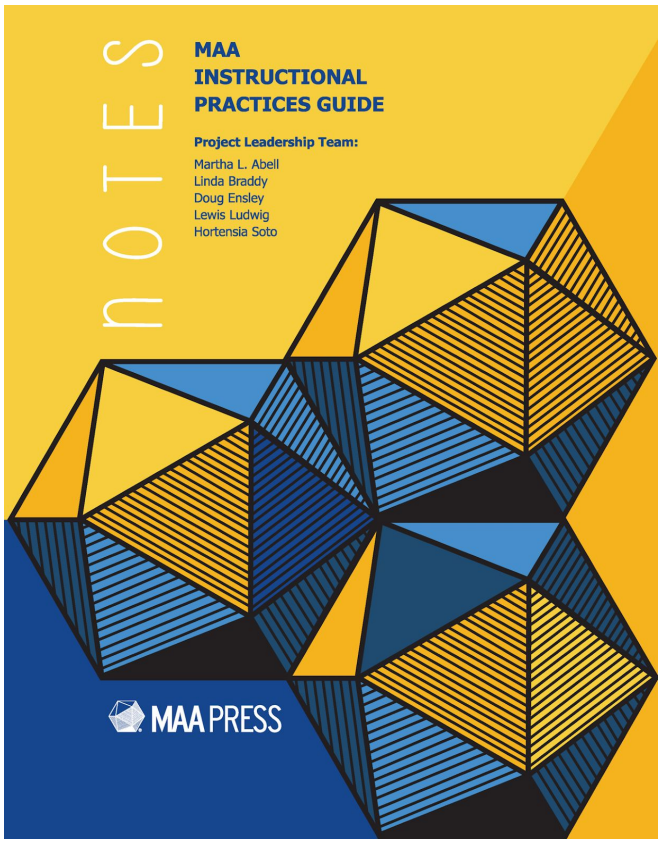
Book Study Guide for

The MAA Instructional Practices Guide



To inspire reform in post-secondary mathematics education, the Mathematical Association of America published the *Instructional Practices Guide* in 2018 to "share effective, evidence-based practices instructors can use to facilitate meaningful learning for students of mathematics" and increase "individuals' access to the opportunities that come with mathematical understanding" (p. vii). The *Instructional Practices Guide* interweaves the cross-cutting themes of **technology** and **equity** in three chapters:

- **Classroom Practices** that foster student engagement and build on high quality mathematical tasks;
- **Assessment Practices**, including formative and summative assessments, that align with learning objectives; and
- **Design Practices**, based on instructional theories, that can help instructors achieve desired learning outcomes.

This book study guide is a way to dig deeper into these ideas individually or as a group.

More information on the *MAA Instructional Practices Guide*, including information on downloading a PDF copy, can be found at www.maa.org.

This *Book Study Guide* was inspired by and follows the format of the National Council of Teachers of Mathematics' (2018) *Book Study Guide for Catalyzing Change in High School Mathematics: Initiating Critical Conversations*.

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How to Use This Book Study Guide

Audience: This book study allows readers to (individually or as part of a reading group) explore issues relevant for all undergraduate instructors of mathematics as well as graduate students, course coordinators, department chairs, and administrators. For reading group leaders, this book study guide can be tailored to fit your group's needs.

Format: The book study guide is designed to support multiple reading group formats including face-to-face, online, or hybrid settings.

Process: Each session in the book study guide is structured around the following five sections to help fully engage readers as they reflect, discuss, and take action.

- **Before You Read:** In this section, we invite the reader to examine their own ideas about the content for that session by individually reflecting on the posed prompts or engaging in conversations with other individuals.
- **While You Read:** In this section, we encourage the reader to record their thoughts and reactions, including any questions that the *Instructional Practices Guide* provokes.
- **After You Read:** In this section, we provide reflection questions for individual reflection or a group discussion. We recommend that reading group leaders select and discuss those questions that are most appropriate for their group.
- **Action Steps:** In this section, we invite the reader to continue their learning and sharing with others by creating and following an action plan related to the content from the reading.
- **Reflection:** In this section, we provide the reader with the opportunity to reflect on their action steps after they implement them.

Launching a Reading Group

If you are using this book study guide to lead a reading group, we have provided you with some suggestions for your first meeting. If you plan to do Session 1 in the first meeting, you may want to skip the Discussion Questions below and move on to Session 1 after introductions and establishing conversation agreements.

Introductions: If your reading group members do not know each other well, spend more time on this. You should introduce yourself first to set an example.

- What is your preferred name and gender pronouns[1]?
- What is your personal context (type of institution, job responsibilities, etc.)?
- What are your individual and/or group goals in undertaking this book study?

Establishing Conversation Agreements: Because some of the discussion questions in this book study guide ask group members to share personal opinions and speak from personal experience, maintaining respectful and equitable conversation is important. We suggest the following agreements as a starting point.

- Be an active listener and listen for understanding
- Focus on impact over intent
- Share talk time
- Be present and aware of your space in the room
- Recognize that everyone in the room has expertise
- Give wait/think time

Have a discussion about what agreements you want to establish. Also, as the leader, consider when and how to revisit these agreements throughout the book study.

Discussion Questions: The following questions are intended to initiate your reading group and frame future critical conversations about the teaching of undergraduate mathematics. Select questions that are most applicable for your group.

1. How does a person learn something new?
2. What concerns you about students' learning of undergraduate mathematics? What concerns you about the teaching of undergraduate mathematics?
3. What do you want students to remember from their undergraduate mathematics classes 5, 10, or 20 years from now, when they may be in a profession they did not expect would require mathematics or in a mathematical profession that doesn't exist yet or doing a kind of mathematics that doesn't exist yet?

[1] https://www.ccsu.edu/lgbt/files/PreferredGenderPronounsForFaculty.pdf

Session 1

Introduction to the *MAA Instructional Practices Guide*

> **Key Idea:** "The *Instructional Practices Guide* aims to share effective, evidence-based practices instructors can use to facilitate meaningful learning for students of mathematics" (p. vii).

For this session, you will read the manifesto and introduction presented at the beginning of the Instructional Practices Guide *(IP Guide). The manifesto is a bold statement of the values and beliefs that the IP Guide is built upon. The IP Guide was "written from the perspective that teaching and learning are forces for social change" and declares that "it is our responsibility to examine the system within which we educate students...find ways to improve that system…help our colleagues improve and to collectively succeed at teaching mathematics to all students so that our discipline realizes its full potential as a subject of beauty, of truth, and of empowerment for all" (p. vii). In the Introduction, the reader is introduced to the overall structure of the IP Guide. The IP Guide is organized into three main chapters: Classroom Practices, Assessment Practices, and Design Practices. While these chapters could be read in succession, they were written so that the reader could read sections independent of each other. Finally, the Introduction ends with a deeper discussion of the two cross-cutting themes, technology and equity, that are integrated throughout the other chapters.*

Either on your own or with another individual, reflect on the following questions before reading.

1. What do you think of when you hear the phrase "evidence-based[2] teaching strategies"?
2. What is the purpose of a manifesto? If you are not familiar with this term, take this opportunity to look it up. Then, as you begin reading the IP Guide, consider

[2] You may want to read the following blog post by Dr. David Chard for a deeper discussion of what it means for instruction to be "evidence based": https://www.understood.org/en/school-learning/partnering-with-childs-school/instructional-strategies/what-does-it-take-for-instruction-to-be-evidence-based

why the author team chose to begin with the Manifesto and why they decided to use this term.

3. Why, in your opinion, is teaching mathematics at the undergraduate level slow to change (given how quickly society changes)?
4. What beliefs, policies, and structures might need to be examined to improve undergraduate mathematics education? Is there anything specific in your personal context that you want to examine?
5. What barriers might be preventing change in your personal context?
6. What critical conversations (about the teaching and learning of mathematics) need to take place in your personal context? Who should be involved in these conversations?

Instructional Practices Guide: Read “Manifesto: A declaration of values” (pp. vii-viii) and “Introduction to this guide” (pp. ix-x)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. In the manifesto it states that “changing teaching practices is hard” (p. vii). If you have tried to make changes to your teaching practices in the past, does this statement ring true? If you have not tried to make changes, why might you anticipate it being hard?
2. Have you taken steps to move beyond your own classroom and advocate for the use of student-centered instructional strategies that promote equitable access to mathematics for all students? If so, what have you done? If not, why not?

The Manifesto declares that “it is our responsibility to examine the system within which we educate students...find ways to improve that system…help our colleagues improve and to collectively succeed at teaching mathematics to all students so that our discipline realizes its full potential as a subject of beauty, of truth, and of empowerment for all” (p. vii). This is a call to action! Therefore, we encourage you to use the action plan below to take a first step individually or as a group to improve the system. There will be more opportunities to reflect on equitable instructional practices in future sessions (e.g., Sessions 2, 5, and 10), so it’s ok if you are unsure about what actions you can take. But everyone can take this opportunity to at least *begin* thinking about how to address issues of equity and access within undergraduate mathematics education!

Action Plan	
(a) What is one thing that you feel confident that you can change today (on your own) to increase equitable access to mathematics for all students in your classroom?	
(b) What is one thing that you would like to change, but you need support in pursuing?	
(c) Identify the people, offices, or other key stakeholders within your context who could help you make a first step to make the change identified in (b). How could you start	

a conversation with them about making this change?	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. After having conversations with key stakeholders, is the change you identified in (b) still a priority for you? If not, what have you learned from the conversations that you've had?
2. What did you try to change and how did it go?
3. What revisions or adaptations might you make to the changes you identified in parts (a) and (b) of the action plan?

We recognize the need to increase equitable access to mathematics as ongoing work for everyone in the broader mathematics community. There will be opportunities to revisit this theme in multiple sessions outlined in this book study guide (e.g., Sessions 2, 5, and 10).

Session 2

Fostering Student Engagement

> **Key Idea:** Students learn better when they are engaged and instructors can promote engagement with a variety of classroom practices.

For this session, you will read the first half of the "Classroom Practices" chapter on "Fostering student engagement". This section highlights specific activities like one minute papers, exit tickets, peer instructions and inquiry-based teaching and learning, designed to promote active classrooms rich with opportunities for students to engage with the content and with each other. It provides suggestions for building community in the classroom and how to respond to the needs of students in real time with just-in-time teaching and responding to student contributions in the classroom. Readers are called to reflect on their teaching with a focus on the first day of class. One can ask "Do your first day activities convey to your students what is important in your class?" This section emphasizes the importance of student to student interactions and teacher to student interactions. The next section will dig deeper into the choice of mathematical tasks that allow for the kinds of productive interactions described here.

Either on your own or with another individual, reflect on the following questions before reading.

1. What do you do on the first day of your course? What are your students doing? What steps do you take to foster an inclusive classroom environment?
2. What kinds of questions do you ask during a typical class? How long do you wait for a response to a question?
3. In what ways do your students engage with each other about their learning in your course?
4. How do you assess learning during the term, and how does that information guide your instruction?

Instructional Practices Guide: Read “CP.1. Fostering student engagement” (pp. 1-26). *Note: This reading is dense, but very important, so take that into account when planning your reading time.*

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. Which strategies from this chapter are you already using? How is it going and where would you like to improve?
2. One of the cross-cutting themes of the IP Guide is increasing equitable access to mathematics for our students. How do the classroom practices described in this section provide opportunities for faculty and the larger mathematics community to increase equitable access? How does this lens inform your thoughts about implementing a strategy described in this section?
3. What needs to shift in your preparation for a single class, unit, or entire course to create opportunities for students to practice persisting through problem solving in your class?

We encourage you to use the action plan below to take a first step to implement a change in your classroom to increase student engagement.

Action Plan	
(a) What is one thing that you can implement in your classroom today to increase engagement for all students?	
(b) What will have to change in your preparation for class to make the in-class change identified in (a) happen?	
(c) How will you determine if the change that you implemented is effective? What kind of feedback will you collect about students' experiences and other possible stakeholders (e.g., TAs, other course staff, a course coordinator)?	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. How was implementing the change you identified in your action plan? How did the implementation compare with how you thought it would go?
2. When did you implement or experiment with the idea that you identified in your action plan? Do you think the timing in the day, week or semester made a difference?
3. What did you learn from the feedback you got from your students or other stakeholders? Was equity addressed as well as engagement in the feedback?
4. How will you tweak the implementation, if at all, and when will you try this again? If the change is ongoing, what are you doing to continue to collect and act on feedback to improve the implementation?

Session 3

Selecting Appropriate Mathematical Tasks

> **Key Idea:** In order for students to actively engage in the learning process, instructors must choose appropriate mathematical tasks. Since there is not one single notion of "appropriateness", several factors must be considered when determining the appropriateness of a task.

Next you will read the second half of the "Classroom Practices" chapter on "Selecting appropriate mathematical tasks". Due to its length, we have split this section across two session. This first session will focus on CP.2.1-CP.2.5, with the remainder of CP.2 being the focus of Session 4. The first portion of CP.2 will address the question: "What makes a mathematical task appropriate?" The question of appropriateness will be approached both intrinsically (by considering how aspects of the task itself make it effective) and extrinsically (by considering how the learning environment affects how well students will learn from the task). These considerations will then be situated within three major theoretical perspectives on human learning. Grounded in these theoretical perspectives, the IP Guide then offers a sequence of questions for an instructor to use when selecting an appropriate mathematical task. The reading closes with the topic of group-worthy tasks, illustrated by a classroom vignette, and discusses the special considerations involved in choosing an appropriate mathematical task for group work.

Either on your own or with another individual, reflect on the following questions before reading.

1. How would you define appropriateness of a task? How do you know when a task is appropriate?
2. Think about a favorite mathematical task that you have used in one of your classes. List a couple of features of the task. Why do you think that this task is appropriate? How do you implement the task? How do you know if it is successful? What do you do if it is not?

Instructional Practices Guide: Read the first half of "CP.2. Selecting appropriate mathematical tasks" (pp. 26-35)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. Vygotsky's notion of the zone of proximal development[3] (ZPD), described on page 28 of the IP Guide, refers to the space between what a student can do without help and what the student cannot do even with help. How should an instructor assess the ZPD of their students for a particular task, and how can they use that information to choose or design appropriate tasks for the class?
2. Ryan and Deci's self-determination theory[4] (SDT), described on pages 28-29 of the IP Guide, proposes that all students possess the need for competence, autonomy, and relatedness. The IP Guide states that "for the purposes of selecting a mathematical task, an instructor should attend to whether a task promotes both competence and autonomy, as well as to whether the social environment of the class promotes relatedness" (p. 29). What are some ways to

[3] Vygotsky, L.S. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.
[4] Ryan, R.M. and Deci, E.L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology, 25*, 54–67.

promote both expertise and the ability for students to locate their expertise within themselves? How might the social environment of the class promote a sense of belonging for all students?

3. What makes a mathematical task appropriate for a group (i.e., “group worthy”)? How is considering the appropriateness of a task for group work similar to and different from considering the appropriateness of a task in general?
4. The fish.net task described in the vignette on page 31-35 of the IP Guide is said to have a “low floor” and a “high ceiling” (p. 30). What do the IP Guide authors mean by this? Why are these considerations important?
5. Many students focus on procedural fluency instead of conceptual understanding. What do you currently do to help students increase their conceptual understanding? What are some ways that you can choose appropriate tasks in the future to increase students’ conceptual understanding?

We encourage you to use the action plan below to take steps towards assessing the appropriateness of a mathematical task. Think about one upcoming task in a course that you teach and interrogate that task using the questions listed here (drawn from the IP Guide). Then implement the task in the course.

Action Plan	
(a) Do you have clearly-stated and concrete learning objectives defined for the lesson in which the task is going to appear, and do students have access to those objectives? If not, state them. Does the task align with the learning objectives? If not, how will you modify it?	
(b) Do you have actionable information about your students’ motivations, attitudes, and readiness for the task? If not how will you acquire it? Based on that information, does the task meet students at their	

level of expertise, level of readiness, and motivation? If not, how will you modify it?	
(c) Is the task well-constructed in terms of building students' intellectual development, competence, and autonomy? Does it leverage the social context of the class to promote relatedness? If not, how will you modify it?	
(d) Does the task have a low floor? If not, how could it be modified to lower the floor? Does it have a high ceiling? If not, how could it be modified to raise the ceiling?	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. What did you learn about your mathematical task during the planning and implementation?
2. How successful was the implementation of your task? How do you know?
3. Now that you have implemented this task in your teaching, revisit Questions 1, 2, and 4 from the "After You Read" section in light of your experience.
4. Based on your experience, what would you do again and why? What would you change for the next implementation and why?

Session 4

Mathematical Communication and Conceptual Fluency

> **Key Idea:** In order to actively engage students in the process of learning mathematical ideas and developing conceptual understanding, procedural fluency goals must also be considered. The traditional lecture format will not allow students to learn material at a meaningful level. Students need the opportunities to create and critique mathematical reasoning.

For this session, you will finish the "Classroom Practices" chapter, which discusses several issues related to the guiding question: "What makes a mathematical task appropriate?" Taking a cue from the Common Core Standards for Mathematical Practices[5]*, the IP Guide discusses mathematical communication. Because errors are an intrinsic, and valuable, part of students learning, the IP Guide discusses error analysis of student work. The IP Guide then considers how teaching using a flipped course design affects the issue of appropriateness of mathematical tasks and examines the relationship between procedural and conceptual fluency. This chapter's brief concluding section ends with a renewed call to teaching in a way that actively engages students, as opposed to traditional lecture format[6].*

Either on your own or with another individual, reflect on the following questions before reading.

1. Is it your experience that students tend to focus on making sense of the mathematics they are learning? Why or why not? What implications does it have? How do we teach for the development of meaning in mathematics?
2. The appropriateness of a task goes beyond the mathematical focus of the task. What tasks do you currently use to develop your students' expertise in

[5] National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards for Mathematical Practices*. Washington, DC: National Governors ASsociation for Best Practices, Council of Chief State School Officers.
[6] Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences, 111*(23), 8410-8415.

mathematical practices? What opportunities do your students have to practice reading, writing, presenting, visualizing, and critiquing others' work?

3. How do you help students embrace mistakes and use them as learning experiences?

Instructional Practices Guide: Read the second half of "CP.2. Selecting appropriate mathematical tasks" (pp. 35-44) and "CP. Conclusion" (pp. 44-45)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. Pages 35-36 the IP Guide contain eight standards for mathematical practice as listed in the Common Core State Standards[7] (CCSS). How well does this list align with your conception of mathematical practice? Is there anything in the list

[7] National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards for Mathematical Practices*. Washington, DC: National Governors ASsociation for Best Practices, Council of Chief State School Officers.

that you would deemphasize? Is there anything missing from the list that you would add?

2. The IP Guide authors say that the CCSS Math Practice #3 (constructing viable arguments and critiquing others' reasoning) "provides an excellent vision of what should be happening in college mathematics classrooms where profound student learning is the goal" (p. 36). Do you agree with the primacy of Math Practice #3 for college mathematics? Why or why not?
3. Why is it important to give students the opportunity to critique other students' work? Why is it important for students to receive critique on their work from other students? How do we develop classroom spaces that are safe and open for mathematical critique from peers?
4. In discussing the role of mistakes in learning, the IP Guide authors say that in mathematics "we need to find ways to embrace mistakes and retask them as learning opportunities, both for us as instructors and for our students" (p. 38). Why are mistakes important for the development of both students and instructors? How do we create a classroom culture that embraces and values mistakes as crucial to the learning process? How does teaching a flipped classroom change the appropriate use of individual space and group space for instruction?
5. On page 42 of the IP Guide, the authors ask the following questions about procedural fluency: "What are the areas of procedural fluency necessary for collegiate mathematics? Does the availability and accessibility of technology influence this discussion? If procedural fluency is desired, how is it developed?" Discuss!
6. What do you currently do to help students increase their conceptual understanding? What are some ways that you can choose appropriate tasks in the future to increase students' conceptual understanding?

Consider one or two ideas from these sections that you could immediately integrate into your teaching that would help promote meaning making, conceptual fluency, and/or willingness to embrace mistakes and retask them as learning opportunities?

Action Plan	
(a) What ideas did you pick and why?	

(b) How will you begin to implement them in your already existing plans?	
(c) When will you do this implementation?	
(d) How will you introduce these ideas to your students?	
(e) How will you determine the effectiveness of your implementation?	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. What went well? How do you know it went well (e.g., what evidence do you have to support the conclusion that it went well)?
2. What would you change for the next implementation and why?

Session 5

Principles of Assessment and Formative Assessment Practices

> **Key Idea:** Assessment practices should inform instructors about their students' learning as well as about their teaching (p. 50) with course learning outcomes on the forefront. Formative assessment practices provide opportunities for instructors to observe their students' growth and proficiency to make appropriate instructional decisions to improve students' learning.

In Section A.P. 1, the IP Guide emphasizes assessment as an essential element of instructional practices that should aim to inform instructors about their students' learning progress towards high-quality goals. Effective assessment practices include providing frequent formal and informal feedback to students about their learning progress as well as evaluating students' growth and proficiency that are related to course learning outcomes. Steen's six principles of assessment (pp. 50-51) serve as useful background when designing such effective assessment practices. In Section AP.1.3., you will read about the importance of integrating two types of assessments: summative and formative. Section AP.2.1 offers important components of formative assessment practices and examples. In Sections AP. 2.2. And AP.2.3, the IP Guide highlights the use of formative assessment practices for addressing course learning outcomes that are related to cognitive and affective domains.

Either on your own or with another individual, reflect on the following questions before reading.

1. What types of assessments do you use in your course? What are your goals of using each type of assessment?
2. What are your course learning outcomes and how do you assess them?
3. What are some informal ways you can assess your students' growth and proficiency during instruction?
4. What kind of feedback do you provide to your students on assessments?
5. How might assessment practices promote or hinder equitable access to mathematics for all students?

Instructional Practices Guide: Read “AP.1. Basics about assessment” (pp. 49-53) and “AP.2. Formative assessment creates an assessment cycle” (pp. 53-59)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. In what ways do you incorporate formative assessment in your instruction? Which of the five strategies listed on page 54 of the IP Guide do you implement and how do you implement them?
2. Compare the wording of your course learning outcomes to the ones listed in the vignette on pages 51-52 of the IP Guide. What are some similarities and differences? How are your learning outcomes and the ones in the vignette related to three domains listed on page 51?
3. Give an example of one way you could repurpose a summative assessment in your course into a formative assessment (see vignettes 1-3 on page 55).
4. What are some ways you could use formative assessment to improve mathematical practices in your course?

5. Consider the assessments you identified prior to reading. Discuss how they align with Steen's six principles. How can you enhance your assessment practices to better align them with these principles?
6. What are some ways you could use formative assessment to influence students' beliefs and motivations in your course?
7. What are some ways that formative assessment could promote equitable access to mathematics for all students?

Identify one or two ways that you could immediately integrate formative assessments in your teaching that would help you evaluate students' growth.

Action Plan	
(a) What formative assessments did you pick and why?	
(b) How will you begin to implement them in your teaching?	
(c) When will you do this implementation?	
(d) How will you introduce these formative assessments to your students?	
(e) How will you determine the effectiveness of your implementation?	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. What did you learn about your students' growth (and proficiency) during the implementation?
2. Now that you have implemented some new formative assessment practices in your teaching, revisit questions 5-7 from the "After You Read" section.
3. What (kind of) feedback did you provide to your students? Reflect on what ideas/beliefs these feedback could have communicated to your students. What else you could have said or done?
4. Based on your experience, what would you do again and why? What would you change for the next implementation and why?

Session 6

Summative Assessment, Mathematical Communication, and Conceptual Understanding

> **Key Idea:** Effective summative assessments should include carefully designed problems that evaluate conceptual understanding and procedural fluency in order to measure one or more learning outcomes. In addition, written assessments, oral presentations, and group projects can be used to evaluate mathematical communication skills.

In Section AP.3, the IP Guide discusses summative assessment, which is defined as assessment that is "conducted with the purpose of evaluating student proficiency with regard to one or more measurable learning outcomes" (p. 59). Summative assessments are often used to determine students' grades and complement instructors' knowledge of students' learning, which is often gathered through formative assessment practices (see Section AP.2). The IP Guide references Huba and Freed's[8] eight characteristics of exemplary assessments (p. 62), which reflect Steen's principles of assessment (see Section AP.1). In addition to these eight characteristics, it is important to also attend to different dimensions of knowledge and cognition (e.g., Bloom's taxonomy[9]) when creating and selecting problems for particular summative assessments. In Section AP.4, you will learn about three types of assessments that can be used to promote student communication (writing assignments, oral presentations, group projects) and read vignettes about these assessments, and practical tips. In Section AP.5, the IP Guide defines conceptual understanding and highlights the importance of attending to it along with procedural fluency. Finally, examples of concept inventories, which are "examinations that test basic but fundamental concepts within a particular subject" (p.71), are discussed in this section.

[8] Huba, M.E., and Freed, J. E. (1999). *Learner-centered assessment on college campuses: Shifting the focus from teaching to learning.* Needham Heights, MA: Allyn and Bacon.
[9] Anderson, L.W., Krathwohl, D.R., Airasian, P., Cruikshank, K., Mayer, R., Pintrich, P., and Wittrock, M. (2001). *A taxonomy for learning, teaching and assessing: A revision of Bloom's taxonomy*. New York, NY: Longman Publishing.

Either on your own or with another individual, reflect on the following questions before reading.

1. List all the things that you take into consideration when assigning course grades. Alternatively, think of all the ways that you have been graded in courses that you have taken as a student.
2. Pick a question from a recent exam that you have given or taken and identify the learning objective that the question measures.
3. How do or might you assess students' mathematical communication skills?
4. How do or might you assess students' conceptual understanding, which involves knowing more than isolated facts and how to apply procedures?

Instructional Practices Guide: Read "AP.3. Summative assessment" (pp. 59-66), "AP.4. Assessments that promote student communication" (pp. 66-71), and "AP.5. Conceptual understanding: What do my students really know?" (pp. 71-75)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. Consider the list you created about your summative assessment practices or experience. Discuss which of the eight characteristics of exemplary assessments (p. 62) identified by Huba and Freed[10] can be observed in your practices or experience. How can you enhance your practices to capture additional ones?
2. Consider the question from a recent exam you picked prior to the reading and the two-dimensional extension of Bloom's taxonomy[11] pictured in the table on page 63 of the IP Guide. Where does your question fit on this table? How can you modify the question so that it fits under a different box?
3. What are some other ways to assess students' mathematical communication? What do you find challenging about assessing this aspect of student learning?
4. Consider the course you are teaching this semester (or alternatively one you would like to teach in the future). Provide one fundamental concept in this course, and one way you could assess conceptual understanding of this (examples provided in Sections AP. 5.2 and AP. 5.3).

Consider one or two ideas from these sections that you could immediately integrate in your teaching that would help in your summative assessment plans or methods to evaluate students' mathematical communication or conceptual understanding.

Action Plan	
(a) What ideas did you pick and why?	
(b) How will you begin to implement them in your already existing plans?	
(c) When will you do this implementation?	
(d) How will you introduce these ideas to your students?	

[10] Huba, M.E., and Freed, J. E. (1999). *Learner-centered assessment on college campuses: Shifting the focus from teaching to learning.* Needham Heights, MA: Allyn and Bacon.
[11] Anderson, L.W., Krathwohl, D.R., Airasian, P., Cruikshank, K., Mayer, R., Pintrich, P., and Wittrock, M. (2001). *A taxonomy for learning, teaching and assessing: A revision of Bloom's taxonomy*. New York, NY: Longman Publishing.

(e) How will you determine the effectiveness of your implementation?	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. What went well? How do you know it went well (e.g., what is some evidence to support the conclusion that it went well)?
2. What would you change for the next implementation and why?

Session 7

Large-Enrollment and Non-Traditional Courses and the Role of Technology

> **Key Ideas:** In both large-enrollment and non-traditional courses (e.g., online), assessments often take on other forms, such as online homework, in-class polling systems, and other technological tools. However, technology should be incorporated into every classroom with careful consideration given to its purpose, goals, and effective implementation.

When used appropriately, online homework is a tool that instructors of large-enrollment and online courses can implement to give them the freedom to focus on developing deeper conceptual understanding and engagement. To promote student engagement and provide real-time formative assessment of mastery in large-enrollment courses, instructors can also use classroom polling systems. In online courses, instructors must be more attentive to issues of academic integrity, but also should use digital tools that provide "possibilities for more personalized, immediate, and engaging assessment experiences" (p. 80). Communicating the learning goals and standards in non-traditional courses helps students to understand expectations and desired outcomes. In the "Technology and instructional practice" section, the authors explain that the motivation behind using technology in teaching is to allow students to actively engage with the concepts they are learning. Instead of asking if technology has a place in our courses, the question should be: "Which technologies can help me accomplish my goals and create an engaging learning environment?" (p. 116). In particular, technology should be used to promote student engagement and accurately reflect the learning goals of a task.

Either on your own or with another individual, reflect on the following questions before reading.

1. What online mathematics homework systems are you familiar with?
2. What are the advantages and disadvantages of online assessments versus paper and pencil? Which do you use as an instructor and why?
3. What has been your experience teaching or taking online courses?

4. What tools and strategies could be used in online courses to engage students in the learning process and conduct assessments?
5. Do you use technology in your courses? Why or why not? If so, how do you incorporate technology into your courses?

Instructional Practices Guide: Read “AP.6. Assessment in large-enrollment classes” (pp.75-78), “AP.7. Assessment in non-traditional classrooms” (pp. 78-82), and “Technology and instructional practice” (pp. 115-122)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. If you have taught a large-enrollment or non-traditional course, share your experience with conducting assessments in these types of courses. In particular, did you use online homework systems, classroom polling systems, or other types of technology for assessments? What went well and what would you do differently next time? If you have not taught these types of courses, seek out someone who has and have a conversation with them about their experience.

2. If you teach lower-level mathematics courses, do you allow your students to use calculators? Why or why not? If you teach upper-division mathematics courses, do you allow the use of computer algebra systems? Why or why not? How does your reasoning align with what the research says about student use of technology (see Section XT.3.)?
3. Reflect on the following quote from the technology section.

   > *"We argue that questions such as 'Does this technology fit into this learning environment?' and 'How should I use this technology in that class?' are the wrong questions with which to begin. Instead, instructors should begin by considering their learning goals and their own comfort level with various technologies, then ask 'Which technologies can help me accomplish my goals and create an engaging learning environment?'" (p. 116)*

   Think of a course you have taught or are currently teaching. What kind of technology could you integrate to help you accomplish your goals and create an engaging learning environment?
4. As you consider integrating technology into your courses, it is important to attend to issues of equity and inclusion. Do your students all have the financial means to purchase the software needed to participate, or are there free versions available that you could use? Do all students all have the necessary hardware (e.g. laptops, smartphones) to successfully use the software you would like to implement? And, how might you prevent feelings of exclusion if a student does not own the requisite software or hardware? Is the technology you are using accessible to students with disabilities? Are there other constraints you should consider as you plan to integrate technology?

In the action plan below, we invite you to explore how you can put the ideas you encountered during your reading to action in your classroom.

Action Plan	
(a) Pick a specific topic that you are going to teach soon in the next few days. Identify one way that you could use technology to help students engage with this topic. *You may find it helpful to look at the*	

examples (e.g., the polling example on page 77 and the differential equations example on page 116) given in IP Guide.	
(b) Develop a quick assessment that you could use during class (e.g., polling system or exit ticket) to evaluate student learning on this topic.	
(c) Write down ideas for how you can refine your lesson plan to address issues of equity and inclusion.	
(d) Finalize and implement your new lesson plan (with the technology and assessment pieces) during class in a way that engages students with the content.	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. Did the lesson go as planned? Why or why not? Were there any technology issues that you did not anticipate? Were there any equity or inclusion issues that you did not anticipate?
2. Based on the data you gathered from your assessment, what can you tell that students have learned? What were they struggling with? Make sure to supportyour claims with specific evidence from the assessment.
3. What went well? What would you do differently next time?
4. Are there other types of technology or assessments that you would like to try out in the same lesson (the next time you teach this topic) or a subsequent lesson?

Session 8

Introduction to Design Practices

> **Key Idea:** There are many aspects to instructional design besides just content and activities. But all design considerations have to start with answering this question: What are my student learning outcomes?

This section of the IP Guide focuses on design practices that are student-centered and result in more active student engagement for all students. The section includes an extensive list of questions instructors should ask themselves at the beginning of a design process. Factors to consider when designing a lesson, a course, or a program include equity, student learning goals, research that supports learning for all, situational factors, learning environments, tasks and activities, homework, formative and summative assessment, and reflective instruction. This section provides recommendations to consider for each of these factors. The section then provides more detail about student learning outcomes. Content, cognitive, and affective goals are discussed and classroom vignettes provide specific examples of designing student learning outcomes with these goals.

Either on your own or with another individual, reflect on the following questions before reading.

1. Many instructors teach the way they were taught or the way they learn best. How do you use student-centered pedagogies to help all students be successful in your courses? Have you changed your teaching practices over your career?
2. Consider a course that you will teach for the first time. How do you prepare before the course begins? What choices do you expect to make during the term? What will you do after the course ends to strengthen your own teaching practice?

Instructional Practices Guide: Read “Design Practices” (pp. 89-100).

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. Consider the list of questions for design provided on pages 90-92. Identify one question (or subquestions) that you address in your course-design process consistently. Identify one question (or subquestions) that you have not attended to in your course-design process yet.
2. On page 90 of the IP Guide, the authors state three beliefs about design: "Designing instruction is more than just planning content…. Instructional design should aim to effect meaningful change…. Design for student-centered learning must be in sync with evidence-based practices." Which one of these three beliefs is most important one for you and why?
3. Section DP.2 states that student learning outcomes should include content, cognitive, and affective goals. For a course you teach, give a couple of examples of each type of these goals. Which type of goal do you think is most overlooked in design? Which do you think is the most difficult for you to articulate and why?
4. On page 98 of the IP Guide, the authors list the four cognitive goals from the CUPM 2015 Curriculum Guide[12]. Which of these four goals do you frequently attend to in your course design and in what ways?

[12] Zorn, P. (Ed.). (2015). *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*. Mathematical Association of America.

5. The IP Guide (p. 98) refers to the definition of affect as “a disposition or tendency or an emotion or feeling attached to an idea or object… [composed] of emotions, attitudes, and beliefs” (Phillip, 2007,[13] p. 259). Some might say that such considerations are irrelevant to mathematical practice and instruction. How would you respond to those individuals?

In this section, we invite you to explore how you can put the ideas you encountered during your reading to action in your instructional design practices.

Action Plan	
(a) Refer to the list of seven questions, and their associated sub-questions, for design on pages 90-92 of the IP Guide. Think of one course that you would like to design or possibly redesign.Consider at least two questions that you have not considered before in your design process and make a plan to incorporate your answers to these questions in your design.	
(b) On pages 92-95 of the IP Guide, the authors pose nine questions related to equity in design framed by Gutiérrez’s Four Dimensions of Equity—Access, Achievement, Identity, and Power. Think of one course that you would like to design or possibly redesign. With this particular course in mind, choose at least two questions from different dimensions and make a plan to	

[13] Philipp, R. A. (2007). Mathematics teachers’ beliefs and affect. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 257-315). United States: Information Age Publishing.

incorporate your answers to them into your course design.	
(c) Compare the original learning outcomes and the redesigned learning outcomes in the table on page 99 of the IP Guide. What is different about the redesigned outcomes? Are they an improvement? Why or why not? Take a list of learning outcomes from the syllabus of a course that you teach, and consider what they might look like with a similar redesign.	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. Reflect on your responses to parts (a) and (b) of the action plan. How did your responses help you improve the course(s) you considered with respect to the content, cognitive, and affective goals of the course?
2. What opportunities and challenges would you find in implementing your responses to part (c) of the plan?
3. How has redesigning the learning outcomes impacted your perspective on the course?

Session 9

Components and Features of Design Practices

> **Key Idea:** Design practices refer to what instructors do before and after they teach. Before they teach, instructors must plan purposely. After they teach, they need to reflect, revise, and modify.

This session continues the discussion of design practices that started in Session 8. It offers additional insight and details in the areas of the learning environment, mathematical activities and interactive discussions, homework, flipped classroom, formative and summative assessment, reflective instruction, and students needing accommodations. The section also considers the challenges and opportunities associated with designing an active student engagement environment. Both specific approaches to overcome challenges and opportunities that improve course outcomes are discussed. The section concludes with brief descriptions of three theories of instructional design that are applicable to mathematics education.

Either on your own or with another individual, reflect on the following questions before reading.

1. What challenges have you personally faced with active engagement design?
2. As a result of active engagement, what improved course outcomes have you and your students experienced?
3. How do you reflect on your teaching? How do you modify and revise your lessons and plans?
4. A common concern about student-centered design is that it sacrifices content coverage. Is this a legitimate objection? If so, how would you address this concern? If not, why not?

Instructional Practices Guide: Read “Design Practices” (pp. 100-111)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. What is the difference between “assigning homework” and “designing homework?” Why might an instructor choose to do the former? Why might an instructor choose to do the latter?
2. How and when do you reflect on the implementation of lessons? Re-read the reflections questions listed on pages 103-104 of the IP Guide. What questions would you add to this list?
3. On pages 105-106 of the IP Guide, the authors list five challenges to instructional design—time to prepare, judging effectiveness, achievement, content coverage, and buy-in. Which of these do you think is the most serious challenge, and why? How would you address this challenge?
4. On page 106 of the IP Guide, the authors say the following about judging effectiveness: “Departments should also withhold early judgement and support instructors as they employ new approaches. Student evaluations may suffer during early implementation of a student-centered design. For example, students may complain that an instructor just stands there and makes them do all the work.” How supportive is your department for pedagogical innovation? Have you

experienced any student complaints about implementation of student-centered design? How did, or would, you handle them?

5. What is the “implicit contract between students and instructor”, on p. 107 of the IP Guide, and how does it potentially affect student buy-in for student-centered instruction?
6. (Optional question for groups with an interest in theories of instructional design.) Pages 109-111 of the IP Guide give three theories of instructional design—backward design, realistic mathematics education, and universal design for learning. Which of these theories most interests you (you would like to learn more about) and why?

In this section, invite the reader to continue their learning and sharing with others by creating and following an action plan related to the content from the reading.

Action Plan	
(a) If you do not currently record your real time reflections after a class, begin a teaching journal. Record your reflections for all of your classes.	
(b) Choose one topic that you would like to teach in an active engagement manner. How would you design the lesson, and what challenges would you face? Think of strategies that would allow you to overcome these obstacles. Teach this lesson.	

After you have implemented your action plan, use the following questions to reflect on the experience.

1. Did recording your reflections help you become more purposeful in thinking about your teaching? Will you continue to use a teaching journal?
2. Were the strategies you thought of to overcome challenges associated with active engagement instruction effective? Were there challenges you had not considered? What would you do differently the next time you teach the lesson?

Session 10

Equity in Practice

> **Key Idea:** Focusing on equity as a process, rather than an end goal, emphasizes the dynamic nature of design practices that operationalize learning of mathematics for all. Our community needs to continuously and "critically examine factors well beyond students' academic preparation and achievement," (p. 122) and be "cognizant of the ways students' social identities impact their participation" (p. 123) in their learning of mathematics.

For this session, your reading will expand on the topic of equity that was introduced earlier in the IP Guide (see Section DP.1.3.). Sections XE.1 and XE.2.1 introduce the need to attend to equity and Gutiérrez's four dimensions, which were introduced in Session 8. In Section XE.2.2, the IP Guide emphasizes the important distinctions between attending to "equality", "equity", and "systemic change". Section XE.3 highlights the ways we can facilitate social discourse to remove deficit narratives about learning mathematics and to foster a sense of belonging and classroom community of mathematicians. Section XE.4 provides a summary of attending to equity through two examples: students with disabilities and developmental mathematics courses. Section XE.4 ends with an emphasis on the positive correlation between instructors' high expectations of students and student success in mathematics, highlighting the importance of anti-deficit perspective and discourse in designing and teaching of mathematics courses.

Either on your own or with another individual, reflect on the following questions before reading.

1. Take the implicit association test on **race** from Project Implicit[14] and reflect on your results.

[14] https://implicit.harvard.edu/implicit/aboutus.html

2. Take another test from Project Implicit and reflect on your results. Why did you decide to take this particular test?
3. Visit and read information on MyPronouns website[15]. How can/did you incorporate these practices in your teaching?
4. Why does equity matter in the teaching and learning of mathematics? Why does equity matter in the practice of mathematics?
5. Reflecting on the previous sessions, what have you already considered in terms of attending to equity in your teaching, assessment and design practices?

Instructional Practices Guide: Read “Equity in Practice” (pp. 122-128) and re-read “DP.1.3. Designing for equity” (pp. 92-95)

Record your thoughts, observations, reactions, and questions. Also record the page number related to each of your responses so that you can easily find and reference the passage related to your notes.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Use the following questions to frame an individual reflection or group discussion.

1. In Session 1, you were asked to reflect on “Why, in your opinion, is teaching mathematics at the undergraduate level slow to change (given how quickly

[15] https://www.mypronouns.org/

society changes)?" After reading this section, how would you answer this question?

2. In Session 1, your action plan had you consider the selection and implementation of a change that would make learning mathematics more equitable in your setting. With your new knowledge from the reading, how might you improve upon your action plan? How can you incorporate the removal of a systematic barrier in your setting (see p. 124)?
3. In Session 2, you were asked to reflect on your actions to foster an inclusive classroom environment on the first day. After reading Section XE, what additional actions might you include or how might you revise the existing ones to set the stage for an inclusive environment?
4. In part (a) of your action plan from Session 2, you were asked to identify one thing that you could implement in your classroom to increase engagement for all. Reflecting on the ideas from Figure 2 on page 124 of the IP Guide, discuss whether your plan looks more like the left image, middle image, or right image.
5. In Session 3, you were asked to consider a favorite mathematical task. In the context of equity design, who may be limited by the task, and how could the task be modified to address this limitation? Who may benefit more from the task, and how could the task be modified so all can reap those benefits?
6. In Session 4, you were asked to consider the practice of having students critique one another's work. While one implementation of this practice can go well in one setting, it can be detrimental in another. What are some ideas that you should attend to in order to promote effective and constructive peer evaluation practices in your setting?
7. In Sessions 5 and Session 6, you were asked to reflect on assessments you use in your courses. After reading Section XE and learning about the constructs presented in Gutiérrez's framework (p. 123), how do you balance your assessment practices to make sure everyone has the opportunity to demonstrate their knowledge and understanding in class? How do the assessment practices relate to the power and identity axis in the framework? How do you think about the balance of the access and achievement axis with the power and identity axis within your assessment practices?
8. In Session 7, we discussed possible uses of technology in our teaching to enhance students' mathematical learning. Discuss the access / technology / equity nexus. In your discussion, remember that this may go beyond just matters of cost or internet connectivity, and could also include physical and learning disabilities.
9. In Session 8, you were asked to pick a course to design or redesign and answer the nine questions on pages 92-95 of the IP Guide.

a. Consider your answers and discuss how your answers (ideas) help your "new" course to navigate from equality to equity ideas discussed in Section XE.2.2 (see Figure 2 on p. 124).
b. Consider your answers (ideas) and reflect on how they align with your setting (student population, institution's, department's stance on equity, institution's, department's and the mathematical program's missions and goals). Reflect on some potential barriers that you might face. What are some ideas that you read in Section XE could help to "remove" or navigate these barriers?

10. In Session 9, we discussed the value of reflecting on our teaching practice, and we considered adding to the list of reflective questions on pages 103-104 of the IP Guide. Given what we have read and discussed about equity, are there further questions that you believe we should consider as part of a reflective practice?
11. What is mathematics? For whom do we define mathematics? Who does mathematics, and who is allowed to do mathematics? And how do your answers to these questions relate to the constructs of the opposing axes on page 123 of the IP Guide?

Revisit your action plan and your reflection from Sessions 5 or 6. Focusing on the four dimensions of equity of Gutiérrez's framework (p. 123), revise your plan to include additional ideas related to each dimension.

Action Plan	
(a) Which action plan did you pick to revise and why?	
(b) How did you revise your action plan? (i.e., What additional ideas did you incorporate to attend to which dimension and why?)	
(c) How (and when) will you implement this revised action plan? To address this question consider: What changes do you need to make	

to your earlier implementation?	
(d) How will you introduce these ideas to your students?	
(e) How will you determine the effectiveness of your implementation?	

After you have implemented your action plan, use the following questions to reflect on the experience. (Note: if you have not had a chance to implement your revised action plan, you may wish to use the questions below to frame a discussion on what you expect to experience.)

1. What went well? How do you know (e.g., what evidence do you have to support the conclusion that it went well)?
2. Which dimensions of equity in your plan worked “better” than the others? How do you know (e.g., what evidence do you have to support the conclusion that it went well)?
3. What would you change for the next implementation, and why?