Teaching Tip: Is This Integral Zero?

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Calculus I students love the integrals

$$\int_{-1}^{1} x^3\,dx, \qquad \int_{-\pi}^{\pi} \sin(x)\,dx, \qquad \int_{0}^{2\pi} \cos(x)\,dx,$$

because they easily see why each is zero. But in Multivariable Calculus when I ask them to evaluate

$$\iiint_E xyz\,dV,$$

where E is the region between between the spheres $\rho = 1$ and $\rho = 4$ and above the cone $\phi = \pi/3$, some students regularly worry because they keep getting 0. Zero is suddenly *numerus non gratus*!

Let's take advantage of this by designing questions which force students to distinguish integrals that yield zero from those that don't. Students will learn how to visualize the interplay between an integrand and the region over which it is integrated, something much more satisfying than writing out pages of iterated integrals. Set the stage by using single variable warm up questions. Ask if they can predict whether this integral is zero:

$$\int_{2}^{8} (x-5)e^{(x-5)^2}\,dx.$$

Follow up with double and triple integral questions like those below. Similar problems can be created for line and surface integrals (both scalar and vector).

Double integrals. For which of the following combinations of function $f(x, y)$ and region of integration D is the double integral zero?

(a) $f(x, y) = x^3\sqrt{y^2+1}$; $D = \{(x, y) : -1 \le x \le 1; -3 \le y \le 2\}$,
(b) $f(x, y) = e^{-xy}$; D is the upper half of the unit circle,
(c) $f(x, y) = x/y^2$; D is the region between the parabola $y = x^2 + 1$ and $y = 10$.

Triple integrals. For which of the following combinations of function $f(x, y, z)$ and region of integration E is the triple integral zero?

(a) $f(x, y, z) = xyz^2$; $E = \{(x, y, z) : -1 \le x \le 1; -2 \le y \le 2; -5 \le z \le 5\}$,
(b) $f(x, y, z) = xyz$; E is the region between concentric spheres of radii 2 and 4,
(c) $f(x, y, z) = x^2 + y^2 + z^2$; E is the upper half of the unit sphere.