Chauvenet Prize

Ezra (Bud) Brown and Matthew Crawford

"Five Families Around a Well: A New Look at an Old Problem," *The College Mathematics Journal*, 49:3 (2018) 162–168. doi.org/10.1080/07468342.2018.1447203

This beautiful paper tells the intriguing story of how a centuries-old linear algebra problem from ancient China came, not only to be generalized, but also to be given a novel combinatorial perspective. The so-called 'well problem' from the two-thousand-year-old *Nine Chapters of the Mathematical Art* has been analyzed extensively, but no one it would seem had previously made the connection between the presence of the number 265 in its solution and a special type of permutation called a derangement. The fact that 265 is also the number of derangements of six items thus provides the key to joining two seemingly disparate strands of mathematics together.

The paper begins with a statement of the problem, which gives rise immediately to a system of linear equations. Using elementary linear algebra to find the least positive integer solutions of the system leads to the first mysterious occurrence of 265. Generalizing the problem then allows the authors to shed light on some intriguing patterns, involving factorials and recurrence relations, leading to the proof of a beautiful new result establishing a link between the generalized well problem and derangements. But other surprises are still in store as the authors conclude by revealing further connections to e, its reciprocal, and a well-known problem in probability dating back to 1708.

This paper is full of subtle and unexpected mathematical connections, engagingly written and attractively presented. The fact that it is the result of a collaboration between a professor and a student further adds to its appeal. Brown and Crawford are to be congratulated on having produced a model of outstanding mathematical exposition.

Response

We are amazed, thrilled, and most grateful to the Chauvenet Prize Committee for this award. The story behind the paper being honored began with Tim Chartier's absorbing book, *Math Bytes*, that contained a puzzle from the ancient Chinese work *The Nine Chapters of the Mathematical Art*. Bud was startled to find something combinatorial about the problem, namely that the length of the longest ladder, 265, is also the number of derangements, or fixed-point free permutations, on six objects. Bud ran into Matt the next morning and quickly got Matt hooked on the problem. They conjectured that for n families, the length of the longest rope is equal to the $(n+1)$st derangement number, and Matt made substantial inroads to our eventual proof. The two of them studied the literature and concluded that this piece of combinatorics, contained in one of the legendary mathematics texts from ancient China, had gone unnoticed until now. We gave a talk at our section meeting, and in the audience was Brian Hopkins, the *College Math Journal* editor, who accepted the (as yet unwritten) paper for the *CMJ*—and he also contributed an observation about the problem that we hadn't noticed. We thank Brian for his excellent editorial work. Finally, our selection by the Chauvenet Prize Committee joins us with such legendary recipients as G. H. Hardy, Marc Kac, Paul Halmos, Peter Lax, and Neil Sloane, and we are honored and proud to be in that number.

Biographical Sketches

Ezra (Bud) Brown grew up in New Orleans, has degrees from Rice and LSU, taught at Virginia Tech for 48 years, and retired in 2017 as Alumni Distinguished Professor Emeritus of Mathematics. He does research in number theory, combinatorics, and the history of mathematics. He enjoys finding and writing about connections between seemingly unrelated mathematical topics, an interest stemming from graduate school

that has never left him. He and the late Richard Guy are the authors of the Carus volume, *The Unity of Combinatorics*, which was published in May 2020.

Bud enjoys baking biscuits, singing (anything from opera to rock'n'roll), playing jazz piano, watching birds, and—since his retirement—traveling with his wife Jo. His favorite number is 265.

Matthew Crawford graduated from Virginia Tech with his BS and MS in mathematics, and is now pursuing his PhD at UNC Chapel Hill. His interests and research include number theory, combinatorics, and ergodic theory. When he is not knee-deep in integers, he is hiking with his wife Elyse and border collie mix Pebbles, getting his energy out on a squash court, playing board games with friends, researching the most famous sports you have never heard of, and drinking chocolate milk.