A Method with linear complexity. The final method uses two parameters to remember the last two Fibonacci numbers, and hence eliminates repetitive computation of the same Fibonacci number.

```
function f(n)
    if n = 1 then
        return 1
    else if n > 1 then
        return g(1, 1, n − 1)
    end if
end function

function g(a, b, n)
    if n = 1 then
        return a
    else
        return g(a + b, a, n − 1)
    end if
end function
```

Since each recursive call reduces the n parameter by 1, and a value is returned when this parameter reaches 1, it is clear that a call to f results in $n-1$ calls to g and 1 call to f. It remains to be shown that F_n is the value that is returned. If $n=1$, then $1=F_1$ is returned by f. If $n=2$, then f calls g with parameters 1, 1, 1, and so $1=F_2$ is returned by g and hence by f. Note that in any first call to g, the first two parameters are $a=F_2=1$, and $b=F_1=1$. Now if we assume that on the kth call of g, the first two parameters are $a=F_{k+1}$ and $b=F_k$, then $a=F_{k+1}$ is returned if the third parameter is 1, and otherwise g is called again with the first two parameters $a+b=F_{k+1}+F_k=F_{k+2}$ and $a=F_{k+1}$. By induction, it follows that F_n will be returned after 1 call to f and $n-1$ calls to g.

References

1. Giles Brassard and Paul Bratley, *Algorithmics, Theory and Practice*, Prentice-Hall, Englewood Cliffs, NJ, 1968, pp. 16–18.
2. Udi Manbar, *Introduction to Algorithms*, Addison-Wesley, Reading, MA, 1989, pp. 46–50.

Distance from a Point to a Plane with a Variation on the Pythagorean Theorem

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In this capsule we give a short and direct derivation of the standard formula

$$|Aa+Bb+Cc+D|/\sqrt{A^2+B^2+C^2} \tag{1}$$

for the distance between a point $P(a,b,c)$ and the plane LMN:

$$Ax+By+Cz+D=0. \tag{2}$$

It avoids the use of dot products of vectors by using a variation on the Pythagorean theorem.

In any right triangle ΔRPN (see Figure 1) if segment RP is perpendicular to NP at P, the lengths r, p and n of the altitudes from the vertices R, P and N respectively satisfy what we call a variation on the Pythagorean theorem:

$$p^{-2} = r^{-2} + n^{-2}.$$

To see this we express the area of ΔRPN in two different ways to obtain $ps/2 = rn/2$, where the distance $RN = s$. Since $s^2 = r^2 + n^2$, substitution yields $p^{-2} = r^{-2} + n^{-2}$. As an aside we note that this means the reciprocals of the altitudes form sides and hypotenuse of another right triangle.

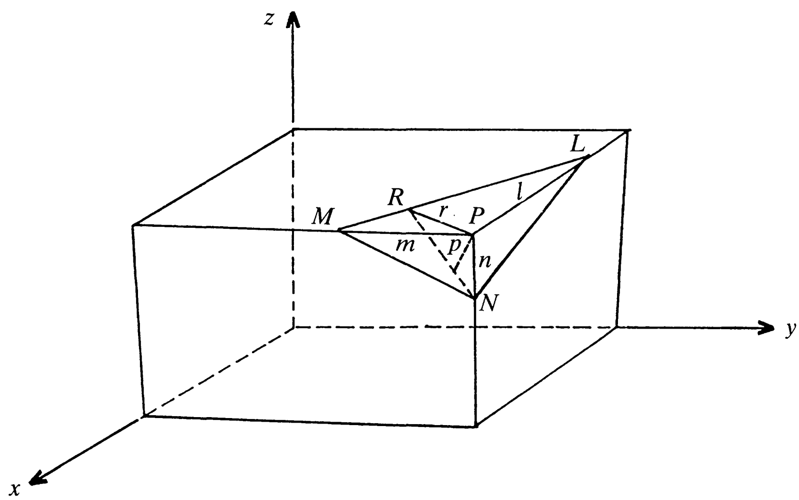

Figure 2

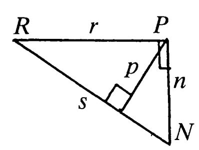

Figure 1

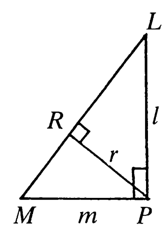

Figure 3

To derive the distance formula we use Figure 2 and assume that $A \neq 0$, $B \neq 0$, and $C \neq 0$. The three lines from $P(a, b, c)$ drawn perpendicular to the planes $z = 0$, $y = 0$, $x = 0$, intersect the plane $Ax + By + Cz + D = 0$ in points

$$N(a, b, c - S/C), \; M(a, b - S/B, c) \quad \text{and} \quad L(a - S/A, b, c)$$

respectively, where $S = Aa + Bb + Cc + D$. Let ΔNPR (see Figures 1 and 2) be any right triangle, right-angle at P, with R as a variable vertex lying on the line ML. Denoting the lengths of the altitudes from the vertices N, P, and R by n, p, and r respectively we have

$$p^{-2} = r^{-2} + n^{-2}, \quad \text{or} \quad p = 1\Big/\sqrt{(1/r^2) + (1/n^2)}\,. \tag{3}$$

Note that p is a variable distance from P to the plane and it varies with R, that is, with distance r. We need to choose R on ML so as to minimize p. From (3) p is minimum when r is minimum, but r is minimized when RP is perpendicular to ML in right triangle ΔMPL (see Figure 3); that is, when

$$r^{-2} = l^{-2} + m^{-2}.$$

Placing this value in (3) gives

$$p^{-2} = l^{-2} + m^{-2} + n^{-2}, \quad \text{or} \quad p = 1\Big/\sqrt{(1/l^2) + (1/m^2) + (1/n^2)}\,. \tag{4}$$

Since

$$n = PN = |S/C|, \qquad M = PM = |S/B| \quad \text{and} \quad l = PL = |S/A|,$$

substituting these values in (4) leads at once to the formula

$$p = |Aa + Bb + Cc + D|\Big/\sqrt{A^2 + B^2 + C^2}\,. \tag{1}$$

An immediate result from (1) when $C = c = 0$ is:

$$p = |Aa + Bb + D|\Big/\sqrt{A^2 + B^2}\,,$$

the distance of point $P(a, b)$ from the line $Ax + By + D = 0$ in the xy-plane.

Reference

A. Sattar Gazdar, Distance from a point to a line, *Mathematics Teacher* 80 (1987) 608–609.

Math as admired from the humanities

"My father happened to remark to me that he never liked mathematics. Since I admired my father very much, it became a point of honor with me not to like mathematics either. I finally squeezed through Solid Geometry. But when, at the age of sixteen, I entered Oberlin College, I found that the authorities felt that one hard course was all anybody ought to be asked to carry. You could take either mathematics or Greek. Of course if you took Greek you were allowed to drop Latin. I did not hesitate a moment. Languages were pie for me. It would have been unfilial to take mathematics. I took Greek, and have never seen a mathematics book since. I have been permitted to glory in the possession of an unmathematical mind."

Robert Hutchins quoted in Harry S. Ashmore, *Unseasonable Truths: The Life of Robert Maynard Hutchins*, Little, Brown & Co., 1989.

Contributed by E. M. Klein, Whitewater, WI.