



**AMC 10-A / AMC 12-A**

8th/58th Annual Contests



Make  
Mathematical  
History

# Teachers' Manual

Tuesday, February 6, 2007

Instructions and Reporting Forms for  
School Contest Managers



Please read this booklet completely upon receipt  
Exams must be administered over a continuous 75-minute period to all students at the same time

**2007 EXAMINATIONS**

AMC10/AMC12 - Tuesday, February 6, 2007

&/or Wednesday, February 21, 2007

AIME - Tuesday, March 13, or Wednesday, March 28, 2007

USAMO - April 24-25, 2007

AMC 8 - Tuesday, November 13, 2007



# The MATHEMATICAL ASSOCIATION OF AMERICA

## American Mathematics Competitions

Steven Dunbar  
AMC Director

To all Contest Managers:

I am very pleased you will be providing the opportunity for your students to participate in the 58<sup>th</sup> annual American Mathematics Contest 12 (AMC 12) or its sister contest, the 8<sup>th</sup> annual American Mathematics Contest 10 (AMC 10). I believe that you and your students will find these contests to be both interesting and challenging. Again this year the Committee on the American Mathematics Competitions and panelists under the leadership of Committee Chairs Professor Douglas Faires for the AMC 10 and Professor David Wells for the AMC 12 developed a total of 4 contests (two AMC 10 and two AMC 12). I extend special thanks to both of them for the effort involved in developing these excellent contests.

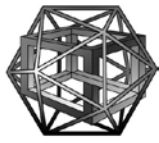
This year, we continue to include in this manual several handouts, included in Section XIV:

- \* Worksheets for contest preparation ([page 14](#))
- \* A handout for parents on why mathematics is important ([page 22](#))
- \* A sample Press Release ([page 23](#))
- \* A Certificate of Participation ([pages 24 & 25](#))

You may reproduce these pages for your students.

Very sincerely,

Steven Dunbar  
*Director*



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# Important Procedures

## Format

There are two official dates for the AMC 10 & AMC 12. Give the AMC 10 and AMC 12 at the same time within each participating school on **TUESDAY, February 6, 2007 (AMC 10-A & AMC 12-A)**, or **WEDNESDAY, February 21, 2007 (AMC 10-B & AMC 12-B)** in a convenient 75-minute interval, preferably in the morning. All four contests consist of 25 questions.

**NOTE:** Each correct answer scores 6 points, a **blank scores 1.5 points** and an incorrect answer scores 0 points.

The AMC 10 and AMC 12 have several questions in common. The students in grades 10 and below should choose between the AMC 10 and AMC 12. Students in grades 11 and 12 may only take the AMC 12. All rules and awards apply to both contests for all schools and students. Any student who missed the exam may take it unofficially, and we will be happy to grade it. Students may take the contest booklets home with them the day of the contest.

## Answer Forms

The AMC 10 and AMC 12 each have their own color coded answer form (AMC 10-orange and AMC 12-red). Contest B answer forms have instructions in black ink. Please be careful to match the correct answer form with the appropriate contest when passing out the papers.

## AIME Qualification

Students who score 100 or above or finish in the top 5% on this AMC 12 or students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 25th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 13, 2007 or Wednesday, March 28, 2007.

## Results

We will send your school's results by email (if available) and first class mail as soon as the answer forms are scored. If you have not received your results from our office within 30 days after the contest, please contact us to verify that your answer forms were in fact received. All student Answer Forms are held for 90 days after the exam date, then they are recycled.

### I. Preliminary Instructions for Administering the AMC 10/AMC 12

1. Inform students far in advance about the date for the AMC 10/AMC 12 and obtain a supply of No. 2 lead pencils.
2. Hand out the student Answer Forms and have the students complete the non-answer sections on the front and back. Have students pay special attention to marking their name and address accurately. Remind them that student names listed in the National Summary come from this form. The AMC Office will not do any editing of the information.
3. The name of the school, city, state and postal code must be stamped or written on each student answer form. This is very important because there is no other school identification on the answer forms. Please do not use stick-on labels to provide this information.
4. Announce that the students may use scratch paper, graph paper, ruler, compass, protractor and four-function, scientific, or graphing calculators. No problems on the contest will require the use of a calculator. However, any non-typewriter keyboard calculator accepted for use on the SAT may be used during the exam. See [http://apps.collegeboard.com/cbsearch\\_code/codeSearchHighschool.jsp](http://apps.collegeboard.com/cbsearch_code/codeSearchHighschool.jsp). Students may not share a calculator.
5. Review past tests and solutions or inform your students how to order copies for themselves. The Publications Order Form can be downloaded from the AMC Web page at [www.unl.edu/amc](http://www.unl.edu/amc), (or call, fax or write to the AMC Office for a copy).
6. Encourage participation by students who have not taken the contest before, especially younger students, but make sure students know what to expect. Let them know about typical scores at your school last year at their grade level. Show students the national statistics in our National Summary of Results and Awards from last year. Tell them to set appropriate goals for themselves.
7. Remind students the day before the contest about the time and place of the AMC 10/AMC 12. Also tell them your plan if the school should suddenly close. All students must take the contest at the same time, either in one group or in separate classrooms under the supervision of a teacher.
8. Make sure you have arranged to follow all the rules and procedures in this manual. **EARLY ADMINISTRATION OF THE CONTESTS IS NEVER PERMITTED**, and will lead to disqualification. To assure the validity of the results we report, we take our rules very seriously.



## II. Instructions For The Day of The AMC 10/AMC 12

If today is not yet **Tuesday, February 6, 2007**. STOP EVERYTHING. Under no circumstances is the contest to be given to anyone before the official day, nor should the contest package(s) be opened before that date.

1. Take the unopened contest package(s) and Certification Form to your Principal (or equivalent) to certify that the package was opened within an hour before the contest. Have the Principal sign the Certification Form at that time.
2. When the students arrive, seat them so they are separated by an empty space, if possible.
3. Hand out the Answer Forms which have been partially completed by the students (AMC 10-orange/AMC 12-red).
4. As you hand out the papers, tell the students not to open the contest booklet. They should then read the entire front cover. Give them 5 minutes to do this.
5. Inform the students to, "Carefully read instructions 3 and 4 on the contest cover." (see [pages 26 and 27](#) of this manual). The AMC 10/AMC 12 has a unique scoring system which has important consequences for guessing. Unless you are fairly sure of the answer, it is better to leave a question unanswered than to guess. Six points are given for a correct answer, 1.5 points for a blank answer and 0 points for an incorrect answer. If a student can reduce the problem to three possible answers, it is advantageous to guess one of the three possible answers. If a student can only reduce to 4 possible answers by eliminating 1 of the possibilities, then it is not advantageous to guess.
6. Inform the participants they may not talk or ask any questions during the contest, and they must do their own work.
7. Remind students they have 75 minutes, then tell them to BEGIN. (See page 6 for student disabilities policy)
8. Students who finish the contest early may be dismissed provided they will be under the supervision of a teacher during the remainder of the contest period.
9. You (and other teachers, if there are many participants) should proctor continually as you would for any important contest. Students whose eyes wander should be warned; students caught copying answers or collaborating must be disqualified. Try to provide as quiet an environment as possible.
10. Announce when there are 30 minutes remaining and when there are 5 minutes remaining.
11. When time is up, tell the students to STOP and have them sign their name in the space provided on the Student Answer Form. Collect the Answer Forms as quickly as possible.
12. Please do not grade the answer forms. They are to be sent to the AMC office for grading. Students may circle their answers on the contest booklet. However, the official answers will be the ones blackened on the answer form.
13. Fan the forms, making sure none are stuck together, place the School ID Form on top. There is only one ID FORM to be used with all the AMC 10/AMC 12 answer forms.

14. YOUR SCHOOL'S CEEB NUMBER IS THE NUMBER WRITTEN ON THE FRONT OF THE AMC 10/AMC 12 REPORT ENVELOPE.
15. Complete the Certification Form (only one form is needed) and place it on top of the School ID Form and answer forms and place all in the Report Envelope. Seal and send it by First Class (trackable recommended) within 24 hours or as soon as possible. Please affix the proper postage before mailing.
16. Please note: After the Answer Forms have been delivered to the school office to be mailed, you may discuss the contest with your students under the following conditions which take into consideration the fact that there will be schools taking the contest in other locations at different times.
  - a. Inform the students that the contest may not be discussed with anyone outside of your school either orally, via email, www, copier or media of any type until after the contest period.
  - b. Students may keep the contest booklets and take them home.

## III. Eligibility

Any student who is officially enrolled in high school (or below) and is taking at least one course at the high school level, and has not graduated, is eligible to take the AMC 10 or AMC 12 or AIME (with qualifying score). *Please note: students in grades 11 & 12 can not take the AMC 10.* However, students in grades 9 & 10 may choose which contest they take.

### Home School Students

Home School Students age 19.5 and under are eligible for AMC 12 and AIME (with qualifying score) and age 17.5 and under are eligible for AMC 10 or AMC 12 and AIME (with qualifying score).

**International Students & Non-Citizens in USA Schools**  
US and Canadian Citizens and International Students residing in the United States (with qualifying scores) are eligible to take the USAMO.

Students learning "English as a Second Language" (ESL) may use a book or electronic dual-language nontechnical dictionary between their native language and English. A student may use the dictionary only the first time that he/she takes the AMC 10/AMC 12. The dictionary must be given to the school contest manager to examine and retain for the 24-hour period preceding the contest. The proctor must announce to other students that the student(s) has/have been given special permission to use the dictionary during the contest.

## IV. Team Score Identification

TO RECEIVE OFFICIAL TEAM STATUS AND AWARDS, A SCHOOL MUST HAVE AT LEAST THREE PARTICIPANTS ON A CONTEST DATE. The team score for a school is the sum of its three highest student scores and will be determined by the AMC Office. The score of USA and Canadian teams is used to determine National School awards. In addition, the team score is used to select the top 60 schools to identify teachers who are eligible for the Edyth May Sliffe Award for

Distinguished High School Teaching.

## V. School Results

The AMC office will send results by email (if available) and first class mail as soon as the answer forms are scored. If you have not received your results from our office within 30 days after the AMC 10/AMC 12 please contact us to verify that your answer forms were in fact received.

If you would like to receive your results by e-Mail, and have not previously sent us your email address, send a message, including your name, school name, address, and CEEB # to:

[hstran@amc.unl.edu](mailto:hstran@amc.unl.edu)

Results are not official until you receive the postal mailed paper copy of your report, and that should be no longer than 30 days after AMC 10/AMC 12.

## VI. Policy Statements

### Early Administration

Administration on an earlier date is NEVER permitted and will lead to disqualification. Such an administration would jeopardize the validity of all scores from other participating schools.

### Official Administration

The AMC 10-A/AMC 12-A will be given officially on **Tuesday, February 6**, The AMC 10-B/AMC 12-B will be given officially on **Wednesday, February 21**. Only official participants, their school and their teacher are eligible for National Awards. In addition, official participants are eligible for all intramural awards and for participation in the AIME.

### Unofficial Administration

If you are unable to give the **Contest A on February 6** because:

- your school is closed,
- your school has an academic conflict,
- the class periods have been shortened due to an assembly or other reason,

then you may give the second version of the contests (AMC 10-B/AMC 12-B) on the second official day, **Wednesday, February 21, 2007** (See Contest B Registration Form on [page 13](#)). You may still take either exam unofficially on later dates, but those contests will not be eligible for state and national awards and will not be eligible for participation in the AIME. Unofficial participants are still eligible for intramural awards.

It is important to note that the only days eligible for official participation are the two official Contest days: **Tuesday, February 6, and Wednesday, February 21, 2007**.

### One Contest per Date

A student may take only one exam on a given day but can participate on both contest dates if the school registers for both contests. The higher score will be used for individual awards.

### Students with Visual or Learning Disabilities

The AMC 10/AMC 12 time limit set by the CAMC for students who are visually impaired or learning disabled is 120 minutes. A teacher or a school administrator may read the questions to the student and mark the answers as directed by the student. The cost of a Braille or Large Print exam is \$7.00 shipping and

handling plus \$1.60 per exam for the AMC 12 and \$1.40 per exam for the AMC 10. **They are mailed separately and must be ordered no later than three weeks before the test.**

### Sickness and Other Special Situations

A student who is sick or on a field trip on the first contest day may register and take the alternate Contest B on **Wednesday, February 21, 2006**. YOU MUST REGISTER FOR CONTEST B if you have not already done so. (see [page 13](#) for a Registration Form).

### Questionable Scores

If it is clear to the Contest Manager from personal observation that a student has cheated, then the Manager must disqualify the student. If the Contest Manager receives an accusation of cheating, or obtains other indirect evidence of cheating, then the Manager must hold back the student's paper and immediately report all the facts of the situation to the AMC Director, who in conjunction with the Chair of the CAMC, will determine what to do. UNDER NO CIRCUMSTANCES may the school decide on its own to accept a questionable score, nor should a school retest the student in question before receiving instructions from the AMC Office.

### Follow-up Inquiries and Reexamination

The results of this contest helps to identify students with unusual mathematical ability. To assure that this purpose is served, the CAMC reserves the right to retest students before deciding whether to grant official status to individual or team scores. Reexamination will be requested when, after an inquiry, there is a reasonable basis to disbelieve a score. Official status will not be granted if a student or school does not agree to a requested retesting.

### Policy for Changes

The CAMC may, from time to time, change the program rules, regulations, awards and conditions of participation in whole or in part. Whenever possible you will be notified of these changes ahead of time.

### Refund/Credit Policy

If your school is unable to take the contests, please use the materials as practice sets for the next year. Do not return them. WE CAN NOT GIVE REFUNDS OR CREDITS AFTER THE CONTEST MATERIALS ARE SHIPPED.

### Request for Student Names Policy

The following statement appears on the student answer forms for the AMC 10 and AMC 12:

- The American Mathematics Competitions (AMC) receives requests from educational institutions and organizations for the names, addresses and grade levels of high scoring students. This information is used for recruiting and academic purposes.*
- Blacken this circle if you give the AMC permission to release this information to these organizations. (Your score will not be affected if you do not blacken the circle.)*

Receiving information is an "opt-in" decision for each individual student.

The AMC handles requests from institutions and organizations

on a case-by-case basis and evaluates each individually for appropriateness. We provide legitimate educational institutions of all levels, both secondary and collegiate/university level, with one-time use of selected names and addresses for postal mailings. We also provide professional and scholarly organizations such as those listed as contributors to the AMC with one-time use of names and addresses for postal mailings, generally for professional or career information.

The only information we provide is the name, address, city, state, and zip code necessary for a postal mailing. We do not list individual scores or awards.

## VII. AIME Instructions

The 25<sup>th</sup> annual American Invitational Mathematics Examination (AIME) will be held on **TUESDAY, March 13, 2007** with a second alternate exam given on the alternate date of **WEDNESDAY, March 28, 2007**. These are the only days the exam may be taken officially. You may give the exam for practice (unofficially), after the official dates. We will be pleased to grade it for you but your students will not be eligible to take the USAMO. The contest is provided free of charge to all those taking the exam on the first date, however those taking the exam on the second alternate date will be charged a processing fee to cover expedited delivery.

### AIME Rules for AMC 10/AMC 12

Students who score 100 or above or finish in the top 5% on this AMC 12 or students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 25th annual American Invitational Mathematics Examination (AIME) on Tuesday, **March 13, 2007** or Wednesday, **March 28, 2007**. PLEASE read the following participation rules to your students as soon as you receive the AMC 10/AMC 12 package so potential AIME students will be able to plan accordingly.

### AIME School Manager

1. The AMC office will include all materials relating to the examination (including instructions for the exam) with your AMC 10/AMC 12 results.
2. All questions or problems concerning the AIME should be directed to the AMC office (800-527-3690).
3. The AIME is a three-hour examination. Each of its 15 questions requires a three digit integer answer and each correct answer will receive one (1) point.
4. Calculators are not allowed.
5. The AIME Answer Forms are sent directly to the AMC office for grading and processing.
6. Each participating school will receive a report of their results, an AIME solution pamphlet, and a list of students who qualify for the USAMO.
7. All AMC 10/AMC 12 procedures for disqualification, follow-up inquiries and reexamination apply to the AIME as appropriate.
8. If you have students who you feel may qualify for the AIME please order prior year AIME exams and solutions for practice now. This way you will have these practice materials on hand when you receive your AMC 10/AMC 12 results.

## Second AIME Testing Date

Situations in which a student may take a second version of the AIME to be held on **WEDNESDAY, March 28, 2007**, keeping their USAMO eligibility open are:

1. School is closed on **March 13th** (i.e. spring break, weather).
2. Student is out of school the entire day due to attendance at an academic/school related event.
3. Student is ill and can not attend school on **March 13th**.

There will be a processing fee for the second AIME as follows: 1-10 students = \$25, 11+ students = \$50. We will need your payment before the answer forms can be graded. A special envelope and payment form will be included with your AIME material, if you have AIME qualifiers. All AIME answer forms must arrive in the AMC office by **March 30, 2007**.

Email requests for the second AIME may be sent to:

AIMEQUAL@AMC.UNL.EDU

or, you can call the AMC office at 1-800/527-3690. Please have your school identification number (CEEB) and charge card information available before calling. E-Mail requests should include the school's CEEB number, and complete mailing address.

Under no circumstances can a student take both AIME's.

## VIII. USAMO Participant Selection

The USA Mathematical Olympiad (USAMO) is a two day, nine-hour, six-question, essay-proof examination. Selection for the USAMO will be explained in the AIME Teacher Manual. The goal is to select about 500 of the top scorers from the prior AIME and AMC 12A, AMC 12B, AMC 10A and AMC 10B contests to participate in the USAMO.

The USAMO is scheduled for **Tuesday and Wednesday, April 24 & 25, 2007** at your school. If you feel you may have a qualifier, please arrange for a space and proctor for these dates.

The top 12 scoring students on the USAMO will be invited to attend an award ceremony held in Washington, D.C., on **May 20-21, 2007**.

## IX. The MOSP Program

The Mathematical Olympiad Summer Program (MOSP) is a 3-week, academic challenge designed to broaden participants' view of mathematics while fostering excitement toward further math study. It is held each year at the University of Nebraska-Lincoln in June-July. Invited students include the top 12 USAMO winners, 12-18 high-scoring USAMO participants, who are current juniors and below, and an additional 30 ninth-grade USAMO participants, grant funding permitting.

Watch for further details to be announced in the **2007 AIME/USAMO Teachers' Manual** and on the AMC website at [www.unl.edu/amc](http://www.unl.edu/amc).

## X. Regions of the AMC 10/AMC 12

The USA and Canada are partitioned into the following regions. National Awards are given to a minimum of 10 high scoring

students and 5 schools (based on the team score) in each of these regions.

#### Region

- 0 Connecticut, Maine, Massachusetts, New Hampshire, Pennsylvania, Rhode Island, Vermont
- 1 New Jersey, New York
- 2 Delaware, District of Columbia, Maryland, North Carolina, South Carolina, Virginia, West Virginia
- 3 Alabama, American Embassy and APO/FPO Schools, Florida, Georgia, Puerto Rico, Virgin Islands
- 4 Indiana, Michigan, Ohio
- 5 Arkansas, Iowa, Kansas, Minnesota, Nebraska, North Dakota, Oklahoma, South Dakota, Wisconsin
- 6 Illinois, Kentucky, Missouri, Tennessee
- 7 Louisiana, Mississippi, Texas
- 8 Alaska, Arizona, Colorado, Guam, Hawaii, Idaho, Montana, Nevada, New Mexico, Oregon, Utah, Washington, Wyoming
- 9 California
- 10 Canada

## XI. Intramural and National Awards

### Intramural Awards

The AMC Office will send you Intramural awards, along with your school results. Your registration fee entitles you to pins, medals, certificates, and a copy of the National Summary of Results and Awards. An order form for additional intramural awards will be included with your results.

1. Winner Pin — given to the top scoring student in your school for both the AMC 10 and AMC 12. Medals will be given for consecutive wins in both contests mentioned above. Details of how medals are awarded will be included with your school's results.
2. Certificate of Distinction — awarded to all students who qualify for the AIME.
3. Honor Roll of Distinction Pin — given to the top 1% of the AMC 12 and to the top 1% of AMC 10 participants.
4. AMC 12 Certificate of Achievement — given to students in grade 10 and below who score 90 or above on the AMC 12 Contests.
5. AMC 10 Certificate of Achievement — given to students in grade 8 and below who score 90 or above on the AMC 10 Contests.
6. School Certificate of Honor — awarded to schools with a team score (AMC 12) of 400 or greater.
7. School Certificate of Merit — awarded to schools with a team score (AMC 12) between 300 and 399, inclusive.

### State/Provincial Awards

Plaques — Top Scoring Student Plaque awarded to a student in each state or province.

Some State/Provincial Directors offer various awards or scholarships in addition to the plaques.

### National Awards

In each of the eleven regions the AMC divides the United States and Canada into, the five schools with the highest team scores

(sum of the highest three scores by participants) are recognized by Donor or Committee Awards.

The nationally highest scoring team will receive the Charles T. Salkind Memorial Silver Cup, a silver Trophy Cup awarded to the school with the highest team score over all, donated by the L.G. Balfour Company, Attleboro, MA.

The second nationally highest scoring team will receive the William H. Fagerstrom Memorial Silver Cup, a silver Trophy Cup awarded to the school with the second highest team score over all, also donated by the L.G. Balfour Company, Attleboro, MA.

In each of the regions in which a silver cup is not awarded the Committee on the American Mathematics Competitions provides a Committee Bronze Cup to the school with the highest team score.

### Awards for Schools not receiving a Trophy Cup

The four remaining schools in each region will receive one of these awards. Distribution of awards is arranged so that schools do not receive duplicates of awards made to them in prior years.

1. CAMC Mathematics Books — in each of the eleven regions, five books are donated by the Committee on the American Mathematics Competitions to one school having a high team score.
2. W. H. Freeman Books — in each of the eleven regions, a set of books, donated by W. H. Freeman Company, San Francisco, CA is awarded to one school having a high team score.
3. Mathematics Magazine — in each of the eleven regions, a one-year subscription is donated by the Committee on the American Mathematics Competitions to two or more schools having a high team score.

### Most Improved Team Score Award

1. Pedagogy Award — awarded to one school in each of the 50 states, US Territories, Military Schools abroad, and Canada with the "Most Improved" Team Score on the AMC 10 and AMC 12.



# XII. Contest A Certification

The AMC 10 and AMC 12 must be administered by a teacher or an adult not associated with or related to any of the participants. The administration of the contest must take place in a public building (e.g. school, library, church). Please send all Answer Forms from your school or group at one time.

The Contest Manager and the Principal, Vice Principal, or Headmaster must sign this form which is to be returned with your student Answer Forms.

### Certification by the Principal, official or person with comparable title:

- (a) I certify that the exam package(s) were retained in their sealed condition within an hour of the start of the contest.
- (b) I accept for our school the rules and procedures described on this page and pages 4-7, and accept that failure to follow these rules and procedures may result in DISQUALIFICATION from official standing of all scores from our school.

Signature \_\_\_\_\_ Time \_\_\_\_\_

Title \_\_\_\_\_ Date \_\_\_\_\_

### Certification by the Contest Manager:

I certify that the following statements are true or that, if there are any exceptions, I have checked the box at the bottom of this page and have listed them on the back of this page. I understand that the absence of either signature from this form, and a consideration of the exceptions may result in DISQUALIFICATION of all scores from our school.

1. The contest was held on TUESDAY, February 6, 2007.
2. The AMC 10-A/AMC 12-A were given at the same time.
3. The participants were continually monitored during the contest, and they were separated by an empty space, if possible.
4. No aids were permitted other than scratch paper, graph paper, ruler, compass, protractor and calculator (see Section I. Item 4).
5. Participants had exactly 75 minutes working time. (See page 6 for Student Disabilities Policy)
6. No students were permitted to proctor or grade the contest.
7. The instructions relating to the opening of the "Complimentary Solutions Envelope" and/or Solution Packets were followed.
8. After the contest, the answer forms were kept secure and no changes were made in the answers.
9. No parent assisted in the administration of the contest.
10. I have followed all the rules as stated in this Teachers' Manual.

Signature \_\_\_\_\_ E-mail (please print) \_\_\_\_\_

Day & Date Test was given \_\_\_\_\_

Name of School \_\_\_\_\_

City \_\_\_\_\_ State/Province \_\_\_\_\_

Telephone ( \_\_\_\_\_ ) \_\_\_\_\_ School CEEB Number \_\_\_\_\_

PLEASE INDICATE THE EXACT NUMBER OF AMC 10-A/AMC 12-A ANSWER FORMS RETURNED FOR GRADING. \_\_\_\_\_

EXCEPTIONS

## Service Questionnaire

Please mark the answers to this questionnaire in the box on the back of the School ID form. The AMC is collecting information about the choices made by students who complete calculus before their senior year and would very much appreciate your response to the following questions.

- Last year (2005-2006), how many students in 11th grade or below finished your school's highest level first-year calculus course?  
 none       1-2       3-5       6-10       11-20       more than 20
- What percentage of the students counted in question 1 are taking a mathematics course this year? The course need not be at your school.  
 less than 50%       50-75%       76-90%       91-99%       100%
- If any of your students take courses beyond first-year calculus, how is this accomplished? Fill in the circle for all that apply.  
 Take a course taught at your school by your teachers       Take a course online  
 Take a course taught at your school by college professors       Self-study  
 Take a course at a nearby college or university       Other
- Which mathematics courses are being taken this year at your school by students who have completed calculus? Fill in the circle for all that apply.  
 Mathematical modeling       Discrete mathematics or combinatorics  
 Linear algebra       More calculus (multivariate, differential equations)  
 Statistics       Other
- If students at your school finish calculus in eleventh grade or earlier, how are they able to accomplish this? Fill in the circle for each method that applies.  
 Students begin their four-course high school sequence in seventh grade or before.  
 Students take two courses in one year.  
 Our school condenses the usual four-course high school sequence into fewer than four courses by omitting some topics.  
 Our school condenses the usual four-course high school sequence into fewer than four courses without omitting any topics.  
 Precalculus/math analysis is taught within the calculus course.  
 Other
- If you have students who finish calculus in eleventh grade or earlier and choose not to take mathematics the next year, why do they make this choice? Fill in the circle for all that apply.  
 Their intended college major doesn't require any mathematics beyond first-year calculus.  
 They prefer to take AP courses in other subjects.  
 There are no advanced mathematics courses available to them at this school.  
 They are not all that interested in mathematics.  
 They are burned out and need a break from mathematics.  
 Other
- Fill in the circle for each activity offered at your school this year to students who have completed calculus.  
 Problem solving course or group       Training for mathematical competitions  
 Tutoring other students       Opportunity for research with college professors  
 Math club       No specific activities are offered
- Would you be interested in receiving materials or training in teaching advanced courses to high school students? If so, fill in the circle for all that apply.  
 Mathematical modeling       Discrete mathematics or combinatorics  
 Linear algebra       More calculus (multivariate, differential equations)  
 Statistics       Other

If you answered "Other" to any questions above, please explain in the space below the answer box.

Is there any question that we haven't asked that you wish we had? Please give your answer in the space provided.

If you would like to contact someone about these issues, please email David Bressoud, Chair-Elect of SIGMAA-TAHSM:  
bressoud@macalester.edu

If you or someone else at your school is interested in joining the MAA's Special Interest Group on Teaching Advanced High School Mathematics, please give us your name and email address at the bottom of the page.

# XIII. Additional Forms used - (if the form you need is not here, please see our web site)

## Additional Bundles Form

Please fill in the information below and FAX your order. The administrator or authorized person of the school agrees to pay the American Mathematics Competitions for the following materials:

School Name \_\_\_\_\_ CEEB # \_\_\_\_\_

Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

Teacher placing the order \_\_\_\_\_

**AMC 10** Contest A Bundles of ten ..... # \_\_\_\_\_ @ \$14/bundle = .....\$ \_\_\_\_\_  
 A Solutions Sets of ten (optional) ..... # \_\_\_\_\_ @ \$ 6/set = .....\$ \_\_\_\_\_

**AMC 12** Contest A Bundles of ten ..... # \_\_\_\_\_ @ \$16/bundle = .....\$ \_\_\_\_\_  
 A Solutions Sets of ten (optional) ..... # \_\_\_\_\_ @ \$ 6/set = .....\$ \_\_\_\_\_

To order either the 2006-2007 AMC 10/12 Math Club Package or the 21<sup>st</sup> Century CD with pdf's of all contests 2001-2005, download the Publications Order Form from the AMC website at [www.unl.edu/amc/](http://www.unl.edu/amc/)

Postage/handling Fee (see chart below) .....\$ \_\_\_\_\_

Total..... \$ \_\_\_\_\_

P.O. Number \_\_\_\_\_

VISA/MC# : \_\_\_\_\_ Address: \_\_\_\_\_

Name (Please Print): \_\_\_\_\_

Exp. Date: \_\_\_\_\_

### AMC ORDERING -- TERMS

1. VISA & MasterCard accepted.

FAX 402-472-6087 or 1-800-527-3690

2. Make checks payable to:  
AMERICAN MATHEMATICS COMPETITIONS

3. PAYMENT IN U.S. FUNDS ONLY.

Please Send Your Order To:

4. U.S.A.:	<u>Order TOTAL</u>	<u>Shipping Charge*</u>
	\$10.00 -- \$40.00	\$7.00
	\$40.01 -- \$50.00	\$9.00
	\$50.01 -- \$75.00	\$12.00
	\$75.01 -- UP	\$15.00

American Mathematics Competitions  
 ATTN: AMC 10/12 Additional Bundles  
 P.O. Box 81606  
 Lincoln, NE 68501-1606

5. OUTSIDE U.S.A.: Add additional \$10 to U.S.A. shipping

\*Orders after **January 26<sup>th</sup>** add \$5.00 additional for 2-day Service. Orders after **February 2<sup>th</sup>** add \$10.00 additional for 1-day Service.

### Proof of Intent to Pay

This document is intended to be used in lieu of pre-payment when calling or faxing in an order. Please indicate if you wish to be billed or will be sending a "check in the mail" (to be received within 2 weeks of order or you will be billed). Mail orders not wishing to be billed should include a check when returning this form. The person who signs this form must be authorized to pay the order that is placed by the teacher.

BILLED

Name of Person Authorized to Pay (please print): \_\_\_\_\_

Signature: \_\_\_\_\_

Title: \_\_\_\_\_

Date: \_\_\_\_\_

# Rescoring Request Form

I would like to have the following student's answer form rescored. I understand that there is a \$5.00 charge for each student answer form rescored

\$ 5.00/each

Student Name \_\_\_\_\_ \$ \_\_\_\_\_  
Contest taken: AMC 10-A AMC 10-B AMC12-A AMC12-B

Student Name \_\_\_\_\_ \$ \_\_\_\_\_  
Contest taken: AMC 10-A AMC 10-B AMC12-A AMC12-B

Grand Total \_\_\_\_\_ \$ \_\_\_\_\_

Teacher's Name \_\_\_\_\_ CEEB # \_\_\_\_\_

School Name \_\_\_\_\_

Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

## Method of Payment:

Check (US funds only) made payable and mailed with this form to the:  
AMERICAN MATHEMATICS COMPETITIONS  
University of Nebraska-Lincoln  
P.O. Box 81606  
Lincoln, NE 68501-1606

Charge to Visa/Mastercard#: \_\_\_\_\_

Name on card (print): \_\_\_\_\_

Signed

Expiration Date: \_\_\_\_\_ Telephone: \_\_\_\_\_

FAX to: 402/472-6087





# 2007 REGISTRATION - B

## AMC 10B & 12B

### FORM B FOR 2/21/07

#### PLEASE PRINT

High School CEEB# (6 digits): \_\_\_\_\_

Contest Manager \_\_\_\_\_

School Name \_\_\_\_\_

School Street Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

( \_\_\_\_\_ )

School Phone # \_\_\_\_\_

#### BILLING ADDRESS (if different from school):

\_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ Zip \_\_\_\_\_

( \_\_\_\_\_ )

Phone # \_\_\_\_\_

E-mail (for sending results) Please Print Clearly \_\_\_\_\_

(Circle appropriate responses, below)

**Type of Group:** Public School \_\_\_\_\_ Private School \_\_\_\_\_ Home School \_\_\_\_\_ Other \_\_\_\_\_

**Grades:** PreK K 1 2 3 4 5 6 7 8 9 10 11 12 13

**School Size:** 0-200 201-400 401-1000 1001+

#### PAYMENT OPTIONS

Do not send payment alone. The Registration Form must be included with your payment option. **Checks sent without appropriate registration information cannot be processed and will be returned to sender.**

Check P.O. # \_\_\_\_\_ Visa \_\_\_\_\_ MasterCard \_\_\_\_\_

Terms - Payment in U.S. Funds only.  
Make checks payable to: **American Math Competitions**

VISA/MC # \_\_\_\_\_

Exp. Date: \_\_\_\_\_

Name (Please Print) \_\_\_\_\_

(Please give an address for mailing the charge receipt in "Billing Address" above)

All orders **Non-Refundable** once shipped.  
Mail along with your payment or Purchase Order to:

**MAA American Mathematics Competitions**  
ATTN: AMC 10/12 Registration  
P.O. Box 81606  
Lincoln, NE 68501-1606

#### REGISTRATION

Pick either U.S. or International Registration

**US REGISTRATION, Contiguous 48 states -- REQUIRED**

♦ One fee covers U.S. 10B/12B Registration & shipping  
(choose 1)

**Registration/Expedited Shipping** ..... \$ 50.00

- ♦ REQUIRED for Short Shipping window
- ♦ 2-day shipping

OR

**Registration/Overnight Shipping** ..... \$ 60.00

- ♦ REQUIRED after January 15 (Jan. 16--Feb 14).
- ♦ Overnight shipping

U. S. REGISTRATION SUBTOTAL ..... \$ \_\_\_\_\_

#### CANADIAN/INT'L REGISTRATION

**Canadian/Int'l Schools, Guam, Alaska, Puerto Rico, Hawaii Registration** ..... \$ 50.00 +  
+ **Shipping** (\$63 maximum)  
# bundles \_\_\_\_\_ @ **\$7.00/bundle** = ..... \$ \_\_\_\_\_ =

CANADIAN/INT'L SUB-TOTAL ..... \$ \_\_\_\_\_

#### CONTEST BUNDLES OF TEN --

♦ **AMC 10B -- 10 copy sets per bundle**

English 10B contests=# \_\_\_\_\_ @ \$14/bundle=.....\$ \_\_\_\_\_ +

Spanish 10B contests=# \_\_\_\_\_ @ \$14/bundle=.....\$ \_\_\_\_\_ +

French 10B contests=# \_\_\_\_\_ @ \$14/bundle=.....\$ \_\_\_\_\_ +

(optional)

English **Solutions** = # \_\_\_\_\_ @ \$ 6/set=.....\$ \_\_\_\_\_ +

♦ **AMC 12B -- 10 copy sets per bundle**

English 12B contests=# \_\_\_\_\_ @ \$16/bundle=.....\$ \_\_\_\_\_ +

Spanish 12B contests=# \_\_\_\_\_ @ \$16/bundle=.....\$ \_\_\_\_\_ +

French 12B contests=# \_\_\_\_\_ @ \$16/bundle=.....\$ \_\_\_\_\_ +

(optional)

English 12B **Solutions** = # \_\_\_\_\_ @ \$ 6/set=.....\$ \_\_\_\_\_ =

CONTEST BUNDLES SUB-TOTAL ..... \$ \_\_\_\_\_

Braille & Large Print Contests are priced individually:

Braille **AMC 10B** # \_\_\_\_\_ @ 1.40/each = .....\$ \_\_\_\_\_ +

LgPrint **AMC 10B** # \_\_\_\_\_ @ 1.40/each = .....\$ \_\_\_\_\_ +

Braille **AMC 12B** # \_\_\_\_\_ @ 1.60/each = .....\$ \_\_\_\_\_ +

LgPrint **AMC 12B** # \_\_\_\_\_ @ 1.60/each = .....\$ \_\_\_\_\_ +

+ Shipping (sent separately) = .....\$ 7.00 =

BRILLE & LARGE PRINT SUB-TOTAL ..... \$ \_\_\_\_\_

#### AMC 10/12 MATH CLUB PACKAGE:

Study Guide, Web materials, see brochure.

Sent separately; available Fall 2006

PACKAGE, INCLUDING SHIPPING:

Contiguous 48 states US = \$ 22.00 = \$ \_\_\_\_\_

Additional postage required for International/Overseas Addresses,  
please email AMC office: amcinfo@unl.edu

**TOTAL ORDER --** ..... \$ \_\_\_\_\_

# XIV. Classroom Accessories

## AMC 10 Student Practice Questions

You will find these and additional problems for the AMC 10 and AMC 12 on AMC's web site: <http://www.unl.edu/amc>, available from the 2007 AMC 10/12 Teacher Manual directory, (<http://www.unl.edu/amc/d-publication/d1-pubarchive/2003-4pub/04tm12/04amc1012tm.html>) or from our Problems page archives (<http://www.unl.edu/amc/a-activities/a7-problems/problem81012archive.html>).

The ratio of Mary's age to Alice's age is  $3 : 5$ . Alice is 30 years old. How many years old is Mary?

- (A) 15            (B) 18            (C) 20            (D) 24            (E) 50

2006 AMC 10 A, Problem #3—  
2006 AMC 12 A, Problem #3— “Put the ratio of their ages into a fraction.”

**Solution (B)** Mary is  $(3/5)(30) = 18$  years old.

**Difficulty:** Easy

**NCTM Standard:** Number and Operations Standard: understand meanings of operations and how they relate to one another

**Mathworld.com Classification:** Number Theory > Arithmetic > Fractions > Ratio

**AMC 10 Student Practice Questions continued**

A player pays \$5 to play a game. A die is rolled. If the number on the die is odd, the game is lost. If the number on the die is even, the die is rolled again. In this case the player wins if the second number matches the first and loses otherwise. How much should the player win if the game is fair? (In a fair game the probability of winning times the amount won is what the player should pay.)

- (A) \$12      (B) \$30      (C) \$50      (D) \$60      (E) \$100

**2006 AMC 10 A, Problem #13—**

**“What is the probability that the player will win?”**

**Solution (D)** Let  $x$  represent the amount the player wins if the game is fair. The chance of an even number is  $1/2$ , and the chance of matching this number on the second roll is  $1/6$ . So the probability of winning is  $(1/2)(1/6) = 1/12$ . Therefore  $(1/12)x = \$5$  and  $x = \$60$ .

**Difficulty:** Medium

**NCTM Standard:** Data Analysis and Probability Standard: understand and apply basic concepts of probability

**Mathworld.com Classification:** Probability and Statistics > Probability > Probability

**AMC 10 Student Practice Questions continued**

Leap Day, February 29, 2004, occurred on a Sunday. On what day of the week will Leap Day, February 29, 2020, occur?

(A) Tuesday (B) Wednesday (C) Thursday (D) Friday (E) Saturday

**2006 AMC 10 B, Problem #16—**

**“In the years from 2004 through 2020, Each Leap Day occurs  $3 \cdot 365 + 366 = 1461$  days after the preceding Leap Day.”**

**Solution (E)** In the years from 2004 through 2020, Each Leap Day occurs  $3 \cdot 365 + 366 = 1461$  days after the preceding Leap Day. When 1461 is divided by 7 the remainder is 5. So the day of the week advances 5 days for each 4-year cycle. In the four cycles from 2004 to 2020, the Leap Day will advance 20 days. So Leap Day in 2020 will occur one day of the week earlier than in 2004, that is, on a Saturday.

**Difficulty:** Hard

**NCTM Standard:** Measurement Standard: understand measurable attributes of objects and the units, systems, and processes of measurement

**Mathworld.com Classification:** Number Theory > Congruences > Modulus



AMC 10 Student Practice Questions continued

Let  $a$  and  $b$  be the roots of the equation  $x^2 - mx + 2 = 0$ . Suppose that  $a + (1/b)$  and  $b + (1/a)$  are the roots of the equation  $x^2 - px + q = 0$ . What is  $q$ ?

- (A)  $\frac{5}{2}$       (B)  $\frac{7}{2}$       (C) 4      (D)  $\frac{9}{2}$       (E) 8

**2006 AMC 10 B, Problem #14—**

**“Write  $x^2 - mx + 2 = 0$  as  $(x - a)(x - b)$ .”**

**Solution (D)** Since  $a$  and  $b$  are roots of  $x^2 - mx + 2 = 0$ , we have

$$x^2 - mx + 2 = (x - a)(x - b) \quad \text{and} \quad ab = 2.$$

In a similar manner, the constant term of  $x^2 - px + q$  is the product of  $a + (1/b)$  and  $b + (1/a)$ , so

$$q = \left(a + \frac{1}{b}\right) \left(b + \frac{1}{a}\right) = ab + 1 + 1 + \frac{1}{ab} = \frac{9}{2}.$$

**Difficulty:** Hard

**NCTM Standard:** Algebra Standard: represent and analyze mathematical situations and structures using algebraic symbols

**Mathworld.com Classification:** Calculus and Analysis > Roots > Root

## AMC 12 Student Practice Questions

Oscar buys 13 pencils and 3 erasers for \$1.00. A pencil costs more than an eraser, and both items cost a whole number of cents. What is the total cost, in cents, of one pencil and one eraser?

- (A) 10      (B) 12      (C) 15      (D) 18      (E) 20

### 2006 AMC 12 A, Problem #9—

**“Let  $p$  be the cost (in cents) of a pencil, and let  $s$  be the cost (in cents) of a set of one pencil and one eraser.”**

**Solution (A)** Let  $p$  be the cost (in cents) of a pencil, and let  $s$  be the cost (in cents) of a set of one pencil and one eraser. Because Oscar buys 3 sets and 10 extra pencils for \$1.00, we have

$$3s + 10p = 100.$$

Thus  $3s$  is a multiple of 10 that is less than 100, so  $s$  is 10, 20, or 30. The corresponding values of  $p$  are 7, 4, and 1. Since the cost of a pencil is more than half the cost of the set, the only possibility is  $s = 10$ .

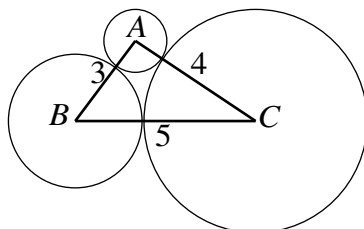
**Difficulty:** Easy

**NCTM Standard:** Algebra Standard: represent and analyze mathematical situations and structures using algebraic symbols

**Mathworld.com Classification:** Algebra > Algebraic Equations > Linear Equation

AMC 12 Student Practice Questions continued

The vertices of a 3–4–5 right triangle are the centers of three mutually externally tangent circles, as shown. What is the sum of the areas of these circles?



- (A)  $12\pi$       (B)  $\frac{25\pi}{2}$       (C)  $13\pi$       (D)  $\frac{27\pi}{2}$       (E)  $14\pi$

**2006 AMC 12 A, Problem #13—**

**“Label the radii. What do we know about the right triangle?”**

**Solution (E)** Let  $r$ ,  $s$ , and  $t$  be the radii of the circles centered at  $A$ ,  $B$ , and  $C$ , respectively. Then  $r + s = 3$ ,  $r + t = 4$ , and  $s + t = 5$ , from which  $r = 1$ ,  $s = 2$ , and  $t = 3$ . Thus the sum of the areas of the circles is

$$\pi(1^2 + 2^2 + 3^2) = 14\pi.$$

**Difficulty:** Medium

**NCTM Standard:** Geometry Standard: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

**Mathworld.com Classification:** Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Triangle

AMC 12 Student Practice Questions continued

An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different points could be the final point?

- (A) 120      (B) 121      (C) 221      (D) 230      (E) 231

**2006 AMC 12 B, Problem #18**— “Each step changes either the  $x$ -coordinate or the  $y$ -coordinate of the object by 1.”

**Solution (B)** Each step changes either the  $x$ -coordinate or the  $y$ -coordinate of the object by 1. Thus if the object's final point is  $(a, b)$ , then  $a + b$  is even and  $|a| + |b| \leq 10$ . Conversely, suppose that  $(a, b)$  is a lattice point with  $|a| + |b| = 2k \leq 10$ . One ten-step path that ends at  $(a, b)$  begins with  $|a|$  horizontal steps, to the right if  $a \geq 0$  and to the left if  $a < 0$ . It continues with  $|b|$  vertical steps, up if  $b \geq 0$  and down if  $b < 0$ . It has then reached  $(a, b)$  in  $2k$  steps, so it can finish with  $5 - k$  steps up and  $5 - k$  steps down. Thus the possible final points are the lattice points that have even coordinate sums and lie on or inside the square with vertices  $(\pm 10, 0)$  and  $(0, \pm 10)$ . There are 11 such points on each of the 11 lines  $x + y = 2k$ ,  $-5 \leq k \leq 5$ , for a total of 121 different points.

**Difficulty:** Hard

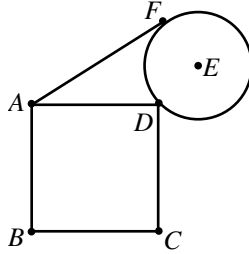
**NCTM Standard:** Measurement Standard: apply appropriate techniques, tools, and formulas to determine measurements

**Mathworld.com Classification:** Discrete Mathematics > Point Lattices > Point Lattice



AMC 12 Student Practice Questions continued

Square  $ABCD$  has side length  $s$ , a circle centered at  $E$  has radius  $r$ , and  $r$  and  $s$  are both rational. The circle passes through  $D$ , and  $D$  lies on  $\overline{BE}$ . Point  $F$  lies on the circle, on the same side of  $\overline{BE}$  as  $A$ . Segment  $AF$  is tangent to the circle, and  $AF = \sqrt{9 + 5\sqrt{2}}$ . What is  $r/s$ ?



- (A)  $\frac{1}{2}$       (B)  $\frac{5}{9}$       (C)  $\frac{3}{5}$       (D)  $\frac{5}{3}$       (E)  $\frac{9}{5}$

2006 AMC 12 A, Problem #17—

“Look at right triangle  $AFE$ .”

**Solution (B)** Let  $B = (0, 0)$ ,  $C = (s, 0)$ ,  $A = (0, s)$ ,  $D = (s, s)$ , and  $E = \left(s + \frac{r}{\sqrt{2}}, s + \frac{r}{\sqrt{2}}\right)$ . Apply the Pythagorean Theorem to  $\triangle AFE$  to obtain

$$r^2 + (9 + 5\sqrt{2}) = \left(s + \frac{r}{\sqrt{2}}\right)^2 + \left(\frac{r}{\sqrt{2}}\right)^2,$$

from which  $9 + 5\sqrt{2} = s^2 + rs\sqrt{2}$ . Because  $r$  and  $s$  are rational, it follows that  $s^2 = 9$  and  $rs = 5$ , so  $r/s = 5/9$ .

OR

Extend  $\overline{AD}$  past  $D$  to meet the circle at  $G \neq D$ . Because  $E$  is collinear with  $B$  and  $D$ ,  $\triangle EDG$  is an isosceles right triangle. Thus  $DG = r\sqrt{2}$ . By the Power of a Point Theorem,

$$9 + 5\sqrt{2} = AF^2 = AD \cdot AG = AD \cdot (AD + DG) = s(s + r\sqrt{2}) = s^2 + rs\sqrt{2}.$$

As in the first solution, conclude that  $r/s = 5/9$ .

**Difficulty:** Hard

**NCTM Standard:** Geometry Standard: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

**Mathworld.com Classification:** Geometry > Plane Geometry > Circles > Circle



# The MATHEMATICAL ASSOCIATION OF AMERICA

## American Mathematics Competitions

February, 2007

Dear Parent or Guardian:

On **February 6, 2007 and/or February 21, 2007** your son or daughter participated in the **58th** annual American Mathematics Competitions contest. This contest has grown from a single city-wide competition in New York City in 1950, organized by the local section of The Mathematical Association of America to a sequence of contests. In 2006, over 250,000 students from over 4,200 US and international schools competed for school, regional, and national awards in this contest and found it fun and rewarding. Top 10, well-known U.S. universities and colleges, including internationally recognized U.S. technical institutions, ask for AMC scores on their application forms. Your students deserve the chance to list these scores on their applications!



Each year the AMC 10 and AMC 12 are on the **National Association of Secondary School Principals Advisory List of Contests and Activities**. The AMC Contests are sponsored by the Mathematical Association of America, and are considered to be such a valuable stimulus to student interest in mathematics that 23 professional societies and organizations, including the National Council of Teachers of Mathematics and those represented below, support the contests with financial contributions.



With these contests, there are awards in each school for the student with the highest score, certificates for high-scoring students in each school, state-wide awards, regional awards, and even national awards. These contests lead to other more selective math contests, even all the way to the USA team sent to the International Mathematical Olympiad, the premier international high school level problem solving contest.

But the real rewards come from challenging each student with mathematics that is new, different, and “outside of the box.” The problems on the contest are hard, but designed to be within reach. Just by participating in the contest your student should still feel accomplishment, because these problems are meant to be more challenging than routinely encountered in mathematics courses.

Mathematics is increasingly important in our technological and scientific age. Taking enough mathematics in high school is the gateway to jobs and careers of all kinds, even those that are not explicitly mathematical, scientific, or technological. We hope that by offering these contests, we can challenge and inspire students to learn more mathematics. We hope that your son or daughter enjoyed the contests, and will continue to take mathematics courses and competitions in high school and beyond.

Sincerely,

Dr. Steven R. Dunbar  
AMC Director

## Publicity

The sample Contest Announcement news release below, should be prepared and distributed to the newspapers, radio and television stations in your region. To make preparation of the news release easier, visit our web site, and download the text from the on-line copy of the AMC 10/12 Teachers' Manual. See the AMC website, or the 2006 Summary of Results for statistics and figures from the 2006 contest. Statistics and figures for the 2007 Contest will be available on our website in March 2007.

(School or School District)  
FOR IMMEDIATE RELEASE

### (School) STUDENTS PARTICIPATE IN NATIONAL MATHEMATICS COMPETITION

(#) students at (School) participated in the 58<sup>th</sup> annual American Mathematics Contest 12, and/or the 8<sup>th</sup> annual American Mathematics Contest 10. The contests were held on Tuesday, February 6, (or Wednesday, February 21, 2007). The students competed for local, regional and national student and school awards. The contest, which covers high school mathematics, is given in participating schools. Its purpose is to spur interest in mathematics and develop talent through the excitement of friendly competition at problem solving in a timed format. In 2006 over 250,000 students from 4,400 schools participated in the AMC 10 & AMC 12 contest including (#) students from (#) schools in (State). Top scorers at (school) were (\_\_\_\_\_, \_\_\_\_\_, etc.).

According to Prof. Steven Dunbar, who serves as Director of the American Mathematics Competitions, the AMC 12 (first offered in 1950), and the AMC 10 (first offered in 2000), are part of a series of contests sponsored each year by The Mathematical Association of America, through their program, the American Mathematics Competitions. The AMC offers the only math competition series in the country leading to the United States of America Mathematical Olympiad (USAMO) and the Mathematical Olympiad Summer Program (MOSP). From this group of students, the AMC sends the highly competitive USA Team to the prestigious annual International Mathematical Olympiad. The AMC program includes :

American Mathematics Contest 8 (AMC8)	Grades 6- 8	November
American Mathematics Contest 10 (AMC 10)	Grades 10 & below	2 dates in February
American Mathematics Contest 12 (AMC 12)	Grades 12 & below	2 dates in February
American Invitational Mathematics Examination (AIME)	All who qualify	2 dates in March
USA Mathematical Olympiad (USAMO)	All who qualify	mid- to late-April
Mathematical Olympiad Summer Program (MOSP)	Qualify thru USAMO	June
International Mathematical Olympiad (IMO)	Top six from USAMO, MOSP	July

The AMC is located at the University of Nebraska - Lincoln. and receives support from the Akamai Foundation, American Mathematical Association of Two Year Colleges, American Mathematical Society, American Society of Pension Actuaries, American Statistical Association, Art of Problem Solving, Awesome Math, Canada/USA Mathpath, Canada/USA Mathcamp, Casualty Actuarial Society, Clay Mathematics Institute, Institute for Operations Research and the Management Sciences, Mu Alpha Theta, National Association of Mathematicians, National Council of Teachers of Mathematics, Pedagoguery, Inc., Pi Mu Epsilon, and the Society of Actuaries. The Contests are given across the U.S.A, Canada, and in many schools abroad.

Details concerning the 2007 AMC 10/12 contests for High School, as well as the rest of AMC's programs are available on the AMC web site: [www.unl.edu/amc/](http://www.unl.edu/amc/).

For further information contact the AMC -- telephone: 800/527-3690, email: [amcinfo@maa.org](mailto:amcinfo@maa.org).



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**Tuesday, FEBRUARY 6, 2007**

8<sup>th</sup> Annual American Mathematics Contest 10

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1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

*Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 25<sup>th</sup> annual American Invitational Mathematics Examination (AIME) on Tuesday, March 13, 2007 or Wednesday, March 28, 2007. More details about the AIME and other information are on the back page of this test booklet.*

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3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
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8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

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