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Word Histories: Melding Mathematics and Meanings

Integrating the history of mathematics into instruction sometimes seems to require taking advanced courses or studying lengthy texts. However, much of mathematics history is reflected in the very words that we use every day. The etymologies, or origins, of mathematics words make a rich resource for deepening students' understanding and appreciation of mathematics, history, and language. They open a window onto the lively history of our science and its connections with other subjects. In this article, we share some of these etymologies, as well as ideas for incorporating them into instruction.

The words of mathematics, like all words, have concrete origins that can be quite intriguing. The fact that *twelve* comes from the Gothic phrase *twa-lif,* meaning *two left,* reflects the deep historical roots of counting with ten (Menninger 1969). The fact that *perpendicular* and *suspend* share the root *pend*, meaning *to hang,* suggests the ancient Mediterranean practice of hanging a weight to construct a right angle to the horizon. The fact that *rational* and *irrational* can refer either to numbers or to beliefs is an artifact of the important role played by ratios of commensurable quantities in Greek mathematics and philosophy. As these examples suggest, words are often preserved fossils from olden times, and digging them up can result in a fascinating discovery of how mathematics developed.

Much of mathematics history is reflected in the words that we use every day

Table 1 shares etymologies for common terms in algebra, geometry, functions, and discrete mathematics. We include related English words because we find that these words are often the essential links that help students grasp or deepen their understanding of the mathematical term. Next, we offer more extensive details about words in three sample categories: the names of the branches of mathematics, the terms for the conic sections, and mathematical words of Arabic origin. Finally, we close by sharing teaching considerations. Throughout, we build extensively from the work of Steven Schwartzman (1994), who has written an excellent resource, *The Words of Mathematics,* as well as from such historical surveys as those by Boyer and Merzbach (1989), Joseph (1991), and Katz (1998).

ROOTS OF BRANCHES OF MATHEMATICS

The names of the disciplines within mathematics are themselves good starting points for etymological explorations.

Algebra comes from the title of an Arabic work written around 825 by the Baghdad mathematician Muhammad ibn Musa al-Khowarizmi. He was a scholar in the caliph's House of Wisdom, perhaps the leading intellectual center in the world at the time. A pair of words in the title of his Arabic text, *Hisab al-jabr w'al-muqabalah,* gave us the word *algebra,* meaning "reunion of broken parts." For example, when solving $5x - 6 = 3 - 2x$, adding $2x$ to both sides was thought of as "reuniting" the like terms. Ibn Khaldun, a medieval Arab historian, once compared solving an algebraic equation to healing broken bones: "The various elements are 'confronted,' and 'broken' portions are 'set' and thus become 'healthy'" (Ibn Khaldun 1958: III: pp. 124–25).

The term *geometry* flowed from the banks of the Nile River. The Nile's flooding every spring required farmers in ancient Egypt to have their lands surveyed and their borders reestablished. "Perhaps this was the way in which geometry was invented, and passed afterwards into Greece," suggested the ancient Greek historian Herodotus (1972, p. 169). Indeed, the Greek word *geometria* is rooted in *geo,* meaning *earth,* and *metron,* meaning *measure*. It shares roots with *geology,* the study of the earth; *geography,* drawing pictures of the earth; and *metronome,* a device that measures time in music. Trigonometry, too, is a measure, in this example, of *trigons,* or *triangles*.

Calculus comes from the Latin word *calx,* meaning *limestone.* Related words include *calcium* and *chalk*. In ancient Rome and in many other cultures,

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TABLE 1

Some High School Mathematics Vocabulary by Strand

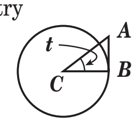
Terms, Roots, Links to Meaning	Related Words	Notes
	ALGEBRA	
polynomial *polus:* many *nomos:* portion, part, or name A polynomial is an expression consisting of many parts or names, that is, terms.	*polygon:* a figure with many angles *polyglot:* someone who speaks many languages (literally, “many tongues”) *denominator:* name of a type of fraction, for example, fifths	Polynomials have many family members: monomials have one term or product, for example, $3ab^2$; binomials have two terms, for example, $3ab^2 + 4a$; trinomials have three terms, for example, $3ab^2 + 4a - 5$.
quadratic *quadratum:* square, from *kwetwer,* meaning four A quadratic equation is one involving x^2, the expression for the area of a square.	*quadrilateral:* a figure with four sides *quadrangle:* a figure with four angles, often an enclosed area of a university	It seems odd that *quadratic* does not refer to a fourth-degree expression. A fourth-degree expression is called *quartic,* from the same root. Instead, *quadratic* refers to the power x^2, the area of the *square* from which the expression is derived.
surd *surdus:* deaf, silent, stupid A surd is an irrational number, or more specifically, an irrational root, such as $\sqrt{23}$. Its exact value is “hidden” and does not “scream out” to us.	*absurd:* ridiculously unsound or unreasonable *surd:* in linguistics, a voiceless sound *sourd:* deaf, muffled	Baghdad mathematician al-Khowarizmi spoke of rational and irrational numbers as “audible” and “inaudible,” respectively. Around 1150, Gherardo of Cremona made the Latin translation *surdus* (Smith 1925, p. 252).
	GEOMETRY	
perimeter *per:* around *meter:* measure The perimeter is the measure around a circle or other figure.	*peripheral:* arrayed around the main thing, for example, peripheral land, peripheral device *diameter:* the measure “across” a figure	In 1706, William Jones chose the Greek letter π (“pi”) to symbolize the ratio of the perimeter and diameter of a circle, because it is the first letter of the ancient Greek word *perimeter* (Boyer and Merzbach 1989).
trapezoid *trapeza:* table, from *tetra* (four) and *ped* (foot) *oid:* looking like A trapezoid is a quadrilateral with two parallel sides.	*trapeze:* a circus stunt apparatus that originally must have had four sides: two ropes, the bottom bar, and a support bar	The ancient Greeks thought that a trapezoid resembled a table. In Europe, a *trapezoid* has no parallel sides, and a *trapezium* is the figure that people in the United States call a *trapezoid*.
tangent *tangens:* touching A line that touches a circle in one point is a tangent line. In calculus, a broader definition is used.	*tangible:* touchable *tax:* to touch someone else’s money	The tangent ratio in trigonometry is the length of the tangent segment *AB* formed by angle *t* with the radius of circle *C*. 
congruent *congruere:* to meet together, agree, correspond Congruent figures correspond in all aspects; they meet or agree in every property. When they are superimposed, they coincide fully.	*congruent:* according to or coinciding An action is said to be congruent with the law when it agrees with the legal statement. *gruel:* a food formed when grain and grinder “meet together”	Numbers are also said to be congruent relative to a certain divisor when they leave the same remainder. For example, 7 and 22 are both “congruent to 2 modulo 5” because they both leave the remainder 2 when divided by 5.
similar *similis:* like or resembling Figures are similar if they have the same shape, with all corresponding measures in the same ratio.	*similitude, ratio of:* the number produced by dividing a length in one of two similar figures by the corresponding length in the other	Beware: everyday English uses *similar* in a broad sense, whereas mathematics uses it in a technical sense. For example, students often apply the everyday meaning of *similar* and think (falsely) that all rectangles are similar.
	FUNCTION	
function *functus:* to perform A function performs a rule on each input to produce a single output.	*functional:* able to work *dysfunctional*, *defunct:* unable to work *perfunctory:* done superficially	The “function game” (or “guess my rule”) makes a nice introduction to the concept of function (Rubenstein 1996).
dependent variable *independent variable* *pend:* hang *de:* down from *in:* not The dependent variable or output of a function “hangs down from” the independent variable.	*pendant:* a piece of hanging jewelry *pendulum:* a weight hanging by a filament or rod *appendage:* a part that is hanging from a body *penthouse:* a room “hanging” from the main living area	Ancient surveyors used a weight, or plumb, hanging from the end of a string to form a line *perpendicular* to the earth’s surface or horizon (“horizontal”).

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TABLE 1—*Continued*

Some High School Mathematics Vocabulary by Strand

Terms, Roots, Links to Meaning	Related Words	Notes
domain *domus:* house or home A function rules over its values like a king over his domain.	*domicile:* a person's home *domestic:* related to a person's or a nation's home *dominate:* to rule, like a feudal master over his home	The domain of a function can be thought of as the set of values with which the function feels "at home." For example, for the reciprocal function $y = 1/x$, zero is a troublemaker that the rule(r) will not tolerate!
range *ring:* to turn, to bend The range of a function is the set of outputs that the rule produces.	*ring:* a band that encircles an area *ranch:* an area for raising animals *rank:* a row, a series, a line of soldiers	The function rings, or encircles, a set of numbers that it produces as output values.
DISCRETE MATHEMATICS		
factorial *factor:* doer, maker, performer Mathematically, a factor is a number that is part of a product. A factorial of a natural number is the product of all the natural numbers from 1 to the number itself, for example, 5 factorial is 1 • 2 • 3 • 4 • 5, or 120.	*factory:* a place where things are made *factor:* something that contributes to the production of a result *manufacture:* to make by hand; obviously, this original meaning has changed over time!	The symbol for factorial is an exclamation mark, signifying the surprise that even small numbers have very large factorials.
combination *com:* together with *bini:* two at a time A combination is a selection of distinct objects from an available pool of objects, without regard to the order in which they are selected.	*combine* (noun): a harvesting machine that performs many functions *binary:* a base-two numeration system *bicycle:* a two-wheeled vehicle	*Combine* originally meant to put two things together; it now means to put any number of things together. The mathematical term *combination* is more restricted: three shirts and four slacks produce twelve "pairings," technically not "combinations."
permutation *mutare:* to change A permutation is an arrangement of distinct objects from an available pool of objects, with the arrangement having a definite order.	*mutation:* a change in genes or other characteristics *commute:* to change places, for example, from home to school, or as in 2 + 7 = 7 + 2	Students frequently confuse permutations and combinations. Making the connection of "permute" with "commute" might help them see that both words focus on changing the order or arrangement.

Calculus *and* abacus *have etymologies that stem from calculations with tokens*

people used marbles or other pebbles (in Latin, *calculi*) to count and to reckon, a process that led to our English word *calculate*. In the 1660s, the noun *calculus* was adopted for the new branch of mathematics that calculates the rate at which quantities vary. The terms *abacus* and *counter,* as for displaying goods in a store, likewise have etymologies that stem from calculation with tokens.

Probability is the study of the likelihood of events. The name derives from the Latin adjective *probus,* meaning *upright* or *honest,* and its cousin *probare,* meaning *to try, to test, to judge*. The original idea was to identify "what is capable of being made good or able to be proved." However, in mathematics something that is *proved* is a necessary consequence of given conditions, whereas something that is *probable* is a likely inference but one that could be false. Related common English words are *probe,* to test by searching or by investigation, and *probation,* a test of a person's character.

Statistics comes from the German term *Statistik,* which around 1800 connoted what we would now call "political science, the study of (political) states." The concept of *state* itself originally referred to the conditions prevailing in a region, its *status* or *standing*—terms that stem from the same Indo-European root. The German political economists excelled in collecting and interpreting data about such states, and so the term *Statistik* evolved to refer more specifically to the data themselves. Today, statistics still tell us "how things stand." For more on the roots of the branches of mathematics, see Rubenstein and Schwartz (1999a).

SPEAKING OF CONICS

Why do *hyperbola,* the conic section, and *hyperbole,* an exaggeration, sound so alike? How about *parabola* and *parable?* Ever wondered what *ellipse* might have to do with *elliptical speech?* In each instance, the similarities stem from the fact that the two words have related Greek origins.

Although Apollonius of Perga (ca. 225 B.C.E.) was famous for making an early comprehensive study of conic sections, the names of those figures were based on the earlier work of the Pythagoreans (Bowsher 1989). In constructing this type of curve, an important length would fall short of, exceed, or match the length of a given segment. The Pythagoreans designated these three results as an *ellipsis,* a *defect;* a *hyperbole,* an *excess;* or a *parabole, alongside*. These three possibilities—also interpretable as "less than, greater than, or equal to"—can be seen in various ways when we investigate conic sections. One way to see them is to picture the cuts, or sections, of the cone with a plane. We start with a right cone, remembering that it has two nappes, as illustrated in **figure 1**. We could cut the cone in at least three different ways, depending on the angle of the cut.

- A cut at an angle *less than* the slant of the cone forms an *ellipse*.
- A cut at an angle *greater than* the slant of the cone forms a *hyperbola*.
- A cut at the *same* angle as the slant of the cone forms a *parabola*.

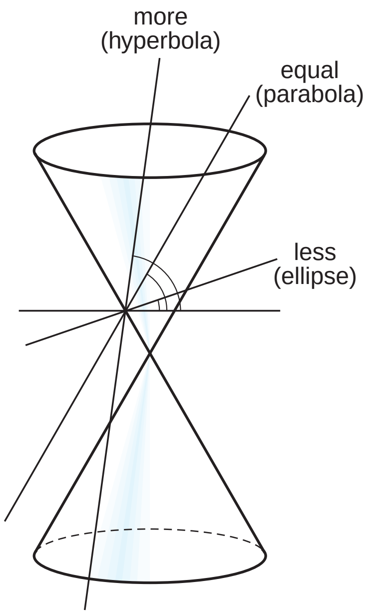

Fig. 1
Slicing a cone at three different angles of inclination (less than, more than, or equal to that of the slant of the cone) yields the three different types of conic sections.

In his noted work in rhetoric, Aristotle built on the same three-way idea to describe various figures of speech.

- To speak in a way that *falls short,* or leaves out important details, is to be *elliptical*.
- To say something that *goes too far,* or exaggerates, is to speak in *hyperbole*.
- To tell a story that *matches,* or parallels, another situation is to tell a *parable*.

All three of these conic names and figures of speech have roots that appear in other everyday words. For example, words with the *hyper* root include *hyperactive* (excessively active), *hypertension* (higher-than-normal blood pressure), and *hypersonic* (beyond the speed of sound). Words with the *para* root include *parallel* (in Euclidean geometry, two lines that remain exactly "alongside" each other by being an equal distance apart at every point), *paragraph* (lines written alongside each other), and *parasite* (something that lives alongside another living thing). Words related to *ellipse* are *ellipsis* (a row of dots indicating omitted items) and *eclipse* of the sun (the phenomenon that occurs when the sun is "left out" by being blocked by the moon).

DECIPHERING "CIPHER"

Anyone who has ever tried doing arithmetic with roman numerals knows what an advance the Hindu-Arabic system was. When numbers are not tallied but are "coded" or *ciphered* according to place value, calculations can be carried out much more efficiently. The philosopher Carra De Vaux, as quoted in Ajram (1992, p. 91), once noted, "By using ciphers, the Arabs became the founders of the arithmetic of everyday life."

Zero is the most important digit in the Hindu-Arabic system

In fact, our term *cipher* is based on the Arabic word for *zero,* the most important digit in the Hindu-Arabic system. For keeping records, people in India had invented a system of place value that used ten written digits, including a zero that they called *sunya,* which is the Sanskrit word for *void*. When Arab conquests reached portions of India in the Middle Ages, Islamic scholars adopted the innovation and later extended it to include decimal fractions. The Arabs translated *sunya* as *sifr,* which had been their word for *empty,* and symbolized this zero with a dot. In 1202, *sifr* was translated into Latin as *cephirum* by Leonardo Fibonacci, an Italian accountant and algebraist, who had picked up the new arithmetic from travels in North Africa and Syria. In Italian, *cephirum* became *zevero* or *zeuero,* which eventually entered English as *zero,* whereas in French the term became *chiffre,* which became the English word *cipher*. A few English speakers still use the term *ciphers* to mean *arithmetic,* but in the United States, it most often refers to a secret code, as in *decipher*.

In formulating the Arab arithmetic, a pivotal role was played in Baghdad by al-Khowarizmi, who, we previously noted, gave us our word *algebra*. His *Book of Addition and Subtraction according to Hindu Calculation* was perhaps the first book that set down efficient ways to calculate with numbers in the new decimal notation. In fact, al-Khowarizmi's name became so inextricably linked to these methods of numerical calculation that its Latin rendering, *algorizmi,* came to denote any textbook about arithmetic procedures. By the time it entered English as *algorism* or *algorithm,* it had come to mean any systematic procedure for solving a problem, whether in arithmetic or elsewhere. Interestingly, the original meaning of *al-Khowarizmi* was simply *the Khowarizmite,* signifying the mathematician's family roots in Khowarizm, a region in what is now Uzbekistan.

Much of the contact between Arabs and Europeans in the Middle Ages depended on trade routes across land and sea. Fibonacci's father, for instance, managed a trading station on the coast of what is now Algeria. Our term *average* seems to be rooted in such commerce. When shipments were damaged or lost at sea, Arab traders used the term *awariyah,* meaning *damaged goods,* to refer to the losses that would be shared by the investors. For example, a loss of 6800 dirhams divided equally among ten investors would mean 680 dirhams per investor. For the Italians with whom the Arabs were trading, *awariyah* came to mean the final result of such a computation, here the 680. They incorporated the term into the Italian language as *avaria,* and the word became *average* in English. Other English terms rooted in Arab trade and navigation include *ream, tariff, tare, carat, caliber, caliper, almanac, zenith, nadir,* and *azimuth.*

Students might be beginning to realize that language has a history

Beyond arithmetic, al-Khowarizmi and his colleagues also had algorithms for solving linear and quadratic equations. In quadratics, they called the unknown quantity *jathr,* meaning *plant root,* because they imagined it as something hidden below the surface. Fibonacci translated the Arabic word *jathr* into the Latin *radic-*, *radix,* which means *branch* or *root* and which is related to such words as *radish, radicalism, eradicate, rutabaga,* and *root* itself. In time, the English word *root* came to mean the solution of any algebraic equation, whether it involved squares, cubes, or other operations entirely. People said that they were solving an equation by "extracting" the root, imagining that they were pulling it out of the ground to make it visible. For more information on the Arabic words of mathematics, see Rubenstein and Schwartz (1999b).

TEACHING CONSIDERATIONS

An excellent source for learning mathematical word origins is Steven Schwartzman's book *The Words of Mathematics* (1994). It is a quick reference that students or instructors can use to learn sources of terms, definitions, literal historical roots, and common related English words. The words of mathematics have roots that come from Latin, Greek, Arabic, Sanskrit, and other languages. Schwartzman shares these origins and their stages of evolution.

Instructors who are familiar with the origins of mathematics terms can introduce them when topics arise. By sharing the related common English words at the same time, connections grow and students enrich both their general knowledge of language and their mathematical fluency. Students who know or are studying other languages can help in identifying more cognates, or related words. Students with an interest or talents in language often very much appreciate the opportunity to apply their linguistic intelligence to mathematics (Gardner 1983). After the etymology strand is established, Schwartzman's book and other dictionaries can be made available in class for students to use whenever a question about a word's origin arises.

Students need some help getting started with etymologies. At the high school level, they might be just beginning to realize that language has a history. A cooperative English teacher might make a good collaborator in helping you and your students begin. The word *geometry* is an easy starter because students know many related words. One strategy is to use a Venn diagram, as shown in **figure 2,** with the two roots and their meanings in each circle. Have students brainstorm related common English words. You could then share a few etymologies of words relevant to current work or have students identify and research the roots of mathematical terms that have piqued their curiosity. The Internet is a good source, too. Etymologies can be incorporated into study-skills activities (UCSMP 1995), can be starting points for students' journal entries (Thiry 1990), can be parts of students' projects, or can be included in student-developed bulletin boards or cartoons. Rubenstein (2000) and Thompson and Rubenstein (2000) contain more suggestions.

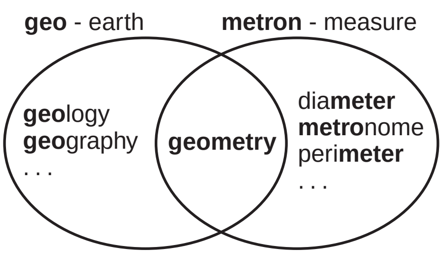

Fig. 2
Venn diagram for roots of "geometry."
Students should be asked to furnish additional words in each category.

Incorporating word origins into our curricula has multiple benefits. Etymologies help students connect mathematical terms with familiar English words, thereby strengthening meaningfulness. Word origins certainly help build common language knowledge. They also allow students with linguistic interests to bring their talents to bear on mathematics. They are also fun; seeing how apparently unrelated words share common origins is intriguing.

Etymologies also help students appreciate the evolution of mathematical terminology. Ideally, students should see that people have struggled with naming and describing mathematical ideas, just as they might be doing themselves. In time, however, we would like them to see that inventing mathematics has been a long-term human endeavor in which they, too, can participate.

Occasionally, opportunities for such student invention arise in our classes. In one of the authors' classes, when students were learning about major and minor arcs in a circle, one student asked whether a term—similar to *complementary* and *supplementary*—exists for a pair of arcs whose sum is 360 degrees. Not knowing such a term, the instructor invited students to invent their own. A few days later someone suggested *circlementary!* It preserved the analogy, as well as pointed to the application of arcs. Such opportunities help students realize that they, too, can be language inventors. When we look back in history to our ancestral mathematicians, we gain inspiration to look ahead at the inventors of tomorrow, our students of today.

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