

# To the Student

Fermat's Last Theorem, a bedeviling open problem in mathematics that remained unsolved for over 300 years, was finally proved in 1995 by Princeton mathematician Andrew Wiles. Professor Wiles describes the work of doing research in mathematics as analogous to exploring a dark mansion [Wi]:

You enter the first room of the mansion and it's completely dark. You stumble around bumping into the furniture but gradually you learn where each piece of furniture is. Finally, after six months or so, you find the light switch, you turn it on, and suddenly it's all illuminated. You can see exactly where you were. Then you move into the next room and spend another six months in the dark. So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of, and couldn't exist without, the many months of stumbling around in the dark that precede them.

"Stumbling around in the dark" is probably not the way you have experienced mathematics. If you are in this course, you are good at mathematics and you have studied quite a lot of it. Your previous mathematics courses have exposed you to various fields of mathematics (e.g., calculus, linear algebra, etc.) that are fully established; you have struggled to master their principles and techniques; and because you study hard, you have been able to use them to arrive at the correct answers to specific, focused questions.

Your previous experience is important and foundational, however it is not the experience of a mathematician doing mathematical research. When doing research, the answers are unknown, and in fact, the right questions to answer may not even be clear. Although mathematicians bring the skills and techniques they have learned previously to every new problem, it also might not be clear which skills and techniques will be helpful, or if new skills and new techniques must be developed.

This book is designed to allow you to experience mathematical discovery in the way that mathematicians do, in the way described by Andrew Wiles.

Each module in this book begins with an exploration. The explorations are open-ended questions posed with little or no guidance on how to proceed. You may feel frustrated when you do not know where to begin or when you are unsure exactly what is required of you; this is the experience of stumbling around a darkened room. In the course of struggling with the mathematical ideas in the modules, you will gain important skills — to come up with appropriate questions and conjectures, to test ideas with examples, to distinguish between a proof and intuition, and to work as part of a team — so that over time you will become more comfortable with openness and uncertainty.

Following each exploration is a brief exposition and a set of exercises. They are designed to build on the intuition and understanding you have developed in the exploration. The answers to many of the questions you grappled with in the exploration will become

clear after you read the exposition and work your way through the exercises.

The exposition and exercises will also increase your skills as an independent reader of mathematics. As you know, reading mathematics is not like reading in other disciplines; you must read slowly, trying examples and testing your understanding of theorems and definitions. Some of the questions found in the exercises are designed to strengthen your mathematical reading skills.

Each module concludes with a project. Like the explorations, the projects involve open-ended questions and little guidance. As in the exploration, you may feel that you are again stumbling around in a dark room, but you will find that you bring a greater depth of understanding and experience to the project as a result of your earlier work in the module.

If you feel unsure and unconfident when learning mathematics in this new, independent, less structured way, you should know that is a normal response. Don't give up! You will be surprised at the skills you will learn, skills that will be applicable to any problem-solving exercise (mathematical or not) in your future. With hard work, you will also find the light switch in some of the dark rooms you have been exploring in the mansion of discrete dynamical systems, and as any mathematician will tell you, there is no more satisfying feeling. So, open yourself up to something new, empowering, and exciting, and enjoy the journey!