

9th United States of America Junior Mathematical Olympiad

Day 1. 12:30 PM – 5:00 PM EDT

April 18, 2018

Note: For any geometry problem whose statement begins with an asterisk (*), the first page of the solution must be a large, in-scale, clearly labeled diagram. Failure to meet this requirement will result in an automatic 1-point deduction.

USAJMO 1. For each positive integer n , find the number of n -digit positive integers that satisfy both of the following conditions:

- no two consecutive digits are equal; and
- the last digit is a prime.

USAJMO 2. Let a, b, c be positive real numbers such that $a + b + c = 4\sqrt[3]{abc}$. Prove that

$$2(ab + bc + ca) + 4\min(a^2, b^2, c^2) \geq a^2 + b^2 + c^2.$$

USAJMO 3. (*) Let $ABCD$ be a quadrilateral inscribed in circle ω with $\overline{AC} \perp \overline{BD}$. Let E and F be the reflections of D over lines BA and BC , respectively, and let P be the intersection of lines BD and EF . Suppose that the circumcircle of $\triangle EPD$ meets ω at D and Q , and the circumcircle of $\triangle FPD$ meets ω at D and R . Show that $EQ = FR$.