

George Pólya Awards

Christopher Catone

“Bringing calculus into discrete math via the discrete derivative,” *The College Mathematics Journal*, 50(1), 21–27. 10.1080/07468342.2019.1530553.

For a sequence of real numbers a_n , its discrete derivative Δa_n is nothing more than the differences between successive terms: $\Delta a_n = a_{n+1} - a_n$. Starting from this extremely simple concept, Catone shows us how to develop an entire theory of discrete differential and integral calculus, with striking similarities to its continuous cousin. There are product, quotient, power and exponential rules; integration is summation; and the “fundamental theorem of calculus” describes telescoping sums. Taylor polynomials appear, and discrete differential equations correspond to recurrence relations.

The article’s title promises to “bring calculus into discrete math,” and indeed the discrete derivative turns out to be very useful for solving commonly encountered discrete problems, such as finding closed-form expressions for sequences and series. (Do you know how to write $\sum_{k=1}^n k \circledast^k$ in closed form? You will after reading this article, and the trick is “summation by parts!”) This article also accomplishes the reverse: by working in a discrete setting, where there are no limits to complicate matters, one can see very explicitly the basic reasons behind the properties of the usual derivative, and understand more deeply why familiar calculus formulas look as they do.

Throughout the article, Catone pays attention not only to the beautiful mathematical ideas illustrated by the discrete derivative, but also how to bring them into the classroom. The article is full of helpful tips on incorporating these ideas into lesson plans and student projects, in either discrete math or calculus courses, with a particular focus on opportunities for active learning and self-discovery. Although the article is addressed to teachers, the elementary nature of its topic, combined with Catone’s clear and straightforward exposition, makes it likewise accessible to students.

This engaging article is a joy to read. All readers of this article, whether beginning students or experienced teachers, and whether interested mainly in calculus or in discrete mathematics, are sure to come away with new insights into both!

Response

As a graduate student, I watched George Polya’s video “Let Us Teach Guessing” and have been an admirer of his work ever since. It is an absolute honor to be chosen for this award that bears his name. *The College Mathematics Journal* is an excellent publication with both original and expository articles. I am happy to be able to contribute to this exposition and highlight a topic that is sometimes forgotten in the undergraduate mathematics curriculum. I am also very humbled to be recognized in a list that includes some of the most esteemed teachers of mathematics of the past half-century. Many thanks to the MAA for this great honor.

Biographical Sketch

Christopher Catone is Associate Professor and Chair of the Mathematics department at Albright College, a liberal arts college in eastern Pennsylvania. He received his BA from The College of New Jersey and his PhD from the University of Colorado Boulder under the direction of Jeanne Clelland. Catone’s interests are in differential geometry, geometry, and pedagogy. He has won teaching awards from Albright College and The University of Colorado and is currently working on an undergraduate geometry textbook. When not doing mathematics, you might find Chris hiking, biking, skiing, playing guitar, or sipping coffee with a classic novel at one of Philadelphia’s many coffee shops.