have a digital watch to keep time but, to be safe, we bring such a watch. In advance of class time, we run through the process several times to establish the time required to drain the cylinder; with our apparatus it consistently took 68 seconds for the water depth above the hole to drop from 10 centimeters to 3 centimeters. Given this, the problem is to predict what the depth will be at times 10, 20, 30, 40, 50, and 60 seconds.

A vital part of the demonstration is to have students participate. We have had no difficulty in getting three volunteers to play all the roles. The timekeeper helps out initially by pouring the water into the leaky bottle while the bottle keeper covers the hole with a finger. Meanwhile, the recorder copies the table of predicted values on the blackboard. As the experiment progresses, the time keeper calls out $\dots 8, 9, \mathbf{10}, \dots, 18, 19, \mathbf{20}, \dots$ and at the appropriate times the bottle keeper estimates out loud the depth reading which can be done with accuracy within a tenth of a centimeter. This person should be cautioned not to look at the predicted values in order to avoid being influenced by them. As the depths are called out the recorder records them on the blackboard. Our experience in trying this in three different classes was that the experimental data were virtually identical to the Torricelli predictions, making for an impressive and memorable demonstration.

A second demonstration we have used involves less apparatus but one item that must be made in advance: a thin "plate" shaped like the interior of a parabola from the vertex back to a line perpendicular to the axis. The basic idea is to determine from first principles, via integral calculus, just where the balance point ought to be. The chance for people to test the result on a physical model gives a "payoff" to the example.

We are still experimenting with ideas along the lines described above. Such topics as Newton's law of cooling, period of a pendulum, differentials for inverse square laws, and spark tapes for velocity/acceleration seem promising, if one can find the right mix of calculus and physical interaction in the classroom.

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Rubberbanding and Holding Out

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In discrete mathematics, one problem of interest is counting bit strings that have some particular property. Two such problems deal with the number of ways that zeros can appear together and the number of ways that they can be separated. Specifically:

Problem 1. How many eight-bit strings with exactly two zeros are there in which the two zeros appear together?

Problem 2. How many eight-bit strings with exactly two zeros are there in which the two zeros *do not* appear together?

To solve the first problem, we "rubberband" the two zeros together and then count the number of ways that six ones can be arranged with the zeros: $C(7,1)$ ways. To solve the second problem, we may use a complement approach. That is, subtract the number of ways the zeros can appear together from the total number of such strings. So the solution to Problem 2 is $C(8,2) - C(7,1)$. Note, however, that this expression is equal to $C(7,2)$. So we might ask how $C(7,2)$ counts the solution to Problem 2 directly. The method is to "hold out" one of the ones and generate a seven-bit string with two zeros. This can be done in $C(7,2)$ ways. Then place the "held out" one directly to the right of the leftmost zero. Note that all such strings with the two zeros apart can be generated this way (remove the one to the right of the leftmost zero), and so this does count all such strings.

This method is easily extended to problems such as the following.

Problem 3. How many ways can five boys and eight girls be arranged in line so that no two boys are next to each other?

Solution. Generate a string of five B's and eight G's with no two B's together. To do this, hold out four G's and generate a string with five B's and four G's. There are $C(9,5)$ ways to do this. Then place a G to the right of the first four B's. Now put a girl in the first G, a girl in the second G, etc. until the girls have been placed. There are 8! ways to arrange the girls. Similarly arrange the boys in one of 5! ways. So the desired solution is

$$C(9,5)8!5! = \frac{9!8!}{4!}.$$

The Making of a Mathematician

"...Hornblower's character was pretty well settled for him. The details had to be filled in. He would be a gangling and awkward man, because that would be an effective contrast with his mental ability, and would offer fuel for the fire of his self-criticism. But he would be an accomplished mathematician; I myself was constitutionally unable to make the leap from the binomial theorem to calculus, and it would be pleasant to have a hero to whom it was easy, especially as some of my close friends had been mathematicians. Yet, of course, in making Hornblower a mathematician I was indulging in shameless wish fulfilment, but it is only today, while writing these lines, that I realize it."

Cecil Scott Forester, *The Hornblower Companion*, Little Boston, 1964.

Contributed by Mark Meyerson, Annapolis, MD.