The Calculus Sequence

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Calculus dominates much of the undergraduate mathematics landscape. In the 2010 fall term, 720,000 students enrolled in single or multivariable calculus in US colleges and universities. It is a gateway course for most students heading into programs in science or engineering. At many institutions, Calculus I is considered the first college-level course in mathematics. At the same time, it is slipping into the high school curriculum. Over 700,000 students studied calculus in high school in 2013–14. Of the 300,000 students enrolled in college mainstream Calculus I each fall, over 50% have completed a course, often a full year, of calculus in high school. At our research universities, the figure is over 70%.[1]

Because of their centrality, calculus courses are under enormous pressure. Enrollments are large, failure rates are high (the average DFW rate in Calculus I is 27%), and many partner disciplines complain that too many students who have completed their study of calculus struggle to use that knowledge. Few colleges or universities have the resources to cut class size or undertake significant restructuring of the course. In those fortunate places where a dean or provost has promised targeted funding to improve calculus instruction, it is seldom clear how those funds would be best spent.

While there is widespread concern and much local experimentation, implementing improvements to such a well-established and fundamental sequence of courses is a slow process. This report deals separately with what is happening in curriculum, instruction, and institutional support, while presenting options and communicating what we know about best practices. It includes some of the findings from MAA's national study of Calculus I, *Characterisitics of Successful Programs in College Calculus* (CSPCC).[2] This report does not include a list of textbooks, partly because there are so many. We do, however, direct the reader toward some of the less traditional textbooks for Calculus.

I. Curricular Options

[1] Data from the CBMS 2010 Statistical Abstract, the National Center for Education Statistics, and the MAA study *Characteristics of Successful Programs in College Calculus*.

[2] Pronounced CriSPiCC. Further information about the study available online. Funded by NSF #0910240. Opinions expressed do not necessarily reflect views of the Foundation.

What is the current thinking about the most important ideas of the Calculus sequence?

The curriculum that is followed from the first course in Calculus I through Several Variable Calculus has been remarkably stable ever since George Thomas produced the first edition of his *Calculus and Analytic Geometry* in 1951. In the late 1980s and early '90s, the Calculus Reform Movement produced several curricula that front-loaded material on differential equations. This came from a recognition that Calculus is about the study and modeling of dynamical systems.[3]

The emphasis within Calculus has traditionally been on derivatives as slopes of tangent lines and integrals as areas—a very static interpretation that makes it difficult for many students to transfer these tools to dynamical situations. Current work on calculus recognizes the central importance of the concept of *covariation*, understanding how change in one of two or more linked variables is reflected in change to the other variable or variables [5].

Many mathematics educators now recognize that it is more useful to see the derivative as the instantaneous rate of change or as a measure of *sensitivity* of change in one variable to change in another, rather than emphasizing its role as a method of finding slopes. Instead of privileging the integral as area, the emphasis should be on measuring *accumulation*. In an echo of Apostol's *Calculus* [1], Patrick Thompson at Arizona State University has found that students grasp the idea of accumulation more readily than that of derivative as a limit of average rates of change, and has been teaching the course with integration coming before differentiation [8].

Recent discussion has also focused on how to define the integral and present the Fundamental Theorem of Calculus (FTC). The traditional approach is to define the definite integral as a limit of Riemann sums and then explain FTC as stating that integration and differentiation are inverse processes. Most students, however, never grasp the formal definition and understand integration as antidifferentiation, thus removing any meaning from FTC. A better approach is to explain FTC as stating the equivalence of two ways of understanding the definite integral: as the change in the value of an antiderivative or as the limit of a summation. This strategy also accords with the historical understanding of this theorem, which, until its name was shortened in the 1960s, was known as the Fundamental Theorem of Integral Calculus (see [2] for more on this history).

What is happening to develop calculus curricula for specific majors?

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[3] This is reflected in the 2009 recommendations for pre-med requirements issued by the Association of American Medical Colleges and the Howard Hughes Medical Institute. Calculus is only referenced indirectly, as the ability to "quantify and interpret changes in dynamical systems." More information on these recommendation is available online .

Only a tiny fraction (about 2%) of college Calculus I students intend to major in the mathematical sciences. Similar low figures—about 7% in each case—hold for the physical sciences and for computer science. Almost certainly, these very low percentages are a result of the fact that many students heading into the mathematical, physical, or computer sciences will start their college mathematics above the level of Calculus I. The point is that the clientele for this course has shifted dramatically over the past couple decades.

Biology majors now make up the largest single group of students taking mainstream Calculus I (30%). (Here "mainstream" describes any Calculus I course that satisfies a Calculus I prerequisite for any subsequent courses in mathematics; it includes honors sections.) For this reason many colleges and universities have created versions of the calculus sequence specifically tailored to their needs. Many of these courses begin with the study of differential equations and strongly emphasize uses of calculus in modeling dynamical systems. The MAA Notes volume *Undergraduate Mathematics for the Life Sciences: Models, Processes, and Directions* [7] provides a sample of curricula now in use. Macalester College's approach to calculus, introducing multivariable topics from the very start and emphasizing calculus as a tool for modeling, is among the courses described in this volume. At St. Olaf College, the "standard" Calculus I course was recently redesigned to emphasize and draw on biological applications.

Engineering is the second largest major (27%) represented among Calculus I students. Here, too, the traditional curriculum may ill suit students' needs, and variant curricula have been developed. West Point's *Core Math* has been in place for a quarter-century; it introduces students to calculus by starting with difference and differential equations. Wright State University has created *Introductory Mathematics for Engineering Applications* as the first mathematics course for prospective engineers. It is taught within Wright State's College of Engineering and Computer Science and serves as both a review of precalculus and an introduction to calculus that is built around problems that arise in engineering.

What is being done to meet the needs of students who enter college with credit for Advanced Placement Calculus, but who may not be well served by being placed into a more advanced course?

Many universities have developed or adopted curricula designed to challenge students who enter with credit for Advanced Placement Calculus. Two innovative approaches are Pomona's *Approximately Calculus* by Shahriar Shahriari, which focuses on the use of calculus as a tool for approximation and brings in other topics, and the University of Pennsylvania's *Coursera* course, *Calculus: Single Variable* by Robert Ghrist. Following very much in the footsteps of Euler and Lagrange, Ghrist's course uses Taylor series from the very beginning. Other approaches introduce elements of real analysis. Spivak's *Calculus* is the traditional choice of textbook for such a course, but there are now other contenders including *Calculus Deconstructed* by Zbigniew Nitecki and *The Calculus Integral* by Brian Thomson. (See [3] for citations and reviews of the Spivak, Nitecki, Shahriari, and Thomson texts, as well as an historical approach by Cates.)

What curricular modifications are common beyond Calculus I?

Several colleges and universities have moved sequences and series out of Calculus II and replaced them with an introduction to multivariable calculus. A project sponsored by the National Science Foundation (NSF), *Resequencing Calculus*, is developing curricular materials for such a course. Grinnell and St. Olaf Colleges have been using this approach for many years. Sequences and series are still important, and there are variety of alternatives to where they first appear in the undergraduate curriculum: in a separate half course, as part of differential equations, in a transition-to-proof course, or delayed until students take real analysis.

The perennial question for Several Variable Calculus is how much vector calculus—especially Green's, Gauss's, and Stokes' theorems—to include. Few are satisfied with the most common solution: to cram these topics into the last few weeks of the course. Many colleges and universities choose to offer a separate course, often designated Advanced Calculus, that includes the topics of vector calculus.

II. Better Pedagogical Approaches to Calculus Instruction

Better pedagogy is advantageous for all mathematics courses, but especially crucial in the calculus sequence because of its pivotal role.

Which fundamental issues shape the choice of pedagogical approach?

Most students say that they want to understand mathematics, but few know what this means or how to achieve it.[4] Part of the problem lies with the traditional lecture format of instruction. Students have difficulty identifying the most important aspects of what they are seeing and hearing. Either they try to record everything, creating notes that are of little use, or they focus on what they imagine to be important, the template solutions. The same is true when students are "studying." They focus on what they know how to do and what they expect will be important on the examinations, learning template solutions. As an MAA study of final exams has revealed,[5] most instructors play to this tendency by giving examinations that require little beyond an ability to master a set of template solutions.

What is *active learning*?

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[4] For an account of what happens in a typical calculus lecture, see David Bressoud, Student attitudes in first-semester calculus, MAA Focus, 14:6-7, 1994.

[5] Tallman, M., Carlson, M. P., Bressoud, D., & Pearson, M. (in preparation). A characterization of calculus I final exams in U.S. colleges and universities. *Journal of Mathematical Behavior*.

What is known to work in helping students learn how to learn is a broad category of pedagogical approaches that are classified as *active learning.* When done well, active learning forces students to engage with the ideas of calculus in a setting where they can be directed and encouraged by the instructor. This is easiest to accomplish in small classes, but it is possible to incorporate active learning into any class. Iowa State uses reading quizzes that are taken before class so that students arrive in class with a context into which to place the instructor's explanations. They also employ clickers and collaborative activities in their large lecture classes so that students are forced to think about what has been said. Group projects undertaken in the recitation sections force students to apply their knowledge to unfamiliar problems and situations. Cornell University has developed a large set of "Good Questions," thought-provoking questions directed at common student misconceptions that can spur active engagement, even in large classes.

The whole point of active learning, helping students to move beyond exclusively procedural knowledge, is lost unless students know that they will be assessed on a more extensive understanding of calculus. One of the common attributes of the most successful calculus programs identified in the MAA calculus study (see Section III) is that they had high expectations for what students could do, and they included a substantial proportion of non-template problems on their examinations.

What is Inquiry Based Learning?

Inquiry Based Learning[6] (IBL) goes a step beyond finding ways of keeping students intellectually engaged in what is happening in class. It also involves students in the process of building toward the big ideas. Over the past decade, the mathematics departments at the Universities of Michigan, Chicago, Texas at Austin, and California at Santa Barbara have engaged in a controlled experiment in the effectiveness of IBL. The Ethnography & Evaluation Research Group at the University of Colorado-Boulder has studied this experiment. Comparing sections taught with and without IBL, they found that IBL improved both the number of subsequent mathematics courses that were taken and performance in those courses [6].

What are the concerns with active learning approaches?

Active learning is not easy. Classroom activities need to be chosen carefully. Mini-lectures need to be thoughtfully prepared. Instructors need to develop experience in identifying where students are struggling and how best to modify instruction to meet their needs. Group projects can be counter-productive unless mechanisms are in place to ensure full participation by all students. Anything done in recitation sections must be closely integrated with what is happening in the classroom. MAA's

[6] The Academy of Inquiry Based Learning provides materials and workshops. The Academy also can connect instructors with mentors and has a limited supply of small grants to help get IBL programs started.

national study of Calculus I, *Characteristics of Successful Programs in College Calculus*, revealed that, while active learning approaches do improve attitudes and retention, their effect is heavily influenced by the basic quality of the teaching. Active learning approaches can be counterproductive when instructors have not built a basic level of rapport and trust with the students.

Fortunately, there is a lot of expertise in the creation and facilitation of active-learning classrooms. The University of Michigan has built a quarter-century of experience in using these techniques and training new faculty in their use. In almost all cases, involvement of or input from faculty with expertise in Mathematics Education has been essential to improving calculus instruction.

What are the options for using online videos and comparable materials, especially for "flipped" classes?

There are two current NSF-sponsored studies of the effects of flipping classes in mathematics, one at Harvey Mudd and the other at the University of Hartford. Hartford's experiment is happening within their calculus courses and involves presenting lectures via online videos and spending class time in small-group problem solving, whole-class discussion, or lab investigations, with mini-lectures provided in class as needed to clarify content and procedures or highlight important conceptual ideas. In fall 2012, Hartford ran a pilot program with half of the classes flipped, half taught in a traditional manner. Preliminary analysis was so encouraging that all of the classes were flipped for spring 2013. Beginning fall 2013, with funding from NSF, they will begin a more extensive and carefully controlled study of the effectiveness of flipped classes. The University of Pennsylvania is also running a similar project under the auspices of the Association of American Universities Undergraduate STEM Initiative.

The University of Hartford is also experimenting with *iPad* sets and multiple projection units to promote more student-to-student discussion and collaborative problem solving. This department also has built databases of questions and curriculum materials that support active learning (see link below). Coordination and constant monitoring lie at the root of what they are able to accomplish.

Those who are not prepared to flip their class can still direct students toward online resources that partially complement and partially replace in-class lectures, as well as providing remediation. Such resources include the Khan Academy videos, MIT OpenCourseware, University of Hartford Material and Paul's Online Math Notes.

What do we know about the use of online homework systems?

One of the tools that can help promote active learning is an online homework system such as *WeBWorK*. This may seem counter-intuitive since what these systems assess most effectively are responses to short-answer template problems. But the use of active learning does not mean that the ability to solve such problems is no longer an important part of learning calculus. Active learning is a strategy for going *beyond* developing proficiency with such problems. Online homework systems enable students to develop this proficiency at their own speed. Most importantly, the feedback they provide immediately informs students and the instructor of what has and has not been understood. At the University of Hartford, this enables instructors to focus class time on addressing student misunderstandings and difficulties. Online homework systems also free the instructor to spend time assessing student ability to tackle deeper and more challenging problems, including application of calculus to unfamiliar contexts, interpretation of answers, and explanation of the reasoning behind a solution.

The AMS Homework Software Survey revealed strongly positive student and instructor responses to the use of these tools and no evidence that it was in any way inferior to hand grading. Today these systems are being used primarily at large universities, simply to handle the homework grading that otherwise would not happen (60% of instructors at research universities and 42% of those at masters universities use online grading), but it is spreading to undergraduate colleges and two-year colleges (27% at undergraduate colleges, 25% at two-year colleges).[7]

What are other common uses of technology?

Many of the active learning techniques described in this section involve technology in the sense of using online resources. Technology in the form of Computer Algebra Systems (CAS) such as *Mathematica*, raise inevitable questions about how much purely procedural knowledge is needed. Here the picture is not clear, and perhaps surprising. MAA's CSPCC calculus study found that while graphing calculators are commonly used, computers as tools for calculating are not. The study also found no measurable impact either from banning or from requiring the use of CAS.

III. Lessons from the MAA Study of Calculus I

MAA's study of *Characteristics of Successful Programs in College Calculus* (CSPCC) consisted of a series of surveys sent to a stratified random sample of colleges and universities in fall 2010 and completed by 213 chairs or calculus coordinators, 502 instructors, and over 14,000 students. Instructors and students were surveyed at both the start and end of the term. Identification of "successful programs" was based on how student affective characteristics changed: confidence, enjoyment of mathematics, intention to continue the study of mathematics, and intention to continue in a major that required at least one additional term of Calculus.

Here we summarize and illustrate some of the insights into best practices gathered from that study and as well as case study visits to a wide variety of calculus programs. The characteristics of successful programs that have been identified fall

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[7] Percentages are from the Fall 2010 CSPCC study.

under three broad categories: Coordination, Monitoring, and Active Learning. Active Learning has been discussed in Section II. This section will consider the other two categories. These two are directed toward issues that go beyond what happens in the individual classroom and involve the entire department or institution.

What is the role of *Coordination* in building a successful calculus program?

Faculty prize their independence, and most will hold tenaciously to their freedom to teach their class the way they want to teach it. This can result in some very innovative and often successful approaches to teaching. The problem is that it is then very difficult to leverage these improvements, to spread them beyond the individual. In a small college or university or with experienced faculty, one can not only allow but encourage such individualization, provided there is some coordination. This could be as simple as periodic observations of each other's classes combined with regular meetings of those who are teaching calculus to share their materials, approaches, and difficulties. When such sharing is built into departmental expectations, it facilitates the dissemination and further development of good ideas. It also can help prevent a class from going off the rails without appearing to target a particular instructor for special attention.

What does coordination look like at larger universities?

At larger institutions, coordination is critical. The most successful programs have a course Coordinator. The Coordinator holds regular meetings in which calculus instructors talk about course pacing and coverage, develop midterm and final exams, and discuss teaching and student difficulties. In addition, the most successful programs have common examinations. In some cases, the homework assignments are coordinated.

It is important to have a Coordinator who is respected by the mathematics faculty and is invested in the program (rather than serving for a semester or a single year). Having a commitment that extends beyond a single year facilitates interaction with other departments and university offices and helps to establish guidelines for handling special situations. At the University of Michigan, these course Coordinators work with a departmental oversight committee that lends credence to the program and assures adherence to the standards and goals of the department.

Part of coordination includes common exams. This is often politically difficult, but it is important. Common exams lead to consistent expectations for instructors in subsequent courses. They inhibit students from selecting the "easy" instructor over one who may have higher expectations, and they allow the department (especially in a department that may have a lot of adjunct faculty or even temporary full-time faculty teaching in the program) to maintain standards and thereby reduce student complaints.

Coordination also includes training and mentoring. This is particularly important for graduate students whether they are taking on a recitation section or teaching their own class, but mentoring and some access to training are also important for adjunct faculty, new faculty, and even experienced faculty who may not have taught calculus for many years. Iowa State's Center for Excellence in Undergraduate Mathematics Education runs seminars for new graduate students as well as all teaching assistants in which they discuss teaching issues, read and discuss case studies in teaching, and discuss problems encountered by assistants.

At the University of Michigan, there is a presumption that all calculus instructors—whether graduate students, adjunct faculty, or new regular faculty—need to know what is expected and to be given guidance and feedback on how they are doing. Instructor training includes pre-semester meetings, weekly meetings, and classroom observations. Mentoring includes availability of coordinators, follow-up visits, feedback, and an openness to new ideas while not allowing an instructor to stray too far from the goals of the department for the course.

What is the role of *Monitoring* in building a successful calculus program?

Monitoring includes attention to local data, issues of placement, and use of Learning Centers. One of the most important attributes of successful calculus programs is attention to what is happening in these classes. This extends from overall monitoring of the success rates of students in these classes to attention to targeted subpopulations including women and first generation college students. It means following the performance of individual students so that interventions can occur before it is too late. One of the tools for monitoring the effectiveness of the calculus program is the Calculus Concept Inventory [**5**], created by Jerry Epstein. This tool has undergone testing and validation and has been used at the University of Michigan. Iowa State has plans to use it to assess its calculus program.

The importance of continual monitoring of the calculus program was illustrated in MAA's *Models that Work* **[9].** The most successful programs are those that are constantly looking to improve their effectiveness, occasionally through major overhauls of what they are doing when it becomes clear that what they are doing is not effective, mostly through continuing small improvements. This requires the regular and consistent collection of information about what actually is happening.

What does Monitoring look like at different types of institutions?

In the most successful programs, someone in the department routinely collects and analyzes data in order to inform and assess program changes. It is essential that departments take on this work themselves. While effective departments almost always work with the Institutional Research Office, they do not rely on this office to know what to collect or how to analyze what is collected. Some of the most useful data include pass rates, grade distributions, persistence, placement accuracy, and success in subsequent courses.

It is not enough to collect data. The department must be prepared to adjust or, sometimes, even radically change what they are doing to counter clearly identified problems. When the mathematics faculty at Macalester College discovered that almost none of the students in Calculus I continued on to Calculus II because they were Biology or Economics majors for whom only one semester of calculus was required and that the students who did take Calculus II—primarily physical science and mathematics majors—were entering with credit for Calculus I, they restructured the first year of calculus. The first semester was turned into a course on dynamical systems that could stand on its own and provide the insights into calculus as tool for modeling that would be useful to Biology and Economics students in their major discipline. At St. Olaf, the Calculus I class has evolved to place much more emphasis on applications, especially those relevant to the life sciences.

It is especially important to monitor the effectiveness of placement programs. The University of Illinois at Urbana-Champaign and Iowa State University have dramatically improved their success rates in Calculus by introducing new placement procedures. In both cases, they have moved to adaptive online testing with opportunities to retake placement exams. This helps to distinguish between those students who knew but have forgotten a critical piece of mathematics and are capable of coming back up to speed on it and those students who never understood it and will be severely disadvantaged if they try to start Calculus without it. Good placement procedures do more than restrict access to Calculus I. They accurately identify which students can succeed in the course, and they help those who are not ready understand where their deficiencies lie.

One of the resources available from MAA is the Calculus Concept Readiness examination. This placement test assesses student understanding of the conceptual underpinnings of calculus.[8] Iowa State is among the universities using materials developed by Marilyn Carlson and her team for the Precalculus Concept Assessment.[9]

What are some of the ways in which students are supported by departments and institutions?

In addition to placing students correctly, it is important to provide support services, especially early in the critical first term of Calculus when habits are established. Problems that are identified early in the term can be addressed in a timely manner through tutors, Learning Centers, or special courses. The University of Michigan offers a half-term, self-paced Precalculus course (Math 110) for those students who start in Calculus I and discover after the first exam that their algebra and precalculus skills are not adequate. This greatly increases the likelihood that misplacement into Calculus I will not delay completion of Calculus I by more than

[8] Additional information available online.

[9] Additional information available online.

one semester.

Moravian College was one of the first to offer a Calculus I course stretched over two terms so that it can include review of precalculus topics as they arise within the calculus course. Worcester Polytechnic Institute also offers such an option. This is a successful alternative to inserting a precalculus class because students are constantly encountering new and more challenging material while getting the support they need.

A hallmark of the most successful programs is the attention paid to the Learning Center. Oklahoma State is one of the universities making a major investment in upgrading their “Mathematics Learning Success Center.” The Learning Center must provide a welcoming environment that students know about and use. An effective center includes training programs for the tutors who will work with students encountering difficulties in Calculus. It is particularly important for the Learning Center to have a strong connection to the calculus instructors. This includes an established mechanism for identifying early in the term those students who are struggling and supplying them with academic support. Swarthmore College has an Academic Support Coordinator for Mathematics, with an office in the Mathematics Department, whose job it is to work with faculty to identify students who need additional support and then to see that they get it. The University of Hartford offers “vampire tutoring,” staffing their learning center from 10pm to midnight, the peak time for students to work on their homework. The key here is adjusting the schedule so that it is convenient for the students, not necessarily the staff.

Conclusions and reflections for the future

From this survey of the landscape of college calculus, we see that this sequence of courses is poised for significant changes driven by several factors:

- The students who, in the past, were well served by the traditional calculus course no longer study calculus in college. They jump directly to more advanced courses.

- The primary audience for Calculus I has shifted to biology majors and others for whom the appropriate emphasis is the investigation of dynamical systems with much greater reliance on computation.

- We know more about how students learn and about the importance of creating an active learning environment in which students are forced to wrestle with the mathematics and build a robust personal understanding.

- The availability of online resources provides opportunities for instructors to spend less time lecturing and more time in active learning, directly engaging students with the critical concepts of calculus.

- With greater attention to the institutional bottom line, there is less tolerance of high failure rates as well as of students who go on without the knowledge or skills needed to succeed in subsequent courses. Placement, support, and effective instruction are more important than ever.

- Many very bright and talented mathematicians and mathematics educators already realize these points and are developing a variety of tools, curricula, and pedagogical approaches to meet the needs these courses serve. At the same time, we are getting better at assessing the effectiveness of these innovations. We now have a much clearer grasp of what works and why it works.

- There is support for reform of undergraduate mathematics instruction coming from influential actors: the White House Office of Science and Technology Policy, the National Academies, and the Association of American Universities among many others. The American Mathematical Society has joined MAA in recognizing the need for more effective undergraduate instruction, and a variety of joint efforts are now underway.

We also now realize that just having a better way of doing something and publicizing it is not sufficient for widespread adoption. We are now beginning to understand obstacles to institutional change and what it takes to facilitate this change.

In 2014, MAA received a grant from NSF, *Progress through Calculus*, #1420389, that will enable it to dig into this process of improving the effectiveness of the entire single variable calculus sequence, to investigate and evaluate departmental efforts to provide better courses, and to establish supportive networks of departments that seek to implement change.

This is an exciting time to be involved in the teaching of calculus. The only certainty is that twenty years from now calculus instruction will be very different. Getting to that place where these courses do a far better job of meeting the needs of our students is an effort that will require our very best minds.

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