

Paul R. Halmos - Lester R. Ford Awards

Paul Ramond

“The Abel-Ruffini Theorem: Complex but Not Complicated,” *The American Mathematical Monthly*, 129:3, 231–245. doi.org/10.1080/00029890.2022.2010494

The unsolvability of the general quintic using radicals is, in the present day, usually taught as a corollary of the general machinery of Galois theory. However, as the author of this paper points out, Abel’s proof of this fact, building on previous work of Ruffini and Lagrange, predates Galois’ famous paper by six years. From a pedagogical perspective, the approach of Abel-Ruffini-Lagrange makes it much clearer how one would be led to consider “symmetries” of a solution, beginning with an analysis of the structure of the solutions for the quadratic, cubic, and quartic. This makes Abel’s proof an ideal introduction to Galois theory, as well as being of interest for its own sake. This paper gives a delightful exposition of Abel’s proof in wonderfully concrete geometrical terms, which is very welcome for those of us who think more easily in pictures than in algebra. As the author mentions, the main idea of this exposition goes back to V. I. Arnol’d and was made more accessible in works by L. Goldmakher and B. Katz. The present author has given an exposition which is both simplified and more self-contained. As such, it is highly recommended as preliminary reading for anyone planning to take, or teach, a course on Galois theory, as well as for anyone curious about what makes the quintic equation so different from the quadratic, cubic, and quartic equations. It is also a wonderful example of how to think about algebraic concepts, such as permutations and commutators, in pictorial terms.

Response

It is a true honour and a very pleasant surprise to receive this prize. The *Monthly* has played a very important role in shaping the researcher that I am today, mostly working in theoretical astrophysics, but always with a strong and keen interest in mathematics. I am thrilled that my elementary proof of this remarkable theorem of Abel and Ruffini’s has been recognised for its geometrical elegance and its simplicity of exposition, and I hope that it will convey to its readers the pleasure I have had writing it.

Biographical Sketch

Paul Ramond is a post-doctoral researcher at the Paris Observatory, working mostly in theoretical astrophysics. His research consists in solving the equations of the general theory of relativity for the motion of black holes and very

dense stars orbiting each other in a cosmic ballet, causing spacetime around them to vibrate in harmony and producing gravitational waves. He also has a strong and keen interest in all branches of science, including mathematics, and has been reading through the Monthly since its early years as a physics student at the university. He is involved in the popularisation of science through various media, and enjoys informing people about the elegance of physics, the intricacies of acoustics and the importance of scientific knowledge in our modern world. But most of all, he enjoys cooking, woodworking and gardening with his wife in their solitary Chelles, in the forest of the beautiful countryside of northern France.