



## Pi Mu Epsilon

Pi Mu Epsilon is a national mathematics honor society with 401 chapters throughout the nation. Established in 1914, Pi Mu Epsilon is a non-secret organization whose purpose is the promotion of scholarly activity in mathematics among students in academic institutions. It seeks to do this by electing members on an honorary basis according to their proficiency in mathematics and by engaging in activities designed to provide for the mathematical and scholarly development of its members. Pi Mu Epsilon regularly engages students in scholarly activity through its *Journal* which has published student and faculty articles since 1949. Pi Mu Epsilon encourages scholarly activity in its chapters with grants to support mathematics contests and regional meetings established by the chapters and through its Lectureship program that funds Councillors to visit chapters. Since 1952, Pi Mu Epsilon has been holding its annual National Meeting with sessions for student papers in conjunction with the summer meetings of the Mathematical Association of America (MAA).



## Mathematical Association of America

The Mathematical Association of America (MAA) is the world's largest community of mathematicians, students, and enthusiasts. Our mission is to advance the understanding of mathematics and its impact on our world. MAA supports undergraduate students through with opportunities to present your work and attend talks at national and section meetings, through the Putnam Mathematical Competition, and through quality publications such as *Math Horizons* magazine, our three math journals, and titles from MAA Press. Students can join the MAA directly as a student member at [maa.org/join](http://maa.org/join) or through their math department if their college or university is an MAA Departmental Member—ask your math faculty members to find out if your school has an MAA Departmental Membership!

## Schedule of Student Activities

Unless otherwise noted, all events are held in the Duke Energy Convention Center.

### Wednesday, July 31

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
8:00 am – 11:00 am	PME Council Meeting .....	Press Room
2:30 pm – 4:00 pm	CUS Meeting .....	Press Room
4:30 pm – 5:30 pm	MAA-PME Student Reception .....	Room 200
5:30 pm – 6:15 pm	Math Jeopardy .....	Rooms 206
6:00 pm – 8:00 pm	Exhibit Hall & Grand Opening Reception .....	Grand Ballroom B
8:00 pm – 8:50 pm	PME J. Sutherland Frame Lecture .....	Grand Ballroom A

### Thursday, August 1

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
7:00 am – 8:00 am	PME Advisors/Enthusiasts Breakfast Meeting .....	<i>tba</i>
8:30 am – 10:45 am	MAA Session #1 .....	Room 210
	MAA Session #2 .....	Room 211
	MAA Session #3 .....	Room 235
	MAA Session #4 .....	Room 251
	PME Session #1 .....	Room 264
	PME Session #2 .....	Room 234
	PME Session #3 .....	Room 236
1:30 pm – 2:20 pm	MAA Chan Stanek Lecture for Students .....	Grand Ballroom A
2:30 pm – 4:25 pm	MAA Session #5 .....	Room 210
	MAA Session #6 .....	Room 211
	MAA Session #7 .....	Room 235
	MAA Session #8 .....	Room 251
2:30 pm – 6:05 pm	PME Session #4 .....	Room 264
2:30 pm – 6:05 pm	PME Session #5 .....	Room 234
2:30 pm – 3:45 pm	PME Session #6 .....	Room 236
4:30 pm – 6:05 pm	MAA Session #9 .....	Room 210
	MAA Session #10 .....	Room 211
	MAA Session #11 .....	Room 235
	MAA Session #12 .....	Room 251
4:00 pm – 5:45 pm	Estimathon! .....	Room 232

## Friday, August 2

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
8:30 am – 10:45 am	MAA Session #13 .....	Room 210
	MAA Session #14 .....	Room 211
	MAA Session #15 .....	Room 235
	MAA Session #16 .....	Room 251
	PME Session #7 .....	Room 264
	PME Session #8 .....	Room 236
10:10 am – 11:30 am	Career Paths in Business, Industry, and Government .....	Room 263
1:30 pm – 2:20 pm	MAA Undergraduate Student Activity .....	Junior Ballroom C
2:30 pm – 4:25 pm	MAA Session #17 .....	Room 210
	MAA Session #18 .....	Room 211
	MAA Session #19 .....	Room 235
	MAA Session #20 .....	Room 251
6:00 pm – 7:45 pm	Pi Mu Epsilon Banquet .....	Junior Ballroom B
8:00 pm – 9:00 pm	MAA Ice Cream Social and Undergraduate Awards Ceremony .....	Junior Ballroom C

## Saturday, August 3

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
8:00 am – 10:00 am	PIC Math Showcase, Student Presentations .....	Junior Ballroom C
8:30 am – 1:00 pm	MAA MathFest Mentoring Workshop for Women (MMWW) .....	Room 201
9:00 am – 10:30 am	USA Problem Solving Competition .....	Room 264
9:00 am – 1:00 pm	Great Talks for a General Audience .....	Junior Ballroom D and Room 211
10:00 am – 11:00 am	PIC Math Showcase, Industry Speakers .....	Junior Ballroom C
12:00 pm – 1:15 pm	PIC Math Showcase, Poster Session .....	Junior Ballroom C

## J. Sutherland Frame Lecture

### ALICE IN NUMBERLAND — ADVENTURES IN CRYPTOGRAPHY, NUMBER THEORY, AND LIFE



**Alice Silverberg**

University of California, Irvine

I will give an account of some of my adventures in the wonderlands of mathematics and cryptography, offering some food for thought on how mathematics can be useful in cryptography, and mentioning some useful things I learned along the way that I wish I had learned sooner.

*The J. Sutherland Frame Lecture is named in honor of the ninth President of Pi Mu Epsilon, who served from 1957 to 1966 and passed away on February 27, 1997. In 1952, Sud Frame initiated the student paper sessions at the annual Pi Mu Epsilon meeting, which is held at the Summer MathFest. He continually offered insight and inspiration to student mathematicians at these summer meetings.*

## Jean B. Chan and Peter Stanek Lecture for Students

### SECRETS OF GRAD SCHOOL SUCCESS



**Mohamed Omar**  
Harvey Mudd College

Around this time of year many rising seniors and even rising juniors are wondering what to do after college, and many contemplate the idea of going to graduate school. Naturally, they seek advice from peers, professors at their college and the internet. In this talk, we'll give some pretty unconventional advice based on the speakers experiences through the same process.

## MAA Undergraduate Student Activity Session

### COLOR ADDITION ACROSS THE SPECTRUM OF MATHEMATICS



**Ron Taylor**  
Berry College

In this talk we consider two family style games whose rules are mathematical in nature, but do not require any explicit mathematics, beyond simple counting, during game play. Both games are based on color mixing rules which yield a nice geometric visual presentation and admit several mathematical interpretations. We will discuss the nature of these color mixing rules, explore the related mathematical structures and see how all of this is related to finger paints and lightbulbs.

## MAA Student Speakers

Name	School	MAA Session
Ebtihal Abdelaziz	Goshen College	2
Tim Ablondi	Centre College	4
Franklin Ajisogun	New York City College of Technology, CUNY	20
Fahmida Akhter	NYC College of Technology	9
Anand	Disha Delphi Public School	19
Natalie Anderson	Xavier University	2
Caira Anderson	Smith College	16
Myles Ashitey	Pomona College	3
Erica Barrett	Williams College	1
Tonia Bell	The American University	20
James Benjamin	McDaniel College	3
Rinni Bhansali	Stanford University	16
Brittany Bianco	Metropolitan State University of Denver	1
Brian Bishop	Pomona College	3
Alexander Black	Cornell University	17
Kendall Bowens	Tuskegee University	4
Blair Boyle	McDaniel College	3
Abby Brickner	Xavier University	2
Isaac Brown	Washington State University	4
Paul Buldak	Lewis University	12
Michael Burgher	University of Colorado Denver	4
Dalton Burke	University of Colorado Denver	4
Addie Buzas	Denison University	15
Victoria Camarena	Smith College	18
Blayne Carroll	Lee University	6
Sam Carryer	Ohio University	9
Ryan Cecil	Duquesne University	10
Harrison Conner	Washington State University	7
Alvaro Cornejo	University of California at Santa Barbara	3
Amanda Cowell	University of Michigan-Dearborn	17
Nicholas Cummings	McDaniel College	13
Kristina Daniels	Fairmont State University	6
Monica Davanzo	University of Central Arkansas	11
Daniel Davidson	Park University	19
Azad Deihim	Wentworth Institute of Technology	15
Travis Dillon	Lawrence University	11
Patrick Dockstader	Dixie State University	15
Liam Doherty	Rowan University	10
Andrea Doty	College of Saint Benedict	16
Albright Dwarka	Hobart and William Smith Colleges	20
Owen Ekblad	University of Michigan at Dearborn	3

### MAA Student Speakers (Continued)

Name	School	MAA Session
Meryem Elbaz	New York City College of Technology, CUNY	7
Iancarlo Espinosa-García	Universidad de Guanajuato	17
Rylee Esplin	Dixie State University	20
Steven Evans	Morehead State University	5
James Evans	University of Wisconsin Stout	7
Annabelle Eycler	Hood College	7
Taylor Fernandes Nunez	Northeastern University	4
Gabriel Flores	Wheaton College	3
Leigh Foster	Metropolitan State University of Denver	1
Rhiana Fox	Lewis-Clark State College	7
Anna Fox	Clemson University	20
Andrew Franklin	Lee University	5
Brady Fritz	Grand Valley State University	16
Ziqi Fu	University of California, Santa Barbara	13
Samuel Fulton	Fort Lewis College	14
Stanley Gao	McDaniel College	1
Sheng Gao	Samford University	15
Jose Garcia	Grand Valley State University	4
Marietta Geist	Carleton College	3
Kayla Gibson	University of Iowa	3
Emil Graf	Williams College	1
Amanda Gulley	Rockhurst University	2
Kayla Harrison	Eckerd College	3
Lamia Hauter	LaGuardia Community College	20
Brett Havertz	Dixie State University	15
Maxwell Hennen	Saint John's University	19
Faith Hensley	Marshall University	14
Samuel Herman	New College of Florida	8
Emma Hiatt	Xavier University	1
Sophie Huebler	University of Oklahoma	20
Adeli Hutton	University of Cincinnati	5
John Iluno	Lee University	6
Robert Imbrie	University of Wisconsin-Milwaukee	10
Dahiana Jimenez	New York City College of Technology, CUNY	7
Benjamin Johnson	Ursinus College	17
Nicholas Joyner	East Tennessee State University	2
Neil Kamath	NYC College of Technology	12
Rihen Khatri	Lee university	6
Amy Kucera	Xavier University	8
Joshua Lang	Stevenson University	16
Gabrielle Langston	New York City College of Technology, CUNY	7
Nicholas Layman	Grand Valley State University	14



### MAA Student Speakers (Continued)

Name	School	MAA Session
Anthony Lazzeroni	Loyola University Chicago	19
Zachary Lippe	Xavier University	8
Abigail Loe	Carleton College	3
Christian Madrigal	Smith College	18
Lan Mai	McDaniel College	13
Sydney Maibach	Fairmont State University	6
Marcella Manivel	Carleton College	11
Guilherme Martins	Lawrence University	1
Matthew McFadden	University of Wisconsin - Milwaukee	16
Justin Merritt	South Dakota State University	9
Christian Miller	Grand Valley State University	12
Rachel Morris	University of Richmond	20
Benjamin Mudrak	Kent State University	19
Rajatava Mukhopadhyay	IISER Thiruvananthapuram	19
Duong Ngo	Liberty University	17
Melissa Novak	The College of New Jersey	5
Lotenna Nwobbi	Harvey Mudd College	4
Dylan O'Connor	University of North Carolina	13
Riley O'Neill	University of St. Thomas	13
Connor Parrow	Hobart and William Smith Colleges	4
Caitlyn Patel	Rollins College	2
Debosmita Pathak	Grinnell College	19
Ashley Peper	University of Wisconsin - Stevens Point	14
Luis Perez	California Lutheran University	16
Rebekah Petrosky	Lee university	6
Moises Ponce	Lee University	6
Christina Pospisil	University of Massachusetts Boston	1
Joyce Quon	California State University, Los Angeles	7
Daniel Ralston	Bowdoin College	13
Ian Ray	Morehead State University	5
Eric Redmon	Lewis University	12
Caleb Robelle	University of Maryland Baltimore County	14
Rachel Roca	Manhattan College	2
Alayna Roesener	Stevenson University	5
Beth Root	Xavier University	8
Daniel Rossano	St. Joseph's College	17
Jacob Roth	Valparaiso University	2
Meagan Scheider	University of Scranton	20
Andrew Schmelzer	CSB/SJU	3
Marisa Schulz	South Dakota State University	9
Shraddha Shankar	Denison University	15
Madelyn Shapiro	University of Puget Sound	18
Sasha-Kay Shrouder	Smith College	18

### MAA Student Speakers (Continued)

Name	School	MAA Session
Luke Shuck	McDaniel College	3
Isaiah Silaski	Texas State University	13
Milena Silva	Carleton College	11
Dilruba Sofia	University of Massachusetts-Dartmouth	6
Yasmine Soofi	NYC College of Technology, CUNY	9
Arame Sow	LaGuardia Community College	18
Nicholas Spanier	Miami University	8
James Sparks	Lewis University	12
Jack Stephens	Santa Clara University	10
William Stowe	Augustana College	3
Kimball Strong	UC Berkeley	1
Andrew Summers	Youngstown State University	4
Ramon Suris-Rodriguez	Towson University	13
Issa Susa	Smith College	16
Kevin Tao	Boston University	15
Yifei Tao	Hobart and William Smith Colleges	20
Cameron Thomas	Morehouse College	4
Delfino Torres	City College of New York	20
Bekkah Trachtenburg	Bellarmino University	18
Emily Twardy	College of Saint Benedict	2
Hikomichi Ueda	Carleton College	10
Enmanuel Valdez	LaGuardia Community College	6
Anthony Valdez	University of Wisconsin - Milwaukee	16
Sarah Van	City College of New York	13
Roman Vasquez	University of Central Florida	13
Cesar Vasquez	Dixie State University	15
James Waldeck	Marshall University	14
Bianka Wang	Saginaw Valley State University	11
Hao Wang	University of Illinois Urbana-Champaign	15
Noelle West	Dixie State University	11
Scott West	California State Polytechnic University, Pomona	20
Alexandria Wheeler	Carthage College	4
David White	Carleton College	10
Jeffrey Wilkinson	Washington and Jefferson College	14
Rachel Wofford	Whitworth University	13
Roman Wong	Washington and Jefferson College	14
Rachel Wood	Lee University	6
Harry Xi	Princeton University	18
Allen Yang	Massachusetts Institute of Technology	13
Bill Zan	University of Texas at Austin	2
Ephrata Zelleke	Goucher College	4
Sharon Zhang	Princeton University	1
Xiaona Zhou	New York City College of Technology, CUNY	15

## Pi Mu Epsilon Speakers

<b>Name</b>	<b>School</b>	<b>PME Session</b>
Emma Armfield	Western New England University	4
Andrew Armstrong	Texas A&M University	1
Jordan Armstrong	United States Air Force Academy	6
Aaron Arnold	Ashland University	8
Meredith Bomers	Hope College	3
Jasmine Brown	Clark Atlanta University	4
Elda Castellon	Saint Peters University	5
Jacquelyn Chapman	Youngstown State University	5
Noah Chicoine	Clarkson University	3
Harris Cobb	Texas A&M University	6
Haley Colgate	Colorado College	7
Maria Cummings	Randolph-Macon College	2
Amanda Cusimano	Xavier University	8
Niyousha Davachi	University of Texas at Arlington	3
Logun DeLeon	Grand Valley State University	5
Keller Dellinger	Lewis University	8
Anthony Dickson	Youngstown State University	7
Kelly Driskell	The University of Tampa	2
Michael English	Clark Atlanta University	7
Jonathan Feigert	Youngstown State University	7
Melanie Ferreri	Wake Forest University	7
Grace Feterl	Augustana University	3
Joshua Forkin	Grand Valley State University	4
Jose Garcia	Grand Valley State University	5
Haile Gilroy	McNeese State University	4
Martha Hartt	Randolph-Macon College	2
Jacob Hines	Hendrix College	2
Yechan Hwang	Hope College	5
Michael Khaimraj	SUNY Fredonia	6
Sam Kottler	Colorado College	2
Nicole Kratz	Augustana University	3
Jackson Krebsbach	Hope College	1
Eric Leu	Hope College	1

### Pi Mu Epsilon Speakers (Continued)

<b>Name</b>	<b>School</b>	<b>PME Session</b>
Xiaomin Li	University of Illinois at Urbana-Champaign	2
Dane Linsky	Hope College	1
Maverick Maynard	South Dakota State University	4
Shannon Miller	Youngstown State University	7
Benjamin Mudrak	Kent State University	5
Janelle Nelson	Howard University	1
Chi Nguyen	Texas A&M University	5
Megan Ott	SUNY Fredonia	4
Marco Pettinato	Lewis University	4
Daniel Plummer	Howard University	5
Jessica Pomplun	Saint Norbert College	4
Lorenzo Riva	Creighton University	3
Kate Sanders	Hendrix College	5
Joshua Schill	Saint Norbert College	4
Aidan Schumann	University of Puget Sound	6
Katrina Teunis	Grand Valley State University	4
Ly Linh Tran	Berea College	8
Bao Van	Saint Norbert College	1
Patrick Ward	Illinois Wesleyan University	8
Ryan Wartenberg	Washington College	4
Jerod Weber	Northern Kentucky University	3
Isaac Weiss	College of Wooster	7
Maia Wichman	Grand Valley State University	8
Caroline Wick	Pepperdine University	1
Angelo Williams	The College of Wooster	5
Page Wilson	Grand Valley State University	8
Everett Yang	Texas A&M University	5
Justin Young	Samford University	2

Room 264

8:30A.M. – 10:45A.M.

8:30–8:45

**Inferring the potentially complex genetic architectures of adaptation, sexual dimorphism and genotype by environment interactions by partitioning of mean phenotypes**

Andrew Armstrong

*Texas A&M University*

Genetic architecture fundamentally affects the way that traits evolve. However, the mapping of genotype to phenotype includes complex interactions with the environment or even the sex of an organism that can modulate the expressed phenotype. Linecross analysis is a powerful quantitative genetics method to infer genetic architecture by analysing the mean phenotype value of two diverged strains and a series of subsequent crosses and backcrosses. However, it has been difficult to account for complex interactions with the environment or sex within this framework. We have developed extensions to linecross analysis that allow for gene by environment and gene by sex interactions. Using extensive simulation studies and reanalysis of empirical data, we show that our approach can account for both unintended environmental variation when crosses cannot be reared in a common garden and can be used to test for the presence of gene by environment or gene by sex interactions. In analyses that fail to account for environmental variation between crosses, we find that line-cross analysis has low power and high false-positive rates. However, we illustrate that accounting for environmental variation allows for the inference of adaptive divergence, and that accounting for sex differences in phenotypes allows practitioners to infer the genetic architecture of sexual dimorphism.

8:50–9:05

**Mathematically Modeling the Persistence of California Red-Legged Frogs**

Caroline Wick

*Pepperdine University*

The California red-legged frog (CRLF) is classified as ‘threatened’ by U.S. Fish and Wildlife with factors such as predation by invasive species, drought, and habitat loss contributing to their decline. Some new populations have been established through reintroductions and natural dispersals. One dispersal in the Santa Monica Mountains led to CRLF washing downstream and reproducing where invasive red swamp crayfish have previously existed but have been removed via intensive crayfish trapping. This trapping may have been critical in enabling the dispersed CRLFs to establish in this stream site. We create a discrete stage-structured model that describes CRLF life history dynamics and includes density-dependent migration. The model is validated against CRLF field data. We expand the model to consider crayfish predation on the aquatic life stages of CRLF and incorporate invasive crayfish trapping. Using equilibrium and sensitivity analysis as well as numerical simulations, we predict local persistence of CRLF and investigate which crayfish trapping regimes most benefit CRLF persistence. Modeling results inform CRLF conservation efforts.

9:10–9:25

**Modeling the Population Dynamics of Diseased Fish**

Bao Van

*Saint Norbert College*

Columnaris Disease is a fatal disease affecting most freshwater fish and fish-farming communities. *Flavobacterium columnare* is the disease-causing agent. Fish become infected through contact with this agent in water. *Flavobacterium columnare* forms into biofilms on the body of fish, affecting respiratory process and motility. As the biofilm grows, it sheds off the host into the water, which results in continued transmission of the disease. *Flavobacterium columnare* is saprophytic, meaning that it does not need a host to survive. Moreover, the biofilm continues to grow on deceased fish. Columnaris disease is unconventional because the deceased members of the population infect the population's healthy members. The conventional SIR models do not capture this phenomenon. In this talk, we present a population model in which the deceased population cannot be removed, or ignored, when predicting infectious transmission. In this talk, we intersect biology, mathematical modeling, and scientific computation to build accurate predictions of the population dynamics of fish infected by Columnaris Disease and compare the results to field data from a fish-farming community.

9:30–9:45

**The Statistical Physics of Intrinsically Disordered Proteins (IDPs)**

Janelle Nelson

*Howard University*

Intrinsically Disordered Proteins (IDPs) are proteins that have no unique folded state but nevertheless perform many important functions inside cells. In particular, IDPs drive phase separation in membrane-less organelles, which store and process biomolecules. Such phase separation by IDPs depends on their thermodynamic properties, which are governed by an individual proteins sequence of amino-acid residues. Stretches of amino acids comprise domains that can fold and interact by forming bonds. For this reason, we created model sequences of these interacting domains to compare their conformational statistics using a generalization of the self-avoiding walk. The conformation of a protein without bonds can be mapped to a self-avoiding walk (SAW) — a sequence of moves on a lattice of vertices that does not visit the same vertex more than once. We used a Recursive Algorithm to enumerate all random walks in which a walk can visit a vertex twice, corresponding to a bond. The purpose of our project is to understand the statistical properties of the spatial conformations adopted by distinct sequences. For each sequence, each random walk has an energy depending on the number of bonds. Our hypothesis is that as the entropy increases, the free energy of a sequence linearly decreases. Using our enumerated states, we calculated thermodynamic quantities like the entropy and free energy to better understand the relationship between sequences, individual protein properties, and phase separation. These quantities will help us to further understand how IDPs drive phase separation to form membrane-less organelles.

9:50–10:05

**A Model of Animal Movement with an Absorbing Interface**

Dane Linsky

*Hope College*

Understanding the impacts of habitat fragmentation on animal populations is an important topic in ecology. Conditions change, sometimes abruptly, at the interface between two patches of different habitats. This can have impacts on the movement choices made by individuals encountering an interface. Our focus was to study how the decision of individuals to rest at an interface impacts the distribution of the population within a one-dimensional landscape. Beginning with a random walk model, we derived an approximating partial differential equation, a diffusion equation model, and boundary conditions that account for the change in behavior at the interface. We numerically solved the diffusion model and showed that it was able to accurately approximate the dispersal patterns resulting from corresponding random walk simulations. Future work will focus on extending the problem to multiple patches with interfaces in between and introducing additional interface behaviors.

10:10–10:25

**Remote Identification of Cloud Forest Landslides: A Machine Learning Approach**

Eric Leu

*Hope College*

Landslides are a major disturbance mechanism in montane tropical rainforests. By opening gaps in the tree canopy and disturbing surface sediment, they play a vital role in maintaining ecological diversity by creating habitat patches that are colonized by pioneer plants. Often triggered by weather or seismic events with a close association to external topographic variables, landslides are often visible in high resolution satellite imagery. We have developed a technique using a supervised machine learning algorithm (Random Forest) to automatically detect landslides and other erosional features in satellite imagery of the cloud forest near Monteverde, Costa Rica. Combining pixel-based spectral information with texture measures and topographic variables (e.g., slope), the algorithm classifies pixels into distinct classes for the separate identification of landslides as well as other features. We used our classifier to identify landslides before and after a major rain event (Hurricane Nate) in order to analyze the impacts of the storm. For future work we will explore the role of weather and other geological variables in determining spatiotemporal patterns of landslide formation.

10:30–10:45

**Dunes and Drones: A Machine Learning Approach to Mapping Dune Vegetation Using Small Unmanned Aerial Systems and Ground-Based Photography**

Jackson Krebsbach

*Hope College*

Sand dune mobility plays a crucial role in maintaining ecological diversity in coastal regions, however migrating dunes may also put human structures at risk. Surface changes in coastal dunes are primarily mitigated by vegetation. Time series of vegetation density maps can assist scientists and resource managers in understanding and managing the factors that influence dune mobility. Small unmanned aerial systems (sUAS) are rapidly emerging as a platform for low-cost, high-resolution aerial surveys. Our goal is to develop techniques to map dune vegetation using sUAS imagery. Our method starts with close-up, ground-based photographs of small patches with varying amounts of vegetation coverage. In these images, which have a much higher ground sampling resolution than the sUAS imagery, pixels are classified using a machine learning algorithm (random forest). The same patches are identified in the sUAS imagery, and vegetation coverage estimates from the classified images are used as a basis for processing the lower resolution imagery. Finally, a vegetation density map for the entire complex is created by stitching the sUAS imagery.

Room 234

8:30A.M. – 10:45A.M.

8:30–8:45

**Minimum distance of locally recoverable codes from algebraic curves**

Sam Kottler

*Colorado College*

An error correcting code, or simply code, is a set of  $n$ -tuples, called codewords. Often, codes are vector spaces over the set of available symbols called the alphabet. Codes can be used for redundancy and error correction when storing or transferring data. There are many ways of measuring how good a code is. One of these, called minimum distance, measures how many errors a code can detect or correct. One specific type of code, in which any position of a codeword can be recovered from a fixed subset of other positions, called a recovery set, is called a locally recoverable code (LRC). An interesting problem with LRCs is called the availability problem, which addresses constructing LRCs with multiple disjoint recovery sets for each position. This talk discusses recent constructions of LRCs from algebraic curves and presents new results improving the bounds on minimum distance of these codes.

8:50–9:05

**Monoid Actions and the Yoneda Lemma**

Jacob Hines

*Hendrix College*

Monoids are one of the most basic algebraic structures, and like all algebraic objects, they have the ability to act on sets via functions called monoid actions. These actions, together with special functions between sets being acted on, called equivariant maps, form a category which we call M-Set. In this presentation, we will uncover a surprising property of this category by carefully applying one of the most important results in the field of category theory.

9:10–9:25

**Exploring special cases of an identity connecting theta series of discriminants  $\Delta$  to  $\Delta p^2$** 

Justin Young

*Samford University*

In the 2016 paper, *On a generalized identity connecting theta series associated with discriminants  $\Delta$  and  $\Delta p^2$* , the author gives a formula which relates theta series of binary quadratic forms with discriminants  $\Delta$  and  $\Delta p^2$ . We find all cases where this identity yields a formula for a theta series corresponding to a single binary quadratic form. From here, we seek to find the corresponding Lambert series and product representation formulas. From these, we can determine the coefficients for each term in the theta series, which is equivalent to finding the total number of representations of  $n$  by the corresponding form.



9:30–9:45

**Almost Beatty Partitions and Optimal Scheduling Problems**

Xiaomin Li

*University of Illinois at Urbana-Champaign*

A Beatty sequence is a sequence of the form  $\{[\alpha n]\}$ , where  $\alpha$  is an irrational number and the bracket denotes the floor function. A remarkable result, called Beatty's Theorem, says that if  $\alpha$  and  $\beta$  are irrational numbers such that  $\frac{1}{\alpha} + \frac{1}{\beta} = 1$ , then the associated Beatty sequences partition the natural numbers. That is, every natural number belongs to exactly one of these two sequences. It is known that Beatty's Theorem does not extend directly to partitions into three or more sets, and finding appropriate analogs of Beatty's Theorem for such partitions is an interesting and wide-open problem, which has applications to optimal scheduling problems. In our research, we introduce the concept of an "almost Beatty sequence" and we construct partitions of the natural numbers into three such almost Beatty sequences that are in some sense closest to actual Beatty sequences.

9:50–10:05

**A Proof of Bertrand's Postulate**

Martha Hartt

*Randolph-Macon College*

In this presentation we prove that there is a prime number between  $n$  and  $2n$  for all natural numbers. This proof is done in a style similar to that of Erdős. However, this proof differs in that it proves the main result for all numbers greater than 32. We will first introduce and prove several lemmas integral to the proof of the main result, then it will prove the main result by means of contradiction. This work was completed as a part of the 2018 SURF program at Randolph-Macon College.

10:10–10:25

**Investigations into the Discrete Arithmetic-Geometric Mean**

Maria Cummings

*Randolph-Macon College*

In this presentation we will discuss the development and derivation of the discrete Arithmetic-Geometric Mean. The continuous Arithmetic-Geometric Mean converges for any two positive real numbers, and its properties extend similarly to negative real numbers and complex numbers. We investigate the extension of the Arithmetic-Geometric Mean to discrete sets of numbers. We first define arithmetic and geometric means modulo  $n$ , with  $n$  belonging to the integers, for discrete sets by applying parallels from the continuous and complex Arithmetic-Geometric Means. Then, we analyze patterns and properties shared with the continuous Arithmetic-Geometric Mean and those independent of its continuous counterpart. Lastly, we pose several questions and conjectures concerning the discrete Arithmetic-Geometric Mean for further research and development on the topic.

10:30–10:45

**Optimal Coherent Partitions: Finding the Balance Between Quantity and Quality of Clusters**

Kelly Driskell

*The University of Tampa*

This study is motivated by clustering, which is a crucial problem related to the organization of social networks and protein networks. To find what makes a “good” cluster, we study networks (graphs) and specific partitions on networks called coherent partitions. A coherent partition is a partition which yields only disconnected subgraphs in the complement. The optimal partition is a partition with the minimum edge cut. Specifically, our goal of this study is to show that any partition with a minimum edge cut (optimal partition) of a graph will always have the same number of components.

Room 236  
8:30–8:45

8:30A.M. – 10:45A.M.

### **Modeling Clarkson University's Electric Energy Purchasing**

Noah Chicoine  
*Clarkson University*

Clarkson University is currently in a contract to purchase all its needed energy from an Energy Supply Company (ESCO) called Constellation. The contract expires at the conclusion of June 2019, allowing Clarkson to purchase power from alternative sources. While Clarkson has been locked into this contract, they have been acquiring credit that can be used to pay for National Grid delivery and commodity. This report examines the projected outcome of three energy-spending strategies Clarkson could adopt in July 2019. We consider plans of action that utilize the university's credit towards National Grid while keeping in mind Clarkson's goal of being 100% renewable by 2025. Our results show that maintaining their current energy contract with Robert Moses Dam would make Clarkson 23% more renewable while limiting the increase of cost compared to buying commodity strictly from National Grid. Based on how little energy from National Grid is produced with renewable resources, Clarkson will have to transition to only buying as much commodity from National Grid as they supply in order to become completely green, forcing them to look for another renewable energy source.

8:50–9:05

### **Developing a General Compton Scattering Cross-Section in Strong Magnetic Fields**

Meredith Bomers  
*Hope College*

Various X-ray space telescopes have detected steady soft X-ray emission originating from highly magnetized neutron stars known as magnetars. Within their magnetospheres, accelerated electrons interact with and boost X-ray photons through the quantum electrodynamic (QED) process, Compton scattering. This is preferred to produce the high-energy tails observed in magnetar thermal X-ray spectra. Through the implementation of Sokolov & Ternov (S&T) spin states, there exist analytic expressions for the spin-dependent lifetimes of excited-state particles in magnetospheres, required to determine the spin-dependent Compton scattering cross section. We propose the development of correct, spin-dependent compact analytic expressions for the Compton cross section to eventually be used in Monte Carlo simulations of magnetars' X-ray emission. With these expressions, we will graphically analyze the specific effects of initial and final spin states as well as the role of polarization in photon intensities. This will allow for more accurate and efficient Monte Carlo modeling of magnetars and may help in understanding distinct features between highly magnetized and conventional gamma-ray pulsars.

9:10–9:25

### **Equations of Mathematical Physics and Lagrangians**

Niyousha Davachi  
*University of Texas at Arlington*

The action of a physical system is defined by the integral over time of a Lagrangian function. This Lagrangian function is then used to determine the behavior of the system by the principle of least action and calculus of variations. Noether's theorem states that every differentiable symmetry of the action of a physical system has a corresponding conservation law. Given this motivation, we derive Lagrangians and a new class of non-standard Lagrangians for equations of mathematical physics including Bessel, Legendre, and Hermite equations. Furthermore, we discuss a new auxiliary condition that must be amended to the calculus of variations for this new class of non-standard Lagrangians.

9:30–9:45

**Global solutions for generalized MHD- $\alpha$  equations**

Lorenzo Riva

*Creighton University*

The Magneto-Hydrodynamic (MHD) equations are a system of differential equation that govern viscous fluids subject to magnetic fields. Since the MHD equations are hard to solve in their particularity, we prefer to study the generalized Magneto-Hydrodynamic alpha (gMHD- $\alpha$ ) system, a generalization which contains diffusion terms that are Fourier multipliers of the form  $||\xi||^\gamma / g(||\xi||)$ . Pennington obtained global solutions for the gMHD- $\alpha$  system of equations subject to a choice of parameters for the multipliers:  $\gamma_1, \gamma_2 > 1$ , with  $\gamma_1 \geq n/3$  and  $\gamma_1 + \gamma_2 \geq n$  in  $\mathbb{R}^n$ ,  $n \geq 3$ . The aim of this project is to apply similar techniques to obtain global solutions in a different case, namely that of  $\gamma_1, \gamma_2 = 2$  and  $\gamma_3 > 1$ .

9:50–10:05

**What Our Teachers Didn't Tell Us About Inverse Functions**

Jerod Weber

*Northern Kentucky University*

Finding an inverse function is often taught by “switching the  $x$  and  $y$ .” However, this approach leads to misconceptions about inverses, especially when applied to a problem with context. In this session, we will discuss an approach that focuses on the conceptual understanding of inverse functions rather than a rote procedure. To develop this conceptual understanding, teachers should use context to introduce inverse functions and use different representations of inverse functions. In addition to this, instead of “switching the  $x$  and  $y$ ,” students should focus on the inverse process as an “undoing” process and pay close attention to the meaning of the dependent and independent variables. If you have struggled to help students understand inverse functions, I hope this session will give you a different way to think about and address inverse functions in your classroom.

10:10–10:25

**Inspiring Youth in STEM through Math Modeling, Part I**

Nicole Kratz

*Augustana University*

Over the last decade, there has been a strong push to illustrate how teachers can engage students in the math modeling process. Knowing this, we created a math modeling competition that utilizes a variety of mathematical techniques. This talk, the first of two, will present the problem, corresponding lesson plans, implementation of a math modeling competition in various schools, and plans regarding the student assessment. This work was supported by the South Dakota Space Grant Consortium 2019.

10:30–10:45

**Inspiring Youth in STEM through Math Modeling, Part II**

Grace Feterl

*Augustana University*

We will continue our discussion of our math modeling competition which focuses on renewable energy sources. Specifically, our competition asks students to analyze the usage of renewable energy sources for a growing city. This talk, the second of two, will cover the mathematics of our chosen modeling problem, the various difficulties student will encounter, and possible solutions. This work was supported by the South Dakota Space Grant Consortium 2019.

Room 264

2:30P.M. – 6:05P.M.

2:30–2:45

**The Combinatorics of Guessing Games with Two Unknowns**

Katrina Teunis

*Grand Valley State University*

A Guessing Game is a game played by two people: a questioner and responder. The responder will choose two numbers from a set previously agreed upon after which the questioner asks a series of questions of the form “How many of the numbers are in the set  $S$ ?” This continues until the questioner knows the pair of numbers chosen. The smallest number of questions required to guarantee finding the chosen pair from the set  $\{1, 2, 3, \dots, n\}$  is denoted  $k_n$ . In our research we have found  $k_n$  for  $n \leq 11$ . We have also found an upper bound on  $k_n$  for all  $n$ , along with an algorithm for asking the questions. We were also able to identify a worst case scenario for this algorithm.

2:50–3:05

**Guessing games with two unknowns and half-lies**

Joshua Forkin

*Grand Valley State University*

Mathematical games similar to Guess Who, guessing games consist of a “questioner” trying to guess a number in  $I_n = \{1, \dots, n\}$  by asking a “responder” if the number falls within a subset of the numbers in  $I_n$ . In this presentation, we look at games in which the questioner is trying to find two unknown numbers in  $I_n$ . In such games, the responder replies with a number corresponding to how many of the unknown numbers are in the subset of numbers guessed. We also introduce the concept of a half-lying in these games. To half-lie, the responder says a number that is one greater than the truthful answer. We present several different strategies and prove the number of guesses they require to assure that the guesser can find the two unknowns and locate the half-lie, if one was used.

3:10–3:25

**Cryptosystems and Modifying the Hill Cipher**

Emma Armfield

*Western New England University*

The Hill Cipher is a block cipher based on a matrix transformation. However, it is vulnerable to cryptanalysis. We will explore a possible modification to the Hill Cipher using circulant matrices and groups of invertible matrices. An  $n \times n$  circulant matrix is formed by rotating each element one to the right in each succeeding row. We will discuss the benefits of using circulant matrices in conjunction with the Hill Cipher. Time permitting, the relationship between circulant matrices and permutation matrices will also be explored.

3:30–3:45

**Constructing Steiner Triple Systems**

Haile Gilroy

*McNeese State University*

The object of this talk is to discuss the Bose Construction, a method for constructing Steiner Triple Systems of order  $v \equiv 3 \pmod{6}$ , and give a brief overview of Steiner Triple Systems, Latin squares, and Quasi-groups.

3:50–4:05

**How not to be lazy: reducing procrastination through two-player task modification**

Jessica Pomplun

*Saint Norbert College*

In 2014, Kleinberg and Oren introduced a graph-theoretic model of a time-inconsistent agent planning a strategy to complete a task. This model is able to predict several real world behaviors including procrastination and abandonment of long-range tasks. In this talk, we investigate an extension to the model which allows two actors to modify the task graph during the agents progress towards task completion.

4:10–4:25

**Variation in Procrastinating over Repeated Tasks**

Joshua Schill

*Saint Norbert College*

In 2014, Kleinberg and Oren introduced a graph-theoretic model of a time-inconsistent agent planning a strategy to complete a task. This model is able to predict several real world behaviors including procrastination and abandonment of long-range tasks. In this talk, we investigate the situation where an agent repeats the same task multiple times and updates its willingness to procrastinate based on how poorly it performed the task in the past.

4:30–4:45

**Triangulations and Tamari Lattices**

Ryan Wartenberg

*Washington College*

Tamari lattices are partial orderings on sets of Catalan objects, most commonly realized using Dyck paths, rooted binary trees, or bracketings. Discovered by Dov Tamari in the 1950s, these structures are closely related to geometric objects known as associahedra. In this talk, we explore the properties and additional structure of Tamari lattices on polygon triangulations. In particular, we notice a natural grading that emerges which corresponds nicely to a known projection from the weak right order on Coxeter groups.

4:50–5:05

**A comparative analysis of the triangle functions and the 3<sup>rd</sup> roots of unity**

Jasmine Brown

*Clark Atlanta University*

A circle is defined as a collection of points  $(x, y)$  that are equidistant from a fixed center point. If  $\theta$  is an angle in standard position,  $(x, y)$  is a point on the terminal side of  $\theta$ , and  $r$  is the radius of the circle, then the circular functions are  $\sin \theta = \frac{y}{r}$  and  $\cos \theta = \frac{x}{r}$ . Suppose we inscribe a regular triangle with  $n = 3$  vertices inside of a unit circle where the first vertex is  $(1, 0)$ . Thus, the remaining vertices of the triangle are  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and  $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ . Each pair of adjacent vertices of the triangle are an equidistant apart with a central angle measuring  $\frac{2\pi}{3}$  radians. The elements of the set  $U_n = \{z \in C | z^n = 1\}$  are called the  $n^{\text{th}}$  roots of unity. In this research we write the solutions that satisfy the equation  $z^3 = 1$  and find the piece-wise equations for the triangular functions. The plots of the triangle functions are constructed, and the three roots of unity are annotated. Further analysis of the triangle functions and the solutions of the 3<sup>rd</sup> roots of unity are captured.

5:10–5:25

**Proof of Feuerbach's Theorem Using Complex Numbers**

Maverick Maynard

*South Dakota State University*

Feuerbach's theorem states that the nine-point circle of triangle ABC is tangent to the three excircles of the triangle as well as its incircle. This will be proven by placing the triangle in the complex plane and then using geometric and algebraic properties of complex numbers.

5:30–5:45

**An Exploration Into the Symmetries of Platonic and Archimedean Solids**

Megan Ott

*SUNY Fredonia*

We will explore the symmetries of Platonic and Archimedean solids using symmetry type graphs. This work is a report on a paper written by Jurij Kovi. We will make liberal use of manipulatives to demonstrate the ideas.

5:50–6:05

**Predictive Modeling and Analysis of Softball Using Linear Algebra-based Ranking Systems**

Marco Pettinato

*Lewis University*

The use of predictive modeling in the analysis of sports data is an exciting but challenging task. There are many mathematically inspired sports ranking systems, but the Colley and Massey Methods are among the most elegant and simple. Additionally, one can incorporate their own systems to build on both the Colley and Massey. One way to improve all these methods for ranking and predicting future outcomes is by introducing weights to these systems. We will share the results of a summer research project that is an expansion of previous research in which we created and tested the predictive power of weighted Colley and Massey Methods using data from softball teams in the NCAA Division II Great Lakes Valley Conference.

Room 234  
2:30–2:45

2:30P.M. – 6:05P.M.

### **Bitcoin, Blockchain Technology, and Secure Hash Algorithms**

Daniel Plummer

*Howard University*

In December 2017, Bitcoin reached its highest value at \$17,900 USD. However, since July 2018 Bitcoin has oscillated between \$3,400 USD and \$4,500 USD. As a result, many question whether Bitcoin and blockchain technologies are stable currencies. Still, not surprisingly, others point to the theft of Bitcoin from Japan's Coincheck which is estimated to be a loss of 860 million yen as an indication that not only are cryptocurrencies insecure financial instruments but their insecurity contradicts blockchain technologies' premise that, "Its stability is based on its secure cryptographic algorithms and immutability of the general ledger." This thesis discusses evidence to the contrary. First, we describe the nature and origins of Bitcoin, blockchain technology, and cryptocurrency. Specifically, we examine how the RSA encryption scheme was proposed for electronic cash and is the foundation of today's banking authentication and data integrity. Second, we examine how Bitcoin initially implemented the Secure Hash Algorithm, SHA-1, as its primary hashing function which was later replaced by SHA-256. Accordingly, we suggest that the SHA-1 algorithm is still secure despite Google and CWI Amsterdam finding a collision in the SHA-1 algorithm through massive computing in 2017. We further discuss how SHA-1 is used in digital signatures, most notably the RSA algorithm, which is frequently used in online banking. Next, we suggest new techniques for further strengthening the algorithm as defined by Halevi and Krawczyk (2006) and endorsed by the National Institute of Standards and Technologies (NIST) which contribute to a robust secure system. Specifically, NIST's recommendation for randomizing hash functions possibly significantly increases digital signature security. Lastly, we point out how human error and weak passwords are the main causes of data breaches and cryptocurrency theft. We conclude by examining how Amazon, AWS, and HARA Token are currently implementing blockchain technology for precision agriculture and rice production in Jakarta, Indonesia.

2:50–3:05

### **Simpson's Paradox in KSU Men's Basketball Statistics**

Benjamin Mudrak

*Kent State University*

Simpson's paradox is a phenomenon in statistics in which a trend is present in two or more distinct categories of data but reverses when the two categories are combined. We show some examples of Simpson's paradox in field goal percentages, which are a combination of 2-point shot percentages and 3-point shot percentages, in Kent State's 2017-2018 men's basketball statistics and investigate what hidden variables might be causing it in each case. We also look into the actual probability of Simpson's Paradox occurring in these statistics.

3:10–3:25

### **An Exploration into Modeling the NBA Draft**

Angelo Williams

*The College of Wooster*

Every year, NBA franchises add top basketball prospects to their team through the NBA Draft. These picks often shape the team for years to come. Our research is an exploration into modeling the NBA Draft. We created various models in an attempt to accurately predict which prospects will perform the best in the NBA. Through our analysis, we found that much of the NBA Draft is just luck. However, we also discovered various factors and statistics that are being undervalued when evaluating a prospect's potential.



3:30–3:45

**A General Algorithm for Constrained Robot Motion Planning**

Everett Yang

*Texas A&M University*

In recent years, robotics has become increasingly important. Robots have the potential to automate tasks that are too dangerous or tedious for a human worker. However, current research lacks a general algorithm for planning the motion for such robots given a high level task. As an example, a robot arm may be tasked with picking and placing an object in some known environment. This problem constrains the robot's "hand" to be at different points in space throughout the execution of the task. Such a constrained motion planning problem is intractable with deterministic algorithms and still hard with state-of-the-art probabilistic methods. These algorithms also unrealistically assume that constraints remain static throughout the task. However, most real-world problems involve different sets of constraints at different times. We attempt to fill this gap by introducing a general framework for constrained motion planning through the extension of a recent approach known as reachable volumes. This work was completed as a part of the 2019 REU at University of Illinois Urbana-Champaign.

3:50–4:05

**Exploring Cannibalization and Market-Basket Analysis for a Furniture Company**

Logun DeLeon

*Grand Valley State University*

A local office furniture company approached us with two separate problems to consider. The first dealt with the recent launch of a new chair, which is at a lower cost than most of their other chairs. They are concerned that the introduction of this low cost chair cannibalized some of their other lines of chairs. We approached this by looking at chair sales over time before and after the launch of the new chair, both as an aggregate and individually. The second problem we studied was to determine which of their products are frequently purchased together. To explore this problem, we performed a market-basket analysis, which is an algorithm for determining probabilities that various products appear together in the same order. With this, we were able to determine common groupings, which our partner will use as the basis for pricing incentives.

4:10–4:25

**Analysis on Children's Healing Center Population**

Jose Garcia

*Grand Valley State University*

The Children's Healing Center of Grand Rapids provides a welcoming social environment for immunocompromised children and their families. The center currently serves about 10% of all eligible families in West Michigan, yet has the resources to serve twice as many families. We investigated the following questions: Can we identify characteristics of members who benefit the most from the center? How do we get eligible members in more quickly after getting referred to our center? We looked for trends within the data that can help the organization grow in an efficient way, all while being an effective resource for the community.

4:30–4:45

**An Investigation of Type I Error for the Two Independent Sample  $t$ -Test with Samples Drawn from Different Non-Normal Population Distributions**

Elda Castellon

*Saint Peters University*

One of the assumptions of the two-sample  $t$ -test is that it should only be applied to pairs of samples if both samples were drawn from normal populations or if the samples are sufficiently large. This research explores the probability of the Type I error of a two-stage hypothesis test performed on two samples, both of which were drawn from different populations, such as the uniform distribution, the exponential distribution, the normal distribution, and the Cauchy distribution. The first step of the two-stage hypothesis test is to apply the Shapiro-Wilk test to test for normality of the samples. The second step is to apply the Mann-Whitney test or the two-sample  $t$ -test, depending on the outcome of the first step. The probability of the Type I errors for different pairs of samples is calculated by running simulations in R.

4:50–5:05

**Is College Even Worth It?: Analysis of Students' Major and Their Opinion on College**

Ye Chan Hwang

*Hope College*

It was investigated whether college students thought college was worth it or not. A random sample ( $n = 50$ ) of undergraduate students was surveyed throughout our campus via email. Each participant was asked several questions including their gender, major, and their opinion on whether college is worth it or not. Using the software R, a model which just considers major was found to be the best model (AIC = 67.11, misclassification = 20%). Although we do not know the  $p$ -value, we can predict that students who are STEM majors are more likely to say that college is worth it than those who are not in the STEM major.

5:10–5:25

**Commonality analysis of context variables for students' mathematical interest**

Chi Nguyen

*Texas A&M University*

Students' mathematical interest refers to engagement, curiosity, enjoyment, optimism, and passion in their learning process. Development of interest in mathematics highly correlated to students' academic performance in mathematics as well as Science, Technology, Engineering, and Mathematics (STEM)-related disciplines. For example, students who have strong interest toward mathematics are willing to show high academic achievement in STEM-related subjects and have positive STEM-related career aspiration. Despite there having been many studies indicating students' mathematics interest, there is a lack of research showing comparison of countries. The current study is to analyze contributions of context variables to students' mathematical interest in Hong Kong and the United States (U.S.). In this study, commonality analysis was conducted with four predictors: gender, learning environment, mathematical talent, and future expectation, and an outcome variable: mathematical interest. Data was extracted from the Trends in International Mathematics and Science Study (TIMSS) 2015 data of 8th grade students in Hong Kong ( $n=4,155$ ) and the U.S. ( $n=10,221$ ). The results revealed that the impacts of predictors to students' mathematical interest in Hong Kong was larger than the U.S. students ( $R^2_{HK}=76.1\%$ ,  $R^2_{US}=71.2\%$ ). All four predictors (gender, learning environment, mathematical talent, and future expectation) statistically significantly impacted on students' mathematical interest in Hong Kong, while three predictors (learning environment, mathematical talent, and future expectation) except gender statistically significantly impacted on students' mathematical interest in the U.S.

5:30–5:45

**Understanding Hendrix’s Impact on Mental Health using Recursive Partitioning**

Kate Sanders

*Hendrix College*

In 2017, a Well-Being survey was conducted at Hendrix College. The study had 531 participants, with similar participation levels between each class year. The question of interest for this analysis was, “How do you feel that the campus environment at Hendrix impacts students’ mental and emotional health?” with a scale of  $-3$  (very negatively) to  $+3$  (very positively). The 443 responses to this question were normally distributed. Using the Recursive Partitioning algorithm implemented by the ‘party’ package in R, multivariate classification was performed on 15 other survey responses. Using the resulting Conditional Inference Tree, we observed that when students do not feel that the administration is listening to mental health concerns, they also view the mental health impact of the college as mostly negative. Students that thought that the administration listened to mental health concerns also saw the impact of the college on student mental health as mostly positive; this trend was stronger with freshmen. These results imply the administration should open more channels for discussing mental health concerns on campus.

5:50–6:05

**iFeel: A Sentiment Analyzer for YSU Surveys**

Jacquelyn Chapman

*Youngstown State University*

Sentiment Analysis is becoming more and more useful with our world’s increasing reliance on technology. Having the capability to analyze any given text to determine polarity is desirable to many large companies, such as Amazon, eBay, and Twitter. This study aims to reveal the polarity, whether positive, negative, or neutral, of survey results provided by Youngstown State University Office of Assessments. The survey comments being analyzed come from the National Survey of Student Engagement that YSU sends out to students. Students have the option to leave comments at the conclusion of this survey. Through this research project, a website has been created that has the capability to analyze these comments to determine individual, group, and yearly polarity. This information will be useful not only for YSU, but also for other universities around the United States. Sentiment Analysis is a relatively new field with endless possibilities in today’s world. Evaluating whether comments are positive or negative could revolutionize the way surveys are conducted and how social media outlets are controlled.

Room 236

2:30P.M. – 3:45P.M.

2:30–2:45

**Surface Volumes in 4 Dimensions**

Michael Khaimraj

*SUNY Fredonia*

This talk will derive the 3-dimensional surface area element in 4 dimensions via the ternary cross product, then apply it to various surfaces of revolution in 4 dimensions.

2:50–3:05

**Products of Matrix Exponentials under Given Constraints**

Harris Cobb

*Texas A&M University*

A matrix exponential can be calculated using the Taylor expansion for  $e(x)$ , just as for scalars. That is, for a matrix  $A$ , its matrix exponential is given by:  $e^{At} = \sum_{n=0}^{\infty} (At)^n/n!$ . Our primary goal is to characterize the set,  $S_{A,B}$ , of matrices of the form  $e^{\lambda_1 A} e^{\gamma_2 B} \dots e^{\lambda_n A} e^{\gamma_n B}$  for some  $n \in \mathbb{N}$  and nonnegative terms  $\lambda_i, \gamma_j$  such that  $\sum_i \lambda_i = 1$  and  $\sum_j \gamma_j = 1$  given square matrices  $A$  and  $B$  of the same size with real entries. We prove periodicity results for  $2 \times 2$  matrices with nonreal eigenvalues when we drop the condition that  $\lambda_i, \gamma_j$  be nonnegative, and investigate the dimension and convexity of  $S_{A,B}$  as well as show connectedness (depending on our matrices  $A, B$ ). We give exact formulas for small cases and give results for projective matrices, and matrices with repeated eigenvalues.

3:10–3:25

**Generalized Devil's Staircase: Random Function Iteration**

Jordan Armstrong

*United States Air Force Academy*

In this work we provide a generalization of the classical Cantor Function. One characterization of the Cantor Function is generated by a sequence of real numbers that starts with a seed value and at each step randomly applies one of two different linear functions. The Cantor Function can be defined as the probability that this sequence approaches infinity. In this work we generalize the Cantor Function to instead use a set of any number of linear functions with integer coefficients. We completely describe the resulting probability function and give a full explanation of which intervals of seed values lead to a constant probability function value. Additionally, we share the initial findings of the natural extension of this work to sets of quadratic functions.

3:30–3:45

**Isomorphism Functions: Nonlinear Functions Which Respect Nonlinear Addition**

Aidan Schumann

*University of Puget Sound*

When it comes to linear functions, what makes addition so special? In this talk, I present a method that allows us to treat most analytic functions which pass through the origin as linear with respect to an isomorphism of addition. In other words, we present a method for (locally) decomposing functions as  $f(x) = \phi^{-1}(a \phi(x))$  for some invertible function  $\phi$  and constant  $a$ . In addition to showing some algebraic results with these “isomorphism functions,” I demonstrate a method for finding their Taylor expansions as well as touch on their application to nonlinear differential equations. When it comes to linear functions, what makes addition so special?

Room 264

8:30A.M. – 10:45A.M.

8:30–8:45

**The Group of Modular Chromatic Intervals and Live Application in Music**

Michael English

*Clark Atlanta University*

An octave is a concept in which the frequency proportion of two pitches is equivalent to two. It is considered to be the most consonant, stable interval to the human ear. In Western cultures, music is traditionally based on the division of an octave into 12 equal parts, called semi-tones. The frequency proportions of a designated starting pitch and the pitches that correspond to respective divisions create the many intervals that are of standard use in Western cultures. A key component of composing various musical works with these intervals lies in the arrangement of pitches as they succeed each other or are sounded simultaneously. Consider the pitches in set  $A = \{E, G, F\#, A, G\#, C, F, D, D\#, C\#, B, B^b\}$  with binary operation,  $\#$ , called the composition of intervals. David Wright in his book, *Mathematics and Music* provides evidence that the binary structure  $\langle A, \# \rangle$  is a group, namely the Group of Modular Chromatic Intervals on 12 pitches with the original row selected as the elements of set A. We prove that  $\langle A, \# \rangle$  is a group by showing that it is isomorphic to  $\mathbb{Z}_{12}$ : the integers modulo 12 under addition. Examples of common mathematical patterns that ascribe to the concept of modular intervals of irregular divisions of an octave are introduced and developed by groups,  $\mathbb{Z}_n$ . A composition for the viola constructed from the Triangular Sequence and Modular Chromatic Intervals is played live.

8:50–9:05

**Change of basis matrices in QSym**

Melanie Ferreri

*Wake Forest University*

A symmetric function is a polynomial which is fixed under any permutation of its variables. The ring of symmetric functions, denoted Sym, is well-studied and has connections to research in algebraic geometry and physics. A quasisymmetric function is a polynomial which remains fixed when the variables with exponent zero are shifted among those with nonzero exponents. The ring of quasisymmetric functions is denoted QSym, and contains Sym. Polynomials in these rings can be written in terms of various bases, and two bases for QSym are the fundamentals and the quasisymmetric Schur functions. Haglund et al. (2011) proved a formula for taking a polynomial in QSym from its representation in the quasisymmetric Schurs to its representation in the fundamentals. In this work, we examine the change of basis in the opposite direction, moving from fundamentals to quasisymmetric Schurs.

9:10–9:25

**Presentations of Common Groups**

Jonathan Feigert

*Youngstown State University*

The presentation of a group uses a subset of elements that generate the entire set, called generators, and some equations that show how these generating elements behave within the group, called relations, to demonstrate the structure of the whole group. We will look at some patterns that arise when using the presentation of groups as well as find some similarities between common groups that become clear when using this notation. This talk is suitable for those who have had an introduction to abstract algebra and group theory.

9:30–9:45

**Irreducible Characters of the Symmetric Group**

Shannon Miller

*Youngstown State University*

Representation theory is an area of mathematics that studies group actions on vector spaces. Character theory is a key tool in understanding representations of finite groups. In this talk, we will explore some aspects of character theory. In particular, we will be looking at the irreducible characters of the Symmetric group, and we will look at several tools that will help create and fill in character tables.

9:50–10:05

**On Inverse Semigroups Associated with Markov Subshifts**

Anthony Dickson

*Youngstown State University*

We will look at the inverse semigroups associated with Markov subshifts. In particular, we look for axioms on an inverse semigroup which tell us that the semigroup is the inverse hull of a Markov subshift. This work was completed as a part of the 2019 REU at UT-Tyler.

10:10–10:25

**Measuring Compactness of Legislative Districts**

Isaac Weiss

*College of Wooster*

We shall explore compactness measures and their relationship to United States legislative districts, including their history of success and failures in the Court system. We will consider traditional compactness measures which have been already implemented, such as the Poslby-Popper measure, as well as some convexity measures. This talk will end with some suggested new measures which could survive the Supreme Court of the United States.

10:30–10:45

**Measuring Gerrymandering: Flaws in Traditional Measures**

Haley Colgate

*Colorado College*

In the United States, many geographically based elections, for bodies like the House of Representatives or state legislature, happen by way of partitioning the populace into voting districts that are won by gaining a plurality of votes. Gerrymandering is the process by which those who have power over the map-drawing process engineer voting district maps to diminish the voting power of those they oppose. Historically, gerrymandering was detected by the “eye-ball test” wherein odd looking districts were thrown out. As computers have begun to excel in constructing heavily slanted maps that look fair, mathematical measures have been developed as a counter. In light of the upcoming census, and associated redistricting, I am conducting an analysis of redistricting in Colorado. This talk will discuss the ideas behind ways of measuring gerrymandering, and discuss where they fall short. This work was supported by the Colorado College Summer Student Collaborative Research Fund.

Room 236

8:30A.M. – 10:45A.M.

8:30–8:45

**Stirling Numbers of Sunflower Graphs**

Page Wilson

*Grand Valley State University*

A Stirling number of the second kind is the number of ways to take  $n$  distinct elements from a set and put them into  $k$  subsets, so that the subsets are non-empty and pairwise disjoint. To get the graphical Stirling number for a graph  $G$ , we add the restriction that any two vertices that are adjacent in  $G$  cannot be in the same subset. The traditional Stirling numbers of the second kind are the graphical Stirling number where the graph is empty. We explore Stirling numbers for sunflower graphs, which are powers of paths joined at a single vertex. We use the relationship between the chromatic polynomial of a graph and the Stirling number of a graph to find the Stirling number for a sunflower graph. Using addition-contraction, we find an interesting connection between the chromatic polynomial and the traditional Stirling numbers.

8:50–9:05

**Doubly Chorded Cycles in Graphs**

Maia Wichman

*Grand Valley State University*

In 1963, Corrádi and Hajnal proved that for any positive integer  $k$  if a graph contains at least  $3k$  vertices and has minimum degree at least  $2k$ , then it contains  $k$  disjoint cycles. This result is sharp, meaning there are graphs on at least  $3k$  vertices with a minimum degree of  $2k - 1$  that do not contain  $k$  disjoint cycles. Their work is the motivation behind finding sharp conditions that guarantee the existence of specific structures, such as cycles, chorded cycles, theta graphs, etc. In this talk, we will explore minimum degree conditions that guarantee a specific number of doubly chorded cycles (these are graphs that contain a spanning cycle and at least two additional edges, called chords). In particular, we will discuss our findings on these conditions and how they fit with previous results in this area.

9:10–9:25

**Solving Instant Insanity**

Aaron Arnold

*Ashland University*

Instant Insanity is a puzzle that was created by the Parker Brothers in 1967. The puzzle consists of four stacked cubes with faces each colored one of four colors. A solution of Instant Insanity is reached when all four different colors can be seen when looking at the stack from the front, left, right and back. We will be looking at how to solve Instant Insanity using graph theory.

9:30–9:45

**How to Avoid Mass Murder with Almost 1/3 Probability**

Ly Linh Tran

*Berea College*

One hundred boxes contain the names of 100 prisoners, one name to a box. Each prisoner gets to inspect at most 50 of the boxes, looking for his or her name. Prisoners who have hunted for their names have no further communication with the other prisoners. No prisoner watches another prisoner hunting for their name. Unless every prisoner finds his or her name, all will be executed. A strategy is described that the prisoners can agree on among themselves before the hunting begins that will give them better than  $1 - \ln 2$  probability of escaping death by execution.

9:50–10:05

**On multidecompositions of complete directed graphs into directed graph pairs of orders 3 and 4**

Patrick Ward

*Illinois Wesleyan University*

A directed graph is a way to encode information about directional relationships among objects. A pair of directed graphs is called a directed graph pair of order  $n$  if they are non-isomorphic, neither has isolated vertices and the union of their edges forms the complete symmetric directed graph on  $n$  vertices. In this project, we are interested in decomposing complete directed graphs into directed graph pairs of order 3 or 4.

10:10–10:25

**Using Graph Theory and Programming to Design Optimal Strategies for DNA Self-Assembly**

Keller Dellinger

*Lewis University*

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods developed with concepts from undergraduate-level graph theory have resulted in significantly increased efficiency. We present our results of creating and modifying existing programs which can be used to apply graph theoretical and linear algebra techniques for designing and testing graphs.

10:30–10:45

**3-cyclic bandwidth and 3-cyclic bandwidth critical graphs**

Amanda Cusimano

*Xavier University*

The *graph cyclic bandwidth problem* can be thought of as identifying the vertices of a graph with the vertices of the cycle graph so that the length of the longest edge is minimized. Such a realization of the graph is called a *cyclic embedding* of the graph and the minimum of all possible cyclic embeddings is called the *cyclic bandwidth of the graph*. We say a graph  $G$  is *k-cyclic bandwidth*, if its cyclic bandwidth is  $k$ . Furthermore, if any proper subgraph of  $G$  has a smaller cyclic bandwidth, we say that  $G$  is *k-cyclic-bandwidth-critical*. In this talk, we explore classes of graphs that are 3-cyclic-bandwidth, and identify the 3-cyclic-bandwidth-critical graphs.



Room 210

8:30–8:45

8:30A.M. – 10:45A.M.

**Generalization Theory of Linear Algebra**

Christina Pospisil

*University of Massachusetts Boston*

An algorithm for multiplying and adding matrices regardless of dimensions via an embedding is presented. An equivalent embedding for a general determinant theory is also investigated. In future work there will be applications to physics and other natural sciences be explored in addition to a promising ansatz to define appropriate inverses for non square matrices.

8:50–9:05

**Automorphisms and Characters of Finite Groups**

Leigh Foster and Brittany Bianco

*Metropolitan State University of Denver*

Representations are special functions on groups that allow us to study abstract finite groups using matrices over the complex numbers. Much of the information about representations can be understood by instead studying the trace of the matrices, giving what we call a character. It turns out that various types of automorphisms (like those of the original group itself and those of any field containing the values of the characters) naturally permute the set of characters of the group. We analyze this phenomenon for a finite matrix group called  $Sp(4, q)$  (the “symplectic group of degree 4”) in the context of some important current conjectures in the field of character theory.

9:10–9:25

**An Exploration of Various Matrices Associated with a Given Graph**

Emma Hiatt

*Xavier University*

Given a graph we associate a matrix by assigning to each pair of vertices a value that would correspond to an entry in the matrix. The most well-known example of a matrix associated with a graph is the adjacency matrix, but other matrices associated with a graph, such as the Laplacian and the Distance matrix (for connected graphs) have also been studied. We explore algebraic properties of these various matrices associated with graphs, and explore in more detail the properties of the distance matrix which is not as well-studied as the other two matrices.

9:30–9:45

**What Does the 2-nacci Fractal Sound Like**

Stanley Gao

*McDaniel College*

The 2-nacci Fractal is constructed from a 2-Fibonacci word which is a string of “0”s and “1”s that is made into a curve by a drawing rule. We analyze the vibrational frequencies of the 2-nacci Fractal by finding the eigenvalues,  $\lambda$ , of a Laplacian on the 2-nacci Fractal. We use two methods, a matrix method and a line segment method.

9:50–10:05

**Gröbner Bases of Determinantal Varieties**

Guilherme Martins

*Lawrence University*

A variety is a set of common solutions to a collection of multivariable polynomial equations. These include, for example, parabolas in  $\mathbb{R}^2$ , lines in  $\mathbb{R}^3$ , quadric surfaces in  $\mathbb{R}^3$ , and so on. Since a fixed variety may be defined by many different finite sets of polynomials, distinguished sets of polynomials, called Gröbner bases, allow us to easily determine whether a variety satisfies a given polynomial equation. These Gröbner bases depend on a choice of monomial ordering. In this project, we begin with a set of polynomials defining a natural family of varieties, and ask: under which orderings does this set of polynomials form a Gröbner basis?

10:10–10:25

**A Distance Function for Rings**

Kimball Strong and Erica Barrett

*UC Berkeley and Williams College*

Given a Noetherian ring with exactly one maximal ideal, we define a distance function on the ring that depends on the maximal ideal. We then construct the completion of the ring with respect to this distance function, and we ask what properties the completion shares with the ring. In this talk, we will present results relating properties of a ring and properties of its completion. This work was supported by NSF Grant DMS1659037, Williams College, and Clare Boothe Luce program, and completed as a part of the SMALL REU.

10:30–10:45

**Prime ideals of a Local Ring and Prime Ideals of Its Completion**

Sharon Zhang and Emil Graf

*Princeton University and Williams College*

The relationship between a local ring and its completion with respect to its maximal ideal is not yet well understood. In this talk, we will present results on the relationship between the prime ideals of a local ring and the prime ideals of its completion. Our results show that this relationship can be surprising. This work was supported by NSF Grant DMS1659037, Princeton University, and Williams College.

Room 211

8:30A.M. – 10:45A.M.

8:30–8:45

**Games with Permutation Groups**

Emily Twardy

*College of Saint Benedict*

We study a variety of two-player combinatorial games on permutation groups. One such example is a game on an equilateral triangle with vertices labeled 1, 2, and 3. The game starts with a “marker” on Vertex 1. The players then take turns moving the marker according to the six symmetries of a the triangle. The game ends when all six symmetries have been used exactly once. The first player wins if the marker ends up on Vertex 1. We will present results and winning strategies for a variety of such games.

8:50–9:05

**Allocation Methods for Partially Defined Games**

Ebtihal Abdelaziz

*Goshen College*

A partially defined cooperative game (*PDG*) consists of a set of players  $N = \{1, 2, \dots, n\}$  and a worth function on a collection  $C$  of non-empty subsets of  $N$ , where  $w(S)$  is the value that can be obtained from the cooperation of the coalition  $S$ . The Shapley value and the nucleolus are well-established fair allocation methods for when  $C$  equals all the non-empty subsets of  $N$ . There are fairness properties associated with each method. The nucleolus is rational, efficient, unbiased, subsidy-free, and consistent. Meanwhile, the Shapley value is marginal-monotone, subsidy-free, unbiased, additive, and efficient.

This paper aims to find fair allocation methods for partially defined cooperative games when  $C$  does not equal all the non-empty subsets of  $N$ . The group-rationality, efficiency, weak-additivity, and consistency of the obtained solution methods are studied. Additionally, we examine the possibility (or impossibility) of finding an allocation method for *PDG* that is monotone, group-rational, and consistent.

9:10–9:25

**A Game Theoretic Analysis of the Weighted Voting System used by the RSU 9 School Board**

Amanda Gulley

*Rockhurst University*

The Mt. Blue Regional School District in Maine (RSU 9) currently uses a weighted voting method that is supposed to adhere to the one-person, one-vote policy that comes from the 14th Amendment to the Constitution’s Equal Protection clause. There are 10 towns on this school board with the 2 largest towns having a majority of the directors and votes on the school board. This situation has left many in the small towns feeling as if their voices are not being heard or represented in school board matters. Some members from the smaller towns have tried to take certain actions to rectify this, such as demanding a legal opinion of the voting method in 2015 to a petition being started in 2018 to change the way voting power is distributed on this school board. In this presentation, we will discuss what weighted voting is, and we will also explain the Shapley-Shubik power index. In particular, we will use this power index to analyze the per capita voting power of this school district, the voting power held by each member of the school board, and how bloc voting affects the distribution of voting power for these towns on the school board.

9:30–9:45

**Mathematical Analysis of Random Sample Voting**

Caitlyn Patel and Rachel Roca

*Rollins College and Manhattan College*

Random Sample Voting (RSV) is a recently proposed voting method that aims to produce more representative election outcomes by restricting voting to a randomly selected sample of the electorate. A key claim of advocates of RSV is that smaller groups are more equipped to make deliberative and informed decisions. In this talk, we will test this claim by using mathematical models and simulation to investigate the potential of RSV to yield better election outcomes. This research was conducted as part of the Summer Mathematics REU at Grand Valley State University.

9:50–10:05

**Quantifying interdependence in voter preferences**

Jacob Roth and Bill Zan

*Valparaiso University and University of Texas at Austin*

In referendum elections, voters are often required to cast simultaneous votes on multiple questions or proposals. The separability problem occurs when the proposals in an election are linked in voters' minds, meaning that their preferences on the outcome of one or more proposals depend on the predicted outcomes of other proposals. In this talk, we will consider ways to quantify the level of interdependence within voter preferences, expanding on previous definitions of separability and considering related combinatorial questions. Our results have potential applications to the development of alternative voting methods and other solutions to the separability problem. This research was conducted as part of the Summer Mathematics REU at Grand Valley State University.

10:10–10:25

**Using Spatial Clustering and Optimization Techniques to Draw Congressional Districts**

Nicholas Joyner

*East Tennessee State University*

Gerrymandering is a problem that is frequently discussed in the media and the court system. This gives rise to the question of how one might go about creating fair and balanced congressional districts. In this presentation, we start with compact congressional districts determined explicitly by population using spatial clustering techniques. We then attempt to minimize the map's efficiency gap, a measure of partisan bias, while maintaining compactness, by using optimization techniques.

10:30–10:45

**A mathematical approach to congressional redistricting**

Abby Brickner and Natalie Anderson

*Xavier University*

In this work, we investigate a genetic algorithm approach for creating optimal U.S. Congressional districts. The project quantifies the quality of redistricting plans according to four criteria: *population equality*, that each district contains approximately the same number of constituents; *competitiveness*, that districts produce as many competitive elections as possible; *compactness*, that districts resemble regular shapes; and *fairness*, that districts do not favor a particular political party. With partisan gerrymandering cases in courts across the country and capturing the attention of the public, our work provides an objective approach to measure how gerrymandered district plans are, and aims at designing fairer plans that preserve communities of interest. Perhaps more importantly with this mathematical framework, we are able to gain a deeper understanding of the redistricting process, as well as the impact it has on American society.

Room 235

8:30A.M. – 10:45A.M.

8:30–8:45

**The Last Digits of Infinity (On Tetrations Over Modular Rings)**

William Stowe  
*Augustana College*

A tetration is defined as repeated exponentiation. As an example, 2 tetrated 4 times is  $2^{2^{2^2}} = 2^{16}$ . Tetrated numbers grow rapidly; however, we will see that when tetrating where computations are performed mod  $n$  for some positive integer  $n$ , there is convergent behavior. We will show that, in general, this convergent behavior will always show up.

8:50–9:05

**Generating Pythagorean Triples of Special Forms**

Andrew Schmelzer  
*CSB/SJU*

The Pythagorean Theorem is an ancient piece of mathematical gold, and it still holds fantastic surprises. Looking at integer solutions of the theorem is of number theoretic interest. We will explore ways to generate primitive Pythagorean triples of special forms en masse, constructing infinite lists of triples using Python/SageMath.

9:10–9:25

**Sums of  $k$ -th powers in ramified  $p$ -adic rings**

Luke Shuck and James Benjamin  
*McDaniel College*

Generalization of Waring's Problem - that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number can be written as the sum of at most  $g(k)$   $k$ -th powers - have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We will examine values of  $g(k)$  for certain ramified extensions of  $\mathbf{Z}_p$ , with specific focus on the case where  $p$  divides  $k$ .

9:30–9:45

**Sums of  $k$ -th powers in Quaternion rings**

Blair Boyle  
*McDaniel College*

Generalization of Waring's Problem – that for every natural number  $k$  there exists an integer  $g(k)$  such that every natural number can be written as the sum of at most  $g(k)$   $k$ -th powers – have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We extend current results on  $g(2)$  for certain Quaternion rings with integer coefficients using a theorem of Watson on indefinite quadratic forms.

9:50–10:05

**A Database of Toroidal Belyi Pairs**

Myles Ashitey, Brian Bishop, and Kayla Gibson

*Pomona College and University of Iowa*

A Belyi map  $\beta : \mathbb{P}^1(\mathbb{C}) \rightarrow \mathbb{P}^1(\mathbb{C})$  is a rational function with at most three critical values; we may assume these are  $\{0, 1, \infty\}$ . A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection:  $\beta^{-1}([0, 1]) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$ . Replacing  $\mathbb{P}^1$  with an elliptic curve  $E$ , there is a similar definition of a Belyi map  $\beta : E(\mathbb{C}) \rightarrow \mathbb{P}^1(\mathbb{C})$ . Since  $E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$  is a torus, we call  $(E, \beta)$  a toroidal Belyi pair. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm:  $\beta^{-1}([0, 1]) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$ . This project seeks to create a database of such Belyi pairs, their corresponding Dessins d'Enfant, and their monodromy groups. This work is part of PRiME (Pomona Research in Mathematics Experience, NSF-1560394).

10:10–10:25

**Minimal Discriminants of Rational Elliptic Curves with Specified Isogeny**

Alvaro Cornejo, Marietta Geist, and Kayla Harrison

*University of California at Santa Barbara, Carleton College, and Eckerd College*

By a rational elliptic curve, we mean a projective variety of genus 1 that admits a Weierstrass model of the form  $y^2 = x^2 + Ax + B$  where  $A$  and  $B$  are integers. For a rational elliptic curve  $E$ , there is a unique quantity known as the minimal discriminant which has the property that it is the smallest integer (in absolute value) occurring in the  $\mathbb{Q}$ -isomorphism class of  $E$ . In 1975, Hellegouarch showed that the elliptic curve  $y^2 = x(x+a)(x-b)$  for relatively prime integers  $a$  and  $b$  comes equipped with an easily computable minimal discriminant. Recently, Barrios extended this result to all rational elliptic curves with non-trivial torsion subgroup. This project gives a classification of minimal discriminant for rational elliptic curves that admit an isogeny of degree  $N \geq 2$ . This work is part of PRiME (Pomona Research in Mathematics Experience) with Alvaro Cornejo, Marietta Geist, and Kayla Harrison with assistance by Alex Barrios and Timothy McEldowney. This work was sponsored by the National Science Foundation (DMS-1560394).

10:30–10:45

**The Modified Szpiro Conjecture and Elliptic Curves with Specified Isogeny**

Owen Ekblad, Gabriel Flores, and Abigail Loe

*University of Michigan at Dearborn, Wheaton College, and Carleton College*

Given three positive, relative prime integers  $a, b, c$  such that  $a + b = c$ , it is rare to have the product of the primes dividing them to be smaller than each of the three. In 1985, David Masser and Joseph Oesterlè made this precise through their celebrated “*ABC* Conjecture.” In 1988, Oesterlè showed that the *ABC* conjecture is equivalent to the modified Szpiro conjecture which states that for each  $\epsilon > 0$  there are finitely many rational elliptic curves  $N_E^{6+\epsilon} < \max\{|c_4^3|, c_6^2\}$  where  $N_E$  is the conductor of  $E$  and  $c_4$  and  $c_6$  are the invariants associated to a minimal model of  $E$ . Recently, Barrios showed that for a rational elliptic curve  $E$ , there is an explicit lower bound which depends only on the torsion subgroup of  $E$ . Our project seeks to create databases of rational elliptic curves in order to study the relationship between the modified Szpiro conjecture and elliptic curves with specified isogeny degree. This work is part of PRiME (Pomona Research in Mathematics Experience, NSF-1560394).

Room 251  
8:30–8:45

8:30A.M. – 10:45A.M.

### Monodromy Groups of Toroidal Graphs

Kendall Bowens, Tayler Fernandes Nunez, and Cameron Thomas

*Tuskegee University, Northeastern University, and Morehouse College*

A graph  $\Gamma = (V, E)$  is a pair consisting of vertices  $V$  and edges  $E$ . Such a graph is said to be planar if it can be embedded  $\Gamma \hookrightarrow S^2(\mathbb{R})$  on the sphere such that its edges do not cross. Similarly, such a graph is said to be toroidal if it can be embedded  $\Gamma \hookrightarrow \mathbb{T}^2(\mathbb{R})$  on the torus such that its edges do not cross. This project seeks to determine the monodromy groups of such graphs. We will discuss some of the challenges of determining the structure of these groups, and present visualizations of group actions on the torus. This work is part of PRiME (Pomona Research in Mathematics Experience, NSF-1560394).

8:50–9:05

### Automated Conjecture Making: Domination on Planar Graphs

Jose Garcia

*Grand Valley State University*

A planar graph  $G = (V, E)$  is a graph that can be embedded in the plane, i.e. it can be drawn in the plane so that no edges intersect except at the vertices. A subset  $S$  of vertices in a graph  $G$  is called a dominating set if every vertex  $v \in V$  is either an element of  $S$  or is adjacent to an element of  $S$ . The domination number of a graph  $G$  is the smallest cardinality of a dominating set; we denote the domination number as  $\gamma(G)$ . Automated conjecture making is the process of having a computer generate conjectures. We investigate the domination number of planar graphs with the use of the automated conjecture making.

9:10–9:25

### Inducibility of Directed Graphs

Dalton Burke and Michael Burgher

*University of Colorado Denver*

A graph  $H$  is an induced subgraph of a graph  $G$  if the graph spanned by a subset of  $|V(H)|$  vertices of  $G$  is isomorphic to  $H$ . For a fixed  $H$ , we study the question which graph  $G$  on  $n$  vertices maximizes the number of induced subgraphs isomorphic to  $H$ . As  $n$  grows large, it is not difficult to show that this number scales with  $\binom{n}{|V(H)|}$ , and this sequence of densities converges. The inducibility of a graph  $H$  is the limit of this sequence. This quantity is known only for very few explicit graphs and digraphs. We use Razborov's flag algebra method to study the inducibility of all directed graphs on up to 4 vertices. In most cases, we fully determine the inducibility and the extremal graphs. This work was supported by NSF Grant DMS-1600483 and the EURECA! program at the University of Colorado Denver.

9:30–9:45

**Results on Neighborhood-Prime Labelings of Graphs**

Tim Ablondi

*Centre College*

A graph labeling on the vertices of a graph is an assignment of integers to the vertices that satisfies specified conditions. A neighborhood-prime labeling of a graph  $G$  is one whose vertices can be labeled from 1 through  $|V(G)|$  in such a way that the neighborhood of any given vertex consists of either only one vertex or has relatively prime labels. In this talk, I provide new neighborhood-prime labelings for spider graphs (trees with only one vertex of degree three or more) and bivalent-free trees (trees with no vertices of degree two). Additionally, I will discuss adding bivalent-free trees and particular caterpillars to preexisting neighborhood-prime graphs. Lastly, I will discuss stacked prism graphs and web graphs. This work was funded by the Center for Undergraduate Research in Mathematics and the National Science Foundation (DMS-1722563).

9:50–10:05

**2-Terminal Node Reliability**

Isaac Brown

*Washington State University*

A two-terminal graph is an undirected graph  $G$  with a vertex set  $V(G)$ , edge set  $E(G)$ , and two specified target vertices in  $V(G)$ . If each vertex of such a graph operates independently with the same fixed probability  $p$ , the two-terminal reliability is the probability that there exists a path between the target vertices. A two-terminal graph is uniformly most reliable if its reliability polynomial is greater than or equal to that of all other two-terminal graphs with the same fixed number of vertices,  $n$ , and edges,  $m$ . In this talk, we present specific values of  $n$  and  $m$  for which no uniformly most reliable two-terminal graph exists, as well as values of  $n$  and  $m$  for which there does exist a uniformly most reliable two-terminal graph. This work was completed as a part of the 2019 REU at University of Texas at Tyler.

10:10–10:25

**Graceful Labeling of Pan Flutes**

Alexandria Wheeler, Lotenna Nwobbi, Connor Parrow, and Ephrata Zelleke

*Carthage College, Harvey Mudd College, Hobart and William Smith Colleges, and Goucher College*

In graph theory, a graceful labeling of a graph with  $n$  edges is the labeling of its vertices with some subset of the numbers 0 through  $n$  such that no label is re-used and if each edge is labeled with the absolute value of the difference of its incident vertices, then each edge is assigned a distinct label from 1 to  $n$ , inclusive. The Graceful Labeling Conjecture, which has been unresolved for more than 60 years, proposes that every tree can be gracefully labeled. In this talk we will discuss a method to gracefully label certain types of trees. This work was done as part of the 2019 REU at Hobart and William Smith Colleges.

10:30–10:45

**Functionally Defined Graphs With a Focus on the Fibonacci Sequence**

Andrew Summers

*Youngstown State University*

This presentation will begin with a brief introduction to graphs and the important related graph theory terminology. Functionally defined graphs, or graphs created by mapping vertices onto a number sequence will then be discussed, with a specific evaluation of graphs created using the Fibonacci Sequence and some of their properties.



Room 210

2:30–2:45

2:30P.M. – 4:25P.M.

**No Talk**

2:50–3:05

**Detecting and Identifying Abnormalities in a Battery’s Electrical Waveform as Dendritic Formation**

Andrew Franklin

*Lee University*

Lithium-ion batteries have become a part of our everyday lives. Unfortunately, these batteries also have the tendency to explode without warning. As Lithium returns to the anode of the battery, the metal tends to plate, forming projections called dendrites. When a dendrite has built up over time to pierce the separator, the battery will short-circuit and explode. Because we are unable to see dendrites form within a battery, there is no method to predict when a dendrite will cause a problem. Oak Ridge National Labs provided data from a circuit, allowing for our investigation of the issue. As voltage is applied to the circuit, the flow of electrons appears as a sine wave. However, as capacitors and diodes are added, these pieces can cause the sine wave to appear more complex. By breaking down the waves into digestible components, a step is made towards being able to match the level of functionality exhibited in a lithium-ion battery with dendrite formation; by being able to fully understand normal-functioning battery graphs, it will then be possible to detect abnormalities in graphs as a sign of a dendrite.

3:10–3:25

**Polynomial Inequalities Handled with Logic**

Steven Evans and Ian Ray

*Morehead State University*

For obvious reasons, many topics treated at the undergraduate level rely heavily on the intuition of the student rather than rigor. For example, when asked to determine the interval(s) where a polynomial satisfies a certain inequality, the technique often provided to the student makes perfect sense, because they already have an intuition about continuity. Surprisingly, by applying the rules of logic, it is entirely possible to solve this problem rigorously and with no reference to theorems of calculus. We illustrate how to do exactly this.

3:30–3:45

**Formative Assessments and the “Testing Effect” in a Mathematics Classroom**

Melissa Novak

*The College of New Jersey*

Richard Bjork coined the term, “testing effect,” when it was discovered that recall activities, such as quizzes, have significant long-term effects on memory. Not much research has been done, however, on the “testing effect” in mathematics classes. As a teaching intern in an Introductory Proof Writing course at The College of New Jersey, I was able to conduct some research on the effect of recall activities on long term retention. TCNJ is a primarily residential liberal arts college, and this course is offered for freshman and sophomore math majors. We implemented both in-class post-lecture quizzes and online quizzes in hopes that, due to the ‘testing effect’, this year’s students will do better overall on the same cumulative final exam than last year’s students. In this presentation, I will review how we added frequent formative assessment to the course and explain how the numerous formative assessments were beneficial to both our students and us educators. I will summarize the results by comparing different segments of the final exam results for this year’s and last year’s classes.

3:50–4:05

**The Efficacy of the Flipped Classroom Technique in Undergraduate Mathematics Education:  
A Review of the Research**

Adeli Hutton

*University of Cincinnati*

The flipped classroom technique has recently been a focus of attention for math instructors and pedagogical researchers. Although research on the subject has greatly increased in recent years, it is still debated whether the flipped classroom technique can significantly increase the success of students in math courses. While there have been meta-analyses that consider the effectiveness of the technique in other STEM fields, there has not yet been a systematic review within undergraduate math education. By analyzing the existing research and compiling the quantitative and qualitative data, this paper examines the efficacy of the flipped classroom technique in undergraduate math courses, ranging from Calculus 1 to proof-based courses, in regards to students' performance, perceptions of the technique, and self-efficacy. This presentation will also cover successful implementation methods, highlight the flipped classroom technique's potential for improving retention of members of underrepresented groups in math by increasing their sense of belonging in the field, and discuss the efficacy of the method in proof-based courses in regards to students' acquisition of sociomathematical norms.

4:10–4:25

**Analyzing the Effects of the Recommendations from the National Council of Teachers of Mathematics  
on Maryland High School Mathematics Education**

Alayna Roesener

*Stevenson University*

This talk focuses on our investigation of the key recommendations of the 2018 study, *Catalyzing Change in High School Mathematics*, published by the National Council of Teachers of Mathematics (NCTM). In particular, we explore the effects of this study on Maryland high school mathematics curricula. *Catalyzing Change* outlines a series of four key guidelines for high schools to implement in order to improve mathematics education including the elimination of student and teacher tracking, focusing on teaching a set of "Essential Concepts", improving teaching practices that are consistent with research-informed methods, and offering continuous, four-year mathematics pathways for all students. We examine these guidelines and see how they have informed the development of high school mathematics curricula around the U.S. and more specifically in Maryland school districts near Stevenson University.

Room 211

2:30P.M. – 4:25P.M.

2:30–2:45

**Mixed Volume of Modified Chemical Reaction Network**

Dilruba Sofia

*University of Massachusetts-Dartmouth*

From a chemical reaction network, we get a system of ordinary differential equations (ODE) and some conservation laws, which are all polynomials in the chemical species concentrations. Finding steady states of the system corresponds to solving this polynomial system. The mixed volume of a chemical reaction network is a volume associated to the Minkowski sum of the Newton polytopes (that is, the convex hulls of the exponent vectors) of the polynomials in the system. The mixed volume of a polynomial system gives an upper bound on the number of non-zero complex solutions of the system. For some chemical reaction networks, the actual number of positive real solutions and the mixed volume are equal. Some chemical reaction networks have reversible reactions and intermediate species. We consider what happens to the mixed volume and the actual number of positive real solutions when we add/remove reactions, or when we add/remove a species. This work was supported by NSF Grant DMS-1757872.

2:50–3:05

**Thermal analysis of PV solar panels**

Enmanuel Valdez

*LaGuardia Community College*

Our objective for this project is to analyze how the change in temperature affects the voltage of flat and curved monocrystalline solar panels. We measure the temperature of the surrounding ambience and of the solar panels subject to weather conditions and use tools like LabQuest and thermostat to collect data. We plan to apply Newton's Law of heating and cooling to determine the rate of change of the temperature and how it is affecting the solar panels. All the data collected will be analyzed through MATLAB using statistical and analytical methods.

3:10–3:25

**Prediction of Dendrite Growth in Lithium Ion Batteries**

Blayne Carroll and John Iluno

*Lee University*

The goal of this project is to increase the safety of lithium ion batteries. Charging and discharging batteries causes significant buildup of dendrites on the electrodes. Enough buildup connects the electrodes and causes a short circuit. Under the direction of our contact at ORNL, we study this dendrite formation and attempt to predict this occurrence. Our group used computer software to recreate a circuit given to us by our contact. This model has a graphical output, which depicts the current through the system at any time. This allows us to look for differences between provided experimental and our generated data. Here we tweak different properties of the circuit and observe the behavior. We then plot the data and fit these curves to an equation, in hopes of predicting a failure. It remains indefinite if the experimental and theoretical data will show any notable differences. We hope to use data analysis to observe the result. This would require an in-depth study on the change in impedance as the voltage changes, by way of Fourier transforms. Thus, we predict that the dendrite growth causes the impedance to change.

3:30–3:45

**Modeling Impedance Change in a Simulated Lithium Ion Battery**

Rebekah Petrosky and Rihem Khatri

*Lee University*

The purpose of this project is to increase the safety of lithium ion batteries. The process of repeatedly charging and discharging this type of battery can cause a significant buildup of Li in the form of dendrites on the porous diodes. Enough buildup connects the electrodes, causing a catastrophic short circuit. We cannot directly measure the dendrite formation, but we are working to understand how it affects the batteries' impedance. Our contact at Oak Ridge National Laboratory has provided data from a battery-like circuit in several environments. To produce the data, he applied a potential difference to the circuit and recorded the impedance using Electrochemical Impedance Spectroscopy. Our focus is on statistical analysis of patterns in the data. We have observed how the impedance of the circuit changes with respect to the independent variables of time and potential. We intend to apply this observation to create a model of battery behavior. Our current analysis suggests that we can model the circuit's impedance over time as a combination of sine waves found by performing Fourier Transforms on the data, and the impedance with respect to DC potential with a polynomial.

3:50–4:05

**Quest for the Mathematically Ideal Font Using Principal Component Analysis**

Rachel Wood and Moises Ponce

*Lee University*

Does the perfect font exist? Most of what we read nowadays is on an electronic screen. Whether it be a computer, smart phone, tablet, or other accessories, much of our daily readings are through these mediums. There is quite a bit of variation in existing fonts, and it can be difficult to differentiate between them all and decide which is most readable and aesthetically pleasing. The goal of this project is to identify characteristics of fonts that make them pleasing or effective in various contexts, and then leverage that knowledge to design better fonts. This will be done primarily using the statistical method known as Principal Component Analysis. The applications of the principal component analysis have been used in countless areas of science, such as facial recognition. Our findings not only pave the way of generating both pleasing and effective fonts via this technique but may be of great use for web designers in the products of our technique.

4:10–4:25

**The Human Error in Cyber Security**

Sydney Maibach and Kristina Daniels

*Fairmont State University*

Email phishing is a serious issue not only for large institutions, but individuals as well. A phishing email is created to try to obtain some kind of sensitive information of the recipient. This research project was performed to determine the success or failure of training techniques to prevent phishing attempts. Various email subjects and training types were used to perform data analysis on the training. We also used files on USBs to phish people as well. We will discuss the types of emails, files, and trainings used along with the data analysis techniques used to assess the data. This work was supported by NASA Shared Services Center Grant 80NSSC19K0178.

Room 235

2:30P.M. – 4:25P.M.

2:30–2:45

**No Talk**

2:50–3:05

**Using Deep Learning to Understand Relationship Between Neural Activity and Behavior**

Annabelle Eyler and Joyce Quon

*Hood College and California State University, Los Angeles*

Several recent papers have shown that activity throughout the brain is strongly correlated with spontaneous motions (e.g. Stringer et al Science, 2019). However, few specifics are known about the relationship between brain activity and motion. In order to better understand this relation, we train and use a deep learning algorithm to identify specific body parts of animals, while brain activity is recorded. Then we use statistical techniques to identify patterns in behavior and compare them with patterns in neural activity recorded at the same time. This work was completed at the 2019 SURIEM REU at Michigan State University.

3:10–3:25

**Time Series Analysis of Wave Elections**

James Evans

*University of Wisconsin Stout*

What is a wave election? It is commonly defined as an election in which one party makes major gains. However, there is little consensus on what constitutes major gains, though a wave election could also be thought of as an election in which the electorate voted fundamentally different than preceding elections. In this study we create a quantitative definition using ideas found in time-series analysis. We apply segmentation algorithms to election data to find the moments when these fundamental shifts in the electorate occur and build an objective definition of a wave election by comparing these results with past accepted wave election occurrences.

3:30–3:45

**Increasing Donor Support for the YWCA in Lewiston, Idaho**

Rhiana Fox and Harrison Conner

*Lewis-Clark State College and Washington State University*

Our research is a study of the practices of donors to the YWCA of Lewiston/Clarkson and how information about these activities can be used to increase the amount of donations received by non-government donors. Data included information about whether the donations were non-monetary or monetary, whether the donor was an individual or organization, and how far the donor was from the YWCA. We take a particularly close look at donor retention, and create a concept of donor regularity to aid in studying how the YWCA can have a regular base of donors. We use methods such as principal component analysis, k-means clustering, and c-means clustering to determine how the YWCA should change their behavior to net more regular donors.

3:50–4:05

**Analysis and Visualizations of the Effects of WeBWorK Homework on Student Performance**

Meryem Elbaz and Gabrielle Langston

*New York City College of Technology, CUNY*

We use real data generated by WeBWorK to investigate possible connections between various measures of online participation and student learning. In particular, we analyze how measures of student learning compare between the groups of students using and not using WeBWorK, and whether the distribution of scores indicate that both groups are sampled from the same population. We also analyze how the measures of student learning compare across factors: gender, class meeting hours, and co-requisite sections. Finally, we analyze and visualize WeBWorK data to better understand the correlations between topics covered in WeBWorK and the corresponding topics on the final exams. We implement the data analysis and visualizations using the tidyverse collection of R packages, following the modern data science framework of analyzing, visualizing and learning from data. This project is supported in part by a MSEIP and Title V Grants from the Department of Education.

4:10–4:25

**Analysis of WeBWorK Homework and How It Correlates to the Final Exam and the Course Grade**

Dahiana Jimenez

*New York City College of Technology, CUNY*

Using real student data generated by WeBWorK, our online homework management system, we investigate to what extent WeBWorK success or participation alone correlates to success on the final exam and the course grade. We also analyze WeBWorK data in order to quantify the relationship between topics addressed in WeBWorK and the corresponding topics on the final exams. We implement the data analysis and visualizations using the tidyverse collection of R packages, following the modern data science framework of analyzing, visualizing and learning from data. This project is supported in part by a MSEIP and Title V Grants from the Department of Education.

Room 251  
2:30–2:45

2:30P.M. – 4:25P.M.

No Talk

2:50–3:05

No Talk

3:10–3:25

**The linear and cyclic bandwidth of a graph**

Zachary Lippe and Amy Kucera

*Xavier University*

The *graph bandwidth problem* can be thought of as placing the vertices of a graph at distinct integer points along the  $x$ -axis so that the length of the longest edge is minimized. Such placement is called a *linear embedding* of the graph and the minimum of all possible embeddings is called the *the bandwidth of the graph*. An extension of this problem is to identify the vertices of a graph with the vertices of the cycle graph so that the length of the longest edge is minimized. Such a realization of the graph is called a *cyclic embedding* of the graph and the minimum of all possible cyclic embeddings is called the *the cyclic bandwidth of the graph*. In this talk, we explore properties of linear and cyclic embeddings of certain graphs and their respective bandwidths.

3:30–3:45

**3-cyclic bandwidth and 3-cyclic bandwidth critical graphs**

Beth Root

*Xavier University*

The *graph cyclic bandwidth problem* can be thought of as identifying the vertices of a graph with the vertices of the cycle graph so that the length of the longest edge is minimized. Such a realization of the graph is called a *cyclic embedding* of the graph and the minimum of all possible cyclic embeddings is called the *the cyclic bandwidth of the graph*. We say a graph  $G$  is *k-cyclic bandwidth*, if its cyclic bandwidth is  $k$ . Furthermore, if any proper subgraph of  $G$  has a smaller cyclic bandwidth, we say that  $G$  is *k-cyclic-bandwidth-critical*. In this talk, we explore classes of graphs that are 3-cyclic-bandwidth, and identify the 3-cyclic-bandwidth-critical graphs.

3:50–4:05

**Orbits of Hamiltonian Paths and Cycles in Complete Graphs**

Samuel Herman

*New College of Florida*

We enumerate certain geometric equivalence classes of undirected Hamiltonian paths and cycles in the complete graph defined on the vertices of a regular polygon. Said classes are orbits in the action of either a dihedral group or a cyclic group on the sets of possible Hamiltonian paths and cycles in these graphs. These orbits are enumerated using Burnside's lemma. The technique used also provides an alternative proof of the formulae found by S. W. Golomb and L. R. Welch which give the number of distinct  $n$ -gons on fixed, regularly spaced vertices up to rotation and optionally reflection.

4:10–4:25

**Minimum Degree Condition for Hamiltonian Cycles in Balanced  $k$ -Partite Graphs**

Nicholas Spanier

*Miami University*

Chen, Faudre, Gould, Jacobson, and Lesniak gave a nearly optimal minimum degree condition which guarantees the existence of a Hamiltonian cycle in a balanced  $k$ -partite graph. In many cases however, this degree condition can be slightly improved. We will discuss our recent result which determines the precise minimum degree condition in all cases.



Room 210

4:30P.M. – 6:05P.M.

4:30–4:45

**Statistical Analysis on a survey from inquiry-based learning students**

Sam Carryer

*Ohio University*

During this past semester, I worked as a learning assistant for an inquiry-based learning Calculus I course. At the end of the Fall 2018 and Spring 2019 semester, the students in the class filled out a survey where they could express their opinions on different aspects of the learning model. This summer, I analyzed the survey data, and looked at how certain aspects of the learning model, such as the class structure, the group-based learning, and the learning assistants correlated to how students felt about the model, the class, their success in the course, and their level of preparedness for Calculus 2. The conclusions that I found about the inquiry-based learning model are certainly very interesting and could be applicable to more math courses in the future.

4:50–5:05

**Exploration of the Lessons Learned by Students Attending Peer-led Workshops**

Fahmida Akhter

*NYC College of Technology*

A study was conducted with students attending additional one-hour per week peer-led workshops, associated with their pre-calculus class. The study focuses on the following research questions: Do peer-led workshops help students become better at problem solving? What are the lessons learned from peer-led workshops that can be useful and applicable in future courses? Data was collected through administering surveys to the students in the peer-led workshop. In this talk, I will discuss the results and the conclusions we made after analyzing the data. This project is supported in part by a MSEIP Grant from the Department of Education.

5:10–5:25

**Student Perspective on Enjoyment in Mathematics Classes**

Yasmine Soofi

*NYC College of Technology, CUNY*

The project will focus on students perspective of enjoyment in mathematics. The research questions that will be explored are: 1) Do students enjoy math classes? If so, what aspects of the classes they find enjoyable and why? If not, what aspects they do not enjoy and why? and 2) What do students think would make the math classes more enjoyable? Data was collected by administering surveys to students from two mathematics courses. In this talk, I will discuss the results and the conclusions we made after analyzing the data. This project is supported in part by a MSEIP Grant from the Department of Education.

5:30–5:45

**Data Driven Analysis to Determine Calculus II Success**

Justin Merritt and Marisa Schulz

*South Dakota State University*

Were you afraid to take Calculus II? In this project, we look at data from the math courses Calculus I and Calculus II (collected from South Dakota State University), including attendance, homework, quiz grades, and test grades. We utilize R programming to interpret our results visually using advanced charts and graphs. We will determine how well students will do in Calculus II based on their scores in Calculus I and which variables are more significant in affecting whether or not the student will be successful.

5:50–6:05

**No Talk**

Room 211

4:30P.M. – 6:05P.M.

4:30–4:45

**A Cost-Efficient Anomaly Detection System for Roads using Machine Learning**

Liam Doherty

*Rowan University*

In this project sponsored by PIC Math, we investigate the problem of road quality evaluation using images measured by a low cost Raspberry Pi computer with a camera module. This cost efficient system is installed on a service vehicle that captures the images and detect anomalies such as cracks, potholes, shadows, or lane markings. Then the images are sent to a server for further classification of the anomalies. To detect and classify them, we use image processing and the multi-layered Perceptron machine learning algorithm. We discuss the steps taken to process the images for training of the machine learning algorithm and demonstrate its performance on a set of images provided by TransDAC LLC: Intelligent Transportation System.

4:50–5:05

**Ultrasound Image Super-Resolution Using Convolutional Neural Networks**

Robert Imbrie

*University of Wisconsin-Milwaukee*

Ultrasound imaging is fast, safe, and inexpensive. However, its applications are limited by a trade-off between resolution and depth. When used at a high frequency, the images are high quality but have trouble penetrating tissues. When used at a low frequency, ultrasounds can image further into the body, but lose image quality, which reduces diagnostic accuracy. By improving overall image quality, ultrasounds could be used in a wider variety of settings. We investigated the super-resolution, in other words, taking an image at one resolution and extrapolating what its features would look like at a higher resolution. In order to do this, we attempted to implement a variety of neural networks already described in literature. We present the results of the investigation and indicate potential future directions.

5:10–5:25

**PDE Based Deep Learning for Geometric Image Data**

Ryan Cecil

*Duquesne University*

Image restoration is the process of estimating uncorrupted images from noisy or blurred ones. Recently, image restoration models have been proposed using the deep learning framework of convolutional neural networks. The Trainable Nonlinear Reaction Diffusion (TNRD) model of Chen and Pock is of specific interest because its architecture mimics a mathematically sound reaction-diffusion equation. Preliminary results have shown that denoising image level line curvature data using a TNRD type model and then reconstructing an image with the denoised curvature data yields more accurate results than denoising the image directly. In this work we are studying how this geometric learning based framework can be extended to situations where the image has undergone linear degradations such as blurring and pixel loss. We are also analyzing how other geometric features of interest can be learned using a deep learning architecture similar to TNRD. This work was supported by NSF Grant DMS1821342.

5:30–5:45

**Semantic geometric reconstruction**

Hiromichi Ueda and David White

*Carleton College*

Suppose you have information (e.g. depth images) about an object's shape from different perspectives, and you want to fuse it all together into a single 3d representation of that object. That fusion is geometric reconstruction. If you stir in extra generic knowledge about the makeup of the object or objects (e.g. it's a tree, or it's a building), it becomes semantic geometric reconstruction. In this research, we apply tools from computational geometry, machine learning, and numerical optimization to create 3d representations of circuit boards from collections of depth images obtained via structured light scanning. We discuss unique difficulties that arise in this particular application and how semantic reconstruction helps address them. This research is supported by the Carleton College Towsley Endowment.

5:50–6:05

**Graduate School Rankings**

Jack Stephens

*Santa Clara University*

Ranking algorithms lie at the heart of many large-scale information systems. These information systems vary greatly, spanning from the ranking of sports teams in a league to the problem of determining the most relevant website to show based on a search query. In this research project, we show that popular sports-ranking algorithms, Massey's method and Colley's method, can be used to rank graduate school mathematics programs in the United States. We have collected admission data from nine-hundred and seventeen students in the United States from a popular graduate school website, GradCafe.com. The data was then loaded into a Python program running Massey and Colley's method in order to accurately rank the top 86 mathematics graduate schools in the United States.

Room 235

4:30P.M. – 6:05P.M.

4:30–4:45

**A Mathematical Model of West Nile Virus:  
The Effect of Interaction Between Humans, Mosquitoes, and Birds**

Noelle West

*Dixie State University*

West Nile virus (WNV) is a mosquito-borne virus that circulates among birds but can also affect humans and horses. Migrating birds carry these viruses from one place to another each year. West Nile virus (WNV) has spread rapidly across the continent resulting in numerous human infections and deaths. Several studies suggest that larval mosquito control measures should be taken as early as possible in a season to control the mosquito population size. Also, adult mosquito control measures are necessary to prevent the transmission of WNV from mosquitoes to birds and humans. To better understand the effective strategy for controlling affected larvae mosquito population, we have developed a mathematical model using a system of first order differential equations to investigate the transmission dynamics of WNV in a mosquito-bird-human community. We also incorporated local temperature data in the model to show the impact of birth and growth rates of the mosquitoes and the migration patterns of birds in disease dynamics. Our model could be used by mosquito abatement centers to determine optimal strategies to efficiently control disease outbreaks.

4:50–5:05

**Entropy of  $S$ -graph shifts**

Travis Dillon

*Lawrence University*

Symbolic dynamics studies the properties of certain collections of infinite strings from an alphabet, called shift spaces. Of particular interest is a shift space's entropy, which is a measure of its complexity. One specific class of shift spaces, called  $S$ -gap shifts, is notable for its rich dynamical and combinatorial structure. In this talk, we generalize  $S$ -gap shifts and give a formula for the entropy of these generalized shifts. This talk assumes no prior knowledge of dynamical systems or symbolic dynamics.

5:10–5:25

**Into Power Functions**

Bianka Wang

*Saginaw Valley State University*

Consider the two curves  $x^n$  and  $x^{1/n}$ , where  $n \geq 2$ ,  $n \in \mathbb{N}$ . We manage to explore interesting patterns related to special areas between the two curves, the centroids of areas between and off the two curves, figures associated with these centroids, and the tangent lines of the curves and their relationship with the maximum vertical distance between them inside the interval  $(0, 1)$ .

5:30–5:45

**Weak Galerkin Finite Element Method for Poisson's Equations on Quadrilateral Grids**

Monica Davanzo

*University of Central Arkansas*

Finite Element Method is a popular method for solving partial differential equations due to its capability of handling complex geometries. The general idea of Weak Galerkin Finite Element Method is to define weak functions and their weak derivatives as distributions. An effective code written in the computing environment MATLAB is used to implement Weak Galerkin Finite Element Method and produce an accurate approximation of the solution to any variation of Poisson equations. The method's reliance on the standard triangular mesh is tested by instead using a quadrilateral mesh. The accuracy and flexibility of the method, due to the change in mesh, is observed and recorded.

5:50–6:05

**Connecting the Dots Over and Over... and Over**

Marcella Manivel and Milena Silva

*Carleton College*

Start with a list of points in the plane. Create a piecewise linear curve by drawing line segments connecting consecutive points in this list. Next create a new list of points by sampling from the piecewise linear curve at evenly spaced intervals of arclength. Iterate this process, using the new list of points produced as the starting point of a new iteration. In this research, we study the properties of this strange discrete curve iteration from different viewpoints: first as a fast algorithm for smoothing and resampling data sampled from curves, second as an iteration defining a discrete dynamical system, and finally, as an approximation to a curve flow obtained via continuum limit. Connecting the dots isn't just for kids anymore! This research is supported by the Carleton College Towsley Endowment and the Carleton Summer Science Fellowship with funding from Carleton College S-STEM (NSF 0850318 and 156018) and North Star STEM Alliance, an LSAMP alliance (NSF 1201983).

Room 251

4:30P.M. – 6:05P.M.

4:30–4:45

**An Interactive Shiny App Exploring the Mathematics of the Lights Out Game**

Neil Kamath

*NYC College of Technology*

We explore the Lights Out game played on an  $n$  by  $n$  grid of switchable lights. When the game starts, a random number or a given pattern of these lights is switched on. Pressing any of the lights will toggle it and the adjacent lights. The goal of the game is to switch all the lights off. Mathematically, solving the classical game amounts to solving a linear system of  $n^2$  equations modulo 2. We implement a solver that gives the solution when the game is solvable. One of the main goals of the project is to develop a web-based, interactive Shiny app that implements the Lights Out game in a way that can be played through the app. This project originated from a REUF research project supported by the American Institute of Mathematics (AIM) in San Jose. It is supported in part by a MSEIP Grant from the Department of Education.

4:50–5:05

**Winnability for the Group Labeling “Lights Out” Game on Complete Bipartite Graphs**

Christian Miller

*Grand Valley State University*

The game “Lights Out!” consists of a  $5 \times 5$  grid of buttons that have two states, on and off. Pushing a button will cause the pushed button and all of the buttons adjacent to it to change their states. We can also represent these states with a number label, either 1 if the button is on or a 0 denoting the button is off. The goal of this game is to be able to turn off all the lights, or in other words, to get all of the buttons to have a state of 0. The rules and setup of the game lends itself well to a graph theory representation. For an arbitrary graph, we can play the Lights Out! game on it if we let the vertices of the graph represent buttons in our original game, with the edges connecting those vertices representing the buttons that are adjacent to each other. This talk will be focused on a slightly modified version of the game’s original rules, with the labels for the vertices coming from the group  $Z_n$ . It is not always possible to win the game. We will be investigating the values of  $n$  for which this group labeling “Lights Out!” game is always winnable when played on various families of graphs, including complete bipartite graphs..

5:10–5:25

**Modeling Fan Graphs in Self-Assembling DNA Using Graph Theory and Linear Algebra**

James Sparks and Paul Buldak

*Lewis University*

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods based on graph theory have resulted in significantly increased efficiency. We present the results of applying graph theoretical and linear algebra techniques for constructing fan graphs.

5:30–5:45

**Modeling Crossed-Prism Graphs in Self-Assembling DNA Using Graph Theory and Linear Algebra**

Eric Redmon

*Lewis University*

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods based on graph theory have resulted in significantly increased efficiency. We present the results of applying graph theoretical and linear algebra techniques for constructing crossed-prism graphs. In particular, we explore various design strategies given two laboratory constraints.

5:50–6:05

**No Talk**

Room 210

8:30A.M. – 10:45A.M.

8:30–8:45

**Bipartite Graphs, Edge Coverings, and Line Segments Defined by Sets**

Roman Vasquez and Rachel Wofford

*University of Central Florida and Whitworth University*

The Hausdorff metric provides a way to measure the distance between sets. From this metric we can build a geometry of sets. In this geometry, a finite configuration  $[A, B]$  is a pair of disjoint finite sets  $A$  and  $B$  that satisfy certain distance conditions. The sets  $A$  and  $B$  form the endpoints of a line segment in this geometry of sets. These line segments have many interesting properties. One specific property is that for a given positive integer  $n$ , there is usually a configuration  $[A, B]$  for which there are  $n$  different sets on the line segment defined by  $[A, B]$  at every distance from  $A$ . We denote this number of sets by  $\#([A, B])$ . We will explain how each configuration  $[A, B]$  can be identified with a bipartite graph  $G$ , and why  $\#([A, B])$  is the same as the number of edge coverings of  $G$ . In this way, each bipartite graph  $G$  defines a line segment in the space of sets. We will share results about  $\#(G)$  for different types of bipartite graphs, which then determines the number of points at each distance on the line segment identified with the graph  $G$ . This research was conducted as part of the 2019 REU program at Grand Valley State University.

8:50–9:05

**Circle Packing on Flat Tori with Two Classes of Circles**

Daniel Ralston and Sarah Van

*Bowdoin College and City College of New York*

The study of maximally dense packings of disjoint circles is a problem in Discrete Geometry. The goal is to find the optimal density and corresponding arrangements of circles in various containers. The optimal densities are known for packings of small numbers of equal circles into hard boundary containers, including squares, equilateral triangles and circles. In this presentation, we will explore packings of small numbers of circles onto a boundaryless container called a flat torus and instead of using equal circles, we use two classes of circles each with a different common radius. Using numerous pictures we will introduce all the basic concepts (including the notion of a flat torus, an optimal packing, and the graph of a packing), illustrate some maximally dense arrangements, and outline the proofs of their optimality. This research was conducted as part of the 2019 REU program at Grand Valley State University.

9:10–9:25

**Euclidean Theorems in Spherical and Hyperbolic Geometries**

Dylan O'Connor and Ramon Suris-Rodriguez

*University of North Carolina and Towson University*

Euclidean, spherical and hyperbolic are three geometries with a common thread. We believe that every Euclidean result has an analogous result in the other two. Many of the basic theorems from Euclidean geometry, like the Pythagorean Theorem and the Law of Cosine and Sines have already been generalized to these other geometries and serve as the tools to help generalize other theorems. In this presentation we will explore the generalization of some more advanced results in Euclidean geometry to both spherical and hyperbolic geometry. The basics of spherical and hyperbolic geometry will be explained. This research was conducted as part of the 2019 REU program at Grand Valley State University.



9:30–9:45

**Exploring Symplectic Geometry**

Allen Yang and Isaiah Silaski

*Massachusetts Institute of Technology and Texas State University*

Symplectic geometry is the geometry of even dimensional smooth manifolds equipped with symplectic structures. A symplectic structure is a closed, nondegenerate 2-form. In this talk we present the research project that we have explored this new geometry and its developments. In addition, we discuss the question “When does one region embed symplectically into another?” which has been studied after the time of Gromov’s Nonsqueezing Theorem. We focused on for special cases, in particular, on 4-dimensional smooth manifolds. This research was conducted as part of the 2019 REU program at Grand Valley State University.

9:50–10:05

**Construction of the  $k$ -Fibonacci Word Fractal**

Nicholas Cummings and Lan Mai

*McDaniel College*

The  $k$ -Fibonacci words are strings of “0”s and “1”s defined by a recursive sequence similar to the Fibonacci sequence. The  $k$ -Fibonacci word fractals are typically constructed by a drawing rule from these words. We explore an alternative method of creating these fractals by exploiting their self-similar structure to define iterated function systems that can produce each fractal from copies of itself. We discuss the applications of these iterated function systems for investigating the properties of the  $k$ -Fibonacci word fractals.

10:10–10:25

**Discrete Geometric Invariants for Bone Fragment Refitting**

Riley O’Neill

*University of St. Thomas*

The reassembly of bones from fragments is a tedious and time-consuming task, but is critical to understanding ancient life. To digitize and automate this process, discrete geometric invariants have been prepared for fragment feature detection, classification, face segmentation, and refitting in MATLAB using CT scan data to construct surface triangulations. Four invariants have been examined thus far: cumulative distance histograms, surface area histograms, spherical volume, and principal curvature, the latter two arising from our new analytic method using boundary integrals. Using Dijkstra’s algorithm and the normalized spherical volume invariant and curvatures, edge detection has now been successfully automated for use in break face segmentation and eventual refitting.

10:30–10:45

**Analysis of Closed Space Curves and Applications on DNA Structure**

Ziqi Fu

*University of California, Santa Barbara*

This research focuses on some classical results regarding the analysis of knotted and unknotted curves in  $\mathbb{R}^3$ . The concepts include local, global geometric properties and topological properties of a closed space curve. Together with the tools of differential geometry and knot theory, we attempt to analyze and quantify the entanglements of knotted curves. In studying circular DNA strands, we draw a parallel between these macro-biomolecules and mathematical curves and knots.

Room 211

8:30–8:45

8:30A.M. – 10:45A.M.

**No Talk**

8:50–9:05

**Fun with Fibonacci Numbers: A Number Guessing Game**

Jeffrey Wilkinson and Roman Wong

*Washington and Jefferson College and Washington and Jefferson College*

Every positive integer has a unique representation as a sum of powers of 2's. To guess a number within the range of 1 to  $n$ , the binary number system gives us an automated strategy that does not require instance responses to questions. Zeckendorf Theorem says that every positive integer can be uniquely written as the sum of non-consecutive Fibonacci numbers with  $F_1 = 1$  excluded. This fact allows us an alternate automated strategy in the number guessing game. We explore the pros and cons between the two systems in the number guessing game. It is a surprise that in certain way, Zeckendorf strings are more efficient than binary strings.

9:10–9:25

**Counting Points on Arbitrary Curves Over Prime Power Rings**

Caleb Robelle

*University of Maryland Baltimore County*

Counting points on algebraic curves over finite fields has applications in cryptography, integer factorization, and coding theory. These applications depend on the ability to quickly determine the number of points on an algebraic curve over some finite field, both chosen to have certain properties. Less is known about efficient algorithms for point counting over prime power rings, particularly for singular curves, where Hensel lifting becomes much more complicated and resolution of singularities becomes too costly. We give the first algorithm to count points on algebraic curves over  $\mathbb{Z}/\langle p^k \rangle$  with complexity polynomial in  $k \log(p)$ . We also provide a new, simpler proof of rationality of Igusa zeta functions for algebraic curves. This work was completed as a part of the 2019 REU at Texas A&M University.

9:30–9:45

**Mate Relationship of Row Cyclic and Diagonally Cyclic Latin Squares**

James Waldeck

*Marshall University*

We study the mate relationships of row cyclic latin squares and diagonally cyclic latin squares. In particular, we are interested in characterizing both these families of latin squares into one larger family. In a **diagonally cyclic latin square**, each right diagonal occurs in cyclic order. In a **row cyclic latin square**, each row is in cyclic order. Two squares are **orthogonal mates** if, when the squares are superimposed, every possible ordered pair of entries appears in the superimposed square. When a collection of latin squares are all mates of each other, we say they are “mutually orthogonal.” We have found a characterization of the matehood relation for odd prime power sized row cyclic latin squares by organizing them into a collection of sets of mutually orthogonal squares. We seek to generalize this characterization to include diagonally cyclic squares and other squares.

9:50–10:05

**Extremal Numbrix Puzzles**

Faith Hensley and Ashley Peper

*Marshall University and University of Wisconsin - Stevens Point*

Numbrix is a puzzle in Parade magazine. The player is given a  $9 \times 9$  grid with some integers between 1 and 81 filled in. The player then needs to fill in the rest of the integers between 1 and 81 so that consecutive integers appear in adjacent cells of the grid. Generalizing this puzzle we consider  $m \times n$  grids with the entries being the integers between 1 and  $mn$ . We say that a set of clues defines a puzzle if there exists a unique solution given those clues. In 2018 Hanson and Nash find the maximum number of clues that fail to define an  $m \times n$  puzzle for all  $m$  and  $n$ . We present our work on their conjecture concerning the minimum number of clues necessary to define a puzzle. This research was conducted as part of the 2019 REU program at Grand Valley State University.

10:10–10:25

**P-tableaux and Marked Tableaux: A Combinatorial Bijection**

Samuel Fulton

*Fort Lewis College*

In representation theory and algebraic combinatorics,  $P_{n,2}$ -tableaux and Marked Tableaux are known to enumerate various objects. In his paper on Eulerian numbers, Stembridge examines Marked Tableaux, which count the multiplicity of irreducibles of certain representations. Shareshian and Wachs examine  $P_{n,2}$ -tableaux in their paper on Chromatic Quasisymmetric Functions and establish an indirect bijection between  $P_{n,2}$ -tableaux and Marked Tableaux. We present a direct combinatorial bijection between these sets of tableaux.

10:30–10:45

**Non-attacking Queen and Rook Placements**

Nicholas Layman

*Grand Valley State University*

In 1848, Max Bezel introduced the problem of placing 8 queens on an  $8 \times 8$  chess board such that none of the queens could attack each other. The generalization of this, the placement of  $n$  non-attacking queens on an  $n \times n$  chess board, is the famous  $n$ -queens problem. A different but similar problem is that of placing non-attacking rooks on a generalized chess board which has connections to restricted permutations and has more general solutions known as compared to its queen counterpart. In this talk, we will discuss our results on the intersection of these problems placing  $n$  pieces (either queens or rooks) on an  $n \times n$  board and other similar, non-attacking chess problems. This work was supported by the 2019 McNair Scholars Program at Grand Valley State University.

Room 235  
8:30–8:45

8:30A.M. – 10:45A.M.

### **Survival Analysis for Per Diem Nurse Staffing**

Azad Deihim

*Wentworth Institute of Technology*

This research was conducted over a three-month period by a group of undergraduate students at Wentworth Institute of Technology to explore solutions to a problem proposed by IntelyCare. IntelyCare is a nursing agency that aims to solve the per diem nurse-staffing issue by matching qualified nursing professionals with health care facilities using an on-demand, tech-enabled, health care staffing platform that serve long-term facilities in six different states. The purpose of the proposed project is twofold: to determine how long a health care facility must wait before a per diem nurse shift is filled, and to draw insights that may help healthcare facilities increase the rate at which shifts are filled. The group's approach employed several statistical, machine learning, and survival analysis models and techniques in order to further study the problem. Using these models and techniques, the results show that it is possible to solve the proposed problem with minute error.

8:50–9:05

### **Incremental Linear Discriminant Analysis for Classification**

Kevin Tao

*Boston University*

A common problem in classification is that data arrives in chunks rather than all at once, and classes may not all be available from early stages. We analyze the incremental linear discriminant analysis (ILDA) model that approaches the situation when data is added to an initial feature space in the form of random chunks. We ran simulations on 8 datasets from the UCI repository and compared the ILDA to the traditional batch LDA and incremental PCA (IPCA) in terms of the efficiency and accuracy of discrimination. We discovered that both incremental methods have similar variability in accuracy, and ILDA seems to offer higher accuracy, and tends to select a lower dimensional eigenspace. Finally, simulations on high dimensional data revealed that ILDA is capable of reaching a higher accuracy, yet it suffer from overly-large eigenspace dimension; whereas, IPCA displays a consistent increasing trend in accuracy as its eigenspace dimension increases. This suggests that although ILDA is perhaps a more optimal method, an algorithm for the selection of eigenspace dimension is critical, whereas IPCA makes for a more promising naive approach to classification.

9:10–9:25

**A Simulation Study of Penalized Regression,  
Model Selection and the Application in Economics and Finance**

Sheng Gao

*Samford University*

In this era of big data, the world is keeping generating large-volume and complex data sets all the time. Modern regression models and statistical technologies which include the lasso, elastic net, and ridge regressions provide new methods of analyzing complex data of high dimension. In this paper, we studied the performances of lasso, elastic net, and ridge and compared with the Ordinary Least Squared(OLS) model in various simulation scenarios. Furthermore, we investigated the effects of different regression criteria including Mean Square Error(MSE), Akaike's information criterion (AIC), Bayesian information criterion (BIC) and AIC correction (AICc) in regression. Last but not least, we applied the lasso regression in the analysis of real economic data and compared the results of different criteria, which expands the nascent field of applications of penalized regression.

9:30–9:45

**Computing Data Trends for Auto Industry using Statistical Model**

Brett Havertz, Patrick Dockstader, and Cesar Vasquez

*Dixie State University*

We will present an algorithm for an auto shop industry that is revolutionizing the services available to customers with transmission problems and has been collecting customer data for years to better understand their market and customer needs. They have noticed that certain factors may have a much higher potential of becoming valuable paying clients based on a raw data set. We created a model that predicts and classifies the clients as high, medium, or low potential, based on the historical data using mathematical and statistical modeling techniques. Furthermore, we demonstrated the model utility using the auto shop data to compute the correlation of customers and customer history. The correlations are then used to compute a conditional probability distribution which will be used to predict an expected rating score. Moreover, our results are validated by comparing the predicted ratings with the actual ratings in test cases from the data set. Our results show that the proposed algorithm is fast, simple, and intuitive, which could be utilized by the auto shop industry in the future.

9:50–10:05

**Predicting college coaching salaries**

Hao Wang

*University of Illinois Urbana-Champaign*

It is well known that college coaching salaries can vary dramatically from one school to another and from one coach to another. In our research we seek to determine which factors are the most relevant in explaining these differences. We build regression models to predict coaching salaries from variables such as the win/loss ratios and ratings and rankings of a team, the athletic budget of the school or program, and the experience and salary history of the coach. We compare the predictions by these models with over 1800 data points of salaries of college football and basketball coaches obtained from the USA Today coaching salaries database.

10:10–10:25

**Lottery Validation**

Xiaona Zhou

*New York City College of Technology, CUNY*

The New York Pick 4 lottery consists of four “randomly” chosen digits from 0 to 9. For this to be fair, each digit should be equally likely to occur. To determine whether this is the case, a Chi-squared goodness of fit test will be applied to historical data. This provides a quantitative way of measuring how well the observed frequency of digits matches our expectations of a fair lottery. We also explore the “Lucky Sum”, which is also a part of the Pick 4. We determine which sum is most likely to occur, and what the odds of winning are if you play the sum most likely to occur. We also developed a computer program, which allows us to find the “Lucky Sum” of any pick. Furthermore, we found the equation that fits the curve approximately. This project is supported in part by a MSEIP Grant from the Department of Education.

10:30–10:45

**Random Walks on the Dictionary**

Shraddha Shankar and Addie Buzas

*Denison University*

The English language dictionary, consisting of over 100,000 words, is a common tool for linguists, writers, and Scrabble players everywhere. We take the English language dictionary and study it from a mathematical perspective by phrasing it as a random walk problem. A random walk on the dictionary would consist of a random walk on a graph where nodes represent words that are connected if they share some property. For example, two words could be connected if one word is in the definition of the other. In order to create and study such graphs, we begin by analyzing random walks on a much smaller subset of words. Within these smaller subsets, we define a few different rules to determine which words are connected. One example of this is where words are connected if they share a common letter. We analyze such properties of the random walks as time to stationarity, hitting time, commute time and cover time. These properties provide understanding towards answering questions such as what percentage of words our random walk covers and where we would have to start to cover a maximum number of words. We talk about our progress towards extending these smaller problems to a random walk on the entire English language dictionary. This research was supported by the Anderson Summer Science Program, conducted at Denison University.

Room 251

8:30–8:45

8:30A.M. – 10:45A.M.

**Predictability and Information Entropy of Musical Compositions**

Joshua Lang

*Stevenson University*

Using concepts from information theory such as entropy, we compute the predictability of several musical compositions. Our work is a parallel of the work done by Claude Shannon in 1948 in which he computed the entropy of a written language which measures the average information produced by letters in a text based on letter and n-gram frequencies. We analyze the predictability and information entropy of various musical compositions and generate new musical compositions based on their n-gram note frequencies.

8:50–9:05

**Applications of Error-Correcting Codes**

Luis Perez

*California Lutheran University*

Error-correcting codes (ECC) are used every day for data transmission. ECC, found in coding theory, use methods to handle possible errors that may arise from electronic noise, to the scratch of a CD. ECC can be used in applications from performing magic tricks to detecting and repairing mutations in DNA sequencing. In addition, error-correcting codes, including Hamming Codes and Reed-Muller Codes, can be viewed through set theory, which gives another perspective to how these ECC work. This research examines Hamming Codes where various ways to explain the methodology behind the detection and correction process are discussed. This research also investigates further applications of the Hamming Code in a team competition and magic tricks through a set theoretic approach and find reasoning as to how these applications are guaranteed to work given properties of the Hamming Code. Moreover, I aim to expand on the applications of the Hamming Code to show its true significance as a perfect ECC.

9:10–9:25

**Solving a Flexible Job Shop Scheduling Problem with Sequence-dependent Setup Times  
Using Parallel Tabu Search and a Genetic Algorithm**

Matthew McFadden and Anthony Valdez

*University of Wisconsin - Milwaukee*

Job shop scheduling problems (JSSP) are among the most intensive NP-hard combinatorial problems with real-world applications. The flexible job shop problem (FJSP) is a generalization of the classical JSSP where each operation can be processed by more than one machine. The FJSP covers two difficulties, namely, machine assignment and operation sequencing. We address the flexible job shop scheduling problem with sequence-dependent setup times under minimization of the total completion time (makespan). Investigation of this problem was risen to address operational efficiency at a local (Milwaukee area) printing company. To solve this problem, we propose a hybrid meta-heuristic method that uses a combination of a genetic algorithm (GA) and parallel tabu search (TS). To evaluate the performance of our algorithm, we compare our results with other methods existing in the literature.

9:30–9:45

**A Trust Model in Bootstrap Percolation**

Rinni Bhansali

*Stanford University*

Bootstrap percolation is a class of monotone cellular automata describing an activation process with certain activation rules. Particularly, in the classical  $r$ -neighbor bootstrap process on a graph  $G$ , a set  $A$  of initially infected vertices spreads by infecting vertices with at least  $r$  already-infected neighbors. Motivated by the study of social interactions on graphs, where vertices represent people and edges represent the relations amongst them, we introduce a novel model named  $T$ -Bootstrap percolation ( $T$ -BP). Here, vertices of the graph  $G$  are given random labels, and the set of initially infected vertices spreads by infecting vertices with at least  $k_i$  already-infected neighbors of each label  $i$ . The Trust Model for Bootstrap Percolation allows one to impose skepticism towards a rumor, as it requires validation by numerous groups in order for the rumor to spread. We describe various properties of this new model (e.g., the critical probability of infection) on several networks. Ultimately, we describe its implications when applied to rumor spread, fake news, and marketing strategies, along with potential future applications in modeling the spread of genetic diseases.

9:50–10:05

**Optimal Control applied to Cancer Vaccine Protocol**

Brady Fritz

*Grand Valley State University*

In our presentation, we will discuss a mathematical model for the administration of a cancer vaccine. The model involves time delays. We will be using software that specializes in optimal control to achieve numerical solutions for optimal administration. We provide analysis of the solutions obtained and build a better understanding of what they signify.



10:10–10:25

**Mathematical modeling of fish populations using flow-kick dynamics**

Andrea Doty

*College of Saint Benedict*

Dynamic modeling of population growth and decay is essential to management of ecosystems. With various changes to the environment affecting birth and death rates of freshwater fish, environmental agencies like the DNR look to predict these alterations to the population via differential equations and mathematical models. In recent years, a new development in mathematics known as flow-kick dynamics has emerged, providing a sense of consistent population flow with drastic kicks from effects like climate change, natural disasters, and licensure dates for harvesting. Here, flow-kick dynamics are used to model the population dynamics of lake fish that are seasonally harvested. Many models exist presently to predict harvesting at constant rates, or time-dependent rates. However, the model exhibiting this flow-kick dynamical shift would provide a more precise depiction of patterns in the current ecosystem, while accurately predicting population increases and decreases in the years to come.

10:30–10:45

**Modeling Population Distributions and Spatial-Temporal Patterns  
of Animal Groups with Producer-Scrounger Behavior**

Caira Anderson and Issa Susa

*Smith College*

Among many animal species, groups form for protection, hunting, and foraging. Patterns emerge from social interactions between the different animals within these groups. In animal groups, some individuals, known as “producers”, search for food (prey) on their own, while others, known as “scroungers”, exploit the producers. This “producer-scrounger” behavior is common in patchy environments where resources are limited. This interaction results in scroungers not receiving as much food as the producers. We will build a mathematical model consisting of partial differential equations that tracks the population sizes of the producers, scroungers, and their prey. We will study the long-term population distributions due to birth, death, and competition, as well as spatial-temporal patterns that emerge from the model.

**Room 210****2:30P.M. – 4:25P.M.**

2:30–2:45

**From human speech to abstract topology: Persistent homology for natural language processor**

Duong Ngo

*Liberty University*

The persistent homology approach formulated from the tools in algebraic topology had provided the field of topological analysis with an invaluable tool for computational methods. This talk would explore the possibility of applying persistent homology to speech analysis through witness complexes construction and delayed embedding. In other words, here I examine the possibility of converting dynamical time-series into  $\mathbb{R}^n$  dimensional space for complexes construction and provide a few algorithms for Betti number calculation.

2:50–3:05

**Merge trees in discrete Morse theory**

Benjamin Johnson

*Ursinus College*

The field of topological data analysis (TDA) seeks to use techniques in topology to study large data sets. The hope is that rather than single quantities that summarize the data, such as mean or standard deviation, information about the data can be learned by studying the overall “shape” of the data. One way to summarize this data is through a merge tree. Merge trees can be thought of as keeping track of certain clusters of data and determining when they merge together. In this talk, we will study merge trees induced by a discrete Morse function on a tree. We will investigate to what extent the original graph can be reconstructed by studying its set of merge trees and compute collections of merge trees given a fixed graph. Depending on the summer's progress, we may begin to look at distances between merge trees, moving towards the question of stability.

3:10–3:25

**Topological evolution of the Game of Life**

Iancarlo Espinosa-García

*Universidad de Guanajuato*

As a new proposal on the study of Cellular Automata, we analyze how the homology of a family of two-dimensional structures in the regular square tessellation of the plane change in time under the rules given by the original Conway's Game of Life. In particular, we study how the rank of the first and the second homology groups evolve in time when the initial structure are elements of two families of shapes of polyominoes. Polyominoes are formed by gluing together by edges finitely many congruent square tiles. The two families of polyominoes that we studied are square polyominoes ( $n$  by  $n$  polyominoes) and worm like polyominoes (polyominoes that have one row and  $n$  columns). Using computational experimentation, we study the possibility of using it as a tool to find oscillators. Besides, we try to understand if their current homology can give us a clue of the form of a previous state of the game. The session will not present a new discovery but open the possibility of a new research path for Cellular Automata. This work was completed as a part of the 2018 SAMMS at Ohio State University.

3:30–3:45

**Using Cubes to Algorithmically Determine the Čech Cohomology of Data Sets**

Daniel Rossano

*St. Joseph's College*

Given a topological space  $X$ , we can compute the Čech cohomology of  $X$  with respect to some open cover  $\mathcal{U}$ . For data sets, this involves centering circles with some radius  $\epsilon > 0$ , referred to as “ $\epsilon$ -balls,” around each point in the data set. Based on the intersections of these  $\epsilon$ -balls, we can present data scientists with information about a given data set, such as what gaps may be present in the set. For large data sets, we can create an algorithm which computes the Čech cohomology for us. However, intersections of three or more  $\epsilon$ -balls is difficult to determine algorithmically. Fortunately, there exists an algorithm utilizing  $\epsilon$ -cubes instead of  $\epsilon$ -balls, which we show is mathematically sound.

3:50–4:05

**The Square Peg Problem for Two Curves**

Alexander Black

*Cornell University*

In 1911, Toeplitz famously conjectured that any simple closed curve in  $\mathbb{R}^2$  inscribes a square. Many variants and simplifications of this problem have been resolved in recent years with the main conjecture still open. We resolve a natural extension of this question to two curves. Namely, it is clearly not true that any two simple closed curves have a square between them, since curves may be too far away from each other. We instead ask whether given any two simple closed curves in  $\mathbb{R}^2$ , there exists a translation of one such that, after translating, the curves have a square between them. We prove this in complete generality and that such a translation exists for any square of sufficiently small side length.

4:10–4:25

**Systole Length and Preservation Under Belt-Sums of the Borromean Rings**

Amanda Cowell

*University of Michigan-Dearborn*

Through the use of Möbius transformations, gluing maps are established for the fundamental polyhedron of the complement of the Borromean rings  $\Omega$ . Then, these transformations are used to describe curves with the base point  $x_0$  belonging to the fundamental group of  $\Omega$ . Taking the homotopy equivalence classes of paths, we find which curves which have systolic length of  $\ell = 2.12255$ . From here, relevant thrice-punctured spheres  $S_i$  are identified and the existence of a systole living in  $\Omega \setminus S$  for each of case of a thrice-punctured sphere is determined. Finally, we reveal an invariant for a particular manifold lacking a curve of length  $\ell$  by showing that it must not include a Borromean rings belt-summand.

Room 211

2:30A.M. – 4:25A.M.

2:30–2:45

**Thermodynamics of Sports**

Arame Sow

*LaGuardia Community College*

Our objective is to model the temperature variations of the body during a workout. We use the thermal camera to record temperatures, a LabQuest and a sensor, and a chronometer to keep track of the time. The logistic and the exponential models are used to analyze the data. Students temperatures will be recorded from the stage of rest, through a warmup, to a complete cool down every 10 minutes. Using the law of cooling and heating by Newton, we will translate the data to differential equations to simulate the future temperature. All the collected data will be analyzed through Maple.

2:50–3:05

**Leprechauns vs. Unicorns: A Competitive Species Model**

Bekkah Trachtenburg

*Bellarmino University*

A competitive species model can show how two species that use the same resources interact with each other given the carrying capacity of their environment. When put into the same area one species will eventually go away while the other thrives, meaning there is not a stable coexistence solution for the two species. We build on this competitive species model to see if we could find a coexistence solution if we add an environmental toxin that affects only one of the species. We want the direct effect of the toxin to be focused in their rate of death. Once we set up our model for competitive species affected by a toxin, we then find the equilibria and their stability. We show that a stable coexistence solution is possible, and we find an example of this. We add a fun twist by using leprechauns and unicorns as the two competing species to help visualize our research. This work was supported by the Clare Boothe Luce Fellowship Program.

3:10–3:25

**Stability Analysis of a Rectangular Strip for 2D Incompressible Fluid Vortices**

Sasha-Kay Shrouder, Christian Madrigal, and Victoria Camarena

*Smith College*

Vortices in 2-Dimensional incompressible fluids modeled by Euler equations with simple shapes such as circles and ellipses are typically stable. Similarly, Beichman and Denisov (2017) showed that a rectangular strip is stable in the periodic domain  $0$  to  $2\pi$ . We will alter the boundary of the strip in the periodic domain, and we will share observations as to how the stability of the vortex changes. We will also observe whether alterations to the strip return to the steady-state strip.

3:30–3:45

**Successive Tightening of the Pinch Points of Logistic Maps and More**

Harry Xi

*Princeton University*

We will explore the stability of the fixed points of the family of logistic maps  $g(x) = ax(1 - x)$  for real parameters  $a$ . From certain values of  $a$ , we will observe that convergence of iterations of the logistic map to those fixed points are nongeometric. For these cases, we will then examine how perturbations of  $g$  from the zero function  $y = 0$ , the positive identity  $y = x$ , and the negative identity  $y = -x$ , translate into rates of convergence to fixed points using differential analysis. Because the resulting differential analysis proves so useful, we will explore how such analysis can predict the trajectory of the size of the Universe in the future. Nonetheless, we will see, however, that if the logistic function  $g$  has an initial perturbation from the negative identity  $y = -x$ , adding higher power perturbations may affect the convergence of iterations to  $p$ . In this respect, we will derive a formula to see which power perturbations may affect convergence and which ones cannot. Finally, we will show that the logistic maps are dynamically paired up to renormalization, and this renormalization leads into the analysis of Julia Sets and related fractals.

3:50–4:05

**Applying the Complete Memory Approximation to Model Euler's Incompressible Equations in Two Dimensions**

Madelyn Shapiro

*University of Puget Sound*

Since the initial publication of Euler's incompressible equations in 1757, the existence of a singularity in the three-dimensional case has remained an open question. Recent innovations in reduced-order modeling, such as the Complete Memory Approximation (CMA), have been implemented with software to investigate this question, yielding results that suggest the presence of a singularity is possible. However, these results have naturally been challenged by the argument that the new techniques used to acquire them have not yet been proven to be effective. By applying the CMA developed by Price and Stinis to the two-dimensional case of Euler's incompressible equations, we further validate the technique by replicating the results of a well-understood system. This research was supported by the Washington NASA Space Grant program.

4:10–4:25

**No Talk**

Room 235

2:30A.M. – 4:25A.M.

2:30–2:45

**Powersum Basis for r-Quasisymmetric Functions**

Anthony Lazzeroni

*Loyola University Chicago*

The algebra of r-quasisymmetric functions (r-QSym) lies between the symmetric functions (Sym) and quasisymmetric functions (QSym). This is true because QSym is the same as 1-QSym and Sym is equivalent to infinity-QSym. Each of these extremal settings has nice bases (i.e., ones that carry combinatorial and representation-theoretic information): the Schur basis and powersum basis for Sym; and Gessel's fundamental basis for QSym. The ultimate goal of the project is to give a combinatorial proof for the fact (due to Garsia and Wallach) that r-QSym is free module over Sym of dimension  $n!$ . Here, we define new powersum functions for r-QSym; show that they do indeed form a basis for all  $r$ ; and explore how they interact with Schur's functions.

2:50–3:05

**Looking into the  $3x + 1$  problem: Matrices, Nature of Graphs, and Dynamic Nature of the problem**

Rajatava Mukhopadhyay and Debosmita Pathak

*IISER Thiruvananthapuram and Grinnell College*

Every odd number may be expressed as an unique sum of non-negative integral powers of 2. It is seen that any number  $p$  of the form  $4x + 1$ , where  $x > 0$  is an integer,  $p$  has a finite stopping time of 2. But for the numbers of the form  $4x1$ , the stopping time cannot be directly calculated as they fluctuate, and they do not show any nice or observable pattern. Inspired by the interesting and random results provided by the  $4x1$  inputs, we aim to study the nature of the output data and their graph with the help of matrices and polynomial vector spaces, and further aim to interpret the dynamic nature of the problem.

3:10–3:25

**Permutation Groups and Error Correcting Codes**

Maxwell Hennen

*Saint John's University*

We will be exploring the usage of permutation bases for error-correcting codes. Using an algorithm discovered by Bailey, we will be using the idea of a permutation basis to correct for errors that occur when an internet signal is transmitted over a power line. We will start off with Dihedral groups and proving some concrete cases, eventually moving up to more abstract cases.

3:30–3:45

**The Equivalence of Tropical Rational Functions and ReLU-Activated Neural Networks**

Benjamin Mudrak

*Kent State University*

We look at the applications of the tropical semi-ring to multi-layer feedforward ReLU-activated neural networks. A feedforward neural network is a neural network without cycles between nodes, and a rectified linear unit (ReLU) is a type of activation function defined as  $y = \max(0, x)$ . Specifically, we show the equivalence of the family of multi-layer feedforward ReLU-activated neural networks to the family of tropical rational maps. This equivalence was originally discovered and proved by Liwen Zhang, Gregory Naitzat, and Lek-Heng Lim in the paper “Tropical Geometry of Deep Neural Networks”.

3:50–4:05

**Straightening Identities in Twisted Multiloop Algebras**

Daniel Davidson

*Park University*

Let  $\mathfrak{g}$  be a complex simple Lie algebra. We examine the vector space formed by tensoring  $\mathfrak{g}$  with the set of multivariable Laurent polynomials,  $\mathfrak{g} \otimes \mathbb{C}[t_1^{\pm 1}, \dots, t_m^{\pm 1}]$ , which is a Lie algebra called the multiloop algebra. We then let a group  $G$ , generated from diagram automorphisms, act on this Lie algebra and consider the set of all elements fixed by this action. We define this set to be the twisted multiloop algebra of  $\mathfrak{g}$ , denoted  $\mathcal{T}_m^G(\mathfrak{g})$ . We have proven some new straightening identities within  $\mathcal{T}_m^G(\mathfrak{g})$  for general  $\mathfrak{g}$  not of type  $A_{2n}$  and for  $\mathfrak{g} = \mathfrak{sl}_3$ . These identities can be used to define integral forms for these Lie algebras, which in turn could be used to investigate the representation theory of these algebras in positive characteristic.

4:10–4:25

**On Sums of Polynomial-type Exceptional Units in  $\mathbb{Z}/n\mathbb{Z}$** 

Anand

*Disha Delphi Public School*

A unit  $u$  in a commutative ring with unity  $R$  is called *exceptional* if  $u - 1$  is also a unit. In this article, we introduce the polynomial notion of exceptional units. For a given polynomial  $f \in \mathbb{Z}[X]$ , and a commutative ring  $\mathbb{Z}/n\mathbb{Z}$  for some integer  $n \geq 2$ , we define an element  $a \in \mathbb{Z}/n\mathbb{Z}$ , which is called a  $f$ -*exunit* if  $\gcd(x, f(a)) = 1$ . We study the representation problem as follows:

For a given element  $c \in \mathbb{Z}/n\mathbb{Z}$ , how many elements  $x$  and  $y$  that are  $f$ -*exunits* such that  $x + y = c$ ? We answer this question for an infinite class of polynomials of a given degree  $m \geq 1$ . More precisely, our main theorem is as follows:

For a given  $r \in \mathbb{N}$  and for all  $i = 1, 2, \dots, r$ , let  $a_i, b_i \in \mathbb{Z}$  such that  $\gcd(a_j t + b_j, a_k t + b_k) = 1$  for all integers  $t$  and  $1 \leq j \neq k \leq r$ . Let  $n \in \mathbb{N}$  be such that  $\gcd(n, \prod_{i=1}^r a_i b_i) = 1$  and let  $f(X) = \prod_{i=1}^r (a_i X + b_i)$ . Then the number of such representations is equal to

$$n \prod_{p|n} \left( 1 - \frac{N^f(p, c)}{p} \right)$$

where  $N^f(p, c) = \#\{l \pmod{p} \mid \text{either } f(l) \equiv 0 \pmod{p} \text{ or } f(c - l) \equiv 0 \pmod{p}\}$ .

This work was carried out when the author was visiting Harish-Chandra Research Institute, Prayagraj, India.

Room 251

2:30A.M. – 4:25P.M.

2:30–2:45

**Forecasting Mosquito Abundance and Assessing Abatement Strategies**

Rylee Esplin

*Dixie State University*

In this study, we designed a mathematical model and compared it with surveillance data for the mosquito's species collected in Washington County, Utah. We investigated the many relationships that can be found within the mosquito abatement center data set consisting of number of mosquitoes trapped, number of treatments, time of year, temperature, humidity, and geographical location and how useful these relationships can be to predict treatment effectiveness. Several statistical analyses are performed to establish the feasible mitigating strategies to control the mosquito population in the southwestern region. Furthermore, our numerical results show that the environment temperature can play a large role in mitigating the outbreak and predict the success rate of the "forecasted" treatment. Our model could be used by mosquito abatement centers and adjusted for local factors to determine optimal strategies to control disease outbreaks most efficiently.

2:50–3:05

**Mathematical Model of Adaptive Immune Response to Chronic Hepatitis B**

Meagan Scheider, Scott West, Tonia Bell, and Anna Fox

*University of Scranton, California State Polytechnic University, Pomona,**The American University, and Clemson University*

Hepatitis B virus (HBV) is one of the most common viral infections, affecting more than 300 million people worldwide. Chronic hepatitis B is a major cause of liver disease, including cirrhosis and hepatocellular carcinoma. The host's HBV-specific immune response plays a significant part in disease progression. To investigate the role of the immune reaction in chronic HBV, we develop a mathematical model using differential equations that account for these responses. We investigate the behavior of this model using linear stability analysis and numerical simulations. This work was completed as a part of the 2019 REU at Hobart and William Smith Colleges.

3:10–3:25

**How stress and anxiety influence sugar metabolism**

Lamia Hauter and Delfino Torres

*LaGuardia Community College and City College of New York*

College students experience stress on daily basis. But this unexpressed feeling of being anxious and nervous may lead to major medical conditions, for example disturbance in the blood sugar level. The purpose of the project is to test how stress hormones including cortisol and epinephrine impact blood sugar level. A Testing Kit will be used to collect data of participants while being in a stressful situation and after returning back to a normal relaxed status. Through differential equations we will construct a mathematical model that expresses a relationship between the blood sugar level and the emotional state of a student.



3:30–3:45

**Implementation of Quartet Distance Consensus Algorithm**

Albright Dwarka

*Hobart and William Smith Colleges*

Using quartet metrization the topology of a species tree can be simulated from gene trees under the multi-species coalescent model. This paper will implement the Quartet Distance Consensus algorithm described in *Topological Metrizations of Trees, and New Quartet Methods, of Tree Inference* by John A. Rhodes. Additionally, we modify the traditional algorithm to increase its speed while maintaining a reasonable margin of error. The increase in efficiency is accomplished by reducing the amount of data that is processed. We studied the algorithm's performance using sample data.

3:50–4:05

**Combinatorics of Semi-Directed Phylogenetic Networks**

Rachel Morris, Yifei Tao, and Sophie Huebler

*University of Richmond, Hobart and William Smith Colleges, and University of Oklahoma*

Phylogenetic tree combinatorics is a well-studied branch of mathematics that gives insight into the evolutionary relationship between species by inferring large trees from sets of smaller trees. However, phylogenetic trees do not take into account events like hybridization or gene flow, thus limiting the accuracy of the depicted relationships. We explore the combinatorics of phylogenetic semi-directed networks, where these gene flow events can be represented as reticulation events that form semi-directed cycles within the networks. We look at how smaller networks are embedded in larger semi-directed networks. In addition, we classify and prove several properties for phylogenetic semi-directed networks.

4:10–4:25

**Instacart Online Grocery Analysis**

Franklin Ajisogun

*New York City College of Technology, CUNY*

Instacart is a company that operates as a same-day grocery delivery service. Customers select groceries through a web application from various retailers and the order is delivered by a personal shopper. Instacart's data science team plays a big part in providing this delightful shopping experience. Currently the data scientists use transactional data to develop models that predict which products a user will buy again, try for the first time, or add to their cart next during a session. This information comes from an open data set study by Instacart, "The Instacart Online Grocery Shopping". This dataset contains a sample of over 3 million grocery orders from more than 200K Instacart users. The focus for this study is to build a probability model to predict what the probability is that an Instacart customer will reorder. To do that we have to understand customers' behavior patterns. We will use data from previous purchases to know when most purchases are made on Instacart and order product data to learn what is a best-selling product. We also developed an interactive web-based Shiny application that implements the data analysis and visualizations that we have created for this project.

## J. Sutherland Frame Lectures

2019	Alice Silverberg	<i>Alice in Numberland — Adventures in Cryptography, Number Theory, and Life</i>
2018	Peter Winkler	<i>The Singular Uniformity of Large Random Systems</i>
2017	Ingrid Daubechies	<i>Bones and Teeth: Analyzing Shapes for Evolutionary Biology</i>
2016	Robin Wilson	<i>Combinatorics - The Mathematics That Counts</i>
2015	Noam Elkies	<i>G-Sharp, A-Flat, and the Euclidean Algorithm</i>
2014	Keith Devlin	<i>Fibonacci and the First Personal Computing Revolution</i>
2013	Gilbert Strang	<i>Matrices I Admire</i>
2012	Melanie Matchett Wood	<i>The Chemistry of Primes</i>
2011	Margaret H. Wright	<i>You Can't Top This: Making Things Better with Mathematics</i>
2010	Nathaniel Dean	<i>Incomprehensibility</i>
2009	Persi Diaconis	<i>The Mathematics of Perfect Shuffles</i>
2008	John H. Conway	<i>The Symmetries of Things</i>
2007	Donald E. Knuth	<i>Negafibonacci Numbers and the Hyperbolic Plane</i>
2006	Donald Saari	<i>Ellipses and Circles? To Understand Voting Problems??!</i>
2005	Arthur T. Benjamin	<i>Proofs that Really Count: The Art of Combinatorial Proof</i>
2004	Joan P. Hutchinson	<i>When Five Colors Suffice</i>
2003	Robert L. Devaney	<i>Chaos Games and Fractal Images</i>
2002	Frank Morgan	<i>Soap Bubbles: Open Problems</i>
2001	Thomas F. Banchoff	<i>Twice as Old, Again, and Other Found Problems</i>
2000	John H. Ewing	<i>The Mathematics of Computers</i>
1999	V. Frederick Rickey	<i>The Creation of the Calculus: Who, What, When, Where, Why</i>
1998	Joseph A. Gallian	<i>Breaking Drivers' License Codes</i>
1997	Philip D. Straffin, Jr.	<i>Excursions in the Geometry of Voting</i>
1996	J. Kevin Colligan	<i>Webs, Sieves and Money</i>
1995	Marjorie Senechal	<i>Tilings as Differential Gratings</i>
1994	Colin Adams	<i>Cheating Your Way to the Knot Merit Badge</i>
1993	George Andrews	<i>Ramanujan for Students</i>
1992	Underwood Dudley	<i>Angle Trisectors</i>
1991	Henry Pollack	<i>Some Mathematics of Baseball</i>
1990	Ronald L. Graham	<i>Combinatorics and Computers</i>
1989	Jean Cronin Scanlon	<i>Entrainment of Frequency</i>
1988	Doris Schattschneider	<i>You Too Can Tile the Conway Way</i>
1987	Clayton W. Dodge	<i>Reflections of a Problems Editor</i>
1986	Paul Halmos	<i>Problems I Cannot Solve</i>
1985	Ernst Snapper	<i>The Philosophy of Mathematics</i>
1984	John L. Kelley	<i>The Concept of Plane Area</i>
1983	Henry Alder	<i>How to Discover and Prove Theorems</i>
1982	Israel Halperin	<i>The Changing Face of Mathematics</i>
1981	E. P. Miles, Jr.	<i>The Beauties of Mathematics</i>
1980	Richard P. Askey	<i>Ramanujan and Some Extensions of the Gamma and Beta Functions</i>
1979	H. Jerome Keisler	<i>Infinitesimals: Where They Come From and What They Can Do</i>
1978	Herbert E. Robbins	<i>The Statistics of Incidents and Accidents</i>
1977	Ivan Niven	<i>Techniques of Solving Extremal Problems</i>
1976	H. S. M. Coxeter	<i>The Pappus Configuration and Its Groups</i>
1975	J. Sutherland Frame	<i>Matrix Functions: A Powerful Tool</i>

## MAA Lectures for Students

2019	Mohamed Omar	<i>Secrets of Grad School Success</i>
2018	Laura Taalman	<i>FAIL: A Mathematician's Apology</i>
2017	David Richeson	<i>Four Tales of Impossibility</i>
2016	Colin Adams	<i>Zombies &amp; Calculus: A Survival Guide</i>
2015	Joseph Gallian	<i>Seventy-Five Years of MAA Mathematics Competitions</i>
2014	Jack Graver	<i>The Founding of Pi Mu Epsilon 100 Years Ago</i>
2013	Frank Morgan	<i>Optimal Pentagonal Tilings</i>
2012	Ivars Peterson	<i>Geometreks</i>
2011	Roger Nelson	<i>Math Icons</i>
2010	Sommer Gentry	<i>Faster, Safer, Healthier with Operations Research</i>
2009	Colm Mulcahy	<i>Mathemagic with a Deck of Cards on the Interval Between 5.700439718 and 806581751709438785716606368564037 66975289505440883277824000000000000</i>
2008	Laura Taalman	<i>Sudoku: Questions, Variations and Research</i>
2007	Francis Edward Su	<i>Splitting the Rent: Fairness Problems, Fixed Points, and Fragmented Polytopes</i>
2006	Richard Tapia	<i>Math at Top Speed: Exploring and Breaking Myths in Drag Racing Folklore</i>
2005	Annalisa Crannell & Marc Frantz	<i>Lights, Camera, Freeze!</i>
2004	Mario Martelli	<i>The Secret of Brunelleschi's Cupola</i>
2004	Mark Meerschaert	<i>Fractional Calculus with Applications</i>
2003	Arthur T. Benjamin	<i>The Art of Mental Calculation</i>
2003	Donna L. Beers	<i>What Drives Mathematics and Where is Mathematics Driving Innovation?</i>
2002	Colin Adams	<i>"Blown Away: What Knot to do When Sailing" by Sir Randolph "Skipper" Bacon III</i>
2002	M. Elisabeth Pate-Cornell	<i>Finding and Fixing Systems' Weaknesses: The Art and Science of Engineering Risk Analysis</i>
2001	Rhonda Hatcher	<i>Ranking College Football Teams</i>
2001	Ralph Keeney	<i>Building and Using Mathematical Models to Guide Decision Making</i>
2000	Michael O'Fallon	<i>Attributable Risk Estimation: A Tale of Mathematical/Statistical Modeling</i>
2000	Thomas Banchoff	<i>Interactive Geometry on the Internet</i>
1999	Edward G. Dunne	<i>Pianos and Continued Fractions</i>
1999	Dan Kalman	<i>A Square Pie for the Simpsons and Other Mathematical Diversions</i>
1998	Ross Honsberger	<i>Some Mathematical Morsels</i>
1998	Roger Howe	<i>Some New and Old Results in Euclidean Geometry</i>
1997	Aparna Higgins	<i>Demonic Graphs and Undergraduate Research</i>
1997	Edward Schaefer	<i>When is an Integer the Product of Two and Three Consecutive Integers?</i>
1996	Kenneth Ross	<i>The Mathematics of Card Shuffling</i>
1996	Richard Tapia	<i>Mathematics Education and National Concerns</i>
1995	David Bressoud	<i>Cauchy, Abel, Dirichlet and the Birth of Real Analysis</i>
1995	William Dunham	<i>Newton's (Original) Method, or, Though This Be Method, Yet There is Madness</i>
1994	Gail Nelson	<i>What is Really in the Cantor Set?</i>
1994	Brent Morris	<i>Magic Tricks, Card Shuffling and Dynamic Computer Memories</i>
1993	Richard Guy	<i>The Unity of Combinatorics</i>
1993	Joseph Gallian	<i>Touring a Torus</i>
1992	Peter Hilton	<i>Another Look at Fibonacci and Lucas Numbers</i>
1992	Caroline Mahoney	<i>Contemporary Problems in Graph Theory</i>
1991	Lester Lange	<i>Desirable Scientific Habits of Mind Learned from George Polya</i>

## **PI MU EPSILON**

### **President:**

Paul Fishback  
Grand Valley State University

### **President Elect:**

Stephanie Edwards  
Hope College

### **Past-President:**

Angela Spalsbury  
Kent State University

### **Secretary-Treasurer:**

Chad Awtrey  
Elon University

### **Councillors:**

Jennifer Beineke  
Western New England University

Darci L. Kracht  
Kent State University

Shannon Overbay  
Gonzaga University

Tom Wakefield  
Youngstown State University

### **Editor, PME Journal:**

Brigitte Servatius  
Worcester Polytechnic Institute

## **MAA Committee on Undergraduate Students**

Pamela Richardson, Chair  
Westminster College

Emily Cilli-Turner  
University of La Verne

James B. Collins  
University of Mary Washington

Janine Janoski  
King's College

Darci L. Kracht, PME Representative  
Kent State University

Emille Davie Lawrence  
University of San Francisco

Aihua Li  
Montclair State University

Sara L. Malec  
Hood College

Rhonda L. McKee, KME Representative  
University of Central Missouri

Stacey Muir  
University of Scranton

Andy Niedermaier  
Jane Street Capital

Eric R. Ruggieri  
College of the Holy Cross

Peri Shereen  
California State University, Monterey Bay

Chasen Grady Smith  
Georgia Southern University

Hortensia Soto, Ex Officio, Associate Secretary  
University of Northern Colorado

Violeta Vasilevska  
Utah Valley University

## **POIANI PME INFINITY AWARD**

The purpose of the Poiani PME Infinity Award is to recognize Pi Mu Epsilon Chapters that “promote scholarly activity in mathematics” through the initiation of extraordinary creative programs integrated with the encouragement of student participation in local and national opportunities to advance their knowledge and communication of mathematics.

The Poiani PME Infinity Award will recognize a chapter that exhibits strength of community and enthusiasm for Pi Mu Epsilon and mathematics among its members, department, and institution including demonstrated sustained commitment to participation in MathFest and other mathematics conferences.

The first Poiani PME Infinity Award will be presented at the PME Banquet.

**CONGRATULATIONS TO THE 2018 ANDREE AWARD WINNERS!**

Jacob Hines  
*Hendrix College*

**Exploring Finite-Time Blow-up of Separable Differential Equations**

Pi Mu Epsilon Journal, Vol. 14, No. 9, Fall 2018

Lara Kassab  
*Lebanese American University*

**Understanding the Center of  $2 \times 2$  Linear Iterative Systems**

Pi Mu Epsilon Journal, Vol. 14, No. 8, Spring 2018

The Richard V. Andree Awards are given annually to the authors of the papers, written by undergraduate students, that have been judged by the officers and councilors of Pi Mu Epsilon to be the best that have appeared in the Pi Mu Epsilon Journal in the past year.

**THE PME JOURNAL—CALL FOR STUDENT PAPERS**

MathFest is all about communication—so is the PME Journal. At the conference, listening is as important as talking. For a journal, readers are as important as authors. The Pi Mu Epsilon Journal readership and authorship is YOU. Send in your mathematical findings under a cover letter and feel the excitement of waiting for the referees' comments. The referees help you become better authors, but they are not, by any means, proof readers or error checkers, so submit only a polished, carefully crafted manuscript. But even if you find improvement unimaginable, most of the time rewriting is required before publication. The process is time consuming but the reward is great. Your paper in print in the PME Journal, now covered by JSTOR, will be accessible forever.

## **THANK YOU TO OUR DONORS AND SUPPORTERS**

Pi Mu Epsilon is grateful for its strong working relationship with the Mathematical Association of America (MAA). The MAA regularly cost-shares various MathFest activities with PME, helps secure spaces for student talks and meetings, and handles all audio-visual needs. Moreover, the MAA is indispensable in assisting with the PME abstract submission and on-line registration systems.

For many years, the National Security Agency (NSA) has provided generous support to help defray costs for Pi Mu Epsilon students who attend MathFest and represent their chapters. Through a series of consecutive grants over a quarter of a century, NSA has provided more than \$250,000 in support. This year NSA Grant H98230-1-0013 has funded subsistence awards to PME speakers and chapter delegates. PME appreciates the NSA's recognition that attending a national conference can have a lasting impact on students and expose them to the mathematics community outside their home institutions.

In addition, Pi Mu Epsilon would like to express its sincere appreciation to the following individuals and organizations for their generous donations and support:

- The American Mathematical Society, the American Statistical Association, Budapest Semesters in Mathematics, the Council on Undergraduate Research, and Bio-SIGMAA for the sponsorship of the Awards for Outstanding Presentations.
- Cambridge University Press, Princeton University Press, The American Mathematical Society Press, and CRC Press for donating book prizes.
- The Youngstown State University Print Shop for printing this abstract booklet at cost.
- Eric Shehadi for his expert help in putting together this abstract booklet.

**Notes:**