Thus

$$\int_0^x \left(1 + y + \frac{ey^2}{2}\right) dy > \int_0^x e^y \, dy > \int_0^x \left(1 + y + \frac{y^2}{2}\right) dy$$

or

$$1 + x + \frac{x^2}{2!} + \frac{ex^3}{3!} > e^x > 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}.$$

Continuing in this manner, we get

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{ex^{n+1}}{(n+1)!} > e^x > 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^{n+1}}{(n+1)!}.$$

Putting $x = 1$ gives

$$e - \frac{1}{(n+1)!} > 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} > e - \frac{e}{(n+1)!}.$$

As $n \to \infty$, the left and right sides tend to e and we obtain the desired result.

Riemann Sums and the Exponential Function

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Every standard calculus textbook contains the derivations for the definite integral of x and x^2 using Riemann sums based on the known results for the sums of the first k integers and the first k squares of the integers. Now that some of the reform calculus projects have moved the exponential function "up front" because of its importance, it is nice to have a comparable derivation for its definite integral. In the process, we can reemphasize some important ideas.

Let $[a, b]$ be any interval and consider a uniform partition of the interval into n subdivisions of size $h = \Delta x = (b - a)/n$. Therefore,

$$\begin{aligned} \int_a^b e^x \, dx &\approx \sum_{k=0}^{n-1} e^{a+kh} h \\ &= he^a \sum_{k=0}^{n-1} (e^h)^k \\ &= he^a \frac{(1 - e^{nh})}{1 - e^h} \\ &= \frac{he^a(e^{b-a} - 1)}{e^h - 1}, \end{aligned}$$

using the sum of the first n terms of a geometric progression and the fact that $b = a + nh$. Therefore, in the limit as $n \to \infty$ and hence $h \to 0$, we obtain

$$\int_a^b e^x \, dx = \lim_{h \to 0} \frac{(e^b - e^a)}{(e^h - 1)/h}.$$

Recognizing that the limit as h goes to 0 of the term in the denominator is precisely the definition of the derivative of e^x at $x = 0$, we immediately conclude that

$$\int_a^b e^x \, dx = e^b - e^a.$$

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On Laplace's Extension of the Buffon Needle Problem

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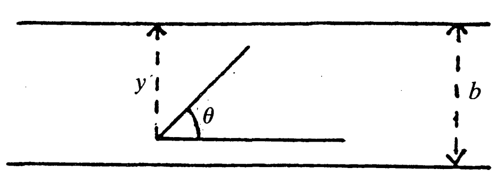
The classical Buffon needle problem is to find the probability that a needle of length n when dropped on a floor made of boards of width b will cross a crack between the boards. This problem can be solved by evaluating a simple single integral. In his extension of the problem, Laplace considered a floor tiled by congruent rectangles and considered the probability of the needle crossing one or two of the cracks between the rectangles. The problem of computing this probability is given as an exercise in some references [**2**, **4**, **13**] the answer is merely stated in others [**1**, **12**], and is discussed in some detail by Solomon [**11**]. The only reference we know of that provides an elementary presentation of the solution of Laplace's problem is in error [**10**], and it is the purpose of this note to provide a complete and correct solution. In our solution, the universe of possible positions in which the needle can fall is modeled on a three-dimensional coordinate system and the problem is solved by some straightforward computations of double integrals. We conclude with a brief survey of the many variations on the Buffon and Laplace needle problems and we suggest some variants to pursue.

The original Buffon needle problem. In the diagram below a needle of length n is depicted on a floor with boards of width b. The distance from the base of the needle to the floorboard "above" the needle is denoted by y. The angle made by the needle with the horizontal is denoted by θ. If we assume that $n < b$, then the universe of positions in which the needle can fall is seen on the following graph with the shaded portion representing the positions where the needle crosses a crack. (The case where $n > b$ is left for the reader.)



$0 \leq y < b$
$0 \leq \theta < \pi$