-Discrete Math 504
Homework Assignment #8
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Date: March 1, 2005

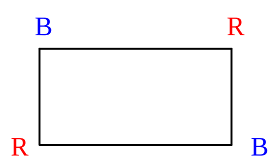
Section 4.6 Exploratory #3 Exercises (2,4,6,10,11)
Section 5.1 Exercises (4,5,6,10,14)
Section 5.2 Exercises (1,2,3,4,5,6,7,10,19,20)

Section 4.6

Exploratory 4.6

3. a. Even Number of Vertices with cycle

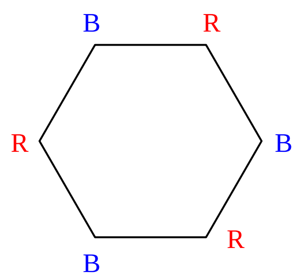
(1) 4 vertices



Chromatic # = 2

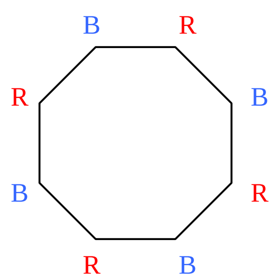
# of Vertices	Chromatic #
4	2
6	2
8	2

(2) 6 vertices



Chromatic # = 2

(3) 8 vertices

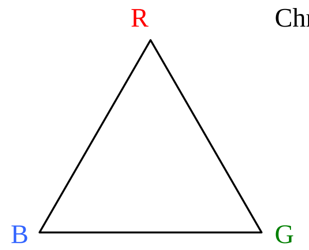


Chromatic # = 2

(4) The Chromatic # of a cycle with an even number of vertices = 2.
You can always alternate 2 colors and there is an even number of vertices to pair.

b. Odd Number of Vertices with cycle

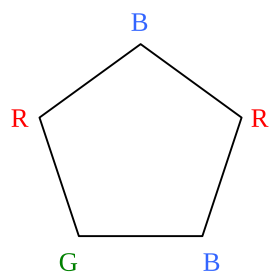
(1) 3 vertices



Chromatic # = 3

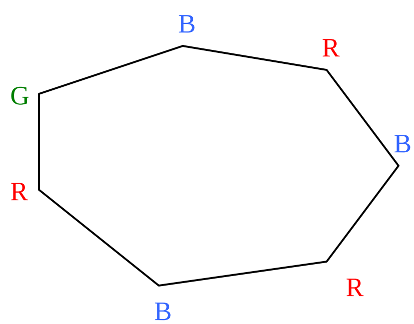
# of Vertices	Chromatic #
3	3
5	3
7	3

(2) 5 vertices



Chromatic # = 3

(3) 7 vertices



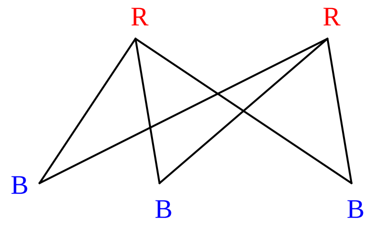
Chromatic # = 3

(4) The chromatic number of a cycle with an odd # of vertices = 3.

(5) The chromatic number of a simple graph that contains a cycle with an odd number of vertices is greater than or equal to 3. (You can always pair up as you go but there will always be one vertex left that cannot be the first color because it connects to that color of first vertex. The leftover vertex cannot be the second color because the vertex before it is that color because of the pairing. So, there must be a third color.

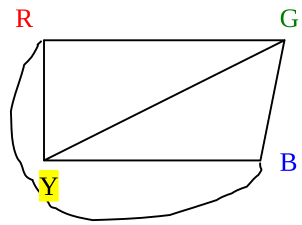
Exercises 4.6

2.



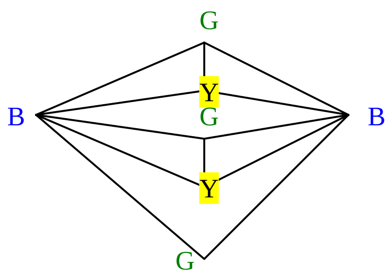
$K_{2,3}$
Chromatic # = 2
Theorem 16 ⇒
Bipartite graph

4.



This is K_4
By Theorem 15
Chromatic # = 4

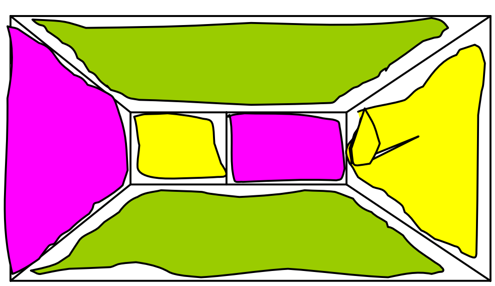
6.



Chromatic # = 3

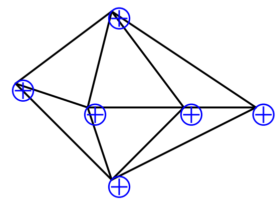
10.

a. Color map

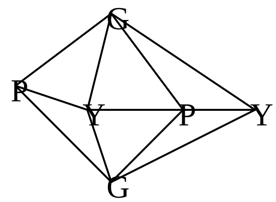


Yellow, Pink, Green
3 colors = Chromatic #

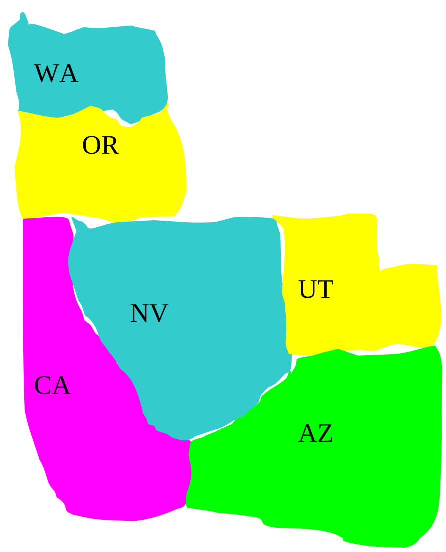
b. Dual Graph ⊕ denotes vertices



c. Coloring of Planar Graph corresponding to map color

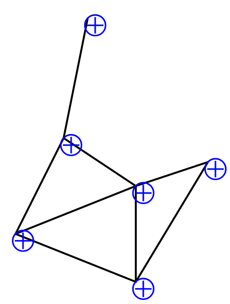


11. a. Color Map (Sorry about my bad drawing – hope I labeled the state right!)

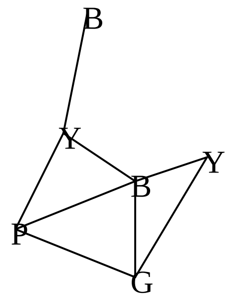


Blue, Yellow, Pink, Green
4 colors = Chromatic #

b. Dual Graph



c. Coloring of Planar Graph corresponding to Coloring of Map



Section 5.1

Exercises 5.1

4. Yellow denotes indegree not equal to out degree

Vertex	In degree	Out degree
a	0	2
b	2	2
c	1	1
d	1	2
e	3	0

5.

Vertex	In degree	Out degree
p	2	2
q	2	2
r	3	1
s	2	1
t	1	4
u	1	1

6. Exercise 4 (By Theorem 2)
weakly connected with 3 vertices (not exactly 2) where indegree ≠ outdegree
So, no Euler path or circuit possible

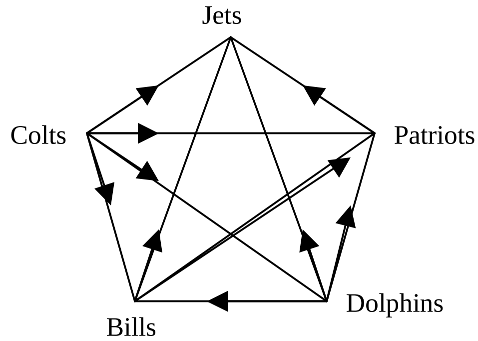
Exercise 5 (By Theorem 2)
Weakly connected with 3 vertices where indegree ≠ outdegree
So, no Euler path or circuit possible.

10.

V	Indegree	Outdegree
a	2	2
b	3	3
c	2	2
d	4	4
e	2	2

Since indegree = outdegree for every vertex, (by Theorem 1) there exists an Euler circuit.
a,d,c,b,c,d,b,a,e,d,e,b,d,a

14. Colts beat every team (my interpretation of what "every other team" means) – with Johnny Unitas
Jets lose to every other team (not with Joe Namath – they did not!)
(a)



(b) Winner if there is a Hamilton path (include all vertices) that begins with it.
Colts (C) is a winner C – D – B – P - J

Vertex Team	Losses Indegree	Wins Outdegree
C	0	4
J	4	0
P	3	1
D	1	3
B	2	2

From vertex B – you can go to: J (cannot go anywhere from there) or
P – J (cannot go anywhere from there)
So, B is not a winner.

From vertex J – you cannot go anywhere. So, J is not a winner.

From vertex P – you can only go to J (nowhere from there). So, P is not a winner.

From vertex D – you can go to: J (nowhere else) or
P to J (nowhere else) or
B which leads to J or to P which leads to J.
So, D cannot get to C $\Rightarrow$ D is not a winner.

(c) yes, there is a unique winner – Colts
See b above for reasoning. Starting at all others, you can never get to C – you get stuck at J.

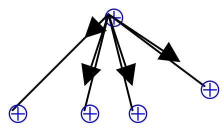
(d) There are no simple circuits because you always land at J and cannot get back to the start.
Therefore, (by Theorem 4) a tournament graph has a unique Hamilton Path.

Final Rankings

# 1	Colt	4-0
# 2	Dolphins	3-1
# 3	Bills	2-2
# 4	Patriots	1-3
# 5	Jets	0-4

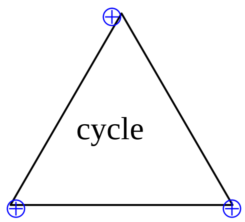
Section 5.2
Exercises 5.2

1. Tree - it is connected with n (number of vertices) = 5 and it has n-1=4 edges. By Theorem 7 ⇒ tree
It can be redrawn to look like a tree:



2. n = 5 vertices and 5 edges ≠ n-1 ⇒ Not a Tree By the contrapositive of Theorem 6 or the equivalent statements in Theorem 7.

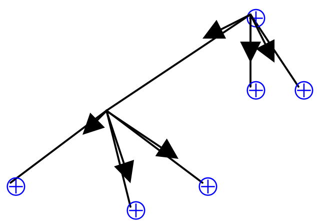
Also, graph contains a cycle ⇒ not a tree



3. Not connected ⇒ Not a Tree

4. Tree - it is connected and V=7, Edges = 7-1 = 6 (By Theorem 7 where statements 1 & 3 are equivalent.

Also, can be redrawn as a tree:



5. Tree - connected and V = 4, Edges = 4-1 = 3 (By Theorem 7 statement 3 = statement 1)

6. Not a Tree – not connected

7.

Problem #	V	E	Tree?
1	5	4	Yes
2	5	5	No
3	6	4	No
4	7	6	Yes
5	4	3	Yes
6	2	0	No

This show examples of confirmation of the contrapositive of Theorem 6

Or supports the equivalency of statements #1 and #3 in Theorem 7. Only tree if E=V-1

10. a.) Asst. Superintendent: Instruction & Superintendent
b.) Asst. Superintendent: Instruction
c.) Director: Budget & Director: Transportation
d.) Superintendent

19. a.) The simple circuit C is a cycle because no vertices are repeated but the initial and terminal vertices.
b.) C is a cycle in the graph because of definition of cycle (no vertices repeated but initial and terminal vertices)
c.) no repeated vertices other than initial and terminal and there are a finite number of vertices
d.) no repeated vertices other than initial and terminal in simple circuit.

20. a.) In a tree, there are no parallel edges because if there were parallel edges between x and y, then that would create a cycle x-y-z. So, going from one vertex to another is unique.
b.) $w_k = u_r = y$ by definition both = y
$w_1 = u_1 = x$ by definition So, both are equal to x.
So, initial to C_1 and C_2 are the same and the terminal to C_1 and C_2 are the same
c.) Because C_1 path is not equal to C_2 path, there exists an integer t with $w_t \neq u_t$
(cannot be the same path) Note: both C_1 and C_2 are simple paths.
Since $w_1 = u_1 = x$, t must be greater than 1.
d.) with path $C_1 \neq C_2$ path, $w_k = u_r = y \Rightarrow$ so, k would be an integer beyond t, w
e.) w_{t-1} follows path C_1 to w_s and then follows C_2 back to w_{t-1} would look like this

$w_{t-1\,-\,\ldots}$ along C_1 … w_s … along C_2 … w_{t-1} This starts (initial) and stops (terminal) at the same vertex and both C_1 and C_2 are simple paths.

f.) C_1 is simple $\Rightarrow$ no edges repeated from w_{t-1} to w_s
C_2 is simple $\Rightarrow$ no edges repeated from w_s to w_{t-1}
So, none of the vertices w_t to w_s in C_1 are vertices in $C_2 \Rightarrow$ no edges repeated.
This must be a simple circuit.
From Exercise 19, graph contains a cycle $\Rightarrow$ it is not a tree. #Contradiction of our hypothesis that it is a tree. So, our assumption that there are 2 distinct simple paths from x to y must be false. Therefore, there is a unique (only 1) simple path from x to y.