideas during the year. Some of the big ideas mentioned were: equivalence, function, transformation, composition, representation, proof and dimension. Michaele Chappell noted weaknesses in spatial visualization among her students and called for increased attention to 3-dimensional geometry during the first two years of college. The book *On the Shoulders of Giants* (Steen, 1990) was suggested as a good source for ideas about big ideas in geometry and other fields.

Conference participants generally concurred with the recommendations in the MET report that emphasized the need for future teachers to develop a deep understanding of the mathematics they will need to teach in schools. Thus, all mathematics teachers from elementary through high school should develop fundamental understanding in four areas: number and operations, algebra and functions, geometry and measurement, and data analysis, statistics and probability. Detailed lists of important topics in each area are given in the MET report. Chapell pointed out that many future teachers of secondary school mathematics taking her methods courses have not studied statistics or probability. Ideally, experiences in these subjects should be part of the first two years of college mathematics for future teachers.

Concern was expressed that the expectations expressed in the MET report for the mathematical preparation of elementary teachers were not strong enough. However, there was general agreement that the mathematics courses taken by elementary teachers, particularly those emphasizing number and geometry, should be completed during the first two years of college and before the student takes courses in the methods of teaching and that the mathematics courses should be coordinated with the methods courses.

Many workshop participants supported the recommendation in the MET report that mathematics in the middle grades (grades 5–9) should be taught by mathematics specialists. Many felt that programs should also be developed for elementary teachers who want to become "mathematics specialists." To develop programs for elementary or middle grades specialists some existing courses would probably have to be redesigned and new courses might have to be created.

Ira Papick described a course being developed at the University of Missouri in algebraic structures for middle grades teachers. In this course Papick introduces students to fundamental notions in number theory (e.g., greatest common divisor, least common multiple, the Euclidean algorithm) through problems taken from NSF-funded middle school curricula. Papick claims that

> *Courses of study relating core middle school algebra curricula to important applied and theoretical topics of university algebra ... empower teachers with new algebraic tools and perspectives, which in turn better prepares them to prompt important questions and enables them to convey a wide spectrum of algebraic ideas. Such courses would provide middle grade mathematics teachers with a strong mathematical foundation and directly connect the mathematics they are learning with the mathematics they will be teaching.*

Tony Peressini and several colleagues have developed a new course that focuses on connections between high school and college mathematics as well as connections within high school mathematics (Usiskin, Stanley Peressini, and Marchisotto, in press). This advanced perspective is also mindful of the historical and conceptual evolution of mathematical theory and school mathematics. For instance, the discussion of the real number system in this course

> *begins by presenting the real number system as it is typically presented in high school—as the set of points on a number line or as the set of decimal numbers. This includes a careful discussion of the decimal representation of the real numbers based on the nested interval property of the number line. Then the algebraic structure of the real number system is placed in the context of algebraic structures, first as a field, then as an ordered field, and finally as a complete ordered field—the structure that is often used to define the real number system in college courses. Finally, the connection returns to the high school setting of the real number system by describing how a decimal representation of each element of a complete ordered field can be constructed.*

The current calculus sequence and other requirements for a mathematics major often result in the study of the mathematics for teaching, such as the courses developed by Papick or Usiskin et al., being taken during the last two years of the undergraduate experience. This may not be enough time to help future teachers develop deep understanding of school mathematics. This concern might be addressed with new models of the mathematics major that allow future teachers to engage in connecting college mathematics to school mathematics earlier in their studies. One option is to design calculus courses for prospective teachers that are set up to make explicit connections to school mathematics. Another option is to offer other courses, say discrete mathematics or geometry, before calculus, where such connections can be made more easily.

## Technology

**Tools for teaching, such as calculators, computers, and physical objects, including manipulatives commonly found in schools, should be available for problem solving in mathematics course taken by prospective teachers.**

Conference participants did not have a thorough conversation about technology, and there were a variety of views about the appropriateness of its use in the school curriculum. However, some examples were discussed. Because technology can generate data so quickly, it can be a useful tool for making conjectures. However, in order to develop understanding, the use of technology should be accompanied by analytic methods or classroom discussion. For instance, in a course in which future elementary teachers study decimals one might use a calculator to determine the sum $.\overline{7}+.\overline{8}$ . Then students could be asked to find the sum using the fraction equivalents of the decimals and compare the results.

When students begin studying the trigonometric functions and limits, having them explore the limit of sin($x$)/$x$ on a calculator can lead to interesting results. Bennett reported that when he did this in his classes, about half of the students came up with the answer 1, while the other half came up with the answer .017. Other participants reported that they had experienced similar results. Analytic techniques verify that the limit of sin($x$)/$x$ equals 1. Thus, students naturally question why calculators give two seemingly different answers. This disparity presents the opportunity to bring up the question of what units are being used, and students have the opportunity to discover, for themselves, a reason to use radian measure for angles.

Spreadsheets and the graphing and table-generating features of graphing calculators can be effectively used to solve problems about functions and families of functions. At present, the study of functions, including the use of exponential functions to describe both population growth and compound interest, is found in many middle and high school curricula. Marjorie Economopoulos and Marian Fox reported that faculty at of Kennesaw State University (KSU) have developed materials using spreadsheets, graphing calculators, and existing school mathematics curricula to help pre-service teachers deepen their understanding of functions. Mathematics courses at KSU also give prospective teachers opportunities to analyze and model data taken from secondary school curricula using the statistical features on a graphing calculator or a statistics package on a computer. Participants also suggested that calculus and other courses for future teachers make use of computer algebra systems to investigate equivalence of expressions and transformations of functions.

Future teachers should be able to use tools such as tiles, cubes, spheres, rulers, compasses, and protractors to deepen their understanding of the mathematics they will have to teach, including 2-D and 3-D geometry and measurement. The use of electronic drawing tools such as the *Geometer's Sketchpad* and *Cabri* is also recommended for use in geometry courses for both elementary and secondary teachers. The dynamic capabilities of these tools allow both students and teachers to test conjectures relatively easily.

## Instructional Techniques

**Mathematics courses for future teachers should provide opportunities for students to learn mathematics using a variety of instructional methods, including many we would like them to use in their own teaching.**

This perspective arose from the belief that instruction in college mathematics classes should involve more than lecture. Instructors should include various techniques for engaging students actively in solving problems. This could include, whenever appropriate, having students solve problems or discuss strategies with a partner or small group, and engaging the whole class in discussion. Instructors and students should be encouraged to solve problems in more than one way, to explain their reasoning, and to describe how the mathematics they are doing today is related to mathematics done earlier. The use of such techniques in undergraduate mathematics courses could help develop the college students' understanding of the mathematical content, and provide an instructional model for the future teachers to emulate.

Proof and justification are an integral part of mathematics in comparison to other sciences. There was virtually universal agreement among workshop participants that reasoning and proof should be a theme in all college mathematics courses, beginning at least with calculus. This is not a call for reintroduction of delta-epsilon proofs into calculus or for an emphasis on axiomatics and formality. Rather, in the introductory courses there should be an emphasis on developing basic mathematical reasoning and communication skills by asking students to explain their thinking or to justify their responses based on definitions. Specific proof techniques need not be taught until later, and at that time students should be given many opportunities to create their own proofs in an effort to promote their own understanding of mathematics. Most elementary mathematics texts do not emphasize reasoning and proof and as a result new materials—in the form of activities, problem sets or entire books—may need to be developed.

Prospective teachers need the opportunity to work on extended problems, perhaps through projects, as well as on short assignments. The content and methods of assessment should be consistent with the content and methods of instruction. Tests, quizzes and projects should measure the students' understanding of the topics emphasized in the course. Questions should ask about representations, connections, reasoning, and problem solving, as well as computations and algorithmic procedures.

Teachers of mathematics courses for future teachers should be familiar with the mathematics content and expectations of elementary schools, and with elementary school children. Mathematics courses designed primarily for future teachers should not be assigned to inexperienced Teaching Assistants unless substantial training and on-going support are provided. More generally, in order to provide high quality mathematics instruction for future K–12 teachers, there needs to be support for the professional development of their college and university teachers. In particular, teaching assistants and instructors should work as apprentices under the supervision of experienced teachers.

Conference participants were generally supportive of the idea that prospective teachers should experience mathematics instruction that engages them in actively doing mathematics on a regular basis. Some suggested that the labeling of small sections as “problem solving labs” rather than “recitations” might signal progress in this direction.

## Instructional Interconnections

Current reform efforts in school mathematics have been influenced by several reports from the National Council of Teachers of Mathematics. The most recent report, *Principles and Standards for School Mathematics* (NCTM, 2000) presents ten standards for mathematics programs in Grades Pre-K to 12. Five standards define the core content (Number, Algebra, Geometry, Measurement and Data Analysis and Probability). Five others define fundamental characteristics of each content area (Problem Solving, Reasoning and Proof, Communication, Representation, and Connections). This report and its predecessor, the *1989 Curriculum and Evaluation Standards for School Mathematics* have generated expectations among many students and teachers that mathematics requires thinking; that mathematics has meaning and application; and that there are multiple approaches to solving problems. Furthermore, since the early 1990s many standards-based school mathematics curricula have been developed and are in use in schools across the United States.

As a result of the increasing spread of the NCTM Standards there is a great need for teachers with deep understanding of this broad range of content. Without such understanding we cannot expect teachers to effectively work with standards-based materials or other challenging mathematics curricula. Several speakers pointed out how at some colleges and universities students seem to be lacking experiences with statistics or probability.

**Faculty involved in the preparation of teachers of mathematics should engage in study and discussion of how people learn mathematics.**

Much research has been done on how people learn mathematics. Both faculty and graduate students from the disciplines of mathematics and mathematics education—groups that include the instructors of all future K–12 teachers—would better serve prospective mathematics teachers if they were well informed about this research and considered its implications for their own teaching.

Two books were recommended by workshop participants for study: *Knowing and Teaching Elementary Mathematics* (Ma, 1999) and *The Teaching Gap* (Stigler & Hiebert, 1999). Ma describes the "profound understanding of fundamental mathematics" of a group of Chinese elementary teachers, and notes how seldom such understanding was seen in a group of elementary teachers in the United States. Stigler and Hiebert compare and contrast teaching mathematics in Grade 8 in classrooms in Germany, Japan, and the United States using data from the TIMSS video study.

At the time of the workshop, the Mathematics Learning Study Committee of the National Research Council had undertaken a review of research on learning and teaching mathematics in Grades K–8 (NRC, 2001). That review has been published recently, and we, the editors of this report, also recommend it for study.

The Mathematics Learning Study committee demonstrated that a diverse set of mathematicians, psychologists, mathematics educators, teachers and others can work together productively on an issue of national importance. Such collaboration is very important, but not always easy to carry out. Recognizing this leads to our last principle.

**Greater communication and cooperation is necessary among all stakeholders in the mathematics preparation of teachers.**

Stakeholders in the preparation of mathematics teachers include faculty in mathematics, mathematics education, and education, instructors at two-year and four-year colleges, and teachers and administrators in public and private schools.

One issue that surfaced during the workshop was the language gap between the stakeholders. For example, terms such as deep understanding, algorithm, and proof evoked questions about their meaning and their use. Sometimes the language gap occurred between mathematicians and mathematics educators, sometimes between university and school teachers. Beliefs about what is important sometimes clashed.

Due in part to this gap, the structure and content of preparation programs for mathematics teachers should arise from a partnership between school and university faculty from all departments. Future teachers deserve well-designed programs and courses within departments of mathematics. Efforts to provide such programs to students could be more successful if common objectives for the education of teachers were established, and methods of meeting those objectives were designed, evaluated and revised with participation from all stakeholders.

Mercedes McGowen reported that approximately 40% of teachers in the United States complete some or all of their mathematics and science courses in community colleges. In view of this statistic, two-year college faculty should be invited to participate in and help organize any future national or local discussions about the mathematical preparation of teachers.

More generally, joint seminars and team teaching be used to foster communication across departments involved in mathematics teacher preparation. Such joint seminars might involve concentrated study of some topic in school mathematics. Mathematicians might help mathematics educators gain new insights about the mathematics. Mathematics educators might help mathematicians gain new insights about student learning. Jack Plotkin from the Department of Mathematics at Michigan State University and Dan Chazan from the Department of Teacher Education at Michigan State recently team-taught a mathematics capstone course for majors focusing on the Fundamental Theorem of Algebra. They recommend that team-teaching should also be considered in introductory courses. Communication within mathematics departments can also be fostered by discussions among instructors who teach the same course. These discussions could provide a forum for sharing information about what they do in their sections, and offer opportunities for instructors to consider how one mathematics course builds on what has been learned in other courses.

Economopoulos and Fox described how the North Metro Mathematics Collaborative fosters communication among schools and colleges in suburban Atlanta. This project, centered in the Mathematics Department at Kennesaw State, is dedicated to supporting mathematics teachers in their pursuit of excellence in instruction. Each year it sponsors many professional development courses and workshops in mathematics led by leaders with national stature who attract hundreds of teachers from kindergarten through college. Other means of fostering communication among instructors of mathematics courses for future teachers include: national and regional workshops similar to this one, as well as sessions at national, regional and state meetings, electronic discussion forums, papers in journals, and on-line professional development opportunities.

Participants were particularly interested in discussing the meaning of deep understanding of big ideas such as equivalence, proof, function, and proportional reasoning. Other topics suggested for study were the development and evolution of these ideas in the school mathematics curriculum and how university courses support the development of deep understanding.

Participants frequently mentioned concerns about the lack of resources and support for work in mathematics departments on courses or programs for future teachers. To meet the goals of the MET report on the preparation of future teachers and to implement recommendations such as those reported here, teacher education will need to become a higher priority within mathematics departments.

## REFERENCES

Conference Board of the Mathematical Sciences. (2001). *The Mathematical Education of Teachers.* Providence, RI and Washington, DC: American Mathematical Society and Mathematical Association of America.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Erlbaum.

Mathematical Association of America. (1991). *A Call for Change*. Washington, DC: MAA.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: The Council.

National Council of Teachers of Mathematics. (1992). *Professional Standards for Teaching Mathematics.* Reston, VA: The Council.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: The Council.

National Research Council (2001). *Adding it up: Helping Children Learn Mathematics.* J. Kilpatrick, J. Swafford & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Steen, L., A., Ed. (1990). *On the Shoulders of Giants: New Approaches to Nume*racy. Washington, DC: National Academy Press.

Stigler, J. W. & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. New York: The Free Press.

Usiskin, Z., Stanley, R., Peressin, A. & Marchisotto, E. (2003*). Mathematics for High School Teachers: An Advanced Perspective*. Needham, MA: Prentice Hall.

# WORKSHOP PARTICIPANTS

**Deborah Ball**, Department of Educational Studies, University of Michigan

**William Barker**, Department of Mathematics, Bowdoin College

**Hyman Bass,** Departments of Educational Studies and Mathematics, University of Michigan

**Susan Beal**, Department of Mathematics, St. Xavier University

**Charlene Beckmann**, Department of Mathematics, Grand Valley State University

**Sybilla Beckmann**, Department of Mathematics, University of Georgia

**Curtis D. Bennett,** Department of Mathematics, Bowling Green State University

**Dawn Berk**, Division of Science and Mathematics Education, Michigan State University

**Keely Burns,** Department of Mathematics, New Trier High School, Winnetka, Illinois

**Joanne Caniglia**, Office of Collaborative Education, Eastern Michigan University

**Suzanne Chapin**, School of Education, Boston University

**Michaele Chappell**, Department of Secondary Education, University of South Florida

**Daniel Chazan**, Department of Teacher Education, Michigan State University

**Herb Clemens**, Department of Mathematics, University of Utah

**Christopher Danielson**, Department of Mathematics, Michigan State University

**Marjorie Economopoulus**, Secondary and Middle Grades Education, Kennesaw State University

**Joan Ferrini-Mundy**, Associate Dean, DSME and Departments of Mathematics and Teacher Education, Michigan State University

**Naomi Fisher,** MER Forum, Department of Mathematics, University of Illinois - Chicago

**Judy Flowers**, Department of Mathematics and Statistics, University of Michigan - Dearborn

**Marian Fox**, Department of Mathematics, Kennesaw State University

**Richard Hill**, Department of Mathematics, Michigan State University

**Stephen Hwang**, Department of Mathematics, Michigan State University

**Nan Jackson**, Department of Mathematics, Lansing Community College

**Gary R. Jensen,** Department of Mathematics, Washington University

**Liz Jones**, Department of Teacher Education, Michigan State University

**Brian Keller**, Department of Mathematics, Michigan State University

**Jeannette Kiernan**, Navigator Elementary School, Pinckney, Michigan

**Karen King**, Department of Mathematics, Michigan State University

**Michael Lehman**, Department of Mathematics, Holt High School, Holt, Michigan

**Eric Knuth,** Department of Curriculum & Instruction, University of Wisconsin

**Valerie Mills**, Office of Instructional Services, Ann Arbor Public Schools

**Diana Lambdin,** School of Education, Indiana University

**Perry Lanier**, Department of Teacher Education, Michigan State University

**Glenda Lappan**, Department of Mathematics, Michigan State University

**Gary Lewis**, Department of Counseling, Educational Psychology, and Special Education, Michigan State University

**Mercedes McGowen**, Department of Mathematics, William Rainey Harper Community College

**Lynn Narasimhan**, Associate Dean, Liberal Arts and Sciences, Department of Mathematical Sciences, De Paul University

**Ira Papick**, Department of Mathematics, University of Missouri

**Tom Parker**, Department of Mathematics, Michigan State University

**Anthony Peressini**, Department of Mathematics, University of Illinois

**Elizabeth Phillips**, Department of Mathematics, Michigan State University

**Jacob Plotkin**, Department of Mathematics, Michigan State University

**Maria Alejandra Sorto**, Department of Mathematics, Michigan State University

**Sheila Sconiers,** COMAP

**Sharon Senk,** Department of Mathematics, Michigan State University

**Kathleen Snook**, Department of Mathematics, U. S. Military Academy

**Patricia Tinto**, School of Education, Syracuse University

**Deborah Zopf**, Department of Mathematics, Henry Ford Community College

## ACKNOWLEDGEMENTS

We wish to thank the following workshop participants for their helpful feedback on earlier drafts of this report: Curt Bennett, Herb Clemens, Gary Jensen, Marjorie Economopolous, and Tom Parker. Thanks also to Bill Berlinghoff, Kevin Clancey, Karen Graham, Libby Krussel, Judy Roitman, Dara Sandow, Annie Selden, and Phil Wagreich for comments made during the Focus Group discussion in New Orleans in January, 2001, and to Richard Askey and Manuel Berriozabal for their thoughtful written comments.

Finally, we would like to thank Lou Anna K. Simon, Provost of Michigan State University, for providing financial support for this workshop, Rebecca Murthum of the Division of Science and Mathematics Education at Michigan State University for making local arrangements, and William Barker, Chair of the CRAFTY subcommittee of the MAA, for the opportunity to host this workshop.