The three general algebraic, yet practical, questions that I had the students work on, and derive a general method of solution for are (in order of difficulty for them):

- Given the FIO2 and the flowmeter settings, what is the total flow?
- Given the total flow $(a + 1) \cdot f$ and the flowmeter setting, determine the delivered FIO2.
- Given the total flow and the desired FIO2, what flowmeter setting should be selected?

On a Common Mnemonic from Trigonometry

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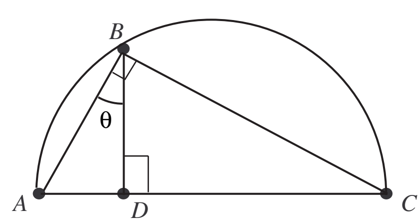
A commonly used mnemonic for the sine of the special angles 0, $\pi/6$, $\pi/4$, $\pi/3$, and $\pi/2$ is

$$\sin\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\} = \frac{\sqrt{\{0, 1, 2, 3, 4\}}}{2}.$$

That is, we take the square roots of the numbers in the sequence 0, 1, 2, 3, 4 and divide by 2. With these numbers we can compute any trig function of any of the special angles between 0 and 2π via elementary identities.



But how is it that the sines of these angles can be obtained via such a simple arithmetic sequence? The angles themselves occur in a pattern to be sure, but not such a simple one.

We will show that θ (see the figure) is one of the special angles, 0, $\pi/6$, $\pi/4$, $\pi/3$, or $\pi/2$, precisely when the ratio $\frac{|AD|}{|AC|}$ is 0, 1/4, 1/2, 3/4, or 1, respectively. To put it another way, if we think of the point D as moving in a straight line from A to C then θ successively reaches each of the special angles $\pi/6$, $\pi/4$, and $\pi/3$ precisely when D is 1/4, 1/2, 3/4, or all of the way to C. (Clearly $\theta = \{0, \pi/2\}$ when $D = \{A, C\}$.)

For clarity and without loss of generality we assume that $|AC| = 1$. Observe that triangles ABD and BCD are similar to each other since both are similar to triangle ABC. Thus by proportionality of similar triangles we have $\frac{|BD|}{|AD|} = \frac{1-|AD|}{|BD|}$ or

$$|BD| = \sqrt{|AD|\,(1 - |AD|)}$$

and it then follows from the Pythagorean Theorem that

$$|AB| = \sqrt{|AD|}.$$

Thus the ratios of the lengths of the sides of triangle ABD are

$$|AD| : |AB| : |BD| = |AD| : \sqrt{|AD|} : \sqrt{|AD|(1 - |AD|)}.$$

When $|AD| = 1/4$ this becomes $1 : 2 : \sqrt{3}$ from which we conclude that triangle ABD is a $30^\circ : 60^\circ$ right triangle, whence $\theta = \pi/6$. A similar argument gives $\theta = \{\pi/4, \pi/3, \pi/2\}$ when $|AD| = \{1/2, 3/4, 1\}$.

The mnemonic is then verified by observing that

$$\sin\theta = \frac{|AD|}{|AB|} = \sqrt{|AD|}.$$

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Introducing the Sums of Powers

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Introduction. Integration is invariably introduced by approximating the area under a curve using the sum of the areas of inscribed or circumscribed rectangles. But unless the number of rectangles is trivially small, the actual summation must be done using a calculator, computer, or by introducing summation formulas such as

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

which, so far as students are concerned, are arbitrary formulas that come from nowhere, formulas that must be memorized or given to them. Proofs of these formulas may be found using induction ([**4**]) but this has the drawback of requiring knowledge of the formula before the validity of the formula can be shown. Methods of deriving the formulas for the sums of the nth powers usually require knowledge of the formulas for some or all of the sums of the kth powers where $k < n$ (see [**1**], [**2**], [**6**]) or identities that are no less obvious to the students than the summation formulas themselves ([**3**]). Here's a genetic way to introduce these summation formulas that gives students a quick lesson in the history of mathematics and prepares them to deal with Taylor series and partial fractions in later courses.

Leibniz's algebraic method. Leibniz's method, which he described in ([**5**], p. 51 and after), is probably the simplest of the historical methods. We'll illustrate it using the sum of the squares of the whole numbers. Let

$$S(x) = 0^2 + 1^2 + 2^2 + 3^2 + \cdots + x^2 \tag{1}$$

Consider the difference $S(x+1) - S(x)$. It should be obvious that this is just the square of the next whole number, or

$$S(x+1) - S(x) = (x+1)^2$$