## Introduction to Limits, or Why Can't We Just Trust the Table?

Allen J. Schwenk (schwenk@wmich.edu), Western Michigan University, Kalamazoo, MI 49008-5152

In a first-semester calculus class, or in any class where limits have just been introduced, students tend to think that they can reliably predict the correct limit for $\lim_{x\to a} f(x)$ by selecting a few well-chosen values for $x$ near $a$. Moreover, they are certain the professor is just being picky by insisting on developing theorems and algebraic methods for "proving" limits.

I find that the following example produces spirited debate about the "correct" limiting value. Ultimately, it convinces the class that *strange things can happen.*

First, I partition the class into nine groups and assign each group a different digit $d$ from $d = 1$ to $d = 9$. Each group is then asked to predict the limit

$$\lim_{x\to 0} \sin \frac{\pi}{x}$$

by completing the following table, where $d$ stands for their assigned digit.

| $x$ | $f(x) = \sin \frac{\pi}{x}$ |
|---|---|
| $0.d$ | |
| $0.0d$ | |
| $0.00d$ | |
| $0.000d$ | |
| $0.0000d$ | |
| $0.00000d$ | |

The curious fact is that eight of the groups will generally conclude that this function has a limit, while the ninth group will think there is no limit. But among the eight groups observing a limit, five get 0, and one each gets the values

$$0.866025403784\ldots$$

$$-\,0.866025403784\ldots$$

$$-\,0.342020143\ldots.$$

Of course the knowledgeable reader understands why this function has no limit, but the selection process has been rigged to lead most students to various faulty answers.

I leave it to you and to your class:

- to deduce what these three curious decimals represent,
- to discover which $d$ gives which "limit," and
- to explain why!

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