A River-Crossing Problem in Cross-Cultural Perspective

MARCIA ASCHER
Ithaca College
Ithaca, NY 14850

1. Introduction Most mathematicians react with interest to the challenge of a logical puzzle. In fact, some story puzzles have become such favorites that many of us cannot even recall where we learned them. Perhaps one of the best known is the puzzle in which a man must ferry across a river a wolf, a goat, and a head of cabbage. The difficulty is that the available boat can only carry him and one other thing but neither the wolf and goat nor goat and cabbage can be left alone together. Story puzzles are simple and accessible because they do not rely on any particular body of knowledge and yet they are mathematical in that a stated goal must be achieved under a given set of logical constraints. Attention to logic, as evidenced by the existence of these puzzles, is not the exclusive province of any one culture or subculture. Here, the river-crossing problem, in African cultures as well as in Western culture, will be used as an explicit example of the panhuman concern for mathematical ideas. Story puzzles are expressions of their cultures and so variations will be seen in the characters, the settings and the way in which the logical problem is framed.

2. Western versions The Western origin of the wolf, goat, and cabbage puzzle is most often attributed to a set of 53 problems designed to challenge youthful minds, "Propositiones and acuendos iuvenes." Although circulated around the year 1000, Alcuin of York (735–804) is said to have authored these as he referred to them in a letter to his most famous student, Charlemagne. The solution given by these works is to carry over the goat, then transport the wolf and return with the goat, then carry over the cabbage, then carry over the goat. A second solution, which simply interchanges the wolf and cabbage, is often attributed to the French mathematician Chuquet in 1484 but is found even earlier in the twelfth century in Germany in the succinct form of Latin hexameter [**1**, **4**, **5**, **23**].

The exact authorship is less important than the fact that the problem was circulated for hundreds of years both orally and in writing. The puzzle is repeated over and over again in mathematical recreation books [**1**, **2**, **7**, **8**, **11**, **15**, **19**, **25**] and has been found as a folk puzzle by collectors of folklore. Sometimes, however, the characters are changed so that a sheep replaces the goat or the trio are replaced by a fox, a fowl, and some corn. Among others, the puzzle is noted in collections of Gaelic, Danish, Russian, Italian, Rumanian, and Black American folklore [**6**, **12**, **20**, **3**, **21**, **9**]. It is one component of a lengthy tale, collected in the French Brittany region [**17**, pp. 208–217], about Jean L'Hébété (the dazed or simple minded) who is eventually given more intelligence by a good fairy in exchange for his wife's solution to her river crossing predicament.

Despite their differences, all of these share the same logical structure: A, B, C must be transported across a river in a boat that can only hold the human rower and one of A, B, or C; neither A nor C can be left alone with B on either shore.

3. African versions Puzzles with the same logical structure are found in Africa among the Tigre (Ethiopia) [**14**, p. 40], on the Cape Verde Islands [**18**] and among the

Bamileke (Cameroon) [**16**]. In the latter, the water is only a stream but the tiger, sheep, and a big spray of reeds have to be walked across individually on the trunk of a fallen tree.

Other related but different problems occur in three regions in Africa. They are similar in that they require a human to transport across a river a predator, its prey, and some food. However, closer examination shows that *they have a distinctly different logical structure*. Now A, B, and C must be transported across a river by a human who can only transport *two* of A, B, C at one time. Neither A nor C can be left alone with B on either shore. What is more, while they superficially share this structure, they contain conscious variations on it.

The most straightforward statement of this is found among the Kabjlie (Algeria) [**10**, p. 246]. In it a man must cross the river with a jackal, goat, and bundle of hay. His solution is to take the jackal and goat, leave the jackal while returning with the goat and then carry across the goat and hay. But, the story continues, another traveler, seeing this, comments that this solution is less efficient than carrying over the jackal and hay and returning for the goat because the goat is being carried on all the trips. "Or", he adds, "did you think that jackal eat hay?" Thus, in the exchange, the traveler points out that a *good* solution should be concerned not only with the *number* of trips but with the lightest load on each trip. And, furthermore, he notes that because the jackal cannot be alone with the goat and the goat cannot be alone with the hay does not imply that the jackal cannot be alone with the hay. In this version of the story there is no boat; the river is sufficiently shallow so that the man can walk across carrying one of the objects under each arm. At first glance this does not seem to affect the logical structure of the problem. But comparison with a Kpelle version shows that it does indeed.

The Kpelle problem is part of a lengthy story [**24**, pp. 445–448]. Set in the northern part of the Kpelle regions (Liberia), the story tells of a king who has a caged cheetah that grabs and eats any fowl near it. The king challenges a suitor for his daughter's hand to transport the cheetah, a fowl, and some rice across the river in a boat that holds one person and two of these. But the king points out, the man cannot control them while rowing the boat, and so in addition to the cheetah and fowl or fowl and rice not being alone together on either shore, they also cannot be together on the boat. The Kpelle problem, therefore, has the additional constraint that neither A nor C can be in the boat with B. The young man tries various solutions and has to appeal to his father for replacement of the fowl and rice when the solutions fail. Eventually he succeeds by carrying over the cheetah and rice and returning for the fowl. Thus, here too, alternative solutions are examined with some ruled out because they involve the cheetah and fowl or the fowl and rice in the boat at the same time.

There is also a Swahili version of the problem. It too is part of a lengthy story of trials [**13**, pp. 19–20, 77–78]. Collected at the turn of the century, the story is set in a sultanate such as was found in Zanzibar until the late 1800's. When a visitor from another region refuses to pay tribute to the sultan, he is confronted with a challenge. If he can carry a leopard, a goat, and some tree leaves to the sultan's son, tribute will be given him and he can remain in the sultanate. The price of failure will be death. The son, of course, lives across a river and the available boat can only hold the traveler and two of the items. The problem, as stated by the traveler, is that he cannot leave *any* two things together on a shore alone. Thus, his solution is that of the Kabjlie with the leaves and leopard interchanged, that is to carry first the leaves and goat, return with the goat, and then carry over the goat and leopard.

These three African problems clearly form a logical unit with slightly differing constraints so that different solutions are appropriate. The logical structure of the basic problem combined with the elaborate stories and the recognition of multiple solutions distinguish these from the predator/prey/food river-crossing problem described for the West.

Still one more African version of the problem is found only among the Ila (Zambia) [**22**, p. 333]. The striking difference is that it involves four items to be transported: a leopard, a goat, a rat, and a basket of corn. The boat can hold just the man and one of these. This problem exemplifies the interrelationship of culture and logical constraints. After considering leaving behind the rat or leopard (and thus reducing the problem to one that can be solved logically), the man's decision is that since both animals are to him as children, he will forego the river crossing and remain where he is!

4. Conclusion The differences in logical structure suggest that the Western and African versions of the problem were independently conceived. Similarity of puzzle goal is not sufficient to imply historical connection. Although the situation depicted seems fanciful if viewed from a twentieth-century, industrial urban setting, the need to get unmanageable items across some water is not uncommon today in other settings and surely was not uncommon during the last thousand years. Even if the puzzles were historically related, the importance is that each group made the problem its own. The case presented here focuses on logic. It is but one example of many that demonstrate that mathematical ideas are of concern in traditional non-Western cultures as well as in the West.

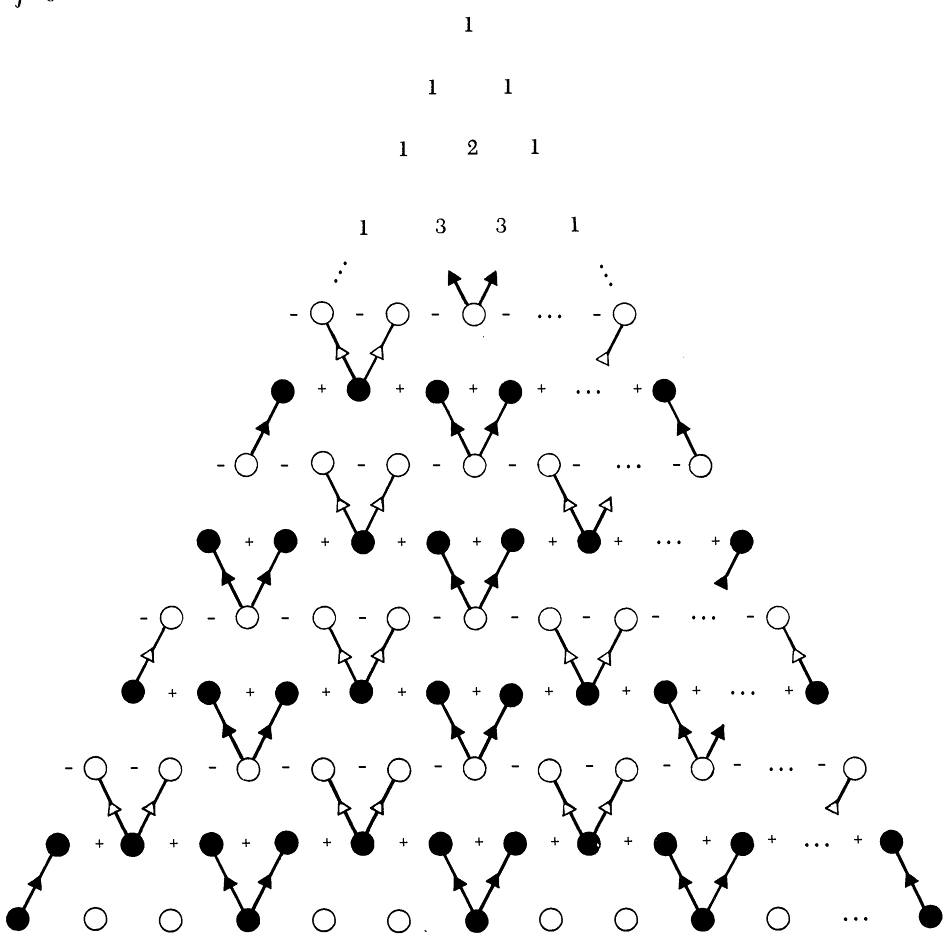
Acknowledgements. Some of the investigation reported here was done while in residence at the Getty Center for the History of Art and the Humanities. I particularly thank Lori Repetti for all her help. Also, I am indebted to Viggo Andreasen, John Harcourt, and Johann Reusch for translations from Danish, Latin, and Transylvanian. My interest in the problem began when reading [**26**].

REFERENCES

1. W. Ahrens, *Mathematische Unterhaltungen und Spiele*, v. 1, B. G. Teubner, Leipzig, 1921, pp. 1–21.
2. W. W. R. Ball, *Mathematical Recreations and Essays*, tenth ed., Macmillian & Co., London, 1922.
3. A. Balladoro, *Folk-lore Veronese: Novelline*, Fratelli Drucker, Verona, 1900.
4. J. Bolte, Der Mann mit der Ziege, dem Wolf und dem Kohle, *Zeitschrift des Vereins für Volkskunde* 13 (1903) 95–96, 311.
5. ______, Der Mann mit der Ziege, dem Wolf und dem Kohlkopf, *Zeitschrift des Vereins für Volkskunde* 33 (1923), 38–39.
6. J. F. Campbell, *Popular Tales of the Western Highlands*, orally collected, with a translation II, Edmonston and Douglas, Edinburgh, 1860.
7. J. Degrazia, *Math is Fun*, Gresham Press, N.Y., 1948, pp. 46–49.
8. H. E. Dudeney, *Modern Puzzles and How to Solve Them*, Frederick A. Stokes Co., N.Y., 1926.
9. A. H. Fauset, Negro folk-tales from the South, *Journal of American Folklore*, XL (1927), 276–292.
10. L. Frobenius, *Atlantis: Volksmarchen der Kabylen*, v. 1, E. Diederichs, Jena, 1921.
11. M. Gardner, ed., *Mathematical Puzzles of Sam Loyd*, Dover Publications, N.Y., 1959.
12. J. Kamp, *Danske Folkeminder, Aeventyr, Folksagen, Gaader, Rim Og Folketro, Samlede Fra Folkemende*, R. Neilsen, Odense, 1877.
13. G. Lademann, *Tierfabeln und andere Erzählungen in Suaheli*, Bank XII, Archiv fur Studium Deutscher Kolonialsprachen, 1910.
14. E. Littman, *Publications of the Princeton Expedition to Abyssinia*, v. 2, E. J. Brill Ltd., Leyden, 1910.
15. E. Lucas, *Recréations Mathématiques*, second ed., Gauthier-Villars et Fils, Paris, 1891, pp. 1–18, 328.
16. S. N. Martin, *Les Devinettes du Cameroun*, Agracam, Yaounde, 1976.
17. A. Orain, *Contes du Pays Gallo*, M. Champion, Paris, 1904.
18. E. C. Parsons, *Folk-lore from Cape Verde Islands*, v. 2, American Folk-lore Society, New York, 1923.
19. E. Riddle, ed., *Recreations in Science and Natural Philosophy: Dr. Hutton's Translation of Montucla's Edition of Ozanam*, Nuttall & Hodgson, London, 1844.

20. D. Sadovnikov, *Riddles of the Russian People*, Ardis Publishers, Ann Arbor, 1986 (trans. from 1876 original).
21. F. Schmitz, *Volkstümliches aus dem Siebengebirge*, Verlag von P. Hanstein, Bonn, 1901.
22. E. W. Smith and A. M. Dale, *The Ila-Speaking Peoples of Northern Rhodesia*, v. 2, Macmillan and Co., London, 1920.
23. F. Tupper, Jr., Riddles of the Bede tradition, *Modern Philology* 2 (1904), 561–572.
24. D. Westermann, *Die Kpelle: ein Negerstamm in Liberia*, Vandenhoeck & Ruprecht, Göttingen, 1921.
25. C. Withers & S. Benet, *The American Riddle Book*, Abelard-Schuman, N.Y., 1954.
26. C. Zaslavsky, *Africa Counts*, Prindle, Weber & Schmidt, Boston, 1973.

Proof without Words: $3\sum_{j=0}^{n}\binom{3n}{3j} = 8^n + 2(-1)^n$, by Inclusion-Exclusion in Pascal's Triangle



$$\sum_{j=0}^{n}\binom{3n}{3j} = \sum_{j=1}^{3n-1}(-1)^{j-1}2^{3n-j} = -2^{3n}\sum_{j=1}^{3n-1}\left(-\frac{1}{2}\right)^{j} = \frac{8^n+2(-1)^n}{3}.$$

—Dean S. Clark
University of Rhode Island
Kingston, RI 02881