W. Thompson, Charleston, S C.	1, 2, 3, 4, 5, 6,	8, 9, 10, 11.
Alex Walsh, New-York,	1, 2, 3, 4, 5.	
Diarius Yankee, Bunker's Hill,	1, 2, 3, 4, 5, 6,	8, 9, 10, 11.
Titus Youle, Portland, Maine,	1, 2, 4,	
N. Young, near Lexington. Ken.	1, 2, 3, 4, 5,	10, 11.

The Prize Medal, (value Six Dollars) has been determined to James Temple, *New-York.*

ARTICLE IX.

A short Disquisition, concerning the Definition, of the word Power, *in Arithmetic and Algebra.*

By G. Baron.

IN the following paper, I contend, that, in the writings of the modern mathematicians, the *powers of numbers*, are inaccurately defined; and that in consequence of this inaccuracy, certain conclusions, in the higher branches of the mathematics, evidently true in their own nature, appear as mysterious and incomprehensible paradoxes. The definition, to which I allude, has generally been expressed, in words to the following effect: *the powers of any number, are the successive products, arising from the continual multiplication of that number, into itself.* Now, if this definition, be considered as perfectly accurate, it must incontrovertibly follow, that, the first power of any number, is produced by multiplying that number, by itself;—that the second power, of a number, is produced, by multiplying the first power, of that number, by the number itself;—that the third power, of a number, is produced, by multiplying the second power, of that number, by the number itself; &c. &c.

Hence $5 \times 5 = 25$, the first power of 5,
$25 \times 5 = 125$, the second power of 5,
$125 \times 5 = 625$, the third power of 5, &c. &c.

The first, second, third, &c. powers of any number, might be conveniently expressed thus, 5^1, 5^2, 5^3, &c; where $5^1 = 5 \times 5 = 25$, the first power of 5; $5^2 = 25 \times 5 = 125$, the second power of 5; $5^3 = 125 \times 5 = 625$, the third power of 5; &c. &c.

In the same manner, the first, second, third, &c. powers of any number x, might be represented, by x^1, x^2, x^3, &c, in which $x^1 = x \times x$, $x^2 = x^1 \times x$, $x^3 = x^2 \times x$, &c. &c. All this is, evidently, in exact conformity, with the common definition, of powers. But it is well known that the word power is not, and cannot be, used in this sense; for, in arithmetic and algebra, every number, is necessarily considered, as equal to the first power, of itself: and hence, since it is impossible, according to the common definition, for any number to be equal to its own first power; it follows, that, this definition is erroneous. Further, it is also well known, that, the product resulting from any number, multiplied by itself, must be considered as the second power, of that number: but according to the common definition, it is only the first power; and hence this definition is again erroneous: in a similar manner, the fallacy of the same definition, might be shown when applied to any other power whatever. Since, therefore, it has been shewn, that, the common definition of powers is universally false, it becomes absolutely necessary, to expel it from the mathematical sciences, and to establish another that shall be truly correct. For the certainty and evidence of these sciences, depend essentially on the precision and accuracy, with which the various terms are defined: every mathematician knows, that, the limits, or bounds of a definition, form the true partition line between science and mystery; and that the moment we pass these limits, we must inevitably, enter the incomprehensible and visionary regions.

From a careful survey, of the application of powers, in every branch of the mathematical sciences, it has long appeared to me, that the following definition, is universally correct. Viz. *The powers of any number, are the successive products, arising from unity, continually multiplied, by that number.* According to this definition, unity multiplied by any number, produces the first power of that number; —the product arising, from the first power of any number, multiplied by the number itself, is the second power of that number;—the product resulting, from the second power of any number, multiplied by the number itself, is the third power of that number; &c. &c.

Hence $\underline{1 \times 5} = 5$, the first power of 5,
$\underline{1 \times 5} \times 5 = 25$, the second power of 5,
$\underline{1 \times 5 \times 5} \times 5 = 125$, the third power of 5,
$1 \times 5 \times 5 \times 5 \times 5 = 625$, the fourth power of 5, &c. &c.

Let now 5^1 denote the first power of 5, or 1×5; 5^2, the second power of 5, or $1 \times 5 \times 5$; 5^3, the third power of 5, or $1 \times 5 \times 5 \times 5$; &c. &c. where the small figures 1. 2. 3. &c. are called the exponents of the first, second, third, &c. powers. In like manner, 7^1, 7^2, 7^3, &c. will represent the first, second, third, &c. powers of 7; and x^1, x^2, x^3, x^4, &c. the first, second, third, fourth, &c. powers of any number x. It may now be necessary to demonstrate the two following propositions.

Prop. I.

The first power of any number, is equal to that number itself.

For, let x represent any number, whatever; then from the definition, and the nature of numbers, $x^1 = 1 \times x = x$.

Prop. II.

The second power of any number, is equal to the product, arising from that number multiplied by itself.

For, by the definition, $x^2 = \overline{1 \times x} \times x = x^1 \times x$; and by Prop. I. $x^1 = x$; consequently $x^2 = x \times x$.

Corollary.

Hence $x^3 = \overline{1 \times x \times x} \times x = x^2 \times x = x \times x \times x$, $x^4 = x^3 \times x = x \times x \times x \times x$, &c. &c. And hence any product, arising from the continual multiplication of a number, into itself, is a power of that number; and the number of factors. used, in such continual multiplication, is equal to the exponent of the power, so produced.

Thus far, every conclusion, drawn from the propounded definition, agrees, exactly, with the common usage of every mathematical author of the present age. Let us, therefore, next inquire, whether the same definition, will not lead us to a clear and intelligible solution, of the mysterious paradoxes, resulting from the common definition, when applied, to what is denominated, the *nothingth* powers of numbers. The word nothing, in a mathematical sense, signifies the privation of some property, which, *necessarily* exists, in the constitution of any thing, considered as the subject of number, or of magnitude. This may be clearly illustrated from the nature of solidity: for, length, breadth, and thickness, necessarily exist together, in the constitution of a solid; and if thickness be abstracted, length and breadth will remain; but the magnitude of this solid, when thus deprived of thickness, will be *nothing*. The word *nothing*, here applied to solidity, evidently signifies, the privation of a property which must necessarily exist, in the constitution of a solid: and hence this *nothing* does not affirm absolute nonentity; for the remaining length and breadth, constitute a superficies; a thing perfect in its own nature, though destitute of solidity.

The learned Mr. Emerson, in the scholium to the 73d* problem of his Algebra, very justly observes, "that, nothing in a mathematical sense, "never signifies absolute nothing; but always "nothing in relation to the object under consider-"ation." If now, this mathematical idea of *nothing*, be applied to our definition, it will be evident that unity, is the nothingth power of any number whatever. For let x denote any number; then by the definition, $1 \times x$, is the first power of x. Hence then, a unit multiplied by x, must, necessarily constitute the first power of any number x; and therefore if the multiplication by x, be abstracted from this first power of x, by means of division; the power will become nothing but the unit will remain: for $\frac{x^1}{x} = \frac{1 \times x}{x} = 1$, and hence it is plain that $x^0 = 1$, when x represents any number whatever. But since the *number* x, is here unlimited with regard to greatness, it follows, that, the nothingth power of an infinite number is equal to a unit.

What, has hitherto been advanced, relates to pure numbers; and it now remains, to apply our

* Mr. Emerson wrote this problem in the *English language*, but it has lately been translated into *nonsense*, by Jared Mansfield of Connecticut, and published in a wonderful work, which this translator calls Mathematical Essays. The indefinitely small *nothings* of Connecticut, are infinitely great absurdites, in the regions of science and common sense.

definition of powers, to abstract quantities. By a pure number, I mean, a definite plurality, or a determinate multitude, of abstract and indivisible units; and by abstract quantity, I understand, magnitude expresssed in numbers, by means of some known measure, considered as a unit, indefinitely divisible. From a certain corresponding analogy, between the quantities of geometrical rectangles, and the products of abstract numbers, mathematicians have linked the former with the latter, subjected quantity, to all the operations of numbers, and proved, that, every proposition, that is true of abstract numbers, is also true of abstract quantities. Hence, then, since, our definition, was universally accurate, in its application to abstract numbers; it will likewise be universally correct, when applied, to abstract quantities. But, in order, to pursue the application of our definition, to quantity in the ultimate extremity of smallness, let us suppose x to represent any fractional quantity; or in other words, let x denote any magnitude, expressed in numbers, by means of some part of its measuring unit: then by the definition $x^1 = 1 \times x$. Let now this multiplication by x, be abstracted; and for the reasons heretofore advanced, we have $x^0 = 1$. Now since x here represents a fractional quantity, independent of any limitation, in respect to smallness; we may therefore suppose x, by means of continual diminution, or decrease, to pass from its present value, through every degree of smallness, until it become *nothing*; then it will be evident, that, during this diminution or decrease of x, x^0 will continue equal to an invariable unit; and that *precisely at the instant*, when x becomes *nothing*, x^0, or $o^0 = 1$. We have therefore proved, immediately from our definition, that, the nothingth power of every number and quantity, is equal to a unit; and, that,

$x^0 = 1$: the truth of which propositions is well known; and hence the definition itself, is universally correct. Indeed, when powers are thus defined, these propositions are self-evidently true, independent of the demonstrations here given: but on the contrary, when powers are considered, as generated according to the common definition; the same propositions, (notwithstanding the demonstrations of modern analysts,) must inevitably appear, as incomprehensible paradoxes. For, if powers be the successive products, resulting from the continual multiplication of any number into itself; such multiplication, must be essential in the production of a power; and where there has been no such multiplication, there can be no power. So long, therefore, as the common definition is retained, it will be absurd to consider a number as the first power of itself. For by that definition any number x multiplied by itself, gives the first power of $x = x^1 = x \times x$; and when this multiplication by x, is abstracted we have $x^0 = x$, which is a manifest absurdity, and utterly repugnant to the well known nature of logarithms. Further, since in adhereing to the same definition it has been shewn, that, $x^0 = x$; it would therefore be impossible for x^0, to be universally equal to a unit. But we know, from the nature of logarithms, and also, from other principles, that x^0, is actually and universally equal to a unit; and hence we infer, that, the inexplicable mysteries, which have long been attributed, to the nothingth powers of numbers and quantities; originated, in a wrong definition of the word *power*.

From what has here been said on the subject, it is plain, that, the powers of any number, form a geometrical series from unity, whose ratio is the number itself; and that the exponents of such a series of powers, form an aritmetical series, from 0, whose common difference is a unit: or in other words, that, the exponents of the powers of any number, are a certain system of the logarithms of their corresponding powers. And hence it follows, that, all the operations of numbers, performed by logarithms, may in a like manner, be done of powers, by means of their exponents. Also, since $x^0 = 1$, whatever be the value of x; of consequence, in every system of logarithms, the logarithm of $1 = 0$.

ARTICLE X.

NEW QUESTIONS to be answered in the next Number.

I. QUEST. 19. *by Niel Gray, New-York.*

Out of an annuity of 1000 dollars per annum, for 10 years, the first payment being due one year hence, the owner desires to know how much he may spend a year; so that his annual savings, with the simple interest arising therefrom, at 7 per cent. per ann. may, at the expiration of this annuity, amount to a sum whose interest at 7 per cent. per ann. shall be equal to the yearly expenditure required.

II. QUEST. 20. *by Alexander Walsh, New-York.*

It is required to determine whether 30 horses can be put into 7 stalls; so that in every stall there may be, either a single horse, or an odd number of horses.

III. QUEST. 21. *by Ebenezer R. White, Danbury Connecticut.*

Given $x = \sqrt[r]{a^m + b^n}$, and $y = \sqrt[r]{a^m - b^n}$, supposing $r = 5$, $m = 9$, $n = 7$, $a = 684{\cdot}588$ and $b = 24{\cdot}5632$; required a general rule, for finding the values of x and y, by a table of common logarithms.