Guidelines for Collegiate Faculty to Teach Mathematics to Blind or Visually Impaired Students

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1. Introduction

The news that a sighted mathematics faculty member in a two- or four-year college program will have a blind student enrolled in a course at any level is all too often greeted with disbelief, panic or resentment. The common assumption is that it is impossible for a blind student to learn or comprehend any mathematical material, given the supposedly highly-visual nature of this subject. This initial reaction immediately places a barrier to the effective teaching of mathematics to a blind student by a sighted faculty member. It is important for us to overcome the difficulties that these negative stereotypes place in the effective teaching and learning of mathematical subjects.

In this document, our aim is to provide a set of guidelines to assist two- and four-year college faculty members to provide meaningful mathematics instruction to blind and visually-impaired students. Although it is generally assumed that the visually-impaired student will encounter less difficulty in learning mathematics because of the possession of a degree of usable vision, this is not necessarily so. We should not assume that this vision is always sufficient for undertaking the study of mathematics or any other discipline. In fact, the successful low-vision student should have acquired all the necessary skills of blindness to work efficiently: use large print if and when that works; use Braille when needed; use synthetic speech on a computer or other note-taking device; use live readers. For these reasons, these guidelines are directed to faculty members who occasionally find themselves teaching mathematics at all levels to blind students, regardless of visual acuity. We will use the terms blind students, blind and visually-impaired students, or students interchangeably.

Students may also wish to read these guidelines. I am suggesting this because students and their instructors must work very closely together to achieve the proper level of communication that will be most beneficial to the student while simultaneously bringing about a more efficient use of the instructor's time.

In these guidelines, we assume that blind students are adequately trained to use the tools of blindness efficiently: Braille reading and writing, synthetic speech on a computer to permit the reading and writing of documents, working with live readers. It is not the responsibility of university faculty members to teach Braille or any other learning or study techniques that blind students need. Instead, it is important for faculty members to think about how mathematics can be communicated without the use of written media. This can be done. In my own experience I have encountered mathematical discussions taking place in situations where pencil and paper pad, or even paper napkins, are not handy. In spare moments, how many of us find ourselves formulating mathematical arguments without writing them down?

This is not to say or imply that mathematics can be done without the aid of written media. I believe that ultimately, we don't understand a mathematical idea until we can write it down. For the blind student or blind mathematician, this means writing it in Braille. While we have heard much about modes of learning (visual or auditory), I cannot believe that mathematics can be done in purely auditory form. However, there is much to be said for the ability of speaking mathematics while writing it down in the classroom. This combination of writing and speaking will enhance the experience for both blind and sighted students.

The guidelines I present here are primarily the result of my own experiences both as a blind student and a professional mathematician working in higher education and in the federal government. They are not the result of extensive surveys or field testing by professional educators or psychologists. As these guidelines gain a wider circulation, others may want to make additions to, or include different perspectives on, these guidelines. Please submit your comments and ideas to me. Some sighted instructors may believe that these guidelines do not cover their particular situation. If this is the case, please contact me directly so that I may be of further assistance to you.

I have written these guidelines to correspond with the two primary functions of mathematics education: the delivery of information and inspiration in the classroom (Section 3), and the demonstration of acquired knowledge by solving problems and taking tests (Section 4). I have included some background information (Section 2) primarily to give sighted faculty members a better understanding of the learning environment in which today's blind or visually impaired student must operate.

As is sometimes practiced, blind students should not be exempted from satisfactorily completing required courses in mathematics and the hard sciences. Although there is much emphasis placed on reasonable accommodations, an exemption from a required course is not a reasonable accommodation. Such exemptions simply deprive students of a learning experience that has been determined to be beneficial for all students.

2. Background

Much of the material included here may be well known to faculty and students. This section may be omitted by such readers.

a. LaTeX

LaTeX was initially invented as a typesetting language for mathematical notation. It is text-based and non-graphical in nature. There are also LaTeX based graphics packages that can be used to draw pictures and diagrams. By typing standard text on a keyboard, one can represent all of the mathematical symbols, from the most elementary to the most advanced.

In recent years, many books on mathematics have been published using LaTeX as the underlying typesetting or mark-up language. This development has been most beneficial to blind readers of these books, as I will explain in the next section.

b. Nemeth Braille Code

The Nemeth Braille code was initially devised around 1950 to give Braille readers a method to represent mathematical symbols in Braille. It has been revised several times to make improvements to earlier versions. Because the Braille cell is made up of only six dots, there are at most sixty-four Braille characters, including the blank or space character that can be produced from a single Braille cell. To allow for the many mathematical characters, the Nemeth code employs well-defined sequences of Braille characters to represent one mathematical symbol. In the Nemeth code there are special symbols that are used to represent complex structures such as subscripted subscripts, subscripted superscripts, complex fractions and other nonlinear mathematical notations.

Despite the complexity of the Nemeth Braille Code, it is extremely useful. Books on mathematical subjects, if they exist in Braille at all, are produced using the Nemeth Code. Given the existence of LaTeX based text books along with LaTeX-Nemeth Braille translation software, it is now much simpler to produce a Braille textbook if it is needed. Despite the existence of liberal copyright laws which permit the production of books in alternative formats, such as Braille, publishers are still reluctant to release their LaTeX source code for fear that this code will end up in the wrong hands, and subsequently be abused. All of the math/science articles in Wikipedia are LaTeX based. As a first introductory step, we may easily produce an article from Wikipedia in Nemeth Braille.

We hasten to add that currently, LaTeX-Nemeth Braille translation software does not permit the conversion of graphic images into tactile ones. Automated conversions present problems which have yet to be solved. The degree of resolution that the human eye can distinguish is much finer than what human fingers can feel. There is also the matter of determining the scale of the conversion. The human fingers require much larger dimensions for a tactile drawing than those required for human eyes. In the sense of touch, there is no equivalent to the distinction of colors in human eyes. Currently the best we can expect of visual graphics translation into tactile graphics images is to produce strictly line drawings. This conversion requires human intervention.

c. Recorded Audio Books vs Live Readers

University-level recorded textbooks have been available from Learning Ally (http://learningally.org) for many years. If a textbook is not available in Learning Ally's catalog, blind students may request Learning Ally to produce an audio edition of that book. Learning Ally solicits volunteers to record that book in its studios located throughout the country.

The selection of audio books on mathematics in Learning Ally's catalog is quite extensive. Historically, producing an audio book has been the quickest and cheapest way to get that book into the hands of blind students. The primary drawback I have found with audio math books is the inconsistency in which equations and formulas are read and the manner in which diagrams are described. Since several volunteers may be used in reading a single title, there is not necessarily consistency in which texts of fractions, subscripted variables and exponents are read. This can leave a listener puzzled and bemused about just what a mathematical expression means. For this reason, I do not recommend audio math books. However, if there are no other alternative formats for a given textbook, the desperate student may have no choice but to borrow the audio version from Learning Ally.

For these reasons it has been most satisfactory for me to have textbooks read by a live reader. My reader and I can agree on the manner in which material is to be read. If I still encounter ambiguities, I can stop the reader and get immediate clarification. I can also make Braille notes of the text as it is being read.

d. Electronic Classroom Tools

Some courses require or permit the use of graphics-based software or graphing calculators as teaching aids in the classroom. These tools may pose vexing problems for the blind student, as they are not always non-visually accessible. If such tools are a required part of a particular course, then faculty and students must cooperate to find acceptable alternatives. It is unwise to list such alternatives in these written guidelines since the field of non-visual access is changing rapidly. There are various websites, listservs, discussion groups and blogs dedicated to non-visual access, some of which are listed at the end of these guidelines in Section 6.

American Printing House for the Blind (www.aph.org) sells the Orion TI-84 Plus Graphing Calculator for $600. It is identical to the standard TI-84 Plus Graphing Calculator, except that synthetic speech and audio graphing features have been added to it. I have no experience working with this calculator. A quick search on google yielded no results for reviews of this product.

The situation may be even bleaker for the symbolic mathematical tools such as, Mathematica, Maple, MyMathLab, SageMath, etc. Some users may have devised roundabout solutions to achieve a degree of nonvisual access to these packages. While these languages are text based, their development environments are not always friendly for the blind user. One common problem area lies in the debugging messages one gets when attempting to compile code written in these languages. For the non-specialist, the student who needs to complete a one or two semester math requirement, the efforts for achieving non-visual access to these tools may be too much for such a student to undertake.

For statistical software, a number of blind users have had success with the R Project (https://www.r-project.org/). However, I don't believe that the use of R is for the faint-hearted. It has a steep learning curve and only people who will be doing serious statistical work should undertake the study of R.

More satisfying at the introductory levels may be the use of Microsoft Excel and the mathematical functions it contains. The developers of the JAWS screen reader have put considerable effort into access to Excel. While graphs generated by Excel may not be reproducible in non-visual form, the mere ability to read table values may be sufficient to give the blind user a good idea about what the data looks like.

If all else fails, there is always access to the output of software tools through the help of a live reader. Your reader simply executes your instructions and describes the results to you.

e. Tactile Graphics

Graphs and other pictorial images have always been troublesome for blind students. Without access to tactile diagrams, blind students have been forced to rely on verbal descriptions of these diagrams. Such verbal descriptions may often be problematic. Sometimes there are simply no words or phrases that can accurately describe a diagram.

A usable system for tactile graphics should satisfy two functions: a read-only function in which a tactile graph or diagram is drawn for a blind student to examine or a read-write function that allows a blind student to draw a new diagram. A read-write function for tactile graphics would permit both functions simultaneously. This means that the tactile diagram should permit erasures so that an existing drawing may be corrected. A major problem for read-write function tactile graphics arises with the use of scribing tools that press down into the drawing medium. This means that to feel a diagram one must turn the drawing sheet over to feel what has been drawn.

For quite some time the Sewell raised-line drawing kit has been marketed as a limited read-write function, right-side up drawing system. It consists of inserting a plastic sheet on a board made up of a sheet of rubber glued onto a hard-surfaced backing. When we drag a pointed scribing tool over this mounted plastic sheet, we create a line or a curve that is raised so that it can be felt right-side up. This has been primarily a read-only function system because there was never an easy way to erase and correct tactile diagrams.

A few years ago a Vermont based company, E.A.S.Y., LLC, developed an improved drawing board, the inTACT Sketchpad, along with an eraser for corrections. The development of the eraser gives the inTACT Sketchpad true read-write functionality. While there are other tactile graphics boards on the market, the inTACT Sketchpad is the only one having true erasing capabilities.

Disclosure: Since the inception of E.A.S.Y., LLC., I have advised company officials on the design of the company's inTACT product line. The company currently keeps me on retainer.

f. The Role of the Disabled Student Services (DSS) Office

These offices exist on virtually every university campus. Their role is to facilitate the learning experience for blind and disabled students. When it comes to mathematics, DSS personnel may attempt to secure Braille textbooks, but not always with complete success given the prohibitive cost of producing Braille. They may attempt to provide "note-takers", people who will take notes during lectures and somehow reproduce these notes in usable form for blind students. DSS officers may also negotiate with instructors about test-taking arrangements. Warning: while DSS officers can be helpful, they should not come between instructor-student communications. In any subject, it is best for students and their instructors to work directly together.

A common complaint that DSS personnel have is that instructors are much too late and too slow in determining their choices of textbooks. Instructors with blind students enrolled in their courses should be mindful of this complaint and do as much as they can to announce their textbook selections early.

3. Your Classroom Delivery

a. The Art of Speaking Mathematics

The greatest criticism that a blind student may have about a sighted math instructor is that not enough of the material that is written during a class lecture is spoken. It is so easy to write something down and simply to point at it when referring back to that particular statement or expression. Often, a lecturer may believe that a formula or other expression is simply too complicated or will take too much time to speak it. There needs to be a balance here. We all recognize that there is only so much time in a lecture to cover the necessary material. Yet, I believe that for the purposes of classroom delivery, we can in certain cases simplify the notation without loss of preciseness. Or, if the notation is too cumbersome for speaking, enough of it can be spoken to give the listener a good enough gist of what is going on to make reasonable sense of the arguments that are being presented. There is an ideal comfort level between not enough speaking and speaking too much. When a blind student understands a style of delivery, he or she can pretty much guess at the items that are not being spoken. This understanding may be acquired by frequent "off-line" chats between student and instructor.

As an illustration, many years ago, I took a course in Lisp programming in which all ordered pairs are set off in parenthesis, (-, -). Needless to say Lisp allows for endless sequences of nested parenthetic expressions within these structures of ordered pairs. Once I understood that all ordered pairs are surrounded by parenthesis, it was not necessary for the instructor to speak out all of the sequences of left and right parenthesis.

b. Written Classroom Notes

As the costs of reproducing printed materials has decreased, the quantity of printed classroom handouts has increased. Gone are the days of mimeographs and ditto sheets. Previously the only recourse to blind students was to have these handouts read by a live reader. Now, if these handouts are produced by LaTeX, students may have these handouts in Braille, or may have these handouts read by synthetic speech on a computer. Mathematical materials translated into synthetic speech may be filled with the same ambiguities that I discussed in Section 2c. These handouts may be enlarged for the low vision student. If handouts cannot be produced in Braille, then they are best read to the student by a live reader.

4. Written Assignments and Taking Tests

a. Doing Math in Braille

When producing Braille notes for personal use, the tendency is to simplify the Nemeth Code for the purposes of convenience and speed. The short-hand Nemeth that I write for myself is definitely context sensitive and filled with inconsistencies. Since I have devised my own short-hand Nemeth, I am the only one who can read and make sense of my short-hand Nemeth notes.

A personalized short-hand Nemeth-Braille system also works for solving problems and completing assignments. A student may also use a sketchpad to produce diagrams that may be needed in the solution of a particular problem, a luxury I did not have in my student days. A sketchpad is simple enough for anyone to use. In an office-hour discussion, an instructor may even use the student's sketchpad to illustrate a point.

There is no Nemeth editing software to help a blind student produce completely accurate Nemeth code. There is no translating software to take a Nemeth-Braille file input to produce LaTeX output. For these reasons the blind student requires considerable time preparing assignments and examinations for sighted instructors. From my personal experience it is most efficient for me to dictate mathematical content to a sighted reader.

A final note on using Braille: Since the beginning of the refreshable Braille Notetakers, BrailleNote, PAC Mate, Braille Sense, etc., many blind students have opted to take classroom and study notes using these devices. These are very powerful tools with helpful features such as spell checkers, online dictionaries, and word processing editors. Their primary drawback is that their Braille displays consists of a single line of Braille containing a maximum off 18 – 40 Braille cells.

Compare this to a standard page of Braille produced by a Braille writer consisting of 40 cells per line and 25 lines. As one writes on a Braille writer, the Braille characters are raised so that it is easy to read what one is typing. The complexity of Nemeth code, multiple Braille cells representing one math character, implies that very little of a mathematical expression is exposed at any given time on a Braille Notetaker. This limitation can definitely impede comprehension of the underlying mathematics. Further, when looking at math in Braille, we want to view an entire expression by using the fingers on both hands, where each hand may rest on a separate Braille line. As an example, consider the problem of checking nested parenthetical expressions.

Some refreshable Braille Notetakers are loaded with full scientific calculators which may be useful to have. In the best of worlds a blind student should possess a refreshable Braille Notetaker as well as a standard Braille writer.

b. LaTeX Again

If a student has a firm grasp of LaTeX it is possible to complete problem-solving assignments and submit answers to examinations by writing answers in LaTeX. Using a LaTeX editor, the student compiles LaTeX source code, and submits readable mathematics. While this sounds simple enough, as we all know, debugging source code in any language can be time consuming. Some instructors may accept the LaTeX source code and may be willing to work through it to evaluate a student's work. The difficulty here for the instructor and the student is to determine whether an error in the student's work is really due to an incorrect solution or to an error in LaTeX coding.

c. Keep it Linear

Some students have had remarkable success by using an editor to type mathematical notation in entirely linear form. With judicious use of parenthesis ( ), square brackets [ ], and curly brackets { }, it is possible to type complex expressions in linear form. Greek alphabet letters may be spelled out instead of representing them as symbols. LaTeX achieves this by spelling out the Greek letter and writing a backslash symbol "\" in front of it. Special symbols denoting functions could be replaced by linear expressions. For example, the integral $\int_a^b f(x)dx$ may be expressed as integral (a,b)f(x)dx. Since there are no agreed-upon conventions for such linearizing, instructor and student must agree on the conventions that will be used here. The linear text equivalent for ∞ may be represented as \infty, adopting the LaTeX convention. To save space, one may adopt a shorter notation such as (00).

d. Using Live Readers

In my student days, pre-technology, when Braille math books were non-existent, I used classmates or math majors ahead of me, to read my text books and to take dictation for my problem assignments and tests. Although there was no money to pay these readers, my classmates saw this reading time as an opportunity for us to study together and to benefit from the ensuing discussions. As they read, I made Braille notes to help me with problem solving assignments and test preparation. I worked out problem solutions in Braille, then dictated these solutions to readers. Tests were completed in the same fashion. My instructors were very cooperative in finding unoccupied spaces for me to do my tests with a reader. Sometimes we just used the instructor's office. It was understood that for tests my reader would scrupulously write down what I dictated orally.

Unfortunately, the use of live readers in any subject is no longer in vogue today. This is a pity, especially in mathematical subjects. The DSS offices are quite explicit about paying for note-takers, people who will take notes for you in class, but not paying for readers in the way I used them in my student days.

Chelsea Cook, a blind graduate in Physics from Virginia Tech in 2015, uses a purely dictation method, which she has developed, for homework and exams in her higher-level classes. Since there is no widely-accepted standard for spoken mathematics, individual students are encouraged to develop their own dictation methods. Obviously, dictation methods of this type are highly context sensitive. They are intended to work only between individual blind students and their live readers.

e. More on Tactile Graphics

With the advent of tactile graphics drawing boards, diagrams from textbooks may be easily drawn to present graphical ideas and constructions to the blind student. Since it is not always easy to automatically render a printed graphic image into a tactile one, graphic images from textbooks must be drawn by hand. This is not to say that a blind student needs to have every graphic image reproduced. My experience has been that, for the most part, I get a good idea of what is represented pictorially from the accompanying textual description.

If drawing a graph or other line diagram is required in a problem assignment or on a test, the blind student may simply use a raised-line drawing board rather than giving verbal instructions to a reader to draw that diagram. Tactile graphs can easily be read by a sighted person.

Since I was never exposed to drawing at an early age, my drawings would be highly inaccurate and very crude. But I think that my drawings would be good enough to convince the harshest examiner that I have mastered the concepts that my drawings represent.

5. Conclusion

Mathematical subject matter is not inherently beyond the capabilities of blind or visually-impaired students. Because of the lack of vision, what is important is the communication channel through which this knowledge is transmitted. The most efficient communication channel is the one which is worked out directly between instructor and student. Instructors can be most helpful by providing out-of-classroom time to their blind students. When thinking about learning arrangements and the communication channel, just remember these watchwords: “keep it simple!” For example, the same methods should be used for both completing homework assignments and taking tests. As I stated in the introduction, if there are matters in these guidelines that I have not covered to fit a particular situation, please get in touch with me.

These guidelines have been written primarily from my own perspective and experience. Others, faculty and students, will have different ideas and experiences that they may wish to share. Therefore, what I envision now for a set of guidelines is not only this article on a website somewhere, but additional space so that more material may be included along with what I have written. As we gather the collective mathematical experiences of blind students and their instructors, the reasons for exempting these students from math requirements will definitely be eliminated.

6. Resources

a. Websites

i. Blindmath listserve, http://nfbnet.org/mailman/listinfo/blindmath_nfbnet.org

ii. National Center for Blind Youth in Science (NCBYS), http://www.blindscience.org/

iii. Independence Science, http://www.independencescience.com

iv. Access2science, http://access2science.com

v. A Selection of Postings from the Blind Math Listserv, http://www.blindscience.org/blindmath-gems-home

b. Publications

i. “Math 2974: Mathematical Visualization,” by Chelsea Cook, unpublished, to request copies, contact cook2010@vt.edu

ii. “The World of Blind Mathematicians,” by Allyn Jackson, http://www.ams.org/notices/200210/index.html

iii. “The Tactile Fluency Revolution: Year Two,” by Al Maneki https://nfb.org/images/nfb/publications/bm/bm14/bm1411/bm141113.htm

iv. “The Dawn of the Age of Tactile Fluency: Let the Revolution Begin!” by Al Maneki https://nfb.org/images/nfb/publications/bm/bm13/bm1310/bm131003.htm

v. “Blind Mathematicians? Certainly!” by Al Maneki https://nfb.org/images/nfb/publications/bm/bm12/bm1207/bm120702.htm

vi. “Can We Erase Our Mistakes? The Need for Enhanced Tactile Graphics,” by Al Maneki https://nfb.org/images/nfb/publications/bm/bm12/bm1206/bm120602.html

vii. “A Simple LaTex Tutorial,” by Al Maneki and Alysia Jeans, unpublished, to request copies, contact apmaneki@earthlink.net

viii. “NFB Math Survey: A Report of Preliminary Results,” by Al Maneki https://nfb.org/images/nfb/publications/bm/bm11/bm1109/bm110909.htm

ix. “Handling Math in Braille: A Survey,” by Al Maneki https://nfb.org/Images/nfb/Publications/bm/bm11/bm1102/bm110208.htm

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