Next, use the tangent function to represent the secant, and differentiate to obtain the usual formula for the derivative of the secant. For the derivatives of the sine and cosine, observe that $\cos x = 1/(\sec x)$ and $\sin x = \tan x \cos x$ for $x \neq (n + \frac{1}{2})\pi$. Differentiate to obtain the usual formulas with this restriction which can be removed by use of the identities

$$\sin x = \cos\left(x - \frac{\pi}{2}\right) \text{ and } \cos x = \sin\left(x + \frac{\pi}{2}\right).$$

Finally, the derivatives of the cotangent and cosecant can be obtained from the derivatives of the sine and cosine in the usual way.

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Riemann Integral of cos x

John H. Mathews and Harris S. Shultz, California State University, Fullerton, CA

Lagrange's identity,

$$\sum_{k=0}^{n-1} \cos k\theta = \frac{1}{2} + \frac{\sin\left(n - \frac{1}{2}\right)\theta}{2\sin\frac{1}{2}\theta},$$

can be verified using mathematical induction and the trigonometric identity,

$$\sin(u+v) - \sin(u-v) = 2\cos u \sin v.$$

We can use Lagrange's identity to obtain a basic calculus formula. Since

$$\sum_{k=0}^{n-1} \frac{x}{n} \cos k\frac{x}{n} = \frac{x}{n}\left[\frac{1}{2} + \frac{\sin\left(n - \frac{1}{2}\right)\frac{x}{n}}{2\sin\frac{x}{2n}}\right]$$

is a Riemann sum for the function $f(t) = \cos t$ on the interval $[0, x]$, we have

$$\int_0^x \cos t\,dt = \lim_{n\to\infty}\left[\frac{x}{2n} + \frac{\frac{x}{2n}}{\sin\frac{x}{2n}}\sin\left(x - \frac{x}{2n}\right)\right] = 0 + (1)(\sin x).$$

That is, we have shown that

$$\int_0^x \cos t\,dt = \sin x$$

without using the fundamental theorem of calculus. Compare [James Stewart, *Calculus*, Brooks/Cole, Monterey, CA, pp. 266–267.]